GRAPE – Dilepton
(Version 1.0)
A Generator for Dilepton Production in ep Collisions

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Abstract: `GRAPE-Dilepton` is a Monte Carlo event generator for dilepton production in ep collisions. The cross-section calculation is based on the exact matrix elements. The Feynman amplitudes were generated by the automatic calculation system `GRACE`. In this version, all the electroweak diagrams at tree level are included except for proton-Z^0 couplings and lepton pair production through photon radiation from the proton. This generator has an interface to `PYTHIA` to get complete hadronic final states.

(The source code is located in `http://www-zeus.desy.de/~abe/grape`.)

1 Introduction

Dilepton[1] production in the electroweak theory is important as a background for various physics analyses like, for example, exclusive J/ψ or Υ production and new physics searches. So far only the generator `LPAIR` [1] has been used in experimental analyses to estimate the dilepton background [3]. `LPAIR` is based on the calculation of the diagrams of the two-photon (2-γ) process [2], the so called *Bethe-Heitler* (BH) process as seen in Fig. 1-(a) without Z^0 contribution. The 2-γ BH process is dominant in most of the phase space. It is, however, expected that in the region of low invariant masses of the dilepton system, internal photon conversion (CO) diagrams as seen in Fig. 1-(b) and Fig. 2-(b) become dominant. In the high mass region, there is an additional interesting process, i.e. Z^0 production which is implemented in the Monte Carlo generator `EPVEC` [4]. `EPVEC`, however, does not include 2-γ BH nor CO diagrams. In the di-electron channel, interference effects of the final state e^-e^- or e^+e^+ are not included, neither in `LPAIR` nor in `EPVEC`.

Here a new Monte Carlo generator for dilepton production based on the electroweak matrix elements in electron/positron-proton (ep) collisions is presented. The Feynman amplitudes in this program were generated by `GRACE` [7] which is an automatic calculation system. The name of `GRAPE` means *GRAce-based generator for Proton-Electron collisions*.

This generator has the following features.

[1]The word *dilepton* represents di-electron, di-muon and di-tau in this paper.

- All diagrams in the electroweak theory at tree level except for proton-Z^0 couplings and lepton pairs generated by photon radiation from the (quasi-)elastic proton are included in the matrix element calculation. In the di-electron channel, interference effects of the final state e^-e^- or e^+e^+ are also included. It is possible to select any sub-set of diagrams in the calculation.
- All fermion masses in this calculation are kept non-zero both in the matrix elements and in the kinematics, which makes it possible to use this program with arbitrary small scattering angles of e^+/e^- and/or small invariant masses of dilepton down to the kinematical limits.
- 3 types of proton vertices are available, which are elastic, quasi-elastic and DIS (Deep Inelastic eq Scattering).
- The effect of Initial State Radiation (ISR) can be included in the cross-section calculation based on the Structure Function method [5].
- Final State Radiation (FSR) and hadronization (in DIS) are performed by `PYTHIA` [6].

In this version, exclusive production of vector mesons (J/ψ, Υ, etc.) decaying to dileptons is not included.

2 Physics Aspects

The relevant processes are classified into 3 categories using the invariant mass of the hadronic system (M_{had}),

$$M_{had}^2 \stackrel{\text{def}}{\equiv} \left\{ (p_{e^\pm(in)} + p_{p(in)}) - (p_{e^\pm} + p_{l^+} + p_{l^-}) \right\}^2 , \tag{1}$$

where $p_{e^\pm(in)}$ and $p_{p(in)}$ are the 4-momenta of the incoming $e^\pm$ and the proton after ISR, respectively. $p_{e^\pm}$ and $p_{l^\pm}$ are those of the scattered $e^\pm$ and the produced lepton before FSR, respectively. The 3 categories are

- $M_{had} = M_p$ (*elastic*),
- $M_p + M_{\pi^0} < M_{had} < M_{cut}$ (*quasi-elastic*) and
- $M_{had} > M_{cut}$ (*DIS*),

where M_p and M_{π^0} are the masses of the proton and the neutral pion, respectively. The default value for M_{cut} is set to 5 GeV.

For the elastic process, the diagrams in Fig. 1 are calculated. The elastic proton vertex is described using a dipole form factor. The general form of the proton-proton-photon ($pp\gamma$) vertex ($\Gamma^{\mu}_{pp\gamma}$) where protons are on-shell can be written as

$$\Gamma^{\mu}_{pp\gamma} = e_p \left(F_1(q^2)\gamma^\mu + \frac{\kappa_p}{2M_p} F_2(q^2) i\sigma^{\mu\nu} q_\nu \right) \tag{2}$$

where e_p indicates the electric charge of the proton, q means the 4-momentum transfer ($= p_{p(out)} - p_{p(in)}$), $F_1(q^2)$ and $F_2(q^2)$ are two independent form factors, κ_p is the anomalous magnetic moment of the proton and M_p is the mass of the proton (see, for example, [8].). The electric and magnetic form factors ($G_E^p(q^2)$ and $G_M^p(q^2)$ respectively) are defined as follows,

$$\begin{pmatrix} G_E^p(q^2) \\ G_M^p(q^2) \end{pmatrix} = \begin{pmatrix} F_1(q^2) & + & \frac{\kappa_p q^2}{4M_p^2} F_2(q^2) \\ F_1(q^2) & + & \kappa_p \ F_2(q^2) \end{pmatrix}. \tag{3}$$

Using the Gordon decomposition and the scaling law,

$$G_E^p(q^2) = G_M^p(q^2)/|\mu_p|, \tag{4}$$

the following formula which is used in this program is obtained,

$$\Gamma^{\mu}_{pp\gamma} = e_p \left(\mu_p G_E^p(q^2)\gamma^\mu - \frac{(p^\mu_{p(in)} + p^\mu_{p(out)})}{2M_p} \frac{\kappa_p}{1 - \frac{q^2}{4M_p^2}} G_E^p(q^2) \right) \tag{5}$$

where $\mu_p = (1+\kappa_p)\mu_B$ and μ_B indicates the Bohr magneton. $G_E^p(q^2)$ is calculated according to the dipole fit,

$$G_E^p(q^2) = \left(1 - \frac{q^2}{0.71\,\mathrm{GeV}^2}\right)^{-2}. \tag{6}$$

The only difference between the elastic and the quasi-elastic processes is the treatment of the proton vertex and the final state hadronic system. The quasi-elastic proton vertex can be described using the hadron tensor in the following form assuming parity and current conservation (for example, see [8]),

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2 \frac{1}{M_p^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right), \tag{7}$$

where $p = p_{p(in)}$. $W_1(q^2, M_{had})$ and $W_2(q^2, M_{had})$ are the electromagnetic proton structure functions. To obtain the cross-section formula, the hadron tensor is contracted with the lepton tensor $L^{\mu\nu}$ which is calculated using amplitudes generated by `GRACE`,

$$d\sigma \sim L_{\mu\nu} W^{\mu\nu}. \tag{8}$$

In this version, the structure functions in `EPVEC` are used for W_1 and W_2. The functions were parameterized fitting the experimental data [9] in the following kinematic region,

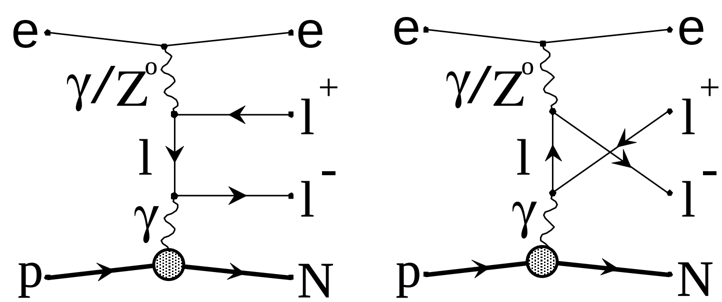
$$Q_p^2 < 16\,\mathrm{GeV}^2 \quad \text{and} \quad M_{had} < 5\,\mathrm{GeV}, \tag{9}$$

where $Q_p^2 = -q^2 > 0$. The exclusive hadronic system is simulated in the event generation step with the method used in the generator `EPSOFT` [10] for proton dissociative processes.

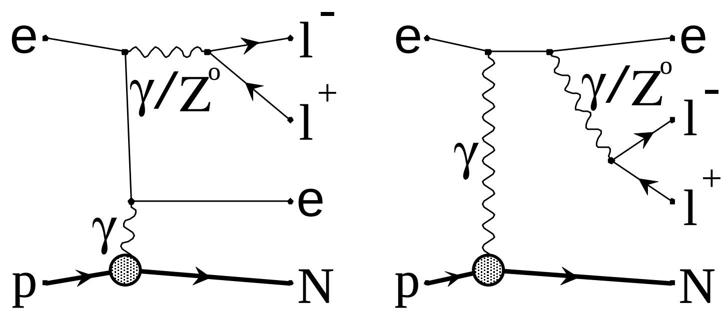
For the DIS process, the Quark-Parton Model (QPM) is used, and the diagrams in Fig. 2 are calculated. The `PDFLIB` [11] is linked to get parton densities. The simulation of the proton remnant and the hadronization are performed by `PYTHIA`. It should be noted that the lowest order calculation in this process is valid only in the region,

$$u \stackrel{\text{def}}{\equiv} |\{p_{q(in)} - (p_{l+} + p_{l-})\}^2| \;\; \gtrsim \;\; 25\,\mathrm{GeV}^2 \tag{10}$$

where $p_{q(in)}$ is the 4-momentum of the incoming quark. The value of u corresponds to the virtuality of the u-channel quark in the diagram in Fig. 2-(b). When it is around or smaller than 25 GeV^2, large QCD corrections are needed, so that the lowest order calculation is not correct as explained in [4].



(a) Bethe-Heitler diagrams



(b) Internal γ-conversion diagrams and Z^o on/off-shell production

Fig. 1: Feynman diagrams included in (quasi-)elastic processes. e=$\{e^+, e^-\}$, $l^\pm$=$\{e^\pm, \mu^\pm, \tau^\pm\}$. N means (dissociated) proton or a nucleon resonance.

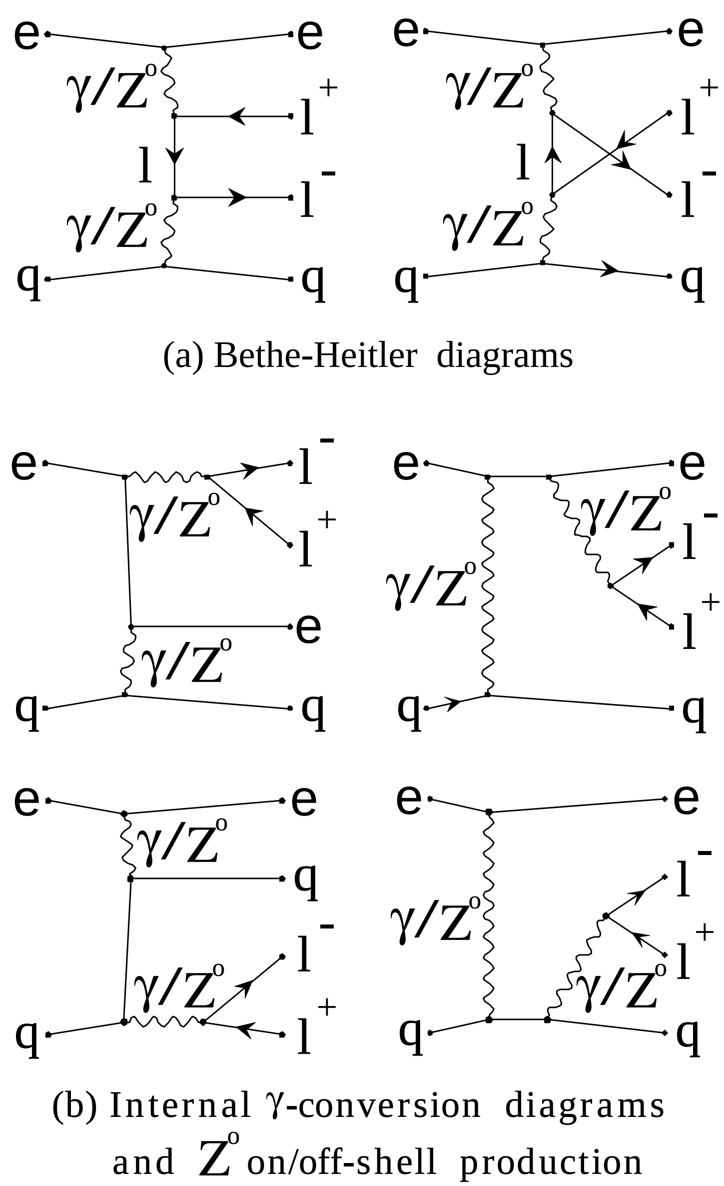


Fig. 2: Feynman diagrams included in DIS processes. $e=\{e^+, e^-\}$, $\mathrm{l}^\pm=\{e^\pm, \mu^\pm, \tau^\pm\}$, $q=\{\overset{(-)}{u},\overset{(-)}{d},\overset{(-)}{s},\overset{(-)}{c},\overset{(-)}{b},\overset{(-)}{t}\}$.

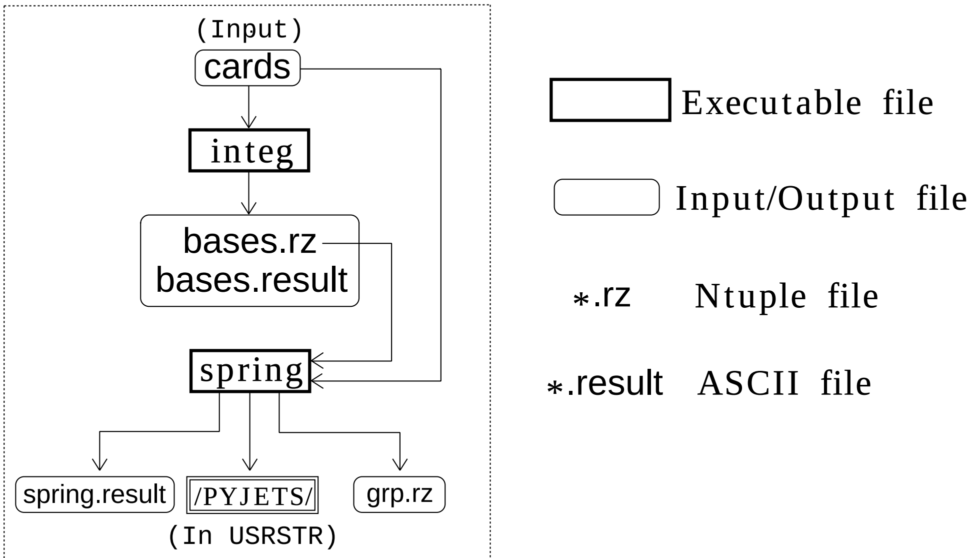


Fig. 3: Flowchart for the program `GRAPE-Dilepton`

3 Program Structure

There are 2 steps to be done by users; the integration step and the event generation step. In the integration step, an effective total cross-section and probability distributions are calculated by the program `BASES` [12] running the executable file `integ`. The resulting information is stored in the file `bases.rz`. In the event generation step by running the executable file `spring`, unweighted events are generated by the program `SPRING` [12] according to the probability distributions in `bases.rz`. You can get the information on the generated events from the common block `/PYJETS/` provided by `PYTHIA` which can be found in the file `usrstr.f`. Program parameters are controlled editing the file `cards` which is read on executing `integ` or `spring`. Fig. 3 shows the flow of the above procedure.

4 Comparison with the Generator `LPAIR`

A check was done by comparing the cross-section with `LPAIR`. Only 2-γ BH diagrams (see Fig. 1-(a) or Fig. 2-(a) without Z^0 contribution) are used in the `GRAPE` calculation. As a sample process we have chosen $e^+p \to e^+X\,\mu^+\mu^-$ with 27.5 GeV positron and 820 GeV proton beams. Good agreement was obtained within the integration precision of 0.1 % as shown in Table 1, where in the run with Cut-I both muons are required to be in $18^o < \theta_\mu < 160^o$ and have $E_\mu > 2\,\mathrm{GeV}$, and in the run with Cut-II the scattered positron is required to be in $15^o < \theta_e < 164^o$ and have $E_e > 5\,\mathrm{GeV}$ in addition to the Cut-I. In the above, θ and E means polar angle and energy, respectively. For quasi-elastic processes, an additional cut of $Q_p^2 < 16\,\mathrm{GeV}^2$ is applied. In DIS processes, GRV94(LO) [13] parton distributions are used with $(\mathrm{QCD\ scale})^2 \equiv |\{p_{q(in)} - p_{q(out)}\}^2| > 4\,\mathrm{GeV}^2$.

	GRAPE	LPAIR
	Elastic (M_{had}=M_p)	
No cut	$(9.742 \pm 0.003) \times 10^4$	$(9.736 \pm 0.003) \times 10^4$
Cut-I	$(6.184 \pm 0.007) \times 10$	$(6.197 \pm 0.007) \times 10$
Cut-II	$(5.009 \pm 0.003) \times 10^{-1}$	$(5.011 \pm 0.003) \times 10^{-1}$
	Quasi-elastic ($M_p+M_{\pi^0}<M_{had}<5\,\mathrm{GeV}$)	
Cut-I	$(2.049 \pm 0.002) \times 10$	$(2.053 \pm 0.002) \times 10$
Cut-II	$(2.136 \pm 0.001) \times 10^{-1}$	$(2.138 \pm 0.002) \times 10^{-1}$
	DIS with GRV94(LO) ($M_{had}>5\,\mathrm{GeV}$)	
Cut-I	$(1.426 \pm 0.001) \times 10$	$(1.429 \pm 0.001) \times 10$
Cut-II	$(2.242 \pm 0.003) \times 10^{-1}$	$(2.241 \pm 0.002) \times 10^{-1}$

Table 1: Cross-section comparison with the generator `LPAIR` (in units of pb) for the process $ep \to eX\,\mu^+\mu^-$ including the BH diagrams. The applied cuts are described in the text.

5 Additional Contributions to 2-γ BH

The main goal of this work is to estimate effects of the other diagrams than 2-γ BH. Here 3 examples are presented for this purpose. In all cases, the beam energies are 27.5 GeV for the positron and 820 GeV for the proton. The label EW denotes results of a calculation including all the diagrams in Fig. 1 or 2.

5.1 Z^0 production in the di-muon channel

For the elastic process $e^+p \to e^+p\;\mu^+\mu^-$, the di-muon mass distribution is shown in Fig. 4. The applied cuts are $15^\circ < \theta_\mu < 164^\circ$ and (transverse momentum) > 5 GeV/c for both muons. The effect of Z^0 can be clearly seen as a resonance peak around its mass. But unfortunately, the current HERA integrated luminosity is yet too small to observe the effect. The Z^0 production cross-section seen in Fig.4 is about 0.002 pb which is consistent with that of `EPVEC`.

5.2 CO effect in μ + jet events

A calculation was done for the process $e^+\,q \to e^+\,q\;\mu^+\mu^-$ with parton densities from CTEQ4L [14] for all light quarks. To select μ + jet events, the following cuts are required,

- for at least one of the muons, $18^\circ < \theta_\mu < 160^\circ$ and $Pt_\mu > 5$ GeV/c,
- for scattered quark, $\theta_q > 10^\circ$ and $Pt_q > 15$ GeV/c,

where Pt indicates transverse momentum. For correct calculations in DIS, the following cuts are applied,

- $M_{had} > 5$ GeV,
- $(\mathrm{QCD\ scale})^2 \equiv |\{p_{q(in)} - p_{q(out)}\}^2| > 3\,\mathrm{GeV}^2$,
- $u > 25\,\mathrm{GeV}^2$ and
- $M_{q\,\mu^+\mu^-} > 5$ GeV.

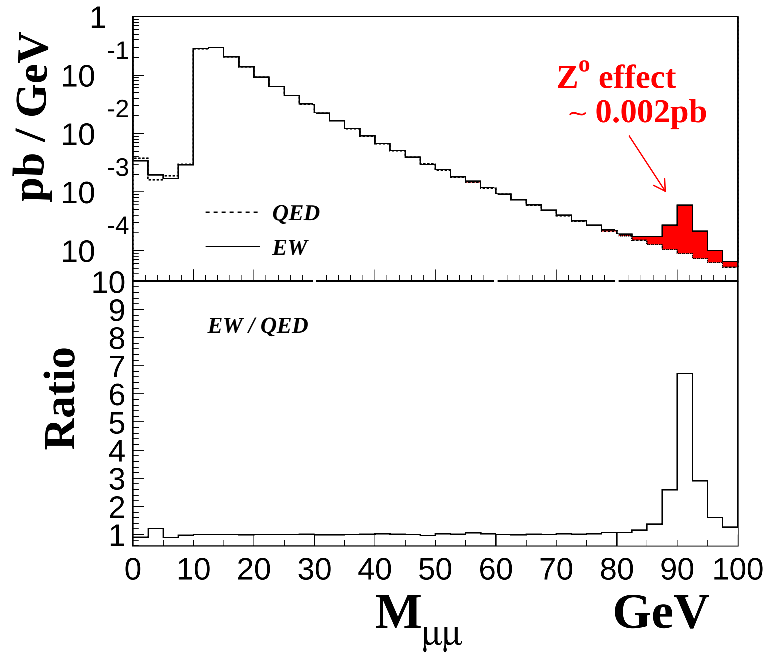


Fig. 4: Di-muon invariant mass distribution for $e^+p \to e^+p\ \mu^+\mu^-$. The applied cuts are described in the text.

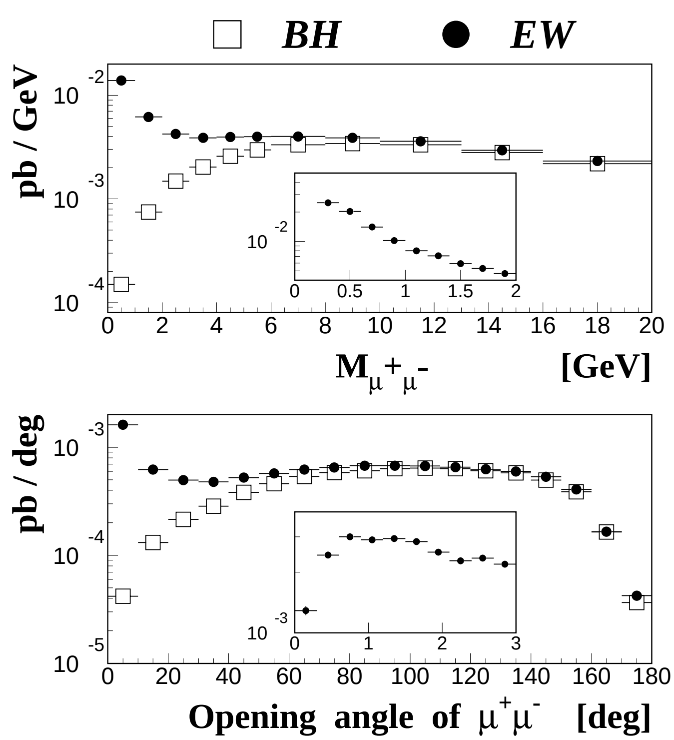


Fig. 5: Di-muon invariant mass and opening angle distributions for $e^+\,q \to e^+\,q\ \ \mu^+\mu^-$. The applied cuts are described in the text.

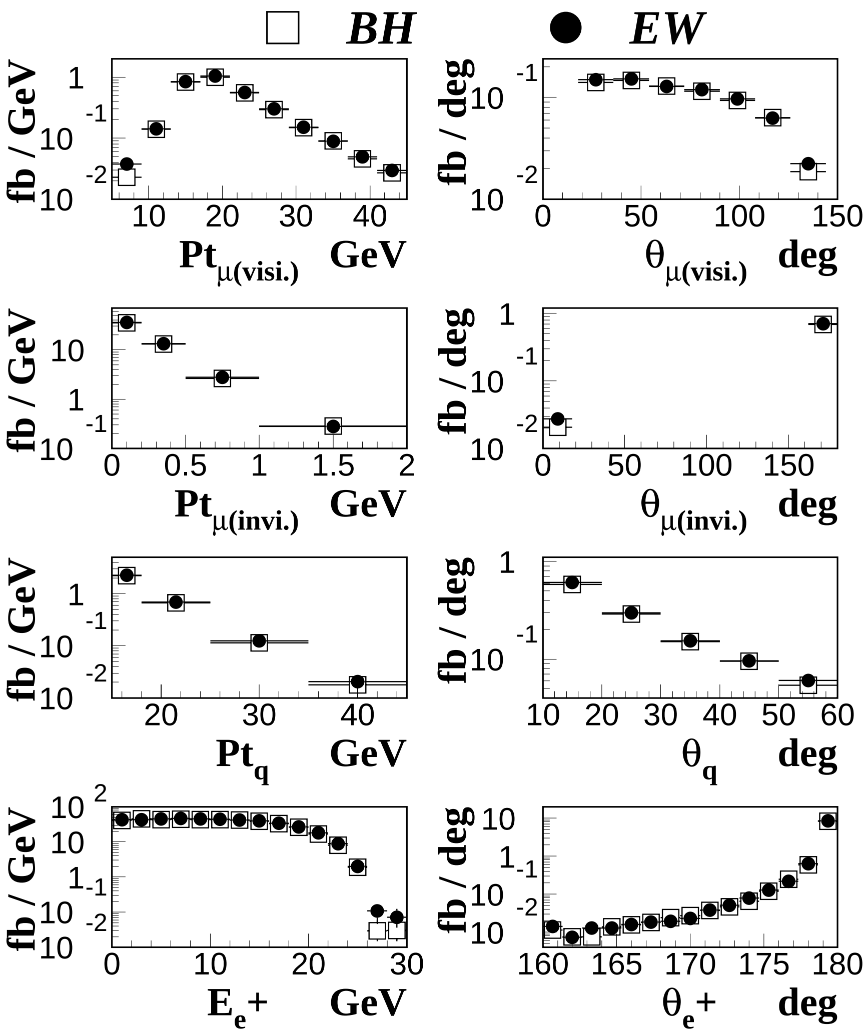


Fig. 6: Distributions of the various kinematical variables for single-μ + jet events. The applied cuts are described in the text.

The cross-section for 2-γ BH is estimated to be 0.0743 ± 0.0003 pb and 0.1061 ± 0.0003 pb for the calculation in the electroweak theory (EW). The difference of 0.03 pb comes from the phase space region with low invariant mass and small opening angle of the di-muon system where CO diagrams are dominant (see Fig. 5.). The cross-section increases as the di-muon mass decreases down to the kinematical limit of twice the muon mass.

5.3 Single-μ + jet events

The additional requirement that one of the two muons should be in $\theta_\mu < 5°$ or $\theta_\mu > 175°$ is applied to the sample in Sec.5.2. It selects events with single-μ + jet. The cross-section for 2-γ BH is estimated to be 12.9 ± 0.1 fb and 13.1 ± 0.1 fb for the calculation in the electroweak theory. The distributions of the various kinematical variables are shown in Fig.6. It can be concluded that there is no other significant contribution than 2-γ BH to this kind of events.

6 Summary

A new Monte Carlo generator for dilepton production in the electroweak theory was presented. It can be used for quantitative estimates of processes which come in addition to the two-photon Bethe-Heitler contributions.

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References

[1] S. P. Baranov, O. Dünger, H. Shooshtari, J.A.M. Vermaseren, *LPAIR — A Generator for Lepton Pair Production*, Proc. of the workshop on *Physics at HERA*, Vol.3, p.1478, Eds: W. Buchmüller and G. Ingelman, DESY(1992)

[2] J.A.M. Vermaseren, *Nucl. Phys.* **B229** (1983) 347

[3] D. Hoffmann and L. Favart, *Lepton Pair Monte Carlo Generators for HERA Physics*, this proceeding.

[4] U. Baur, J.A.M. Vermaseren, D. Zeppenfeld, *Nucl. Phys.* **B375** (1992) 3

[5] J. Fujimoto, M. Igarashi, N. Nakazawa, Y. Shimizu, K. Tobimatsu, *Suppl. Prog. Theor. Phys.* **100** (1990) 285

[6] T. Sjöstrand, *Comp. Phys. Comm.* **82** (1994) 74

[7] T. Ishikawa, T. Kaneko, K. Kato, S. Kawabata, Y. Shimizu and H. Tanaka, *KEK Report 92-19* (1993)

[8] F. Halzen and A. D. Martin, *Quarks and Leptons: An Introductory Course in Modern Particle Physics*, John Wiley & Sons, 1984

[9] A. Bodek et al., *Phys. Rev.* **D20** (1979) 1471

[10] M. Kasprzak, *ZEUS-note 95-069*

[11] H. Plothow-Besch, *Comput. Phys. Commun.* **75** (1993) 396

[12] S. Kawabata, *Comput. Phys. Commun.* **88** (1995) 309

[13] M. Glück, E. Reya, A. Vogt, *Z. Phys.* **C67** (1995) 433

[14] H. L. Lai, J. Huston, S. Kuhlmann. F. Olness, J. Owens, D. Soper, W. K. Tung, H. Weerts, *Phys. Rev.* **D55** (1997) 1280