Neutrino Masses in a Two Higgs Doublet Model

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Outline

- Introduction.
- Neutrino masses in the presence of right-handed neutrinos.
- Neutrino masses in the presence of right-handed neutrinos and one *extra* Higgs doublet.
- Comparison between the 2RHN-1HDM and the 1RHN-2HDM.
- Mixing angles: θ_{13} and deviation from maximal atmospheric mixing.
- Conclusions

Introduction

Present status in the determination of neutrino parameters:

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\mathrm{eV}^2]$	$7.59^{+0.20}_{-0.18}$	7.24–7.99	7.09–8.19
Δm^2_{31} $[10^{-3}\mathrm{eV}^2]$	$2.50^{+0.09}_{-0.16}$ $-(2.40^{+0.08}_{-0.09})$	$2.25-2.68$ $-(2.23-2.58)$	$2.14-2.76$ $-(2.13-2.67)$
$\sin^2\theta_{12}$	$0.312^{+0.017}_{-0.015}$	0.28–0.35	0.27–0.36
$\sin^2\theta_{23}$	$0.52^{+0.06}_{-0.07}$ 0.52 ± 0.06	0.41–0.61 0.42–0.61	0.39–0.64
$\sin^2\theta_{13}$	$0.013^{+0.007}_{-0.005}$ $0.016^{+0.008}_{-0.006}$	0.004–0.028 0.005–0.031	0.001–0.035 0.001–0.039
δ	$\left(-0.61^{+0.75}_{-0.65}\right)\pi$ $\left(-0.41^{+0.65}_{-0.70}\right)\pi$	$0-2\pi$	$0-2\pi$

Schwetz, Tortola, Valle
arXiv:1108.1376

WMAP, $0\nu2\beta \rightarrow m_\nu \lesssim 0.5$ eV

No information about CP violation or about the neutrino mass spectrum

Even with this limited information about the neutrino sector, we can already notice some features:

- Neutrino masses are tiny, $m_\nu \lesssim \mathcal{O}$ (0.1 eV)
- Two large mixing angles ($\theta_{\text{atm}} \simeq \pi/4$, $\theta_{\text{sol}} \simeq \pi/6$)

  One small mixing angle ($\theta_{13} \simeq 0$)

$$U_{lep} \simeq \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

- The two heaviest neutrinos present a mild mass hierarchy

$$\Delta m^2_{atm} = m_3^2 - m_1^2 \longrightarrow m_3 = \sqrt{\Delta m^2_{atm} - m_1^2}$$

$$\Delta m^2_{sol} = m_2^2 - m_1^2 \longrightarrow m_2 = \sqrt{\Delta m^2_{sol} - m_1^2}$$

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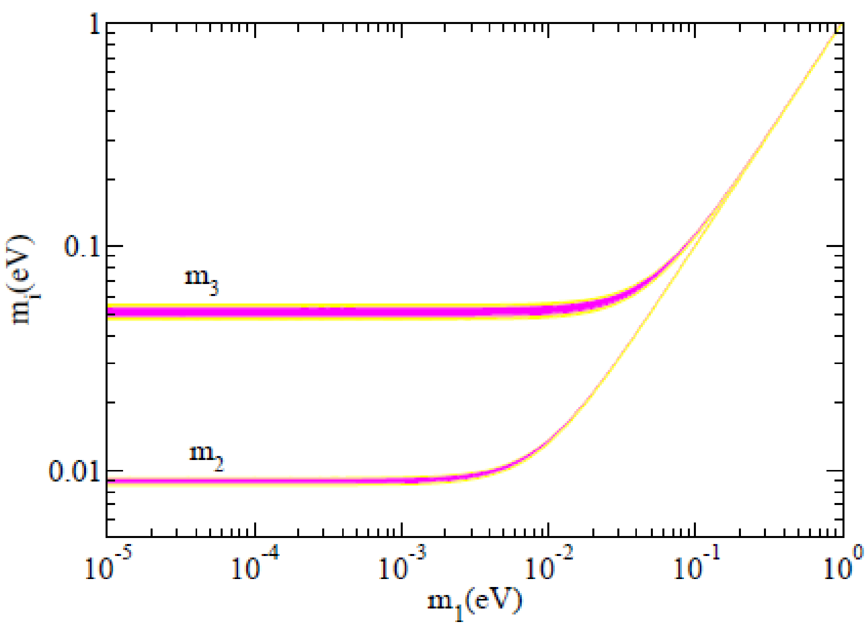

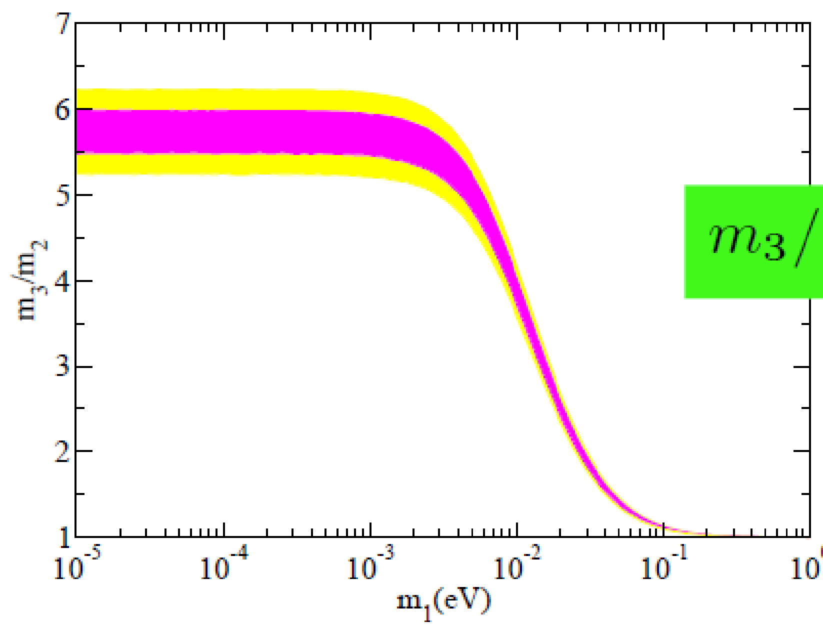

$m_3/m_2 \lesssim 6$

Compare to the quark sector

$$\left.\begin{aligned} m_u &= 1.7 \text{ to } 3.3 \text{ MeV} \\ m_c &= 1.27^{+0.07}_{-0.09} \text{ GeV} \\ m_t &= 172.0 \pm 0.9 \pm 1.3 \text{ GeV} \end{aligned}\right\} \begin{aligned} m_t/m_c &\simeq 140 \\ m_c/m_d &\simeq 500 \end{aligned}$$

$$\text{vs.} \quad m_3/m_2 \lesssim 6$$

$$\left.\begin{aligned} m_d &= 4.1 \text{ to } 5.8 \text{ MeV} \\ m_s &= 101^{+29}_{-21} \text{ MeV} \\ m_b &= 4.19^{+0.07}_{-0.09} \text{GeV} \end{aligned}\right\} \begin{aligned} m_b/m_s &\simeq 41 \\ m_s/m_d &\simeq 20 \end{aligned}$$

$$|U_{\text{CKM}}| = \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.973 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix} \quad \text{vs.} \quad |U_{\text{lep}}| \simeq \begin{pmatrix} 0.82 & 0.56 & 0 \\ 0.41 & 0.56 & 0.71 \\ 0.41 & 0.56 & 0.71 \end{pmatrix}$$

Compare also to the charged lepton sector

$$\left.\begin{aligned} m_e &= 0.51 \text{ MeV} \\ m_\mu &= 106 \text{ MeV} \\ m_\tau &= 1.78 \text{ GeV} \end{aligned}\right\} \begin{aligned} m_\tau/m_\mu &\simeq 17 \\ m_\mu/m_e &\simeq 208 \end{aligned}$$

The neutrino sector presents a completely different pattern

Any model of neutrino masses should address the following questions:

- **Why tiny masses?**
- **Why large mixing angles?**
- **Why mild mass hierarchy?**

And preferably, the model should be testable

A very popular neutrino mass model: the (type I) see-saw model

Add to the Standard Model at least two right-handed neutrinos

$$-\mathcal{L}^{\nu} = (Y_{\nu})_{ij}\bar{l}_{Li}\nu_{Rj}\tilde{\Phi} - \frac{1}{2}M_{\mathrm{M}ij}\bar{\nu}^{C}_{Ri}\nu_{Rj} + \mathrm{h.c.}$$

$$\Big\Downarrow \; M_{\mathrm{Maj}} \gg M_Z$$

$$-\mathcal{L}^{\nu,\ \mathrm{eff}} = \frac{1}{2}\kappa_{ij}(\bar{l}_{Li}\tilde{\Phi})(\tilde{\Phi}^T l^C_{Lj}) + \mathrm{h.c.}$$

$$\kappa = (Y_\nu M_{\mathrm{M}}^{-1} Y_\nu^{\ T}) \Longrightarrow \mathcal{M}_\nu = \frac{v^2}{2}\kappa$$

Very compelling explanation
to the small neutrino masses

Furthermore, this model predicts charged lepton flavour violation:

BR($\mu \rightarrow e\gamma$)~10^{-57}, in *excellent* agreement with experiments.

Can the type I see-saw mechanism accommodate a mild mass hierarchy?

The high energy theory, spanned by $\{Y_\nu, M_{maj}\}$, depends on 18 parameters

The low energy theory, spanned by $\{\mathcal{M}_\nu\}$, depends on 9 parameters

There is a lot of freedom at high energies

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The answer is yes. In fact the see-saw mechanism can accommodate anything

$$Y_\nu = \frac{1}{\langle \Phi^0 \rangle} U^*_{\rm lep} \sqrt{D_m} R^T \sqrt{D_M}$$

$$R = \begin{pmatrix} \hat{c}_2\hat{c}_3 & -\hat{c}_1\hat{s}_3 - \hat{s}_1\hat{s}_2\hat{c}_3 & \hat{s}_1\hat{s}_3 - \hat{c}_1\hat{s}_2\hat{c}_3 \\ \hat{c}_2\hat{s}_3 & \hat{c}_1\hat{c}_3 - \hat{s}_1\hat{s}_2\hat{s}_3 & -\hat{s}_1\hat{c}_3 - \hat{c}_1\hat{s}_2\hat{s}_3 \\ \hat{s}_2 & \hat{s}_1\hat{c}_2 & \hat{c}_1\hat{c}_2 \end{pmatrix}$$

$$D_M = \mathrm{diag}(M_1, M_2, M_3)$$

But there is a price...

The price is that the resulting Yukawa coupling could be "weird"

For example, taking $M_1=10^9$ GeV, $M_2=10^{11}$ GeV, $M_3=10^{13}$ GeV and $R(z_1=2i, z_2=0, z_3=0)$, one obtains the matrix

$$Y_\nu = \begin{pmatrix} 1.9\times 10^{-4} & 0.011 & 0.11i \\ -8.6\times 10^{-5} & 0.012-0.031i & 0.32+0.12i \\ 8.6\times 10^{-5} & -0.012-0.031i & 0.32-0.12i \end{pmatrix}$$

Which reproduces, *by construction*, the low energy neutrino data (m_3=0.05 eV, m_2=0.0083 eV, $\sin^2\theta_{12}$=0.3, $\sin^2\theta_{23}$=1, and $m_1=m_2/6$, θ_{13}=0 and no CP violation)

However, the eigenvalues are

$$\left.\begin{aligned} y_3 &= 0.50 \\ y_2 &= 1.3\times 10^{-3} \\ y_1 &= 2.2\times 10^{-4} \end{aligned}\right\} \quad \begin{aligned} y_3/y_2 &= 379 \\ y_2/y_1 &= 6 \end{aligned}$$

This Yukawa coupling does not seem to be generated by the same mechanism that generates Y_u, Y_d, Y_e (whatever it is...)

A more interesting question is not whether the see-saw can accommodate the data, but whether the see-saw can accommodate the data *with our present (very limited) understanding of the origin of flavour.*

Can the see-saw mechanism accommodate the oscillation data when the neutrino Yukawa couplings are hierarchical?

Not so easy... The see-saw mechanism tends to produce very large neutrino mass hierarchies Casas, AI, Jimenez-Alburquerque

"Naïve see-saw" (no mixing)

$$m_1 \sim \frac{y_1^2}{M_1}\langle\Phi^0\rangle^2, \quad m_2 \sim \frac{y_2^2}{M_2}\langle\Phi^0\rangle^2, \quad m_3 \sim \frac{y_3^2}{M_3}\langle\Phi^0\rangle^2 \qquad \frac{m_3}{m_2} \sim \frac{y_3^2}{y_2^2}\frac{M_2}{M_3}$$

- Assume hierarchical Yukawa couplings
  $y_1 : y_2 : y_3 \sim 1 : 20 : 20^2$ (down-type quark Yukawas)
  $y_1 : y_2 : y_3 \sim 1 : 300 : 300^2$ (up-type quark Yukawas)

- For the right-handed neutrino masses, we don't know

Hierarchy in ν_R as in Y_ν

$$\frac{m_3}{m_2} \sim 20 - 300$$

Degenerate ν_R

$$\frac{m_3}{m_2} \sim 400 - 90000$$

far from $\frac{m_3}{m_2} \lesssim 6$

A more rigorous analysis shows that generically

$$\frac{m_3}{m_2} \gtrsim \frac{y_3^2}{y_2^2}\frac{M_3}{M_2}$$

Hierarchical ν_R $\quad \frac{m_3}{m_2} \gtrsim \mathcal{O}(10^{3-7})$

Degenerate ν_R $\quad \frac{m_3}{m_2} \gtrsim \mathcal{O}(10^{2-5})$

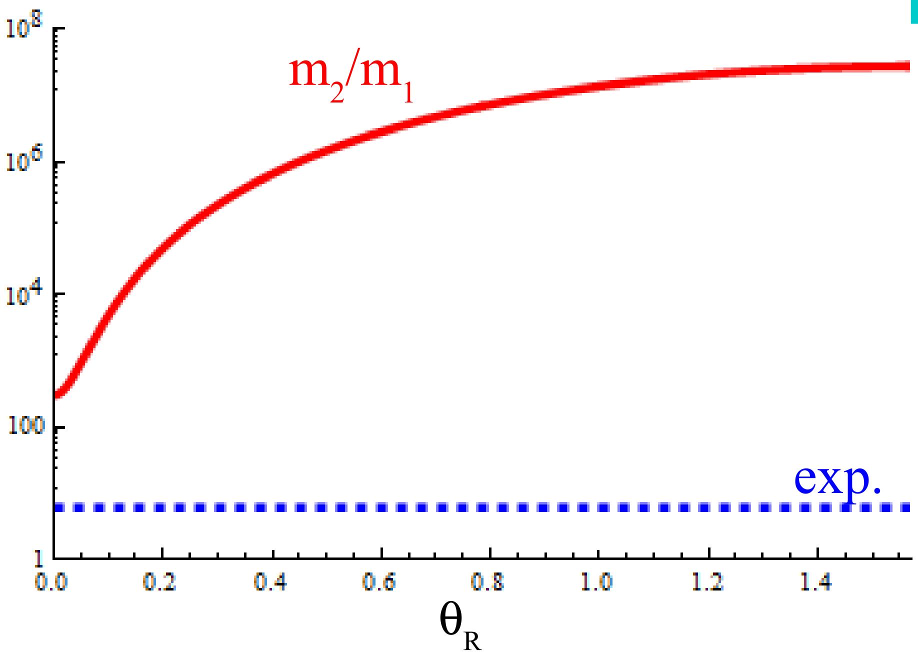
far from $\frac{m_3}{m_2} \lesssim 6$

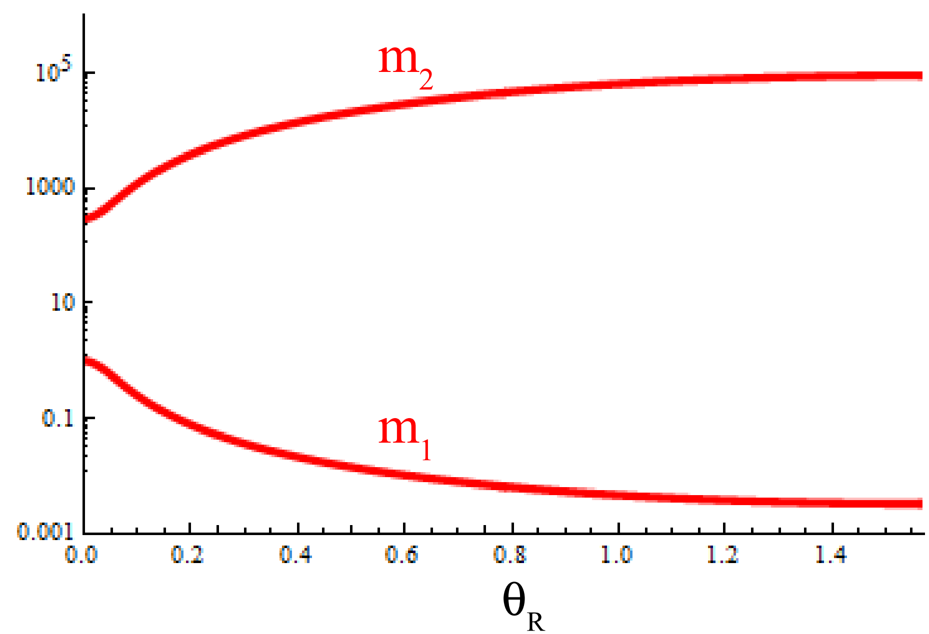
Consider a model with just two right-handed neutrinos

Assume
$$\boxed{\begin{aligned} y_1 : y_2 &= 1:300 \\ M_1 : M_2 &= 1:300 \end{aligned}}$$
(inspired by the hierarchy in the up-quark sector)

The neutrino mass hierarchy depends just on the mixing angle in the right-handed sector, θ_R, and on the Majorana phase α.

$\alpha=0$





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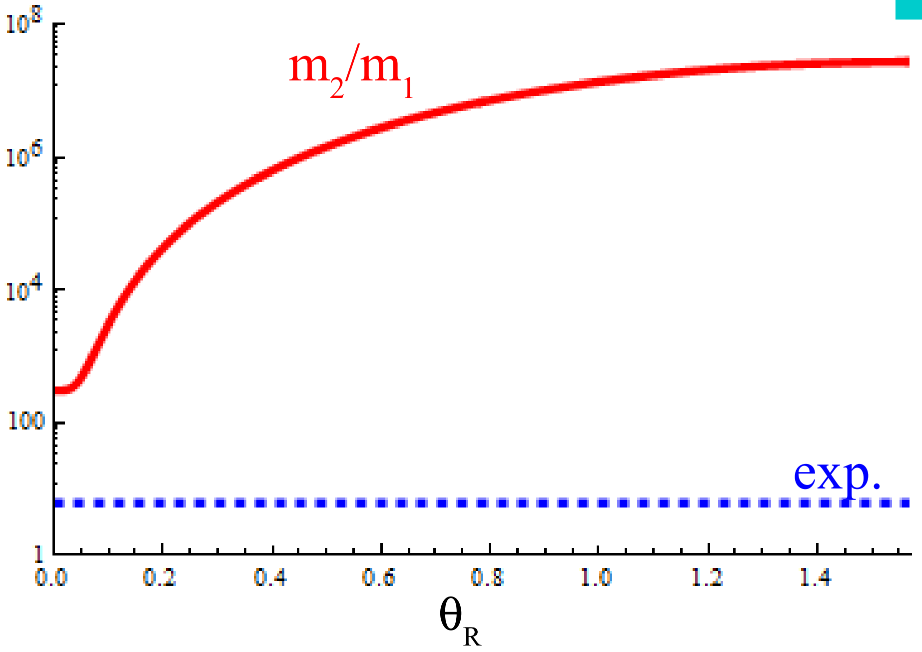

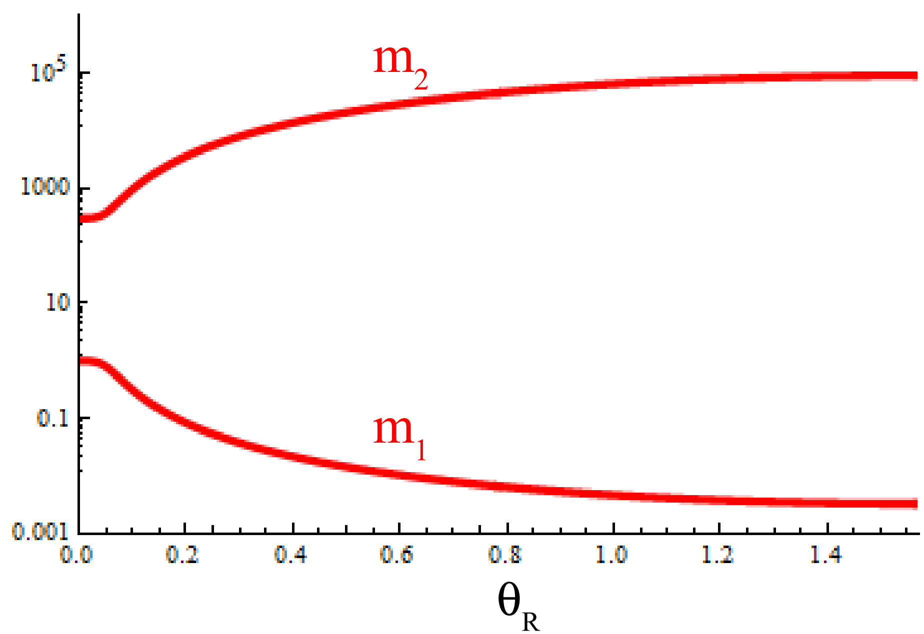

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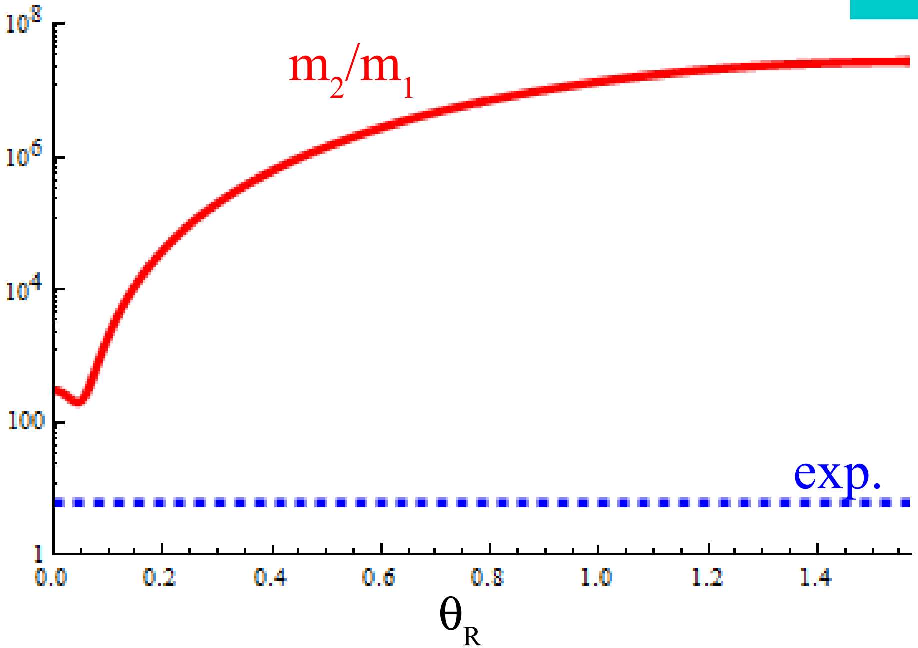

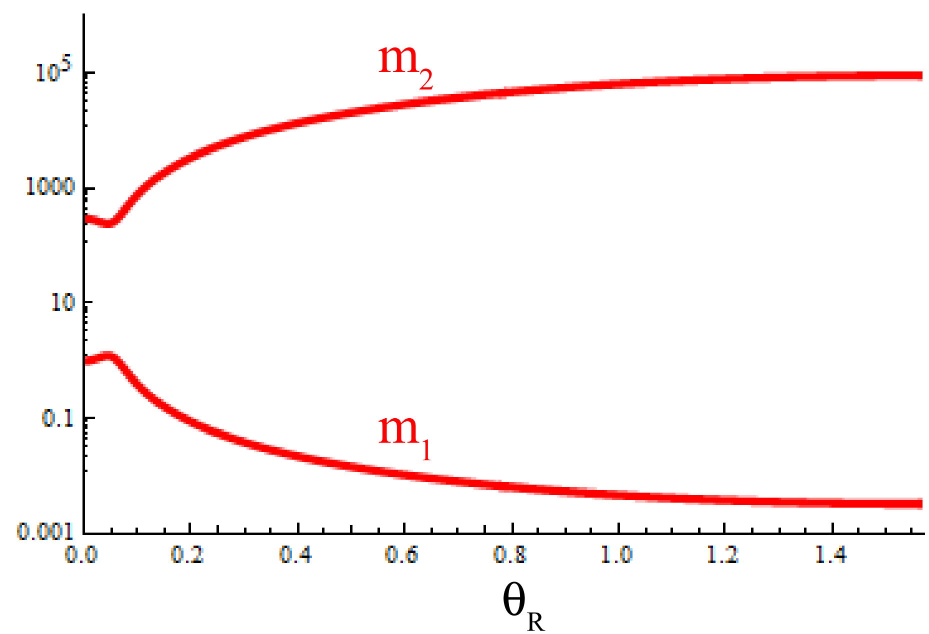

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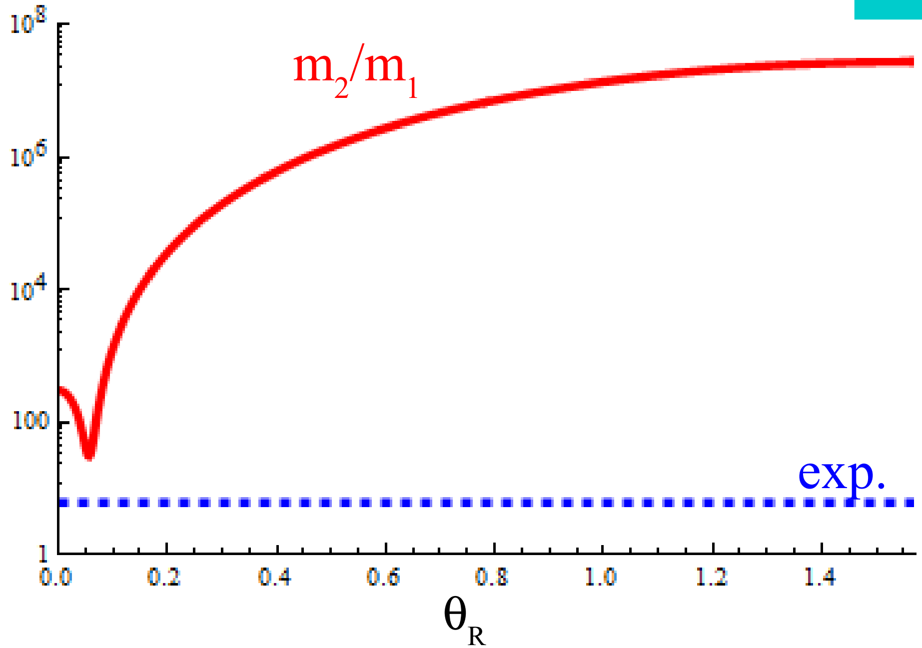

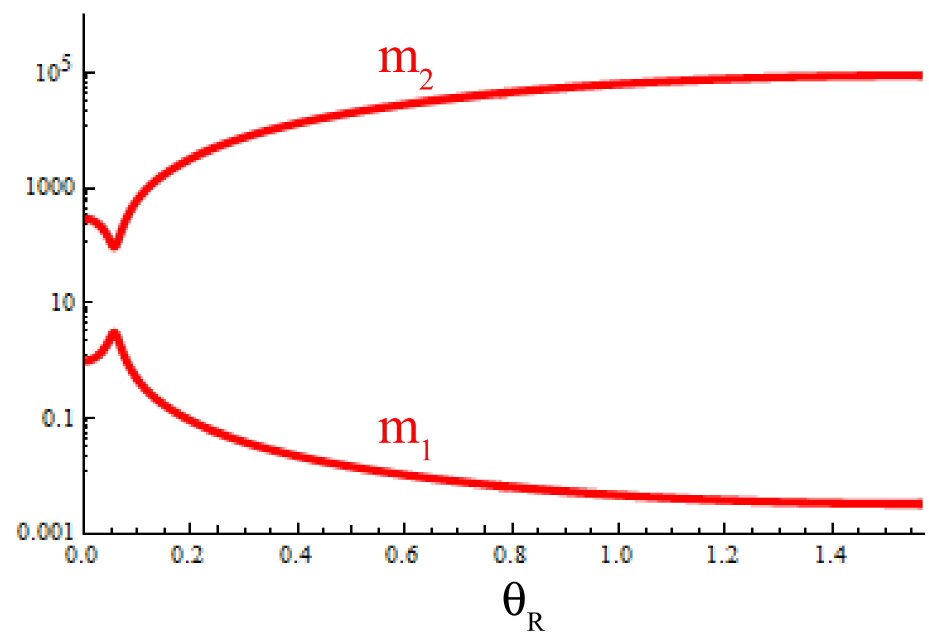

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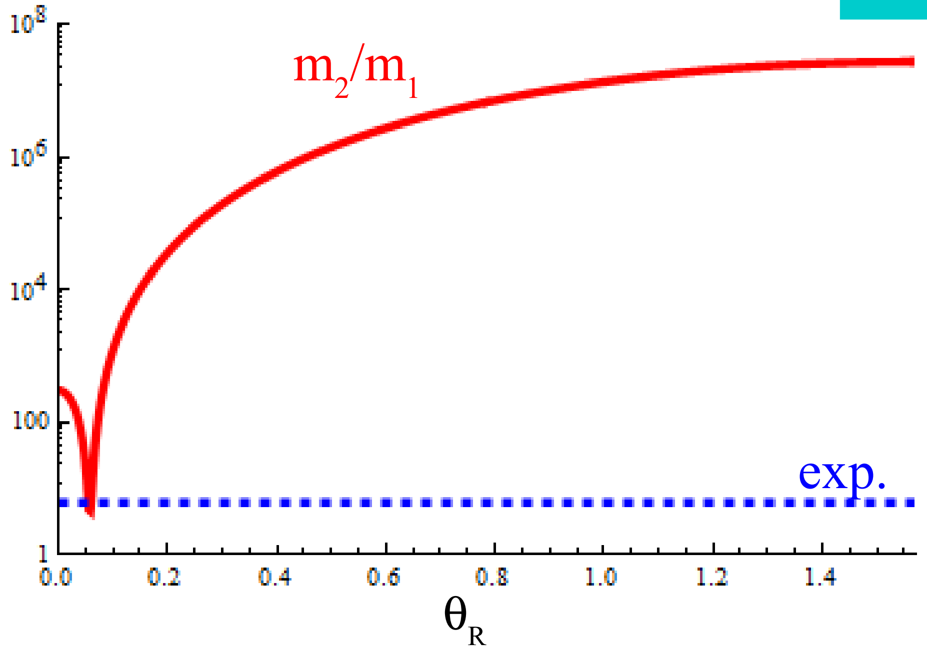

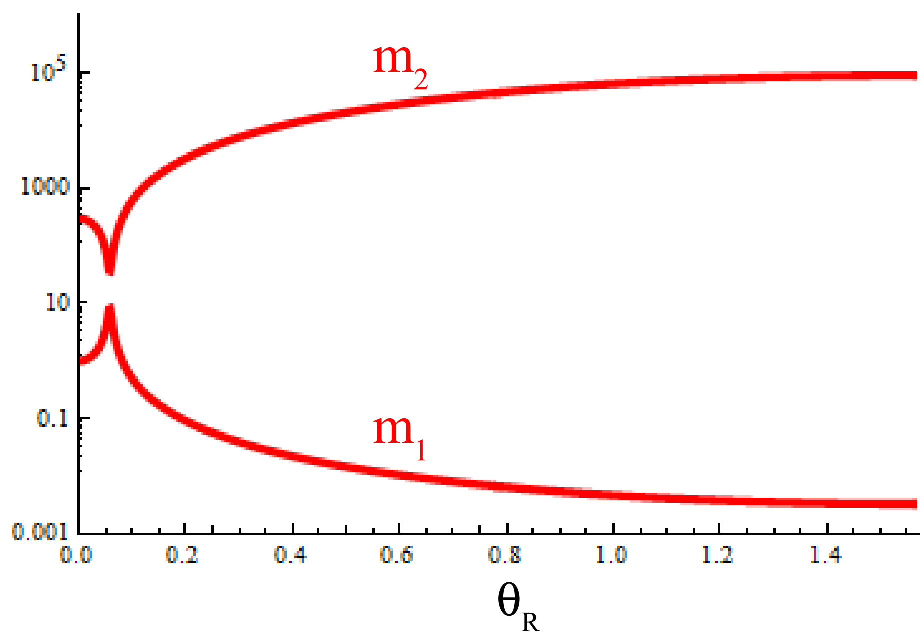

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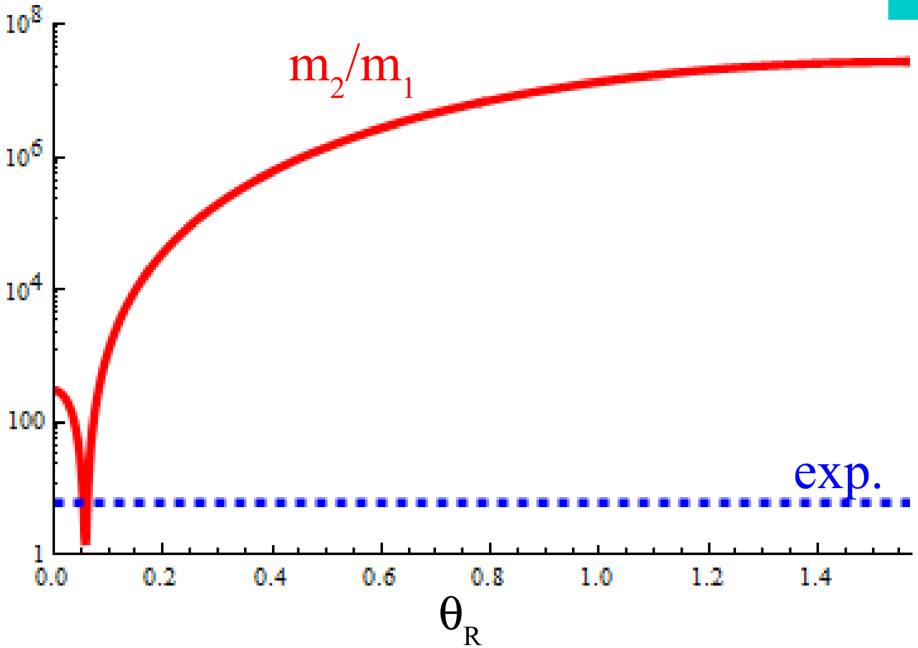

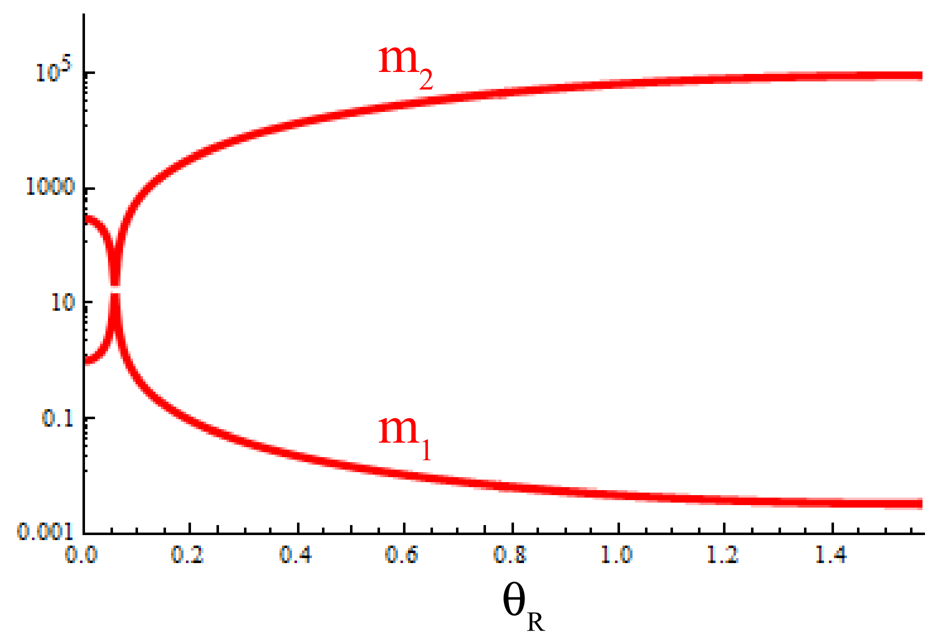

The see-saw mechanism (with two right-handed neutrinos) with hierarchical Yukawa eigenvalues and RH masses can accommodate the observed neutrino mass hierarchy, but *only for very special choices of parameters.*

$$\boxed{\cos^2\theta_R \simeq \frac{M_2 y_2^2 - M_1 y_1^2}{(M_1+M_2)(y_2^2-y_1^2)} \qquad \text{and} \qquad \alpha \simeq \pi/2}$$

$\alpha=\pi/2$

There are two situations where the see-saw mechanism can naturally generate a mild neutrino mass hierarchy (without tunings):

"naive see-saw" $$\frac{m_2}{m_1} \sim \frac{y_2^2}{y_1^2}\frac{M_1}{M_2} \lesssim 6$$

Very mild hierarchies

$y_1 \simeq y_2$

$M_1 \simeq M_2$

$y_1 : y_2 = 1 : 2$

$M_1 : M_2 = 1 : 6$

Very strong hierarchy in RH masses

$M_1 : M_2 \simeq y_1^2 : y_2^2$

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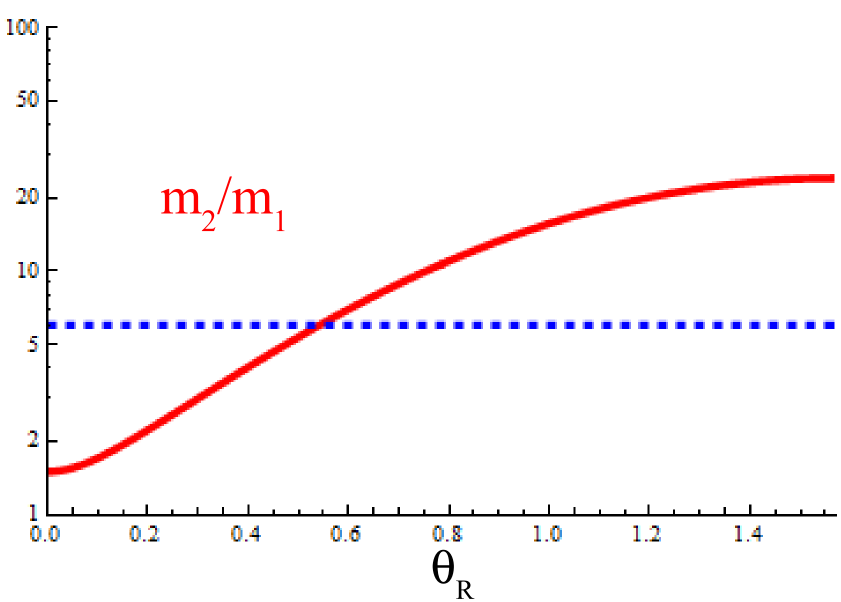
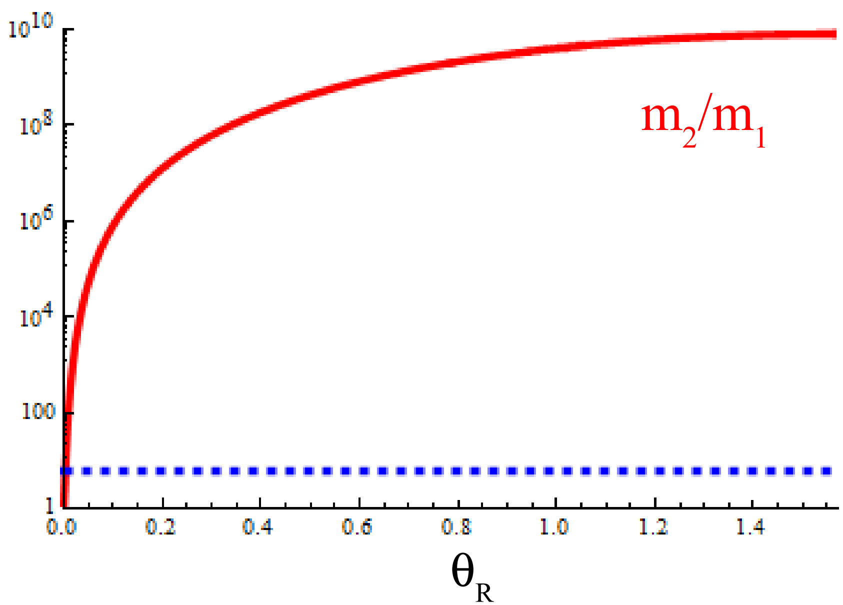


$\alpha=0$



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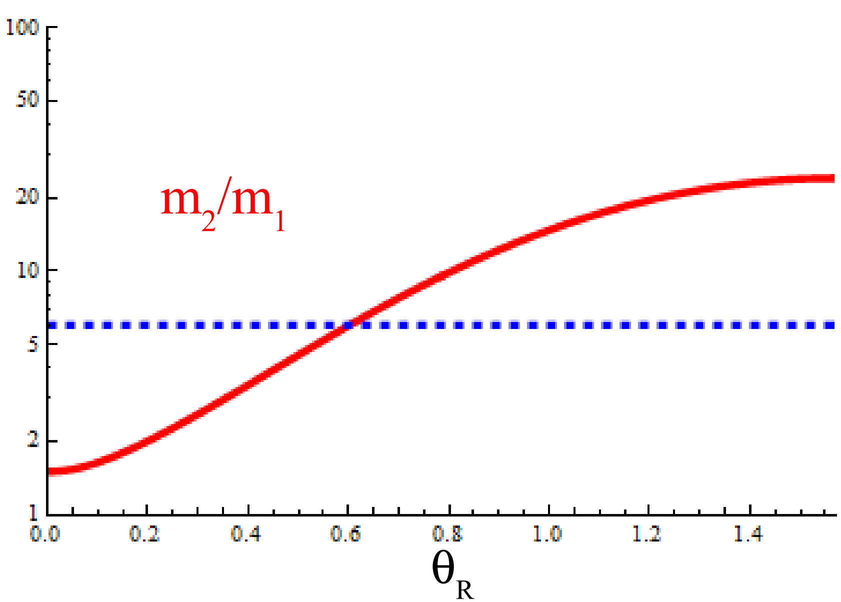

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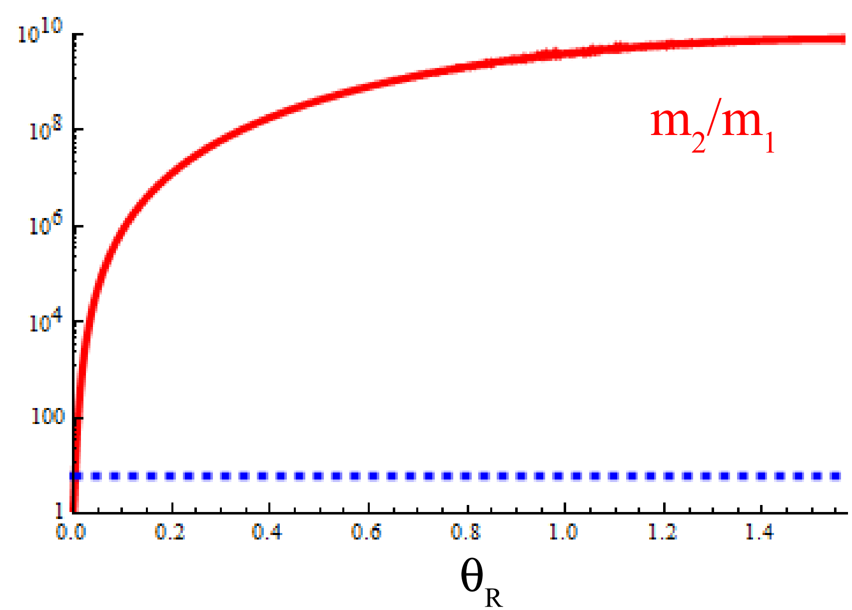

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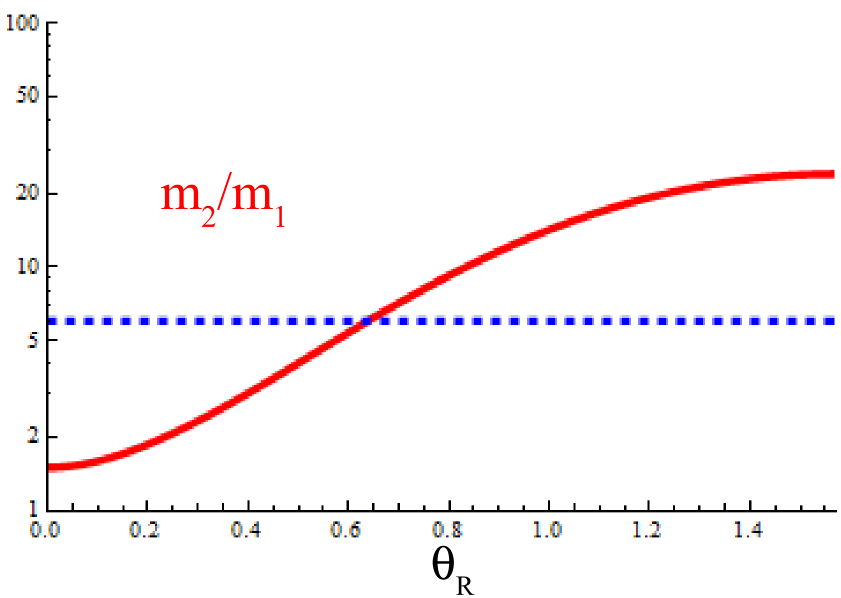

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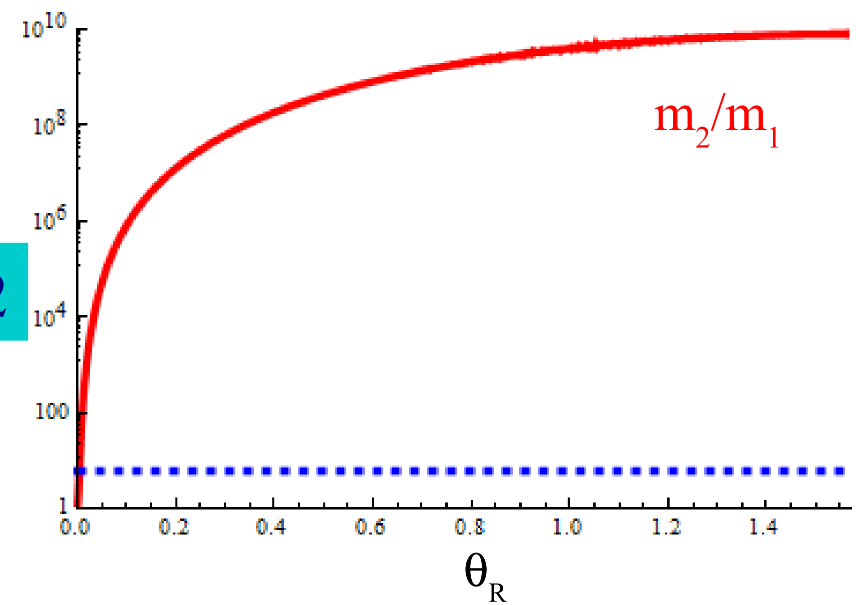

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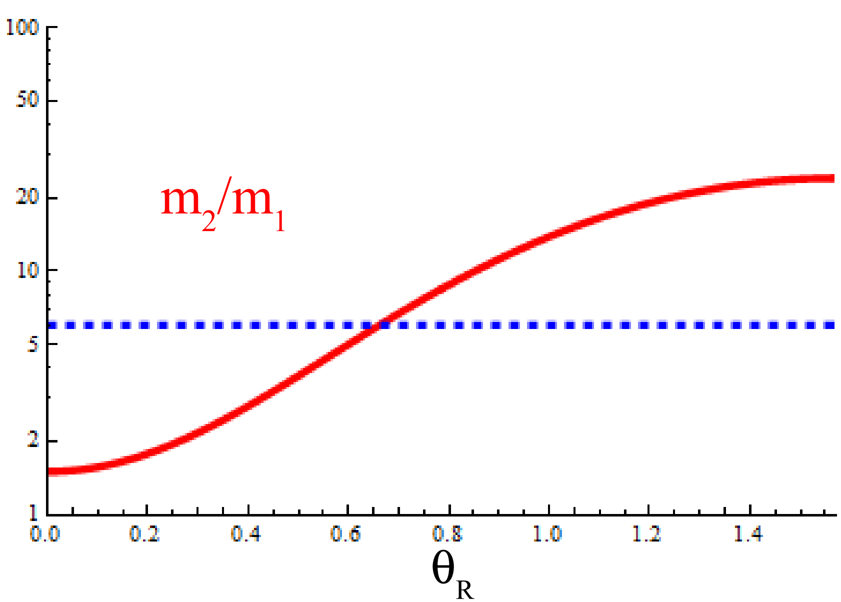

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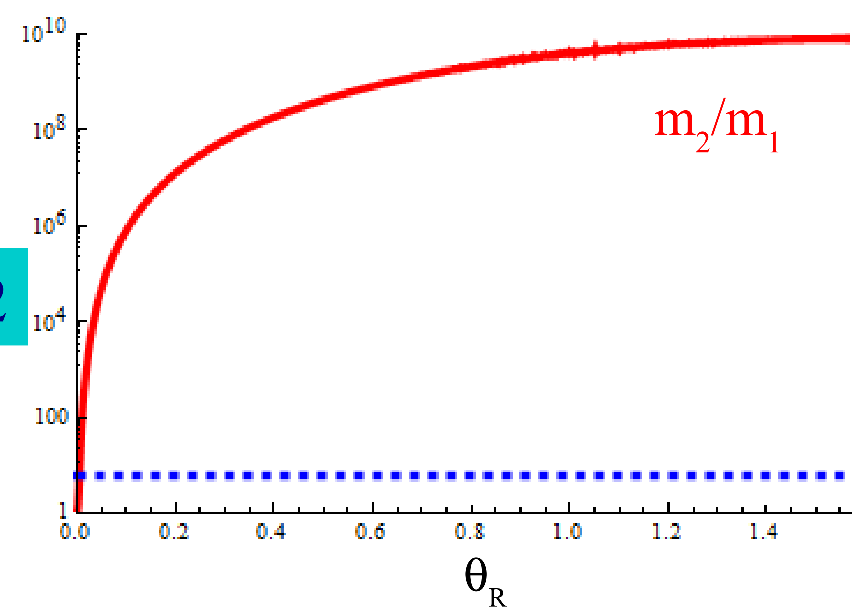

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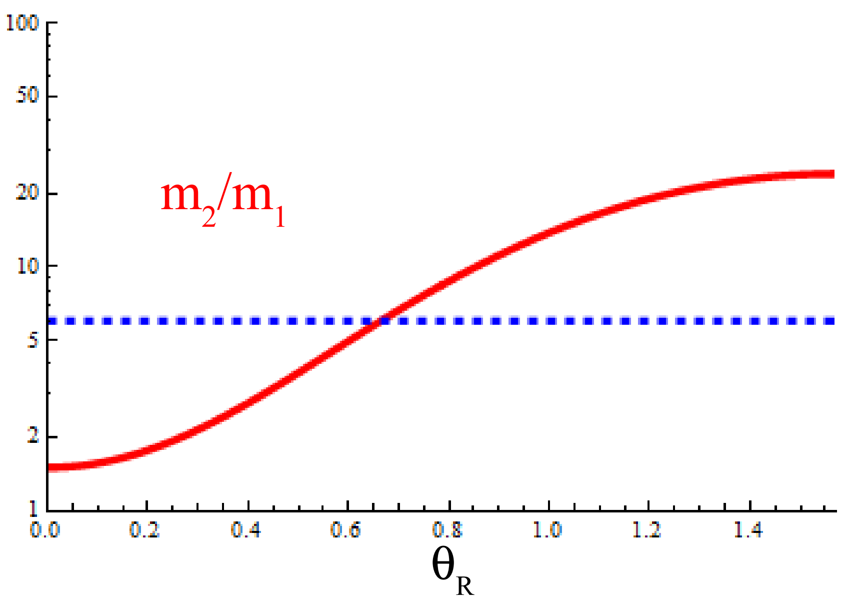

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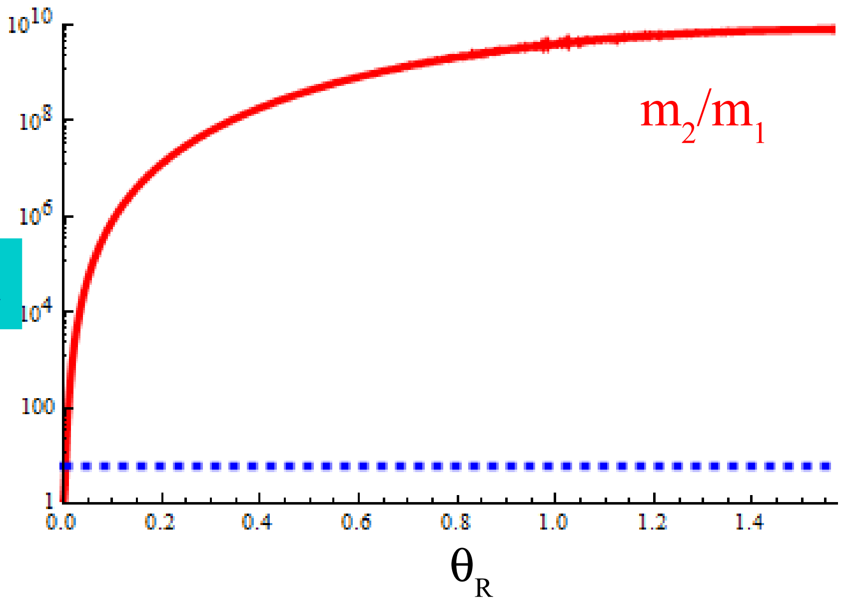

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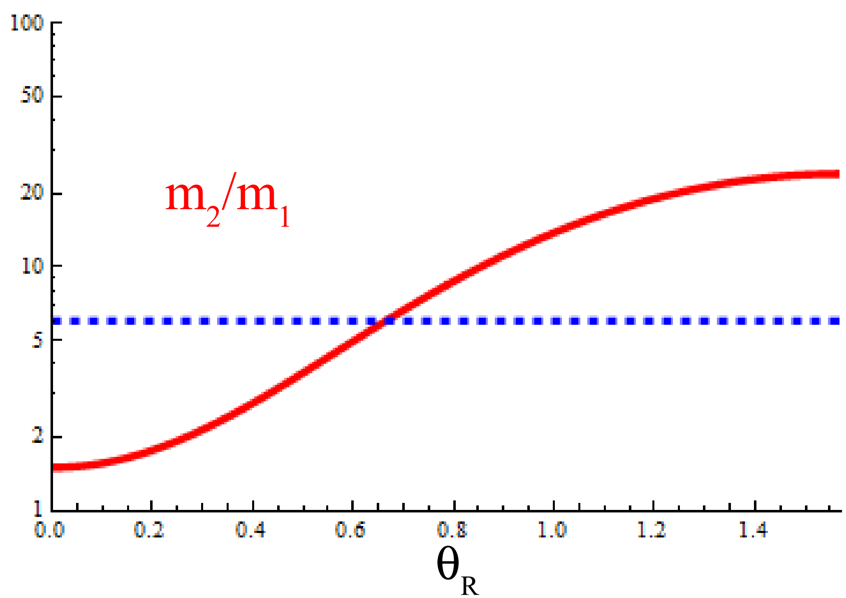

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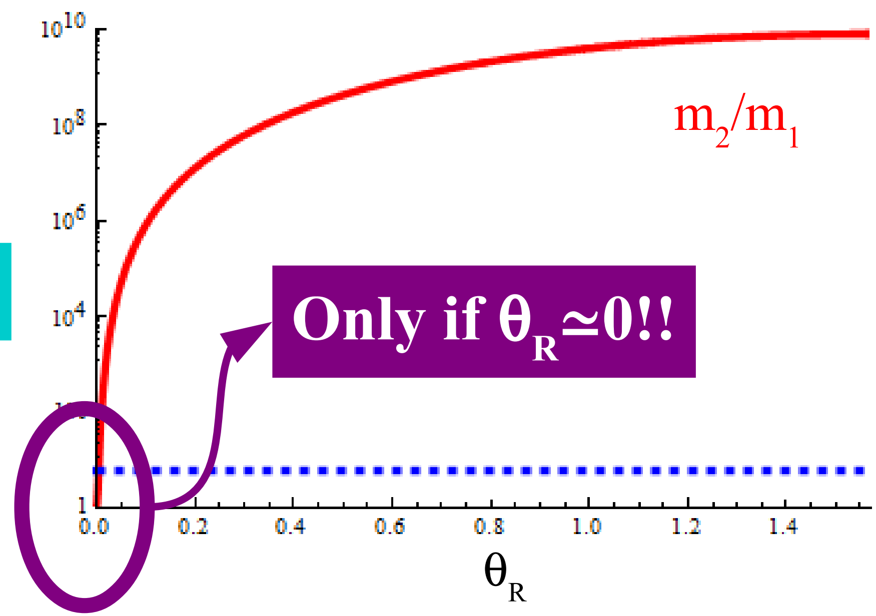

The case with three right-handed neutrinos

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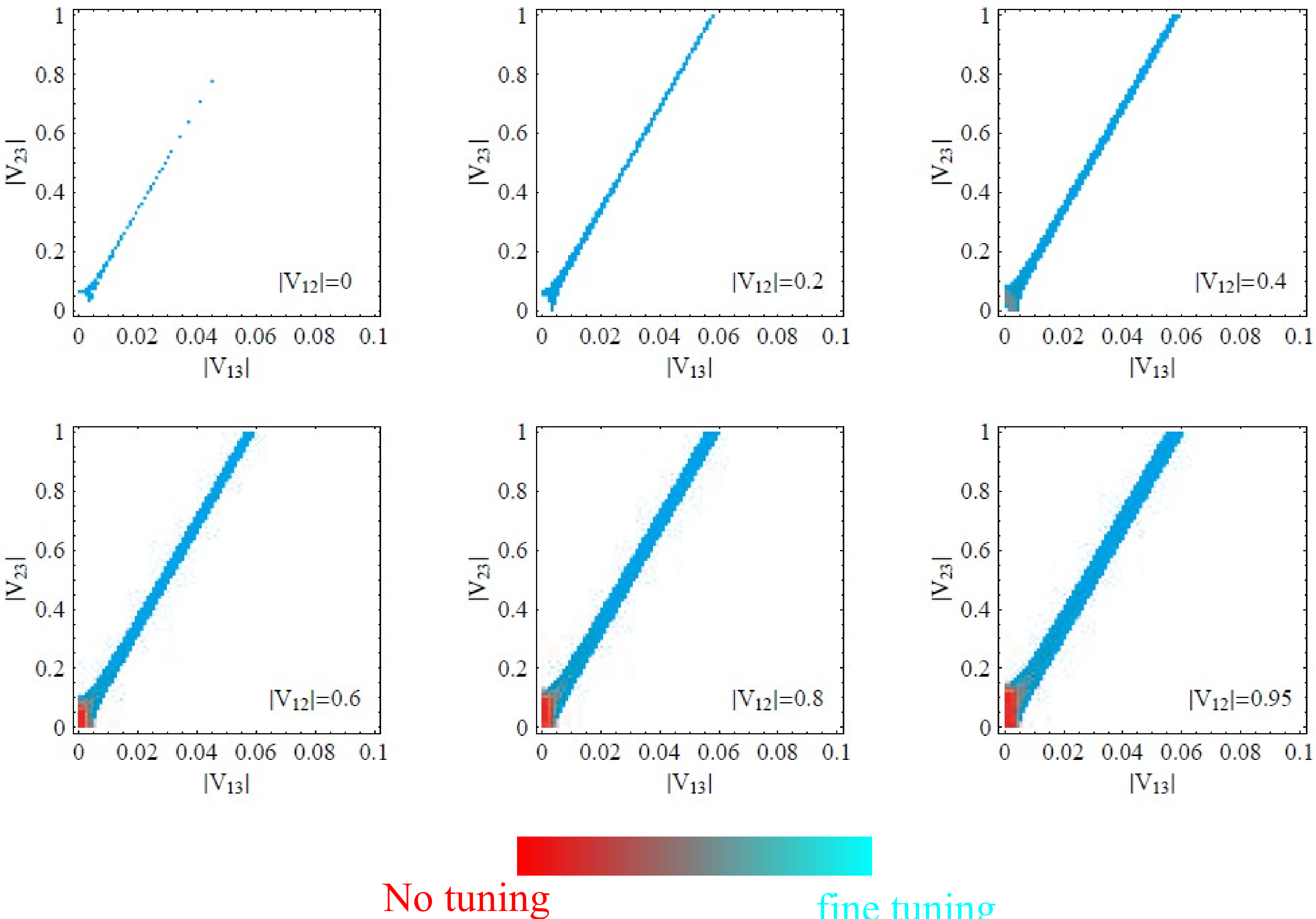
$$M_1 : M_2 : M_3 = 1 : 300 : 300^2$$



The see-saw mechanism generates a neutrino mass hierarchy much larger than the observed experimentally, except:

- When the Yukawa eigenvalues and right-handed masses present a mild mass hierarchy.

- In the case of hierachical Yukawa eigenvalues, for very special choices of the parameters.

The see-saw mechanism provides a very compelling explanation to the smallness of neutrino masses, while keeping all the successes of the Standard Model. However, it fails to provide a compelling explanation to why the neutrino mass hierarchy is so mild.

A possible solution: Introduce a second higgs doublet

With a second higgs doublet, quantum corrections can soften the neutrino mass hierarchy.

Even if at tree level m_3/m_2 is very large, as generically expected in the (standard) see-saw mechanism, the quantum corrections can generate $m_3/m_2{\sim}6$.

Neutrino masses in the see-saw model extended with one extra Higgs

Consider the Standard Model extended by right-handed neutrinos and at least one extra Higgs doublet (no ad-hoc discrete symmetries)

$$-\mathcal{L}^{\nu} = (Y_\nu^a)_{ij} \bar{l}_{Li} \nu_{Rj} \tilde{\Phi}_a - \frac{1}{2} M_{\mathrm{M}ij} \bar{\nu}^C_{Ri} \nu_{Rj} + \mathrm{h.c.}$$

$$\Big\Downarrow \quad M_{\mathrm{Maj}} \gg m_H, M_Z$$

$$-\mathcal{L}^{\nu,\ \mathrm{eff}} = \frac{1}{2} \kappa_{ij}^{ab} (\bar{l}_{Li} \tilde{\Phi}_a)(\tilde{\Phi}_b^T l^C_{Lj}) + \mathrm{h.c.}$$

$$\kappa^{ab}(M_1) = (Y_\nu^a M_{\mathrm{M}}^{-1} Y_\nu^{b\,T})(M_1)$$

Work in the basis where only Φ_1 acquires a vev

$$\mathcal{M}_\nu(M_1) = \frac{v^2}{2} \kappa^{11}(M_1)$$

The neutrino mass matrix is affected by quantum corrections below M_1

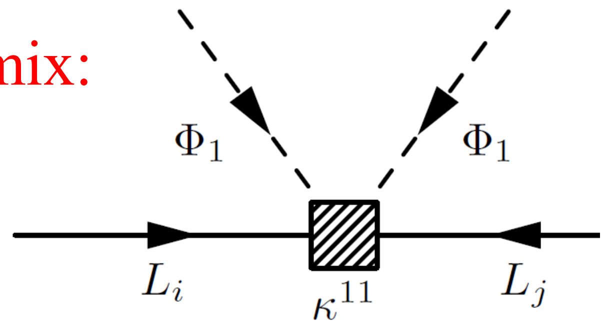
RGE effects

Quantum effects generate a correction to the coefficient of the dimension 5 operator which generates neutrino masses:

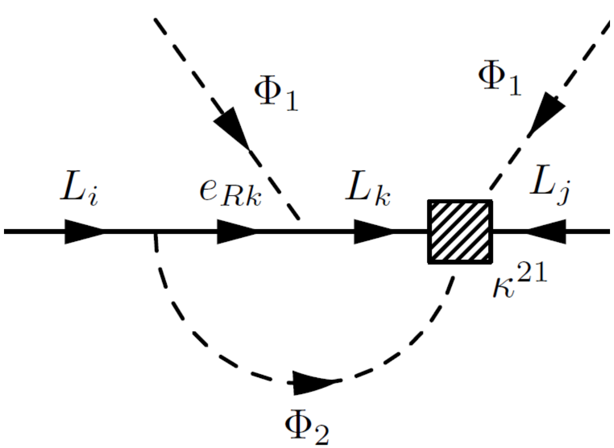
$$\delta\kappa^{11} \simeq B_{1a}\kappa^{a1} + \kappa^{1a}B^T_{1a} + b\kappa^{22}$$

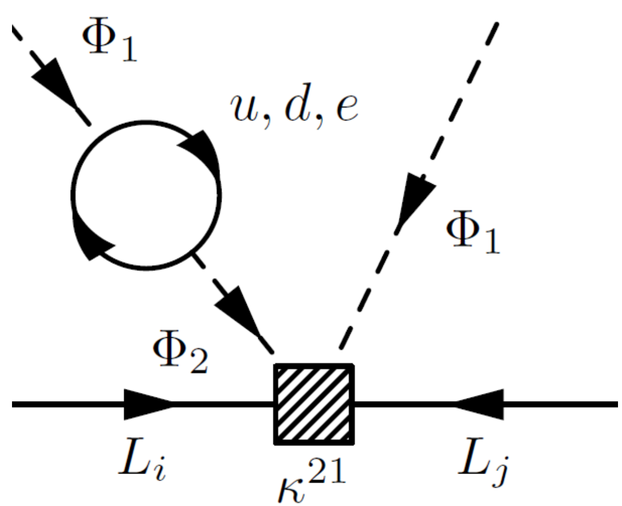
Grimus, Lavoura

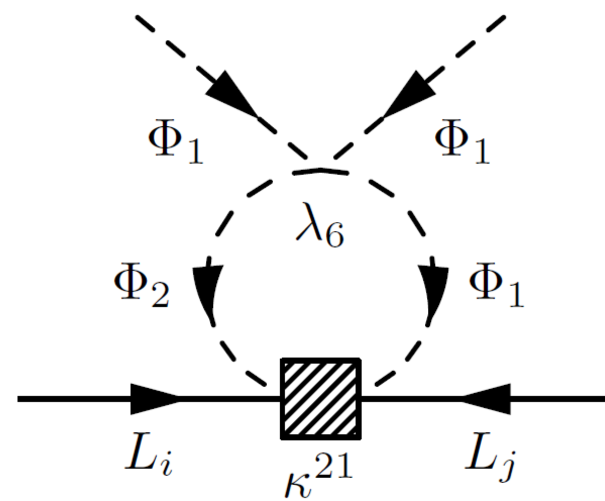
Different operators mix:



Is corrected by $B_{12}\,\kappa^{21}$







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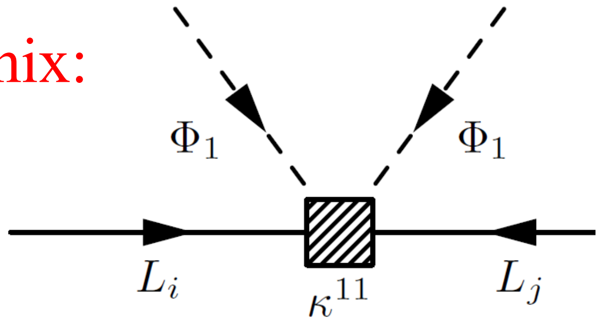
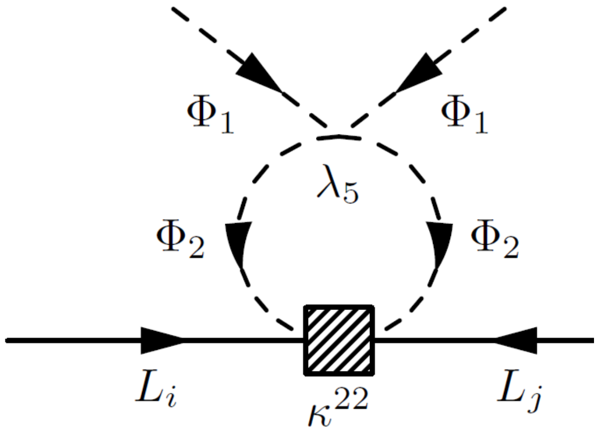


Is corrected by $b\,\kappa^{22}$



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$$\delta\kappa^{11} \simeq B_{1a}\kappa^{a1} + \kappa^{1a}B^T_{1a} + b\kappa^{22}$$

Grimus, Lavoura

Different operators mix.

Compare to the correction in the "one Higgs doublet model":

$$\delta\kappa \simeq B\kappa + \kappa B^T$$

New qualitative features?

To highlight the new features, consider a model with one right-handed neutrino and two Higgs doublets (no ad-hoc discrete symmetries imposed):

$$-\mathcal{L}^{\nu} = (Y_\nu^1)_i \bar{l}_{Li} \nu_R \tilde{\Phi}_1 + (Y_\nu^2)_i \bar{l}_{Li} \nu_R \tilde{\Phi}_2 - \frac{1}{2} M_{\rm Maj} \bar{\nu}_R^C \nu_R + \text{h.c.}$$

$$\Big\Downarrow \quad M_{\rm Maj} \gg m_H, M_Z$$

$$-\mathcal{L}^{\nu,\ \text{eff}} = \frac{1}{2} \kappa_{ij}^{ab} (\bar{l}_{Li} \tilde{\Phi}_a)(\tilde{\Phi}_b^T l_{Lj}^C) + \text{h.c.}$$

Work in the basis where only Φ_1 acquires a vev

$$\mathcal{M}_\nu(M_{\rm Maj}) = \frac{v^2}{2} \kappa^{11}(M_{\rm Maj})$$

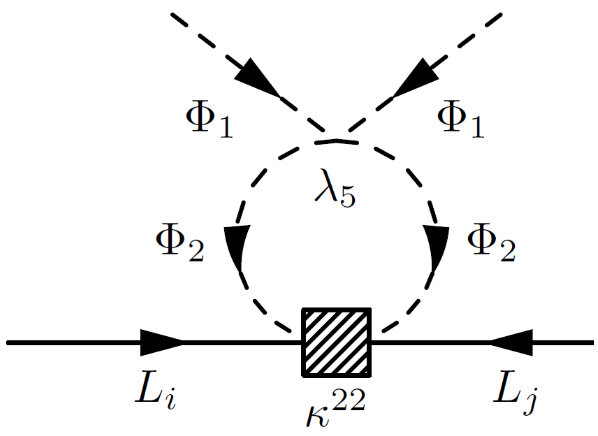
Rank 1. At tree level

$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2 M_{\rm Maj}}$$

$$m_2,\ m_1 = 0$$

RGE effects

$$\delta\kappa^{11} \simeq B_{1a}\kappa^{a1} + \kappa^{1a}B^T_{1a} + b\kappa^{22}$$



$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\rm maj}} + \text{small corrections}$$

$$m_2 = \frac{1}{16\pi^2}\frac{|\lambda_5| v^2}{M_{\rm maj}}\left[|Y_\nu^2|^2 - \frac{|Y_\nu^{2\dagger}Y_\nu^1|^2}{|Y_\nu^1|^2}\right]\log\frac{M_{\rm maj}}{m_H}$$

$$m_1 = 0$$

A second neutrino mass is generated from the same right-handed neutrino mass scale $M_{\rm maj}$ → a mild mass hierarchy might be naturally accommodated.

$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\rm maj}}$$

$$m_2 = \frac{1}{16\pi^2}\frac{|\lambda_5| v^2}{M_{\rm maj}}\left[|Y_\nu^2|^2 - \frac{|Y_\nu^{2\dagger}Y_\nu^1|^2}{|Y_\nu^1|^2}\right]\log\frac{M_{\rm maj}}{m_H}$$

Neutrino mass hierarchy:

Assume:

- M_{maj} large, to implement the see-saw mechanism $m_H << M_{maj}$ (e.g m_H=100 GeV-1 TeV)
- Neutrino Yukawa couplings misaligned (new sources of flavour violation are required)
- $|Y_\nu^1| \sim |Y_\nu^2|$
- $\lambda_5 \sim O(1)$

$$\boxed{\left|\frac{m_2}{m_3}\right| \simeq \frac{|\lambda_5|}{8\pi^2}\frac{|Y_\nu^2|^2}{|Y_\nu^1|^2}\log\left(\frac{M_{\rm maj}}{m_H}\right) \simeq 0.2}$$

$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\rm maj}}$$

$$m_2 = -\frac{1}{16\pi^2}\frac{|\lambda_5| v^2}{M_{\rm maj}}\left[|Y_\nu^2|^2 - \frac{|Y_\nu^{2\dagger} Y_\nu^1|^2}{|Y_\nu^1|^2}\right]\log\left(\frac{M_{\rm maj}}{m_H}\right)$$

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- Neutrino Yukawa couplings misaligned (new sources of flavour violation are required)

$$|Y_\nu^1| \sim |Y_\nu^2|$$

Yukawa couplings to the same generation of right-handed neutrinos (more details later)

- $\lambda_5 \sim \mathcal{O}(1)$

$$\boxed{\left|\frac{m_2}{m_3}\right| \simeq \frac{|\lambda_5|}{8\pi^2}\frac{|Y_\nu^2|^2}{|Y_\nu^1|^2}\log\left(\frac{M_{\rm maj}}{m_H}\right) \simeq 0.2}$$

$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\rm maj}}$$

$$m_2 = -\frac{1}{\ldots}\frac{|\lambda_5| v^2}{\ldots}\left[\ldots \frac{|Y_\nu^{2\dagger}Y_\nu^1|^2}{\ldots}\right] \ldots \left(\frac{M_{\rm maj}}{m_H}\right)$$

LEPTON FLAVOUR VIOLATION?
ELECTRIC DIPOLE MOMENTS?

Neutrino mass hierarchy:

Assume:

- M_{maj} large, to implement the see-saw mechanism $m_H << M_{maj}$ (e.g m_H=100 GeV-1 TeV)
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$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\rm maj}}$$

LEPTON FLAVOUR VIOLATION?
ELECTRIC DIPOLE MOMENTS?

Neutrino mass hierarchy:

Assume:

- M_{maj} large, to implement the see-saw mechanism $m_H \ll M_{maj}$ (e.g m_H=100 GeV-1 TeV)
- Neutrino Yukawa couplings misaligned (new sources of flavour violation are required)
- $|Y_\nu^1| \sim |Y_\nu^2|$
- $\lambda_5 \sim O(1)$

Logarithmic dependence with m_H, while the rate for $\mu \to e\gamma$ decreases as m_H^4

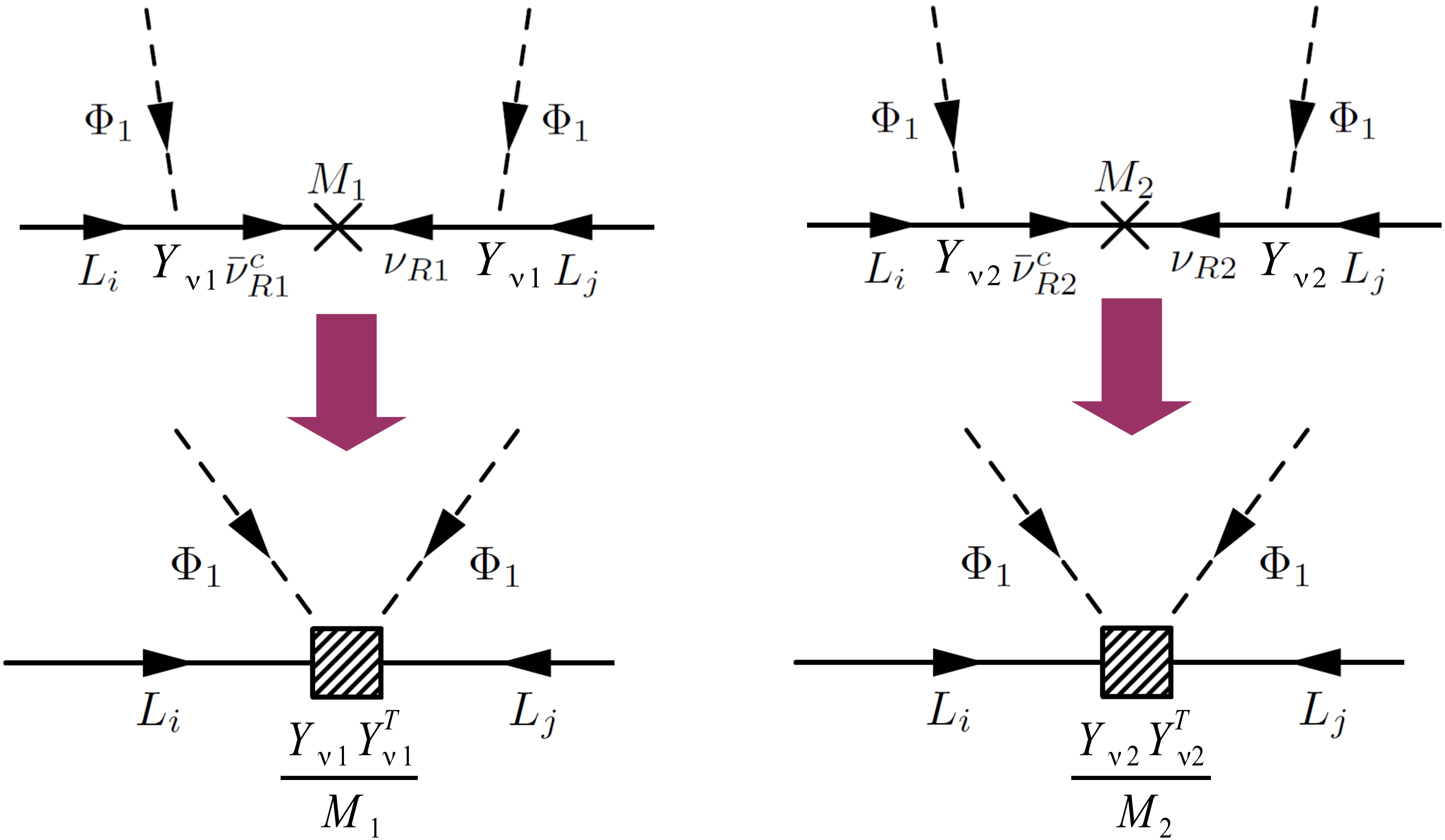
$$\boxed{\left|\frac{m_2}{m_3}\right| \simeq \frac{|\lambda_5|}{8\pi^2}\frac{|Y_\nu^2|^2}{|Y_\nu^1|^2}\log\left(\frac{M_{\rm maj}}{m_H}\right) \simeq 0.2}$$

Message to take home:

The Standard Model extended with ≥1 right-handed neutrino and ≥1 Higgs doublet can naturally explain the smallness of neutrino masses and the existence of a mild mass hierarchy, without jeopardizing any of the successes of the Standard Model, since all extra particles decouple at low energies.

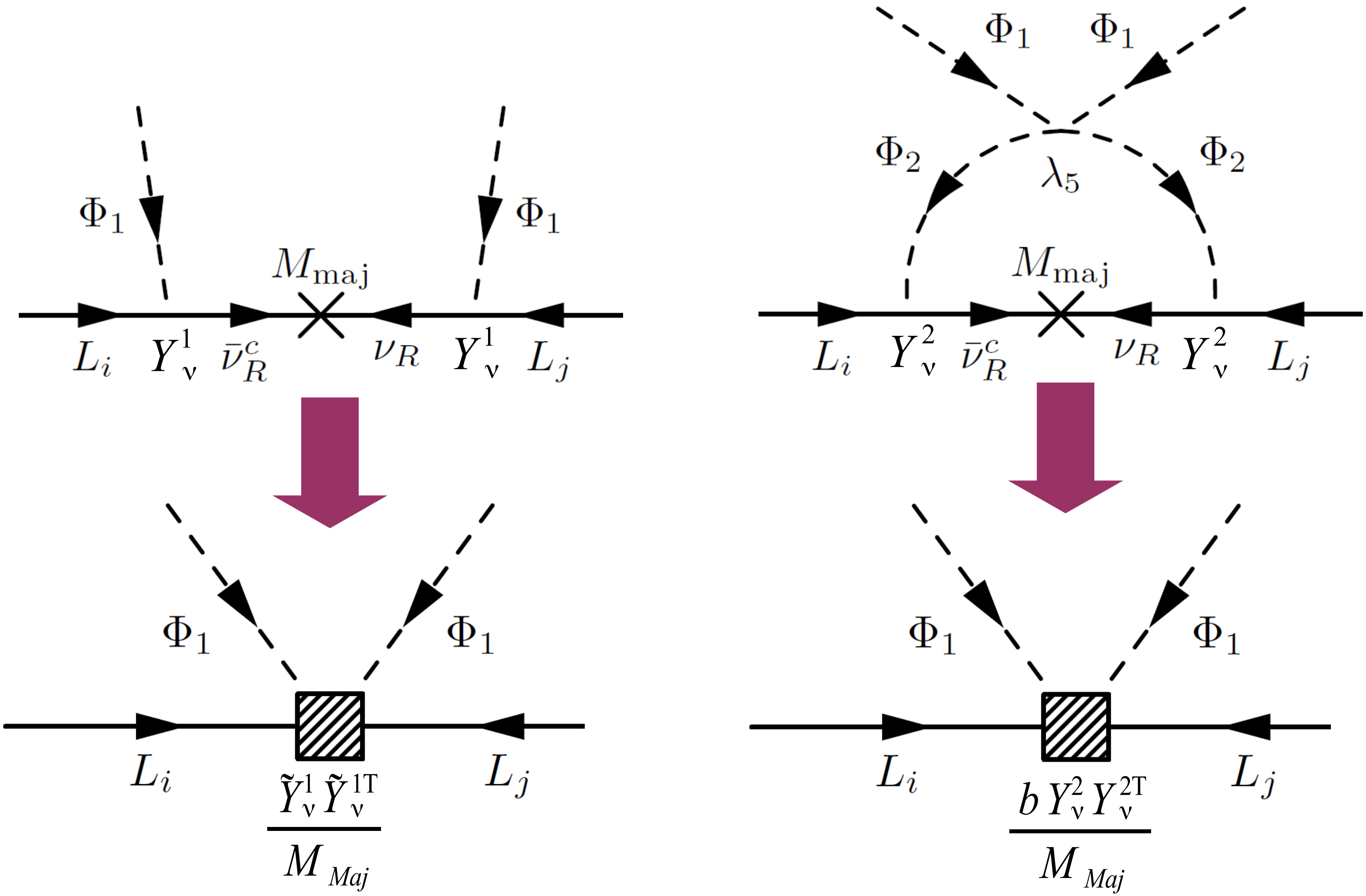
Comparison to the two right-handed neutrino model

Effective theory of the 2RHN-1HDM



Comparison to the two right-handed neutrino model

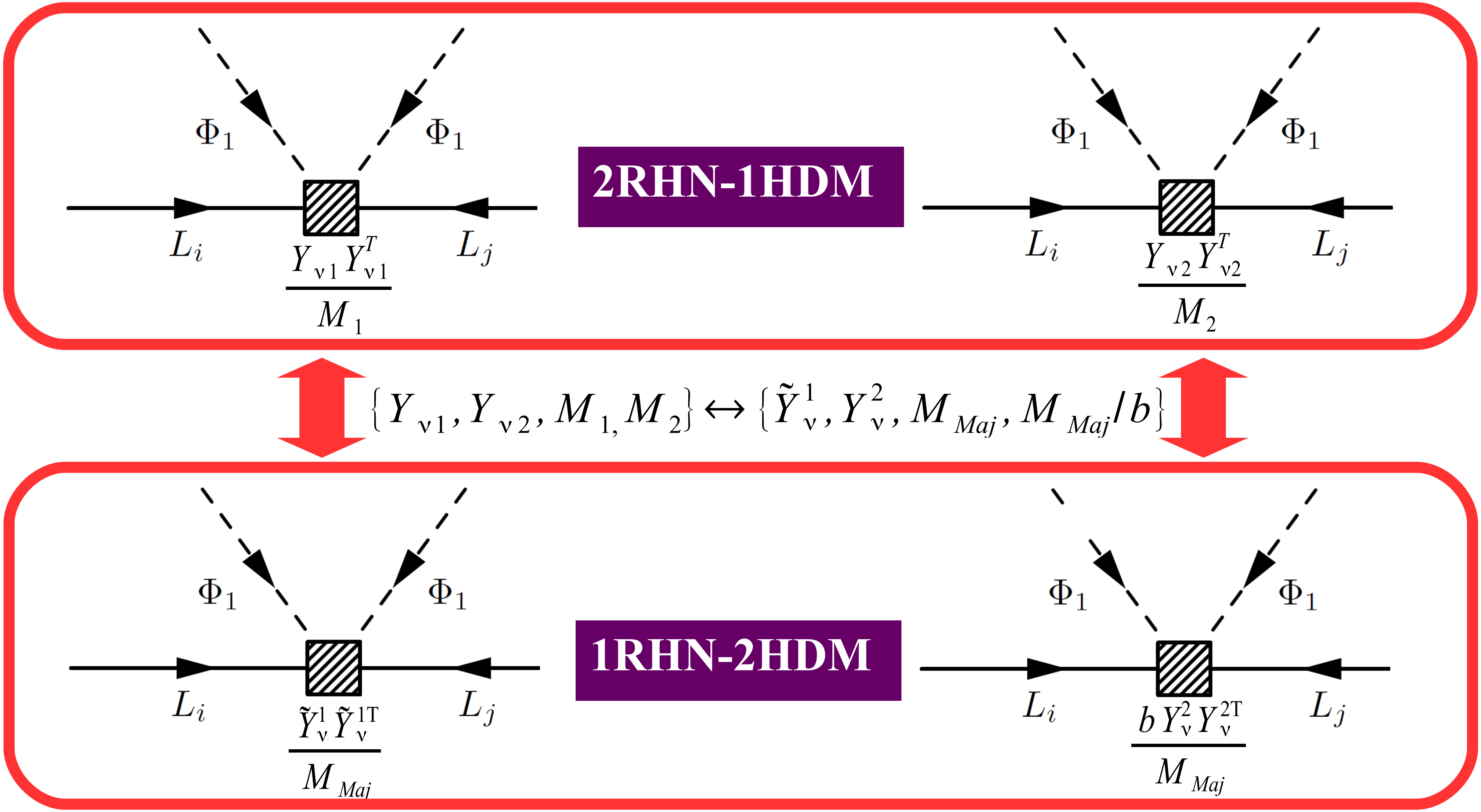
Effective theory of the 1RHN-2HDM



$$\tilde{Y}^1_\nu = Y^1_\nu + B_{1a} Y^a_\nu$$

The effective theories are identical

The effective theories are identical



However, there are important differences in the way the can generate the mild neutrino mass hierarchy.

First part of the talk: the 2RHN-1HDM can generate a neutrino mass hierarchy in agreement with experiments when:

- When the Yukawa eigenvalues *and* right-handed masses present a mild hierarchy.

- When there are hierachical Yukawa eigenvalues, only for very special choices of the parameters.

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2RHN-1HDM

$$\{Y_{\nu 1}, Y_{\nu 2}, M_{1,} M_{2}\}$$

Yukawa couplings to *different* generation of RH neutrinos.
Possibly hierarchical eigenvalues

Two different right-handed neutrino masses.
Hierarchy completely unknown

First part of the talk: the 2RHN-1HDM can generate a neutrino mass hierarchy in agreement with experiments when:

- When the Yukawa eigenvalues *and* right-handed masses present a mild hierarchy. implausible

- When there are hierachical Yukawa eigenvalues, only for very special choices of the parameters. tuned?

2RHN-1HDM

$$\{\underbrace{Y_{\nu 1}, Y_{\nu 2}}, \underbrace{M_{1}, M_{2}}\}$$

Yukawa couplings to *different* generation of RH neutrinos.
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2RHN-1HDM 1RHN-2HDM

$$\{Y_{\nu 1}, Y_{\nu 2}, M_{1,} M_{2}\} \leftrightarrow \{\underbrace{\tilde{Y}^{1}_{\nu}, Y^{2}_{\nu}}, \underbrace{M_{Maj}, M_{Maj}/b}\}$$

$$b = -\frac{\lambda_5}{8\pi^2}\log\left(\frac{M_{\rm Maj}}{m_H}\right)$$

Yukawa couplings to *the same* generation of RH neutrinos.
Eigenvalues expected to be similar

The "equivalent" right-handed neutrino masses naturally have a mild hierarchy.

First part of the talk: the 2RHN-1HDM can generate a neutrino mass hierarchy in agreement with experiments when:

- When the Yukawa eigenvalues *and* right-handed masses present a mild hierarchy.

Generic situation in 1RHN-2HDM

- When there are hierachical Yukawa eigenvalues, only for very special choices of the parameters.

2RHN-1HDM 1RHN-2HDM

$$\{Y_{\nu 1}, Y_{\nu 2}, M_1, M_2\} \leftrightarrow \{\underbrace{\tilde{Y}^1_\nu, Y^2_\nu}, \underbrace{M_{Maj}, M_{Maj}/b}\}$$

$$b = -\frac{\lambda_5}{8\pi^2}\log\left(\frac{M_{\rm Maj}}{m_H}\right)$$

Yukawa couplings to *the same* generation of RH neutrinos. Eigenvalues expected to be similar

The "equivalent" right-handed neutrino masses naturally have a mild hierarchy.

A remarkable difference with respect to the two right-handed neutrino model:

Possibly, new phenomena at low energies, apart from neutrino masses

- Extra scalars at the LHC, if they are not too heavy.
- LFV processes could be observable, if not too suppressed by m_H.

$$\mathrm{BR}(\mu \to e\,\gamma) = \frac{8\alpha^3}{3\pi^3}\frac{|Y^2_{e12}|^2 + |Y^2_{e21}|^2}{|Y^1_{e22}|^2}\left| f\!\left(\frac{m_t^2}{m_h^2}\right)\cos\alpha - \frac{Y^2_{u33}}{Y^1_{u33}}\frac{m_t^2}{m_H^2}\log^2\frac{m_t^2}{m_H^2}\right|^2$$

Paradisi
Hisano, Sugiyama, Yamanaka

Could be present at tree level.
If not, generated radiatively by the neutrino Yukawa couplings

A more realistic model: 3RHN+2HDM

If the neutrino Yukawa couplings have hierarchical eigenvalues, at tree level one generically expects:

$$m_3^{(0)} \gg m_2^{(0)} \gg m_1^{(0)}$$

The radiative corrections induce

$$\delta m_2 \sim m_3^{(0)} \times \mathcal{O}(0.1) \gg m_2^{(0)}$$

In the 3RHN+2HDM one naturally obtains:

$$\boxed{\begin{array}{l} \dfrac{m_2}{m_3} \sim 0.2 \\ \dfrac{m_1}{m_2} \ll 1 \end{array}}$$

Mixing angles

New flavour structures in κ^{22} and Y_e^{2} modify, through quantum corrections, the flavour structure of the neutrino mass operator κ^{11} and the charged lepton Yukawa coupling Y_e^{1}.

Leptonic mixing matrix

$$\delta U_\kappa = U^{(0)} T$$

Charged lepton Yukawa coupling

$$l_L \to V_e^L l_L$$

$$\delta U_{Y_e} = (V_e^L - \mathbb{1})^T U^{(0)}$$

Summing up both contributions

$$\boxed{U^{(1)} = V_e^{L\,T} U^{(0)} + U^{(0)} T}$$

Mixing angles: effect on θ_{13} and θ_{23}

New flavour structures in κ^{22} and $Y_e^{\,2}$ can induce radiatively a non-vanishing θ_{13} and a deviation from maximal atmospheric mixing.

$$\delta U_{13} = -\frac{1}{16\pi^2}\frac{Y^{2*}_{\nu 1}}{|Y^1_\nu|}\left[3\mathrm{Tr}(Y_u^{1\dagger}Y_u^2 + Y_d^1 Y_d^{2\dagger}) + 2\lambda_6^* + 2\lambda_5^*\frac{Y_\nu^{2\dagger}Y_\nu^1}{|Y_\nu^1|^2}\right]\log\frac{M_{\mathrm{maj}}}{m_H}$$
$$+\frac{1}{16\pi^2}\frac{(Y_\nu^{1\dagger}(Y_e^1)^{-1}Y_e^{2\dagger})_1}{|Y_\nu^1|}\left[3\mathrm{Tr}(Y_u^{2\dagger}Y_u^1 + Y_d^2 Y_d^{1\dagger})\right]\log\frac{M_{\mathrm{maj}}}{m_H}$$

Similar to m_2/m_3 → δU_{13} can easily be ~ 0.2

Mixing angles: effect on θ$_{13}$ and θ$_{23}$

New flavour structures in κ^{22} and $Y_e^{\,2}$ can induce radiatively a non-vanishing θ_{13} and a deviation from maximal atmospheric mixing.

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Similar to m_2/m_3 → δU_{13} can easily be ~ 0.2

Mixing angles: effect on θ_{13} and θ_{23}

New flavour structures in κ^{22} and Y_e^2 can induce radiatively a non-vanishing θ_{13} and a deviation from maximal atmospheric mixing.

Additional effects if the cut-off Λ is larger than M_{maj}, through the quantum effects from the neutrino Yukawa couplings Y_ν^1, Y_ν^2.

$$\delta U_{13} = -\frac{1}{16\pi^2}\frac{Y_{\nu 1}^{2*}}{|Y_\nu^1|}\Big\{ \Big[3\mathrm{Tr}(Y_u^{1\dagger}Y_u^2 + Y_d^1 Y_d^{2\dagger}) + \mathrm{Tr}(Y_\nu^2 Y_\nu^{1\dagger}) + 2Y_\nu^{1\dagger}(Y_e^1)^{-1}Y_e^{2\dagger}Y_\nu^1\Big]\log\frac{\Lambda}{M_{\mathrm{maj}}}$$
$$+\Big[3\mathrm{Tr}(Y_u^{1\dagger}Y_u^2 + Y_d^1 Y_d^{2\dagger}) + 2\lambda_6^* + 2\lambda_5^*\frac{Y_\nu^{2\dagger}Y_\nu^1}{|Y_\nu^1|^2}\Big]\log\frac{M_{\mathrm{maj}}}{m_H}\Big\}$$
$$+\frac{1}{16\pi^2}\frac{(Y_\nu^{1\dagger}(Y_e^1)^{-1}Y_e^{2\dagger})_1}{|Y_\nu^1|}\Big\{\mathrm{Tr}(Y_\nu^{2\dagger}Y_\nu^1)\log\frac{\Lambda}{M_{\mathrm{maj}}} + 3\mathrm{Tr}(Y_u^{2\dagger}Y_u^1 + Y_d^2 Y_d^{1\dagger})\log\frac{\Lambda}{m_H}\Big\}$$

Mixing angles: effect on θ_{13} and θ_{23}

Under well motivated assumptions, there is a correlation between the radiatively generated θ_{13} and the radiatively generated deviation from maximal atmospheric mixing.

Neglect the effects of the charged lepton Yukawa couplings. Then,

$$U^{(1)} = V_e^{L\,T} U^{(0)} + U^{(0)} T$$

$V_e^L \approx 1$

$$U_{i3} = (1+\epsilon_3) U_{i3}^{(0)} + \epsilon_2 U_{i2}^{(0)}$$

$U_{13}^{(0)} \simeq 0, \; U_{23}^{(0)} \simeq 1/\sqrt{2}$

$$\frac{U_{23}}{U_{33}} - 1 \simeq 2\sqrt{2} U_{13}$$

Mixing angles: effect on θ$_{13}$ and θ$_{23}$

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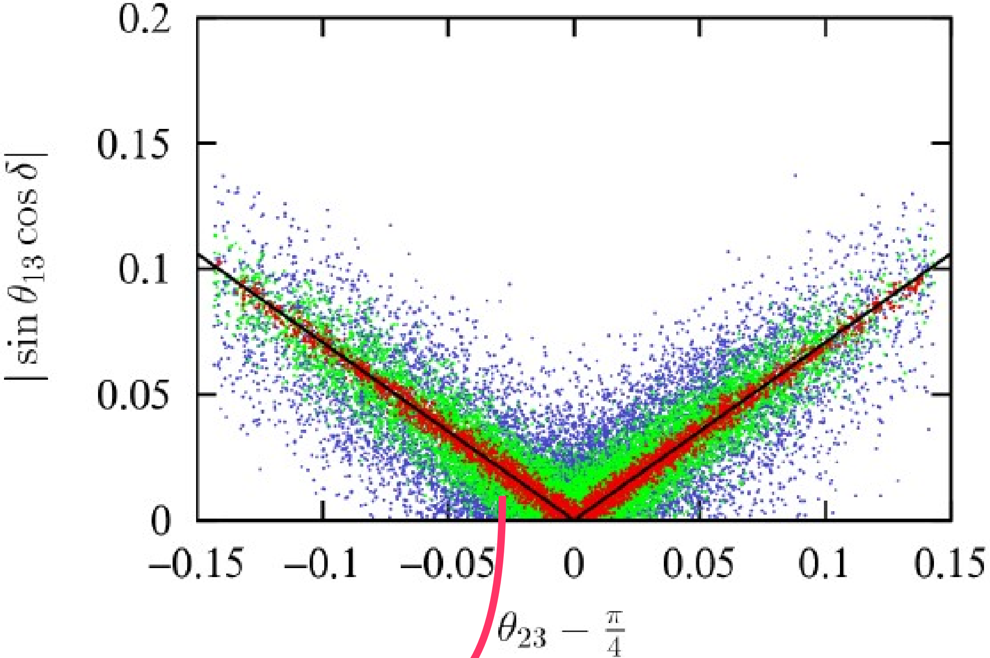
$U_{13}^{(0)} \simeq 0,\ U_{23}^{(0)} \simeq 1/\sqrt{2}$

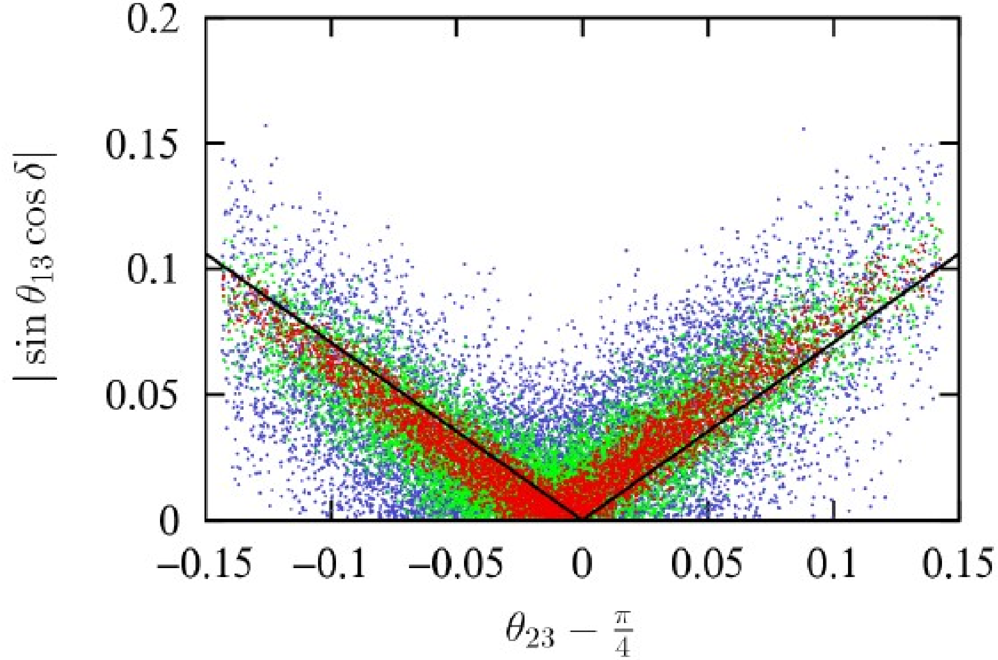
$$\boxed{\left|\theta_{23} - \frac{\pi}{4}\right| \simeq \sqrt{2}\,|\sin\theta_{13}\cos\delta|}$$

Mixing angles: effect on θ_{13} and θ_{23}

$\Lambda = M_{maj}$



$\Lambda = 10^{18}$ GeV



$$\theta_{23} - \frac{\pi}{4} \simeq \sqrt{2}\sin\theta_{13}\cos\delta$$

Some speculations about the mixing angles

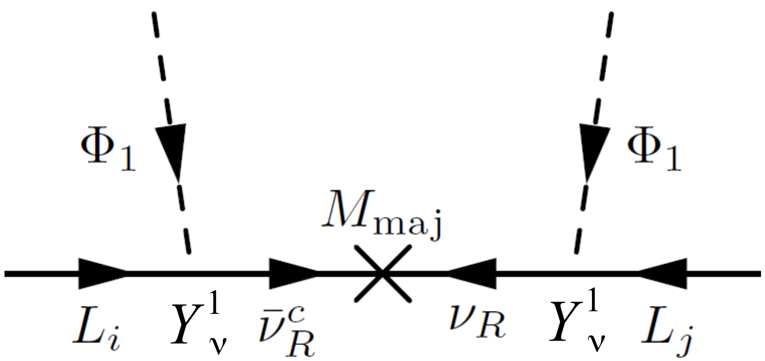
The third column of the leptonic mixing matrix seems to follow a pattern, at least at lowest order: $\theta_{13}=0$, $\theta_{23}=\pi/4$.

$$U_{i3} \approx \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

The second column does not seem to follow any pattern: the solar mixing angle is neither minimal nor maximal.

$$U_{i2} \approx \begin{pmatrix} O(1) \\ O(1) \\ O(1) \end{pmatrix}$$

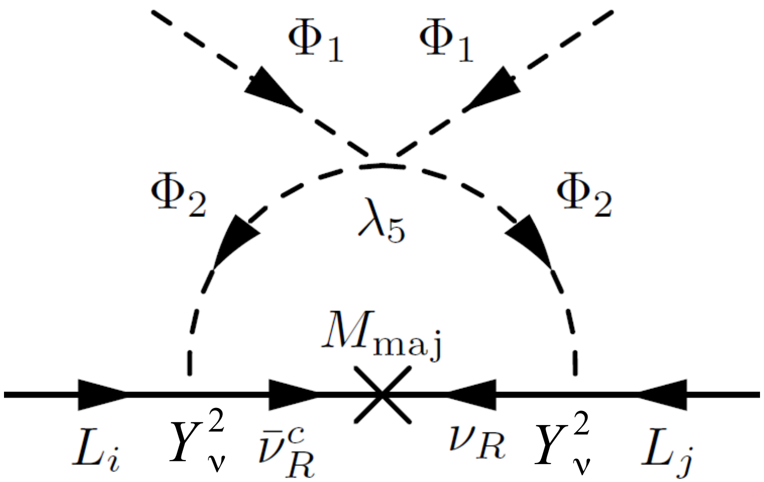
In the 1RHN-2HDM



Third column of U_{lep}

Possibly with a pattern

(if Y_ν^1 has a pattern)



second column of U_{lep}

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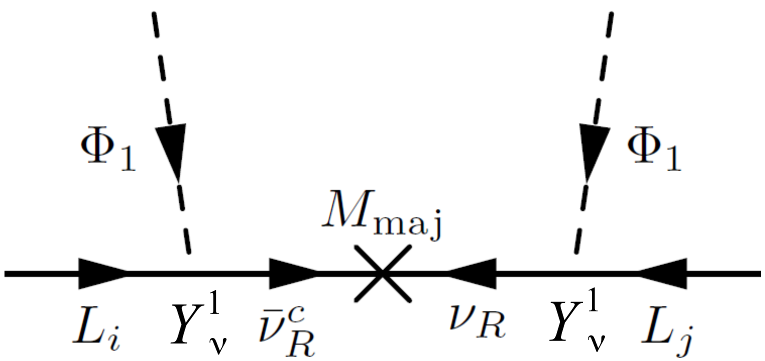
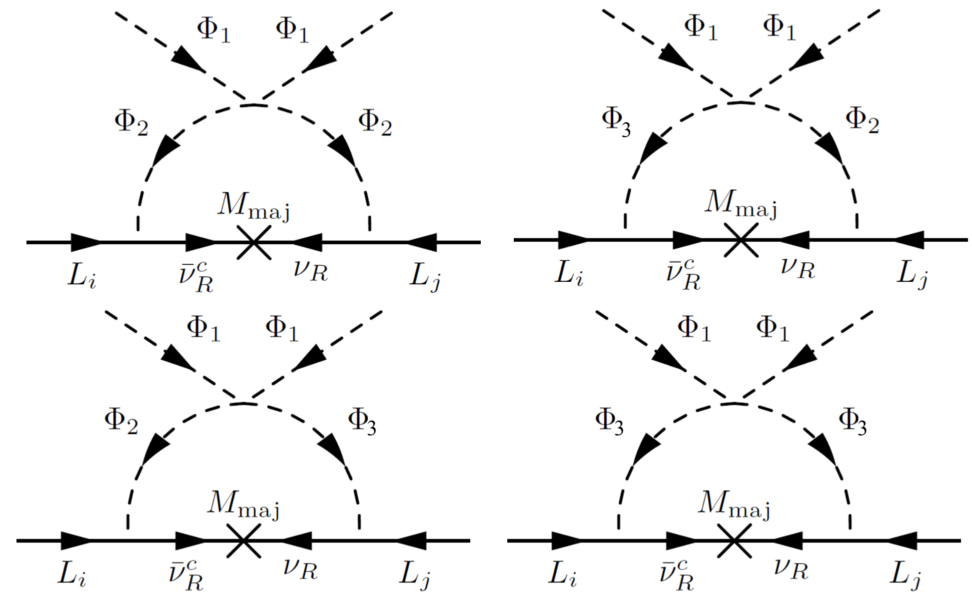
In the 1RHN-3HDM,
(more higgs doublets!)



Third column of U_{lep}
Possibly with a pattern
(if Y_ν^1 has a pattern)



second column of U_{lep}
Even if each Yukawa coupling had an structure, the combination of them gives a “structureless” U_{i2}.

Some speculations about the mixing angles

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In the 1RHN-3HDM,
(more higgs doublets!)

Third column of U_{lep}
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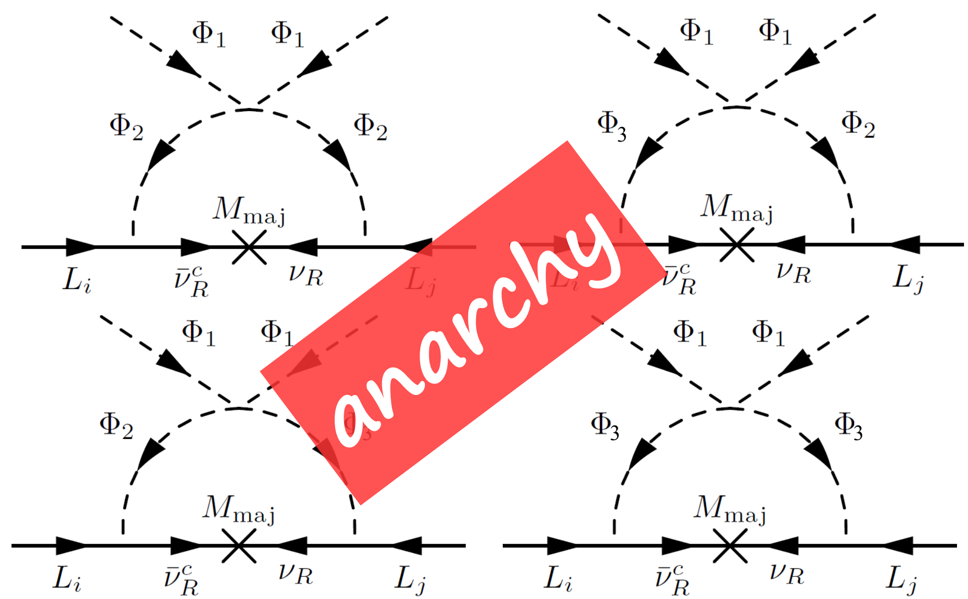
(if Y_ν^1 has a pattern)



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Even if each Yukawa coupling had an structure, the combination of them gives a "structureless" U_{i2}.

Conclusions

	SM	SM + heavy RH neutrinos	SM + Heavy RH neutrinos + *heavy* higgs doublet
Flavour, CP, EWPD			
Tiny neutrino masses			
Mild ν mass hierarchy			
Neutrino Mixing angles			
Baryogenesis			
Dark matter			
Strong CP problem			
Hierarchy problem			
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Thank you for your attention!