GRAPE

GRAce for Proton-Electron interactions

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in collaboration with

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—— Outline ——

[1] Introduction
[2] The GRACE System
[3] Methods of GRAPE
[4] Program Structure
[5] Examples of Calculations
[6] Summary and Prospects

[1] Introduction

The actual motivation of the GRAPE project

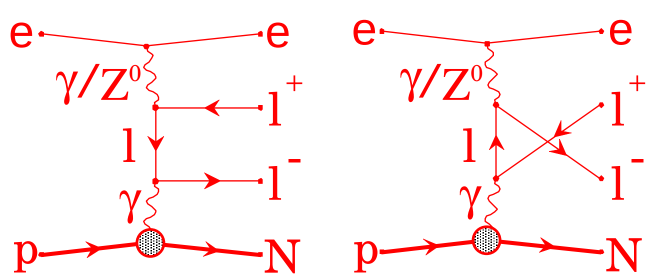
$||$

Need of a new dilepton generator for HERA physics

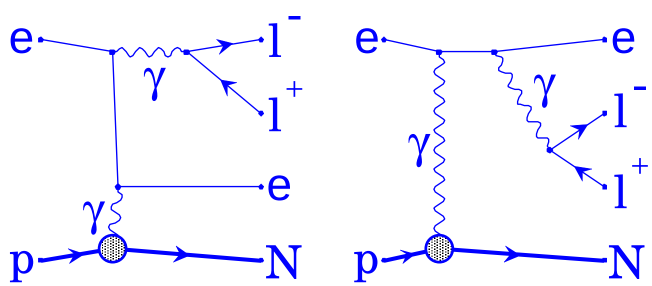
($\longrightarrow$ EW diagrams)

Significant background for
exclusive J/ψ,Υ productions, CC, LFV, W production, etc.

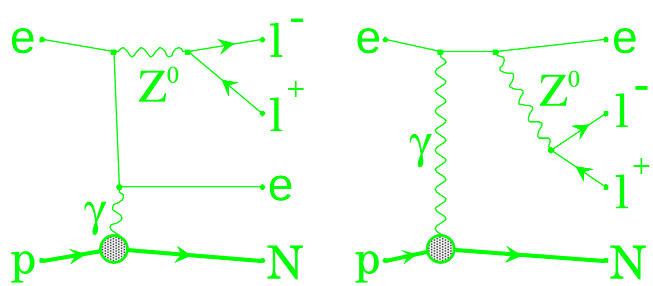
Electroweak(EW) Dilepton Production



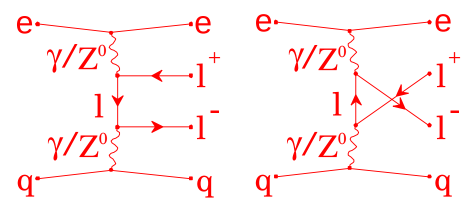
(a) Bethe-Heitler (type) diagrams



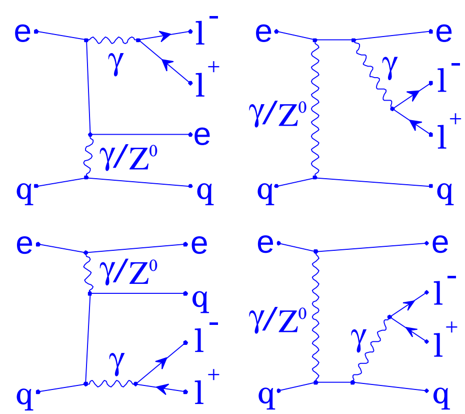
(b) QED Compton type diagrams



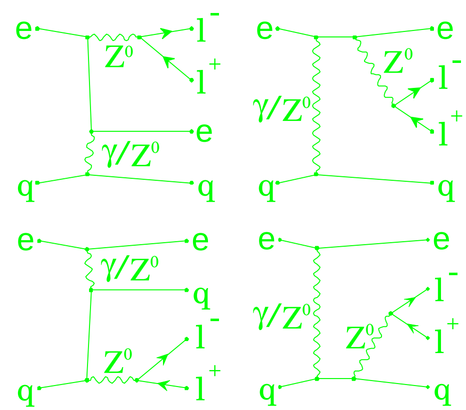
(c) Z^0 on/off-shell production



(a) Bethe-Heitler (type) diagrams



(b) QED Compton type diagrams



(c) Z^0 on/off-shell production

Existing generators for dilepton production in ep collisions

	LPAIR*	TRIDENT†
Calculation	Exact ME with numerically stable formula to avoid gauge cancellations	Exact ME with REDUCE
Numerical stability	Stable at any phase space point	Unstable at low scattering angles
Included diagrams	Bethe-Heitler (BH) of two-pthon	BH+CO (QED) including ee interference in ee channel
Weight	Unweighted	Weighted

$\star, \dagger$: Please see PHYSICS at HERA vol.3

We want to have an event generator with

- the exact ME calculation,
- all related diagrams,
- and numerical stableness.

We suffer from

- a large number of diagrams (at most 48),
- various singularities,
- numerical cancellations.

The automatic calculation by GRACE

[2] The GRACE System

— *Automatic calculation of Feynman amplitudes* —

Successful experience in e^+e^- physics (eg. LEP2)

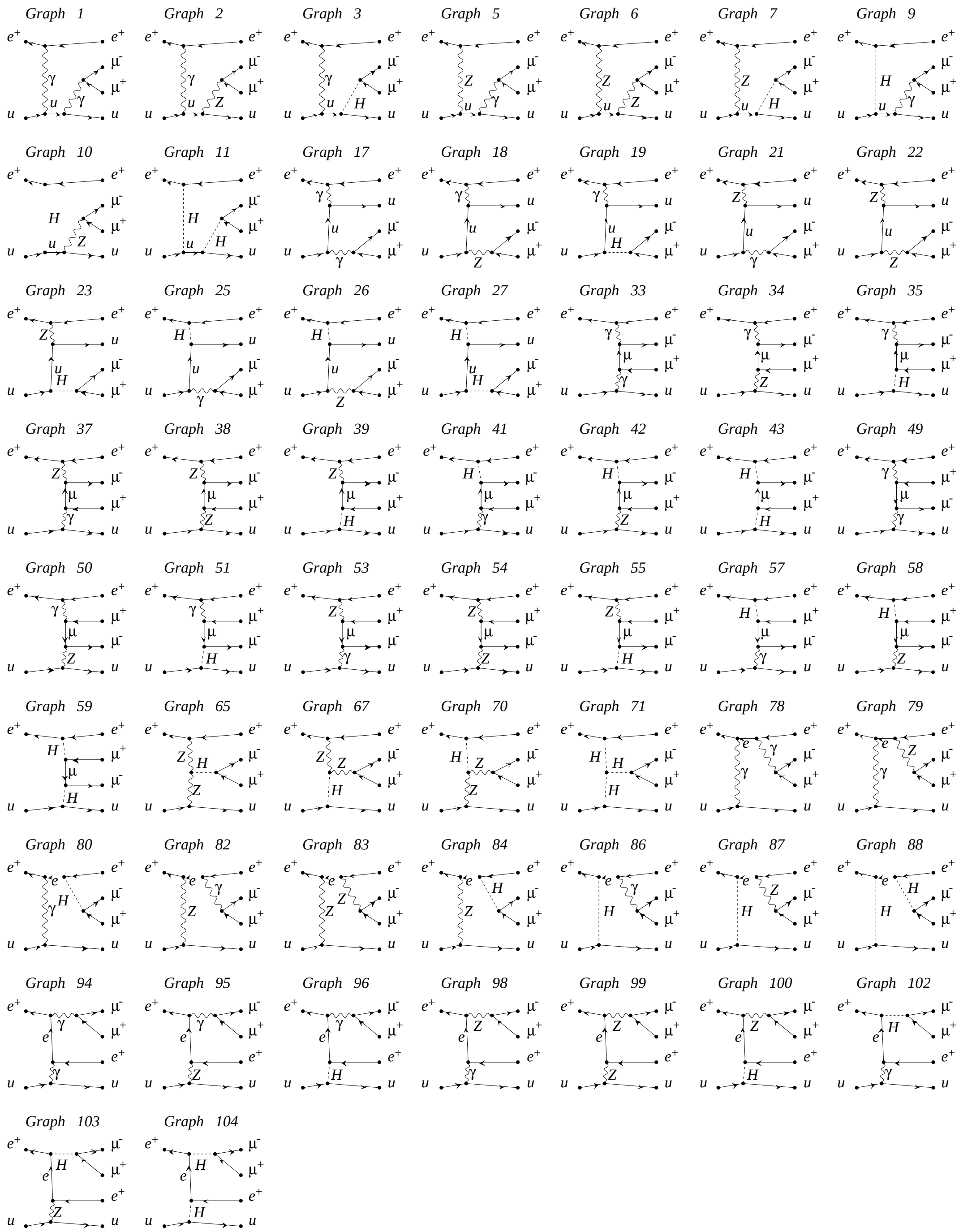
1. *Specification of a model file, order of perturbation, and initial/final state particles*
2. *Generation of all Feynman diagrams*
3. *Generation of FORTRAN source code to calculate the Feynman amplitudes* $\Longrightarrow$ *Exact ME calculation*
4. *Integration, event generation by BASES/SPRING program* $\Longrightarrow$ *Unweighted events*

In an input file

- Model : SM
- EW=4, QCD=0
- Initial= $\{e^+,\, u\}$
- Final= $\{e^+,\, u,\, \mu^+,\, \mu^-\}$

# of generated diagrams

- 109 in covariant gauge
- 58 in unitary gauge



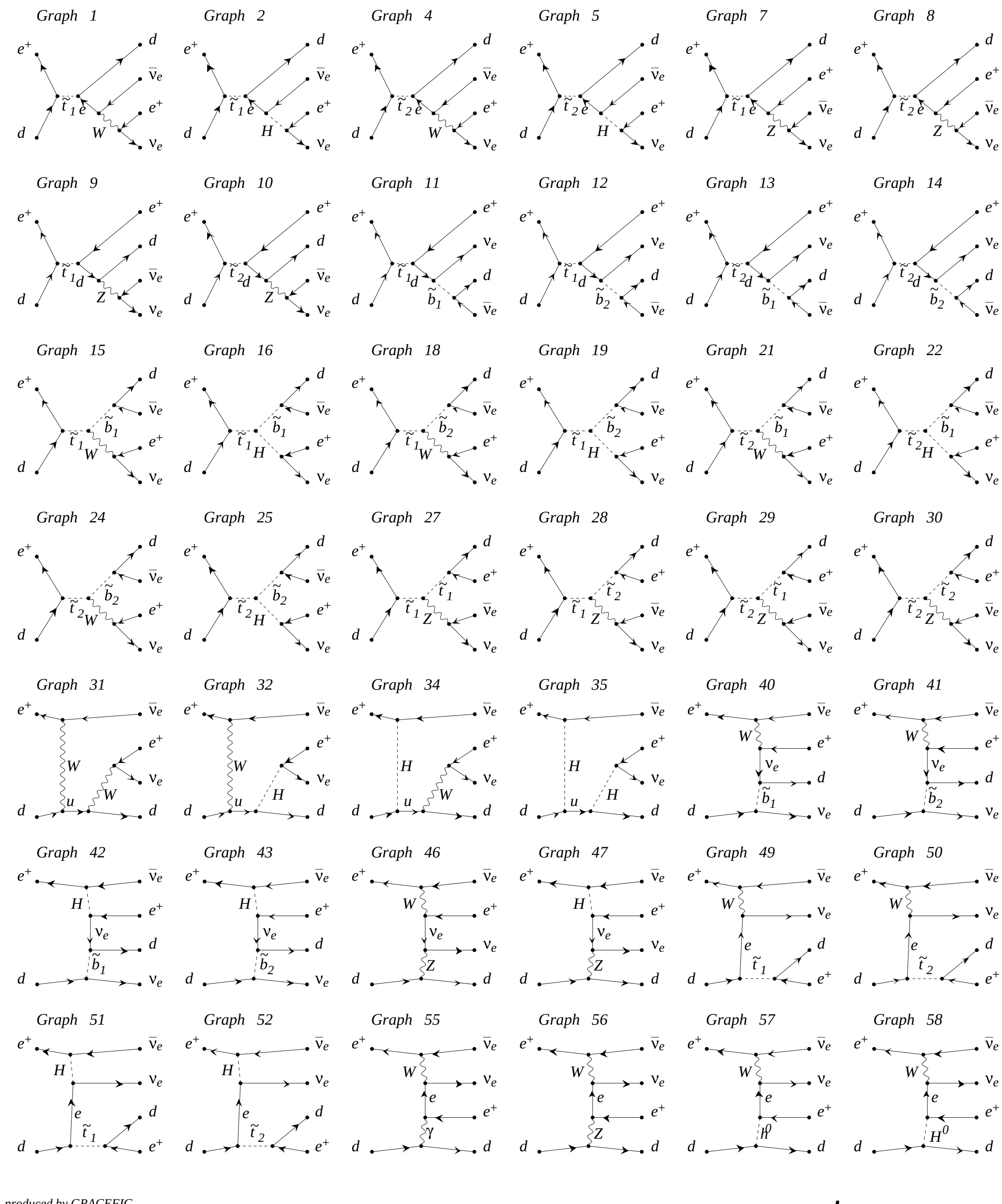
produced by GRACEFIG

In an input file

- Model : MSSM with $\not{R}_p$
- EW=4, QCD=0
- Initial= $\{e^+, d\}$
- Final = $\{e^+, d, \nu_e, \bar{\nu}_e\}$

# of generated diagrams

- 237 in covariant gauge
- 164 in unitary gauge



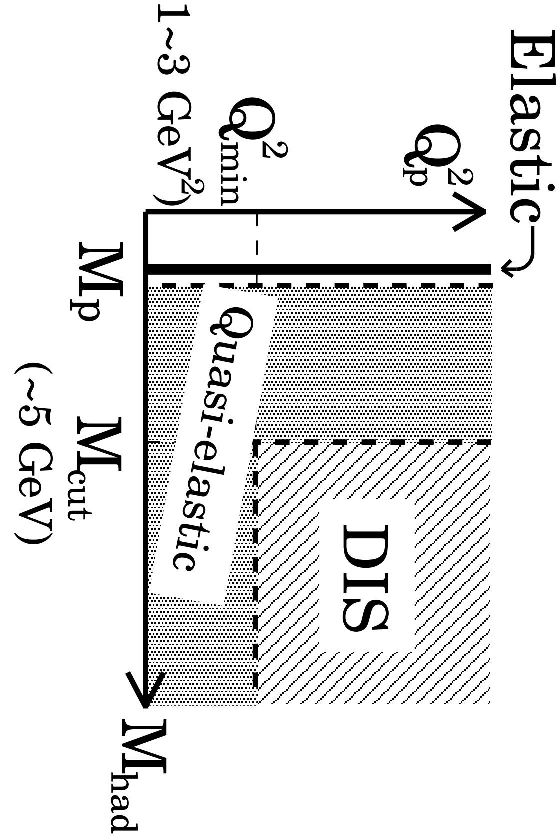
etc...

[3] Methods of GRAPE

So far GRACE has only fundamental particles.

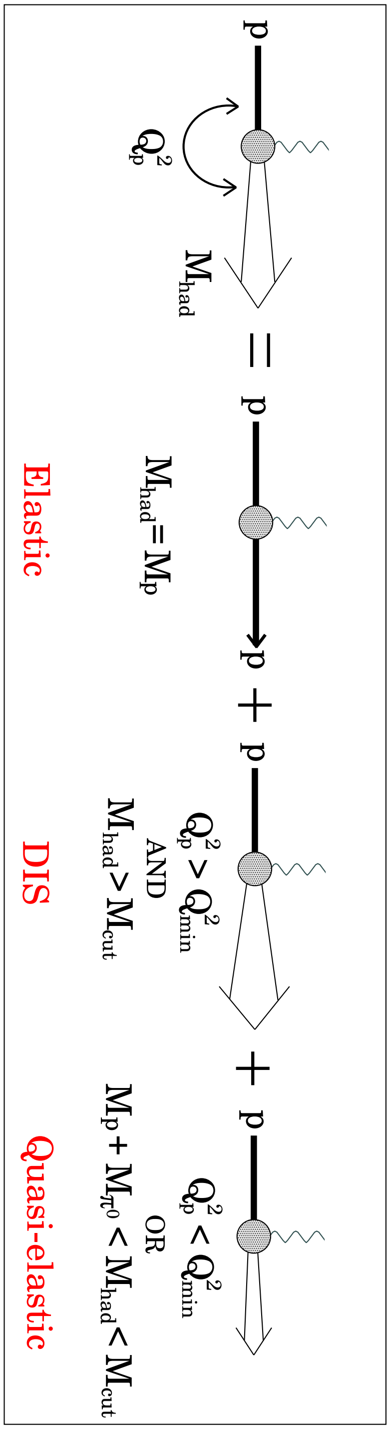
$\Longrightarrow$ **Calculation of the Proton Vertex**

$$e^{\pm}p \rightarrow e^{\pm}X\ l^{+}l^{-}$$



$$Q_p^2 \stackrel{\rm def}{\equiv} -\left\{p_{e^{\pm}(in)} - (p_{e^{\pm}} + p_{l^+} + p_{l^-})\right\}^2$$

$$M_{had}^2 \stackrel{\rm def}{\equiv} \left\{(p_{e^{\pm}(in)} + p_{p(in)}) - (p_{e^{\pm}} + p_{l^+} + p_{l^-})\right\}^2$$



Elastic process

$(M_{had} = M_p)$

- New particle '*proton' was added into GRACE as a fundamental particle.*
- *Definition of* $pp\gamma$ *vertex*

$$\Gamma^{\mu}_{pp\gamma} = e_p \left(F_1(q^2)\gamma^{\mu} + \frac{\kappa}{2M_p} F_2(q^2) i\sigma^{\mu\nu} q_{\nu} \right)$$

κ : Anomalous magnetic moment of proton
$F_1(q^2), F_2(q^2)$: Independent formfactors

$$\begin{pmatrix} G_E(q^2) \\ G_M(q^2) \end{pmatrix} = \begin{pmatrix} F_1(q^2) & + & \frac{\kappa q^2}{4M_p^2} F_2(q^2) \\ F_1(q^2) & + & \kappa \ F_2(q^2) \end{pmatrix}$$

Dipole-Formfactor

$$G_E(q^2) = \frac{1}{\left(1 - \frac{q^2}{0.71}\right)^2} = \frac{G_M(q^2)}{\mu_p}$$

DIS process

$(Q_p^2 > Q_{min}^2)$
AND
$(M_{had} > M_{cut})$

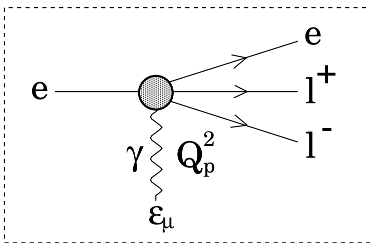
- eq scattering ($\leftarrow$ GRACE amplitudes)
- Parton density function ($\rightarrow$ Kinematics)
- Interfaced to PYTHIA to get complete hadronic final states

$$\boxed{\sigma_{ep\to eXl^+l^-}(s) = \sum_i^{quarks} \int dx\, f_i(x)\ \sigma_{eq_{(i)}\to eq_{(i)}l^+l^-}(\tilde{s})}$$

Quasi-elastic process

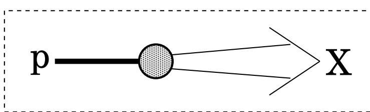
$$(Q_p^2 < Q_{min}^2)$$
OR
$$(M_p + M_{\pi^0} < M_{had} < M_{cut})$$

- General form of the proton current
- Structure functions from the experimental data



$\longleftarrow$ calculated by GRACE

$\equiv l_\mu \Longrightarrow L_{\mu\nu} \sim l_\mu l_\nu^*$

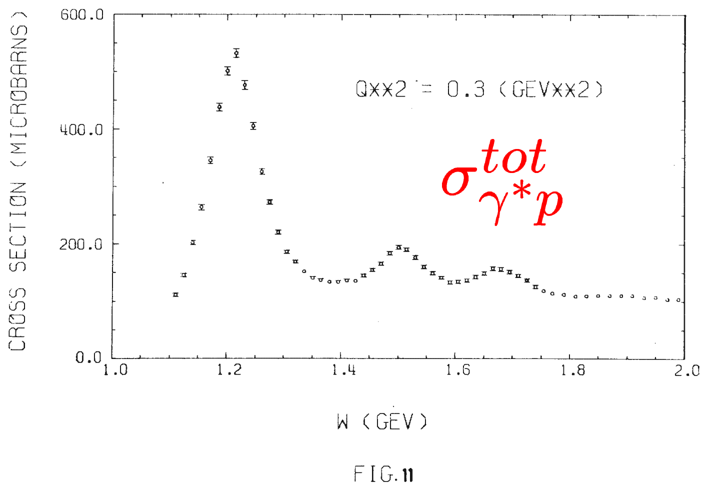
$\sum_X \Big|$  $\Big|^2 \Longrightarrow W^{\mu\nu}$

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2 \frac{1}{M_p^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right)$$

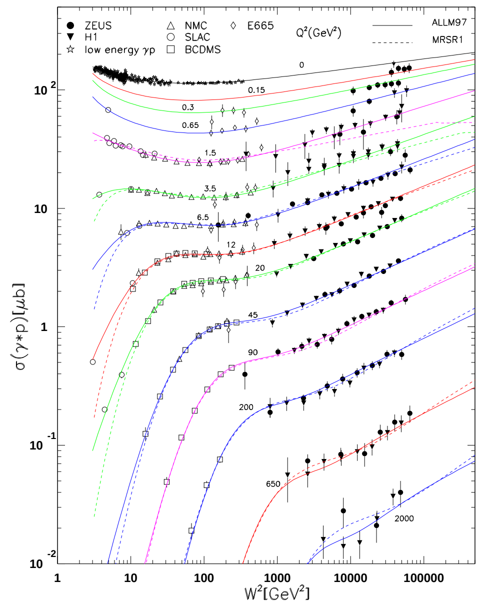
$$\boxed{d\sigma \sim L_{\mu\nu} W^{\mu\nu}}$$

Two sets of the parameterization for W_1, W_2
are used in the cross-section calculation.

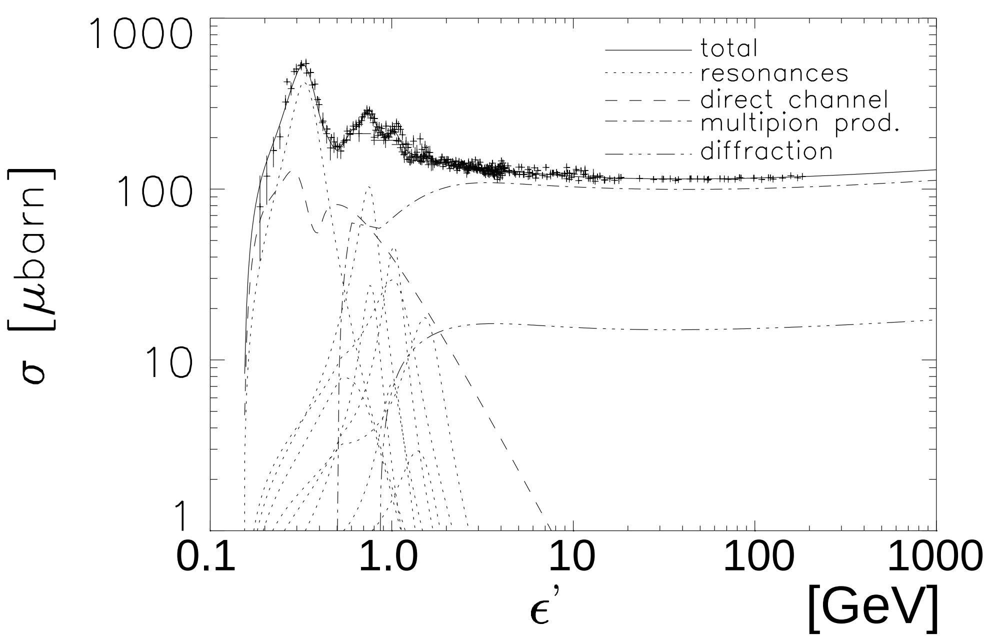
Brasse et al. (M_{had}<2 **GeV)**



ALLM97 (M_{had}>2 **GeV)**

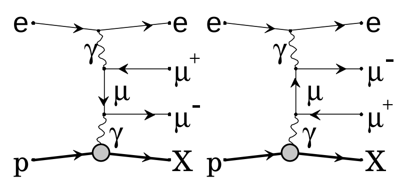


The exclusive hadronic final state is simulated
by
SOPHIA
as a result of the real-photon and proton collision



Cross Section Comparisons with LPAIR

Process : $ep \to eX\mu^+\mu^-$
(at HERA energy)
with Bethe-Heitler only



Detector cuts

- Cut(1) — $15^\circ < \theta_\mu < 164^\circ$, $E_\mu > 2\,\text{GeV}$
  (for both muons)
- Cut(2) — $15^\circ < \theta_\mu < 164^\circ$, $E_\mu > 2\,\text{GeV}$
  (for both muons)
  & $15^\circ < \theta_e < 164^\circ$, $E_e > 4\,\text{GeV}$
  (for scattered positron)

Stableness of the GRACE calculation

Elastic

	GRAPE	LPAIR
No cut	$9.742(\pm 0.003) \times 10^4$	$9.736(\pm 0.003) \times 10^4$
Cut(1)	$8.493(\pm 0.005) \times 10$	$8.496(\pm 0.008) \times 10$
Cut(2)	$6.094(\pm 0.008) \times 10^{-1}$	$6.091(\pm 0.005) \times 10^{-1}$

(in unit of pb)

DIS

	GRACE	LPAIR
No cut	$9.463(\pm 0.002) \times 10^2$	$9.464(\pm 0.002) \times 10^2$
Cut(1)	$3.651(\pm 0.005) \times 10$	$3.649(\pm 0.004) \times 10$
Cut(2)	$4.311(\pm 0.005) \times 10^{-1}$	$4.313(\pm 0.004) \times 10^{-1}$

(in unit of pb)

Quasi-elastic

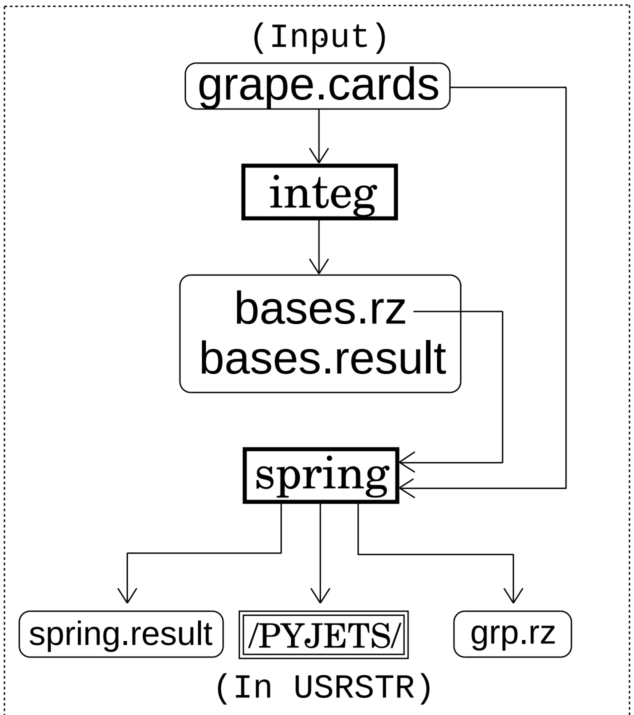
	GRACE	LPAIR
No cut	$7.029(\pm 0.003) \times 10^3$	$7.025(\pm 0.002) \times 10^3$
Cut(1)	$4.855(\pm 0.005) \times 10$	$4.846(\pm 0.004) \times 10$
Cut(2)	$4.254(\pm 0.004) \times 10^{-1}$	$4.255(\pm 0.004) \times 10^{-1}$

(in unit of pb)

Good Agreement in all cases

within statistical error of $\sim$ 0.1 %

[4] Program Structure



Executable file

Input/Output file

$*$.rz Ntuple file

$*$.result ASCII file

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```
C ======================================================================
C << KF-code of the Lepton Beam >>   (11:electron, -11:positron)
KFLBEAM     -11
C ======================================================================
C << Polarization of the Lepton Beam >>
C             (1)    (2)    (3)
EPOL           0.     0.     0.
C (1) Degree of the polarization(>=-1&<=+1)
C (2) Direction of the polarization vector (polar     angle in degree)
C (3) Direction of the polarization vector (azimuthal angle in degree)
C ======================================================================
C << Momenta of the Beam Particles(lepton/proton) in MeV/c >>
C              |||                                  ^^^^^
EBEAM        27520
PBEAM       820000
C              |||
C ======================================================================
C << Process in the Proton Vertex >>   (1:elastic, 2:quasi-elastic, 3:DIS)
PROCESS        1
C ======================================================================
C << Produced Lepton-pair >>   (1:di-e, 2:di-mu, 3:di-tau)
LPAIR          2
C ======================================================================
C << ISR for the Beam Lepton >>  (0:off, 1:on)
ISR            1
C ======================================================================
C ######################################################################
C ======================================================================
C << Electroweak Dilepton Production >>
C   1:BH(direct), 2:BH, 3:QED, 4:EW(-Higgs), 5:EW(+Higgs),
C   13:CO_only, 14:Z0_only, 15:Higgs_only
GRASEL         3
C ======================================================================
C ######################################################################
C ======================================================================
C << BASES Parameters >>
ITMX1          4
ITMX2         10
NCALL   1200000
C ======================================================================
C ######################################################################
C ======================================================================
C << SPRING Parameters >>
NGEN         100
NMOD        1000
C ======================================================================
C << PYTHIA parameters >>
PSISR          1
PSFSR          1
PSBRA          2
PSSUP          0
PYDECAY        1
PRIPT          1
C ======================================================================
C ######################################################################
C ======================================================================
C << Mass Range for the Hadronic System >>
MHAD           1.08   5.
C ------------
C << Q2 Range for the Ptoron Vertex >>
Q2P            0.     1.E20
C ------------
C << Cuts for each Final-state Particle >>
C            <p/q>  <e+->  <l-+>  <l+->
THMIN          0.     0.     0.     0.
THMAX        180.   180.   180.   180.
EMIN           0.     0.     0.     0.
EMAX           1.E20  1.E20  1.E20  1.E20
PMIN           0.     0.     0.     0.
PMAX           1.E20  1.E20  1.E20  1.E20
PTMIN          0.     0.     0.     0.
PTMAX          1.E20  1.E20  1.E20  1.E20
C ------------
C << Mass cuts >>
MASSLL         0.     1.E20
               0.     1.E20
C ======================================================================
```

1

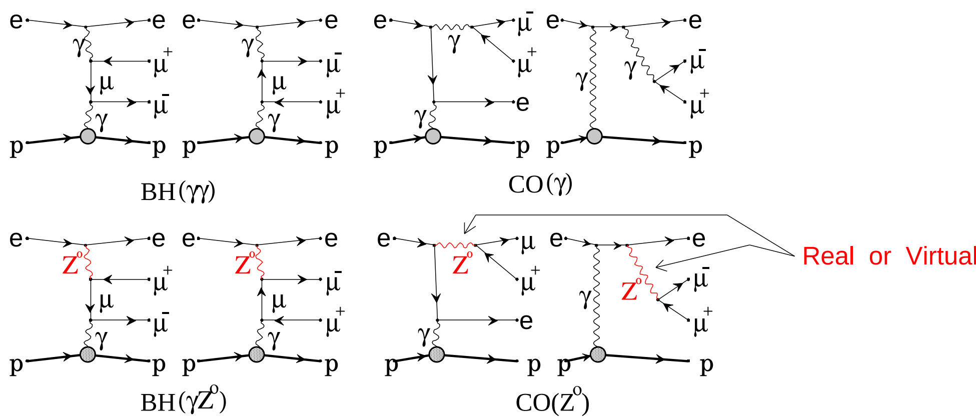
[5] Examples of Calculations

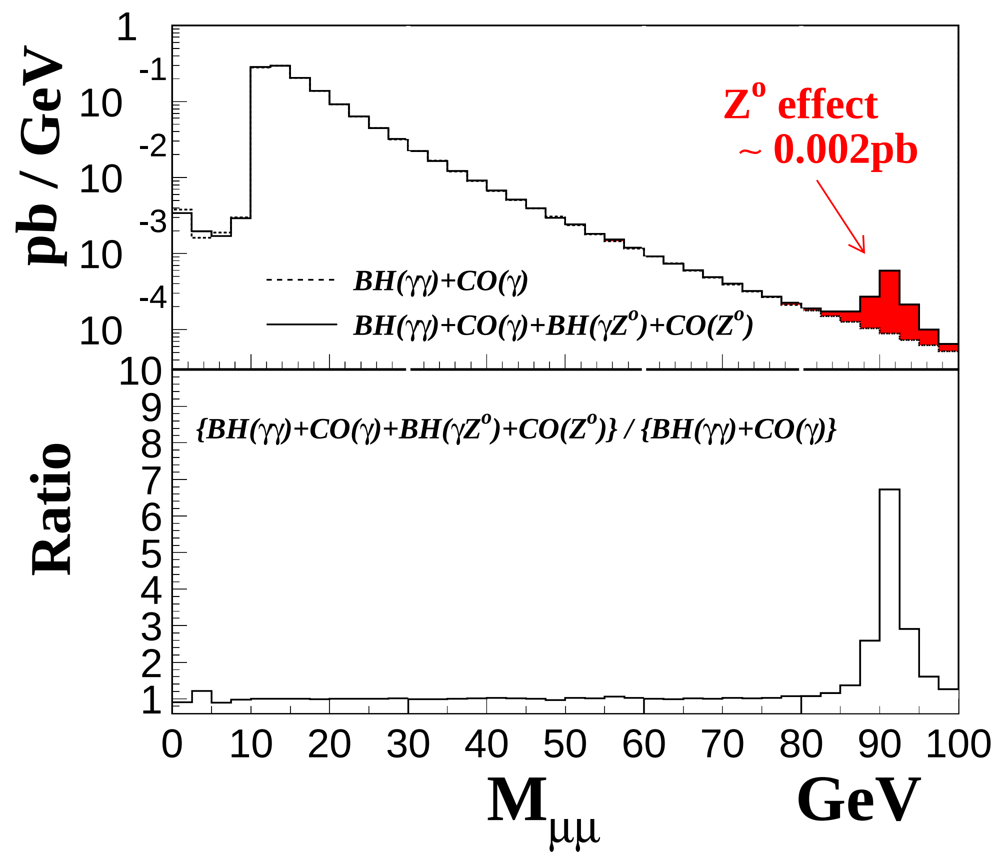
Z^o Effect

$$e^+ p \longrightarrow e^+ p\ \mu^+ \mu^-$$ (at HERA energy)

Detector cuts (2μ visible)

$15^\circ < \theta_\mu < 164^\circ$, $P_t > 5$ GeV/c
(for both muons)



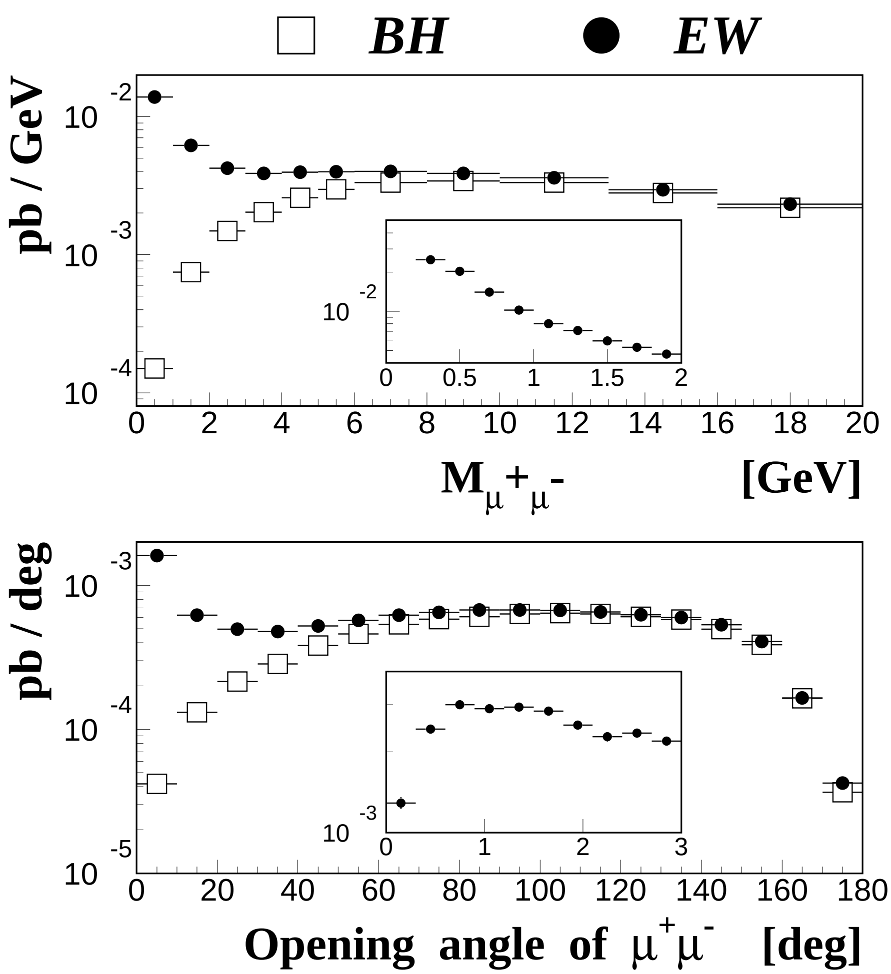


CO Effect

$$e^+ q \longrightarrow e^+ q\ \mu^+ \mu^- \quad \text{(at HERA energy)}$$

Detector cuts

$18^\circ < \theta_\mu < 160^\circ$ & $P_t > 5$ GeV/c
(for at least one muon) &
$Pt_q > 15$ GeV & $\theta_q > 10^\circ$



[6] Summary and Prospects

- The methods and the interface for the general proton vertex have been established.

  $\longrightarrow$ Dilepton production (`GRAPE-Dilepton generator`)

  $\longrightarrow$ QED Compton

  $\longrightarrow$ Sbottom-W production

- New processes can be (will be) easily included not only for ep but also for pp, $p\bar{p}$ collisions.

  $\longrightarrow$ Future works