LATTICE SIMULATION OF B PHYSICS USING NONRELATIVISTIC QCD

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We present results from our studies of leptonic and semileptonic decay processes of B mesons using lattice QCD. For the heavy quark, we use the Nonrelativistic QCD (NRQCD) action at various masses around the b-quark mass. The light quarks are described with the Wilson action. We investigate the heavy quark mass dependence of the form factors and the decay constant. The renormalization factors for the form factors and the decay constant are included at one loop level. In this approach, we find the $O(1/m_Q^2)$ correction is small for the decay constant. Clear signals of the form factors are observed, which enables us to study the q^2 dependence.

1 INTRODUCTION

The physics concerning the hadrons including b-quarks is so called "B physics", which is important to the precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements in the Standard Model (SM). The CKM matrix describes the quark mixing among the generations due to the difference of its weak eigenstate and mass eigenstate, and is expressed as a 3×3 unitary matrix in the frame work of the SM. Verification of its untarity leads to not only the verification of the SM, but also the search for "new physics".

Especially the CKM matrix elements, V_{ub}, V_{cb}, V_{td}, and V_{ts}, which are related to the "B physics", still have large uncertainties in spite of match efforts with various approaches. In principle, these matrix elements can be determined from the measurement of B meson weak decay processes. There, however, are theoretical uncertainties, which are main source of the uncertainties of the CKM matrix elements, in hadron weak decay amplitudes because of the non-perturbative effect of strong interaction. The string interaction is described by Quantum Chromo Dynamics (QCD). For example $|V_{ub}|$ and $|V_{tb}^* V_{td}|$ are extracted from the experiments via following theoretical equations

$$d\Gamma(B \to \pi l \nu) \propto |V_{ub}|^2 |f^+(q^2)|^2, \tag{1}$$

$$dM_d \equiv |M_{B_d^0} - M_{\bar{B}_d^0}| \propto f_{B_d}^2 B_{B_d} |V_{tb}^* V_{td}|^2, \tag{2}$$

where

$$\langle \pi(\boldsymbol{p}_\pi)|\bar{q}\gamma_\mu b|B(\boldsymbol{p}_B)\rangle = \left(p_B + p_\pi - q\frac{m_B^2 - m_\pi^2}{q^2}\right)_\mu f^+(q^2) + q_\mu \frac{m_B^2 - m_\pi^2}{q^2} f^-(q^2), \tag{3}$$

$$B_B = \langle \bar{B}^0 | \bar{b}\gamma_\mu(1-\gamma_5)q\bar{b}\gamma^\mu(1-\gamma_5)q|B^0\rangle / \left(\frac{8}{3} f_B^2 M_B^2\right), \tag{4}$$

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 b|B(\boldsymbol{p}_B)\rangle = if_B p_{B\mu}, \tag{5}$$

and $d\Gamma$ is the differential branching ratio of the exclusive $B \to \pi$ semileptonic decay and dM_d is the mass difference between B_d^0 and $\bar{B}_d^0$. The branching ratio and the mass difference are experimentally measured from the B meson decay processes and the $B_d^0 - \bar{B}_d^0$ mixing respectively. The $B \to \pi$ semileptonic decay form factors $f^+(q^2)$, $f^-(q^2)$, the B meson decay constant f_B, and the bag parameter B_B are theoretically calculable, but still have large uncertainties because of QCD non-perturbative effects.

Lattice QCD (LQCD) enables us to calculate non-perturbatively hadron decay amplitudes from the first principle of QCD. Many efforts have been devoted to the calculations of these hadron weak decay amplitudes using LQCD. Since available lattice cut off a^{-1} ($\sim$ 3 GeV) is smaller than a b-quark mass ($\sim$ 5 GeV), the largeness of b-quark mass leads to the large systematic errors such as $O(am_b)$ and has been an obstacle to the reliable calculations of the decay amplitudes in standard LQCD. One method to avoid this situation is usage of Lattice Nonrelativistic QCD (NRQCD) to describe the b-quark. NRQCD is designed to remove the large mass of b-quark through the Foldy-Wouthuysen-Tani (FWT) transformation of relativistic QCD, which transformation is a systematic $1/m_b$ expansion and reproduce the relativistic QCD results in a low energy scale. Although removing the large systematic error $O(am_b)$, the $1/m_b$ expansion introduces a new systematic error coming from the truncation of the $1/m_b$ expansion. It is important to investigate the truncation effects in the frame work of Lattice NRQCD.

Our final purpose is to calculate the form factors, the decay constants, and the bag parameters of the B mesons using LQCD in order to precisely determine the CKM matrix elements from B experiments.

In this paper, we focus on the applicability of NRQCD for the calculation of the exclusive $B \to \pi$ semileptonic decay form factors $f^+(q^2)$, $f^-(q^2)$ [2] and the truncation effect of the B meson decay constant f_B [1]. We use NRQCD and Wilson actions for heavy quark and light quark respectively. The q^2 dependence of the form factors is especially important, because the available region of q^2 in experiments is differ from that in LQCD calculation. The $1/m_B$ dependence of the form factors and the decay constant are also important connected with the truncation effects of NRQCD. The results on the dependence in lattice simulations are often compared to the prediction of the Heavy Quark Effective Theory (HQET). Much of our interest are poured into the $1/m_B$ dependencies of the form factors and the decay constant and how we can extend its reliable q^2 region of the form factors in LQCD to bring close to the experimentally available region.

This paper is organized as follows. In section 2 we will summarize the numerical parameters and methods to calculate the decay amplitudes and briefly introduce Lattice NRQCD. Three types of Lattice NRQCD actions are used in our simulation. One is an action used in the calculation of the form factors, which involves through $O(1/m_Q)$ terms and leads simple structure evolution equation to reduce the cost of computational times. Others are actions used in the calculation of the decay constant. The evolution equation of these actions is complicated compared to the former action. To investigate the truncation effect, we prepare the $O(1/m_Q)$ and $O(1/m_Q^2)$ actions for the decay constant. In section 3 we show the simulation results on the truncation effect, the

$1/m_B$ dependence of the B meson decay constant, the simulation results on the $B \to \pi$ semileptonic decay form factors, and the q^2 dependencies. Finally we conclude the status of our simulation.

2 SIMULATION

The numerical simulation is carried out on a $16^3 \times 32$ lattice with 120 quenched gauge configurations generated with the standard Wilson gauge action at $\beta = 5.8$. Each configuration is generated with the pseudo-heat-bath method and fixed to the Coulomb gauge. An inverse lattice spacing a^{-1} is 1.71(6) GeV determined from ρ meson mass. We use the Wilson and NRQCD actions for light quark and heavy quark to describe the heavy-light systems, respectively. The propagators of Wilson light quark are solved with the ILU-BiCGStab method. We employ three hopping parameters for Wilson light quark, which are related to the light quark mass, as $\kappa = 0.15770$, 0.1585, and 0.1600, which values roughly lie on a range $[m_s, 2m_s]$, where the strange quark mass is $m_s \sim 150$ MeV. Taking the chiral limit at which pion mass vanish, we found that the critical hopping parameter is $\kappa_{\rm cr} = 0.16346(6)$. The mean field improvement is applied for both of the NRQCD actions and the current operators as the replacement of $U_\mu \to U_\mu/u_0$ using the average value of the single plaquette $u_0 = \langle {\rm Tr} U_{\rm plaq}\rangle/3 = 0.867994(13)$.

The NRQCD actions used in our simulation are as follows.

$$\begin{aligned} S &= \sum_{t,\boldsymbol{x}} Q(\boldsymbol{x},t)\left[Q(\boldsymbol{x},t) - \left(1-\frac{aH_0}{2n}\right)^n \left(1-\frac{a\delta H}{2}\right)\right. \\ &\qquad \left. \times U_4^\dagger \left(1-\frac{a\delta H}{2}\right)\left(1-\frac{aH_0}{2n}\right)^n Q(\boldsymbol{x},t-1)\right], \quad \text{type } A, \end{aligned} \tag{6}$$

and

$$\begin{aligned} S &= \sum_{t,\boldsymbol{x}} \left[Q(\boldsymbol{x},t+1)\left(1-\frac{aH_0}{2n}\right)^{-n} U_4 \left(1-\frac{aH_0}{2n}\right)^{-n} Q(\boldsymbol{x},t)\right. \\ &\qquad \left. -Q(\boldsymbol{x},t)(1-a\delta H)Q(\boldsymbol{x},t)\right], \quad \text{type } B, \end{aligned} \tag{7}$$

where $H_0 = -\boldsymbol{\Delta}^{(2)}/[2M_0]$, M_0 is a heavy quark mass, and Q is the effective 2-component spinor field. The type A action is used for the decay constant f_B and the type B for the form factors $f^+(q^2)$, $f^-(q^2)$. Two types of the interaction terms are employed as $\delta H = -g\boldsymbol{\sigma}\cdot\boldsymbol{B}/[2M_0]$ for both of the type A and B, and

$$\begin{aligned} \delta H &= -\frac{(\boldsymbol{\Delta}^{(2)})^2}{8(M_0)^3} - \frac{g}{2M_0}\boldsymbol{\sigma}\cdot\boldsymbol{B} + \frac{ig}{8(M_0)^2}(\boldsymbol{\Delta}\cdot\boldsymbol{E} - \boldsymbol{E}\cdot\boldsymbol{D}) \\ &\quad -\frac{g}{8(M_0)^2}\boldsymbol{\sigma}\cdot(\boldsymbol{\Delta}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{\Delta}) + \frac{a^2\boldsymbol{\Delta}^{(4)}}{24M_0} - \frac{a(\boldsymbol{\Delta}^{(2)})^2}{16n(M_0)^2}, \quad \text{for the type } A. \end{aligned} \tag{8}$$

Heavy quark propagators are produced through the evolution equations derived from these NRQCD actions. Since the cost in calculating the evolution equation of NRQCD is much less than that of Wilson, relativistic action, the equation can be solved on a work station, which is one of the advantages of the usage of NRQCD for heavy quark.

The current operators, of which expectation values become the form factors or the decay constant, are

$$A_4 = \bar{\psi}_l \gamma_5 \gamma_4 \psi_h, \text{ for the decay constant, and } V_\mu = \bar{\psi}_l \gamma_\mu \psi_h, \text{ for the form factors,} \tag{9}$$

where ψ_l is the light quark 4-component spinor field, and ψ_h is the heavy quark 4-component spinor field which consists of the effective 2-component spinor field Q via the inverse FWT transformation as

$$\psi_h = \left(1 - \frac{\boldsymbol{\gamma}\cdot\boldsymbol{\Delta}}{2M_0} + \frac{\boldsymbol{\Delta}^{(2)}}{8(M_0)^2} + \frac{g\boldsymbol{\Sigma}\cdot\boldsymbol{B}}{8(M_0)^2} - \frac{ig\gamma_4\boldsymbol{\gamma}\cdot\boldsymbol{E}}{4(M_0)^2}\right)\begin{pmatrix} Q \\ 0 \end{pmatrix}. \tag{10}$$

We employ the heavy quark field including the terms up to $O(1/M_0)$ for the form factors, and up to both of $O(1/M_0)$ and $O(1/(M_0)^2)$ for the decay constant in order to see the truncation effect.

We perturbatively calculate the renormalization parameters, which are required in converting lattice results to continuum values, for the current operators and both of the type A and type B actions through $O(1/M_0)$ at $O(\alpha)$. Since we calculate the only multiplicative part of the renormalization parameters for the current operators, the operator mixing problem is not in settlement, which is one of the largest systematic errors in LQCD. We will not argue about this matter in this paper [3].

We take a standard procedure using correlation functions to calculate the decay amplitudes. The two-point correlation function for B meson is

$$\begin{aligned} C_{PP}(t) &= \sum_{\boldsymbol{x}} \langle P_B(\boldsymbol{x},t) P_B^\dagger(\boldsymbol{0},0)\rangle = -\sum_{\boldsymbol{x}} \langle \mathrm{Tr}[G_q^\dagger(\boldsymbol{x},t;\boldsymbol{0},0) G_Q(\boldsymbol{x},t;\boldsymbol{0},0)]\rangle \\ &\underset{t\gg 1}{\Longrightarrow} \frac{1}{2M_B} |\langle 0|P_B(\boldsymbol{0},0)|B\rangle|^2 \exp(-dM_B t), \end{aligned} \tag{11}$$

where P_B is a interpolating operator for pseudoscalar B meson, G_q and G_Q are the propagator of light quark and heavy quark respectively. It is need another correlation function to obtain the decay constant.

$$\begin{aligned} C_{A_4P}(t) &= \sum_{\boldsymbol{x}} \langle A_4(\boldsymbol{x},t) P_B^\dagger(\boldsymbol{0},0)\rangle = -\sum_{\boldsymbol{x}} \langle \mathrm{Tr}[G_q^\dagger(\boldsymbol{x},t;\boldsymbol{0},0)\gamma_4 G_Q(\boldsymbol{x},t;\boldsymbol{0},0)]\rangle \\ &\underset{t\gg 1}{\Longrightarrow} = \frac{f_B}{2Z_A} \langle B|P_B^\dagger(\boldsymbol{0},0)|0\rangle \exp(-dM_B t), \end{aligned} \tag{12}$$

where Z_A is the renormalization parameter for the decay constant. Fitting these correlation functions with single exponential functions and combining the fitting results, we can obtain the decay constant f_B[1]. The following three-point correlation function yields the $B \to \pi$ weak decay amplitude with the helps of the two-point correlation functions of B meson and pion [2].

$$C_\mu(\boldsymbol{p},\boldsymbol{k};t_f,t_s,t_i) = \sum_{\boldsymbol{x}_f,\boldsymbol{x}_s} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_f} e^{-i(\boldsymbol{k}-\boldsymbol{p})\cdot\boldsymbol{x}_s} \langle P_B(\boldsymbol{x}_f,t_f) V^\dagger(\boldsymbol{x}_s,t_s) P_\pi(\boldsymbol{x}_i,t_i)\rangle \tag{13}$$

$$= -\sum_{\boldsymbol{x}_f,\boldsymbol{x}_s} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}_f} e^{-i(\boldsymbol{k}-\boldsymbol{p})\cdot\boldsymbol{x}_s} \langle \mathrm{Tr} G_q^\dagger(\boldsymbol{x}_f,t_f;\boldsymbol{x}_i,t_i) G_Q(\boldsymbol{x}_f,t_f;\boldsymbol{x}_s,t_s)\gamma_\mu G_q(\boldsymbol{x}_s,t_s;\boldsymbol{x}_i,t_i)\rangle$$

$$\underset{t_f\gg t_s\gg t_i}{\Longrightarrow} \propto \langle B(\boldsymbol{p})|V_\mu^\dagger(\boldsymbol{0},0)|\pi(\boldsymbol{k})\rangle e^{-E_B(t_f-t_s)} e^{-E_\pi(t_s-t_i)}.$$

3 RESULTS

We calculate the decay constant using the type A action (6) and the current operator A_4 with terms up to $O(1/M_0)$ and $O(1/M_0^2)$. Figure 1 shows the pseudoscalar meson decay constant multiplied by the square root of the meson mass as a function of inverse meson mass. The error bars stand for the statistical error only. Comparing the $O(1/M_0^2)$ result to the $O(1/M_0)$ result, we find that the difference at the B meson mass is small, which indicates that the truncation effects are indeed small. While the effects of the perturbative matching is large, which suggests that the 2-loop calculation for the renormalization parameters is required for the precise determination of the decay constant. Taking into account the various systematic errors, we quote the following estimation from the present work for the B meson decay constant in the quenched approximations [1].

$$f_B = 184 \pm 7 \pm 5\text{MeV}, \qquad (14)$$

with the discretization and perturbative errors of about 20% each, of which estimations are based on the conservative estimations, where the first error is the statistical error and the second error is the truncation error.

The $B \to \pi$ semileptonic decay form factors $f^+(q^2)$, $f^-(q^2)$ are calculated using the type B action (7) and the current operator V_μ with terms up to $O(1/M_0)$. Figure 2 shows the q^2 dependence of the form factors at $M_0 = 2.6$, $\kappa = 0.1570$, and $= 0.1600$. The solid lines are the results of fitting with the single pole function: $f(q^2) = 1/(q^2 - M_{\text{pole}}{}^2)$. Our calculation can cover one third of the whole kinamatically allowed region. We expect that there is the overlap region between the experimentally available region, which is around the $q^2 = 0$, and the result of LQCD using NRQCD. The pole fit is consistent with the kinematical constraint: $f^+(q^2 = 0) = f^+(q^2 = 0)$, and it seems to be well reproduced the behavior of the pole dominance model. The signals become noisy toward the $q^2 = 0$ point because of the fluctuation of the higher momentum states of pion. In Addition, the systematic error, $O(a\Lambda_{\text{QCD}})$, from the usage of Wilson quark for light quark would grow large. In order to obtain the physical form factors, we need to extrapolate the form factors to the chiral limit. Unfortunately the fluctuation is too large to evaluate the q^2 dependence after the chiral limit, which requires more high statistical calculation for concrete discussion.

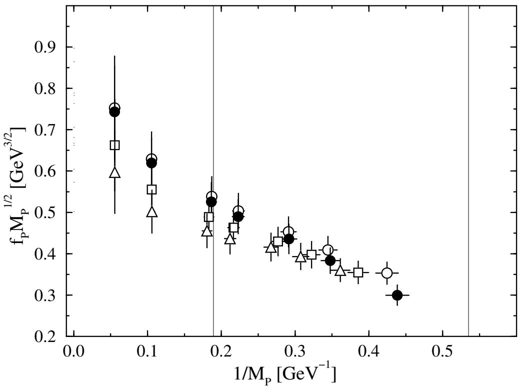

Figure 1: The inverse pseudoscalar meson mass $1/M_{\text{P}}$ dependence of the chirally extrapolated decay constant $f_{\text{P}} M_{\text{P}}^{1/2}$ in physical unit. Open symbols: $O(1/M_0)$ results, solid symbols: $O(1/M_0^2)$. Circles are the results with tree-level matching, squares and triangles are with 1-loop matching on the scale of $aq^* = \pi$ and $aq^* = 1$ respectively. The vertical solid lines around $1/M_{\text{P}} = 0.2$ and $= 0.5$ correspond to the inverse B meson mass and the D meson mass respectively.

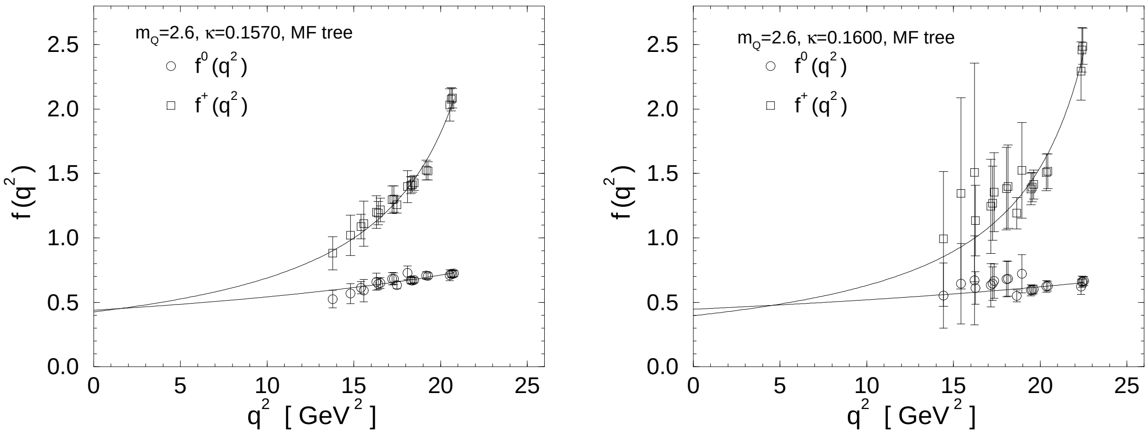

Figure 2: The q^2 dependence of the form factors $f^+(q^2)$,$f^-(q^2)$. $m_Q = 1.5 \sim 4.5$ GeV which corresponds to b-quark, $\kappa = 0.1600 \sim 300$ MeV, $\kappa = 0.1600 \sim 150$ MeV which corresponds to s-quark.

4 CONCLUSION

We have calculated the $B \to \pi$ semileptonic decay form factors $f^+(q^2)$, $f^-(q^2)$ and decay constant f_B. we find the $O(1/m_Q^2)$ correction is small for the decay constant. Clear signals of the form factors are observed with finite light quark mass. In the chiral limit, however, the signals become noisy because of the lack of statistics. One of the largest systematic error in the both of our results is the discretization error, which error is caused by the usage of Wilson quark for light quark. This error could be removed to employ Clover action instead of Wilson action, and we are planning the simulation using the Clover action with high statistics. We conclude that NRQCD can serve the simulation of "B physics", and hope that our simulation contribute to the comprehension of the SM through the study of the CKM matrix element.

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