Testing NRQCD factorization at the LHC

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In collaboration with Mathias Butenschön
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Outline

Heavy Quarkonia

Heavy quarkonia: Bound states of heavy quark and its antiquark.

- Charmonia ($c\overline{c}$) and Bottomonia ($b\overline{b}$)
- Top decays too fast for bound state.

Charmonium spectrum ($\mathbf{c\overline{c}}$):

$n\,^{2S+1}L_J$	Name	Mass
$1\,^1S_0$	η_c	2980 MeV
$1\,^3S_1$	J/ψ	3097 MeV
$1\,^3P_0$	χ_{c0}	3415 MeV
$1\,^3P_1$	χ_{c1}	3511 MeV
$1\,^1P_1$	h_c	3526 MeV
$1\,^3P_2$	χ_{c2}	3556 MeV
$2\,^1S_0$	η_c'	3637 MeV
$2\,^3S_1$	ψ'	3686 MeV

- 1974: Discovery of J/ψ: First observation of heavy quarks
- Long lifetime of $c\overline{c}$: Spectrum and radiative transitions seen $\Longrightarrow$ Potential models
- Calculation of energy spectrum: Challenge for lattice QCD.
- Production and decay rates: One of first applications for perturbative QCD.

Production and Decay Rates of Heavy Quarkonia

The classic approach: Color-singlet model

- Calculate cross section for $c\bar{c}$-pair in physical color-singlet (= color neutral) state. In case of J/ψ: $c\bar{c}[^3S_1^{[1]}]$
- Then multiply by J/ψ wave function or its derivative at origin.
- Leftover infrared divergences at P wave quarkonia.
  $\Longrightarrow$ Theoretically inconsistent

Nonrelativistic QCD (NRQCD):

- 1995: Rigorous effective field theory by Bodwin, Braaten, Lepage
- Based on factorization of soft and hard scales
  (Scale hierarchy: $Mv^2, Mv \ll \Lambda_{\mathrm{QCD}} \ll M$)
- Theoretically consistent: No leftover singularities.
- Can explain hadroproduction at Tevatron

J/ψ Production within NRQCD

Factorization theorem: $\sigma_{J/\psi} = \sum_n \sigma_{c\bar{c}[n]} \cdot \langle O^{J/\psi}[n] \rangle$

- n: Every possible Fock state, including color-octet states.
- $\sigma_{c\bar{c}[n]}$: Production rate of $c\bar{c}[n]$, calculated in perturbative QCD.
- $\langle O^{J/\psi}[n] \rangle$: Long distance matrix elements (ME): describe $c\bar{c}[n] \to J/\psi$, universal, extracted from experiment.

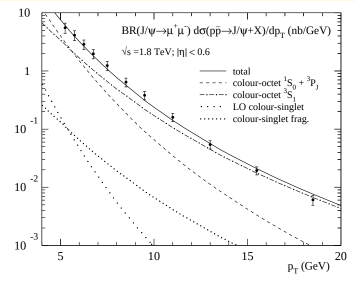
Scaling rules: MEs scale with relative velocity v ($v^2 \approx 0.2$):

scaling	v^3	v^7	v^{11}
n	${}^3S_1^{[1]}$	${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^3P_{0/1/2}^{[8]}$	$\ldots$

- Double expansion in v and α_s.
- Leading term in v ($n = {}^3S_1^{[1]}$) equals color-singlet model.

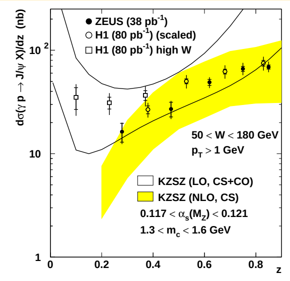
Production of J/ψ: NRQCD vs. Experiment

Hadroproduction at Tevatron:



- Color octet states important
  $\Longrightarrow$ Great success for NRQCD

Photoproduction at HERA:

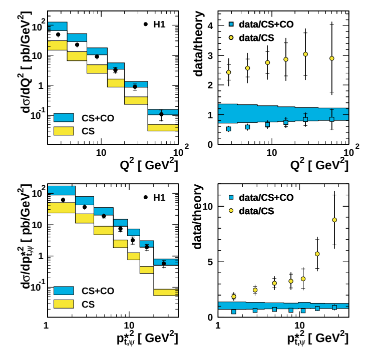


- MEs from fits to Tevatron data.
- Importance of color octet unclear

Our work: NRQCD calculation for photo- and hadroproduction and e^+e^- annihilation at NLO
$\Longrightarrow$ Establish universality of long distance matrix elements.

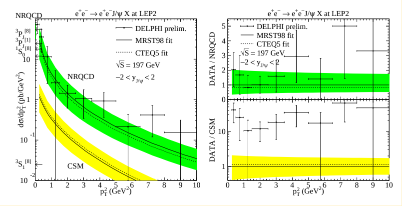
Production of J/ψ: NRQCD vs. Experiment (cont.)

Electroproduction at HERA:



BK, Zwirner, NPB(2002)

Two-Photon Collisions at LEP II:

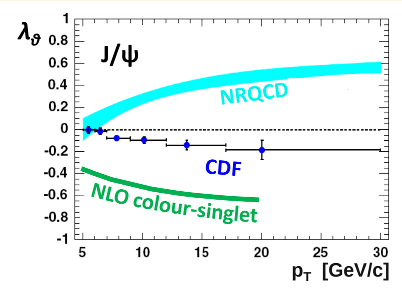


Klasen, BK, Mihaila, Steinhauser, PRL(2002)

Evidence of color-octet mechanism at LO

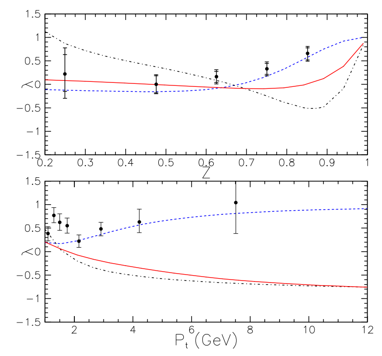
Production of J/ψ: NRQCD vs. Experiment (cont.)

Polarization at Tevatron:



Braaten, BK, Lee, PRD(2000);
Gong, Wang, PRL(2008)

Polarization at HERA:



Chang, Li, Wang, PRD(2009)

$$\frac{d\sigma}{d\cos\vartheta} \propto 1 + \lambda_\vartheta \cos^2\vartheta$$

Puzzling situation: LO NRQCD and NLO CSM fail

$\Longrightarrow$ How about NLO NRQCD?

Production of J/ψ: Summary of Calculations

Hadroproduction:

	$^3S_1^{[1]}$	$^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_{0/1/2}^{[8]}$
Born	Baier, Rückl (1980)	Cho, Leibovich (1996)
NLO	Campbell et al. (2007)	Butenschön, BK (2010) Ma et al. (2010)

Photoproduction:

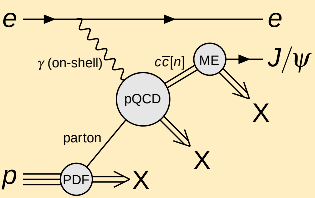
	$^3S_1^{[1]}$	$^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_{0/1/2}^{[8]}$
Born	Berger, Jones (1981)	Ko, Lee, Song (1996)
NLO	Krämer et al. (1995)	Butenschön, BK (2009)

Open question of ME universality:

- NLO NRQCD calculation: after 14 years!
- Difficulty: virtual corrections to P states

Direct J/ψ Production

Factorization formulas: (e.g. photoproduction)



- Convolute partonic cross sections with proton PDFs:

$$\sigma_{\text{hadr}} = \sum_i \int dx\, f_{i/p}(x) \cdot \sigma_{\text{part,i}}$$

- NRQCD factorization:

$$\sigma_{\text{part,i}} = \sum_n \sigma(\gamma i \to c\bar{c}[n] + X) \cdot \langle O^{J/\psi}[n] \rangle$$

Amplitudes for c$\bar{\text{c}}$[n] production by projector application, e.g.:

$$A_{c\bar{c}[{}^3S_1^{[1/8]}]} = \varepsilon_\alpha \operatorname{Tr}[C\, \Pi^\alpha A_{c\bar{c}}]\,|_{q=0}$$

$$A_{c\bar{c}[{}^3P_J^{[8]}]} = \varepsilon_{\alpha\beta} \frac{d}{dq_\beta} \operatorname{Tr}[C\, \Pi^\alpha A_{c\bar{c}}]\,|_{q=0}$$

- $A_{c\bar{c}}$: Amputated pQCD amplitude for open $c\bar{c}$ production.
- q: Relative momentum between c and $\bar{c}$.

Main Difference to Previous Calculations

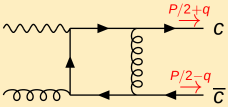
Virtual corrections: Two different approaches:

- First loop integration, then projectors: (Previous publications)
  - − Loop integrals Coulomb divergent.
- First projectors, then loop integration: (Our method)
  - + No Coulomb singularities.
  - + One scale less in loop integration.
  - − Loop integrals not standard form.

Where do Coulomb divergences come from?

- Projectors: Relative momentum $q \to 0$.
- Scalar diagrams with gluon between external c and $\overline{c}$, e.g.:

$I(q) \equiv$



$$\lim_{q\to 0} I(q) = \frac{A}{q^2} + \frac{B}{\varepsilon} + C$$

But: $I(0) = \frac{B}{\varepsilon} + C$

- $\Longrightarrow$ No Coulomb singularities in dimensional regularization!

Organisation of our Calculation

FeynArts: Generate Feynman diagrams

Mathematica script: Apply color projectors. Evaluate color factors with FeynCalc.

FORM: Apply spin projectors. Treat squared amplitudes.

Two methods for virtual corrections:

FORM: Perform our tensor reduction
$\Longrightarrow$ Scalar integrals

FORM: Use Integration by parts relations
$\Longrightarrow$ Master integrals (uses AIR)

FORM: Cancel scalar products by denominators. Neg. propagator powers. Directly apply IBP relations (uses AIR).
$\Longrightarrow$ Master integrals
(Not for ${}^1S_0$ spin singlet state)

Mathematica script: Simplify results due to kinematics (Size dramatically reduced)

Analytical check: Results of two methods equal. All divergences cancel.

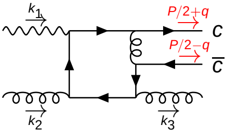
Numerical evaluation:

FORTRAN: Phase space integrations with VEGAS. Phase space slicing method.

Virtual Corrections: Tensor Reduction

Partonic subprocesses: $\gamma g \to c\bar{c}[n] + g$ and $\gamma q \to c\bar{c}[n] + q$

Example loop diagram:



Application of projectors:

- P-States: Derivatives $\Longrightarrow$ double powers of propagators
- Setting $q = 0$ $\Longrightarrow$ propagators with linear dependent momenta
- Example: $\displaystyle\int d^D Q \frac{Q^\mu Q^\nu}{Q^2\left[(Q+\frac{P}{2}+q)^2-m^2\right]\left[(Q-\frac{P}{2}+q)^2-m^2\right]^2}$

Standard Passarino-Veltman formulas do not work:

- Because naively: $D^{\mu\nu}(\frac{P}{2}, -\frac{P}{2}, -\frac{P}{2}, 0, m, m, m)$
  $\Longrightarrow$ Gram determinants zero (by which we must divide)
- Denner-Dittmaier methods [NPB **734**, 62] also do not apply here.

Tensor Reduction: Our Formulas

Example: $$\int d^D Q \, \frac{Q^\mu Q^\nu}{Q^2 \left[(Q+\frac{P}{2})^2 - m^2\right] \left[(Q-\frac{P}{2})^2 - m^2\right]^2}$$

Our approach:

- Classify integrals according to number of independent momenta, not according to number of propagators.
- Tensor decomposition for the example:
$$B^{\mu\nu}(\frac{P}{2}, 0, m; -1, m; 2, 1) = g^{\mu\nu} B_{1,0} + \frac{1}{4} P^\mu P^\nu B_{0,2}$$

We derived extension of Passarino-Veltman formulas valid for:

- Arbitrary number of Lorentz indices
- Up to five propagators of which one can have a double power
- One propagator momentum may depend linearly on the others.

Therefore: Can reduce all our tensor integrals to scalar ones!

Integration by Parts: The Method

IBP relations:
$$0 = \frac{\partial}{\partial Q^\mu} Q^\mu f(Q, p_1, \ldots, p_n)$$
$$0 = \frac{\partial}{\partial Q^\mu} p_i^\mu f(Q, p_1, \ldots, p_n)$$

- $f(Q, p_1, \ldots, p_n)$: Integrand of dim. reg. Feynman integral
- Q: Loop momentum, p_i: External momenta

Reduction of scalar integrals:

- Application of IBP relations leads to relations between different integrals of one topology.
- A topology means integrals differing only in propagator powers.
- Linear combinations of these integral relations can express one integral in terms of others with lower powers of the propagators.
  $\Longrightarrow$ Reduction to set of master integrals.

Integration by Parts: Our Application

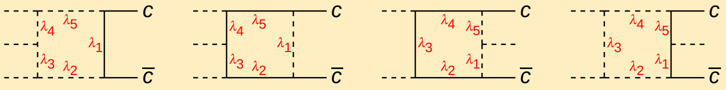
Normally:

- IBP mainly applied in multi loop calculations.
- Normal one loop diagrams are already master integrals.

In our case:

- IBP works fantastically because:
  Linear dependent propagator momenta, double powers.
- Find reduction formulas with AIR (uses Laporta algorithm).

Our four topologies. Every scalar integral belongs to one:



- Propagator powers: $\lambda_i = -4, -3, -2, -1, 0, 1, 2$
- Dashed lines: Photons, gluons or light quarks

Integration by Parts: The Results

After the reduction:

- 14 master integrals (4 boxes, 5 triangles, 4 bubbles, 1 tadpole)
- Full analytic results known.

One feature:

- Resulting expressions contain terms like: $\frac{1}{\varepsilon} \cdot B(\ldots)$
  with B a tadpole or bubble master integral

Consequences:

- Must expand corresponding master integrals up to $O(\varepsilon)$.
- Information about what are UV- and what IR-divergences is lost!
- $\Longrightarrow$ In order to distinguish between UV and IR divergences: Extract UV divergences before IBP is applied. (No problem.)

Cancellation of Divergences

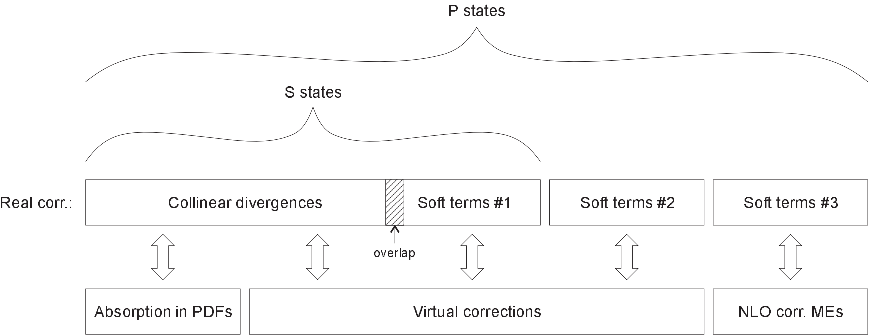
UV-divergences: Cancellation within virtual corrections:

- Loop integrals
- Charm mass renormalization
- Strong coupling constant renormalization
- Wave function renormalization of external particles

IR-divergences: Cancellation between:

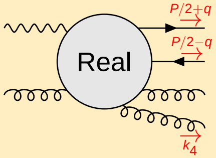
- Virtual corrections (loop integrals + wave function renormal.)
- Soft and collinear parts of real corrections
- Universal part absorbed into proton and photon PDFs
- Radiative corrections to long distance matrix elements

Overview over IR Singularity Structure

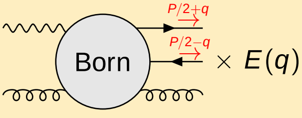


Structure of Soft Singularities

Soft limits of the real corrections:



$$\xrightarrow[k_4 \text{ soft}]{k_4 \to 0}$$



$$A_{\text{soft}}(q) = A_{Born}(q) \cdot E(q)$$

S and P states: Soft #1 + Soft #2 + Soft #3 terms:

$$A_{\text{soft,s}} = A_{\text{soft}}(0) = A_{\text{Born,s}} \cdot E(0)$$

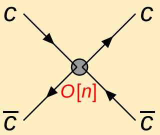
$$A_{\text{soft,p}} = A'_{\text{soft}}(0) = A_{\text{Born,p}} \cdot E(0) + A_{\text{Born,s}} \cdot E'(0)$$

$$|A_{\text{soft,s}}|^2 = |A_{\text{Born,s}}|^2 \cdot E(0)^2$$

$$|A_{\text{soft,p}}|^2 = |A_{\text{Born,p}}|^2 \cdot E(0)^2 + 2\,\text{Re}\, A^*_{\text{Born,s}} A_{\text{Born,p}} \cdot E(0) E'(0) + |A_{\text{Born,s}}|^2 \cdot E'(0)^2$$

Radiative Corrections to Long Distance MEs

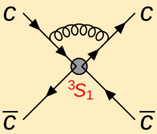
In NRQCD: Long distance MEs = $c\bar{c}$ scattering amplitudes:

$\langle O^{J/\psi}[n] \rangle =$ 

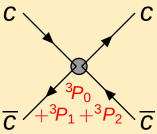
$O[n]$ = 4-fermion operators

$(n = {}^3S_1^{[1]}, {}^1S_0^{[8]}, {}^3S_1^{[8]}, {}^3P_{0/1/2}^{[8]}, \ldots)$

Corrections to $\langle \mathbf{O}^{J/\psi}[{}^3\mathbf{S}_1^{[1/8]}] \rangle$ with NRQCD Feynman rules:



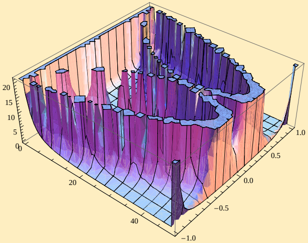
$$+ \text{ similar diagrams } \propto \frac{4\alpha_s}{3\pi m_c^2}\left(\frac{1}{\varepsilon_{\mathrm{UV}}} - \frac{1}{\varepsilon_{\mathrm{IR}}}\right) \cdot$$



- UV singularity cancelled by renormalization of 4-fermion operat.
- IR singularity cancels soft #3 terms of p states!

Real Corrections: Phase Space Slicing

Example: Squared amplitude for $\gamma + g \to c\overline{c}[^3S_1^{[8]}] + d + \overline{d}$:



- Infrared divergences: Cannot do complete integration numerically.
- Collinear and soft limits: Phase space and $|M|^2$ factorizes $\Longrightarrow$ Analytical D dimensional integration possible!

(Plotted against $(k_d + k_{\overline{d}})^2$ and $\cos\theta(c\overline{c}, d)$ in d-$\overline{d}$ rest frame for $s = 100$ GeV^2, $t = -20$ GeV^2.)

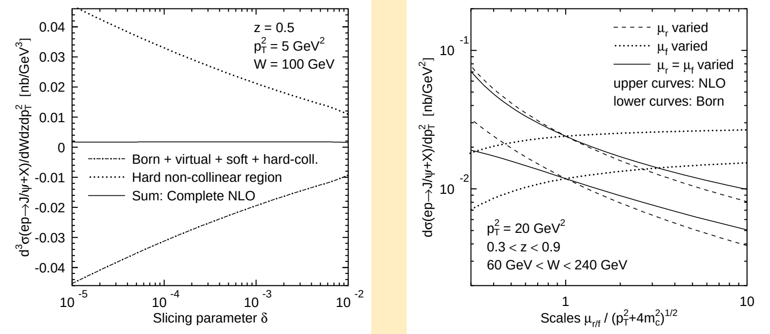
Idea: Split integration into two regions:

1. $\delta s > 100(k_i \cdot k_j)^2$ or $\delta\sqrt{s} > 2E_{3/4}$: Analytical integration.
2. $\delta s < 100(k_i \cdot k_j)^2$ and $\delta\sqrt{s} < 2E_{3/4}$: Numerical integration.

Both contributions: $\log\delta$ terms. These terms cancel for small δ!

Parameter Dependences

Dependence on slicing parameter and unphysical scales:



- Phase space slicing works!
  $\Longrightarrow$ Check on our kinematics and soft / collinear limits
- Dependence on renormalization and factorization scale:
  $0.7 \lessapprox \sigma/\sigma_0 \lessapprox 1.6 \quad \text{if} \quad 0.5 < \mu_r/\mu_0 = \mu_f/\mu_0 < 2.$

Global fit at NLO in NRQCD

Fit CO LDMEs to all available world data on J/ψ inclusive production:

type	$\sqrt{s}$	collider	collaboration	reference
pp	200 GeV	RHIC	PHENIX	PRD82(2010)012001
$p\overline{p}$	1.8 TeV	Tevatron I	CDF	PRL97(1997)572; 578
$p\overline{p}$	1.96 TeV	Tevatron II	CDF	PRD71(2005)032001
pp	7 TeV	LHC	ALICE, ATLAS, CMS, LHCb	talks; arXiv:1011.4193
γp	300 GeV	HERA I	H1, ZEUS	EPJ25(2002)25; 27(2003)173
γp	319 GeV	HERA II	H1	EPJ68(2010)401
$\gamma\gamma$	197 GeV	LEP II	DELPHI	PLB565(2003)76
e^+e^-	10.6 GeV	KEKB	BELLE	PRD79(2009)071101

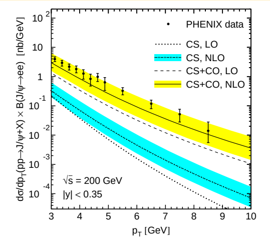
- Best fit values:

10^{-2} GeV^{3+2L}	PRL106(2011)022003	new
$\langle \mathcal{O}(^1S_0^{[8]}) \rangle$	4.50 ± 0.72	4.97 ± 0.44
$\langle \mathcal{O}(^3S_1^{[8]}) \rangle$	0.312 ± 0.093	0.224 ± 0.059
$\langle \mathcal{O}(^3P_0^{[8]}) \rangle$	-1.21 ± 0.35	-1.61 ± 0.20

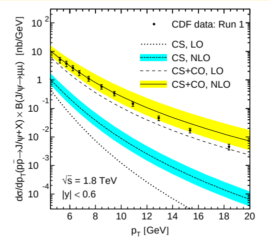
- $\chi^2/\text{d.o.f.} = 857/194 = 4.42$ for default prediction
- $\propto v^4 \langle O_1(^3S_1) \rangle \rightsquigarrow$ NRQCD velocity scaling rules $\surd$

Comparison with RHIC and Tevatron

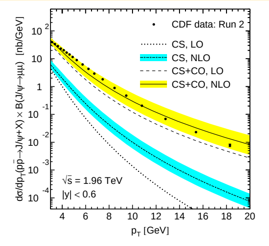
RHIC PHENIX



Tevatron I CDF



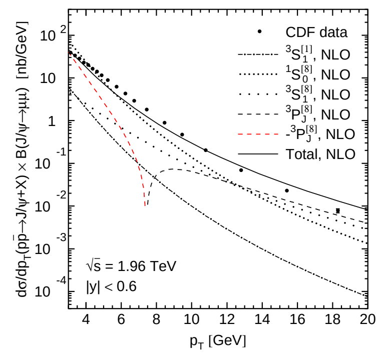
Tevatron II CDF



- Data well described by CS+CO at NLO.
- CS orders of magnitudes below data.

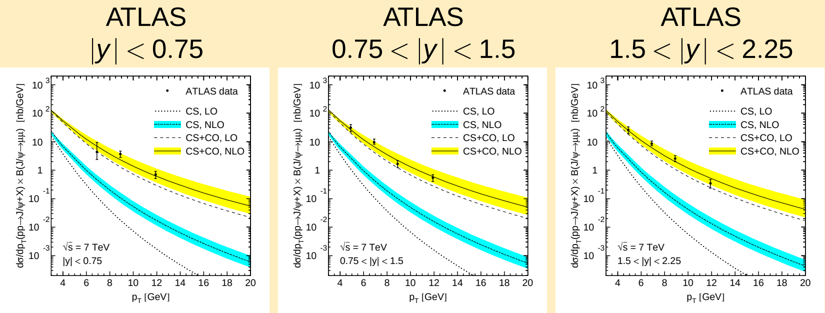
Comparison with Tevatron (cont.)

Relative importance of CO processes:



- Short-distance $\sigma(c\bar{c}[^3P_J^{[8]}]) < 0$ for $p_T \gtrsim 7$ GeV.
- But: Short-distance cross sections and LDMEs unphysical (NRQCD scale and scheme dependence) $\rightsquigarrow$ No problem!

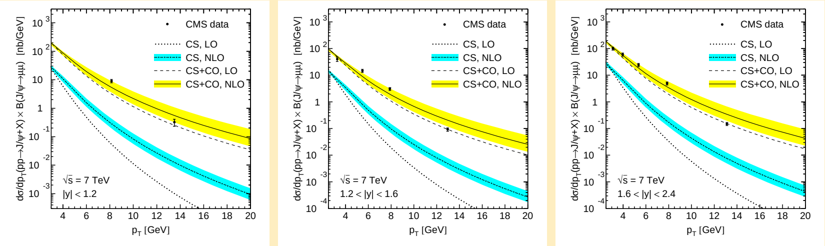
Comparison with ATLAS at LHC



- Data well described by CS+CO at NLO.
- CS orders of magnitudes below data.

Comparison with CMS at LHC

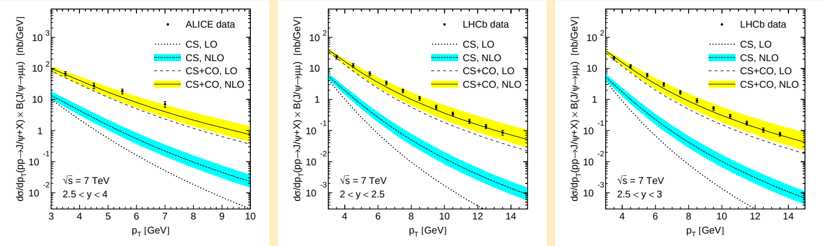
CMS $|y| < 1.2$ | CMS $1.2 < |y| < 1.6$ | CMS $1.6 < |y| < 2.4$



- Data well described by CS+CO at NLO.
- CS orders of magnitudes below data.

Comparison with ALICE and LHBb at LHC

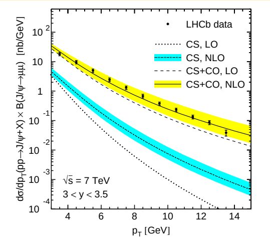
ALICE $2.5 < y < 4.0$ | LHCb $2.0 < y < 2.5$ | LHCb $2.5 < y < 3.0$



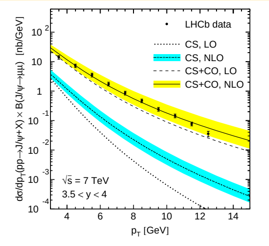
- Data well described by CS+CO at NLO.
- CS orders of magnitudes below data.

Comparison with LHBb at LHC (cont.)

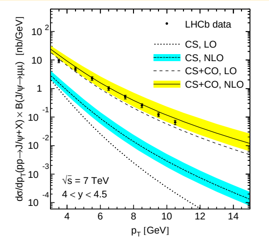
LHCb
$3.0 < y < 3.5$



LHCb
$3.5 < y < 4.0$

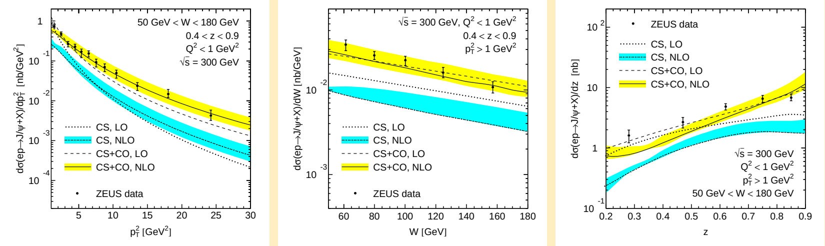


LHCb
$4.0 < y < 4.5$



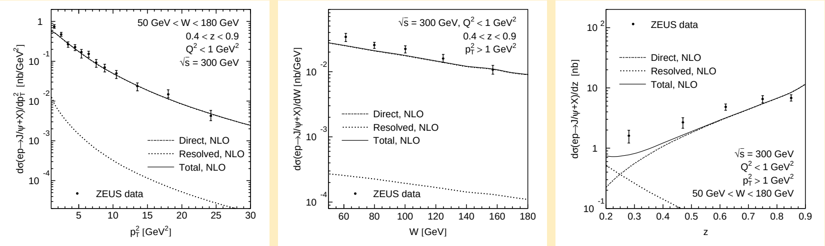
- Data well described by CS+CO at NLO.
- CS orders of magnitudes below data.

Comparison with ZEUS at HERA I (1)



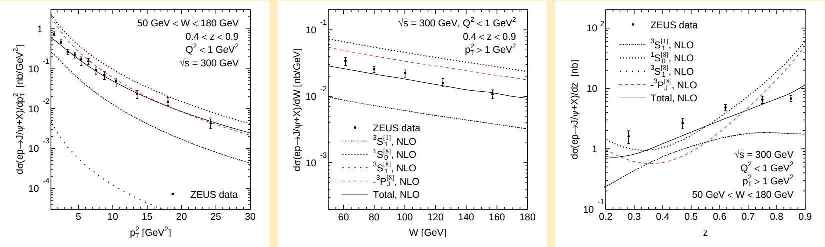
- $W =$ γp CM energy.
- $z =$ fraction of γ energy going to J/ψ in p rest frame.
- Singularity for $z \to 1$ eliminated by shape function in SCET.
- Data well described by CS+CO at NLO.
- CS factor of 3–5 below the data.

Comparison with ZEUS at HERA I (2)



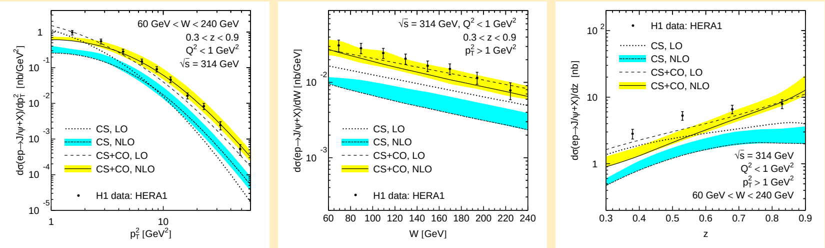
- Data for $0.4 < z < 0.9$ exhausted by direct photoproduction.
- Resolved photoproduction only relevant for $z \lesssim 0.4$.

Comparison with ZEUS at HERA I (3)



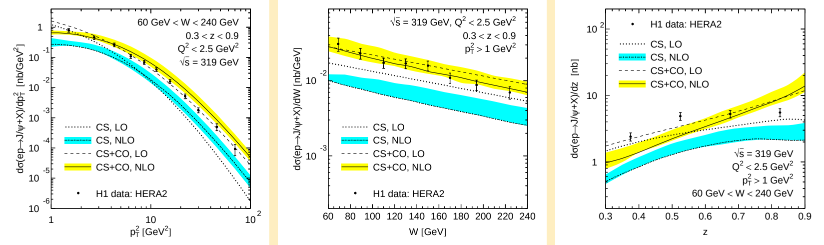
- $\langle \mathcal{O}(^3P_0^{[8]}) \rangle < 0 \rightsquigarrow {}^3P_0^{[8]}$ contribution negative.
- Negative interference with $^1S_0^{[8]}$ contribution beneficial.
- $^3S_1^{[8]}$ contribution negligible here.

Comparison with H1 at HERA I



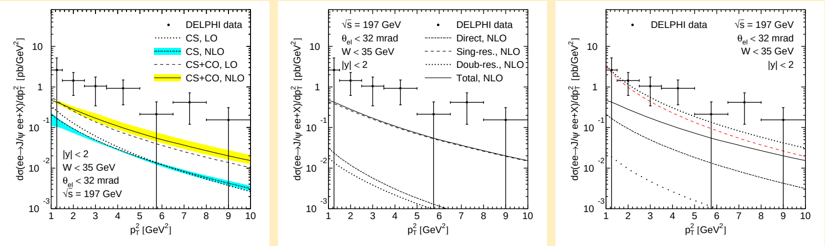
- Data well described by CS+CO at NLO.
- CS factor of 3–5 below data.

Comparison with H1 at HERA II



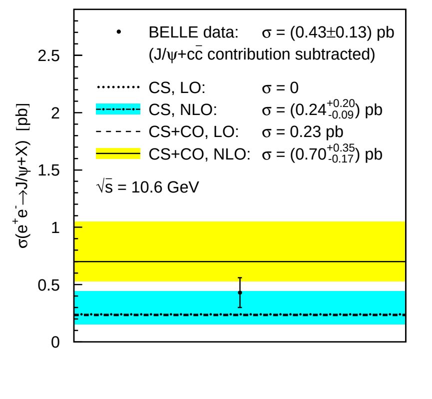
- Data well described by CS+CO at NLO.
- CS factor of 3–5 below the data.

Comparison with DELPHI at LEP II



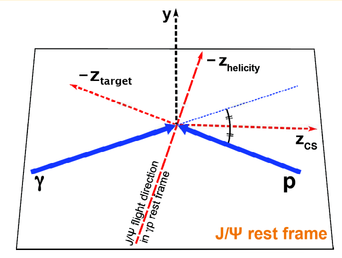
- Agreement with NRQCD at NLO worse than 2002 with LO.
- Just 16 DELPHI events with $p_T > 1$ GeV.
- No results from ALEPH, L3, OPAL.
- Data exhausted by single-resolved contribution.

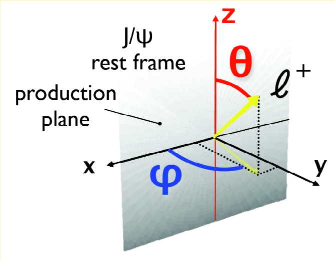
Comparison with BELLE at KEKB



- At NLO, both CSM and NRQCD agree with data.

Polarized J/ψ photoproduction





Decay angular distribution:

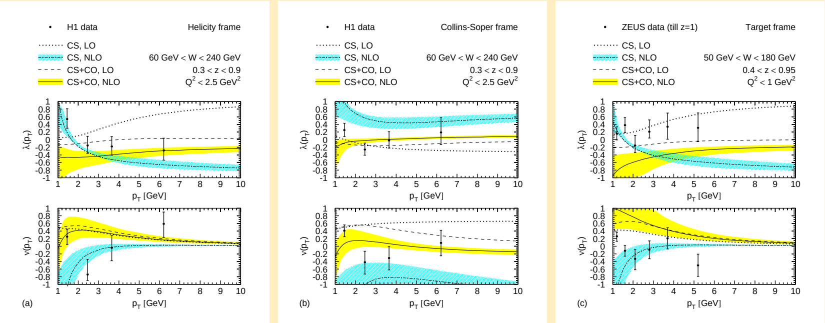
$$\frac{d\Gamma(J/\psi \to l^+l^-)}{d\cos\theta\, d\phi} \propto 1 + \lambda\cos^2\theta + \mu\sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)$$

Polarization observables in spin density matrix formalism:

$$\lambda = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \qquad \mu = \frac{\sqrt{2}\mathrm{Re}\, d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}}, \qquad \nu = \frac{2d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}}$$

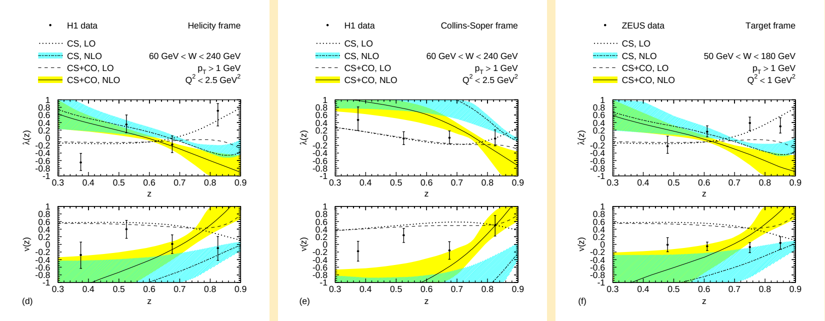
$\lambda = 0, +1, -1$: unpolarized, transversely and longitudinally porarized.

Comparison with H1 and ZEUS



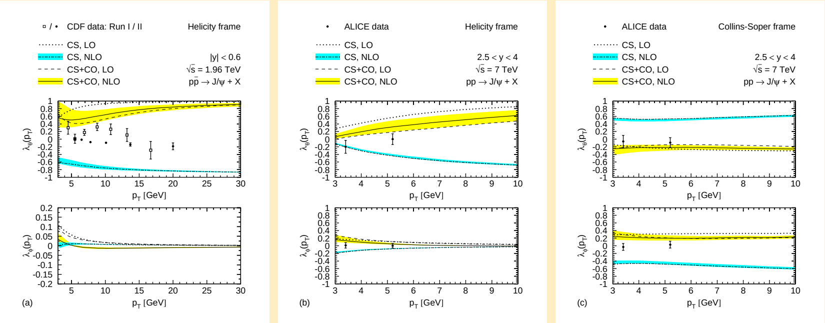
- No z cut on ZEUS data $\rightsquigarrow$ diffractive production included.
- Perturbative stability in NRQCD higher than in CSM.
- J/ψ preferrably unpolarized at large p_T.

Comparison with H1 and ZEUS (cont.)



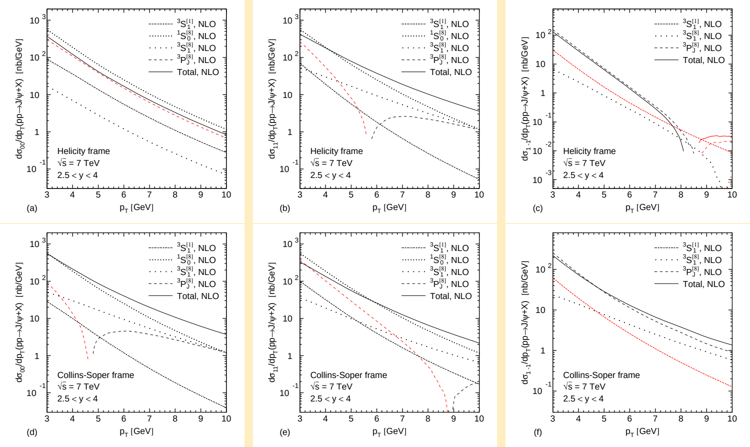
- Large scale uncertainties due to low cut $p_T > 1$.
- Overall χ^2 w.r.t. default prediction more than halved by going from CSM to NRQCD.

Comparison with CDF and ALICE



- CDF I and II data mutually inconsistent for $p_T < 12$ GeV.
- CDF J/ψ polarization anomaly persits at NLO.
- 4/8 ALICE points agree w/ NLO NRQCD within errors, others $< 2\sigma$ away.

Decomposition for ALICE



- $d\sigma_{\mathrm{unpol}} = d\sigma_{00} + 2d\sigma_{11}$; $d\sigma_{1,-1}$ auxiliary.
- Previously unknown $^3P_J^{[8]}$ NLO correction significant.

Summary

- NRQCD provides rigorous factorization theorem for production and decay of heavy quarkonia; predicts:
  - existence of CO states;
  - universality of LDMEs.
- Previous LO tests not conclusive.
- Here: first global analysis of unpolarized-J/ψ world data at NLO.
- Hadro- and photoproduction: striking evidence of NRQCD vs. CSM.
- $\gamma\gamma$ scattering, e^+e^- annihilation: not conclusive yet.
- Contributions from feed-down and B decays throughout small against theoretical uncertainties.
- Hadroproduction data alone cannot reliably fix all 3 CO LDMEs and give misleading results for their linear combinations; cf. Ma et al., PRL106(2011)042002; PRD84(2011)114001; MB & BK, AIPConfProc1343(2011)409.

Summary (cont.)

- Case for NRQCD less strong in polarized J/ψ photoproduction at HERA.
- Polarized J/ψ hadroproduction at Tevatron in severe conflict with NLO NRQCD, while first LHC data nicely agree.
- Physics opportunities with LHC at 7 TeV (and above) include verification/falsification of NRQCD factorization in charmonium and bottomonium yield and polarization!

Backup Slides

Tensor Reduction: Definitions

Idea: Derive general formulas for arbitrary number of Lorentz indices.

Tensor decomposition of most general B-function:

$$\int \frac{d^D Q \quad Q^{\mu_1} Q^{\mu_2} \cdots Q^{\mu_P}}{\left[Q^2 - m_0^2\right]^y \left[(Q+p)^2 - m_1^2\right] \left[(Q+ap)^2 - m_5^2\right]^x} \qquad (x \leqq 0;\, y \leqq 1)$$

$$\equiv B^{\mu_1 \cdots \mu_P}(p, m_0, m_1; a, m_5; x, y) = \sum_{n=0}^{P\,\mathrm{DIV}\,2} \left\{g^n p^{P-2n}\right\}^{\mu_1 \ldots \mu_P} B_{n,P-2n}$$

Here, $\left\{g^n p^{P-2n}\right\}^{\mu_1 \ldots \mu_P}$ is the sum of all different terms when distributing the P Lorentz indices over the n g's and the $(P-2n)$ p's.

Example: $$B^{\mu\nu\rho\sigma} = p^\mu p^\nu p^\rho p^\sigma B_{0,4} + \left(g^{\mu\nu} p^\rho p^\sigma + g^{\mu\rho} p^\nu p^\sigma + g^{\mu\sigma} p^\nu p^\rho + g^{\nu\rho} p^\mu p^\sigma + g^{\nu\sigma} p^\mu p^\rho + g^{\rho\sigma} p^\mu p^\nu\right) B_{1,2} + \left(g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}\right) B_{2,0}$$

Tensor Reduction: Step 1

Step 1: Multiply with $g_{\mu_1\mu_2}$ and p_{μ_1} and compare Lorentz structure:

Reduction formulas for B-functions:

$$B_{r+1,s} = \frac{1}{D+2r+s-1}\left[B_{r,s}(\not{0}) + m_0^2 B_{r,s} - Z_{r,s+1}\right]$$

$$B_{r,s+1} = \frac{1}{p^2}\left[Z_{r,s} - s\,B_{r+1,s-1}\right]$$

with

$$Z_{i,j} = \frac{1}{2}\left[B_{i,j}(\not{1}) - B_{i,j}(\not{0}) - \left(p^2 - m_1^2 + m_0^2\right)B_{i,j}\right]$$

Here, $B_{i,j}(\not{0})$ and $B_{i,j}(\not{1})$ are the coefficients of the tensor structure $\{g^i p^j\}^{\mu_1\cdots\mu_{2i+j}}$ after deleting the 0th/1st propagator.

Tensor Reduction: Step 2

Tensor coefficients for integrals with deleted propagators:

$$B_{r,s}(\not{1}) = \begin{cases} x=0,\, s>0: & 0 \\ x=0,\, s=0: & A_r(m_0;0,0,y) \\ x>0: & a^s\, B_{r,s}(ap,m_0,m_5;1,m_5;x-1,y) \end{cases}$$

$$B_{r,s}(\not{0}) = \begin{cases} y>1: & B_{r,s}(p,m_0,m_1;a,m_5;x,y-1) \\ y=1,\, x=0: & (-1)^s\, A_r(m_1;m_5,x,1) \\ y=1,\, x>0: & \sum_{\alpha=0}^{s} \binom{s}{\alpha}(-1)^{\alpha}(a-1)^{s-\alpha} \\ & \quad \times B_{r,s-\alpha}((a-1)p,m_1,m_5;1,m_5;x-1,1) \end{cases}$$

- Normal Passarino-Veltman: $x=0$ and $y=1$.
- C- and D-functions (two and three independent external momenta): We derived formulas as well, are more complicated.
  $\Longrightarrow$ Can reduce all occurring tensor integrals to scalar ones.

Gamma 5: Occurrence

Where do we have γ_5?

- Diagrams are pure QCD, free of γ_5.
- But γ_5 in projector onto $c\bar{c}$ spin singlet states (like $^1S_0$):

Amplitude for $c\bar{c}[^1S_0]$ production: $A_{c\bar{c}[^1S_0]} = \mathrm{Tr}\left[C\,\Pi_0\,A_{c\bar{c}}\right]\Big|_{q=0}$

- $A[c\bar{c}]$: Original diagrams with charm spinors amputated
- $\Pi_0 = (2m)^{-3/2}\left(\not{P}/2 - \not{q} - m\right)\gamma_5\left(\not{P}/2 + \not{q} + m\right)$
- Trace taken over c-spinline.

$\Longrightarrow$ Long trace with γ_5: $\mathrm{Tr}\left[\gamma_5\,\gamma_\alpha\,\gamma_\beta\,\gamma_\gamma\,\gamma_\delta\,\gamma_\varepsilon\,\gamma_\zeta\,\gamma_\eta\,\gamma_\theta\right]$

γ_5 in D dimensions:

- $\{\gamma_5, \gamma^\mu\} = 0$ and $\mathrm{Tr}\left[\gamma_5\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta\right] = 4i\varepsilon^{\alpha\beta\gamma\delta}$ incompatible.
(Ambiguous results when changing order of rule application)

Gamma 5: 't Hooft-Veltman-Breitenlohner-Maison

't Hooft-Veltman scheme: (= Breitenlohner-Maison scheme)

- Definition: $\gamma_5 := \frac{i}{4!}\,\varepsilon_{\mu\nu\rho\sigma}\,\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$
- γ's D-dimensional, but ε 4-dimensional!
- $\{\gamma_5,\gamma^\mu\} = 0$ no longer valid!
- Our application: No γ_5 related counterterms.

't Hooft-Veltman in our calculation:

1. $A_{c\bar{c}[^1S_0]} \propto \mathrm{Tr}\left[\gamma_5\,\gamma_{\mu_1}\ldots\gamma_{\mu_n}\right] \to \frac{i}{4!}\,\varepsilon^{\alpha_1\alpha_2\alpha_3\alpha_4}\,\mathrm{Tr}\left[\gamma_{\alpha_1}\gamma_{\alpha_2}\gamma_{\alpha_3}\gamma_{\alpha_4}\gamma_{\mu_1}\ldots\gamma_{\mu_n}\right]$
2. In the end: $A^*A \propto \varepsilon^{\alpha_1\alpha_2\alpha_3\alpha_4}\varepsilon^{\beta_1\beta_2\beta_3\beta_4} \to -\begin{vmatrix} \tilde{g}^{\alpha_1\beta_1} & \cdots & \tilde{g}^{\alpha_1\beta_4} \\ \vdots & & \vdots \\ \tilde{g}^{\alpha_4\beta_1} & \cdots & \tilde{g}^{\alpha_4\beta_4} \end{vmatrix}$
3. $\tilde{g}^{\mu\nu}$ 4-dimensional: $\tilde{g}_{\mu\nu}g^{\nu\rho} = \tilde{g}_\mu{}^\rho$, $\tilde{g}_{\mu\nu}p^\nu = \tilde{p}_\mu$, $\tilde{g}_{\mu\nu}\tilde{g}^{\mu\nu} = 4$.