Belle 衝突点領域に於ける 放射光ワット量分布の計算法の開発

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KCG Seminar

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Introduction

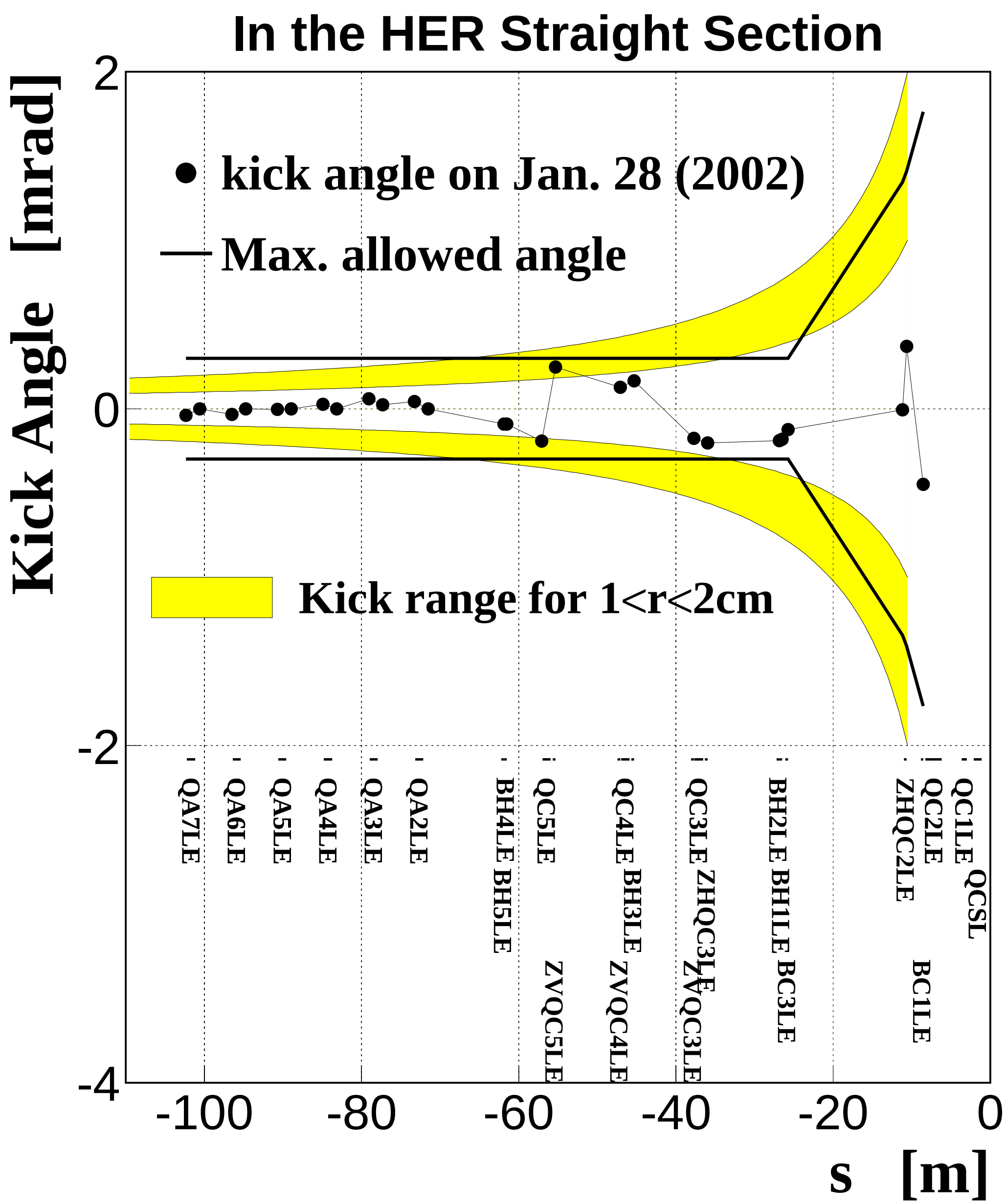
Motivation

- SVD1.0 was killed by the synchrotron radiation (SR) from HER.
  - ⟶ Limitation for the HER steering magnets
  - ⟶ Some current values are close to the limits. (→ figure)
- Smaller-radius IR beampipe (r = 2.0 cm → 1.5 cm → 1.0 cm ?)
- Larger beam currents for higher luminosities → more SR

A new idea has been proposed by Hitoshi Yamamoto:

Fitting **BPM**-measured values → **Real** beam orbit
→ **Wattage distribution** in the IR → Online alarm for SR

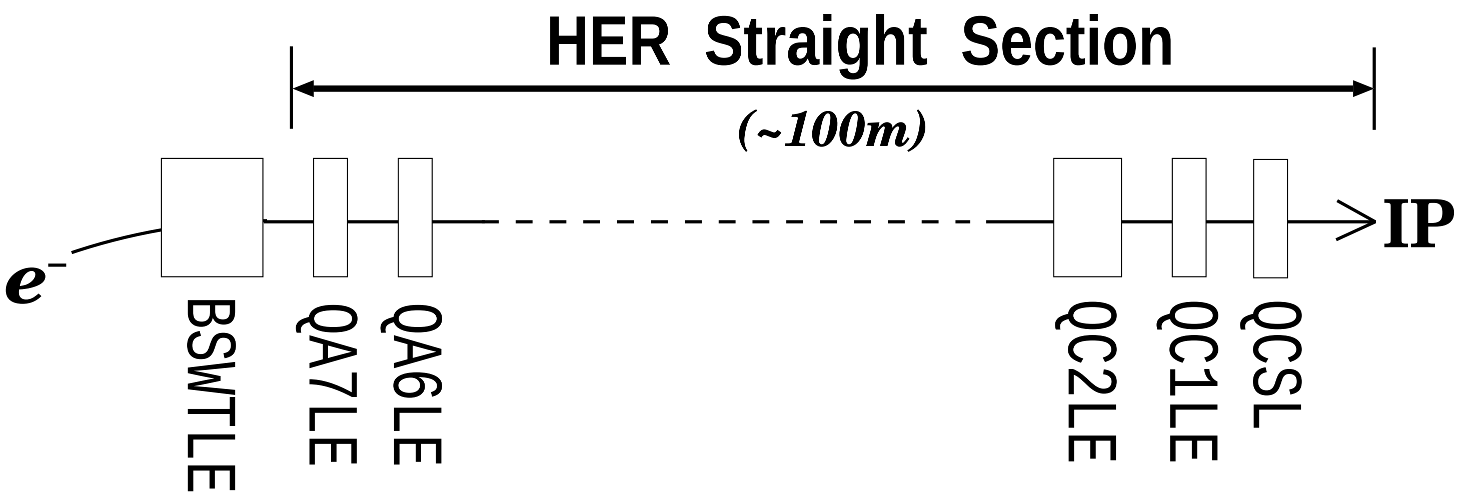
- LOOSER LIMITATION for the HER steering magnets
- PRIOER NOTICE for detector damages


In the HER Straight Section
kick angle on Jan. 28 (2002)
Max. allowed angle
Kick range for 1<r<2cm
Kick Angle [mrad]
s [m]
2
0
-2
-4
-100
-80
-60
-40
-20
0
QA7LE
QA6LE
QA5LE
QA4LE
QA3LE
QA2LE
BH4LE BH5LE
QC5LE
ZVQC5LE
QC4LE BH3LE
ZVQC4LE
QC3LE ZHQC3LE
ZVQC3LE
BH2LE BH1LE BC3LE
ZHQC2LE
BC1LE
QC2LE
QC1LE
QCSL

Calculation Method

[I] Orbit Calculation

- In the HER straight section (see ↓)
- Using a linear approximation ⟶ **transfer matrices**
- Implemented in my own program



≪ Transfer Matrices ≫

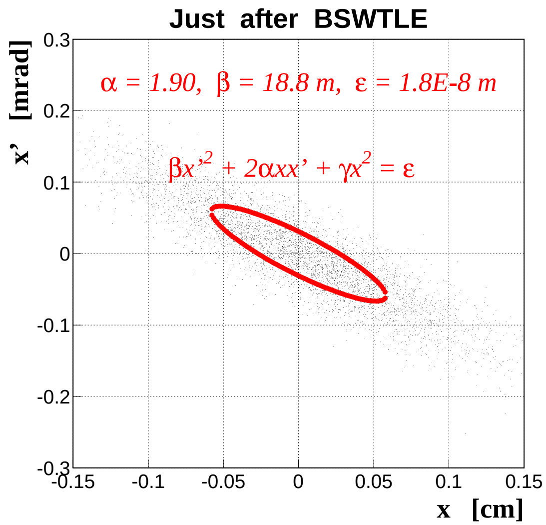
$$\text{Orbit}: \mathbf{X}(s) = \left(\begin{array}{c} x(s) \\ x'(s) \\ y(s) \\ y'(s) \\ \hline \mathbf{1} \end{array}\right) \qquad \text{Drift space}: \mathbf{R}_{\text{drift}}(L) = \left(\begin{array}{cccc|c} 1 & L & & & \\ & 1 & & & \\ & & 1 & L & \\ & & & 1 & \\ \hline & & & & \mathbf{1} \end{array}\right)$$

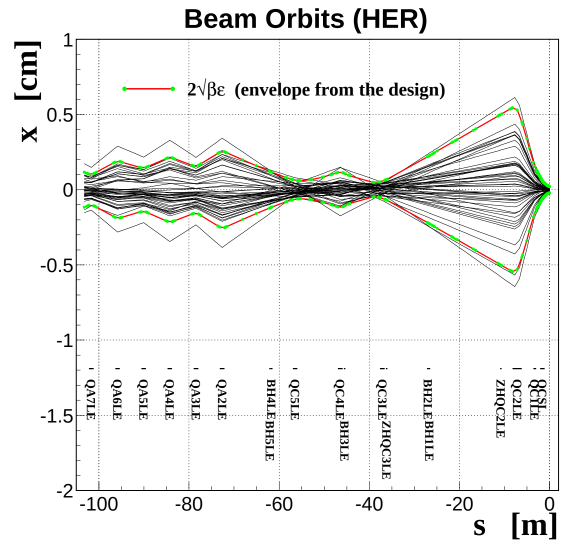
$$\text{QUAD}: \mathbf{R}_{\text{quad}} = \left(\begin{array}{cccc|c} \cos(kL) & k^{-1}\sin(kL) & & & \\ -k\sin(kL) & \cos(kL) & & & \\ & & \cosh(kL) & k^{-1}\sinh(kL) & \\ & & k\sinh(kL) & \cosh(kL) & \\ \hline & & & & \mathbf{1} \end{array}\right)$$

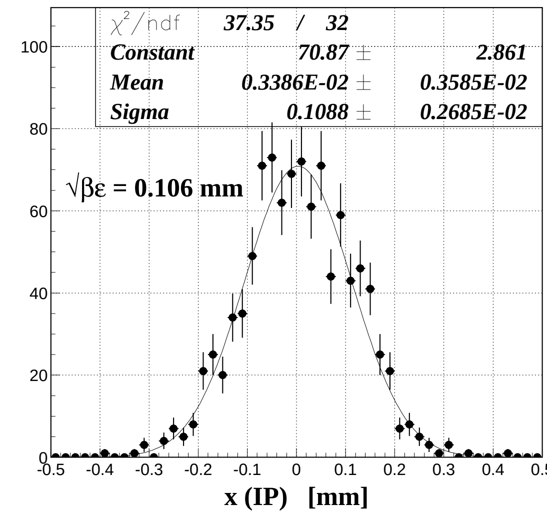
$$\text{BEND}: \mathbf{R}_{\text{bend}} = \mathbf{R}_{\text{drift}}(L/2) \times \left(\begin{array}{cccc|c} 1 & & & & \\ & 1 & & & \theta_x \\ & & 1 & & \\ & & & 1 & \theta_y \\ \hline & & & & \mathbf{1} \end{array}\right) \times \mathbf{R}_{\text{drift}}(L/2)$$

$$\text{Skew QUAD}: \mathbf{R}_{\text{skewQ}} = \mathbf{R}_{\text{rot}}(-45^\circ) \times \mathbf{R}_{\text{quad}} \times \mathbf{R}_{\text{rot}}(+45^\circ)$$

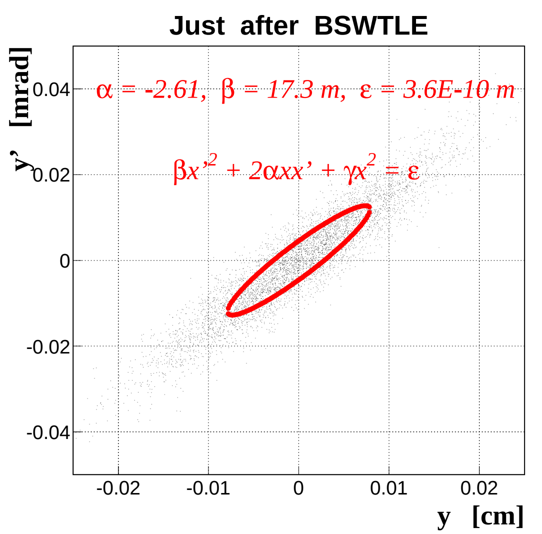
Check of the Transfer-Matrix Implementation [1]

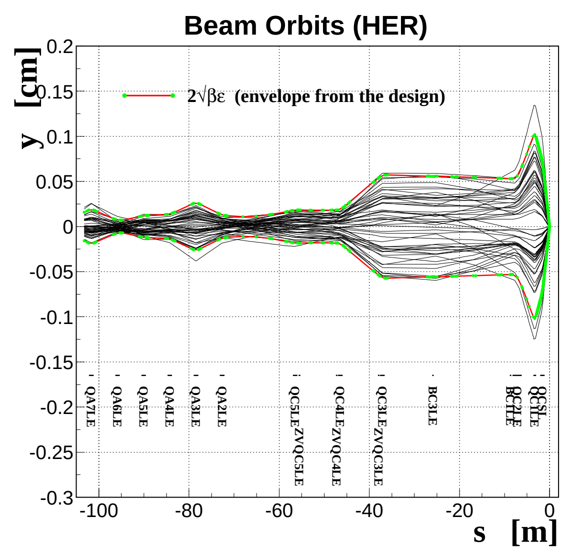


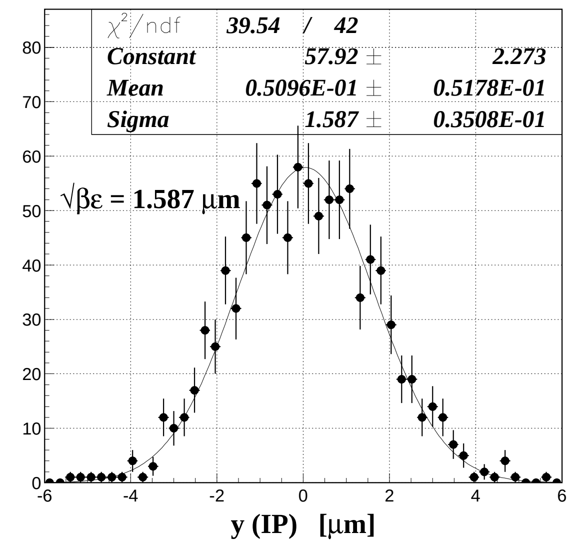




BSWTLE e^- ———————————————> IP







Check of the Transfer-Matrix Implementation [2]

Numerical comparison with SAD on QC3LE

```
 In[61]:= TransferMatrix["QC3LE","LX086E.1"]
Out[61]:=
  {1.0415110062367,1.094542869142296,0,4.440892098501E-16}
  {.077425177671287,1.0415110062367,3.469446951954E-18,0}
  {3.382710778155E-17,-3.53883589099E-16,.959055549847622,1.065412001995326}
  {-2.60208521397E-18,2.775557561563E-17,-.075287731090181,.959055549847622}
```

$$\frac{R_{ij} - R_{ij}^{(SAD)}}{|R_{ij}^{(SAD)}|} = \begin{pmatrix} 0 & +4\times 10^{-15} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\times 10^{-15} & -3\times 10^{-14} \\ 0 & 0 & 0 & -2\times 10^{-15} \end{pmatrix}$$

Check of the Transfer-Matrix Implementation [3]

Numerical comparison with SAD on QKELE

```
 In[62]:= TransferMatrix["QKELE","LC51LAE"]
Out[62]:=
  {1.000001092692837,.372300042338456,-.00256049953636,-2.68166492929E-4}
  {1.3572130039052E-5,1.000001092692837,-.013755037460226,-.00256049953636}
  {-.00256049953636,-2.68166492929E-4,1.000001092692837,.37230004233846}
  {-.013755037460226,-.00256049953636,1.3572130039019E-5,1.000001092692837}
```

$$\frac{R_{ij} - R_{ij}^{(SAD)}}{|R_{ij}^{(SAD)}|} = \begin{pmatrix} -4\times10^{-14} & -2\times10^{-14} & -8\times10^{-14} & +6\times10^{-13} \\ -3\times10^{-11} & -4\times10^{-14} & 0 & -8\times10^{-14} \\ -8\times10^{-14} & +6\times10^{-13} & -4\times10^{-14} & -3\times10^{-14} \\ 0 & -8\times10^{-14} & +3\times10^{-11} & -4\times10^{-14} \end{pmatrix}$$

[II] Orbit Fitting

- Input parameters: $\begin{cases} \text{BPM values from M002QC1} \sim \text{M013QA7} \\ k_0, k_1 \ (\rightarrow \text{optics}) \end{cases}$

- Floating parameters: $\begin{pmatrix} x_{\text{ini}} \\ y_{\text{ini}} \end{pmatrix}$, $\begin{pmatrix} x_{\text{IP}} \\ y_{\text{IP}} \end{pmatrix}$

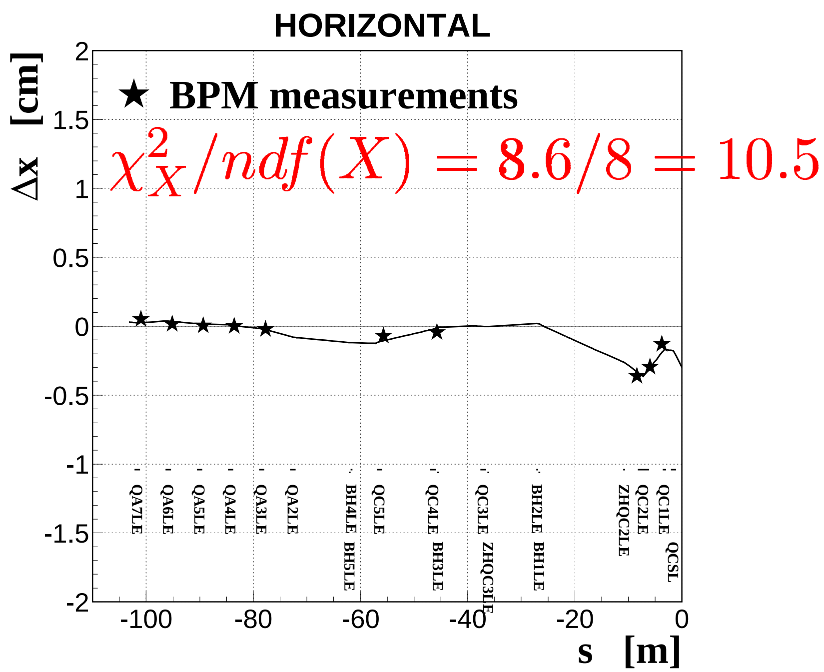
- $$\chi^2 \equiv \sum_{j:\text{BPM}} \left\{ \left(X_j^{(\text{BPM})} - X_j^{(orbit)} \right)^2 + \left(Y_j^{(\text{BPM})} - Y_j^{(orbit)} \right)^2 \right\} / \sigma^2$$

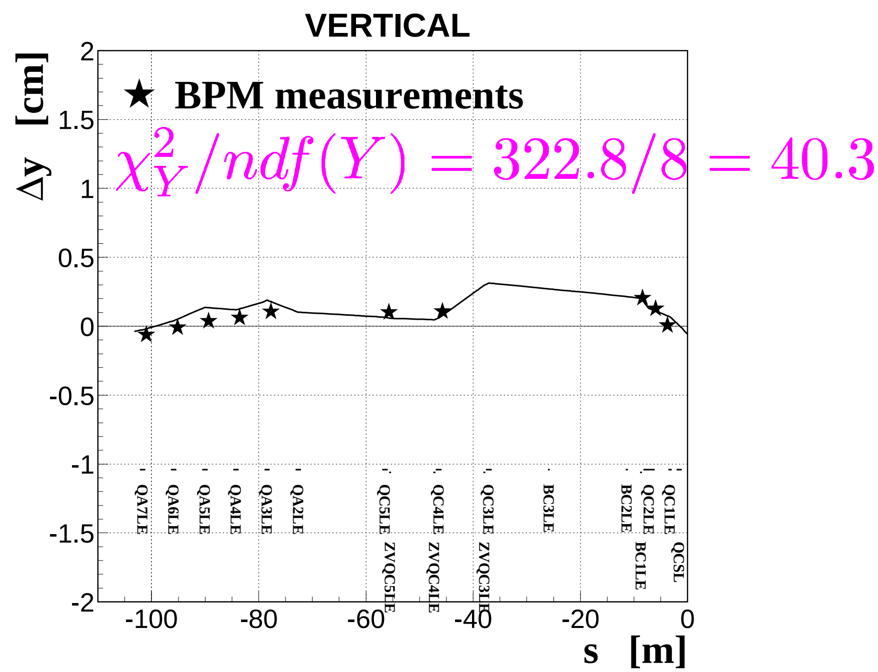
- Minimization using `MINUIT`

- $$\frac{\chi^2}{ndf} = \frac{\chi_X^2 + \chi_Y^2}{ndf(X) + ndf(Y)} = \frac{\frac{\chi_X^2}{ndf(X)} \frac{1}{ndf(Y)} + \frac{\chi_Y^2}{ndf(Y)} \frac{1}{ndf(X)}}{\frac{1}{ndf(X)} + \frac{1}{ndf(Y)}}$$

Fitting Example

- BPM resolution: 100 μm   (including offset uncertainties)
- Data on Nov. 7 (2001)

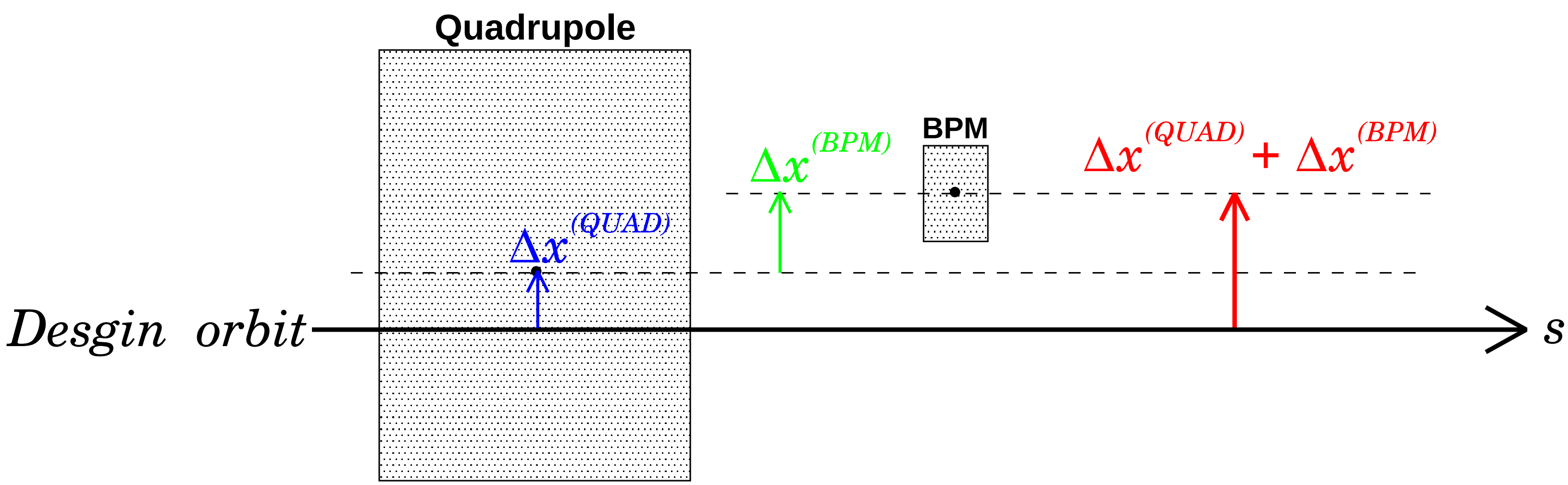




Failure with $X_{IP} \sim -3$ mm & $\chi^2/ndf \gg 1$

Need Some Offset Corrections!

Offset Definition



$$\chi^2 = \sum_{i:orbits} \sum_{j:\mathrm{BPMs}} \Big[\Big\{ \underline{\Big(X_{j,i}^{(\mathrm{BPM})} + \Delta X_j^{(\mathrm{QUAD})} + \Delta X_j^{(\mathrm{BPM})} \Big)} - X_{j,i}^{(orbit)} \Big\}^2 + \Big\{ \underline{\Big(Y_{j,i}^{(\mathrm{BPM})} + \Delta Y_j^{(\mathrm{QUAD})} + \Delta Y_j^{(\mathrm{BPM})} \Big)} - Y_{j,i}^{(orbit)} \Big\}^2 \Big] / (100\mu m)^2$$

Offset Determination

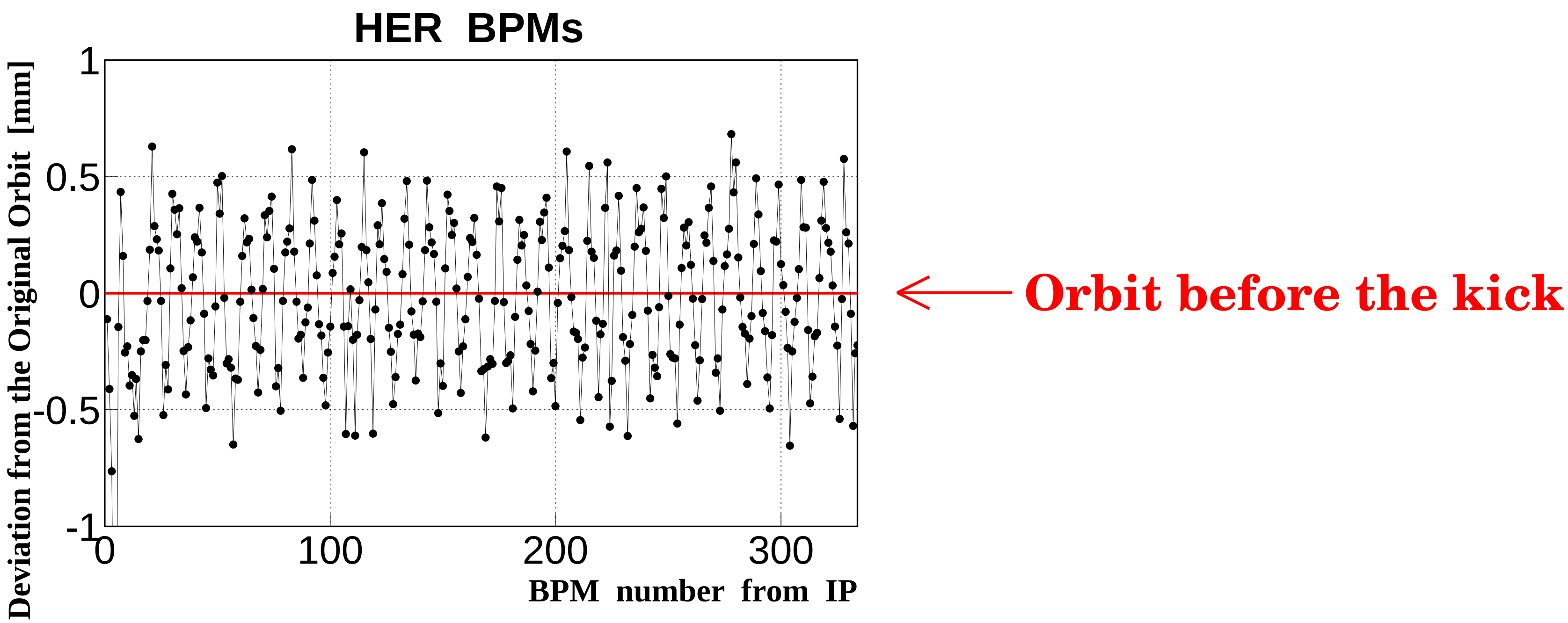
Using **BETARAW** sample: one of the following six sets of steering magnets ON

$$\left\{\begin{array}{c}\texttt{ZHQFRNE1}\\ \texttt{ZVQDRNE1}\end{array}\right\}\left\{\begin{array}{c}\texttt{ZHQFRNE2}\\ \texttt{ZVQDRNE2}\end{array}\right\}\left\{\begin{array}{c}\texttt{ZHQFROE1}\\ \texttt{ZVQDROE1}\end{array}\right\}\left\{\begin{array}{c}\texttt{ZHQFROE2}\\ \texttt{ZVQDROE2}\end{array}\right\}\left\{\begin{array}{c}\texttt{ZHQX6E1}\\ \texttt{ZVQX5E1}\end{array}\right\}\left\{\begin{array}{c}\texttt{ZHQX6E2}\\ \texttt{ZVQX5E2}\end{array}\right\}$$

$$s = \sim 710\text{m}, \quad \sim 730\text{m}, \quad \sim 2220\text{m}, \quad \sim 2240\text{m}, \quad \sim 1460\text{m}, \quad \sim 1540\,\text{m}$$

$\longrightarrow$ An oscillation around the original orbit

$\longrightarrow$ Six different orbits are obtained.



≪ Floating Parameters ≫

- Starting and ending points of orbits:

$$\left\{ \begin{pmatrix} X^i_{ini} \\ Y^i_{ini} \end{pmatrix}, \begin{pmatrix} X^i_{IP} \\ Y^i_{IP} \end{pmatrix} \right\}$$

$$\left(i = 1,\ 2,\ 3,\ 4,\ 5,\ 6\ : \mathrm{orbits} \right)$$

- Offsets of BPMs and QUADs
- **Scales of bendings:**

$$\theta^{(NEW)}_{kick} = (\mathbf{1} + \mathbf{a}) \times \theta^{(ORG)}_{kick}$$

Common among all the orbits

→ **Global Fit**

If we float all,

➡ $>$ 100 floating parameters
➡ Large offsets ($>$ 1 cm)
➡ Multi-solution
➡ Not positive-definite Hessian matrix

Strategy

Fitting Parameters

BPM **offsets** **`(10)`** | **QUAD** **offsets** **`(11)`** | **BEND** **scales** **`(5or6)`**

`0 or 1 floated; Others fixed.` | **`0 or 1 floated; Others fixed.`** | **`0 or 1 floated; Others fixed.`**

Global Fits for All the Combinations

Criteria

Choose the best χ^2 *!*

Criteria

- ⇒ **MINUIT convergence with a positive-definite Hessian matrix**
- ⇒ $\left|\text{\textbf{BPM offsets}}\right| <$ **1 cm**
- ⇒ $\left|\text{\textbf{QUAD offsets}}\right| <$ **1 cm** (alignment)
- ⇒ $\left|\text{\textbf{BEND rescaling factor}}\right| <$ **20%** (kick angle)
- ⇒ $|X_{IP}| < 1$ **mm** & $|Y_{IP}| < 1$ **mm**

We find a solution

- With the smallest modification
- Within the scope of the numerical approach

BETARAW sample on Jun. 4, 2001

≪Horizontal≫

	$X1$	$X2$	$X3$	$X4$
floating BPM	M006QC4LE	M006QC4LE	M006QC4LE	M006QC4LE
floating QUAD	QC2LE	QC2LE	QC2LE	QC2LE
floating BEND		BH12LE	BH45LE	ZHQC5LE
$\chi^2_X/ndf(X)$	3.5	3.4	3.4	3.5
|BPM offset| [mm]	$+0.56 \pm 0.04$	$+0.60 \pm 0.05$	$+0.62 \pm 0.05$	$+0.50 \pm 0.07$
|QUAD offset| [mm]	$+0.62 \pm 0.04$	$+0.61 \pm 0.04$	$+0.61 \pm 0.04$	$+0.62 \pm 0.04$
|BEND scale| [%]		$+3.5 \pm 3.2\%$	$+4.2 \pm 2.1\%$	$-6.3 \pm 4.9\%$
IP offset [mm]	$\sim -0.7 \pm 0.1$	$\sim -0.6 \pm 0.1$	$\sim -0.7 \pm 0.1$	$\sim -0.6 \pm 0.1$
M005QC3 offset [mm]	-0.85 ± 0.04	-0.88 ± 0.05	-0.83 ± 0.04	-0.88 ± 0.05
M008QA2 offset [mm]	-0.85 ± 0.05	-0.82 ± 0.06	-0.94 ± 0.07	-0.80 ± 0.06

BETARAW sample on Jun. 4, 2001

≪Vertical≫

	$Y1$	$Y2$	$Y3$
floating BPM	M002QC1LE	M002QC1LE	M003QC2LE
floating QUAD	QA2LE	QA2LE	QA2LE
floating BEND	ZVQC3LE	ZVQC5LE	ZVQC5LE
$\chi^2_Y/ndf(Y)$	0.7	0.7	1.1
$\vert$BPM offset$\vert$ [mm]	$+0.64 \pm 0.06$	$+0.74 \pm 0.06$	-0.52 ± 0.05
$\vert$QUAD offset$\vert$ [mm]	$+0.85 \pm 0.03$	$+0.79 \pm 0.03$	$+0.77 \pm 0.03$
$\vert$BEND scale$\vert$ [%]	$+17 \pm 1\%$	$-12 \pm 1\%$	$-16 \pm 1\%$
IP offset [mm]	$\sim -0.57 \pm 0.003$	$\sim -0.54 \pm 0.001$	-0.54 ± 0.001
M005QC3LE offset [mm]	-0.49 ± 0.09	-1.12 ± 0.05	-1.38 ± 0.05
M008QA2LE offset [mm]	-0.56 ± 0.05	-0.50 ± 0.05	-0.48 ± 0.05

BETARAW sample on Oct. 24, 2001

≪Horizontal≫

	$X1$	$X2$	$X3$	$X4$
floating BPM	M006QC4LE	M006QC4LE	M006QC4LE	
floating QUAD	QC2LE	QC2LE	QC2LE	
floating BEND		BH12LE	BH45LE	
$\chi^2_X/ndf(X)$	3.4	2.8	3.4	
|BPM offset| [mm]	$+0.63 \pm 0.04$	$+0.78 \pm 0.05$	$+0.59 \pm 0.05$	
|QUAD offset| [mm]	$+0.58 \pm 0.04$	$+0.52 \pm 0.04$	$+0.58 \pm 0.04$	
|BEND scale| [%]		$+16 \pm 3\%$	$-3.5 \pm 2.1\%$	
IP offset [mm]	$\sim -0.8 \pm 0.1$	$\sim -0.7 \pm 0.1$	$\sim -0.8 \pm 0.1$	
M005QC3LE offset [mm]	-0.74 ± 0.04	-0.86 ± 0.05	-0.76 ± 0.04	
M008QA2LE offset [mm]	-1.01 ± 0.05	-0.84 ± 0.06	-0.93 ± 0.07	

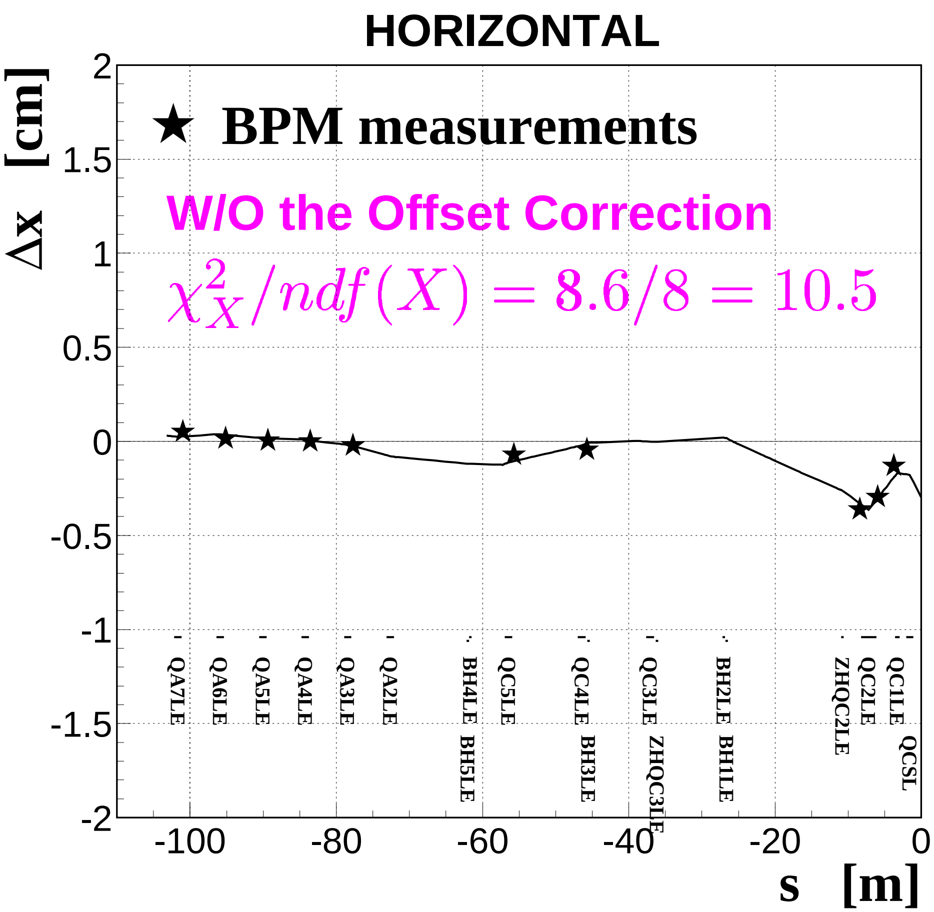
BETARAW sample on Oct. 24, 2001

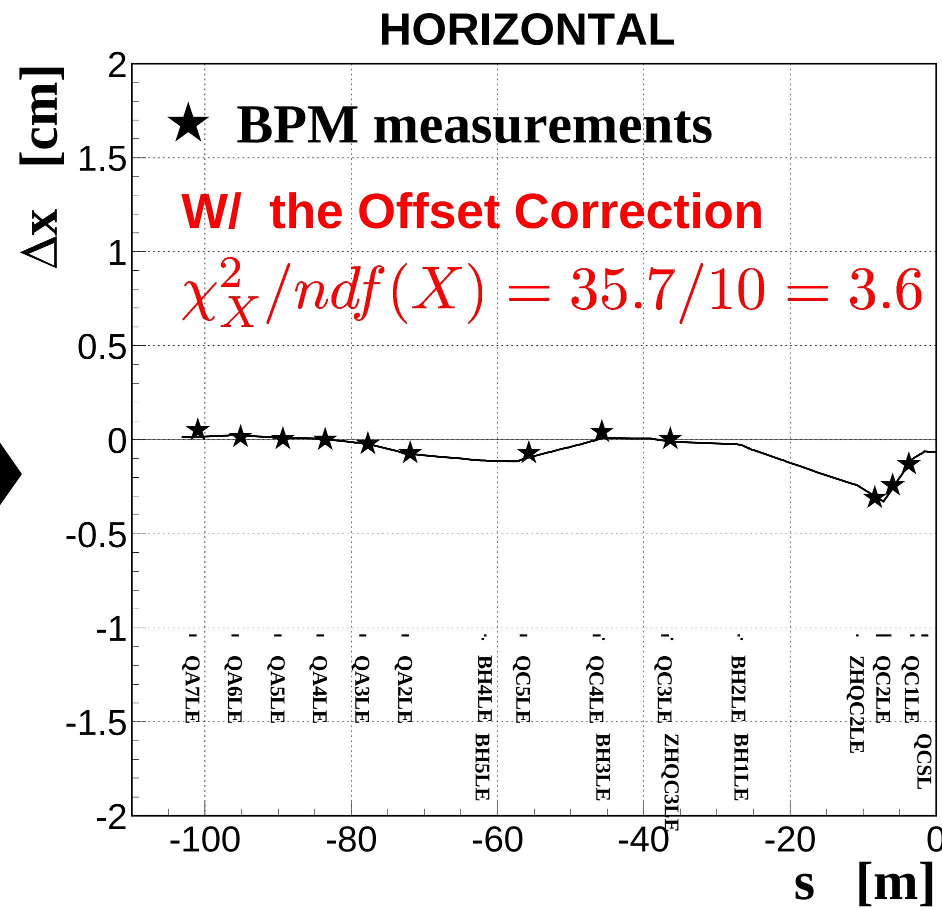
≪Vertical≫

	$Y1$	$Y2$	$Y3$
floating BPM	M002QC1LE	M002QC1LE	M002QC1LE
floating QUAD	QA2LE	QA2LE	QA3LE
floating BEND	ZVQC3LE	ZVQC5LE	ZVQC5LE
χ^2_Y/ndf_Y	0.4	0.5	0.7
|BPM offset| [mm]	$+0.74 \pm 0.06$	$+0.85 \pm 0.06$	$+0.85 \pm 0.06$
|QUAD offset| [mm]	$+0.66 \pm 0.03$	$+0.60 \pm 0.03$	-0.21 ± 0.01
|BEND scale| [%]	$+15 \pm 1\%$	$-12 \pm 1\%$	$-19 \pm 1\%$
IP offset [mm]	$\sim -0.63 \pm 0.003$	$\sim -0.60 \pm 0.001$	-0.60 ± 0.001
M005QC3LE offset [mm]	-0.32 ± 0.09	-1.02 ± 0.05	-1.01 ± 0.05
M008QA2LE offset [mm]	-0.51 ± 0.05	-0.45 ± 0.05	-0.10 ± 0.04

Results of the Offset Correction

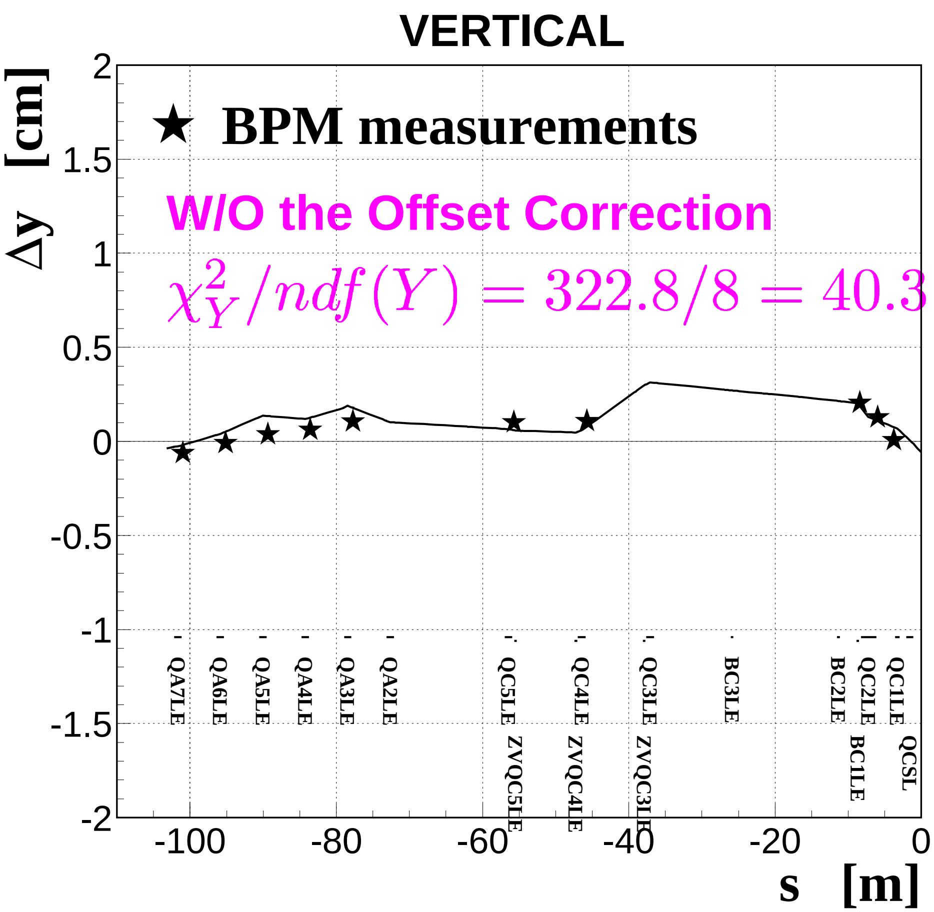
(Horizontal case)

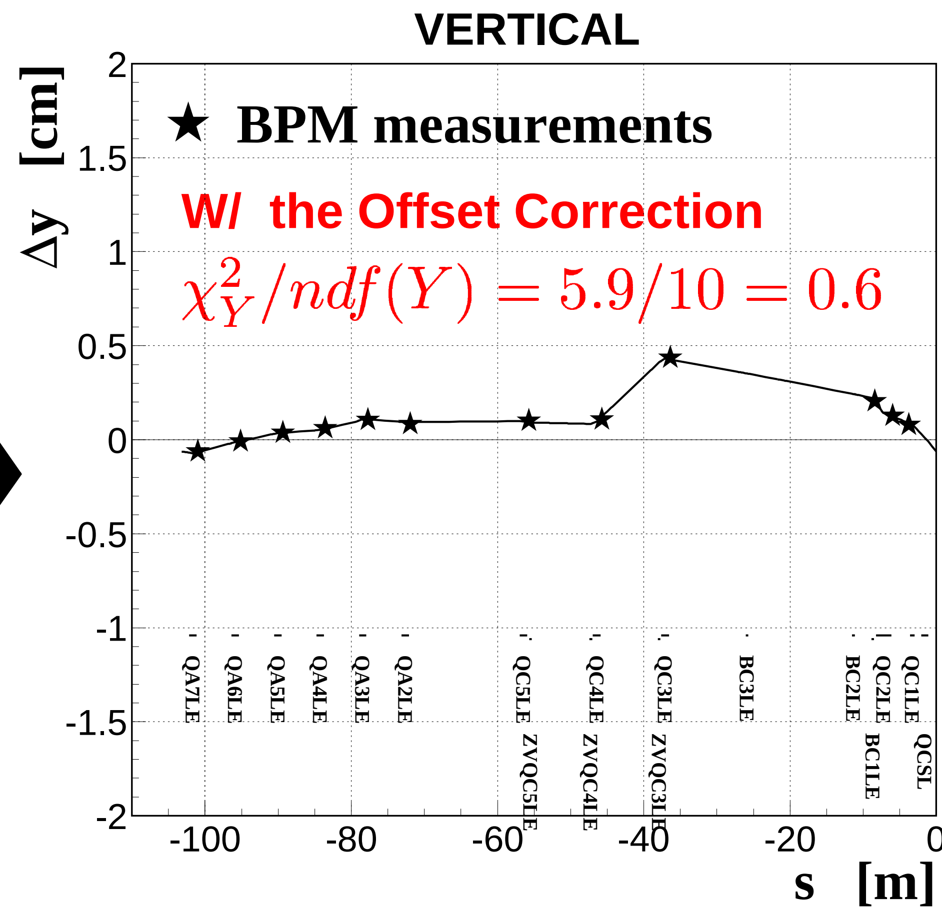




Results of the Offset Correction

(Vertical case)



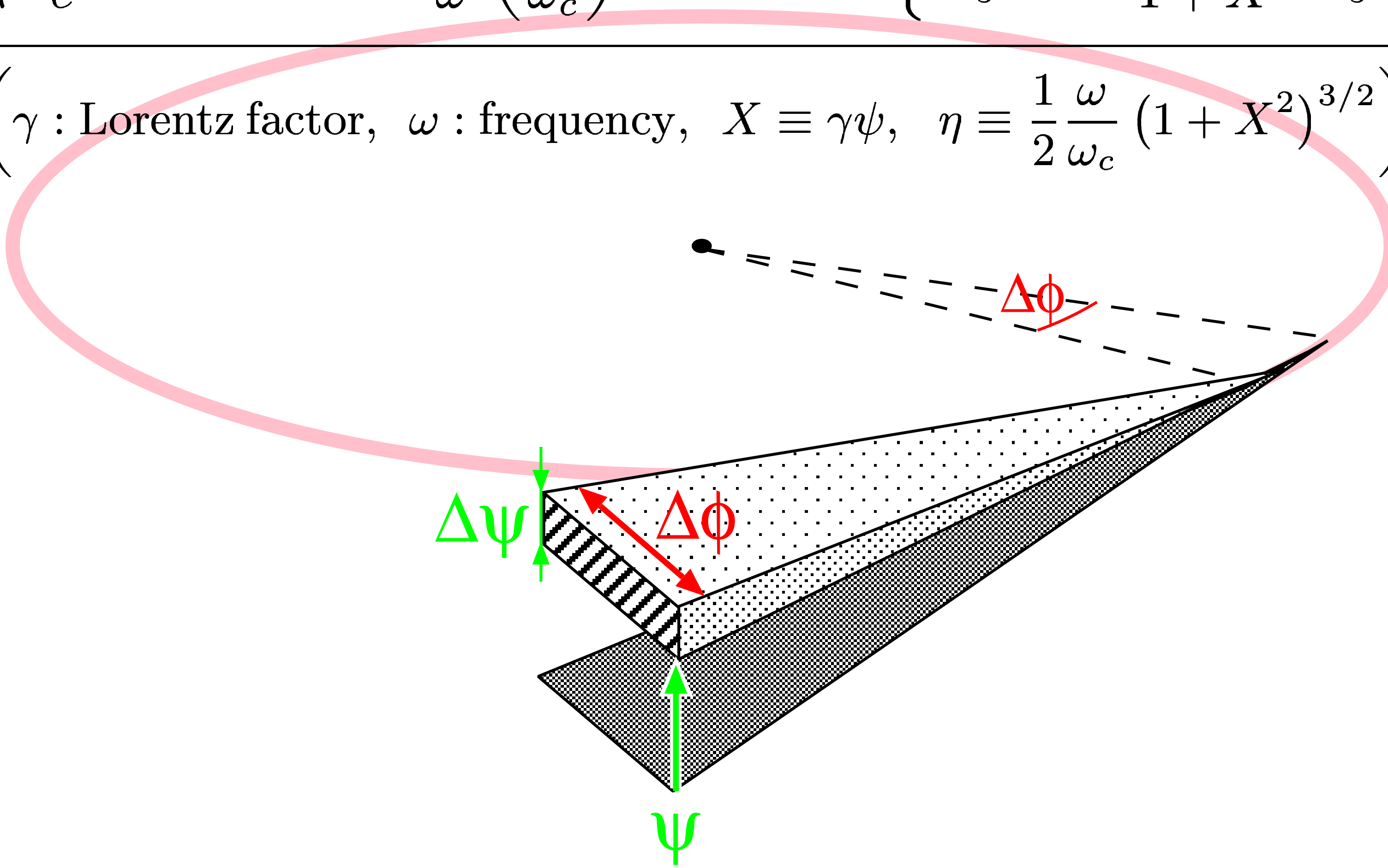


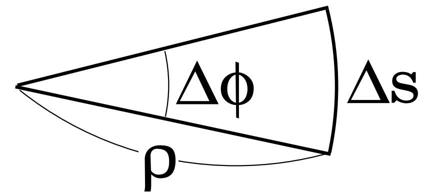
[III] Wattage Calculation

Using the following analytical formula (Phys. Rev. **75** (1949) 1912) :

$$\Delta W = \frac{3\alpha}{4\pi^2}\frac{I}{e}\gamma^2 \times \Delta\omega\Delta\phi\Delta\psi\,\frac{1}{\omega}\left(\frac{\omega}{\omega_c}\right)^2 \left(1+X^2\right)^2 \left\{K^2_{\frac{2}{3}}(\eta) + \frac{X^2}{1+X^2}K^2_{\frac{1}{3}}(\eta)\right\}\hbar\omega$$

$$\left(\gamma : \text{Lorentz factor},\quad \omega : \text{frequency},\quad X \equiv \gamma\psi,\quad \eta \equiv \frac{1}{2}\frac{\omega}{\omega_c}\left(1+X^2\right)^{3/2}\right)$$





$$\Delta\phi \cong \frac{\Delta s}{\rho(s)}$$

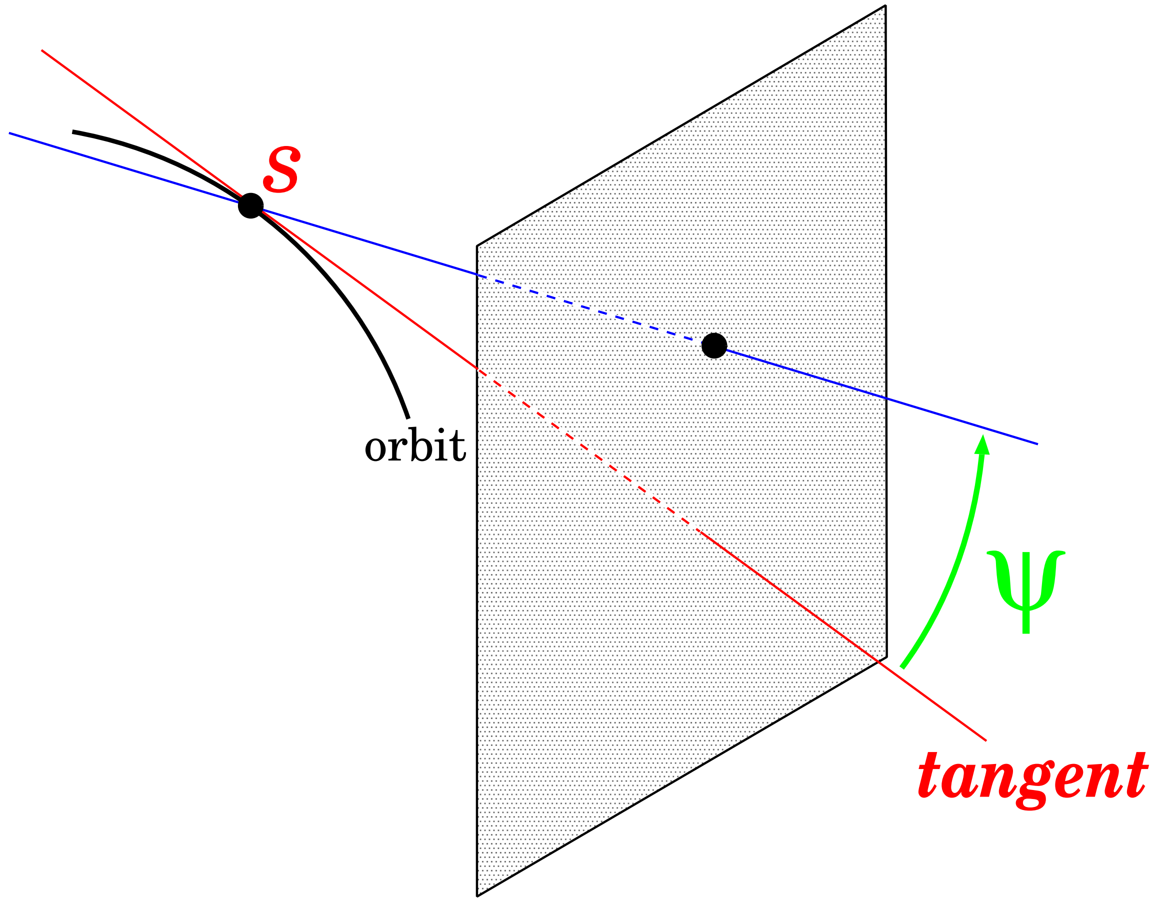
$$\sum \Delta s \rightarrow \int ds \quad \text{(integration along the orbit)}$$

$$W = \frac{3\alpha}{4\pi^2}\frac{I}{e}\gamma^2 \iiint d\omega ds d\psi \frac{1}{\omega}\left(\frac{\omega}{\omega_c}\right)^2 \left(1+X^2\right)^2 \left\{K_{\frac{2}{3}}^2(\eta) + \frac{X^2}{1+X^2}K_{\frac{1}{3}}^2(\eta)\right\} \frac{\hbar\omega}{\rho(s)}$$

↑

3D-integration is performed using BASES**.** (MC integration)

Projection to a Plane



Mapping: $(s, \psi) \;\rightarrow\;$ point in the plane

Check of the Wattage Calculation

by comparing with the SAD calculation

Start with the same lattice model file (`/ldata/KEKB/KCG/HER/herfq270.sad`)

↓

For a certain component (e.g. QC3LE)

↓ ↓

My Calculation | SAD

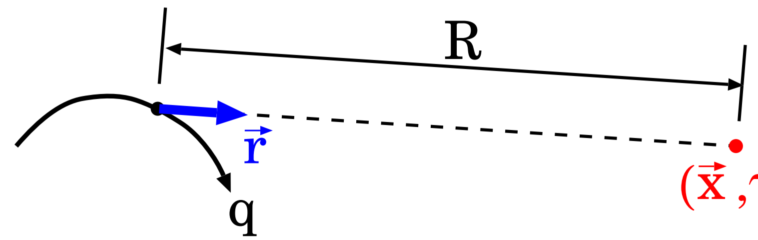
↓ ↓

Wattage distribution in the XY plane at $Z = -8\,\text{cm}$

Exact Calculation using SAD

starting with the **Feynman-Heaviside** formula

$$\vec{E}(\vec{x},\tau) = \frac{q}{4\pi\epsilon_0}\left[\frac{\vec{r}}{R^2} + \frac{R}{c}\frac{d}{d\tau}\left(\frac{\vec{r}}{R^2}\right) + \frac{1}{c^2}\frac{d^2}{d\tau^2}\vec{r}\right]$$



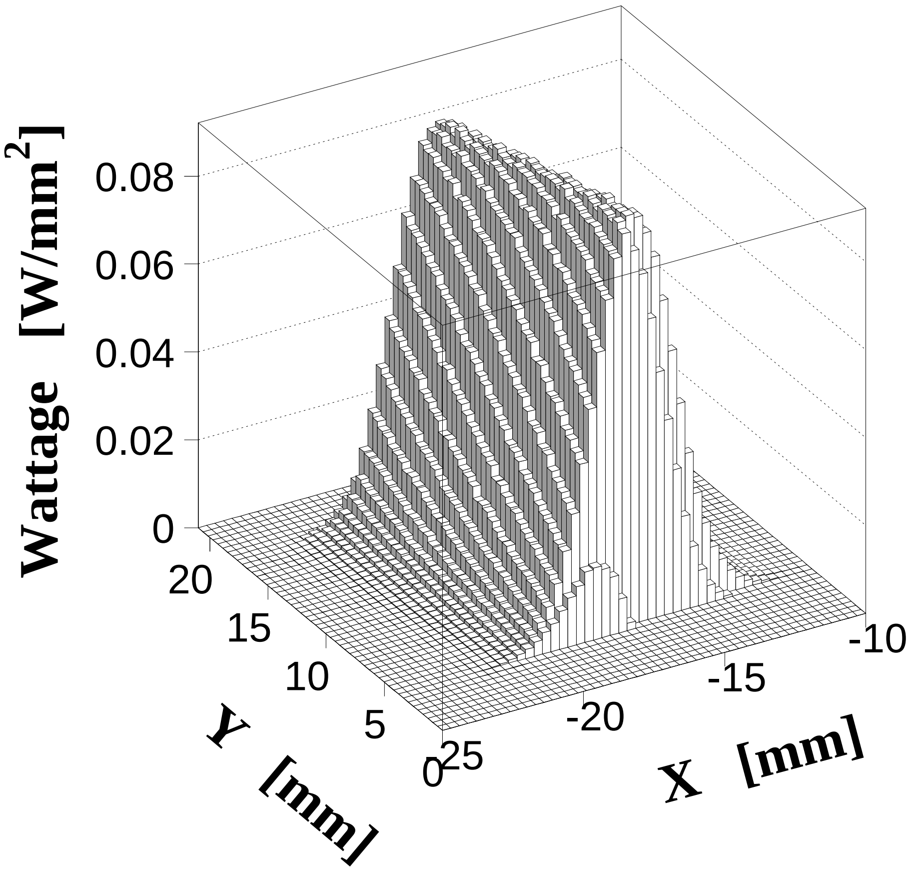
$$\vec{E}'(\vec{x},\tau) = \frac{1}{2\pi}\int_{\omega_{min}}^{\omega_{max}} \hat{\vec{E}}(\vec{x},\omega)\, e^{-i\omega\tau} d\omega, \quad \vec{H}' = \vec{r}\times\vec{E}'$$

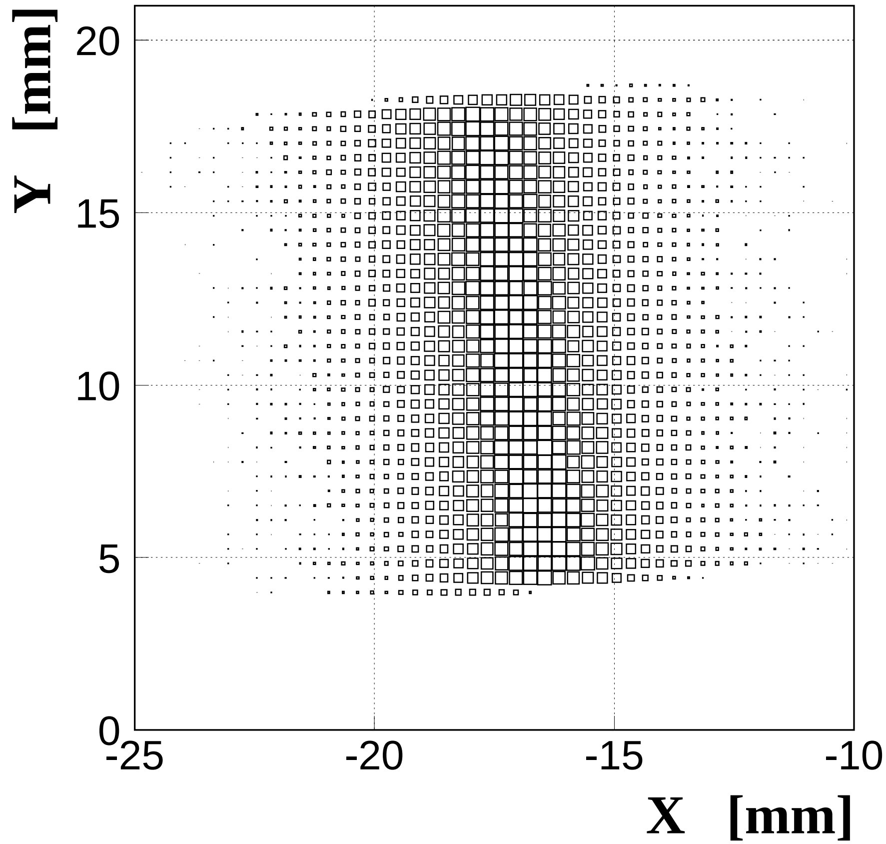
Poynting vector: $\vec{S} = \vec{E}' \times \vec{H}'$

$$W = \int\left\{\int_{-\infty}^{+\infty}\vec{S}(\vec{x},\tau)\,d\tau\right\}\cdot\vec{n}\,d\Omega \times \frac{I_{beam}}{e}$$

Example of My Calculation on QC3LE

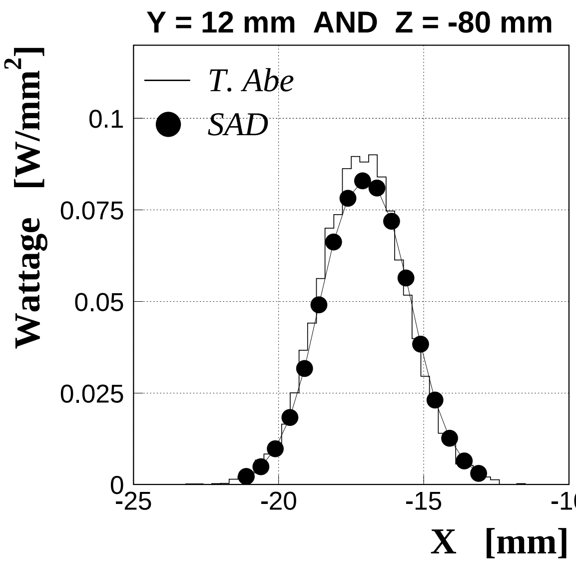
at $Z = -8\,\mathrm{cm}$

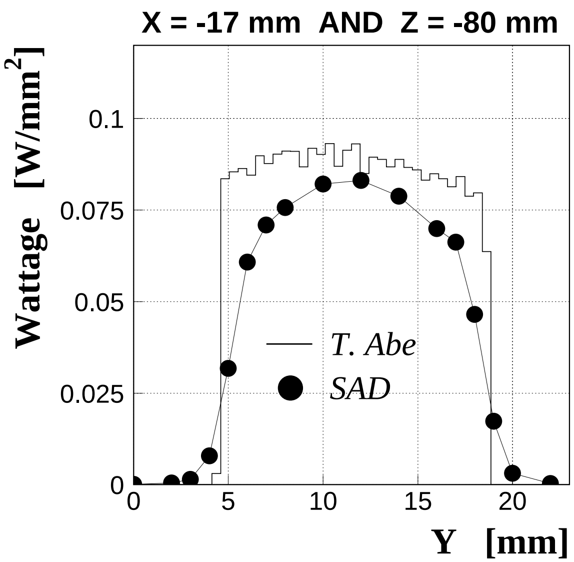




(X, Y, Z): Belle coordinate system

$I_{\mathrm{HER}} = 780\,\mathrm{mA},\ 0.1 < E_\gamma < 1\,\mathrm{keV}$

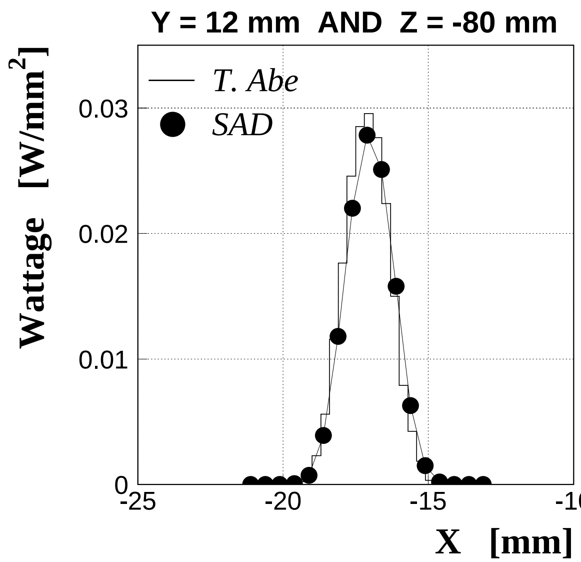


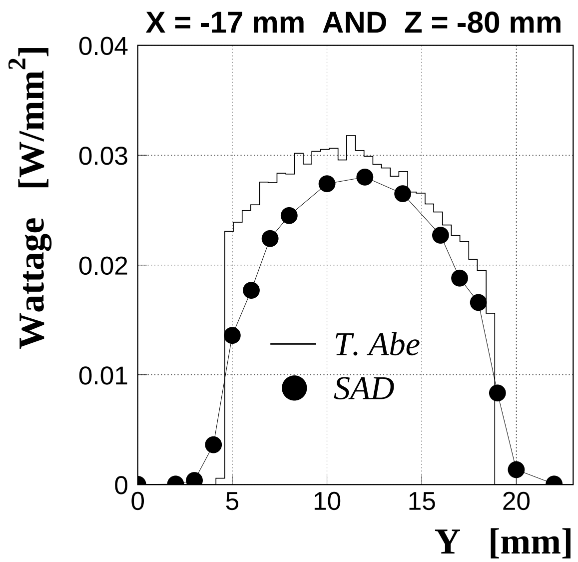


4.7 W (T.Abe)

4.0 W (SAD)

$I_{\mathrm{HER}} = 780\,\mathrm{mA},\ 1 < E_\gamma < 2\,\mathrm{keV}$



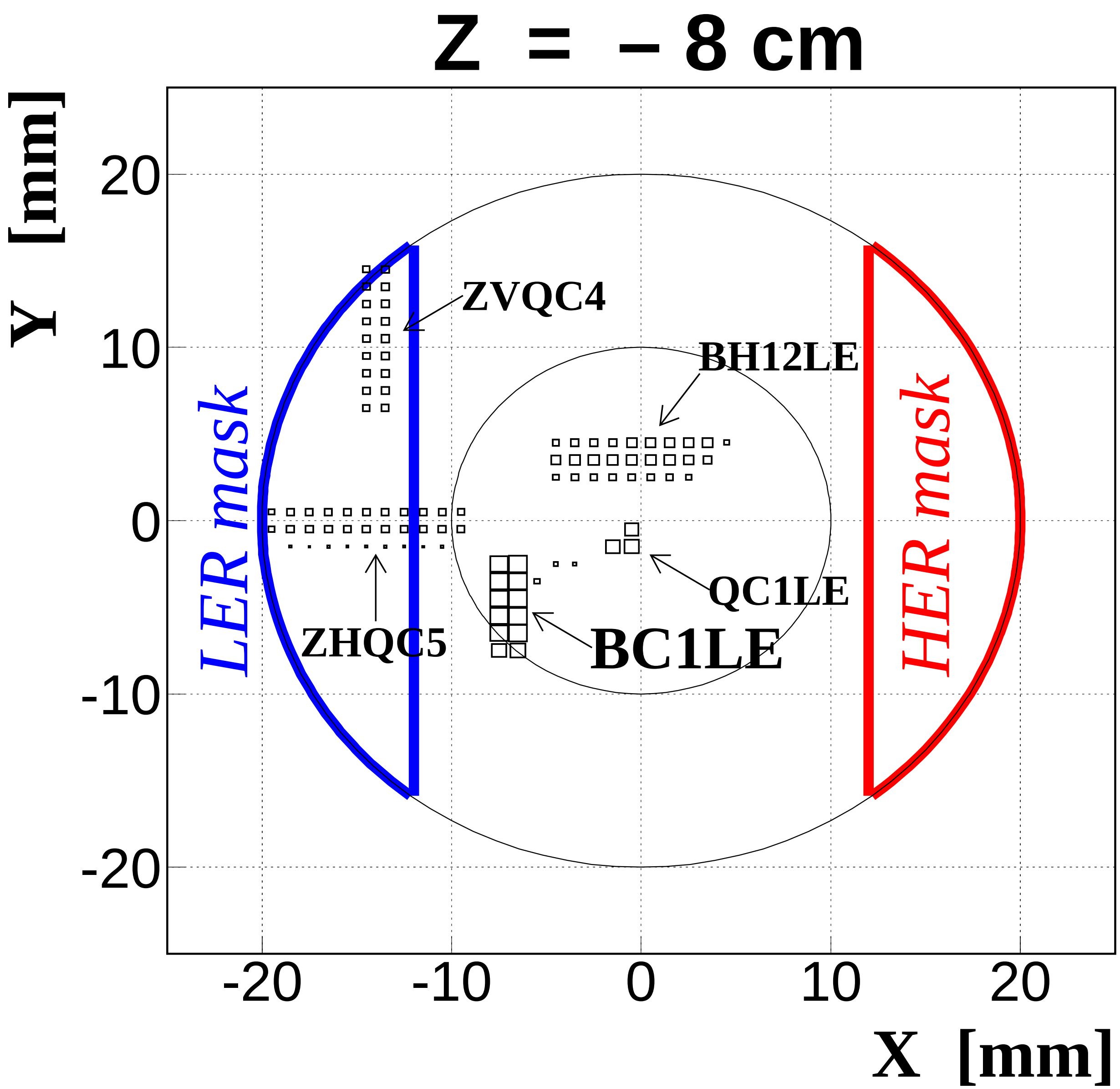


0.85 W (T.Abe)

0.75 W (SAD)

Conclusions on the Wattage Calculation

- **My geometry calculation is successful.**
- **Good agreements on ψ distributions**
- **Small differences on ϕ distributions:**
  $\rightarrow \sim 10\%$ **higher than SAD results for $E_\gamma \sim 1$ keV.**



HER SR

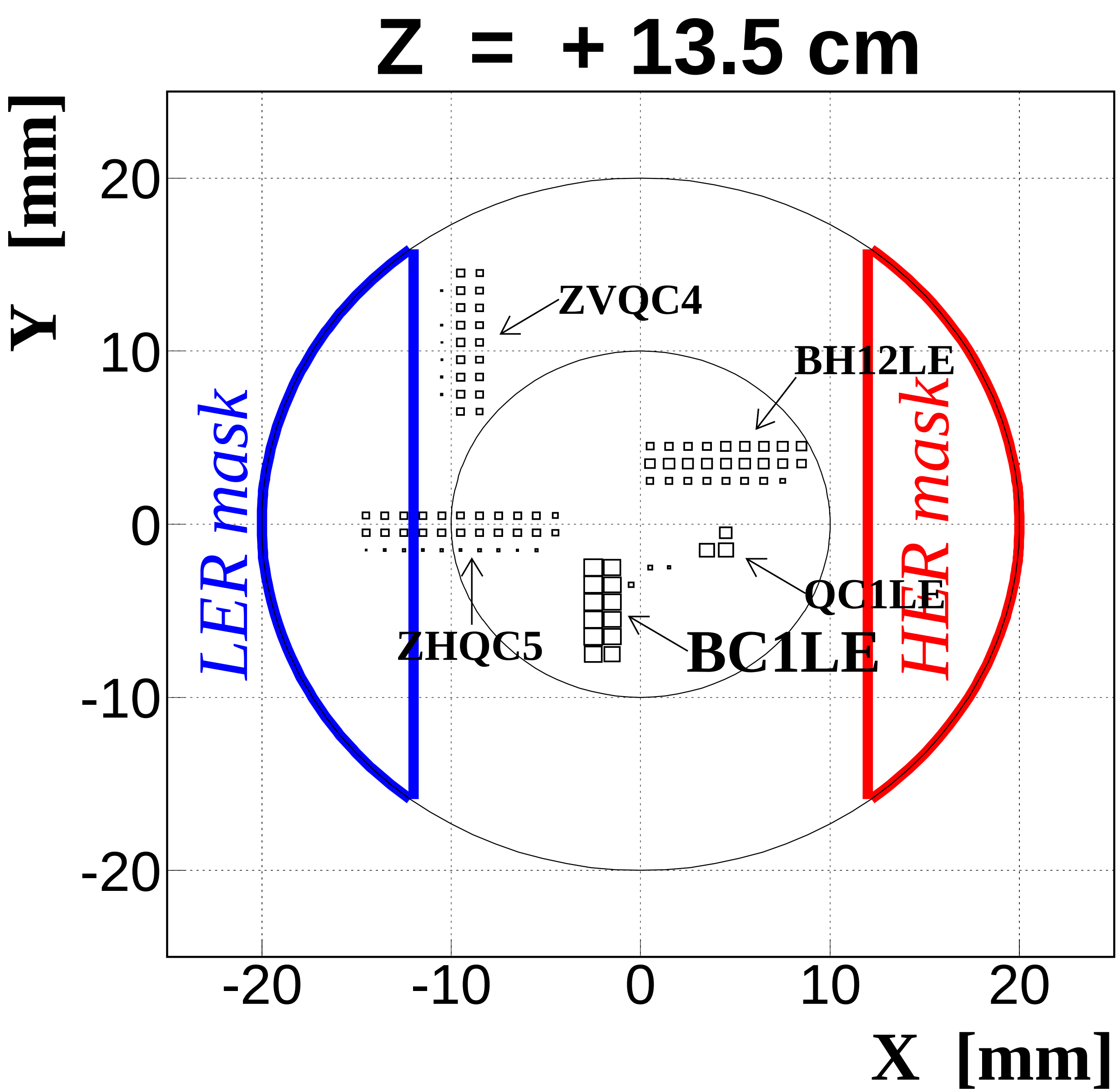
(2001.11.07.13:00:00)

$I_{HER} = 780$ mA

$E_\gamma > 10$ keV

$\Delta W_{MC}/W < 1\%$

Magnet	Wattage
BC1LE	0.1
BH1LE	0.9×10^{-6}
BH2LE	2.0×10^{-6}
ZVQC4	3.5×10^{-7}
ZHQC5	3.5×10^{-7}
QC1LE	1.1×10^{-4}
QC2LE	1.1×10^{-8}



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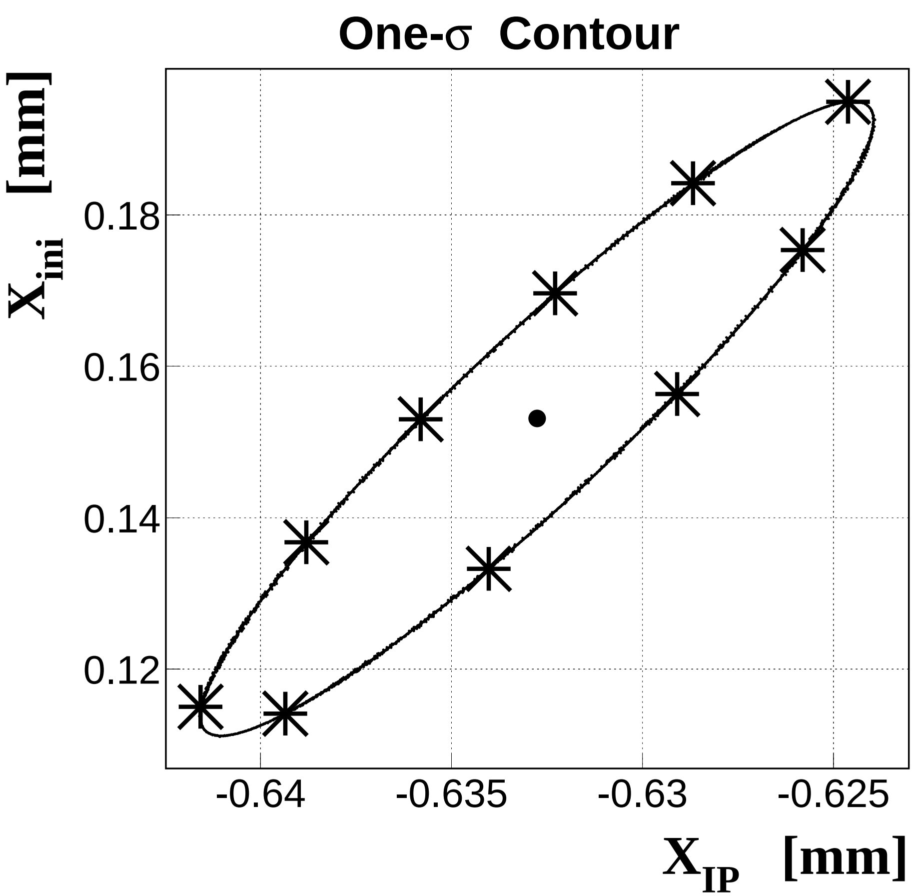
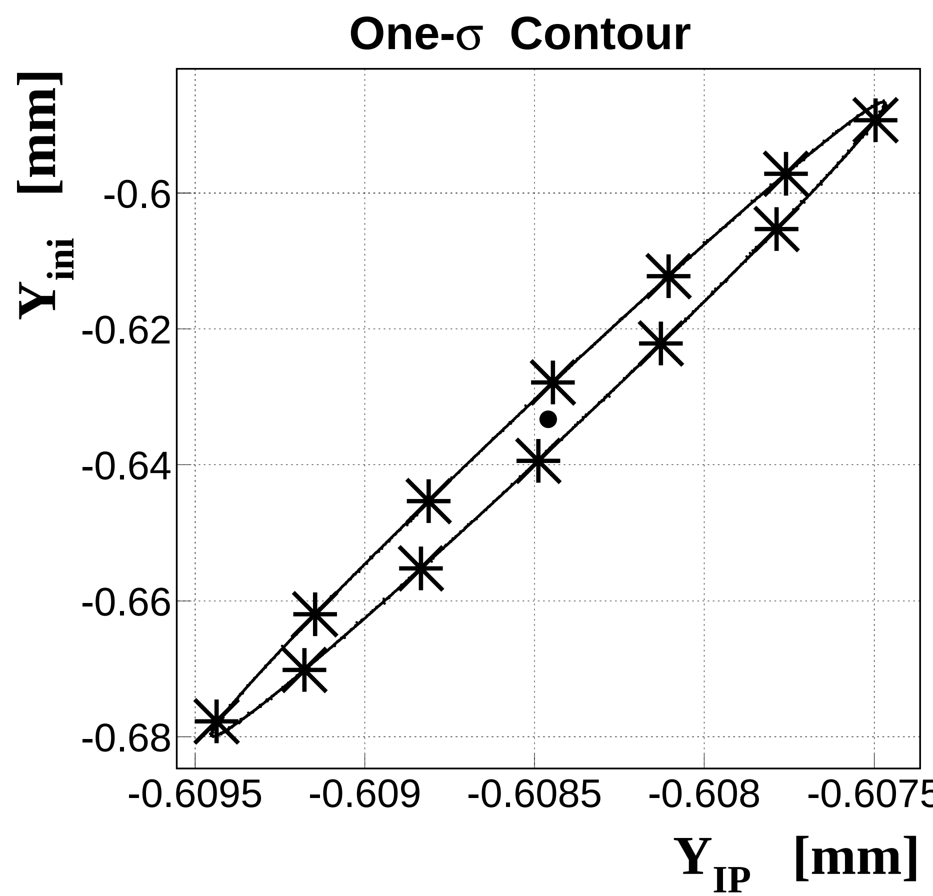
Error Estimation

Errors on the BPM **resolutions** $\oplus$ **offset** uncertainties are estimated.

- Using the data on Nov. 7 (2001) at 1:00PM
- If $\chi^2/ndf > 1 \Rightarrow \sigma_{BPM}^{(new)} \equiv \sigma_{BPM}^{(org)} \times \sqrt{\chi^2/ndf}$
  so that $\chi^2(n^{ew})/ndf \equiv 1$.

The values at the asterisks are used.



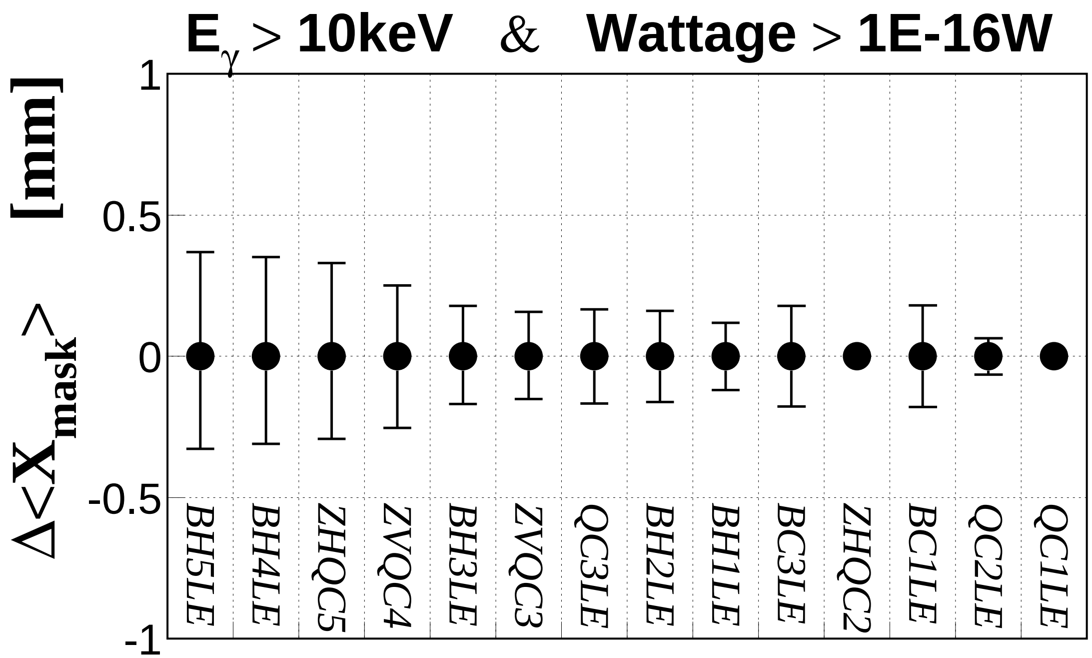


Position Errors

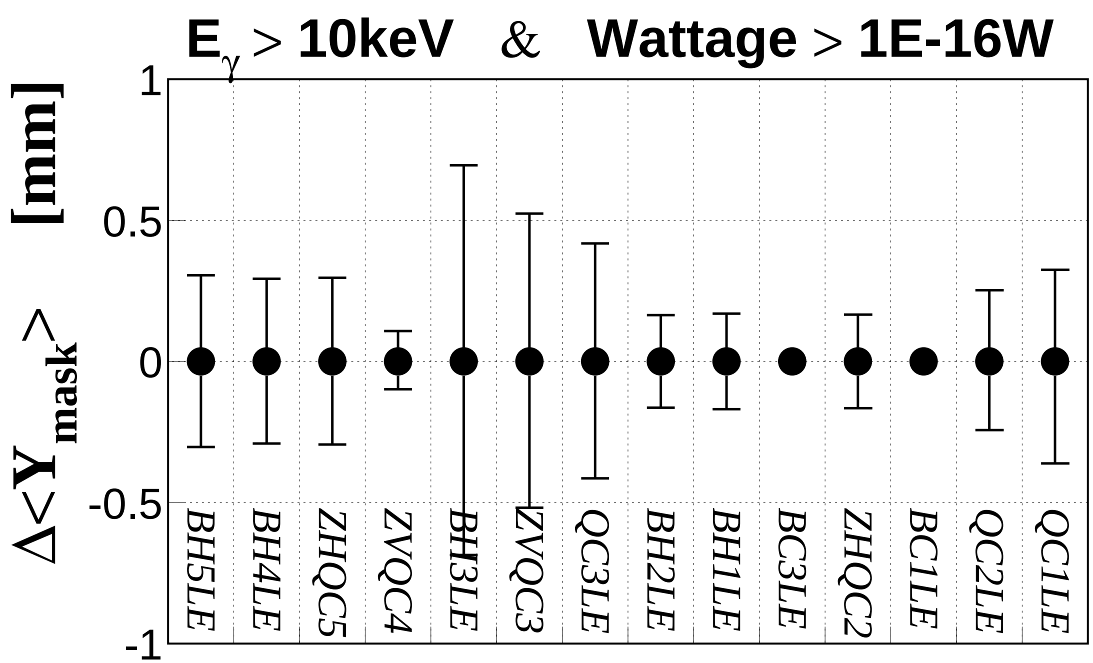
for

$$< X_{mask} > \equiv \frac{\int\int X \frac{d^2W(X,Y)}{dXdY} dXdY}{\int\int \frac{d^2W(X,Y)}{dXdY} dXdY}$$

$$< Y_{mask} > \equiv \frac{\int\int Y \frac{d^2W(X,Y)}{dXdY} dXdY}{\int\int \frac{d^2W(X,Y)}{dXdY} dXdY}$$

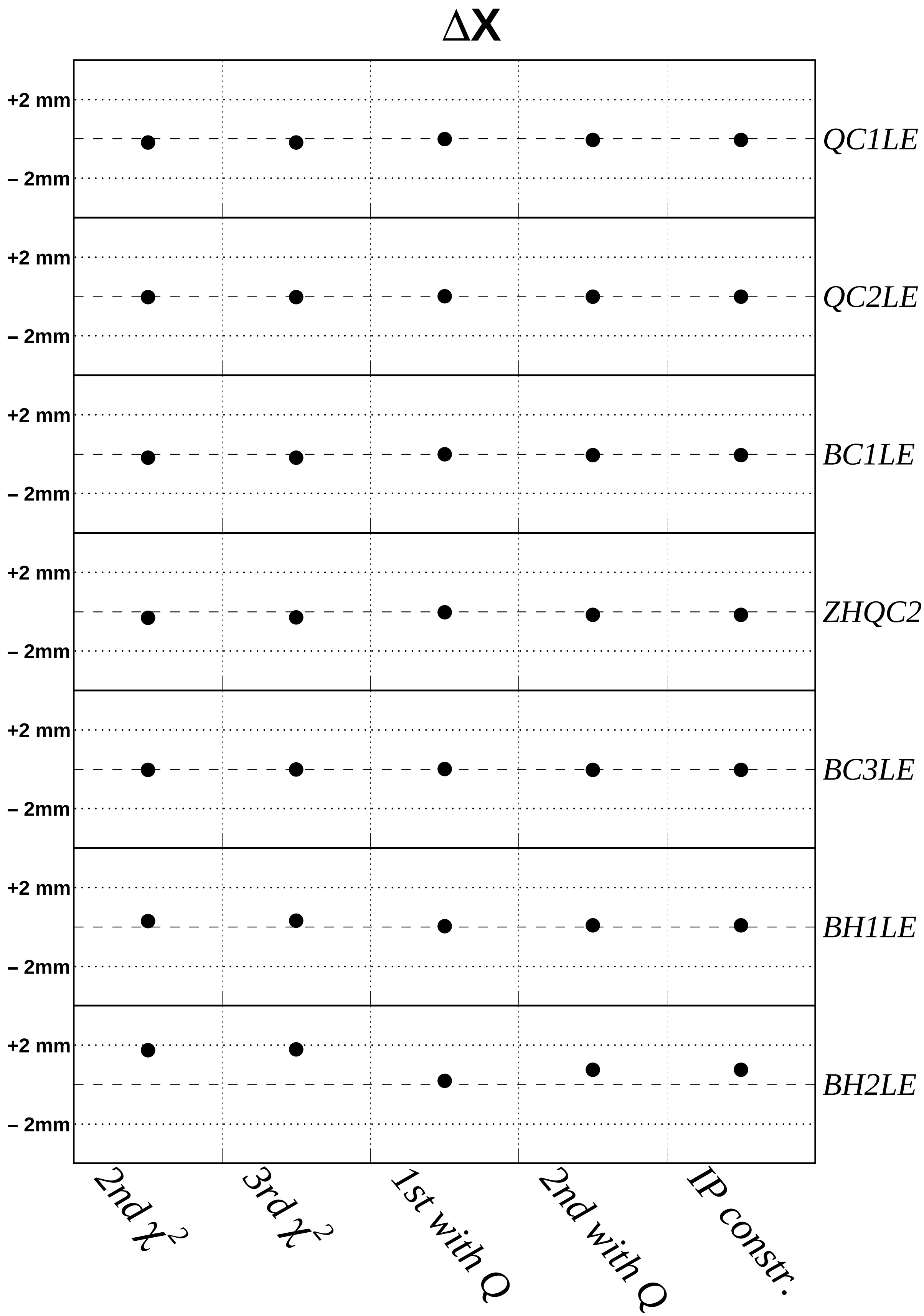


$$|\Delta X| \cong 0.2\,\mathrm{mm}$$

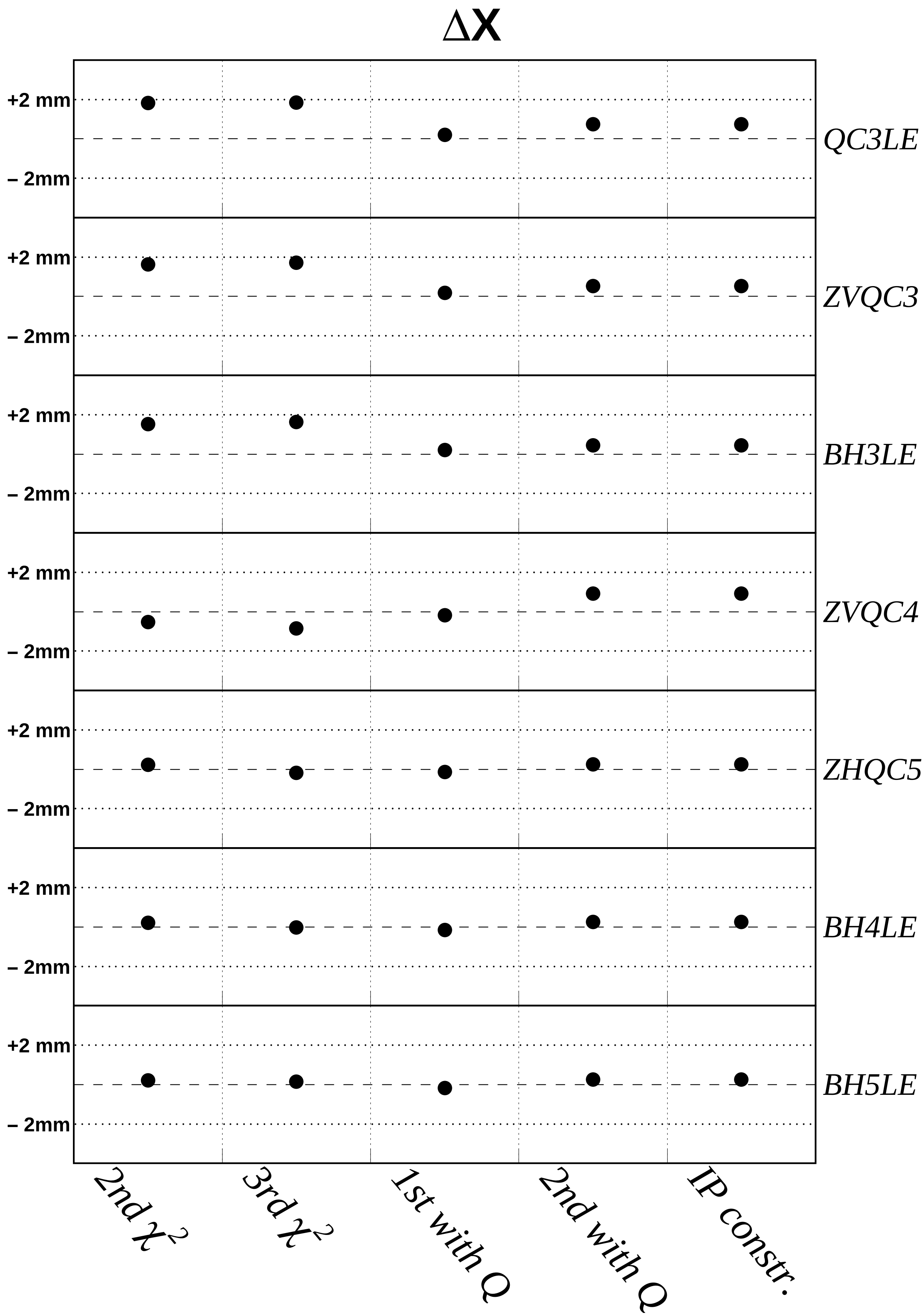


$$|\Delta Y| \cong 0.3\,\mathrm{mm}$$

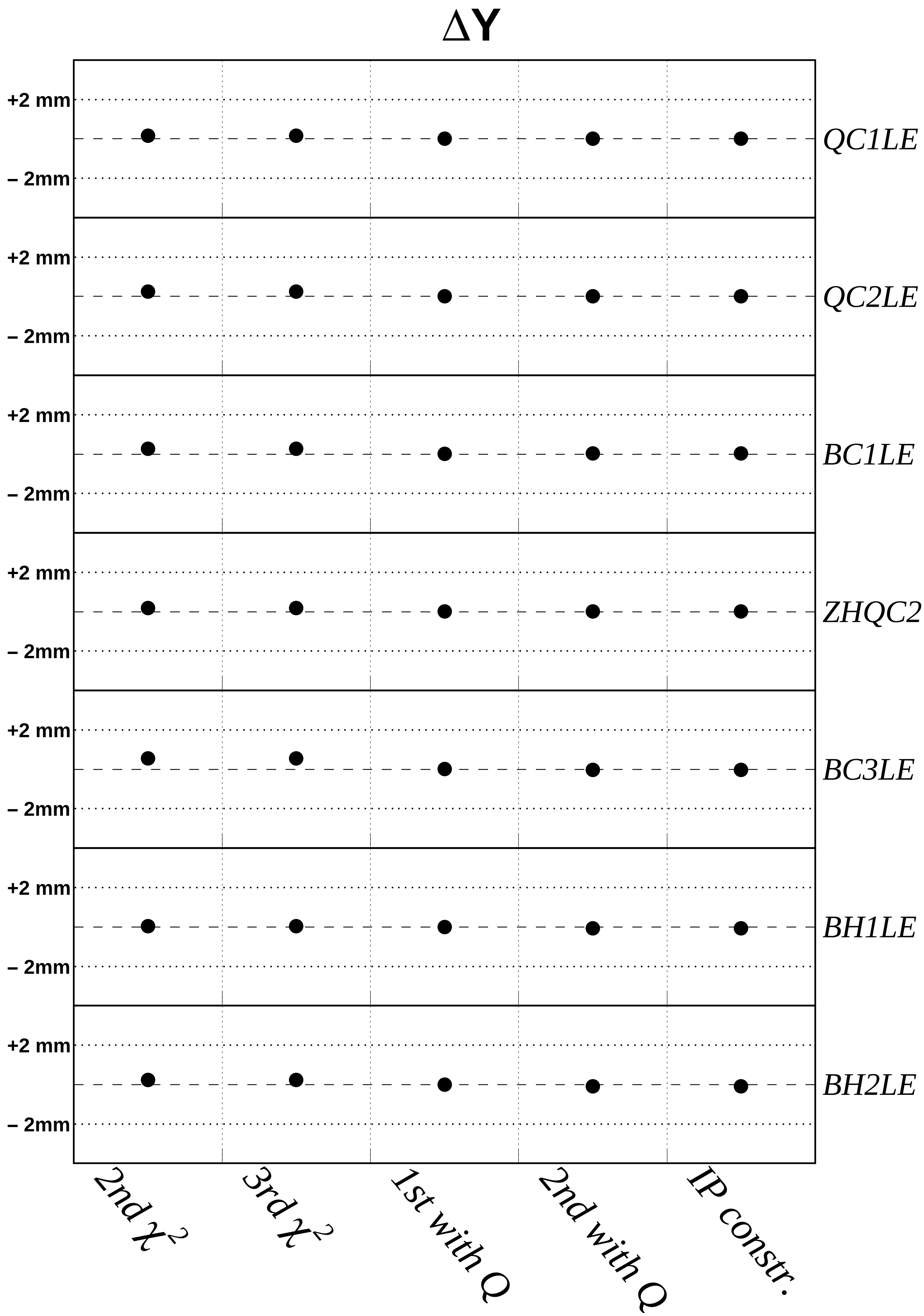
Horizontal Position Systematics (1)



Horizontal Position Systematics (2)



Vertical Position Systematics (1)



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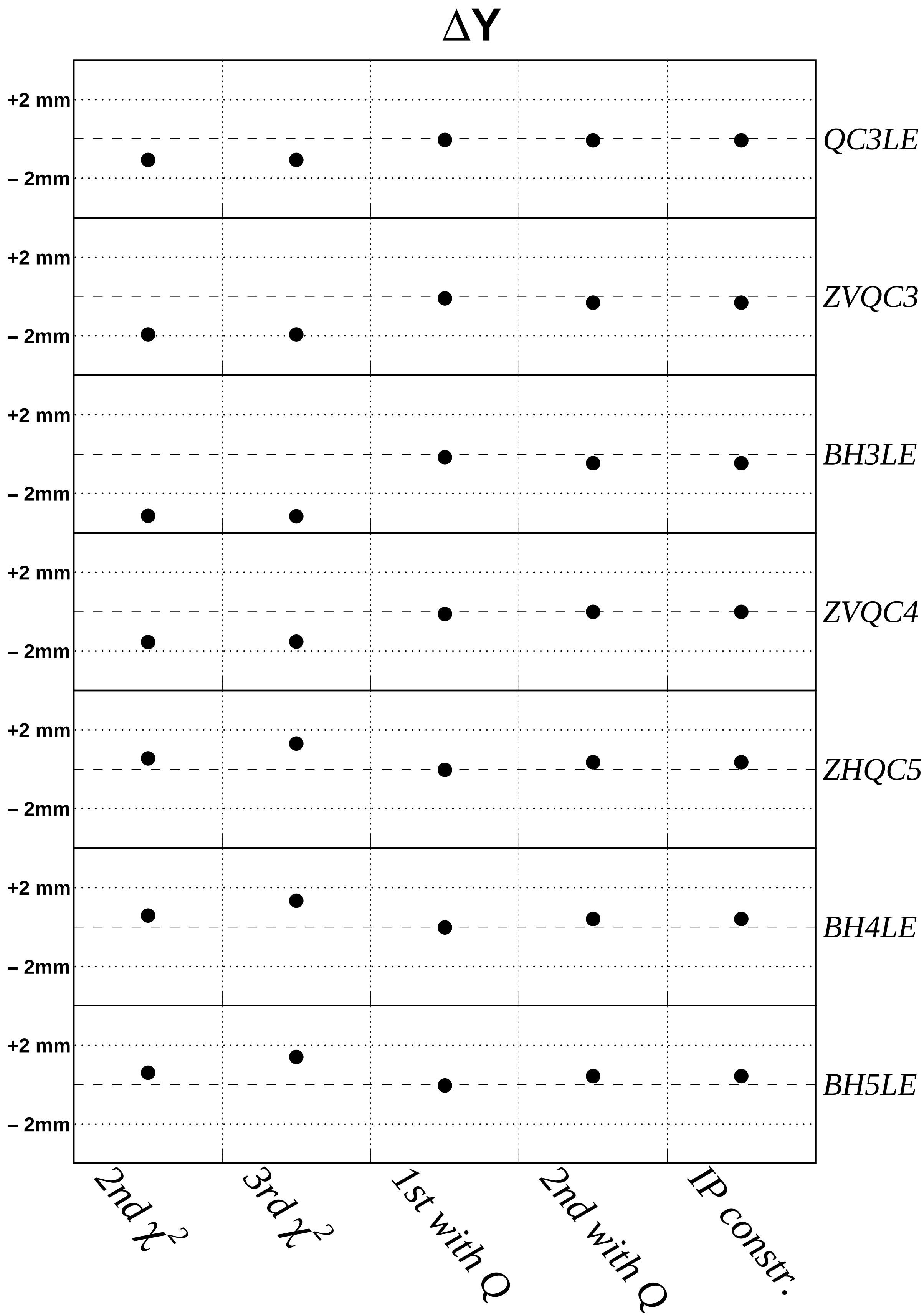
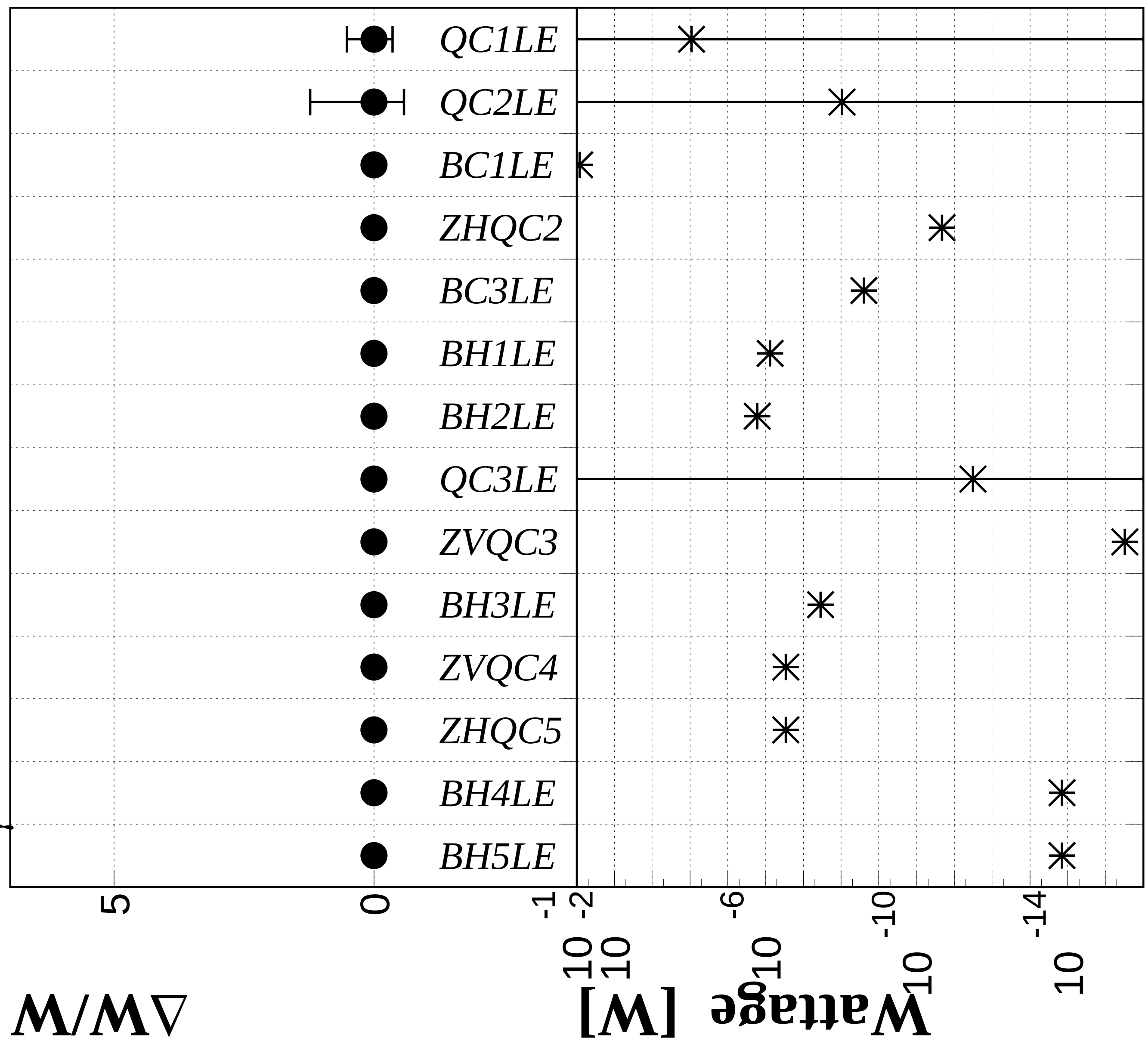
Vertical Position Systematics (2)



Wattage Errors on X

E_γ > 10keV & Wattage > 1E-16W



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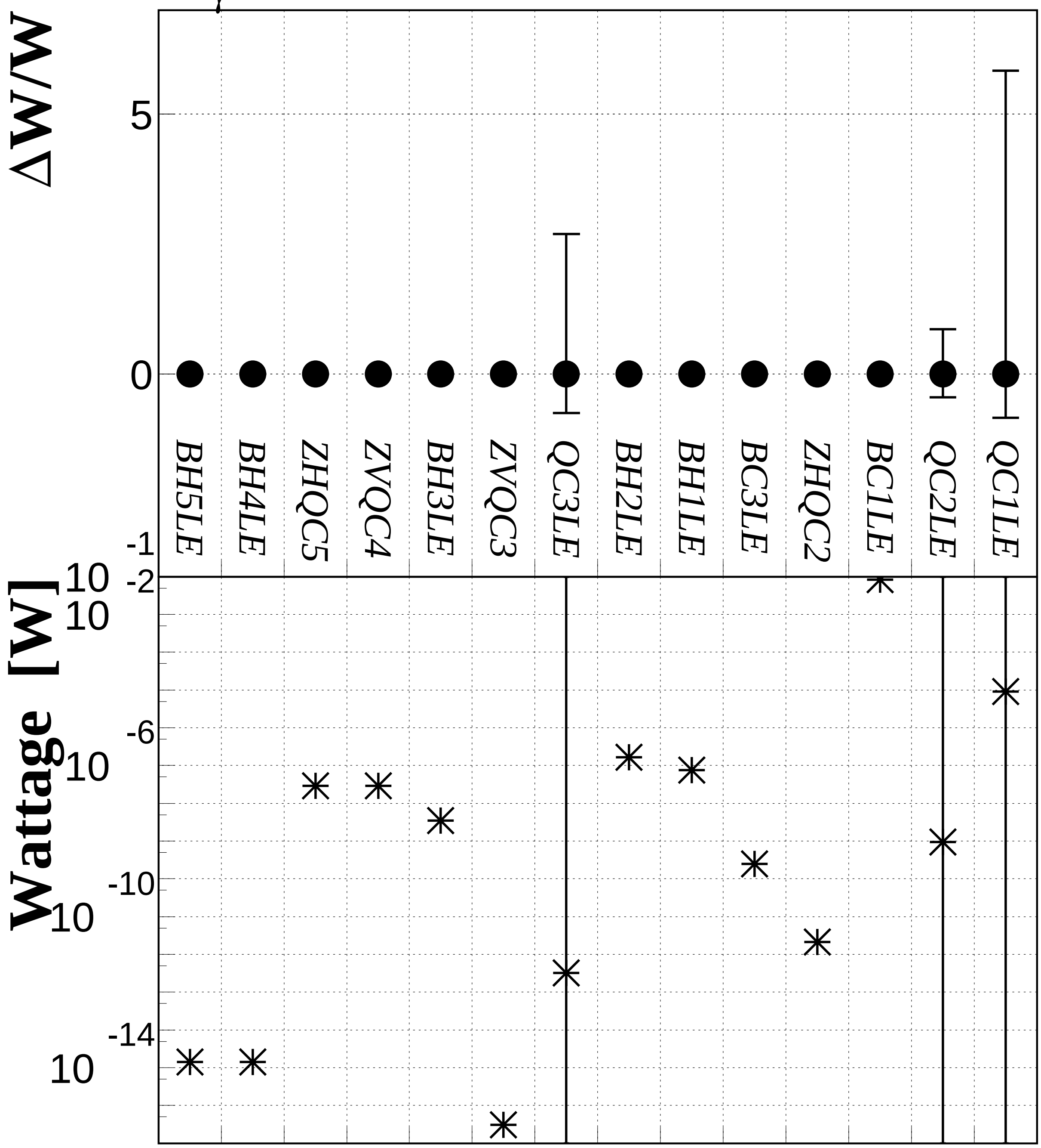
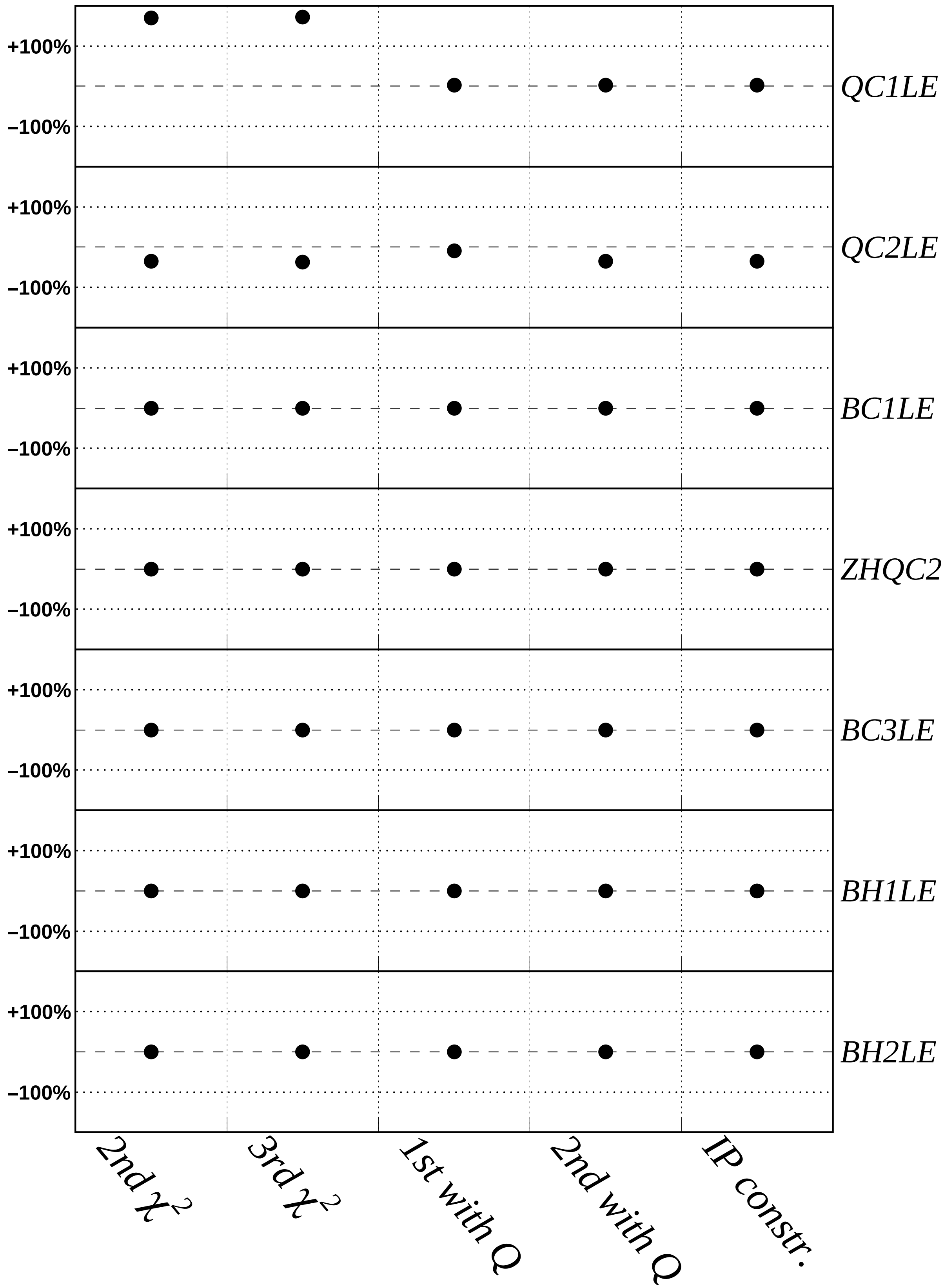
Wattage Errors on Y

$E_\gamma > 10$keV & Wattage > 1E-16W



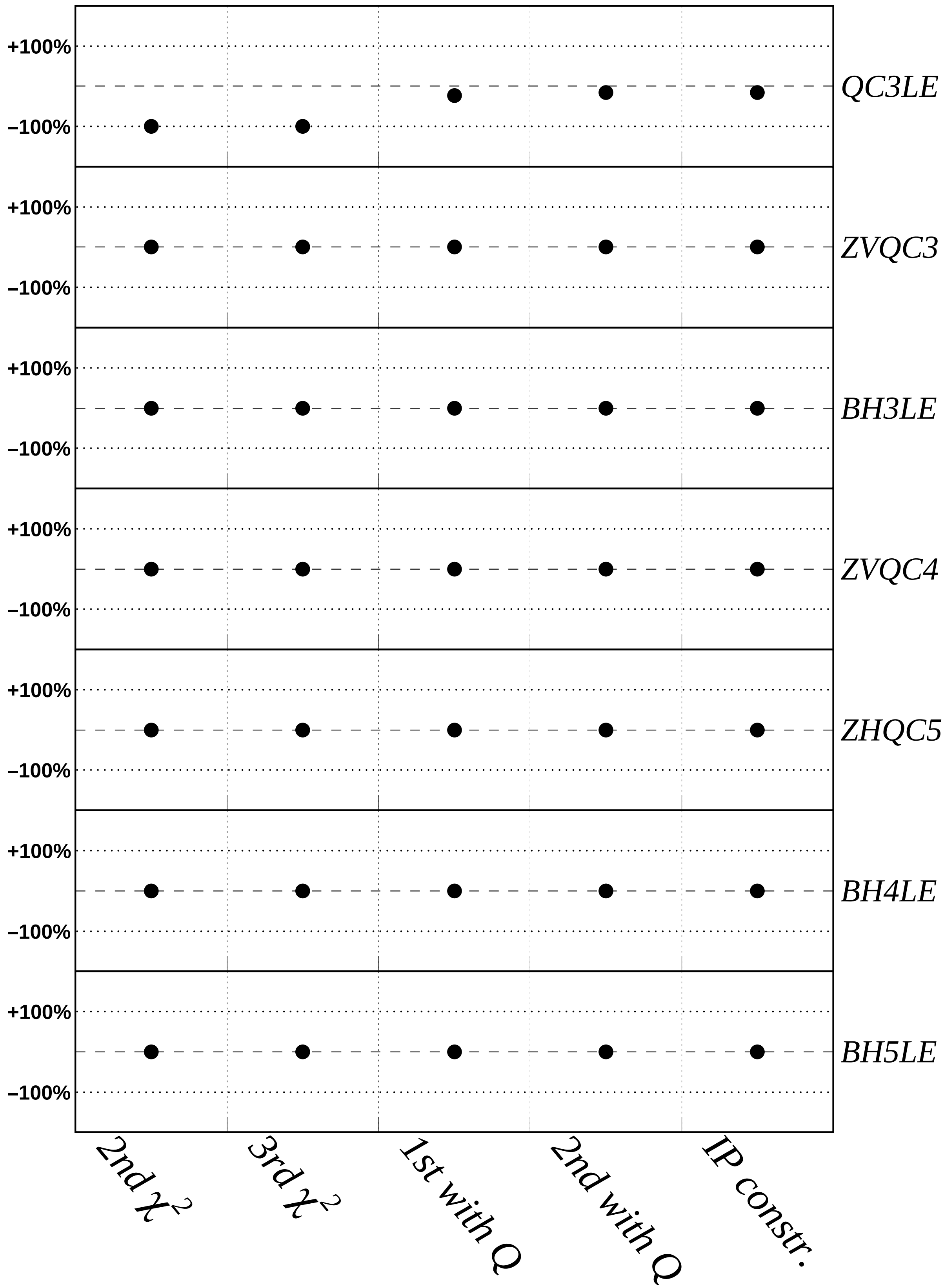
Wattage Systematics (1)

ΔW / W



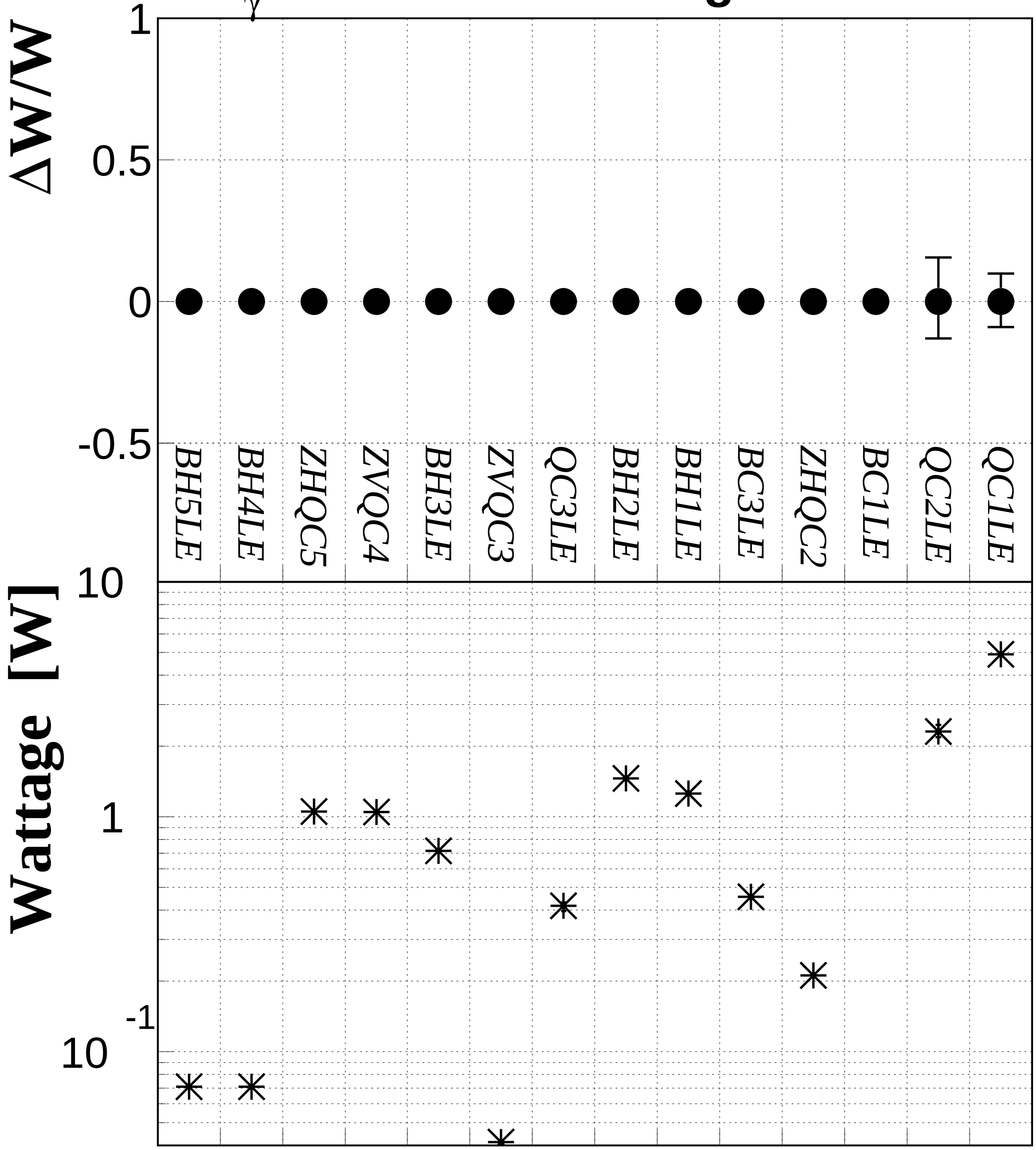
Wattage Systematics (2)

ΔW / W



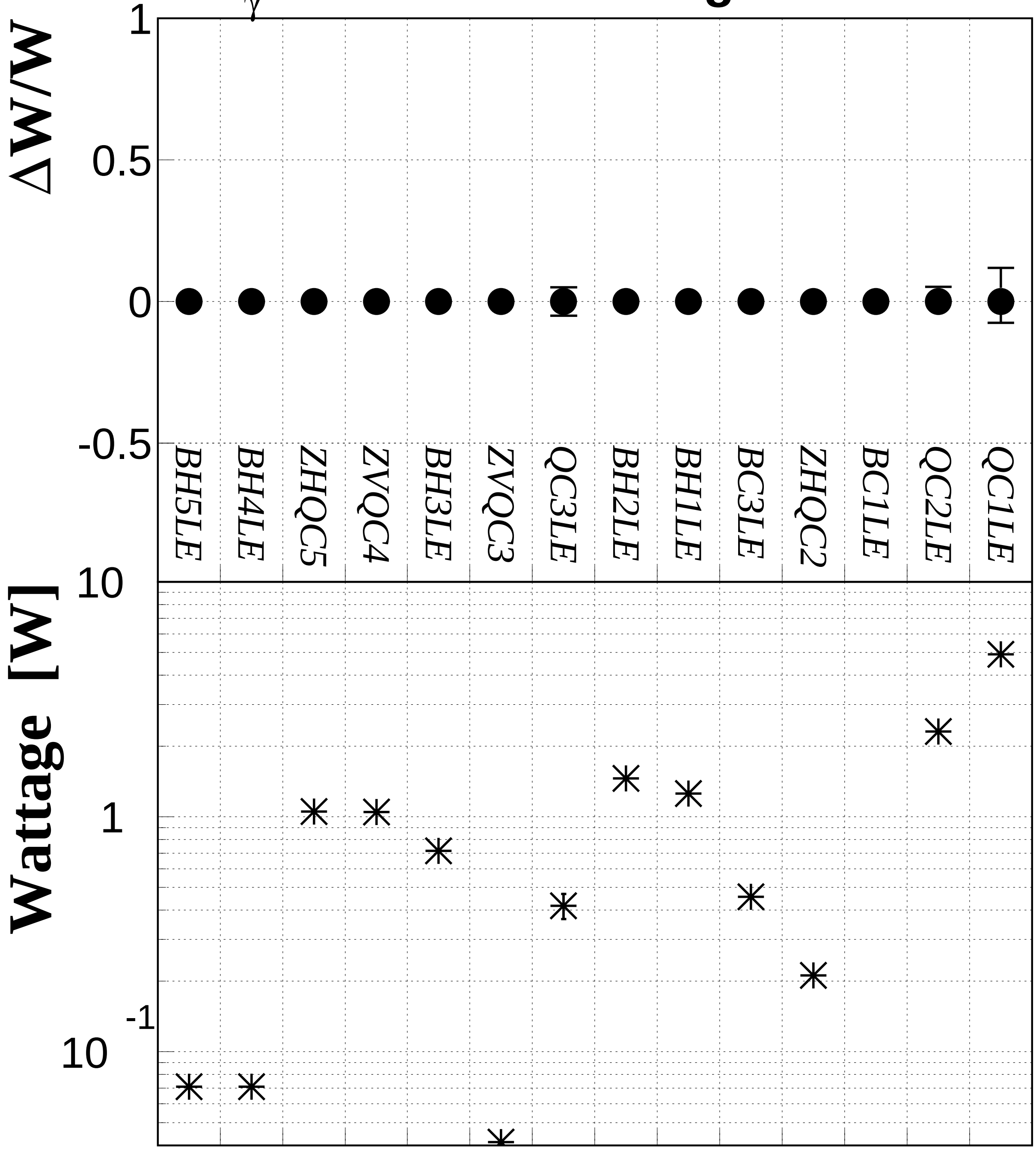
Wattage Errors on X

$E_\gamma > 1$keV & Wattage $>$ 4E-2W



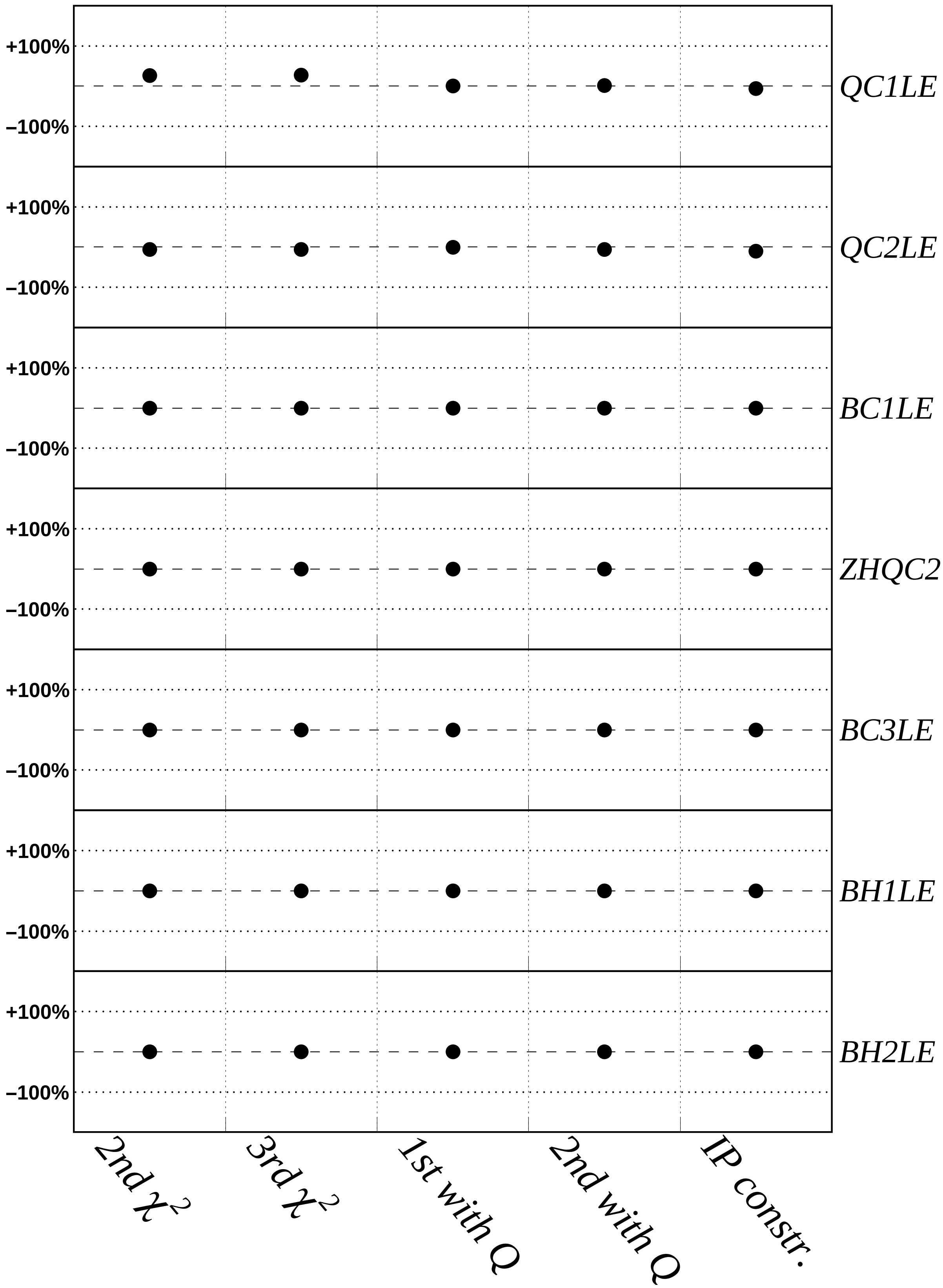
Wattage Errors on Y

$E_\gamma > 1$keV & Wattage > 4E-2W

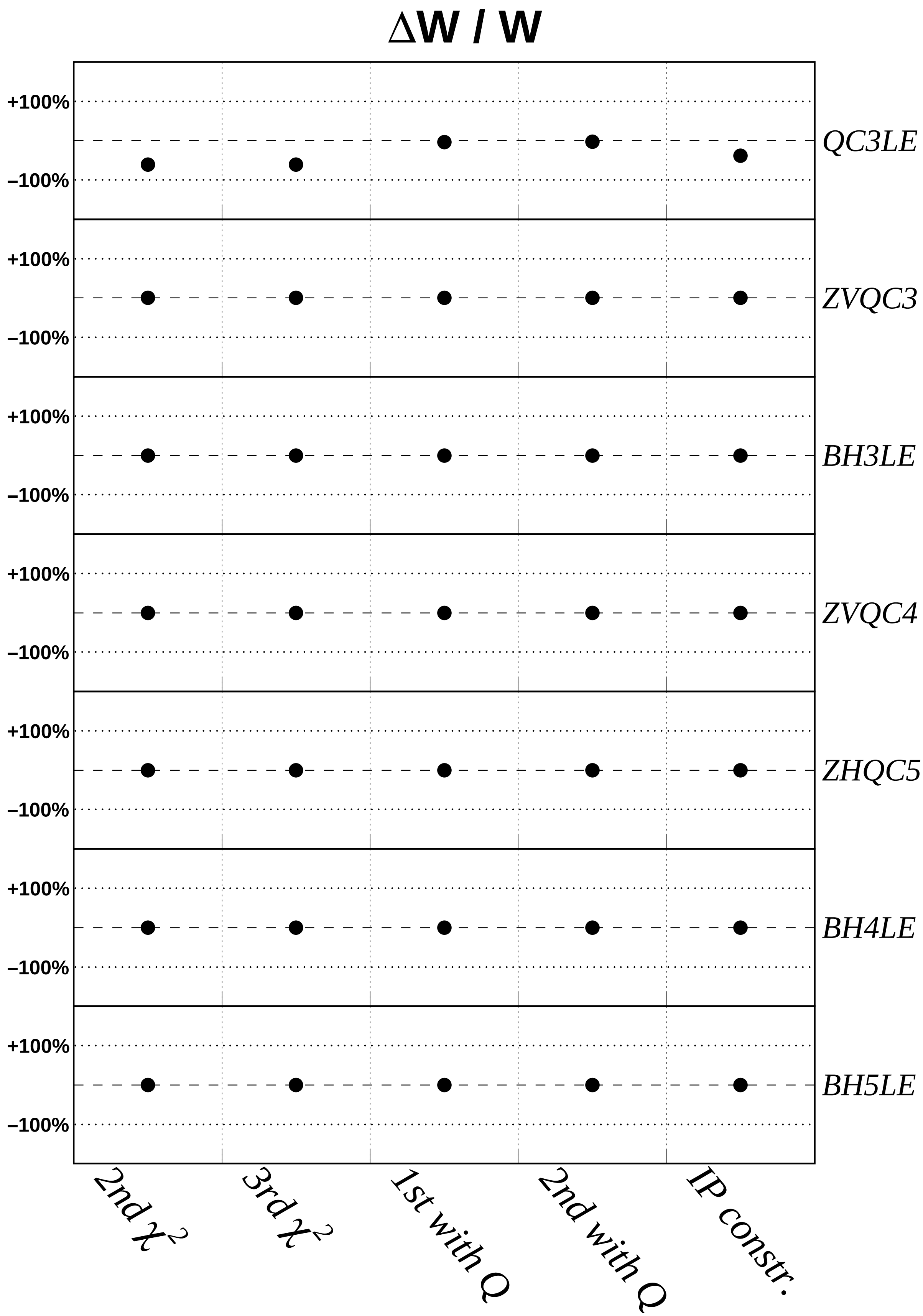


Wattage Systematics (1) ($E_\gamma > 1$ keV)

ΔW / W



Wattage Systematics (2) ($E_\gamma > 1$ keV)



Summary

Methods of the real orbit and wattage calculations have been established.

⇓

Wattage distributions in the IR

- My program has been checked comparing with the SAD results both on the orbit and wattage calculations.
- Error estimations have been also done:
  - $|\Delta < X > |, |\Delta < Y > | \sim 1$ mm
  - $\Delta W/W$: $10 \sim 30\%$ for $E_\gamma > 1$ keV,
    $2 \sim 5$ for $E_\gamma > 10$ keV

Future Works

- **For online monitoring**
  - Calculation speed: $3 \sim 4$ sec / (fit & MC-integrations) $\rightarrow$ OK
  - Input parameters (BPM, k) are already available from the Belle NSM.
- **Beam size** $\rightarrow$ to be implemented
- **Transformation to the corresponding radiation dose** $\rightarrow$ not yet
  - Interface to EGS4
- **Improvement of the offset determination algorithm**
  - Based on the current method
  - To reduce the wattage errors
  - More understanding on the real orbit calculation is needed.