Determination of Parton Distribution Functions in Nuclei

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Introduction

- parton distributions in the nucleon
- parton distributions in nuclei

  EMC effect, shadowing

Parametrization of nuclear parton distributions

1. functional form of $f_i^A(x)$
2. constraints: A dependence, conservation laws
3. χ^2 fitting to experimental data
4. results: optimum nuclear parton distributions

Summary

Purposes

Determination of parton distributions

- unpolarized distributions in the nucleon

  3 major groups (CTEQ, GRV, MRS)

- polarized distributions in the nucleon

  several groups (GS, GRSV, ..., AAC)

- distributions in nuclei

  no χ^2 analysis! (see Eskola, Kolhinen, Ruuskanen)

  → for understanding nuclear mechanisms in the high-enegy region

  → for heavy-ion physics

  → re-examination of $q_V(x)$ in the nucleon

Current status on unpolarized distributions in the nucleon

Parton distributions are determined by fitting various experimental data.

- electron/muon $\mu + p \rightarrow \mu + X$
- neutrino $\nu_\mu + p \rightarrow \mu + X$
- Drell-Yan $p + p \rightarrow \mu^+\mu^- + X$
- direct photon $\mu/p + p \rightarrow \gamma + X$
- ...

(1) assume parton distributions at Q_0^2 (~1 GeV^2)

e.g. $f_i(x,Q_0^2) = A_i\, x^{\alpha_i} (1-x)^{\beta_i} (1+\gamma_i x)$

where $i = u_v, d_v, \bar{u}, \bar{d}, \bar{s}, g$

(2) calculate structure functions at experimental Q^2 points

(3) then, A_i, α_i, β_i, γ_i are determined in comparison with data

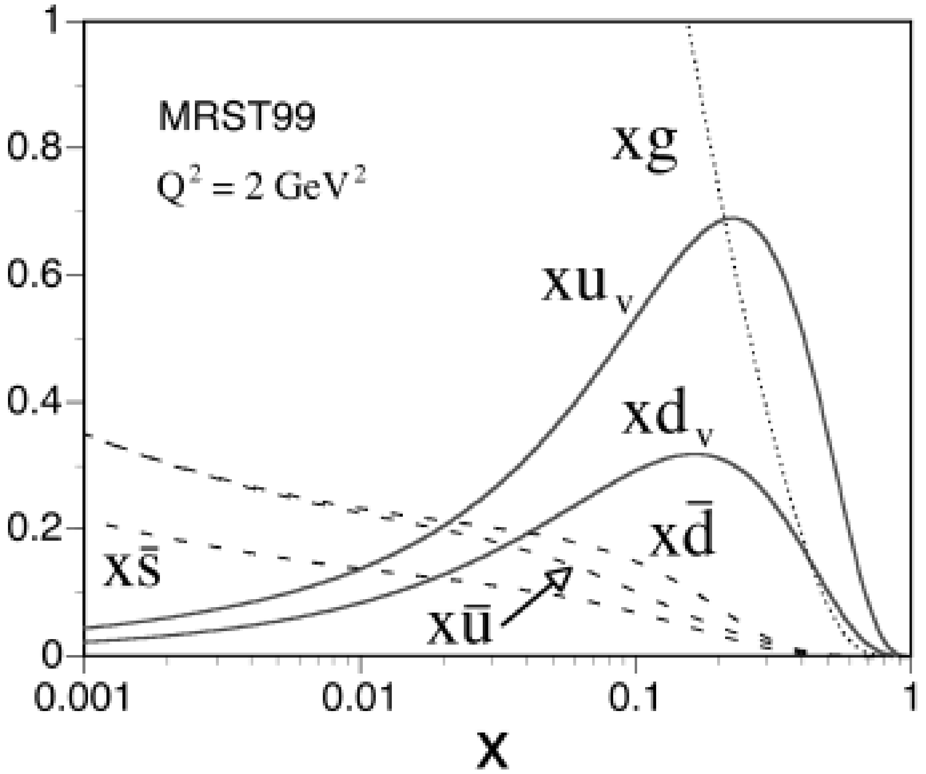
Available data for determining parton distributions

(Ref. MRST, hep/ph-9803445)

Process/ Experiment	Leading order subprocess	Parton behaviour probed
DIS ($\mu N \to \mu X$) $F_2^{\mu p}, F_2^{\mu d}, F_2^{\mu n}/F_2^{\mu p}$ (SLAC, BCDMS, NMC, E665)*	$\gamma^* q \to q$	Four structure functions $\to$ $u+\bar{u}$, $d+\bar{d}$, $\bar{u}+\bar{d}$, s (assumed $=\bar{s}$),
DIS ($\nu N \to \mu X$) $F_2^{\nu N}, xF_3^{\nu N}$ (CCFR)*	$W^* q \to q'$	but only $\int xg(x, Q_0^2)dx \simeq 0.35$ and $\int(\bar{d}-\bar{u})dx \simeq 0.1$
DIS (small x) F_2^{ep} (H1, ZEUS)*	$\gamma^*(Z^*)q \to q$	λ ($x\bar{q} \sim x^{-\lambda_S}$, $xg \sim x^{-\lambda_g}$)
DIS (F_L) NMC, HERA	$\gamma^* g \to q\bar{q}$	g
$\ell N \to c\bar{c}X$ F_2^c (EMC; H1, ZEUS)*	$\gamma^* c \to c$	c ($x \gtrsim 0.01$; $x \lesssim 0.01$)
$\nu N \to \mu^+\mu^- X$ (CCFR)*	$W^* s \to c$ $\hookrightarrow \mu^+$	$s \approx \frac{1}{4}(\bar{u}+\bar{d})$
$pN \to \gamma X$ (WA70*, UA6, E706, ...)	$qg \to \gamma q$	g at $x \simeq 2p_T^\gamma/\sqrt{s} \to$ $x \approx 0.2-0.6$
$pN \to \mu^+\mu^- X$ (E605, E772)*	$q\bar{q} \to \gamma^*$	$\bar{q} = ...(1-x)^{\eta_S}$
$pp, pn \to \mu^+\mu^- X$ (E866, NA51)*	$u\bar{u}, d\bar{d} \to \gamma^*$ $u\bar{d}, d\bar{u} \to \gamma^*$	$\bar{u}-\bar{d}$ $(0.04 \lesssim x \lesssim 0.3)$
$ep, en \to e\pi X$ (HERMES)	$\gamma^* q \to q$ with $q = u, d, \bar{u}, \bar{d}$	$\bar{u}-\bar{d}$ $(0.04 \lesssim x \lesssim 0.2)$
$p\bar{p} \to WX(ZX)$ (UA1, UA2; CDF, D0)	$ud \to W$	u, d at $x \simeq M_W/\sqrt{s} \to$ $x \approx 0.13$; 0.05
$\to$ **$\ell^\pm$ asym** (CDF)*		slope of u/d at $x \approx 0.05-0.1$
$p\bar{p} \to t\bar{t}X$ (CDF, D0)	$q\bar{q}, gg \to t\bar{t}$	q, g at $x \gtrsim 2m_t/\sqrt{s} \simeq 0.2$
$p\bar{p} \to$jet $+ X$ (CDF, D0)	$gg, qg, qq \to 2j$	q, g at $x \simeq 2E_T/\sqrt{s} \to$ $x \approx 0.05-0.5$

Recent unpolarized distributions

- CTEQ5, Eur. Phys. J. C12 (2000) 375
- GRV98, Eur. Phys. J. C5 (1998) 461
- MRST99, Eur. Phys. J. C14 (2000) 133

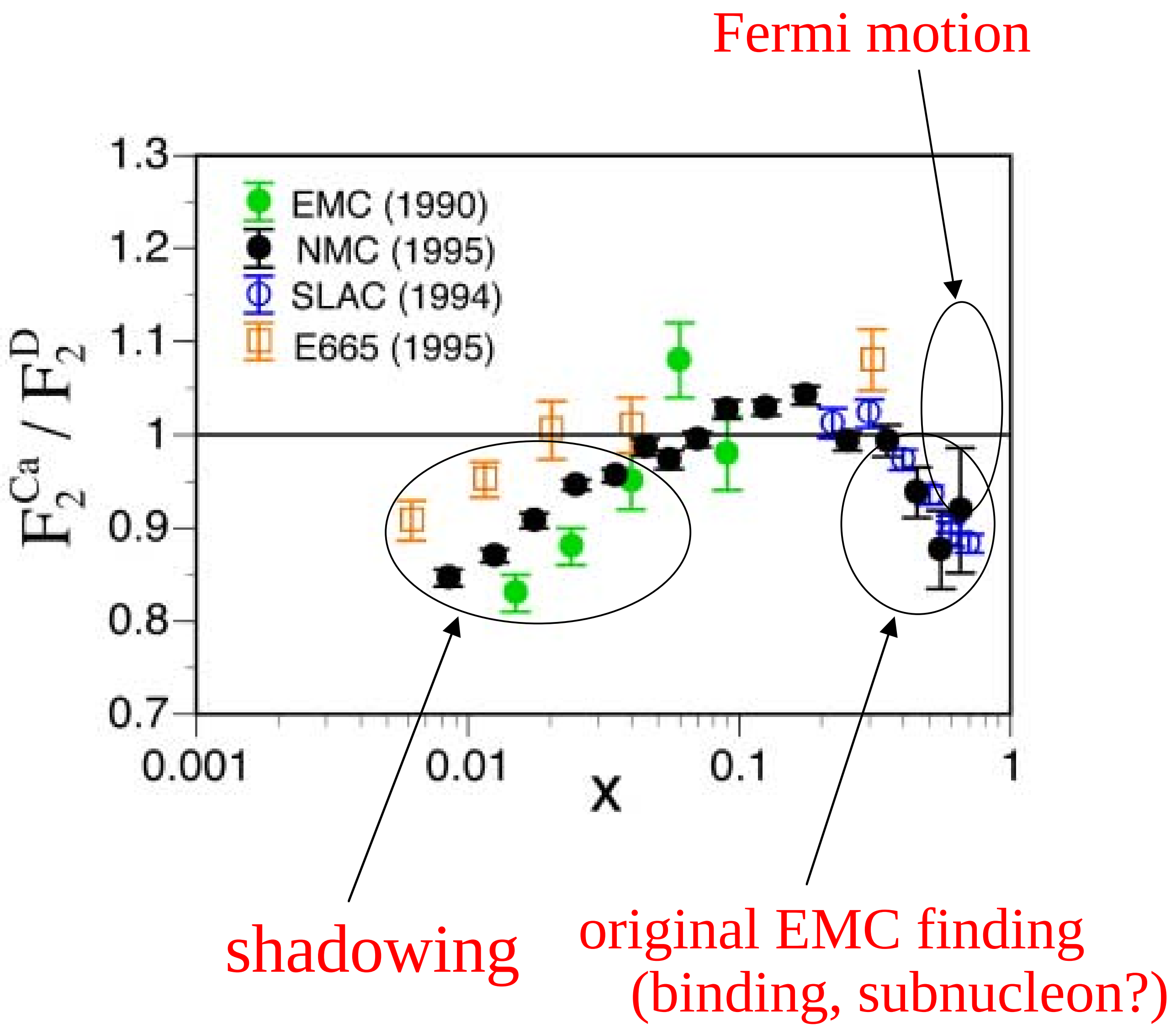


Unpolarized distributions are well known in the nucleon.

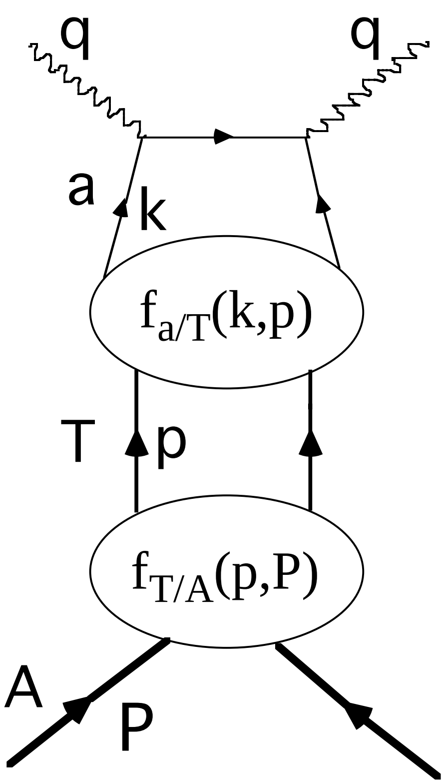
• parton distributions in nuclei

Nuclear modification of F_2^A / F_2^D is well known in electron/muon scattering.

$$F_2^A = \frac{1}{9}\, x \left[4\, u_v(x) + d_v(x)\right]_A + \frac{2}{9}\, x\, S_A(x)$$



Convolution



$$f_{a/A}(q^2,P\cdot q) = \sum_T \int \frac{d^4p}{(2\pi)^4}\, f_{a/T}(p,q)\, f_{T/A}(P,p)$$

$$\Downarrow$$

$$f_{a/A}(x,Q^2) = \sum_T \int_{x_A}^{1} dy_A\, f_{a/T}\left(\frac{x_A}{y_A}\right) f_{T/A}(y_A)$$

Q^2 rescaling model, $\cdots$

nuclear binding, nuclear pion, $\cdots$

Parametrization of

nuclear parton distributions

without relying on models!

- Nuclear parton distributions (per nucleon)
  if there *were* no modification

$$A\,u^A = Z\,u^p + N\,u^n, \quad A\,d^A = Z\,d^p + N\,d^n$$

Isospin symmetry: $u^n = d^p \equiv d\,,\ \ d^n = u^p \equiv u$

$$\rightarrow u^A = \frac{Z\,u + N\,d}{A}, \quad d^A = \frac{Z\,d + N\,u}{A}$$

- Take into accont the nuclear modification by the factors $w_i(x,A)$

$$u_v^A(x) = w_{u_v}(x,A)\ \frac{Z\,u_v(x) + N\,d_v(x)}{A}$$

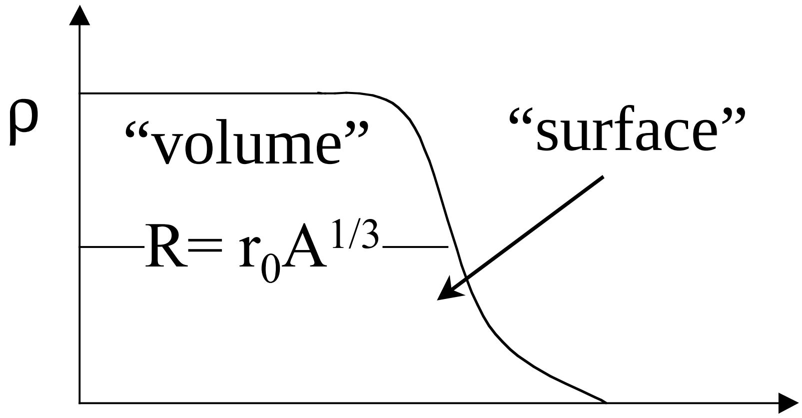
$$d_v^A(x) = w_{d_v}(x,A)\ \frac{Z\,d_v(x) + N\,u_v(x)}{A}$$

$$\bar{q}^A(x) = w_{\bar{q}}(x,A)\ \bar{q}(x)$$

$$g^A(x) = w_g(x,A)\ g(x)$$

A dependence

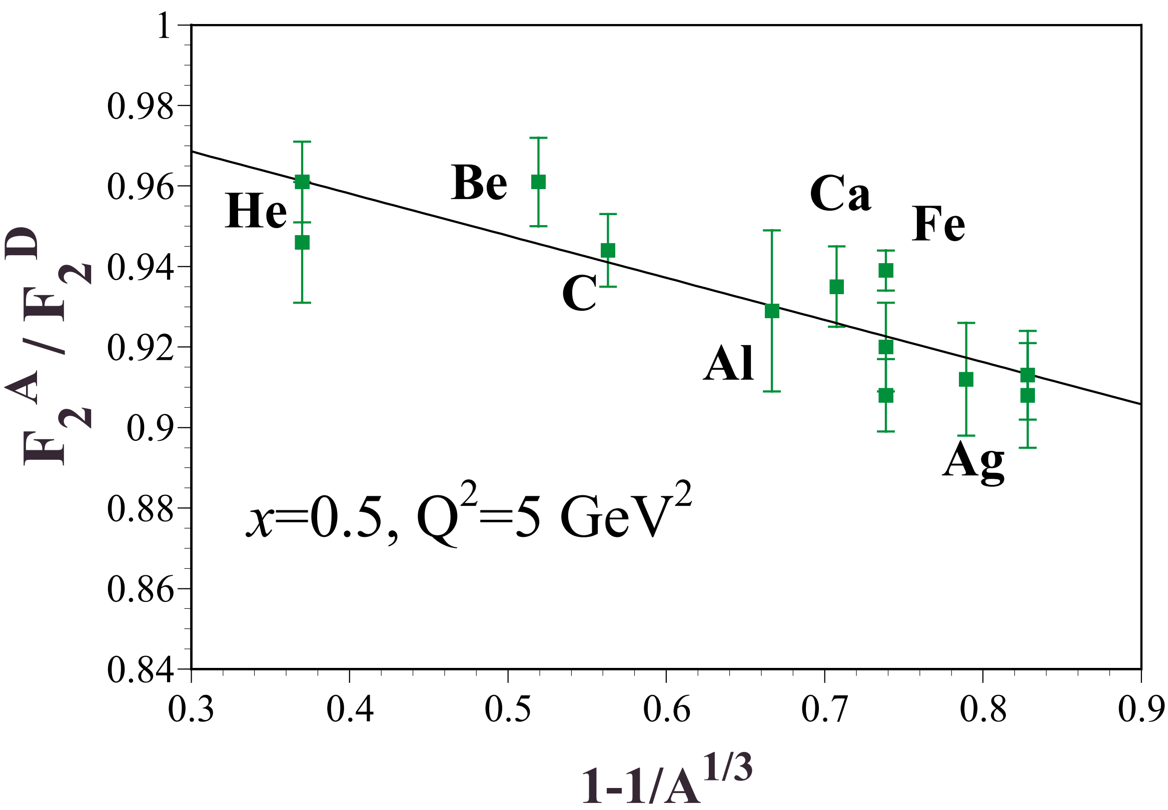
Ref. I. Sick and D. Day, Phys. Lett. B 274 (1992)



roughly speaking $\sigma_A = A\,\sigma_V + A^{2/3}\,\sigma_S$

$$\rightarrow \frac{\sigma_A}{A} = \sigma_V + \frac{1}{A^{1/3}}\,\sigma_S$$

$\sim \frac{1}{A^{1/3}}$ dependence



Functional form of $w_i(x,A)$

$$f_i^A(x) = w_i(x,A)\, f_i(x), \quad i = u_v, d_v, \bar{q}, g$$

first, assume the A dependence as $1/A^{1/3}$

then, use

$$w_i(x,A) = 1 + (1-1/A^{1/3}) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}}$$

a_i, b_i, c_i, d_i, β_i: parameters to be determined by χ^2 analysis

Fermi motion: $\dfrac{1}{(1-x)^{\beta_i}} \rightarrow \infty$ as $x \rightarrow 1$ if $\beta_i > 0$

Shadowing: $w_i(x \rightarrow 0, A) = 1 + (1-1/A^{1/3})\, a_i < 1$

Fine tuning: b_i, c_i, d_i

Constraints

- **Nuclear charge**

$$Z = A \int dx \left[\frac{2}{3}(u^A - \bar{u}^A) - \frac{1}{3}(d^A - \bar{d}^A) - \frac{1}{3}(s^A - \bar{s}^A) \right]$$

$$= A \int dx \left(\frac{2}{3} u_V^A - \frac{1}{3} d_V^A \right)$$

- **Baryon number**

$$A = A \int dx \, \frac{1}{3} (u_V^A + d_V^A)$$

- **Momentum**

$$A = A \int dx \, x \, (u_V^A + d_V^A + 6 \, \bar{q}^A + g^A)$$

Three parameters can be determined by these conditions.

Structure function in leading order

$$F_2^A = \sum_i e_i^2 \, x \, (q_i^A + \bar{q}_i^A) = x \left(\frac{4}{9} u_V^A + \frac{1}{9} d_V^A + \frac{12}{9} \bar{q} \right)$$

$$f_i^A(x) = w_i(x,A) \cdot f_i(x)$$

$$w_i(x,A) = 1 + (1 - 1/A^{1/3}) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}}$$

parameters: $\cancel{a_{u_V}}$, $\cancel{a_{d_V}}$, b_V, c_V, $\xcancel{d_V}$, β_V,
$a_{\bar{q}}$, $b_{\bar{q}}$, $c_{\bar{q}}$, $\xcancel{d_{\bar{q}}}$, $\cancel{\beta_{\bar{q}}}$, $\cancel{a_g}$, $\bcancel{b_g}$, c_g, $\xcancel{d_g}$, $\cancel{\beta_g}$

: nuclear charge, baryon #, momentum

: $d_V = d_{\bar{q}} = d_g = 0$ in the quadratic fit

: $d_g = 0$ in the cubic fit

: $\beta_{\bar{q}} = \beta_g = 1$: $\bar{q}$ & g cannot be determined in the large x region at this stage

: $b_g = -2\,c_g$, g(x) shape cannot be determined at medium and large x

7 or 9 parameters for 309 data points

Experimental data on F_2^A/F_2^D

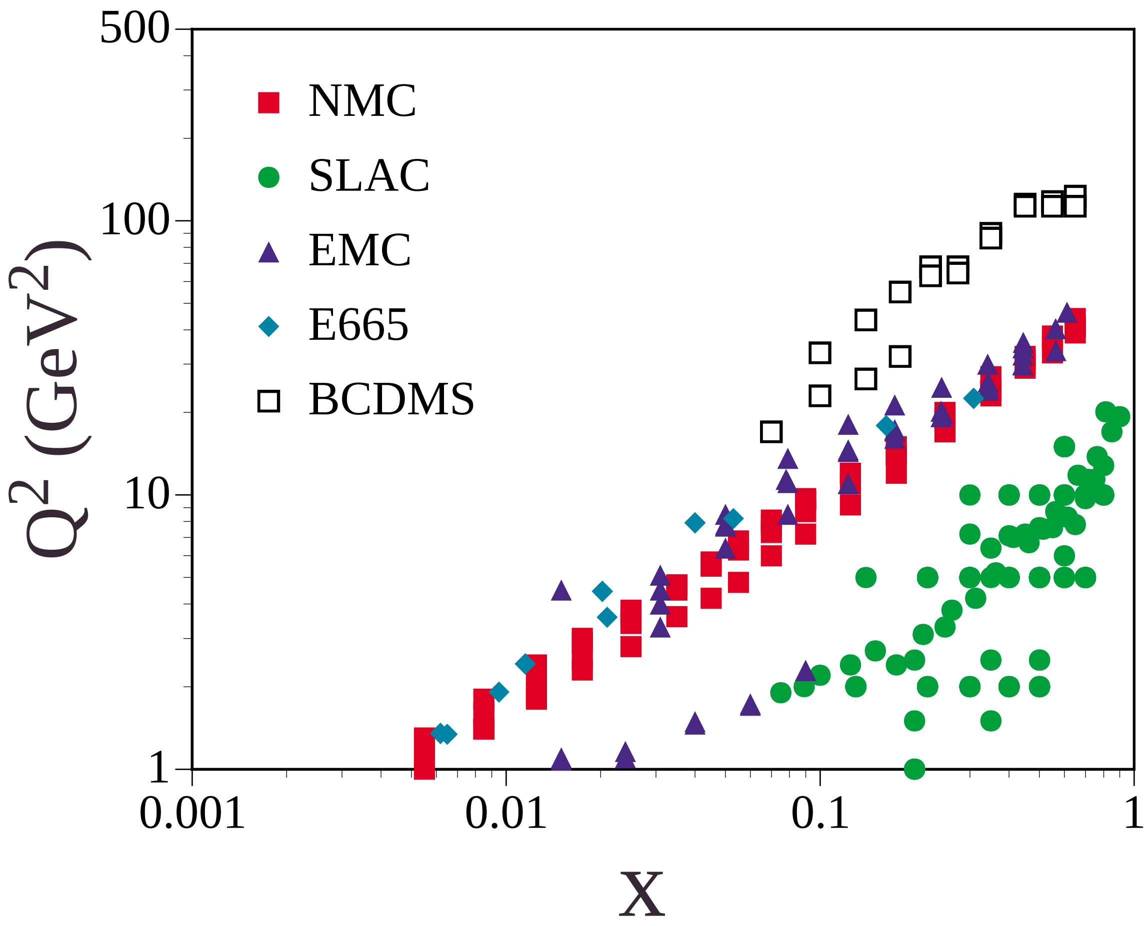
NMC: He, Li, C, Ca

SLAC: He, Be, C, Al, Ca, Fe, Ag, Au

EMC: C, Ca, Cu, Sn

E665: C, Ca, Xe, Pb

BCDMS: N, Fe



Analysis conditions

- parton distributions in the nucleon

  MRST98 - LO (Λ_{QCD}=174 MeV)

- Q^2 point at which the parametrized distributions are defined: $Q^2 = 1$ GeV^2

- used experimental data: $Q^2 \geq 1$ GeV^2

- total number of data: 309

- number of flavor: $n_f = 3$

- subroutine for the χ^2 analysis: CERN - Minuit

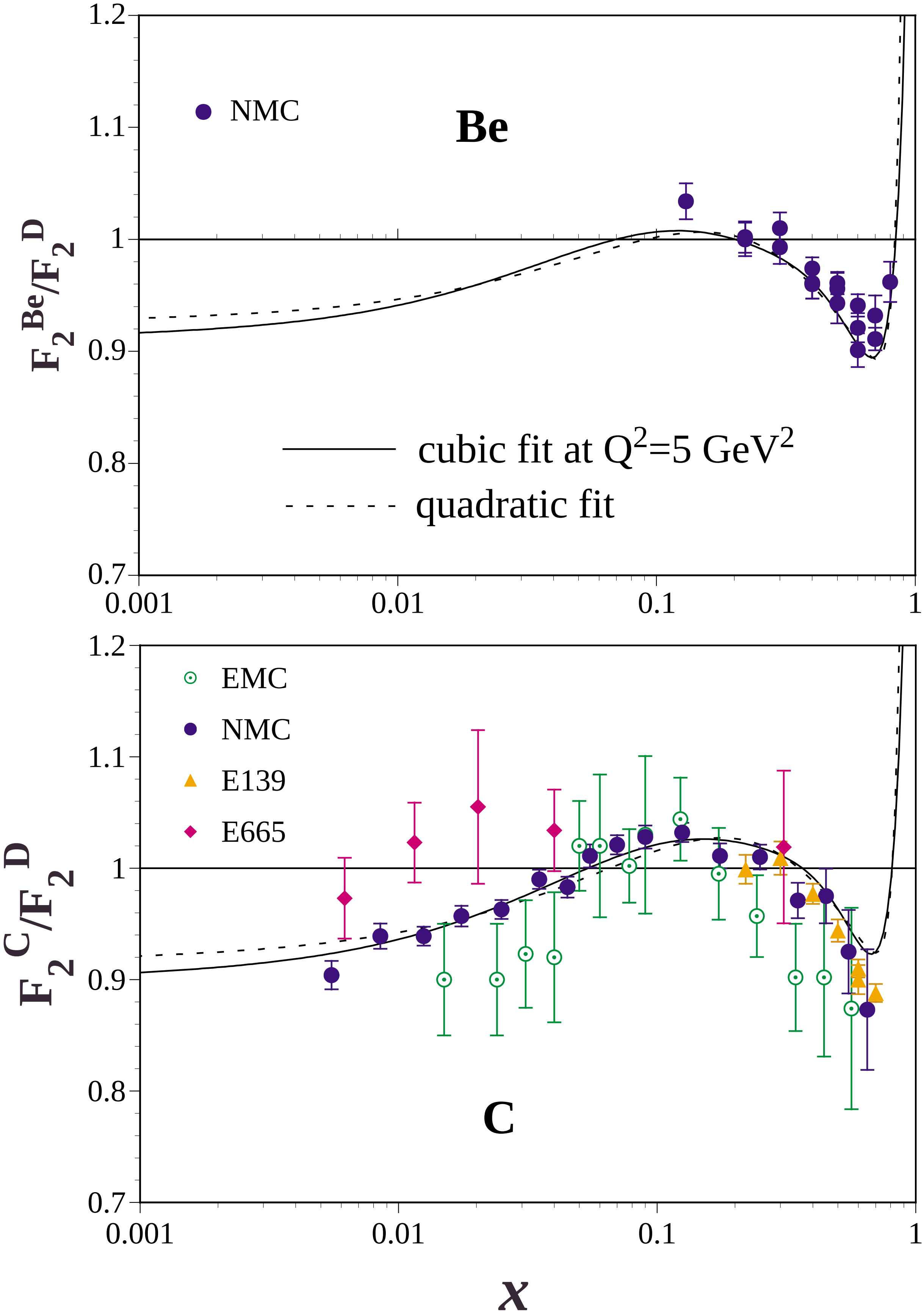
$$\chi^2 = \sum_i \frac{(R_i^{data} - R_i^{calc})^2}{(\sigma_i^{data})^2}$$

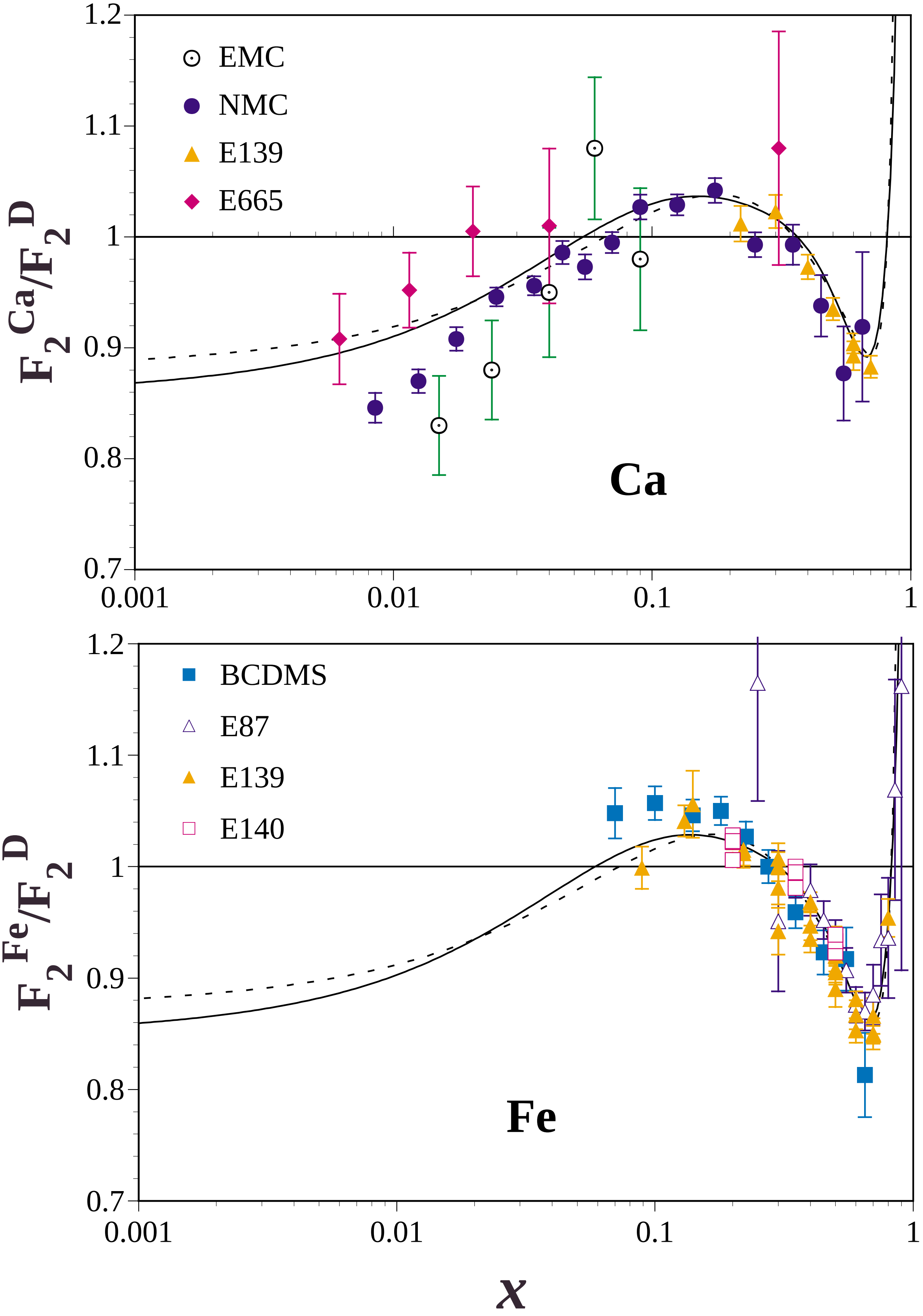
$$R = \frac{F_2^A}{F_2^D}, \qquad \sigma_i^{data} = \sqrt{(\sigma_i^{sys})^2 + (\sigma_i^{stat})^2}$$

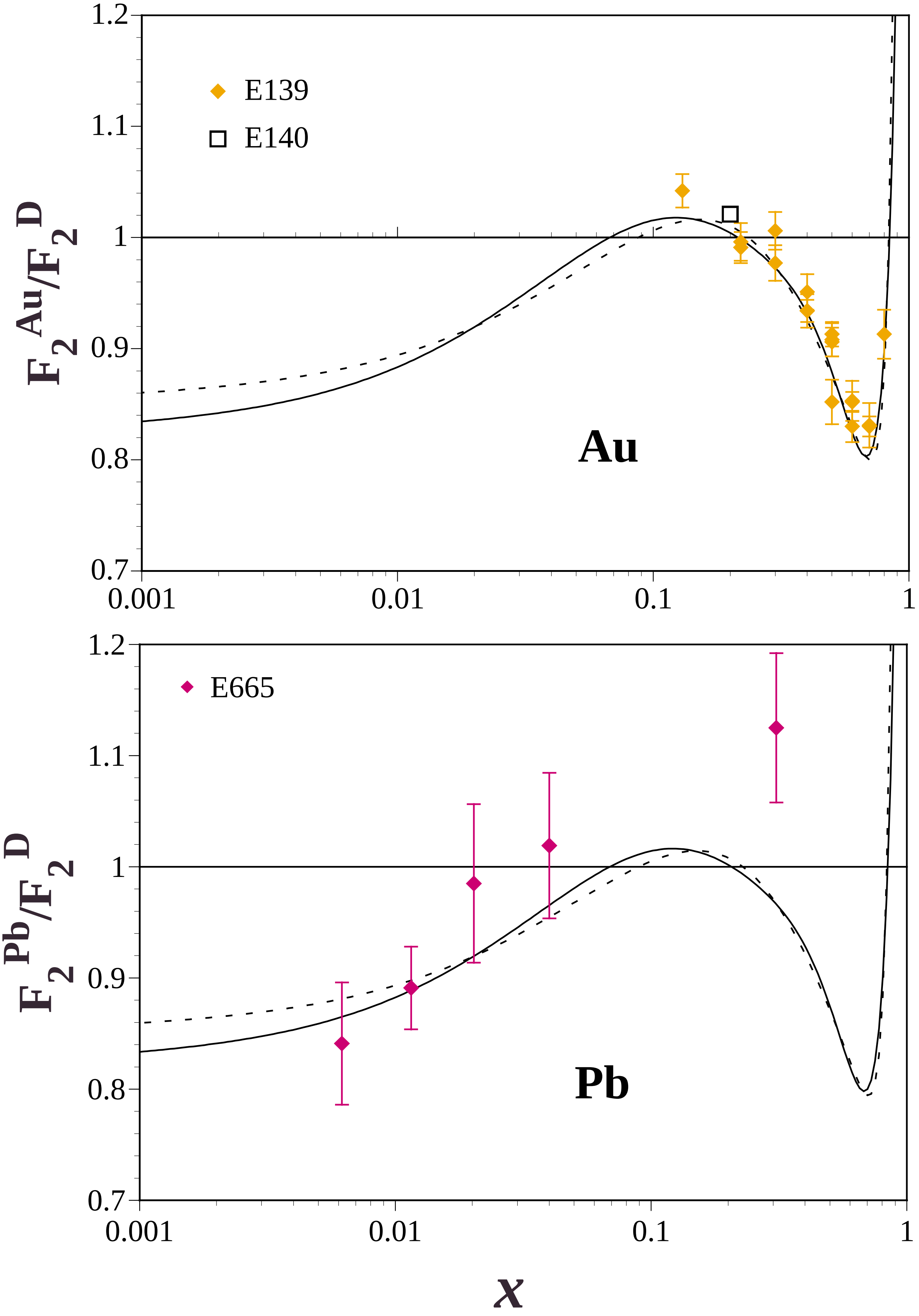
$\rightarrow$ obtained χ^2_{min}/d.o.f. = 583.7 / 302 (quadratic)

= 546.6 / 300 (cubic)

Analysis results



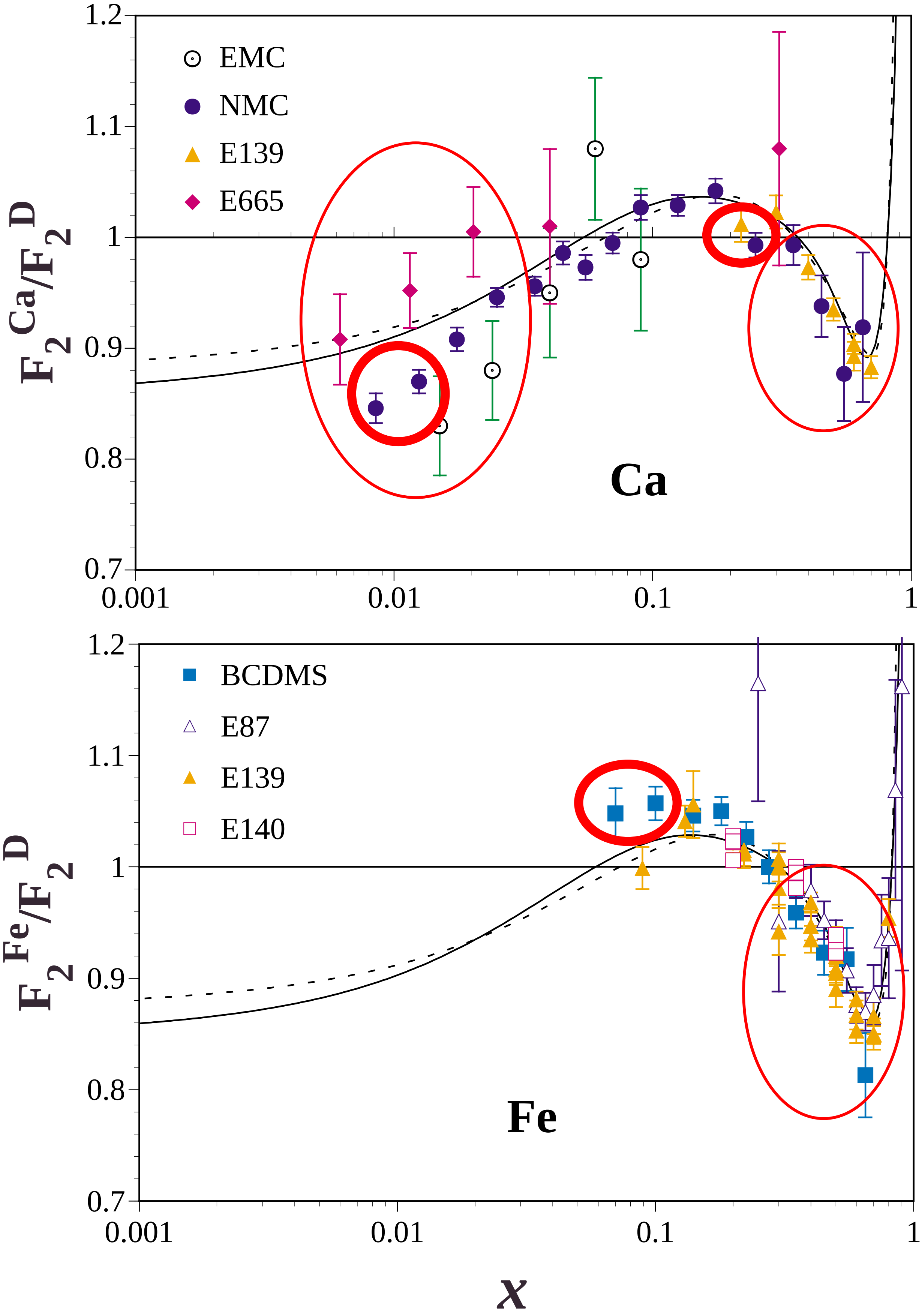

EMC
NMC
E139
E665
Ca
F_2^{Ca}/F_2^D
BCDMS
E87
E139
E140
Fe
F_2^{Fe}/F_2^D
x
1.2
1.1
1
0.9
0.8
0.7
0.001
0.01
0.1
1



1.2
1.1
1
0.9
0.8
0.7
E139
E140
Au
0.001
0.01
0.1
1
F_2^{Au}/F_2^D
E665
Pb
F_2^{Pb}/F_2^D
x

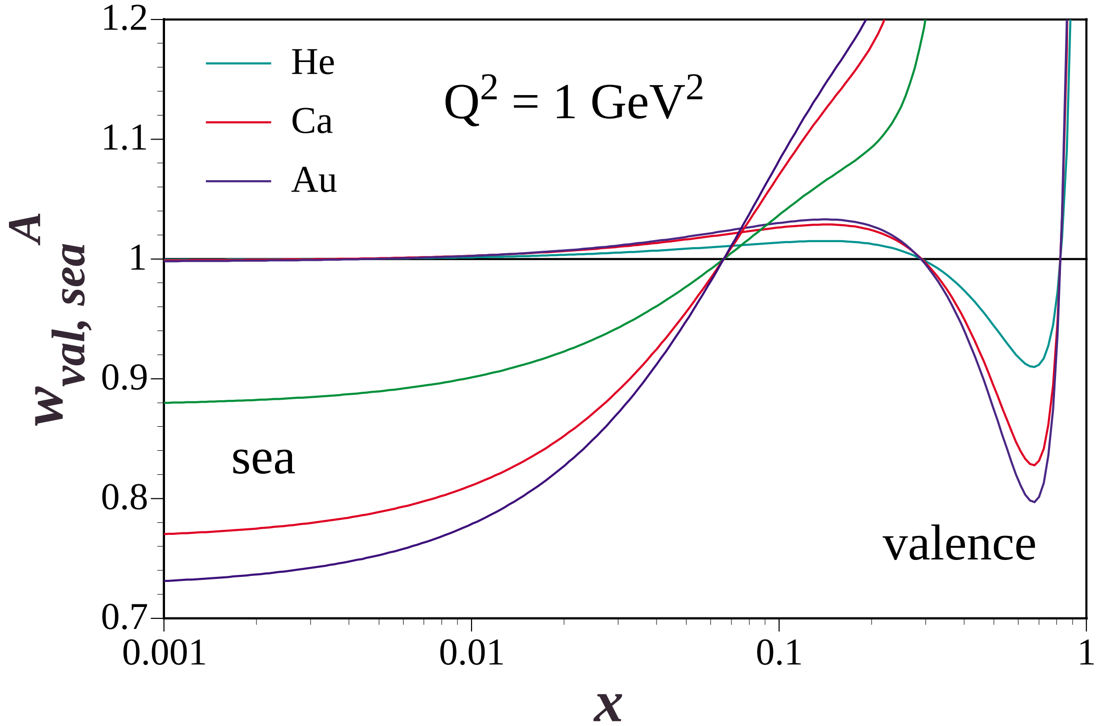
χ^2 contributions

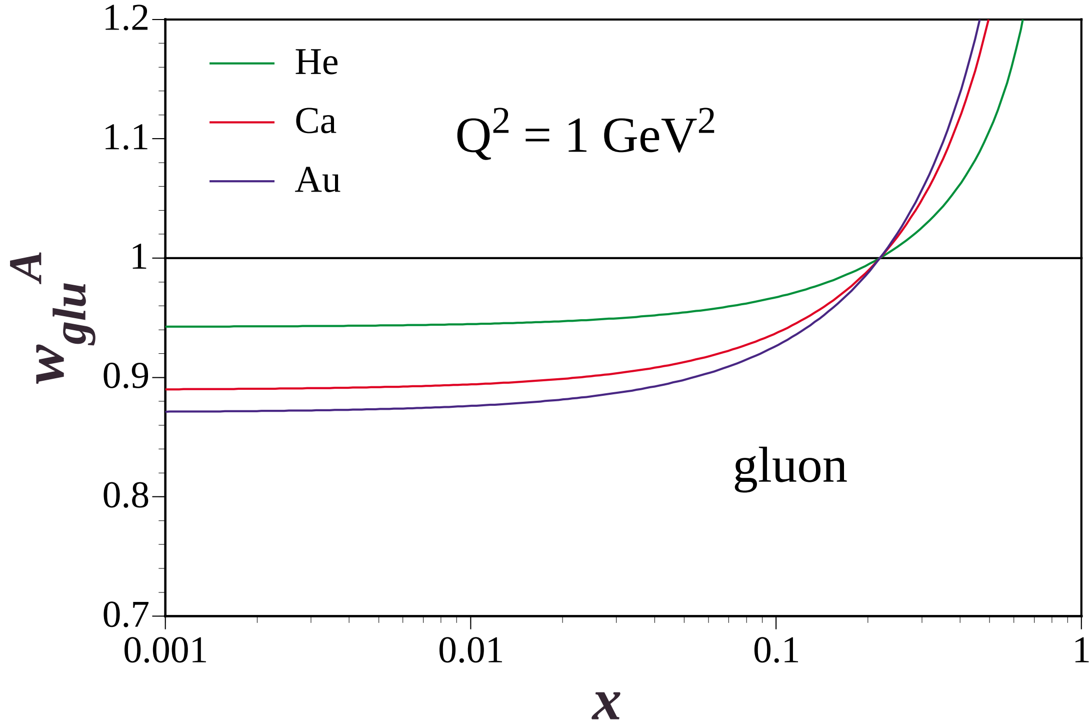
A	# of data	$\chi2$ quad.	$\chi2$ cubic
He	35	55.6	54.5
Li	17	45.6	49.2
Be	17	39.7	38.4
C	43	97.8	88.2
N	9	10.5	10.4
Al	35	38.8	41.4
Ca	33	72.3	69.7
Fe	57	115.7	92.7
Cu	19	13.7	13.6
Ag	7	12.7	11.5
Sn	8	14.8	17.7
Xe	5	3.2	2.4
Au	19	55.5	49.2
Pb	5	7.9	7.6
total	309	583.7	546.6



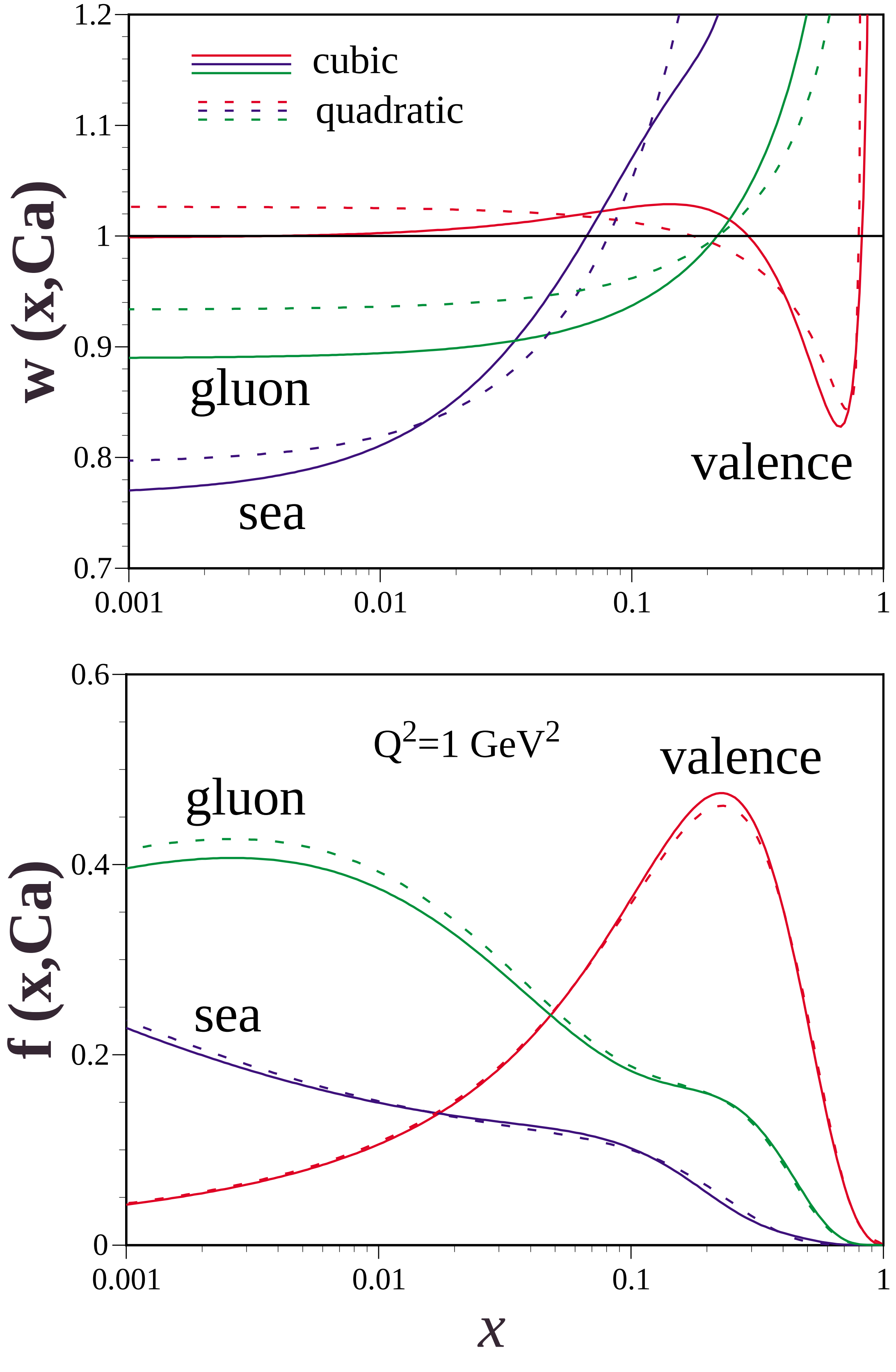
EMC
NMC
E139
E665
Ca
F_2^{Ca}/F_2^D
BCDMS
E87
E139
E140
Fe
F_2^{Fe}/F_2^D
1.2
1.1
1
0.9
0.8
0.7
0.001
0.01
0.1
1
x

Weight functions $w_i(x,A)$ for He, Ca, Au





Weight functions and parton distributions for Ca



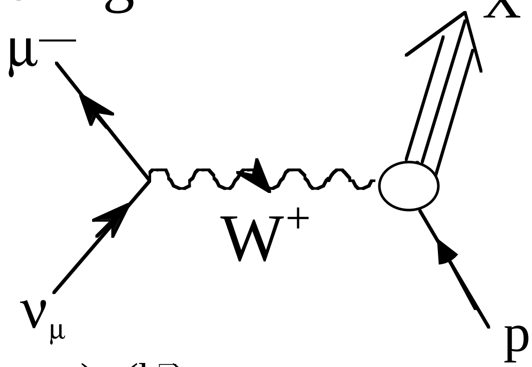
Determination of each distribution

Valence quark

note: $q_v \equiv q - \bar{q}$

neutrino deep inelastic scattering

$$\nu_\mu + p \to \mu^- + X$$



$$L^{\mu\nu} = \sum_{\text{spins}} \bar{u}(k)\, \gamma^\mu (1-\gamma_5)\, u(k) \cdot \bar{u}(k')\, \gamma^\nu (1-\gamma_5)\, u(k')$$

$$= 2\left[k^\mu k'^\nu + k^\nu k'^\mu - g^{\mu\nu} k\cdot k' + i\varepsilon^{\mu\nu\rho\sigma} k_\rho k'_\sigma \right]$$

$$W_{\mu\nu} = -W_1 \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + W_2 \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right)$$

$$\boxed{-\frac{i}{M} W_3\, \varepsilon_{\mu\nu\rho\sigma}\, p^\rho q^\sigma}$$

$$\frac{d\sigma^{W^\pm}}{dE'\, d\Omega} = \frac{G_F^2 E'^2}{2\pi^2 (1+Q^2/M_W^2)^2}$$

$$\times \left[W_2 \cos^2\frac{\theta}{2} + 2W_1 \sin^2\frac{\theta}{2} \mp \frac{E+E'}{M} W_3 \sin^2\frac{\theta}{2} \right]$$

parity violation: $W_3 \propto \sigma_L - \sigma_R$, $F_3 = \nu W_3$

in parton model

$$F_3^{\nu p} = 2(d + s - \bar{u} - \bar{c}), \quad F_3^{\bar{\nu} p} = 2(u + c - \bar{d} - \bar{s})$$

$$F_3^p = \frac{F_3^{\nu p} + F_3^{\bar{\nu} p}}{2} = u_v + d_v \quad \text{if } s=\bar{s},\ c=\bar{c}$$

Sea quark

e/μ scattering

$$F_2^p = \frac{4}{9}\,x\,(u+\bar{u}) + \frac{1}{9}\,x\,(d+\bar{d}+s+\bar{s})$$

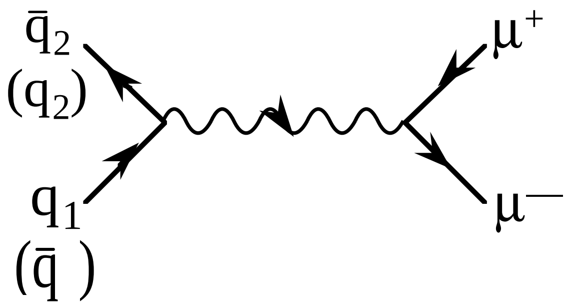
$$F_2^n = \frac{4}{9}\,x\,(d+\bar{d}) + \frac{1}{9}\,x\,(u+\bar{u}+s+\bar{s})$$

$$F_2^N = \frac{F_2^p+F_2^n}{2} = \frac{5}{18}\,x\,(u+\bar{u}+d+\bar{d}) + \frac{2}{18}\,x\,(s+\bar{s})$$

$$= \underline{\frac{5}{18}\,x\,V} + \underline{\frac{4}{18}\,x\,S} \quad \text{if SU(3) symmetric sea}$$

Drell-Yan (lepton-pair production)

$$p_1 + p_2 \rightarrow \mu^+\mu^- + X$$



$$d\sigma \propto q(x_1)\bar{q}(x_2) + \bar{q}(x_1)q(x_2)$$

at large $x_F = x_1 - x_2$

projectile target

$$d\sigma \propto q_v(x_1)\bar{q}(x_2)$$

$\bar{q}(x_2)$ can be obtained if $q_v(x_1)$ is known

Gluon

scaling violation of F_2

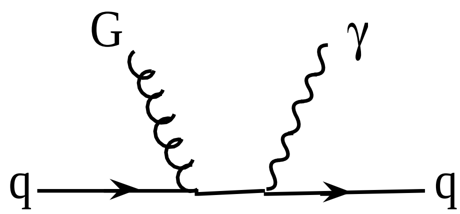
$$\frac{\partial}{\partial(\ln Q^2)} q_i(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right) q_i(y,Q^2) + P_{qG}\left(\frac{x}{y}\right) G(y,Q^2) \right]$$

$$\frac{\partial}{\partial(\ln Q^2)} G(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{Gq}\left(\frac{x}{y}\right) q_i(y,Q^2) + P_{GG}\left(\frac{x}{y}\right) G(y,Q^2) \right]$$

at small x $\quad \dfrac{\partial F_2}{\partial(\ln Q^2)} \approx \dfrac{10\ \alpha_s}{27\pi} G$

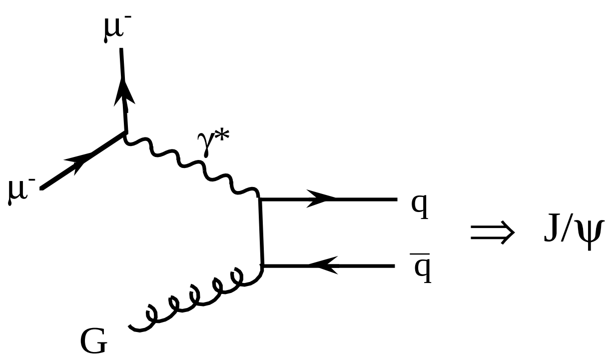
direct photon

$$p + p \to \gamma + X$$



J/ψ production

$$\mu + p \to \mu + J/\psi + X$$



Future possibilities

- Valence: F_3 at ν factories
  F_2 at eRHIC, HERA-eA
- Antiquark: Drell-Yan at RHIC, LHC,
  Fermilab
  F_2 at eRHIC, HERA-eA
- Gluon: direct γ etc. at RHIC, LHC
  $dF_2/\ln(Q^2)$ at HERA-eA

Summary

- first χ^2 analysis

  for the nuclear parton distributions

  Computer codes could be obtained from

  http://www-hs.phys.saga-u.ac.jp.

- nuclear PDFs are still premature

  $\rightarrow$ need analysis refinement

- reasonably good fit with

  χ^2_{min} / d.o.f. = 1.93 (quad.), 1.82 (cubic)

- $g^A(x)$? $\bar{q}^A(x)$ at medium x ?

- $q_V(x)$ in the nucleon is determined

  by ν-Fe scattering at this stage

  $\rightarrow$ reinvestigate with nuclear modification

- relation to heavy-ion physics ?
- future experiments ?