RESIDUAL BEAM MOTION DRIVEN BY THE NOISE AT TRANSVERSE FEEDBACK

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Abstract

The analysis on excitation of beam motion by noise of transverse feedback systems is performed. The result shows that high resolution beam position monitors of micro meter resolution for a single passage of bunches is required to reduce the amplitude of this motion to acceptable level, sub-micro meters for recent high brightness storage rings.

INTRODUCTION

Bunch-by-bunch feedback systems are widely used at storage rings to suppress beam instabilities. The most of recent systems employ digital feedback processors and the schematic diagram of the digital transverse bunch-by-bunch feedback system at the SPring-8 storage ring [1,2] is shown in Fig. 1. The positions of bunches are detected by a beam position monitor (BPM), and turn-by-turn and bunch-by-bunch position data are stored and processed by a digital feedback processor [3] to produce kick to damp betatron oscillation of bunches. For bunch-by-bunch feedback systems, the required bandwidth is more than one half of the bunch rate. In SPring-8 case, the bunch rate is 508.6MHz and the bandwidth of the feedback is more than 254MHz and noise caused by such wide bandwidth degrade the position resolution of BPMs. In the following, we analyze of the effect of this finite resolution of BPMs on the feedback.

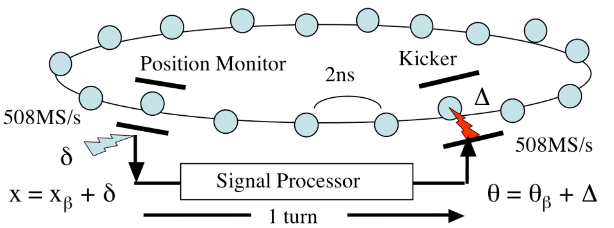

Figure 1: Schematic diagram of a bunch-by-bunch feedback system. Bunch rate and bunch spacing of the SPring-8 storage ring are 508MHz and 2ns. The bunch-by-bunch, turn-by-turn position data are stored and processed by a signal processor to produce a signal for kicker. The noise at a beam position monitor excites a feedback system and produce unwanted kick on a beam.

BUNCH-BY-BUNCH FEEDBACK

The required kick at n-th turn to damp the oscillation is

$$\theta_n = -\frac{2T_0}{\tau_{FB}} x'_n \qquad (1)$$

where T_0 and τ_{FB} are the revolution period of a ring and the feedback damping time, and x'_n is the angle of the beam at n-th turn. To produce x'_n from turn-by-turn position data of bunches, x_{n-k} (k=1,2,...), FIR filters are used in digital feedback processors.

FIR filter

A FIR filter is a one of digital filters and has the form of

$$y_n = \sum_{k=1}^{M} a_k x_{n-k} \qquad (2)$$

where x_n are the data of n-th sampling and y_n is the n-th output of the filter, and M is the number of taps.

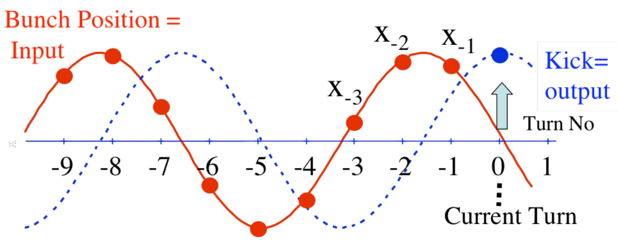

Figure 2: Turn-by-turn bunch positions (solid line) for the input data to FIR filter and required output of FIR filter (dashed line) to damp betatron oscillation shown in Eq. 3.

As shown in Fig.2, the required FIR filter for feedback systems is

$$-\beta x'_n = \sum_{k=1}^{M} a_k x_{n-k} \qquad (3)$$

where β is the beta function at the BPM and used for the normalization of the FIR filter. As shown in Fig. 2, the frequency dependence of the FIR filter should be

$$\sum_{k=1}^{N} a_k e^{-ik\phi} = e^{-i\frac{\pi}{2}} \qquad (4)$$

to fulfill the requirement of Eq. 3 where ϕ is the target betatron phase advance per turn. From Eq. 1 and Eq. 3, the kick can be expressed as

$$\theta_n = \frac{2T_0}{\tau_{FB}} \frac{1}{\beta} \sum_{k=1}^{M} a_k x_{n-k} . \qquad (5)$$

In this paper, we do not show the detail of methods to make such filters that fulfill Eq. 3. One of methods is shown in Ref. [1,2].

Position Error by Noise

To analyze the effect of the noise, we will apply similar discussion on radiation excitation.

We assume that the position error by noise, δ_n, is random:

$$\langle \delta_k \delta_{k'} \rangle = \delta_{k,k'} \langle \delta^2 \rangle, \quad \langle \delta_k \rangle = 0 \quad . \qquad (6)$$

Using the position data with noise, $x_k+\delta_k$, the kick by feedback shown by Eq. 5, is

$$\theta_n = \frac{2T_0}{\tau_{FB}\beta}\sum_{k=1}^{M} a_k\left(x_{n-k}+\delta_{n-k}\right)$$
$$= -\frac{2T_0}{\tau_{FB}}\left(x'_n + \frac{1}{\beta}\Delta_n\right) \qquad (7)$$

where we set

$$\Delta_n = -\sum_{k=0}^{M} a_k \delta_{n-k}\,. \qquad (8)$$

We assume that the angle of a beam has a format the neighbourhood of (n-N)-th turn:

$$x'_{n'} = A\cos n'\phi \quad (n' \sim n-N) \quad . \qquad (9)$$

This evolves to n-th turn with the kick in Eq. 7 as

$$x'_n = A\cos n\phi + \sum_{k<n}^{n-N}\theta_k \cos(n-k)\phi \quad . \qquad (10)$$

This can also be expressed as

$$x'_n = \left(A+\Delta A(n)\right)\cos\left(n\phi+\psi(n)\right) \quad . \qquad (11)$$

where θ_k is the stochastic, and ΔA and ψ are the functions of θ_k, so that those are also stochastic. Using Eq. 7 for θ_k, the second term of the r.h.s. of Eq. 10 is

$$\sum_{k\le n}^{N}\theta_k\cos(n-k)\phi$$
$$= -\sum_{k\le n}^{N}\frac{2T_0}{\tau_{FB}}\left(x'_k+\frac{1}{\beta}\Delta_k\right)\cos(n-k)\phi \qquad (12)$$

We assume that the evolution of the amplitude is small during one period of betatron oscillation and we can use the approximation of $x'(k) \cong A\cos k\phi$ in r.h.s. of Eq. 12 and we have

$$\sum_{k\le n}^{N}\theta_k\cos(n-k)\phi$$
$$= -\frac{2T_0}{\tau_{FB}}\sum_{k\le n}^{N}\left(A\cos k\phi\cos(n-k)\phi + \frac{1}{\beta}\Delta_k\cos(n-k)\phi\right)$$
$$= -\frac{2T_0}{\tau_{FB}}\left(\frac{1}{2}NA\cos n\phi + \frac{1}{\beta}\sum_{k\le n}^{N}\Delta_k\cos(n-k)\phi\right) \qquad (13)$$

where we also used the approximation:

$\sum_{k\le n}^{N}\cos^2 k\phi \cong \frac{1}{2}N$ and $\sum_{k\le n}^{N}\cos k\phi\sin k\phi \cong 0$

assuming that N is much larger than the fractional tune of the betatron oscillation.

Then from Eq. 10 and Eq. 13, we have

$$x'_n =$$
$$= \left(1-\frac{T_0}{\tau_{FB}}NA\right)\cos n\phi - \frac{2T_0}{\tau_{FB}}\frac{1}{\beta}\sum_{k\le n}^{N}\Delta_k\cos(n-k)\phi \qquad (14)$$

The first term of r.h.s. of Eq. 14 shows the damping and second term shows the effect of the noise.

Taking the ensemble average of ${x'_n}^2$ in Eq. 14, we have

$$\left\langle {x'_n}^2\right\rangle = A^2\left(1-\frac{T_0}{\tau_{FB}}N\right)^2\cos^2 n\phi$$
$$+\left(\frac{2T_0}{\tau_{FB}}\right)^2\frac{1}{\beta^2}\sum_{k\le n}^{N}\sum_{k'\le n}^{N}\left\langle\Delta_k\Delta_{k'}\right\rangle\cos(n-k)\phi\cos(n-k')\phi$$
$$= A^2\left(1-\frac{T_0}{\tau_{FB}}N\right)^2\cos^2 n\phi + \left(\frac{2T_0}{\tau_{FB}}\right)^2\frac{1}{\beta^2}I_n \qquad (16)$$

where

$$I_n = \sum_{k\le n}^{N}\sum_{k'\le n}^{N}\left\langle\Delta_k\Delta_{k'}\right\rangle\cos(n-k)\phi\cos(n-k')\phi \qquad (17)$$

and we used $\left\langle\Delta_k\right\rangle = -\sum_{i=0}^{M}a_i\left\langle\delta_{k-i}\right\rangle = 0$.

Then we have, with Eq. 4 and Eq. 6,

$$I_n = \sum_{k\le n}^{N}\sum_{k'\le n}^{N}\sum_{i=0}^{M}\sum_{i'=0}^{M}a_i a_{i'}\left\langle\delta_{k-i}\delta_{k'-i'}\right\rangle\cos(n-k)\phi\cos(n-k')\phi$$
$$= \sum_{k\le n}^{N}\sum_{k'\le n}^{N}\sum_{i=0}^{M}\sum_{i'=0}^{M}a_i a_{i'}\left\langle\delta^2\right\rangle\delta_{k-i,k'-i'}\cos(n-k)\phi\cos(n-k')\phi$$
$$= \sum_{k\le n}^{N}\sum_{i=0}^{M}\sum_{i'=0}^{M}a_i a_{i'}\left\langle\delta^2\right\rangle\cos(n-k)\phi\cos(n-k+i-i')\phi$$
$$= \left\langle\delta^2\right\rangle\sum_{k\le n}^{N}\sum_{i=0}^{M}\sum_{i'=0}^{M}a_i a_{i'}\frac{1}{2}\left\{\cos 2(n-k)\phi + \cos(i-i')\phi\right\}$$
$$\cong \frac{1}{2}N\left\langle\delta^2\right\rangle\sum_{i=0}^{M}\sum_{i'=0}^{M}a_i a_{i'}\cos(i-i')\phi$$
$$= \frac{1}{2}N\left\langle\delta^2\right\rangle\left(\sum_{i=0}^{M}a_i\cos i\phi\sum_{i'=0}^{M}a_{i'}\cos i'\phi + \sum_{i=0}^{M}a_i\sin i\phi\sum_{i'=0}^{M}a_{i'}\sin i'\phi\right)$$
$$= \frac{1}{2}N\left\langle\delta^2\right\rangle\left\{\left(\sum_{i=0}^{M}a_i\cos i\phi\right)^2 + \left(\sum_{i=0}^{M}a_i\sin i\phi\right)^2\right\}$$
$$= \frac{1}{2}N\left\langle\delta^2\right\rangle\left|\sum_{k=0}^{M}a_k e^{ik\phi}\right|^2 = \frac{1}{2}N\left\langle\delta^2\right\rangle \quad . \qquad (18)$$

where we used $\left|\sum_{k=0}^{M}a_k e^{ik\phi}\right|^2 = 1$ with Eq. 4

Now we have

$$\left\langle {x'_n}^2\right\rangle =$$
$$A^2\left(1-\frac{T_0}{\tau_{FB}}N\right)^2\cos^2 n\phi + \left(\frac{2T_0}{\tau_{FB}}\right)^2\frac{1}{\beta^2}\frac{1}{2}N\left\langle\delta^2\right\rangle \;.(19)$$

Using Eq. 11, we have

$$\left\langle\left(A+\Delta A\right)^2\right\rangle\cos^2(n\phi+\psi) =$$
$$A^2\left(1-\frac{T_0}{\tau_{FB}}N\right)^2\cos^2 n\phi + \left(\frac{2T_0}{\tau_{FB}}\right)^2\frac{1}{\beta^2}\frac{1}{2}N\left\langle\delta^2\right\rangle. \;(20)$$

We take a time average for several oscillation period and we have

$$\frac{1}{2}\left\langle (A+\Delta A)^2 \right\rangle = \frac{1}{2}A^2\left(1-\frac{T_0}{\tau_{FB}}N\right)^2 + \left(\frac{2T_0}{\tau_{FB}}\right)^2 \frac{1}{2\beta^2}N\left\langle \delta^2 \right\rangle$$

$$\cong \frac{1}{2}A^2 - \frac{1}{2}A^2\frac{2T_0}{\tau_{FB}}N + \left(\frac{2T_0}{\tau_{FB}}\right)^2 \frac{1}{\beta^2}\frac{1}{2}N\left\langle \delta^2 \right\rangle \qquad (21)$$

with the assumption, $\frac{T_0}{\tau_{FB}}N << 1$.

Then the evolution of the amplitude of the betatron oscillation is

$$\left\langle \frac{d\left(A^2\right)}{dt} \right\rangle = \frac{\left\langle (A+\Delta A)^2 \right\rangle - A^2}{NT_0} =$$

$$-\frac{2}{\tau_{FB}}A^2 + \left(\frac{2T_0}{\tau_{FB}}\right)^2 \frac{1}{\beta^2 T_0}\left\langle \delta^2 \right\rangle . \qquad (22)$$

If we include radiation damping or growth by instabilities, the equation is

$$\left\langle \frac{d\left(A^2\right)}{dt} \right\rangle = -\frac{2}{\tau}A^2 + \left(\frac{2T_0}{\tau_{FB}}\right)^2 \frac{1}{\beta^2 T_0}\left\langle \delta^2 \right\rangle \qquad (23)$$

where τ is the total damping time with feedback, radiation damping and growth by instabilities. We treat A as definite value at n-N turn. However, as we see, the value A evolves stochastically with ΔA and become stochastic after many damping time. At the equilibrium, the value of Eq. 23 should be 0 and we have expectation value:

$$\left\langle A^2 \right\rangle = \frac{2\tau T_0}{\tau_{FB}{}^2}\frac{1}{\beta^2}\left\langle \delta^2 \right\rangle \qquad (24)$$

or

$$\sigma_{x'}{}^2 = \frac{1}{2}\left\langle A^2 \right\rangle = \frac{\tau T_0}{\tau_{FB}{}^2}\frac{1}{\beta^2}\left\langle \delta^2 \right\rangle \qquad (25)$$

for r.m.s. of divergence of the beam at the feedback and

$$\sigma_x = \frac{\sqrt{\tau T_0}}{\tau_{FB}}\sigma_\delta \qquad (26)$$

for r.m.s. of beam size where $\sigma_\delta = \sqrt{\left\langle \delta^2 \right\rangle}$ is the resolution of the BPM in r.m.s.

SPring-8 CASE

We will apply the result to the SPring-8 storage ring. The parameters of the SPring-8 storage ring are listed in Table 1.

Table 1: Parameters of the SPring-8 storage ring

parameter	symbol	value	unit
Revolution Period	T_0	4.79	μs
Bunch rate / RF acc. freq	f_{RF}	508.58	MHz
Beam size at feedback	σ_H/σ_V	301 / 6	μm
Average Current	I	100	mA

From Eq. 26, we have to reduce total damping time, τ, by feedback to be much faster than growth time of instabilities to keep σ_x small, and we set τ_{FB} ~ 0.5ms. With this damping time, Eq. 26 shows $\sigma_x = 0.1\sigma_\delta$. A Monte-Carlo simulation confirms this relation.

The residual amplitude of the betatron oscillation should be reduced to one tenth of the beam size. The beam size at the feedback is 6μm hence required resolution of a BPM, σ_δ, is 6μm.

The button type BPM of SPring-8 for correction of slow beam motion is 0.5μm with bandwidth ~1kHz. Scaling of the resolution to bandwidth, B is $\sqrt{B}$ and the expected resolution at B ~ 250MHz that required for the feedback is 250μm. This value is 50 times larger than required.

We developed a new shorted-stripline type BPM for the SPring-8 feedback [4] that produces tens times higher signal than button types, and its position resolution is 5μs r.m.s. for a single passage of a bunch with 0.24nC charge. Also we use 12-bit ADCs for the front-end of our feedback processor to achieve sub mm resolution with dynamic range of 0.5mm that is required to control the turbulence on the stored beam produced at injection.

CONCLUSION

We derived a formula for the amplitude of residual motion excited by a feedback driven by a noise at beam position detection. The result shows that the special high-resolution beam position monitors are required to suppress the amplitude of these motions to nano meter region.

REFERENCES

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