SAGA-HE-169-00
RIKEN-AF-NP-378
TMU-NT-00-1
hep-ph/0012280

December 7, 2000

Determination of polarized parton distribution functions

M. Hirai*, H. Kobayashi†, and M. Miyama‡
(Asymmetry Analysis Collaboration)

*Department of Physics, Saga University, Saga 840-8502, Japan
†RIKEN BNL Research Center, Upton, NY 11973, U.S.A.
‡Department of Physics, Tokyo Metropolitan University, Tokyo 192-0397, Japan

Talk given at the 14th International
Spin Physics Symposium
Osaka, Japan, October 16 – 21, 2000
(talk on October 20, 2000)

*98td25@edu.cc.saga-u.ac.jp, †hyuki@bnl.gov, ‡miyama@comp.metro-u.ac.jp,
Information on their research is available at http://spin.riken.bnl.gov/aac/.

to be published in proceedings

Determination of polarized parton distribution functions

M. Hirai [1] , H. Kobayashi [2] , and M. Miyama [3]
(Asymmetry Analysis Collaboration)

[1] *Department of Physics, Saga University, Saga 840-8502, Japan*
[2] *RIKEN BNL Research Center, Upton, NY 11973-5000, U.S.A.*
[3] *Department of Physics, Tokyo Metropolitan University, Tokyo 192-0397, Japan*

Abstract. We study parametrization of polarized parton distribution functions in the α_s leading order (LO) and in the next-to-leading order (NLO). From χ^2 fitting to the experimental data on A_1, optimum polarized distribution functions are determined. The quark spin content $\Delta\Sigma$ is very sensitive to the small-x behavior of antiquark distributions which suggests that small-x data are needed for precise determination of $\Delta\Sigma$. We propose three sets of distributions and also provide FORTRAN library for our distributions.

INTRODUCTION

Experimental data on polarized structure functions g_1 have been accumulated for the last several years. The data with the proton, deuteron, and ^{3}He targets are now available and new data are expected to be given by RHIC at BNL, COMPASS at CERN, and other facilities in the near future. In the light of such progresses and future projects, we should summarize present knowledge of polarized parton distribution functions by using all available data at this stage. For this purpose, we formed the group called Asymmetry Analysis Collaboration (AAC) and tried to determine the polarized parton distribution functions.

In this paper, we explain our analysis to determine optimum polarized distributions by χ^2 fitting to the experimental data of spin asymmetry A_1. In addition to the discussions based on the work in Ref. [1], we introduce a FORTRAN library for our distributions as a recent progress.

1) 98td25@edu.cc.saga-u.ac.jp
2) hyuki@bnl.gov
3) miyama@comp.metro-u.ac.jp

PARAMETRIZATION

We determine initial polarized parton distributions at $Q^2 = 1.0$ GeV^2 ($\equiv Q_0^2$). Considering the counting rule, we adopt the following functional form:

$$\Delta f_i(x, Q_0^2) = A_i\, x^{\alpha_i}\, (1 + \gamma_i\, x^{\lambda_i})\, f_i(x, Q_0^2). \tag{1}$$

Here, Δf_i and f_i represent the polarized and unpolarized parton distributions, respectively and A_i, α_i, γ_i, and λ_i are free parameters. These parameters are constrained by the positivity condition, $|\,\Delta f_i(x)\,| \le f_i(x)$. We simply apply it not only for the leading-order (LO) case but also for the next-to-leading-order (NLO) case although it is valid only in LO. Furthermore, we assume SU(3) flavor-symmetric sea because we don't have enough data to extract flavor dependence of polarized sea. Under this assumption, the first moments of Δu_v and Δd_v can be fixed by the axial charges for octet baryon, F and D. Therefore, we should determine Δu_v, Δd_v, $\Delta \bar{q}$, and Δg distributions and total number of the parameters becomes 14.

As the experimental data to which the parameters are fitted, we chose spin asymmetry A_1 instead of g_1 since A_1 is closer to direct observable in experiment rather than g_1. A_1 is given by

$$A_1(x, Q^2) \simeq \frac{g_1(x, Q^2)}{F_1(x, Q^2)} = g_1(x, Q^2)\frac{2x[1 + R(x, Q^2)]}{F_2(x, Q^2)}, \tag{2}$$

where, R is the ratio of longitudinal to transverse cross-sections. In our analysis, A_1 is calculated by using F_2 obtained by GRV98 distributions, g_1 obtained by our parametrized distributions, and R taken from the SLAC-1990 analysis. Then, $\chi^2 = \sum[A_1^{\rm data}(x, Q^2) - A_1^{\rm calc}(x, Q^2)]^2/[\sigma^{data}(x, Q^2)]^2$ is calculated and is minimized by the subroutine MINUIT. Here, A_1^{data}, A_1^{calc}, and σ^{data} indicate the experimental A_1, calculated A_1, and experimental error, respectively.

RESULTS AND DISCUSSIONS

From the χ^2 analysis, we obtain the results with $\chi^2 = 322.6$ in the LO case, and $\chi^2 = 300.4$ in the NLO case for 375 data points. The NLO χ^2 is significantly smaller than the LO one. It suggests that the NLO analysis is necessary for precise analysis. The obtained A_1 for the proton and for the neutron at $Q^2 = 5.0$ GeV^2 are shown in Fig.1 with the experimental data. Although the comparison is not straightforward because the data are taken at various Q^2, the obtained parameters reproduce the data well both in the LO and the NLO case. Figure 2 show the obtained polarized parton distribution functions at $Q^2 = 1.0$ GeV^2. In our analysis, the first moment of Δu_v (Δd_v) is fixed by positive (negative) value and the obtained distribution becomes positive (negative). Furthermore, antiquark and gluon distributions become negative and positive, respectivly.

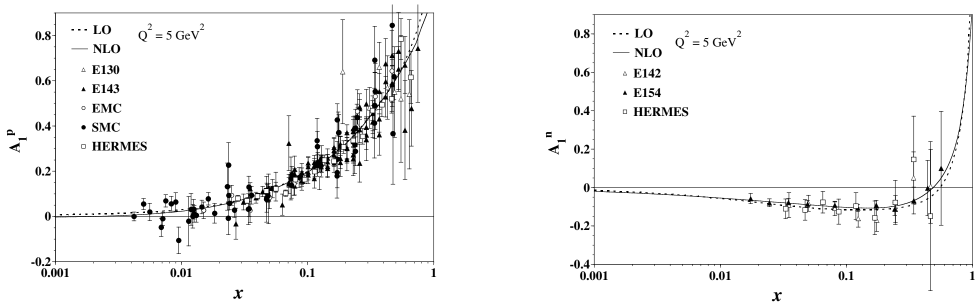

FIGURE 1. Spin asymmetries A_1 for the proton (left figure) and neutron (right figure).

For these distributions, quark spin content becomes $\Delta\Sigma = 0.201$ for LO and $\Delta\Sigma = 0.051$ for NLO at Q^2=1.0 GeV^2. The NLO $\Delta\Sigma$ is significantly smaller than the LO one. It is also smaller than the generally quoted values in other analyses that range in $0.1 \sim 0.3$. The difference of $\Delta\Sigma$ comes mainly from the difference of $\Delta\bar{q}$ in the small-x region. Because we don't have a small-x data at this stage, we should fix the small-x behavior of $\Delta\bar{q}$ from theoretical suggestions. We perform χ^2 fitting by fixing the parameter $\alpha_{\bar{q}}$, which controls the small-x behavior of $\Delta\bar{q}$, to $\alpha_{\bar{q}} = 1.0$ and 1.6 according to the predictions of the Regge theory and the perturbative QCD. These values are rather larger than the value $\alpha_{\bar{q}} = 0.32 \pm 0.22$ which is obtained by the previous NLO analysis. As a result, we get $\Delta\Sigma = 0.241$ ($\chi^2 = 305.8$) for $\alpha_{\bar{q}} = 1.0$ and $\Delta\Sigma = 0.276$ ($\chi^2 = 323.5$) for $\alpha_{\bar{q}} = 1.6$. The obtained $\Delta\Sigma$ are larger than the previous NLO result and are within the usually quoted range. From these results, we find that $\Delta\Sigma$ is very sensitive to the small-x behavior of the antiquark distributions. In order to determine the quark spin content precisely, the data in the small-x region are needed. Because χ^2 for the $\alpha_{\bar{q}} = 1.0$ case is close to the one for the previous NLO, this set can be also considered as a good parametrization. Therefore, we propose three sets of distributions, LO, NLO with free $\alpha_{\bar{q}}$ (NLO-1), and NLO with $\alpha_{\bar{q}} = 1.0$ (NLO-2), as AAC polarized distribution functions.

Finally, we briefly comment on the error estimate of the obtained distributions [2]. The uncertainties of our distributions can be estimated by using the error

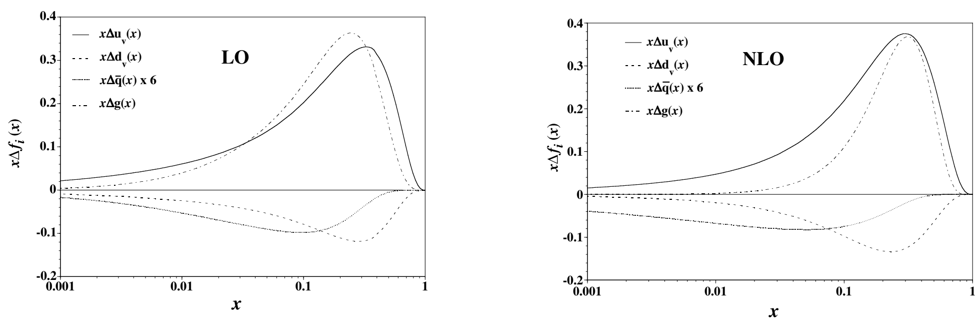

FIGURE 2. Obtained polarized distributions at $Q^2 = 1.0$ GeV^2.

matrix which is obtained by MINUIT. The 1-σ boundary of the distribution F is given by

$$(\delta F)^2 = \sum_i \sum_j \frac{\partial F}{\partial a_i} V_{ij} \frac{\partial F}{\partial a_j}, \tag{3}$$

where, V_{ij} is the error matrix element for parameters a_i and a_j. This analysis is in progress and will be reported elsewhere.

LIBRARY

For practical application of our distributions, we provide a library program which is given as FORTRAN subroutine AACPDF(ISET, Q2, X, POLPDF, STRUCT). The users should call this subroutine in their programs with the input parameters ISET, Q2, and X. This library contains three sets of distributions, LO, NLO-1, and NLO-2, and the parameter ISET designates which set is used. When ISET=1, 2, or 3, the returned values are those in the LO, NLO-1, or NLO-2, respectively. The parameters Q2 and X specify Q^2 and Bjorken-x at which the distributions are calculated. The allowed ranges are 1.0 GeV^2 $\leq Q^2 \leq 10^6$ GeV^2 and $10^{-9} \leq x \leq 1.0$. The obtained values are returned by the arrays POLPDF(-3:3) and STRUCT(3). The available polarized parton distribution functions and the structure functions are listed in Table 1. It should be noted that the returned values are the distributions and the structure functions multiplied by x.

The distribution values are obtained by interpolating the grid data which are provided as DATA statements in the subroutine. The number of the grid points is 23 for the variable Q^2 and 68 for the variable x. Because the Q^2 dependence of the distributions is almost linear function of $t \equiv \ln Q^2$, we simply use the linear interpolation of t for the variable Q^2. If t' is in the range $t_i \leq t' < t_{i+1}$, the distributions at $t = t'$ are approximated by

$$f(t') \approx \frac{t_{i+1} - t'}{t_{i+1} - t_i} f(t_i) + \frac{t' - t_i}{t_{i+1} - t_i} f(t_{i+1}). \tag{4}$$

TABLE 1. Available distributions and structure functions.

I	POLPDF(I)	STRUCT(I)
-3	$\bar{s}$ quark $[x\Delta\bar{s}(x,Q^2) = x\Delta s(x,Q^2))]$	—
-2	$\bar{d}$ quark $[x\Delta\bar{d}(x,Q^2) = x\Delta d_{sea}(x,Q^2)]$	—
-1	$\bar{u}$ quark $[x\Delta\bar{u}(x,Q^2) = x\Delta u_{sea}(x,Q^2)]$	—
0	gluon $[x\Delta g(x,Q^2)]$	—
1	u-valence quark $[x\Delta u_v(x,Q^2)]$	$xg_1(x,Q^2)$ for proton
2	d-valence quark $[x\Delta d_v(x,Q^2)]$	$xg_1(x,Q^2)$ for neutron
3	s quark $[x\Delta s(x,Q^2) = x\Delta\bar{s}(x,Q^2)]$	$xg_1(x,Q^2)$ for deuteron

On the other hand, the x dependence of the distributions is rather complicated. Therefore, we use the cubic spline interpolation for the variable x. If x' is in the range $x_i \leq x' < x_{i+1}$, the distributions at $x = x'$ are approximated by

$$f(x') \approx f(x_i) + B_i(x' - x_i) + C_i(x' - x_i)^2 + D_i(x' - x_i)^3. \tag{5}$$

Here, B_i, C_i, D_i are the spline coefficients which can be calculated by the grid data. By using these interpolation methods and the grid data, the AACPDF returns the values of the polarized parton distribution functions and the structure functions g_1 at the specified Q^2 and x point. We verified that the distributions obtained by the library reproduce well the results in Ref. [1].

AAC library can be downloaded from our web page [3]. The subroutine AACPDF is in the file " aac.f ". This file also includes the subroutine SPLINE and the function ISERCH which are used in the AACPDF.

SUMMARY

The polarized parton distribution functions have been determined by χ^2 fitting to A_1 experimental data. As a result, we found that the NLO χ^2 is significantly smaller than the LO one. It implies that the NLO analysis is important. In the NLO analysis, the obtained $\Delta\Sigma$ is rather smaller than the usually quoted values, and the differences come from the small-x behavior of $\Delta\bar{q}$. In order to fix $\Delta\Sigma$ precisely, the small-x measurements are required. We propose three sets of AAC distributions, LO, NLO-1 ($\alpha_{\bar{q}}$: free), and NLO-2 ($\alpha_{\bar{q}} = 1.0$ fixed). The FORTRAN library is provided for calculating the AAC distributions numerically. This library is useful for practical application of the AAC distributions and available at our web site [3].

ACKNOWLEDGEMENTS

M.H. and M.M. were supported by the JSPS Research Fellowships for Young Scientists and by the Grant-in-Aid from the Japanese Ministry of Education, Science, and Culture. This talk is based on the work with Y. Goto, N. Hayashi, H. Horikawa, S. Kumano, T. Morii, N. Saito, T.-A. Shibata, E. Taniguchi, and T. Yamanishi.

REFERENCES

1. Asymmetry Analysis Collaboration, Y. Goto, et al., *Phys. Rev.* **D 62**, 034017 (2000).
2. Asymmetry Analysis Collaboration, research in progress.
3. http://spin.riken.bnl.gov/aac/