

Studies of parton distributions at a neutrino factory

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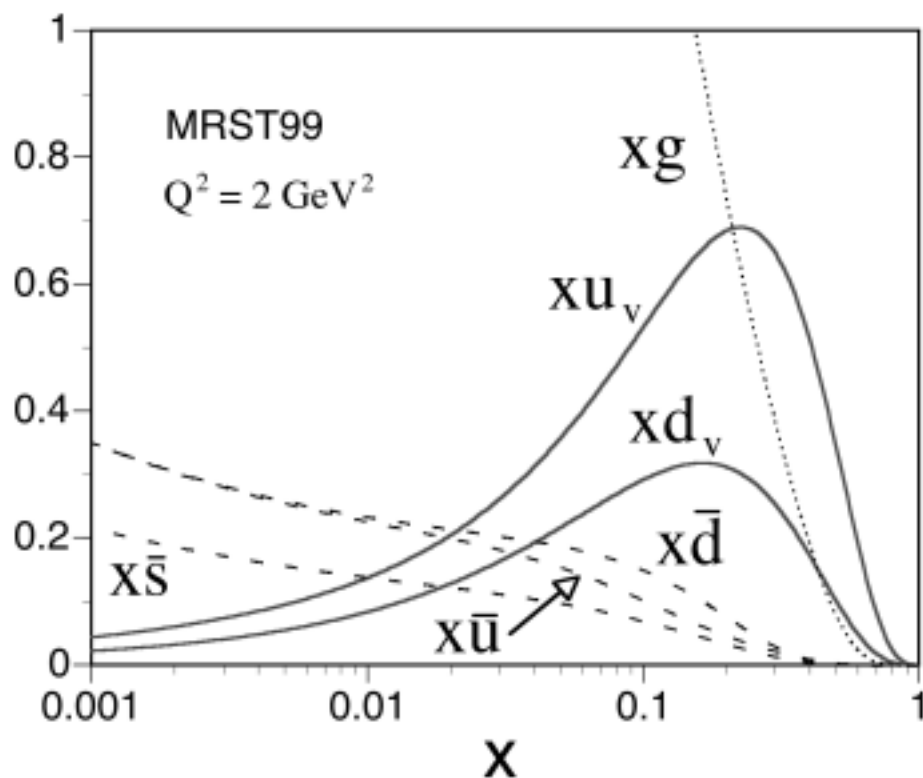
(3) Other topics at a ν factory

- polarized distributions
- $\bar{u}/\bar{d}$, $\Delta\bar{u}/\Delta\bar{d}$ asymmetries
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Recent unpolarized distributions

- CTEQ5, Eur. Phys. J. C12 (2000) 375
- GRV98, Eur. Phys. J. C5 (1998) 461
- MRST99, Eur. Phys. J. C14 (2000) 133



Neutrino data are important for determining valence-quark distributions in the “*nucleon*”.

However, the used CCFR target is iron!?

What can be done at a ν factory?

- unpolarized distributions in the *nucleon*

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dE_{\square} d\Omega} = \frac{G_F^2 E'^2}{2 \pi^2 (1+Q^2/M_W^2)^2} \times \left[\frac{F_2}{\nu} \cos^2 \frac{\theta}{2} + \frac{2F_1}{\nu} \sin^2 \frac{\theta}{2} \mp \frac{E+E_{\square}}{M^2} F_3 \sin^2 \frac{\theta}{2} \right]$$

in parton model

$$F_2 = 2 \times F_1$$

$$F_2^{\nu p} = 2 \times (d + s + \bar{u} + \bar{c})$$

$$F_2^{\bar{\nu} p} = 2 \times (u + c + \bar{d} + \bar{s})$$

$$F_2^{\nu n} = 2 \times (u + s + \bar{d} + \bar{c})$$

$$F_2^{\bar{\nu} n} = 2 \times (d + c + \bar{u} + \bar{s})$$

$$xF_3^{\nu p} = 2 \times (d + s - \bar{u} - \bar{c})$$

$$xF_3^{\bar{\nu} p} = 2 \times (u + c - \bar{d} - \bar{s})$$

$$xF_3^{\nu n} = 2 \times (u + s - \bar{d} - \bar{c})$$

$$xF_3^{\bar{\nu} n} = 2 \times (d + c - \bar{u} - \bar{s})$$

parity-violation
terms



$$F_3^{\nu(p+n)/2} + F_3^{\bar{\nu}(p+n)/2} = 2 (u_v + d_v) + 2 (s - \bar{s}) + 2 (c - \bar{c})$$

$$F_3^{\nu(p+n)/2} - F_3^{\bar{\nu}(p+n)/2} = 2 (s + \bar{s}) - 2 (c + \bar{c})$$

valence-quark distributions

kinematical range

suppose $E_\nu = 30 \text{ GeV}$

$$x = \frac{Q^2}{2 M q_0}$$

$$x_{\min} = \frac{\min(Q^2)}{2 M \max(q_0)} = \frac{1}{2 \cdot 1 \cdot 30} = 0.017$$

where $\min(Q^2) \sim 1 \text{ GeV}^2$, $\max(q_0) = E_\nu$

Fe target $\rightarrow$ p, d, ^3He targets
with intense ν beam



accurate determination of u_ν and d_ν
in the “*nucleon*”



accurate determination of $\bar{q}$ and g



important for finding any exotic
signature beyond current theoretical
framework

Parton Distributions in Nuclei

References

Nuclear parametrization, F_3

- M. Hirai, SK, and M. Miyama,
hep-ph/0103208, to be published in Phys. Rev. D.
- R. Kobayashi, SK, and M. Miyama,
Phys. Lett. B354, 465 (1995).

Purposes

Determination of parton distributions

- unpolarized distributions in the nucleon
3 major groups (CTEQ, GRV, MRS)
- polarized distributions in the nucleon
several groups (GS, GRSV, ..., AAC)
- distributions in nuclei
no χ^2 analysis! (see Eskola, Kolhinen, Ruuskanen)

→ for understanding nuclear mechanisms
in the high-energy region

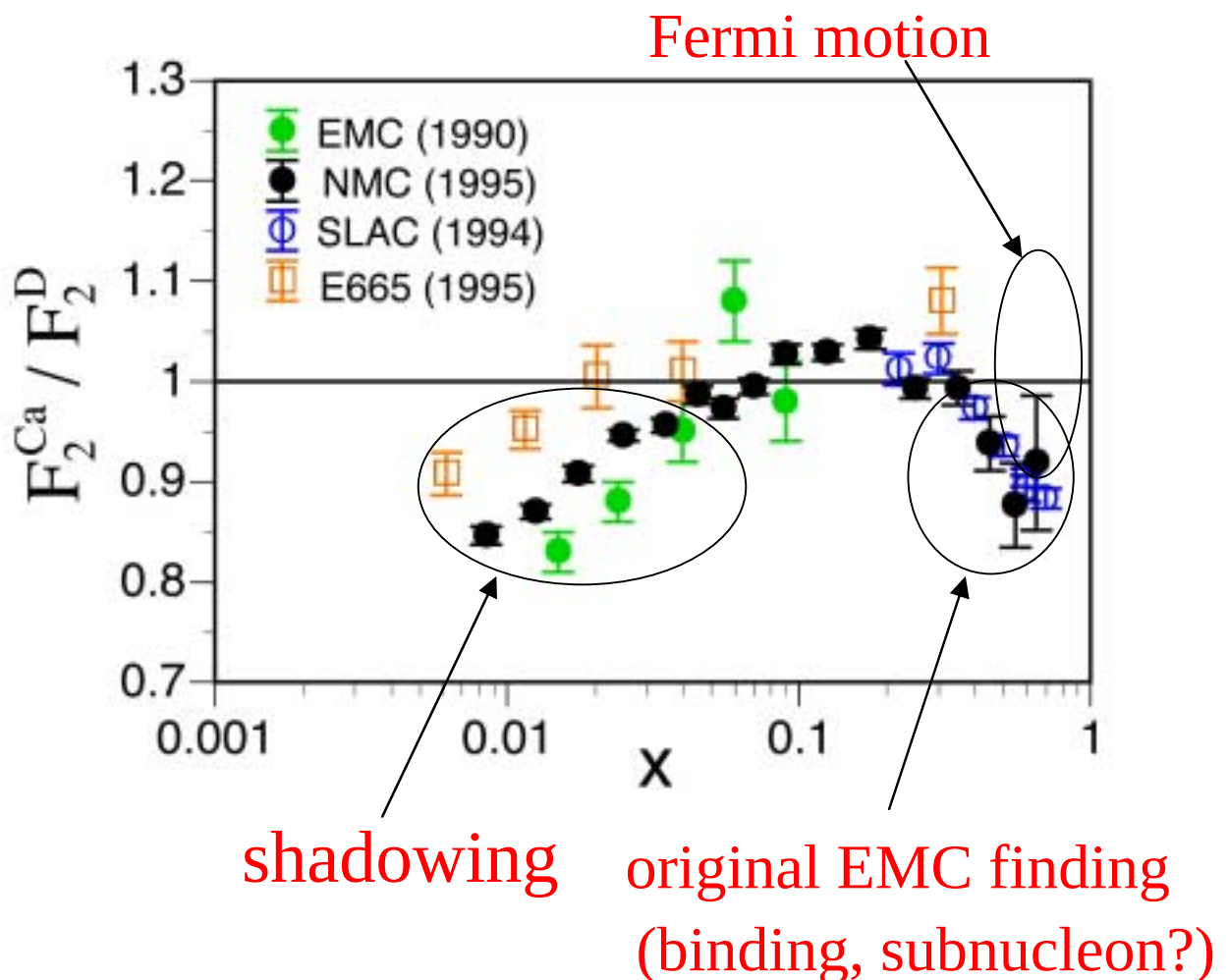
→ for heavy-ion physics

→ re-examination of $q_v(x)$ in the nucleon

- parton distributions in nuclei

Nuclear modification of F_2^A / F_2^D is well known in electron/muon scattering.

$$F_2^A = \frac{1}{9} \times \left[4 u_v(x) + d_v(x) \right]_A + \frac{2}{9} \times S_A(x)$$



However, little is known about F_3 (namely valence-quark) modification at small x due to lack of accurate deuteron data.

Parametrization of nuclear parton distributions

1. functional form of $f_i^A(x)$
2. constraints: A dependence,
conservation laws
3. χ^2 fitting to experimental data
4. results: optimum nuclear parton
distributions

Ref. hep-ph/0103208 (Phys. Rev. D)

- Nuclear parton distributions (per nucleon)
if there *were* no modification

$$A u^A = Z u^p + N u^n, \quad A d^A = Z d^p + N d^n$$

Isospin symmetry: $u^n = d^p \equiv d$, $d^n = u^p \equiv u$

$$\rightarrow u^A = \frac{Z u + N d}{A}, \quad d^A = \frac{Z d + N u}{A}$$

- Take into account the nuclear modification
by the factors $w_i(x, A)$

$$u_v^A(x) = w_{u_v}(x, A) \frac{Z u_v(x) + N d_v(x)}{A}$$

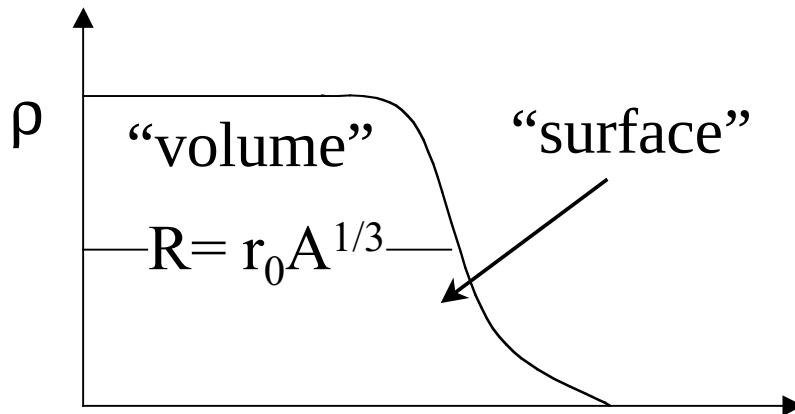
$$d_v^A(x) = w_{d_v}(x, A) \frac{Z d_v(x) + N u_v(x)}{A}$$

$$\bar{q}^A(x) = w_{\bar{q}}(x, A) \bar{q}(x)$$

$$g^A(x) = w_g(x, A) g(x)$$

A dependence

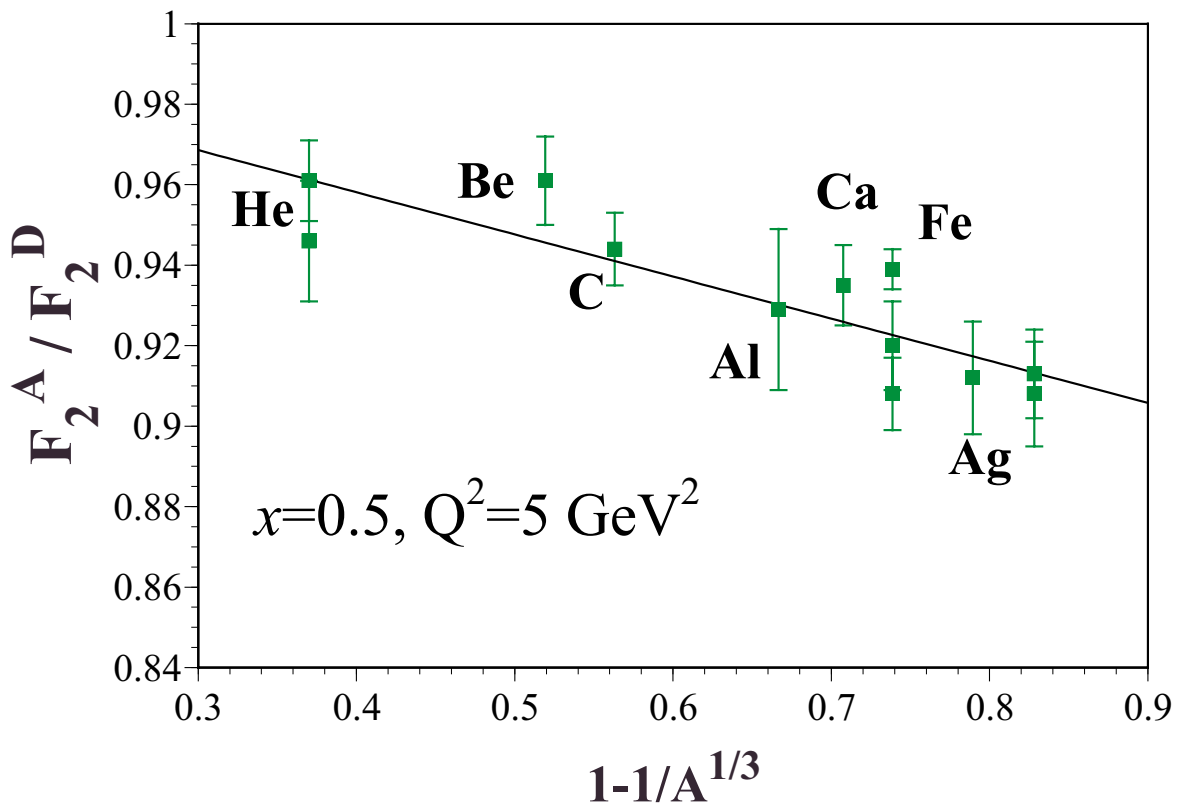
Ref. I. Sick and D. Day, Phys. Lett. B 274 (1992)



roughly speaking $\sigma_A = A \sigma_V + A^{2/3} \sigma_S$

$$\rightarrow \frac{\sigma_A}{A} = \sigma_V + \frac{1}{A^{1/3}} \sigma_S$$

$\sim \frac{1}{A^{1/3}}$ dependence



Functional form of $w_i(x,A)$

$$f_i^A(x) = w_i(x,A) f_i(x), \quad i = u_v, d_v, \bar{q}, g$$

first, assume the A dependence as $1/A^{1/3}$

then, use

$$w_i(x,A) = 1 + (1 - 1/A^{1/3}) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1 - x)^{\beta_i}}$$

$a_i, b_i, c_i, d_i, \beta_i$: parameters to be determined
by χ^2 analysis

Fermi motion: $\frac{1}{(1 - x)^{\beta_i}} \rightarrow \infty$ as $x \rightarrow 1$ if $\beta_i > 0$

Shadowing: $w_i(x \rightarrow 0, A) = 1 + (1 - 1/A^{1/3}) a_i < 1$

Fine tuning: b_i, c_i, d_i

Constraints

- **Nuclear charge**

$$\begin{aligned} Z &= A \int dx \left[\frac{2}{3}(u^A - \bar{u}^A) - \frac{1}{3}(d^A - \bar{d}^A) - \frac{1}{3}(s^A - \bar{s}^A) \right] \\ &= A \int dx \left(\frac{2}{3} u_v^A - \frac{1}{3} d_v^A \right) \end{aligned}$$

- **Baryon number**

$$A = A \int dx \frac{1}{3} (u_v^A + d_v^A)$$

- **Momentum**

$$A = A \int dx x (u_v^A + d_v^A + 6 \bar{q}^A + g^A)$$

Three parameters can be determined by these conditions.

Experimental data on F_2^A/F_2^D

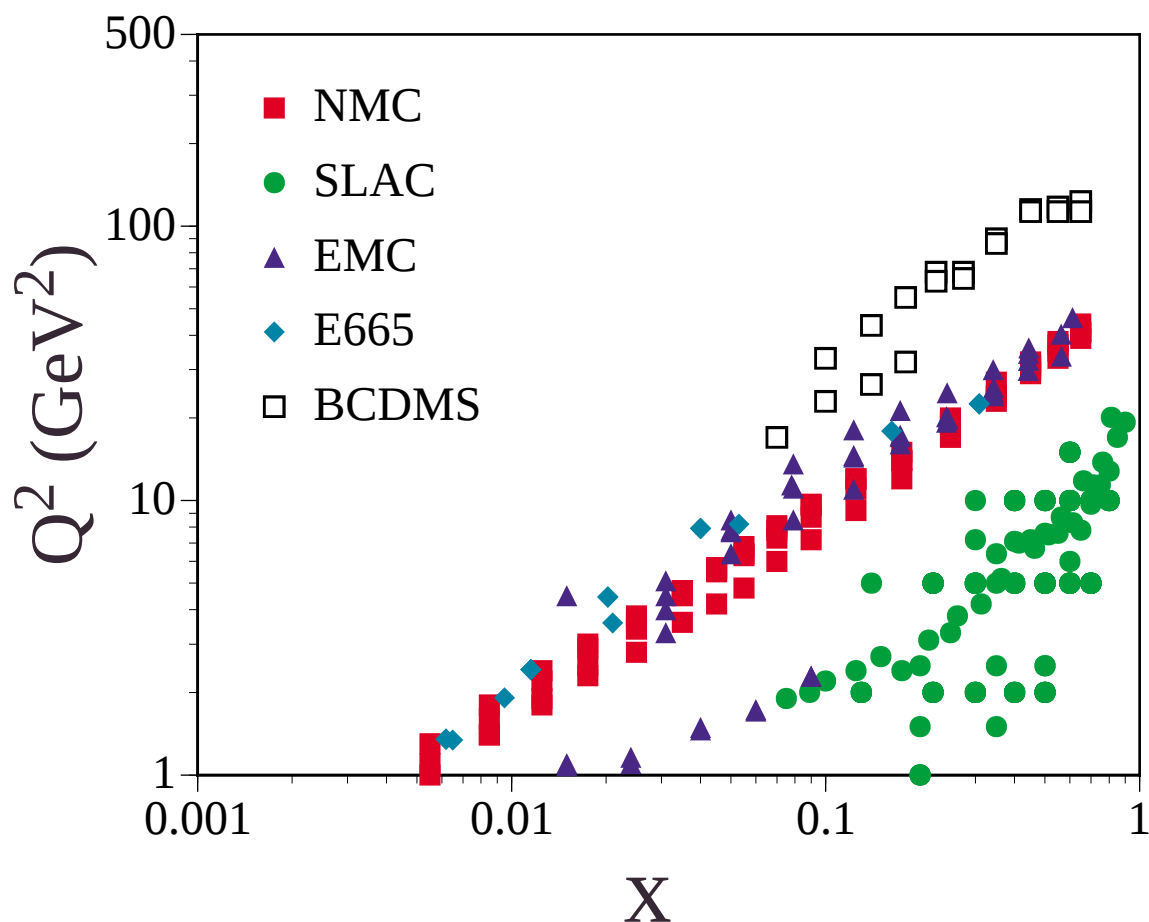
NMC: He, Li, C, Ca

SLAC: He, Be, C, Al, Ca, Fe, Ag, Au

EMC: C, Ca, Cu, Sn

E665: C, Ca, Xe, Pb

BCDMS: N, Fe



Analysis conditions

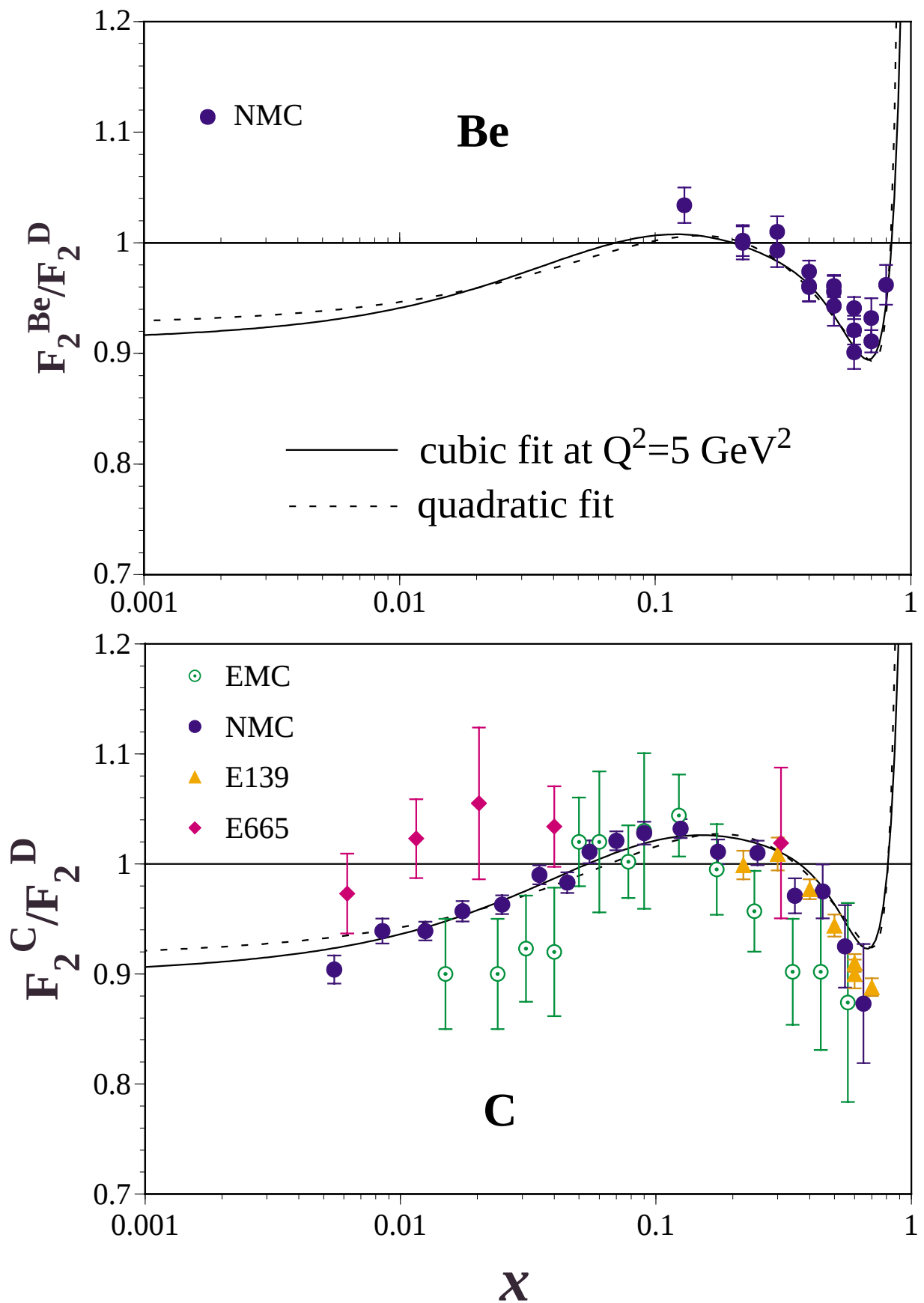
- parton distributions in the nucleon
MRST98 - LO ($\Lambda_{\text{QCD}}=174 \text{ MeV}$)
- Q^2 point at which the parametrized distributions are defined: **$Q^2 = 1 \text{ GeV}^2$**
- used experimental data: **$Q^2 \geq 1 \text{ GeV}^2$**
- total number of data: **309**
- number of flavor: **$n_f = 3$**
- subroutine for the χ^2 analysis: **CERN - Minuit**

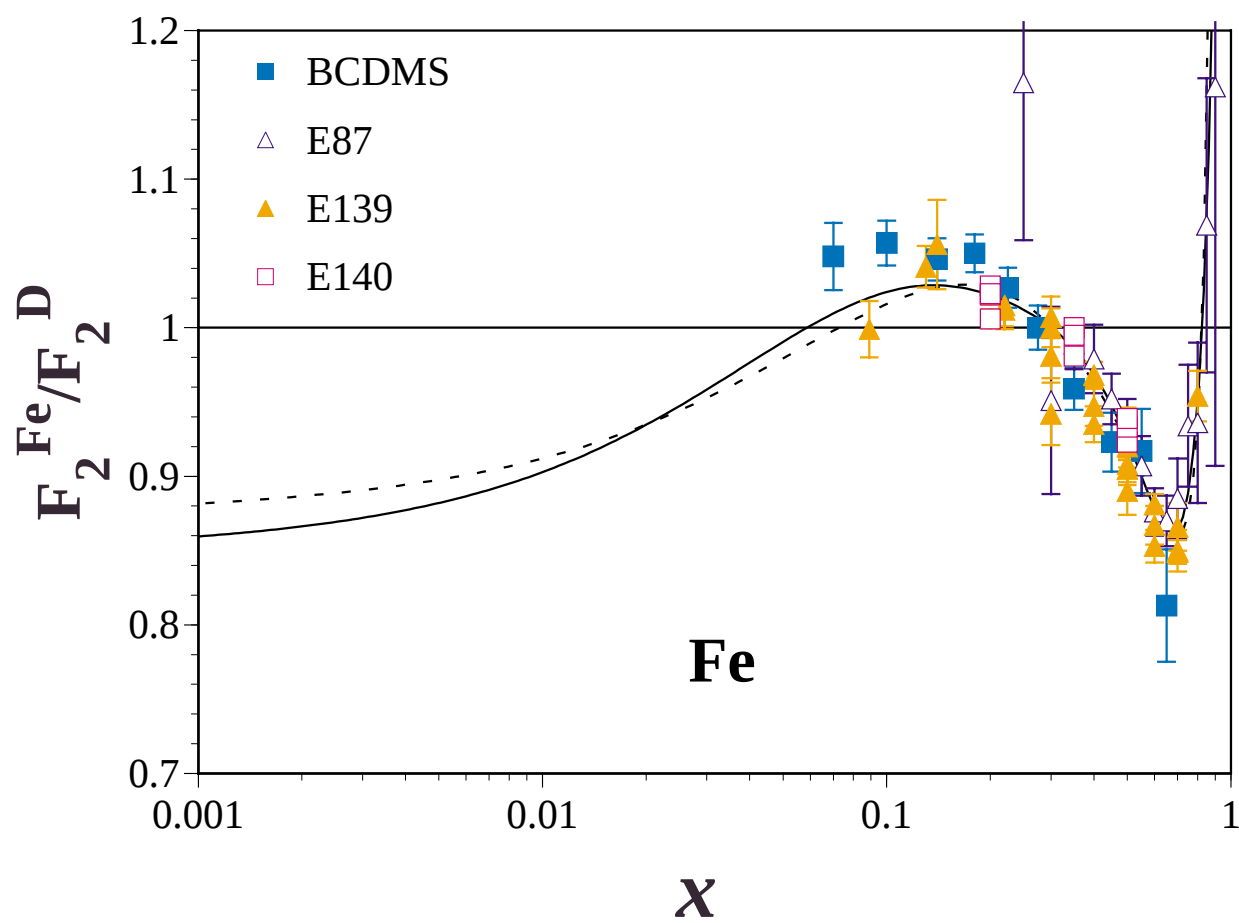
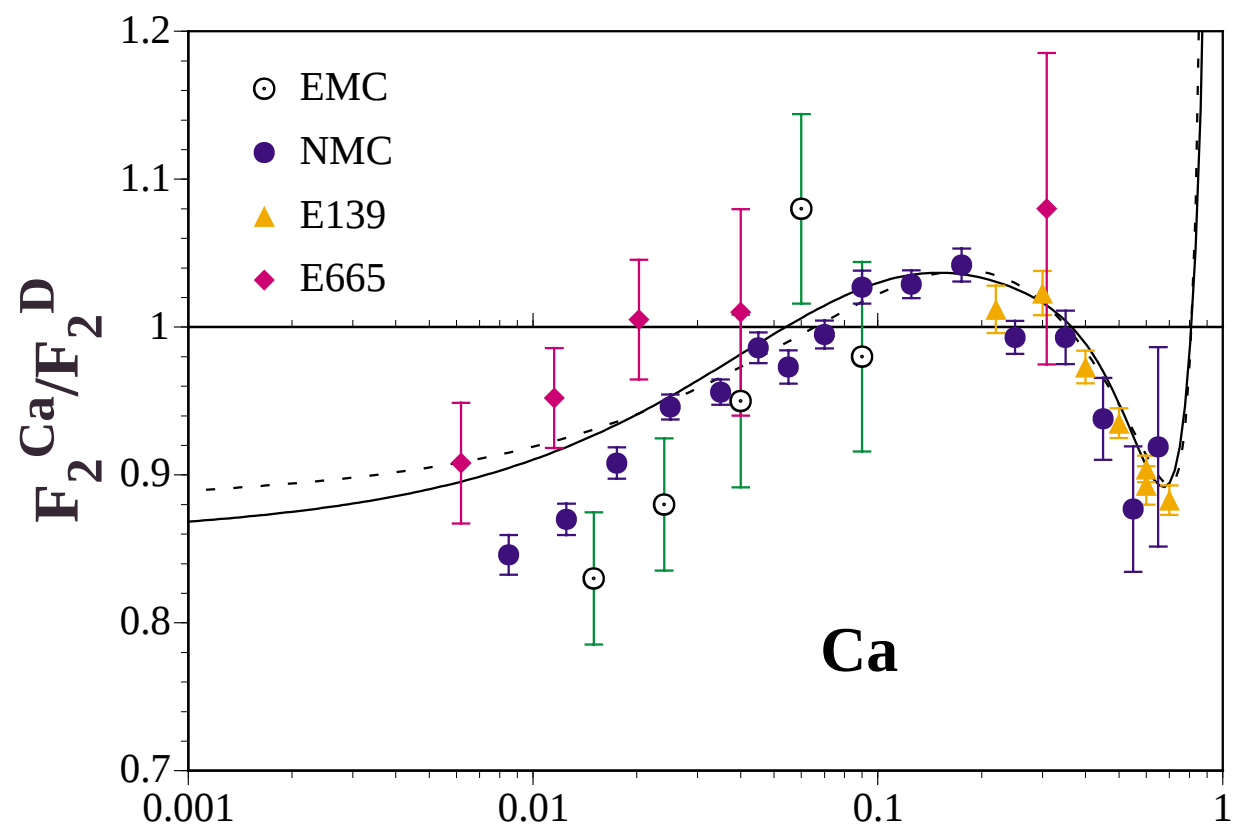
$$\chi^2 = \sum_i \frac{(R_i^{\text{data}} - R_i^{\text{calc}})^2}{(\sigma_i^{\text{data}})^2}$$

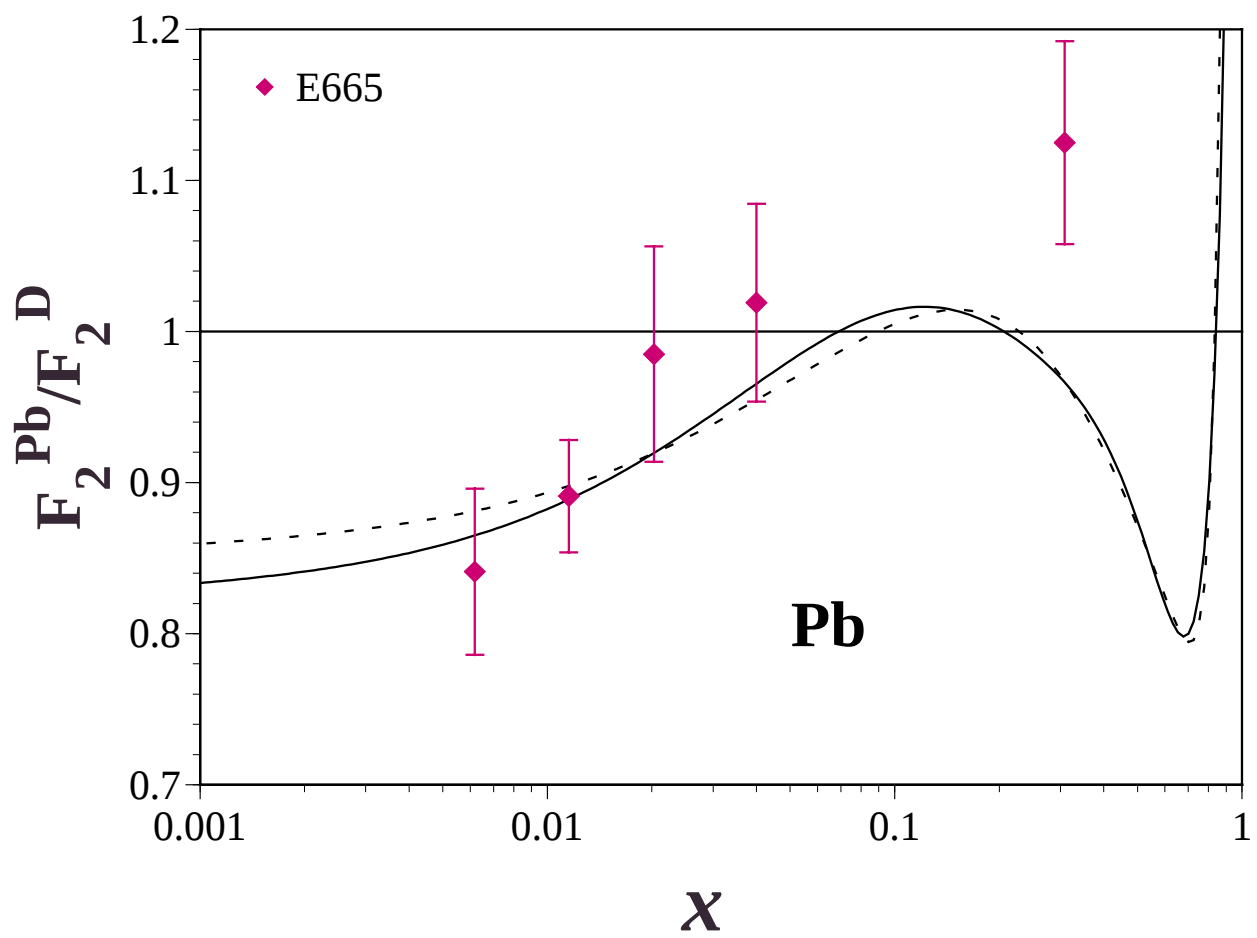
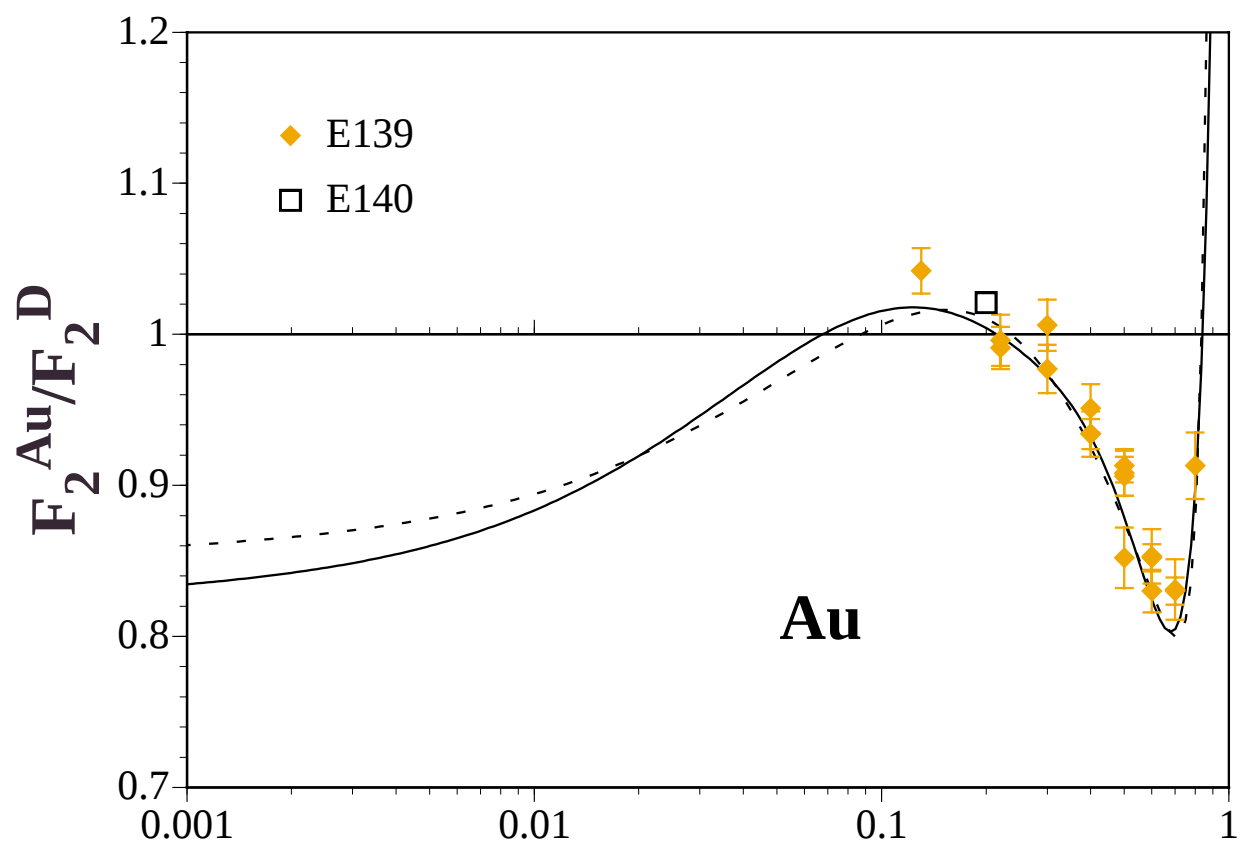
$$R = \frac{F_2^A}{F_2^D}, \quad \sigma_i^{\text{data}} = \sqrt{(\sigma_i^{\text{sys}})^2 + (\sigma_i^{\text{stat}})^2}$$

→ obtained $\chi_{\text{min}}^2/\text{d.o.f.} = 583.7 / 302$ (quadratic)
= 546.6 / 300 (cubic)

Analysis results



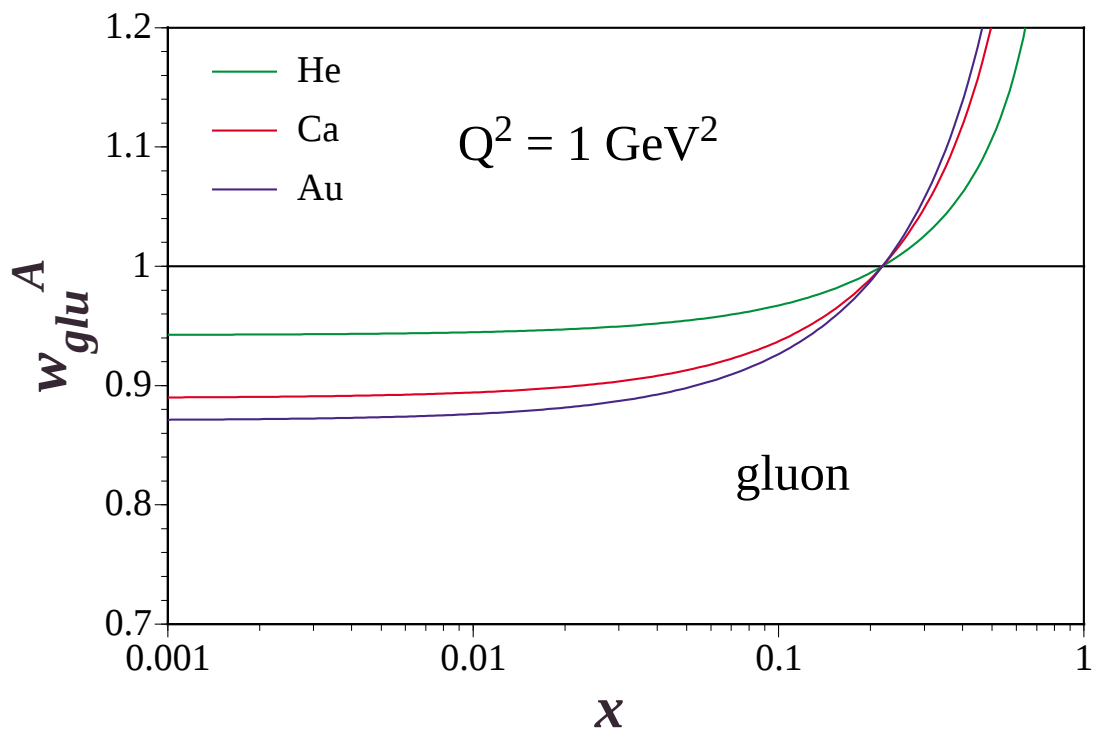
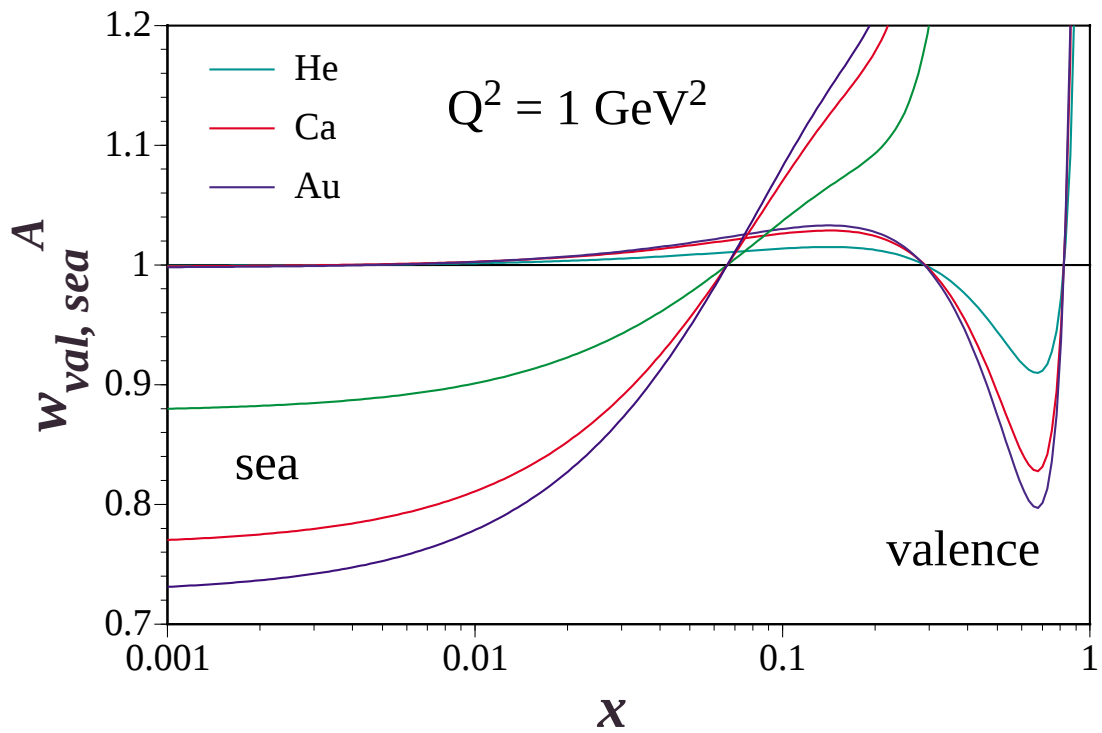




Weight functions $w_i(x,A)$

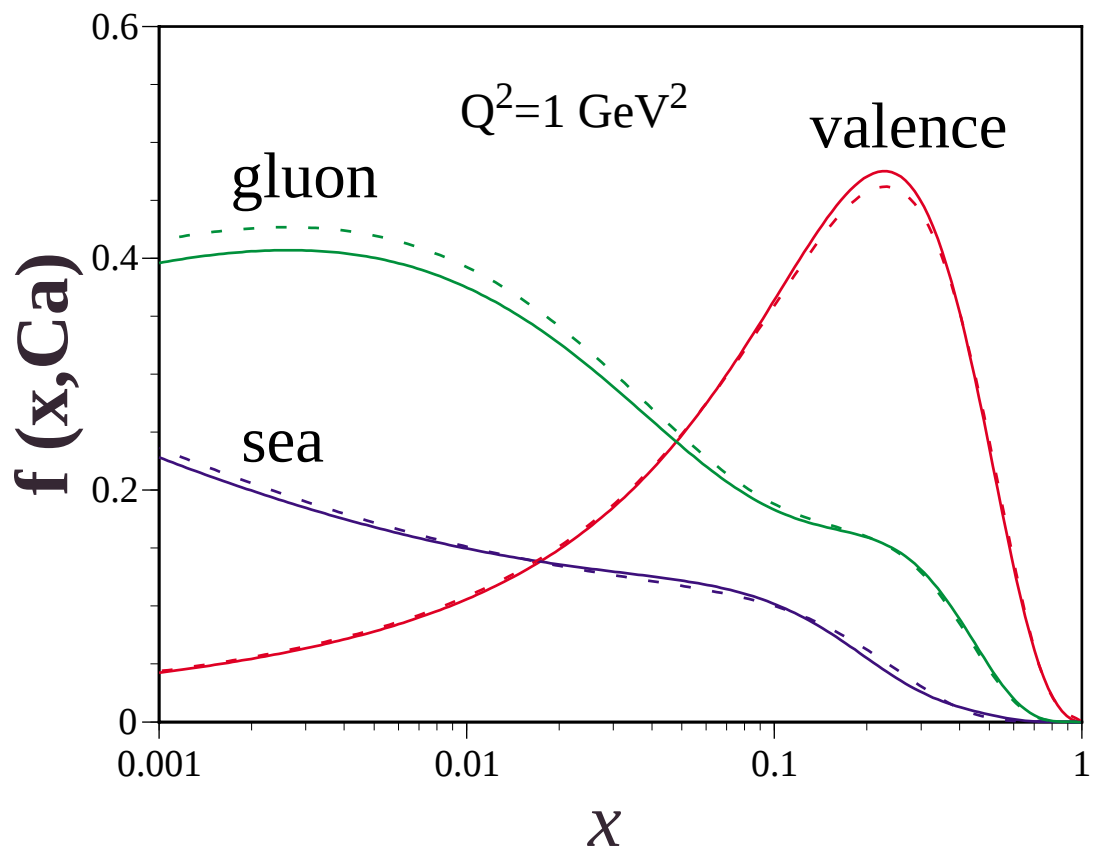
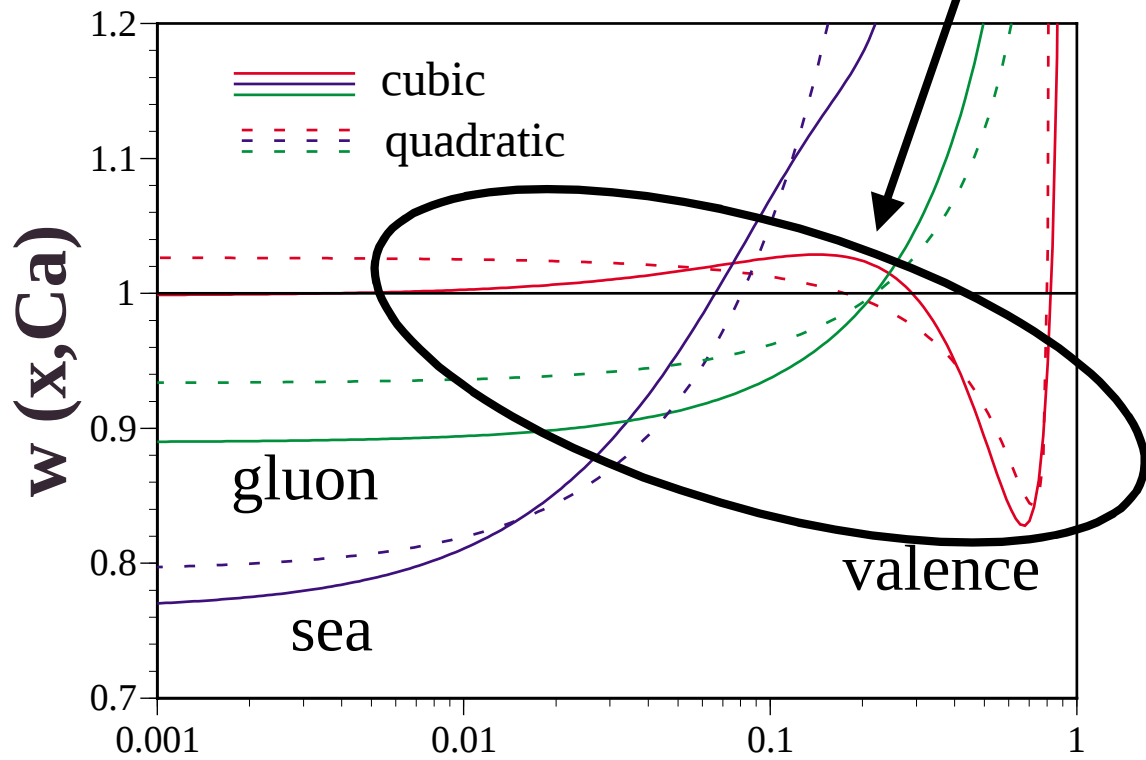
for He, Ca, Au

“cubic fit”



Weight functions and parton distributions for Ca

v factory



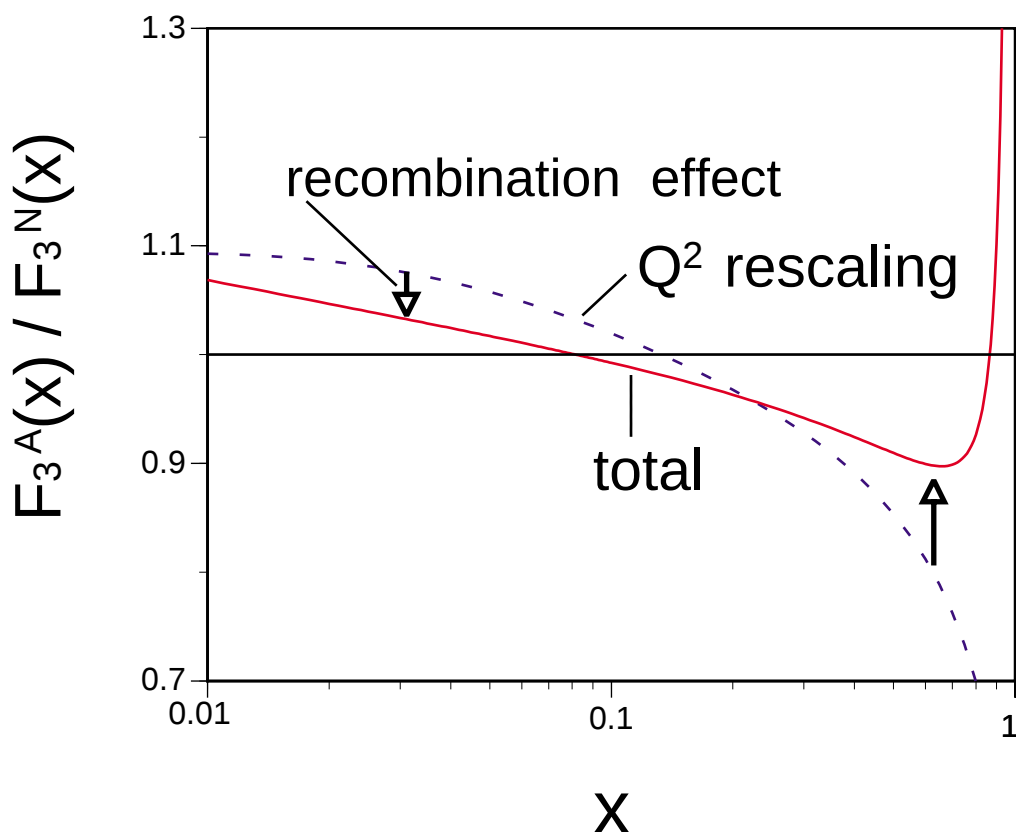
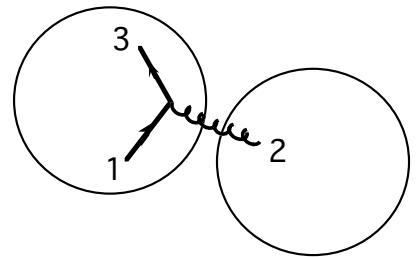
Theoretical predictions of F_3^A/F_3^D

(1) Q^2 rescaling & recombination

Q^2 rescaling: $V_A(Q^2) = V_N(\xi_A Q^2)$

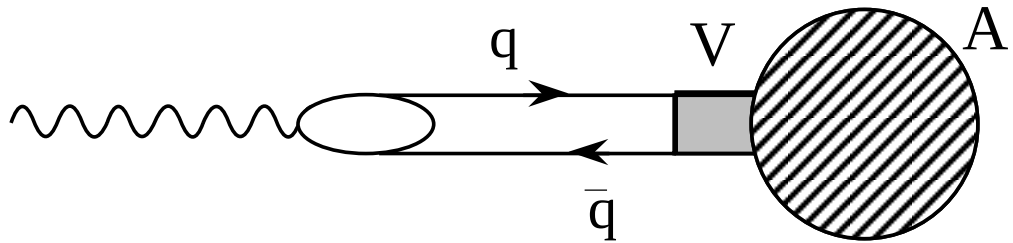
Recombination:

$$\Delta q_3(x_3) \propto p_1(x) p_2(x) \Gamma(x_1, x_2, x_3)$$

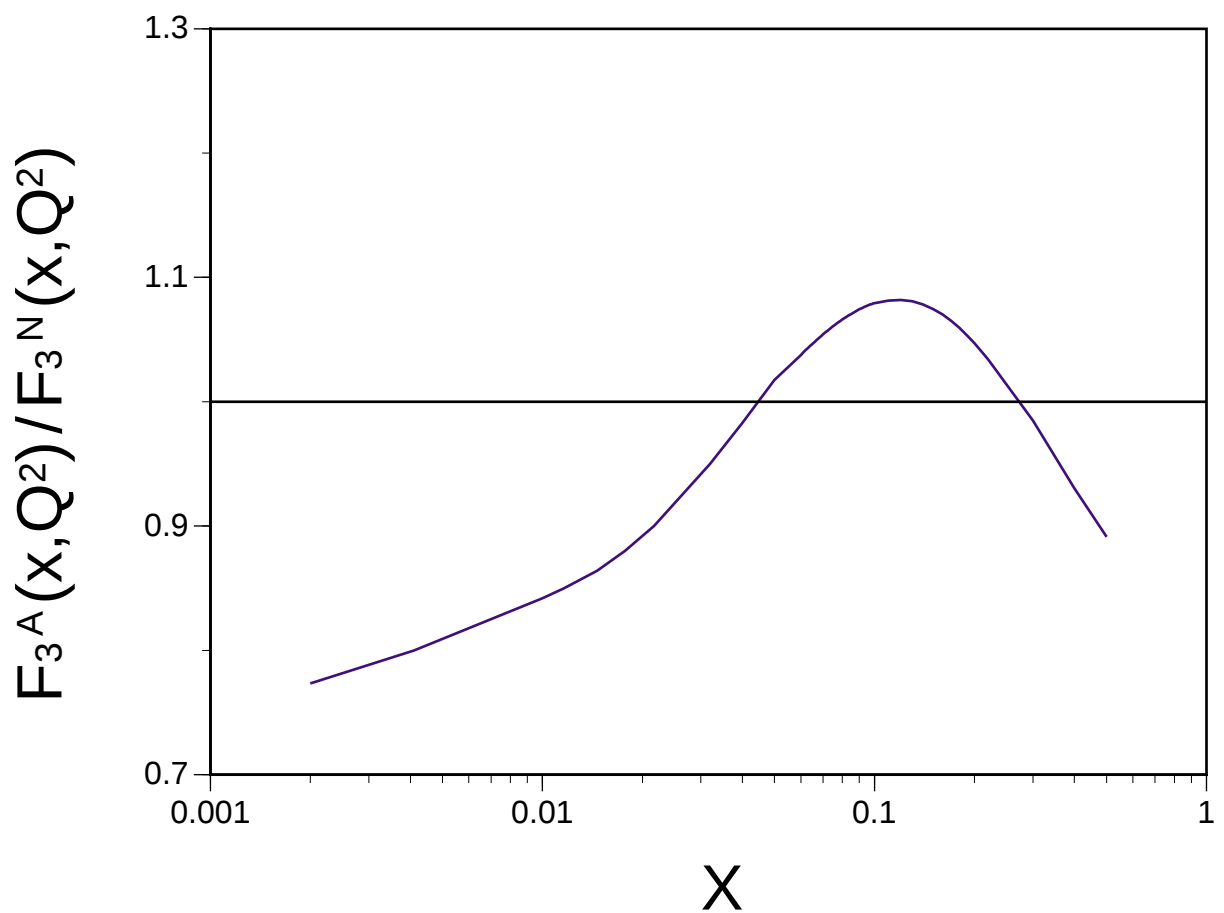


anti-shadowing ?

(2) vector-meson type



e.g. Frankfurt-Liuti-Strikman, PRL, 65, 1725 (1990)



shadowing ?

Valence-quark distributions in nuclei

Nuclear modification of F_3 cannot be investigated at this stage due to lack of accurate deuteron data.

accurate F_3^A/F_3^D data are valuable

- for determining the shadowing model
- for determining accurate nuclear parton distributions → application to heavy-ion physics

Summary of nuclear parametrization

- first χ^2 analysis

for the nuclear parton distributions

Computer codes could be obtained from

<http://www-hs.phys.saga-u.ac.jp>.

- reasonably good fit with

$$\chi_{\min}^2 / \text{d.o.f.} = 1.93 \text{ (quad.)}, 1.82 \text{ (cubic)}$$

- $g^A(x)$? $\bar{q}^A(x)$ at medium x ?

- need analysis refinement

- $q_v(x)$ in the nucleon is determined

by ν -Fe scattering at this stage

→ reinvestigate with nuclear modification

→ ν -N scattering (ν factory)

- relation to heavy-ion physics ?

- future experiments ?

Selected Topics on Parton Distributions in the Nucleon

References

Polarized distributions

- Y. Goto et al. (AAC), Phys. Rev. D62 (2000) 034017.
- B. Lampe and E. Reya, Phys. Rep. 332 (2000) 1.

Light-antiquark flavor asymmetry, Isospin

- SK, Phys. Rep. 303 (1998) 183.

- polarized parton distributions

status of proton-spin issue

polarized e/ μ -proton scattering

→ measurement of g_1

$$g_1^{\text{LO}} = \frac{1}{2} \sum_i e_i^2 (\Delta q_i + \Delta \bar{q}_i)$$

proton, deuteron, ^3He g_1 data
with isospin symmetry

→ valence and sea polarization
 $\Delta u_v, \Delta d_v, \Delta \bar{q}$

quark spin content

$$\Delta \Sigma = \Delta u_v + \Delta d_v + 6 \cdot \Delta \bar{q}$$

experimentally $\int_0^1 dx \Delta \Sigma(x) \approx 0.1 - 0.3$

rest of the spin ???

Initial parton distributions

$$\Delta f_i(x, Q_0^2) = A_i x^{\alpha_i} (1 + \gamma_i x^{\lambda_i}) f_i(x, Q_0^2)$$

$$(i=u_v, d_v, \bar{q}, g) \quad A_i, \alpha_i, \gamma_i, \lambda_i : \text{free}$$

positivity $|\Delta f_i(x, Q_0^2)| \leq f_i(x, Q_0^2), \quad |A_i| \leq 1$

flavor-symmetric distributions at initial Q_0^2

$$\Delta \bar{u}(x) = \Delta \bar{d}(x) = \Delta \bar{s}(x)$$

first moments η_{uv}, η_{dv} are fixed

$$F = 0.463 \pm 0.008, \quad D = 0.804 \pm 0.008$$

$$\eta_{uv} = 0.986, \quad \eta_{dv} = -0.341$$

γ_{uv} and γ_{dv} are determined so as

to satisfy these conditions.

$$\eta_i = \int_{x_{\min}}^1 \Delta f(x, \gamma) dx,$$

We determine 14 parameters by the χ^2 fitting !

$$A_{uv}, \alpha_{uv}, \lambda_{uv}, A_{dv}, \alpha_{dv}, \lambda_{dv},$$

$$A_{\bar{q}}, \alpha_{\bar{q}}, \gamma_{\bar{q}}, \lambda_{\bar{q}}, A_g, \alpha_g, \gamma_g, \lambda_g$$

χ^2 fitting to the data

[proton, neutron (^3He), deuteron]

$$A_1 = \frac{\sigma_{1/2}^T - \sigma_{3/2}^T}{\sigma_{1/2}^T + \sigma_{3/2}^T} \approx \frac{g_1}{F_1} = g_1 \frac{2x(1+R)}{F_2}$$
$$\left(R(x, Q^2) = \frac{F_L}{2xF_1} = \frac{F_2 - 2xF_1}{2xF_1} \right)$$

$$\chi^2 = \sum_i \frac{\left(A_{1,i}^{\text{data}} - A_{1,i}^{\text{cal}} \right)^2}{\left(\sigma_i^{\text{data}} \right)^2}$$

which is minimized by the subroutine MINUIT

We analyzed with the following conditions.

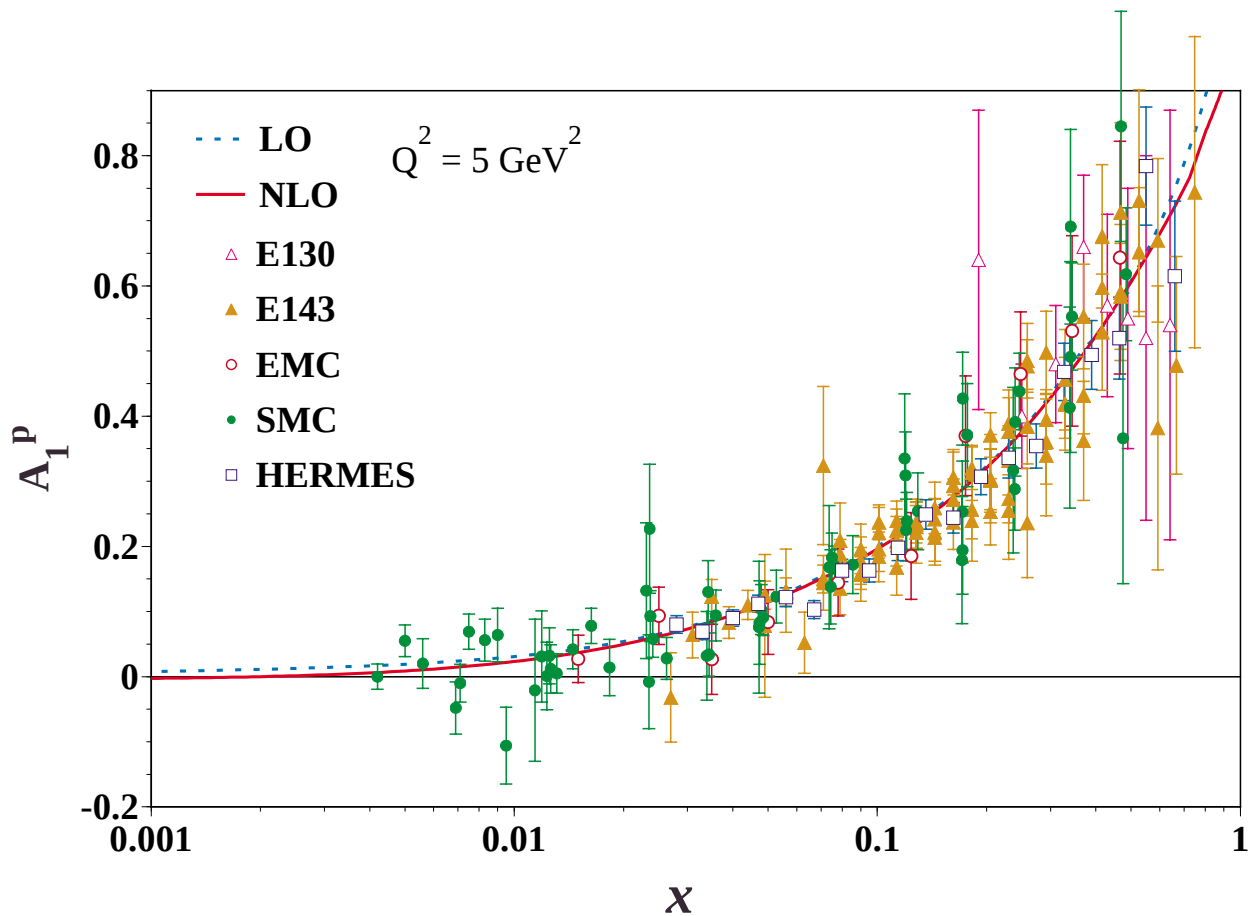
- **Unpolarized PDF** **GRV98**
- **Initial Q^2** **$Q_0^2 = 1 \text{ GeV}^2$**
- **Number of flavor** **$N_f = 3$**
- **Λ_{QCD}** **$\Lambda_{\text{LO}} = 204 \text{ MeV}$**
 $\Lambda_{\text{NLO}} = 299 \text{ MeV}$

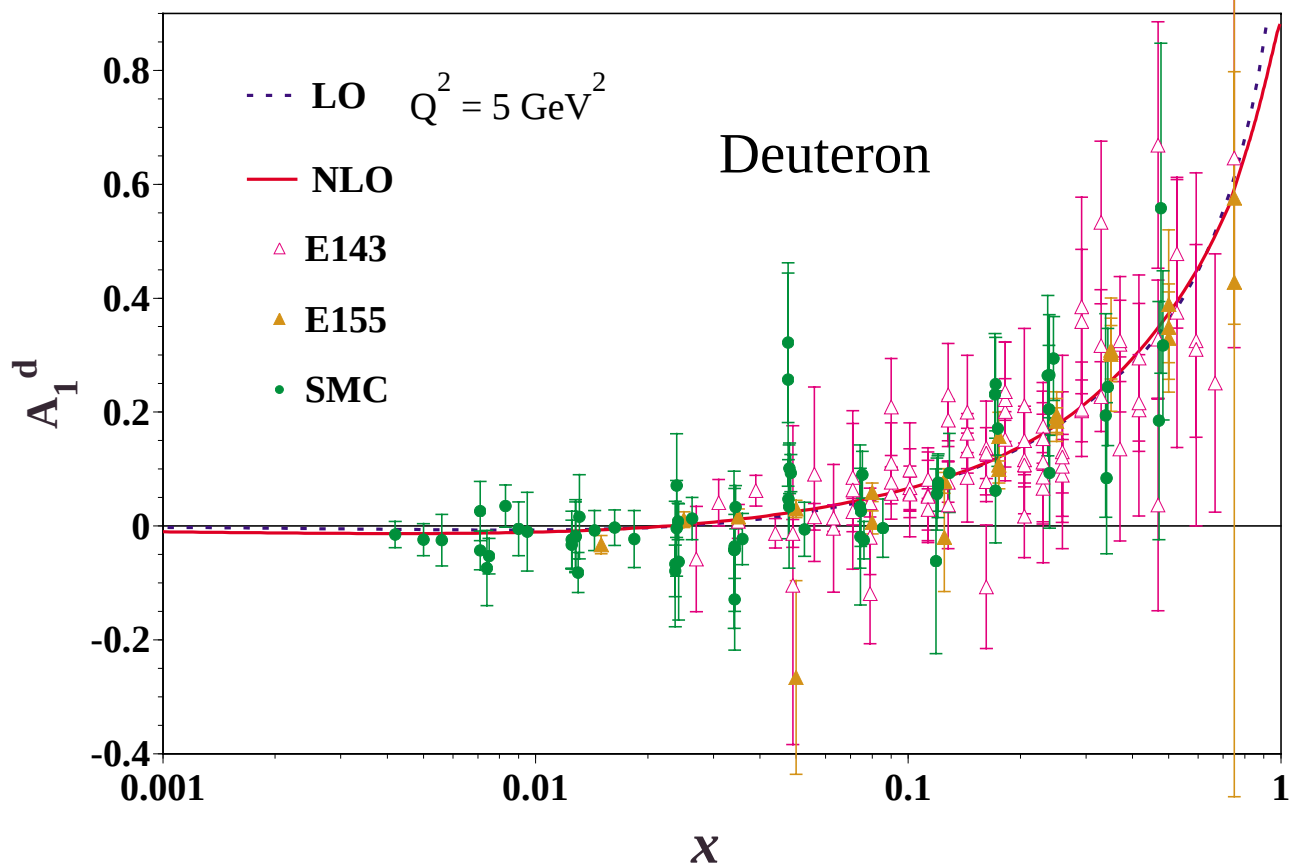
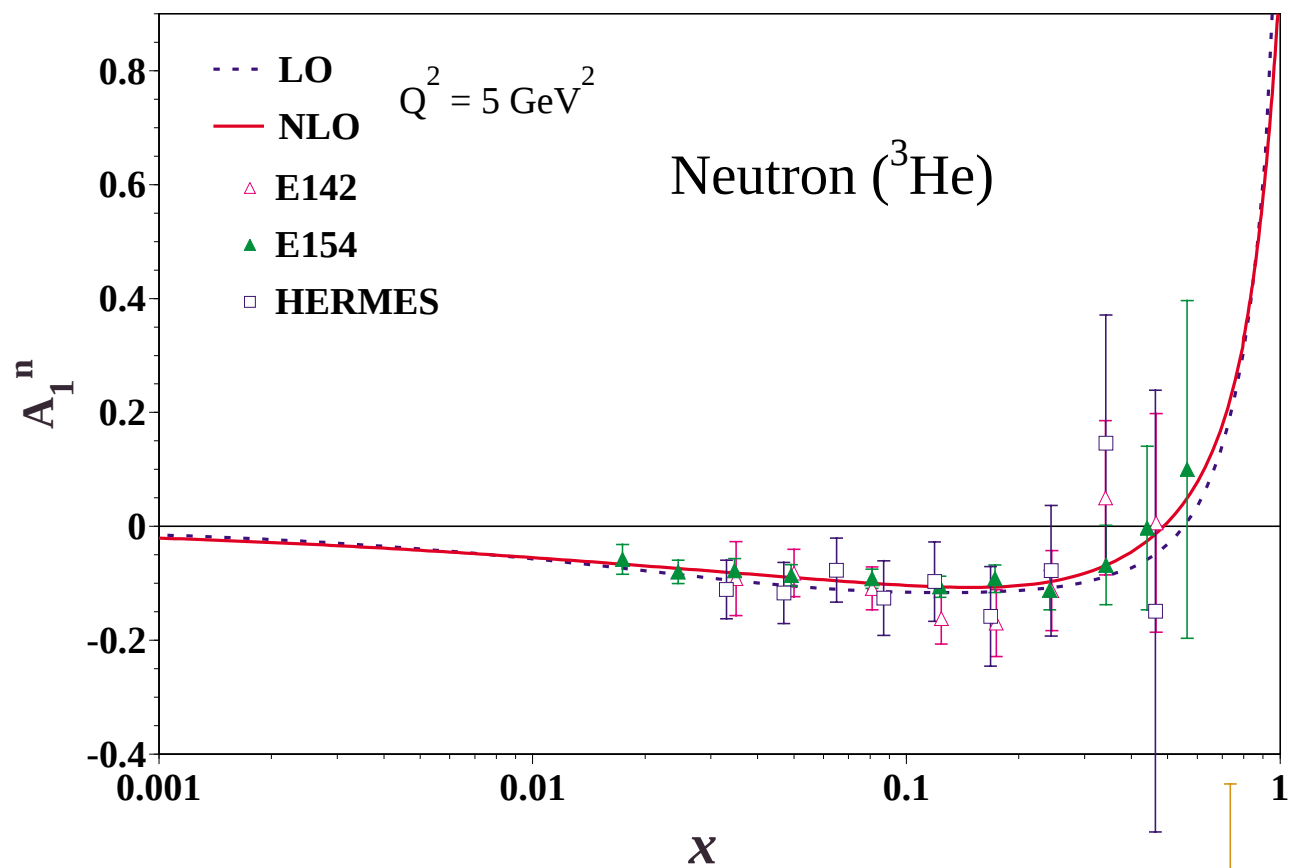
- Results -

Total χ^2 LO $\chi^2/\text{d.o.f.} = 322.6 / 360$
 NLO $\chi^2/\text{d.o.f.} = 300.4 / 360$

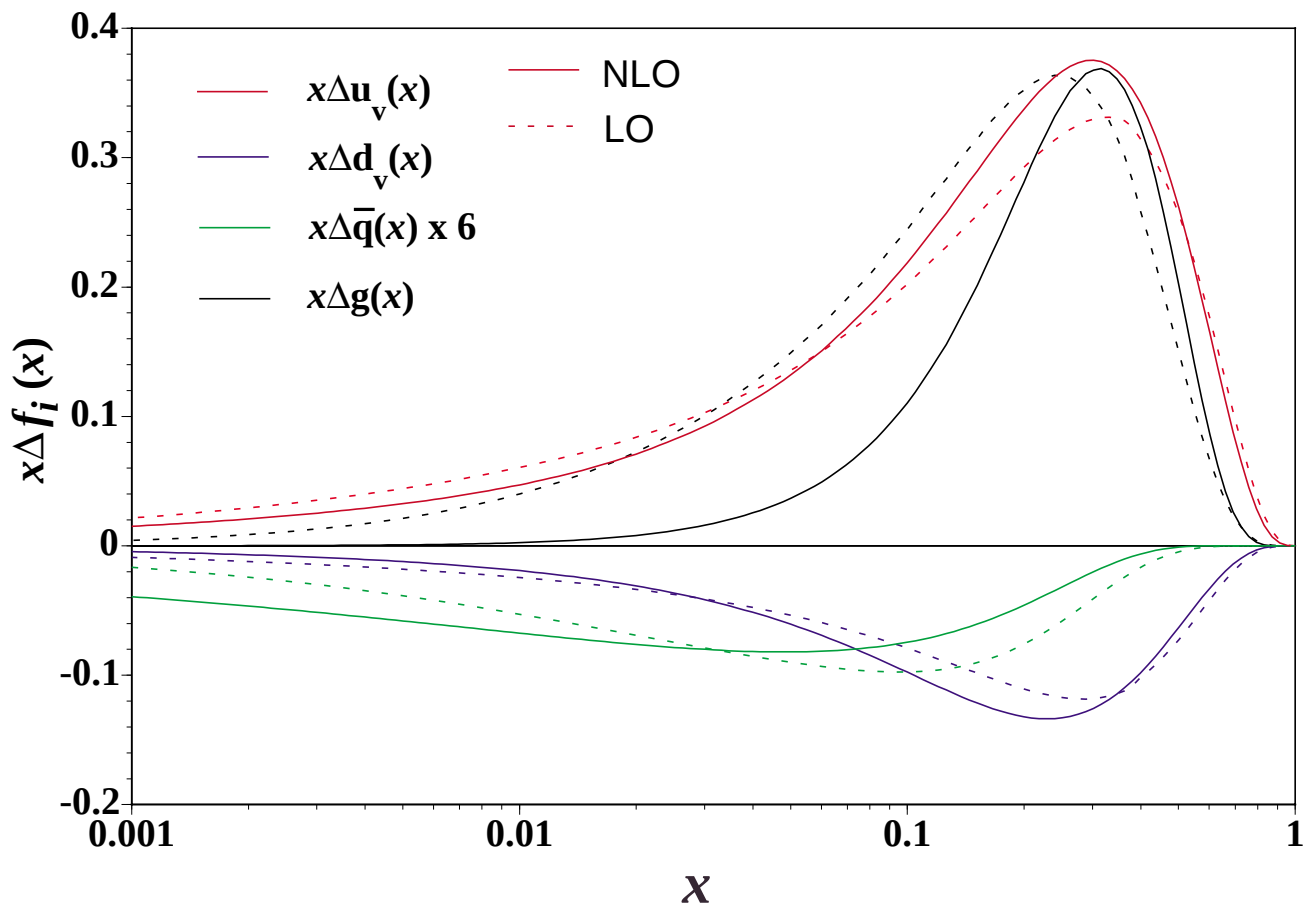
Total data 375

Spin asymmetry A_1^p





Parton distributions ($Q^2=1 \text{ GeV}^2$)



Role of ν measurements

$$\frac{d^2(\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow})_{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 M E_\nu}{\pi} \left\{ \pm y \left(1 - \frac{y}{2} - \frac{xyM}{2E} \right) x g_1 \mp \frac{x^2 y M}{E} g_2 \right. \\ \left. + y^2 x \left(1 + \frac{xM}{E} \right) g_3 + \left(1 - y - \frac{xyM}{2E} \right) \left[\left(1 + \frac{xM}{E} \right) g_4 + g_5 \right] \right\}$$

new parity-violating
polarized structure functions

in parton model

$$g_{4+5}^{\nu p} / 2x = g_3^{\nu p} = -(\Delta d + \Delta s - \Delta \bar{u} - \Delta \bar{c})$$

$$g_{4+5}^{\bar{\nu} p} / 2x = g_3^{\bar{\nu} p} = -(\Delta u + \Delta c - \Delta \bar{d} - \Delta \bar{s})$$



$$g_3^{\nu p} - g_3^{\bar{\nu} p} = -(\Delta u_v + \Delta d_v) \\ -(\Delta s - \Delta \bar{s}) - (\Delta c - \Delta \bar{c})$$

$$g_3^{\nu(p+n)} - g_3^{\bar{\nu}(p+n)} = -2(\Delta s + \Delta \bar{s}) + 2(\Delta c + \Delta \bar{c})$$

Polarized valence-quark and sea-quark
(strange, charm) distributions can be
investigated in detail.

also $\nu p \rightarrow \mu^- \mu^+ X$ ($\mu^- c \rightarrow X, c \rightarrow \mu^+ X'$)

for finding $s / (\bar{u} + \bar{d})$

Quark spin content at a ν factory

e/μ scattering $\rightarrow \Delta\Sigma = 0 \sim 30\%$

- not uniquely determined
- issue of using low-energy data

for fixing $\int dx \Delta u_v, \int dx \Delta d_v$

ν scattering

$$g_1^{\nu p} = \Delta d + \Delta s + \Delta \bar{u} + \Delta \bar{c}$$

$$g_1^{\nu p} = \Delta u + \Delta c + \Delta \bar{d} + \Delta \bar{s} \quad \text{in parton model}$$

$$\begin{aligned} g_1^{\nu p} + g_1^{\bar{\nu} p} &= (\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d}) \\ &\quad + (\Delta s + \Delta \bar{s}) + (\Delta c + \Delta \bar{c}) \end{aligned}$$

$$\text{in LO} \quad \int dx (g_1^{\nu p} + g_1^{\bar{\nu} p}) = \Delta\Sigma$$

$$\text{in NLO} \quad = \Delta\Sigma - n_f \frac{\alpha_s}{2\pi} \Delta G$$

independent determination of
quark spin content $\Delta\Sigma$!

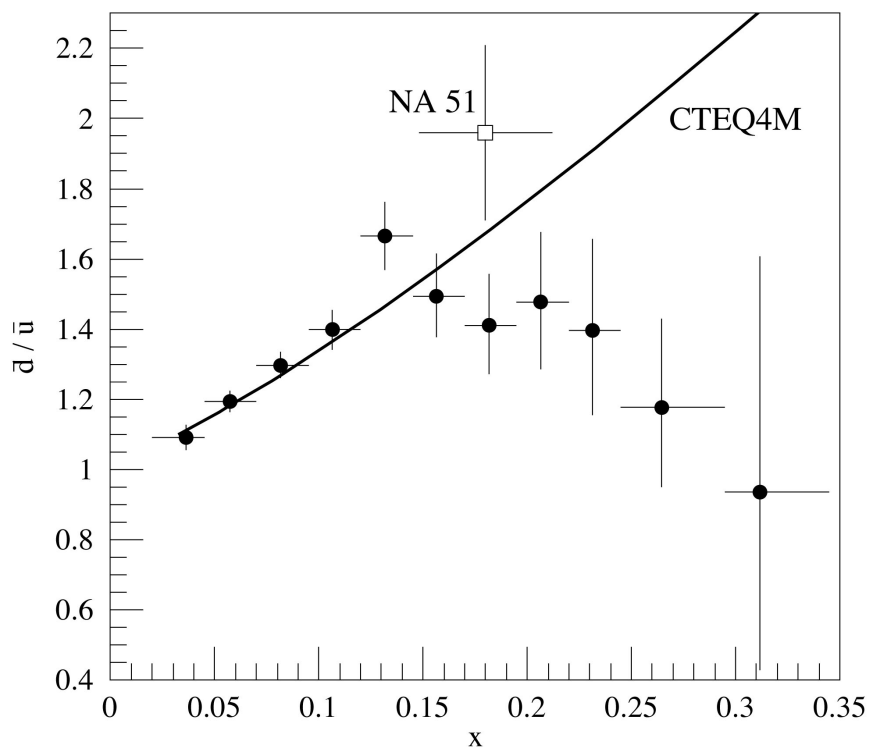
$\bar{u}/\bar{d}$, $\Delta\bar{u}/\Delta\bar{d}$ asymmetries at a ν factory

introduction

- **Gottfried sum** (NMC $S_G = 0.235 \pm 0.026$)

$$\begin{aligned} S_G &= \int_0^1 \frac{dx}{x} [F_2^{\mu p}(x) - F_2^{\mu n}(x)] \\ &= \frac{1}{3} + \frac{2}{3} \int_0^1 dx [\bar{u}(x) - \bar{d}(x)] \end{aligned}$$

- **Drell-Yan p-n asymmetry** (NA51, E866)



(from hep-ex/9803011)

- $\bar{u}/\bar{d}$ asymmetry

$$\bar{u} - \bar{d} = \frac{1}{4}(F_2^{\nu p}/X - F_3^{\nu p}) - \frac{1}{4}(F_2^{\nu n}/X - F_3^{\nu n})$$

see SK, Phys. Rep. 303 (1998) 183 -- Sec 5.5

- $\Delta\bar{u}/\Delta\bar{d}$ asymmetry

$$\Delta\bar{u} - \Delta\bar{d} = \frac{1}{2}(g_1^{\nu p} + g_3^{\nu p}) - \frac{1}{2}(g_1^{\nu n} + g_3^{\nu n})$$

If the neutrino data are accurate enough,
the $\bar{u} - \bar{d}$ and $\Delta\bar{u} - \Delta\bar{d}$ distributions
can be extracted in principle.

Isospin symmetry violation at a ν factory

see SK, Phys.Rep. -- Sec 4.5 & 5.6

The isospin symmetry is usually taken for granted in parton distributions.

$$u_n = d_p \equiv d, \quad \bar{u}_n = \bar{d}_p \equiv \bar{d}$$

$$d_n = u_p \equiv u, \quad \bar{d}_n = \bar{u}_p \equiv \bar{u}$$

$$s_n = s_p \equiv s, \quad \bar{s}_n = \bar{s}_p \equiv \bar{s}$$

...

Although we expect

isospin violation $\sim O(\alpha) = \text{a few \%}$,
this "common sense" has to be tested.

$$\begin{aligned} & \int \frac{dx}{x} (F_2^{\nu p} + F_2^{\bar{\nu} p} - F_2^{\nu n} - F_2^{\bar{\nu} n}) \\ &= 4 \int dx [(\bar{u} + \bar{d} + \bar{s} + \bar{c})_p - (\bar{u} + \bar{d} + \bar{s} + \bar{c})_n] \\ &= 0 \quad \text{if isospin symmetry can be applied} \\ &\neq 0 \quad \text{could be a signature of violation} \end{aligned}$$

Polarized distributions & other topics

At this stage, $\int dx \Delta u_v(x)$, $\int dx \Delta d_v(x)$ are fixed by semi-leptonic decays.

- not undoubtedly obvious whether such small- Q^2 data can be used in connection with DIS data

↓ independent determination
at a ν factory

understand accurate polarized valence-quark distributions

→ accurate determination of antiquark distributions

- determination of quark spin content
- $\bar{u}/\bar{d}$ & $\Delta\bar{u}/\Delta\bar{d}$ asymmetries
- test of isospin symmetry

Summary

In general, v measurements are important for finding the non-singlet part of structure functions and $s(x)$. However, if the data are accurate enough, there are other interesting topics.

- accurate p, d, ^3He data
 - valence-quark distributions in the *nucleon*
 - nuclear modification of $V(x)$
 - determination of shadowing model
- polarized distributions and other topics
 - valence-quark contributions to nucleon spin
 - new polarized structure functions
 - determination of quark spin content
 - (polarized) light-antiquark flavor asymmetry
 - (polarized) strange and charm distributions
 - isospin symmetry