

## Nuclear modification difference between $u_v$ and $d_v$ distributions and its relation to NuTeV $\sin^2 \theta_W$ anomaly

M. Hirai,<sup>1,\*</sup> S. Kumano<sup>†,2,‡</sup> and T.-H. Nagai<sup>2,§</sup>

<sup>1</sup>*Institute of Particle and Nuclear Studies, KEK, 1-1, Oho, Tsukuba, Ibaraki, 305-0801, Japan*

<sup>2</sup>*Department of Physics, Saga University, Saga, 840-8502, Japan*

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We investigate a possible nuclear correction to the NuTeV measurement of the weak-mixing angle  $\sin^2 \theta_W$ . In particular, a nuclear modification difference between  $u_v$  and  $d_v$  distributions contributes to the NuTeV measurement with the iron target. First, the modification difference is determined by a  $\chi^2$  analysis so as to reproduce nuclear data on the structure function  $F_2$  and Drell-Yan processes. Then, taking the NuTeV kinematics into account, we calculate a contribution to the  $\sin^2 \theta_W$  determination. In addition, its uncertainty is estimated by the Hessian method. Although the uncertainty becomes comparable to the NuTeV deviation, the effect is not large enough to explain the whole NuTeV  $\sin^2 \theta_W$  anomaly at this stage. However, it is difficult to determine such a nuclear modification difference, so that we need further investigations on the difference and its effect on the NuTeV anomaly.

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### I. INTRODUCTION

Weak-mixing angle  $\sin^2 \theta_W$  is one of fundamental constants, and there are various experimental methods for determining it [1]. They include experiments on atomic parity violation, left-right and forward-backward asymmetries in electron-positron annihilation,  $W$  and  $Z$  masses, elastic and inelastic neutrino scattering, parity-violating electron scattering, and polarized Møller scattering.

The NuTeV collaboration announced that their neutrino-nucleus scattering data indicate a significant deviation from the standard model. If the neutrino-nucleus scattering data are excluded, the average value is  $\sin^2 \theta_W = 0.2227 \pm 0.0004$  [2] in the on-shell scheme. The NuTeV value,  $\sin^2 \theta_W = 0.2277 \pm 0.0013$  (stat)  $\pm 0.0009$  (syst) [3], is significantly larger than the average of the other data. This difference is called “NuTeV anomaly”. Independent experiments are in progress to measure the angle by the Møller scattering [4] and parity-violating electron scattering [5]. It is also interesting to find  $Q^2$  dependence of  $\sin^2 \theta_W$  from various experiments. From these measurements, we expect that the experimental situation will become clear. However, since it is an important fundamental constant, fast clarification of the NuTeV deviation is needed.

There are a number of papers which intended to explain the deviation. Although there may be new mechanisms beyond the standard model [6], we should inves-

tigate all the possibilities within the present theoretical framework. Next-to-leading-order effects are not a source of the deviation [7]. There are proposals for conventional explanations [8]. First, because the used target is the iron nucleus instead of the isoscalar nucleon, it is natural to explain it in terms of nuclear corrections [9–12]. However, there is still no clear interpretation at this stage. Second, the difference between strange and antistrange quark distributions should contribute to the deviation [13–16]. However, it cannot be uniquely determined at this stage. For example, a CTEQ analysis result [15] differs from a NuTeV result [16]. Third, there are proposals that the anomaly could be explained by isospin violation [17], which could be related to the nuclear correction in Ref. [11] as shown in Sec. IV B.

In this paper, we investigate a conventional explanation in terms of a nuclear correction. We study the details of the nuclear effect in Ref. [11], where it was pointed out that the nuclear modification difference between the  $u_v$  and  $d_v$  distributions contributes to the anomaly. Because such a nuclear modification is unknown, possible differences are simply estimated by considering baryon-number and charge conservations [11]. It is the purpose of this paper to clarify this kind of nuclear effect by a  $\chi^2$  analysis with many nuclear data by assigning explicit parameters for the difference. We also estimate uncertainties of the difference by the Hessian method, and they are especially important for discussing an effect on the  $\sin^2 \theta_W$  anomaly.

This paper is organized as follows. In Sec. II, we show a nuclear modification effect on  $\sin^2 \theta_W$  by using the Paschos-Wolfenstein relation. Next, a  $\chi^2$  analysis method is explained in Sec. III for determining the modification difference between  $u_v$  and  $d_v$ . Analysis results and an actual effect on the NuTeV  $\sin^2 \theta_W$  are discussed in Sec. IV. These studies are summarized in Sec. V.

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<sup>†</sup>Address after Jan. 1, 2005: Institute of Particle and Nuclear Studies, KEK, 1-1, Oho, Tsukuba, Ibaraki, 305-0801, Japan. URL: <http://research.kek.jp/people/kumanos/>

\*Electronic address: [mhirai@post.kek.jp](mailto:mhirai@post.kek.jp)

<sup>‡</sup>Electronic address: [shunzo.kumano@kek.jp](mailto:shunzo.kumano@kek.jp)

<sup>§</sup>Electronic address: [tnagai@post.kek.jp](mailto:tnagai@post.kek.jp)

## II. CORRECTIONS TO PASCHOS-WOLFENSTEIN RELATION

Neutrino and antineutrino scattering data are used for extracting the weak mixing angle by the NuTeV collaboration. In discussing various corrections to the NuTeV measurement, it is theoretically useful to find the corrections to the Paschos-Wolfenstein (PW) relation [18]:

$$R^- = \frac{\sigma_{NC}^{\nu N} - \sigma_{NC}^{\bar{\nu} N}}{\sigma_{CC}^{\nu N} - \sigma_{CC}^{\bar{\nu} N}} = \frac{1}{2} - \sin^2 \theta_W, \quad (1)$$

where  $\sigma_{CC}^{\nu N}$  and  $\sigma_{NC}^{\nu N}$  are charged-current (CC) and neutral-current (NC) cross sections, respectively, in neutrino-nucleon deep inelastic scattering.

The various correction factors to the PW relation are discussed in Ref. [11]. Because they are important for our analysis, an outline is discussed in this section. The iron target is used in the NuTeV experiment, so that nuclear corrections should be properly taken into account. The correction investigated in this paper is the nuclear modification difference between the  $u_v$  and  $d_v$  distributions. Nuclear modifications of the valence-quark distributions,  $u_v$  and  $d_v$ , are expressed by  $w_{u_v}$  and  $w_{d_v}$ , which are defined by

$$\begin{aligned} u_v^A(x, Q^2) &= w_{u_v}(x, Q^2, A, Z) \frac{Z u_v(x, Q^2) + N d_v(x, Q^2)}{A}, \\ d_v^A(x, Q^2) &= w_{d_v}(x, Q^2, A, Z) \frac{Z d_v(x, Q^2) + N u_v(x, Q^2)}{A}. \end{aligned} \quad (2)$$

Here,  $u_v^A$  and  $d_v^A$  are the up- and down-valence quark distributions, respectively, in a nucleus;  $u_v$  and  $d_v$  are the distributions in the proton. The variable  $x$  is the Bjorken scaling variable,  $Q^2$  is defined by the momentum transfer  $q$  as  $Q^2 = -q^2$ ,  $Z$  is the atomic number of the target nucleus, and  $A$  is its mass number. In Ref. [19], the weight functions  $w_{u_v}$  and  $w_{d_v}$  are defined only at the fixed  $Q^2$  point,  $Q^2=1$  GeV<sup>2</sup>; however, they are defined at any  $Q^2$  in the present research. As mentioned in Ref. [20], the form of Eq. (2) is not appropriate at  $x \rightarrow 1$  because the nuclear distributions do not vanish at  $x = 1$ . However, it is more practical at this stage to use the form due to the lack of large- $x$  data. In any case, the large  $x$  ( $> 0.8$ ) region does not contribute to the anomaly because of the NuTeV kinematics. For investigating the difference between the modifications, we define a function

$$\varepsilon_v(x) = \frac{w_{d_v}(x, Q^2, A, Z) - w_{u_v}(x, Q^2, A, Z)}{w_{d_v}(x, Q^2, A, Z) + w_{u_v}(x, Q^2, A, Z)}. \quad (3)$$

The function  $\varepsilon_v$  depends also on  $A$ ,  $Z$ , and  $Q^2$ , but these factors are abbreviated for simplicity.

Writing the cross sections in terms of the nuclear parton distribution functions (PDFs) in the leading order of  $\alpha_s$ , we obtain a modified PW relation with small correction factors. Expanding the modified relation in terms

of the correction factors and retaining only the leading correction of  $\varepsilon_v$ , we obtain

$$\begin{aligned} R_A^- &= \frac{1}{2} - \sin^2 \theta_W \\ &- \varepsilon_v(x) \left\{ \left( \frac{1}{2} - \sin^2 \theta_W \right) \frac{1 + (1-y)^2}{1 - (1-y)^2} - \frac{1}{3} \sin^2 \theta_W \right\} \\ &+ O(\varepsilon_v^2) + O(\varepsilon_n) + O(\varepsilon_s) + O(\varepsilon_c). \end{aligned} \quad (4)$$

Here,  $O(\varepsilon)$  indicates a correction of the order of  $\varepsilon$ , and the variable  $y$  is given by the energy transfer  $q^0$  and neutrino energy  $E_\nu$  as  $y = q^0/E_\nu$ . The derivation of Eq. (4) is found in Ref. [11]. The details are investigated for the first correction factor  $\varepsilon_v(x)$  in this paper. Other corrections come from neutron excess, strange-antistrange asymmetry ( $s - \bar{s}$ ), and charm-anticharm asymmetry ( $c - \bar{c}$ ). They are denoted  $O(\varepsilon_n)$ ,  $O(\varepsilon_s)$ , and  $O(\varepsilon_c)$  in Eq. (4). The  $\varepsilon$  factors are defined by  $\varepsilon_n = (N-Z)(u_v - d_v)/[A(u_v + d_v)]$ ,  $\varepsilon_s = s_v^A/[w_v(u_v + d_v)]$  and  $\varepsilon_c = c_v^A/[w_v(u_v + d_v)]$  with  $w_v = (w_{d_v} + w_{u_v})/2$  and  $q_v^A \equiv q^A - \bar{q}^A$ . The correction factor  $\varepsilon_v$  is related to the isospin violation as shown in Sec. IV B. The studies of these contributions to  $\sin^2 \theta_W$  should be found elsewhere. In deriving Eq. (4), differential cross sections are used instead of the total ones for taking the PW ratio. The equation is useful for finding theoretical possibilities; however, it is practically limited for estimating a numerical effect because NuTeV kinematical conditions should be taken into account. This point is discussed in Sec. IV. Investigating the modified PW relation, we find that the  $\varepsilon_v(x)$  factor contributes to the  $\sin^2 \theta_W$  measurement in the neutrino scattering. We explain the distribution  $\varepsilon_v(x)$  and its effect on the  $\sin^2 \theta_W$  determination in the following sections.

## III. $\chi^2$ ANALYSIS METHOD FOR DETERMINING $\varepsilon_v(x)$

The nuclear modification difference between the distributions  $u_v$  and  $d_v$  is not known at this stage. There are few theoretical guidelines for calculating the difference. Of course, the valence-quark distributions are constrained by the baryon-number and charge conservations, so that there should be some restrictions to their nuclear modifications. From this consideration, two possibilities were proposed for  $\varepsilon_v(x)$  in Ref. [11]. However, the conservation conditions are not enough to impose  $x$ -dependent shape of these modifications. It is, therefore, an appropriate way is to show the modification difference by analyzing available experimental data.

We express the difference by a number of parameters, which are then determined by a  $\chi^2$  analysis of nuclear data. The difference between the valence-quark modifications is denoted

$$\Delta w_v(x, Q^2, A, Z) = w_{u_v}(x, Q^2, A, Z) - w_{d_v}(x, Q^2, A, Z), \quad (5)$$

and their average is  $w_v$  as used in the previous section. The difference at  $Q^2=1$  GeV<sup>2</sup> ( $\equiv Q_0^2$ ) is expressed by four parameters,  $a'_v$ ,  $b'_v$ ,  $c'_v$ , and  $d'_v$ :

$$\Delta w_v(x, Q_0^2, A, Z) = \left(1 - \frac{1}{A^{1/3}}\right) \times \frac{a'_v(A, Z) + b'_v x + c'_v x^2 + d'_v x^3}{(1-x)^{\beta_v}}. \quad (6)$$

For the functions  $w_v$ ,  $w_{\bar{q}}$ , and  $w_g$ , which indicate nuclear modifications of average valence-quark, antiquark, and gluon distributions, we use the ones obtained by a recent global analysis in Ref. [19]. The reason of selecting the functional form of Eq. (6) is discussed in Ref. [20]. We briefly explain the essential point. The  $A$  dependence of the modification is assumed to be proportional to  $1 - 1/A^{1/3}$  by considering nuclear volume and surface contributions to the cross section [21]. It should be noted that such  $A$  dependence could be too simple to describe the distributions. The factor  $1/(1-x)^{\beta_v}$  is introduced so as to explain the large- $x$  Fermi-motion part. Looking at typical data of  $F_2^A/F_2^D$  ratios, we find that a cubic function seems to be appropriate for the  $x$  dependence. However, we should aware that an appropriate functional form is scarcely known for  $\Delta w_v$ .

The parameters  $a'_v$ ,  $b'_v$ ,  $c'_v$ , and  $d'_v$  are determined by analyzing experimental data for structure-function ratios  $F_2^A/F_2^{A'}$  and Drell-Yan cross-section ratios  $\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$ . The details of the experimental data sets are discussed in Ref. [19]. The  $F_2^A/F_2^{A'}$  data are from the European Muon Collaboration (EMC) [22], the SLAC-E49, E87, E139, and E140 Collaborations [23], the Bologna-CERN-Dubna-Munich-Saclay (BCDMS) Collaboration [24], the New Muon Collaboration (NMC) [25], the Fermilab-E665 Collaboration [26], and HERMES Collaboration [27]. The Drell-Yan data are taken by the Fermilab-E772 and E866/NuSea Collaborations [28]. These data are for the nuclei: deuteron, helium-4, lithium, beryllium, carbon, nitrogen, aluminum, calcium, iron, copper, krypton, silver, tin, xenon, tungsten, gold, and lead. The total number of the data is 951. Because the small- $x$  data are taken in the small  $Q^2$  region, our leading-twist analysis could be affected especially at small  $x$  [19].

One of these parameters is fixed by baryon-number or charge conservation, and  $a'_v$  is selected for this fixed quantity. In addition, the parameter  $\beta_v$  is taken as the same value in the analysis of Ref. [19] ( $\beta_v = 0.1$ ). Therefore, there are three free parameters in the analysis. The total  $\chi^2$  is defined by

$$\chi^2 = \sum_j \frac{(R_j^{data} - R_j^{theo})^2}{(\sigma_j^{data})^2}, \quad (7)$$

where  $R_j$  indicates the ratios,  $F_2^A/F_2^{A'}$  and  $\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$ . The theoretical ratios  $R_j^{theo}$  are calculated by evolving the nuclear PDFs with the valence-quark modification difference in Eq. (6) by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations to experimental  $Q^2$

points. The total  $\chi^2$  is minimized by the subroutine MINUIT [29]. Running the subroutine, we obtain optimized values for the parameters and also a Hessian matrix for error estimation. The uncertainty of the weight function  $\Delta w_v(x, Q^2, \hat{\xi})$  is calculated with the Hessian matrix:

$$[\delta \Delta w_v(x, Q^2)]^2 = \Delta \chi^2 \sum_{i,j} \left( \frac{\partial \Delta w_v(x, Q^2, \xi)}{\partial \xi_i} \right)_{\xi=\hat{\xi}} \times H_{ij}^{-1} \left( \frac{\partial \Delta w_v(x, Q^2, \xi)}{\partial \xi_j} \right)_{\xi=\hat{\xi}}, \quad (8)$$

where a parameter is denoted  $\xi_i$  and the optimized point is denoted  $\hat{\xi}$ . The  $\Delta \chi^2$  value is taken so that the confidence level is the one- $\sigma$ -error range for the normal distribution [19]. Because the parameter number is three, we have  $\Delta \chi^2 = 3.527$ . This uncertainty estimation is especially important in discussing an effect on the NuTeV anomaly.

## IV. RESULTS

### A. Valence-quark modification difference between $u_v$ and $d_v$

We explain  $\chi^2$  analysis results. From the analysis, we obtain  $\chi_{min}^2/d.o.f.=1.57$ . The parameters are defined at  $Q^2=1$  GeV<sup>2</sup>, and their optimum values obtained by the fit are shown in Table I. We note that the constant  $a'_v$  is fixed by one of the baryon-number and charge conservations, and it depends on nuclear species. Several nuclei ( $D, C, Ca, Fe, Ag, Au$ ) are selected for listing its values. It is almost constant from a small nucleus ( $a'_v(D) = 0.00612$ ) to a large one ( $a'_v(Au) = 0.00615$ ). The parameters  $b'_v$ ,  $c'_v$ , and  $d'_v$  are determined by the analysis. We notice huge errors which are almost an order of magnitude larger than the optimum parameter values. This fact indicates that the determination of  $\Delta w_v$  is almost impossible at this stage. However, it is important to show an uncertainty range of  $\Delta w_v$  in order to compare with the NuTeV anomaly.

TABLE I: Parameters obtained by the analysis. The constant  $a'_v$  depends on nuclear species. Its value is listed for typical nuclei.

parameter	value
$b'_v$	$-0.0905 \pm 0.4044$
$c'_v$	$0.239 \pm 1.300$
$d'_v$	$-0.193 \pm 1.126$
$a'_v(D)$	0.00612
$a'_v(C)$	0.00612
$a'_v(Ca)$	0.00612
$a'_v(Fe)$	0.00613
$a'_v(Ag)$	0.00614
$a'_v(Au)$	0.00615

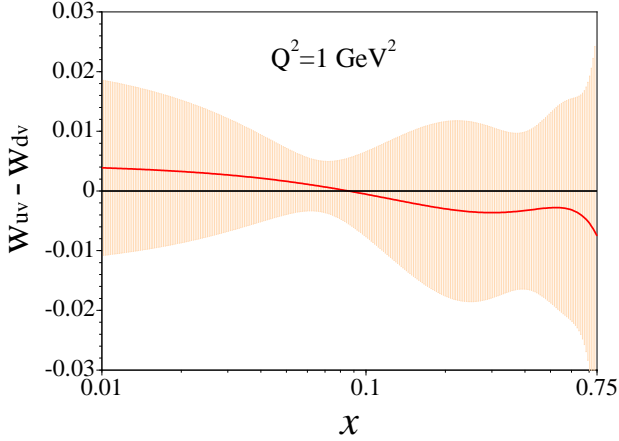


FIG. 1: (Color online) Nuclear modification difference  $\Delta w_v = w_{u_v} - w_{d_v}$  between  $u_v$  and  $d_v$  for the iron nucleus. The curve indicates the obtained  $\Delta w_v$  at  $Q^2=1 \text{ GeV}^2$ , and the shaded area shows the uncertainty range calculated by the Hessian method.

The difference  $w_{u_v} - w_{d_v}$  is plotted as a function of  $x$  at  $Q^2=1 \text{ GeV}^2$  in Fig. 1 by using the optimum parameters. The solid curve is the distribution obtained by the  $\chi^2$  analysis. The shaded area is the one- $\sigma$  uncertainty range which is calculated by the Hessian method in Eq. (8). Although the distribution is positive at small  $x$  and it becomes negative at large  $x$ , its functional form is not clear if the uncertainties are considered. Because the uncertainty band is much larger than the distribution itself, we obviously need future experimental efforts to find an accurate distribution.

### B. Effect on $\sin^2 \theta_W$

The average  $Q^2$  of the NuTeV neutrino and antineutrino scattering experiments is about  $Q^2=20 \text{ GeV}^2$ . The valence-quark distributions in the iron nucleus are obtained at  $Q^2=1 \text{ GeV}^2$  in the previous section. The nuclear PDFs as well as the nucleonic PDFs are evolved to the ones at  $Q^2=20 \text{ GeV}^2$  by the DGLAP evolution equations. Then, the nuclear modifications  $w_{u_v}$  and  $w_{d_v}$  are calculated at  $Q^2=20 \text{ GeV}^2$  by using Eq. (2), and the obtained function  $\varepsilon_v(x)$  is shown in Fig. 2.

The solid curve is the distribution obtained by the current analysis, and its uncertainties are shown by the shaded area. The previous results in Ref. [11] are also shown in the figure. These distributions are proposed as the ones which satisfy the baryon-number and charge conservations. The dashed curve indicates the distribution by the model-1, in which the conservation integrands are assumed to vanish in the leading order of the correction factors. The dotted curve indicates the distribution by the model-2, in which the global  $\chi^2$  analysis results for the nuclear PDFs in Ref. [20] are used. Three curves are much different; however, they are certainly within the error band. It suggests that they should be consistent

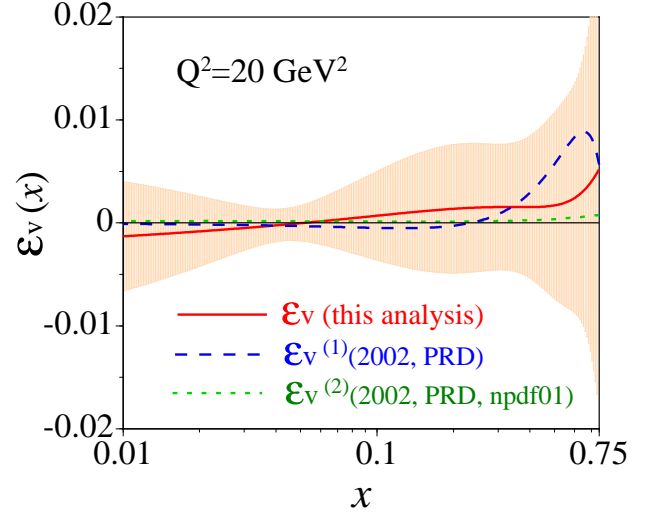


FIG. 2: (Color online) The function  $\varepsilon_v(x)$  at  $Q^2=20 \text{ GeV}^2$  for the iron nucleus. The solid curve indicates the current analysis result, and the shaded area shows the uncertainty range. The dashed and dotted curves indicate the model-1 and model-2 results, respectively, in Ref. [11].

within the uncertainties.

Because a Monte-Carlo code, instead of the PW relation, is used in the experimental analysis for extracting  $\sin^2 \theta_W$ , it is not straightforward to compare theoretical results with the NuTeV deviation. Obviously, NuTeV kinematical conditions should be taken into account. In particular, few data exist in the large- $x$  region, where  $\varepsilon_v(x)$  is large. Fortunately, such kinematical factors are provided in Ref. [13]. In order to use them, our PDFs should be related to the NuTeV convention [30]. The up- and down-valence quark distributions per nucleon are defined by

$$\begin{aligned} x u_v^A &= w_{u_v} \frac{Z x u_v + N x d_v}{A} = \frac{Z u_{vp}^* + N u_{vn}^*}{A}, \\ x d_v^A &= w_{d_v} \frac{Z x d_v + N x u_v}{A} = \frac{Z d_{vp}^* + N d_{vn}^*}{A}, \end{aligned} \quad (9)$$

where the middle expressions with  $w_{u_v}$  and  $w_{d_v}$  are our notations, and the PDFs with the asterisk (\*) are the NuTeV expressions. Using Eq. (9), we find that the distribution  $\varepsilon_v(x)$  is related to the isospin-violating distributions:

$$\begin{aligned} \delta u_v^* &= u_{vp}^* - d_{vn}^* = -\varepsilon_v (w_{u_v} + w_{d_v}) x u_v, \\ \delta d_v^* &= d_{vp}^* - u_{vn}^* = +\varepsilon_v (w_{u_v} + w_{d_v}) x d_v. \end{aligned} \quad (10)$$

Therefore, according to the NuTeV convention of the nuclear PDFs, we have been investigating isospin-violation effects in the nucleon and their nuclear modifications by the function  $\varepsilon_v(x)$ . However, we should aware that such isospin-violation effects could possibly include other nuclear effects which may not be related to the isospin violation. In any case, using the NuTeV functionals in Ref.

[13], we need to calculate the integral

$$\Delta(\sin^2 \theta_W) = - \int dx \{ F[\delta u_v^*, x] \delta u_v^*(x) + F[\delta d_v^*, x] \delta d_v^*(x) \}, \quad (11)$$

for estimating a correction to the NuTeV measurement. All the NuTeV kinematical effects are included in the functionals  $F[\delta u_v^*, x]$  and  $F[\delta d_v^*, x]$  which are provided in Fig. 1 of Ref. [13]. It should be noted that our sign convention of the correction  $\Delta(\sin^2 \theta_W)$  is opposite to the NuTeV one.

Using Eqs. (10) and (11) together with the distribution  $\varepsilon_v(x)$  obtained in Sec. IV A, we calculate the effect on the NuTeV value. The results are shown in Fig. 3 as a function of  $Q^2$ . The deviation  $\Delta(\sin^2 \theta_W)$  is shown by the solid curve, and the shaded area corresponds to the one- $\sigma$ -error range. For comparison, the NuTeV deviation 0.0050 is shown by the dashed line in the figure. We notice that the correction is not strongly dependent on  $Q^2$ . Even if the uncertainty range is considered, the contribution is smaller than 0.0050. In fact, calculating the correction at  $Q^2=20 \text{ GeV}^2$ , which is approximately the average  $Q^2$  of NuTeV measurements, we obtain

$$\Delta(\sin^2 \theta_W) = 0.0004 \pm 0.0015. \quad (12)$$

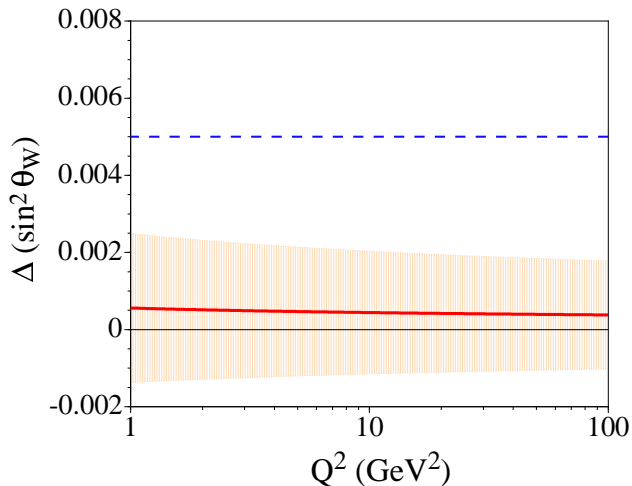


FIG. 3: (Color online) Effects on the NuTeV  $\sin^2 \theta_W$  determination as a function of  $Q^2$  by taking into account the experimental kinematics. The curve indicates the  $\chi^2$  analysis result, and the shaded area shows the uncertainty range. The NuTeV deviation 0.0050 is shown by the dashed line.

It indicates that the whole NuTeV anomaly could not be explained by the nuclear modification difference  $w_{u_v} - w_{d_v}$  at this stage. However, we should be careful

that the uncertainty estimation depends on the analysis conditions. For example, a certain  $A$  and  $x$  dependent functional form is assumed in the  $\chi^2$  analysis and such uncertainties are not included in Fig. 3 and Eq. (12). The situation is the same as the nucleonic PDF case, for example in Ref. [31], where such functional uncertainties are not also estimated. In addition, the uncertainties due to other parameter errors in Ref. [19] are not included. There is a possibility that the uncertainty 0.0015 could be underestimated.

Because of these issues, it is, strictly speaking, too early to exclude the mechanism for explaining the NuTeV anomaly. However, if the NuTeV anomaly were to be explained solely by the nuclear modification of the PDFs, the magnitude should be a factor of ten larger than the global analysis result. Furthermore, the obtained error is a factor of three smaller than the NuTeV discrepancy. Therefore, it is unlikely at this stage that the anomaly is explained by such an effect according to the analysis of nuclear data. In any case, because the difference  $w_{u_v} - w_{d_v}$  cannot be determined from current experimental data, we need future experimental efforts to find it. Because it is related to nuclear valence-quark distributions, possibilities are neutrino scattering experiments at future neutrino facilities such as MINER $\nu$ A [32] and neutrino factories [33].

## V. SUMMARY

We have extracted the difference between nuclear modifications of  $u_v$  and  $d_v$  by the  $\chi^2$  analysis of nuclear  $F_2$  and Drell-Yan data. We found that it is difficult to determine it at this stage due to the lack of data which are sensitive to the difference. Such a difference contributes to the NuTeV determination of  $\sin^2 \theta_W$  because the iron target was used in the experiment. Taking the NuTeV kinematics into account, we estimated the effect on the  $\sin^2 \theta_W$ . It is not large enough to explain the whole NuTeV deviation. However, the nuclear modification difference cannot be accurately determined at this stage, we need further efforts to find it and its effect on the NuTeV determination of  $\sin^2 \theta_W$ .

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