Flavor changing amplitudes in the littlest Higgs model with T-parity

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March 16, 2009
“The 2nd general meeting of the "Working Group on the Interplay between Collider and Flavour Physics" at CERN

Flavour and Electroweak symmetry breaking

- There should be something new at the TeV scale that is related to physics of the electroweak symmetry breaking.
- How flavour observables are sensitive to new physics depends on scenarios.
  - Tree vs. Loop
  - Mass scale of new physics
  - New source of flavour mixing vs. MFV
- SM has a special feature on flavour mixing through CKM and PMNS matrices.
Little Higgs Models with T parity

- The Higgs doublet field is a part of pseudo-NG bosons associated with a symmetry breaking dynamics at about 10 TeV.
- The quadratic divergence of the Higgs mass term is cancelled by extra-gauge bosons and a heavy top quark partner at the one loop level. (a solution to the “little hierarchy problem”).
- Electroweak precision measurements still put a strong constraints mostly due to tree-level exchange of extra-gauge bosons.
- The original model is extended to possess T-parity, so that no dangerous diagrams exist for electroweak constraints. Masses of new particles can be below 1 TeV.
The littlest Higgs Model with T-parity (LHT)

Electroweak symmetry breaking

SU(5)/SO(5) non linear sigma mode

\[ \Sigma = \xi \Sigma_0 \xi^T \]

\[ \langle \Sigma \rangle = \Sigma_0 = \begin{pmatrix}
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{pmatrix} \]

24-10=14 Nambu-Goldstone bosons are expressed by

\[ \xi = \exp(i\Pi/f) \]

f~O(1)TeV

SM Higgs doublet, \( H_{SM} \)

T parity: \( \Pi \rightarrow -\Omega \Pi \Omega : \Omega = \text{diag}(1, 1, -1, 1) \)

Only \( H_{SM} \) is T-even.
Gauge symmetries

Global:
\[ SU(5) \uparrow \Rightarrow \quad SO(5) \uparrow \]

Gauged:
\[ [SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2 \quad \Rightarrow \quad SU(2) \times U(1) \]

SM electroweak

T-parity:
\[ g_1 = g_2, \quad g'_1 = g'_2 \]

\[ W_1, B_1 \leftrightarrow W_2, B_2 \]

T-odd gauge bosons \((W_H, Z_H, A_H)\) \(\sim\) mass 0(f)

T-even gauge boson = SM gauge boson

Gauge-NG Lagrangian
\[ \mathcal{L}_{NG} = \frac{f^2}{8} \text{tr} \left[ (\mathcal{D}^\mu \Sigma^\dagger)(\mathcal{D}_\mu \Sigma) \right] . \]

\[ \mathcal{D}_\mu \Sigma = \partial_\mu \Sigma - i \left[ g(\mathcal{W}_\mu \Sigma + \Sigma \mathcal{W}^T_\mu) + g'(\mathcal{B}_\mu \Sigma + \Sigma \mathcal{B}^T_\mu) \right] \]
Fermion sectors

In addition to the heavy top partner for the little Higgs mechanism, mirror fermions for SU(2) doublets have to be introduced to assign the T-parity.

\[
q_1 = \begin{pmatrix} u_1 \\ d_1 \end{pmatrix}_L \quad \quad q_2 = \begin{pmatrix} u_2 \\ d_2 \end{pmatrix}_L
\]

T-parity:

\[q_1 \leftrightarrow -q_2\]

T-even

\[q_{SM} = \frac{1}{\sqrt{2}}(q_1 - q_2)\]

T-odd

\[q_{HL} = \frac{1}{\sqrt{2}}(q_1 + q_2)\]
Right-handed heavy doublet

In order to provide gauge-invariant mass terms for mirror quarks/leptons right-handed doublet fermions have to be introduced.

SU(5) embedding

\[
\psi_1 = \begin{pmatrix} -i\sigma^2 q_1 \\ 0 \\ 0 \end{pmatrix}
\]

\[
\psi_2 = \begin{pmatrix} 0 \\ 0 \\ -i\sigma^2 q_2 \end{pmatrix}
\]

\[
\psi_R = \begin{pmatrix} * \\ * \\ -i\sigma^2 q_H R \end{pmatrix}
\]

\[
\psi_R \rightarrow U\psi_R
\]

\[
\xi\psi_R \rightarrow V\xi U^\dagger U\psi_R = V\xi\psi_R
\]

\[V \in SU(5), \quad U(\Pi, V) \in SO(5)\]
Mirror fermion mass terms

\[ L_\kappa = -\kappa^{ij} f \left( \bar{\psi}^2_i \xi + \bar{\psi}^1_i \sum_0 \Omega \xi^{+}_i \Omega \right) \psi_R^i + H.c. \]

This terms gives mass terms for mirror quark doublets.

\[ \kappa^{ij} \] is a new source of flavor mixing. 

After diagonalization of the fermion mass matrices, flavour changing are induced in the gauge boson-fermion verteces.

Out of three mixing matrices, two are independent.
Flavour changing neutral current processes

- FCNC and LFV processes have been studied in LHT.
- We have reevaluated FCNC amplitudes and found that the left-over logarithmic divergence is cancelled by new contributions due to an extra term in the $Z-u_{HR}$ vertex.
K-$\rightarrow$ $\pi$ $\nu\nu$ process

We have calculated one loop $Z$ penguin and box contributions in the ‘tHooft-Feynman gauge.

T-even contributions
SM + T even heavy top loop. Proportional to the SM CKM factor.
(MFV-type contribution)

$$\lambda_k = (V_{CKM}^*)_{ki} (V_{CKM})_{kj}$$

T-odd contributions
Vanish at $f\rightarrow$ infinity. Mirror fermion contributions should decouple in this limit.
$O(\nu^2/f^2)$ contributions can be sizable because they depend on a new mixing factor.

$$\xi_k = (V_{Hd}^*)_{ki} (V_{Hd})_{kj}.$$
O(ν^2/f^2) contributions come from expansion of $\xi = \exp(i\Pi/f)$ around the vacuum. (Hyper order in non-linear sigma model.)

Box diagrams: finite

Z-penguin diagram

The left diagram gives a divergent contribution from the $ν^2/f^2$ term.
A new $O(\nu^2/f^2)$ contribution from the $Z-u_R$ vertex.

Extra contribution from the left diagram cancel the divergence. Extra terms arise because $u_R$ is a part of non-linear representation of SU(5).

$$\psi'_R = \xi \psi_R$$

$$\mathcal{L}_{\text{kin}}(q_{HR}) = \frac{1}{2} \bar{\psi}'_R \gamma^\mu i \gamma^\nu \psi'_R + (T\text{-parity conjugate})$$

$$= \frac{1}{2} \bar{\psi}_R \left[ i \partial^\mu + \xi^\dagger (g \hat{W} + g' \hat{P}_\mu \psi_R) \xi + (i \xi^\dagger \phi \xi) \right] \psi_R + \text{c.c.}$$
\[ \mathcal{L}_{[d\nu]}^{\text{eff}} = C_{[d\nu]LL}^{ijlm}(\bar{d}^i \gamma^\mu L d^j)(\bar{\nu}^l \gamma_\mu L \nu^m) \]

\[ C_{[d\nu]LL}^{ijlm} = -\frac{g^4}{(4\pi)^2 m_W^2} \left[ \delta_{lm} \left( \sum_k \lambda_k X_{SM}(x_k) + \lambda_t \tilde{X}_{\text{even}} \right) + \sum_{k,n} \lambda_{nH}^{H\nu} \xi_k J^{\nu\nu}(z_k, y_n) \right] \]

\[ x_k = m_{u_k}^2 / m_{W_L}^2, \quad z_k = m_{u_H}^2 / m_{W_H}^2, \quad y_n = m_{e_H}^2 / m_{W_H}^2 \]

T-even heavy top

SM

T-odd

Without the new contribution (\( \Lambda = 4\pi f \))

Full contribution

Drop the divergent term by hand
Example of numerical results.

\[ f = 1 \text{TeV}, \ m_{T^+} = 1.34 \text{TeV} \]
\[ \text{Re}[(V_{Hd})^*_{31}(V_{Hd})_{32}] = 0 \]
\[ \text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \]

\[ \xi_3 = (V_{Hd})_{31}(V_{Hd})^*_{32}, \ \text{Br(exp)} = (1.5^{+1.3}_{-0.9}) \times 10^{-10}. \]
The deviation form the SM can be still large after canceling the divergence. More importantly, the FCNC process is predictable within the effective Lagrangian without reference to physics at the cutoff scale.
Summary

- We have reevaluate FCNC amplitudes in the Littlest Higgs model with T-parity, and found that there is no UV-cutoff dependence.

- The Branching ratios of K->pnn processes can be significantly different from the SM predictions in this model.

The absence of the divergence is confirmed by recent papers.

- Altmannshofer, Ball, Bharucha, Buras, Straub & Wick, arXiv:0811.1214 [hep-ph] ($B \rightarrow K^\ast \mu^+\mu^-$);

- del Águila, Ilana & Jenkins, arXiv:0811.2891 [hep-ph] ($\mu \rightarrow e\gamma$ and $\mu \rightarrow eee$).