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# Structure functions in the polarized Drell-Yan processes with spin-1/2 and spin-1 hadrons: I. general formalism

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We discuss general formalism for the structure functions which can be investigated in the polarized Drell-Yan processes with spin-1/2 and spin-1 hadrons. To be specific, the formalism can be applied to the proton-deuteron Drell-Yan processes. Because of the spin-1 nature, there are new structure functions which cannot be studied in the proton-proton reactions. Imposing Hermiticity, parity conservation, and time-reversal invariance, we find that 108 structure functions exist in the Drell-Yan processes. However, the number reduces to 22 after integrating the cross section over the virtual-photon transverse momentum $\vec{Q}_T$ or after taking the limit $Q_T \to 0$. There are 11 new structure functions in addition to the 11 ones in the Drell-Yan processes of spin-1/2 hadrons. The additional structure functions are associated with the tensor structure of the spin-1 hadron, and they could be measured by quadrupole spin asymmetries. For example, the structure functions exist for "intermediate" polarization although their contributions vanish in the longitudinal and transverse polarization reactions. We show a number of spin asymmetries for extracting the polarized structure functions. The proton-deuteron reaction may be realized in the RHIC-Spin project and other future ones, and it could be a new direction of next generation high-energy spin physics.

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## I. INTRODUCTION

Spin structure of the proton has been investigated through the polarized deep inelastic lepton scattering. A mysterious aspect of spin physics was revealed by the European Muon Collaboration (EMC) experimental result in 1988 [1]: almost none of the proton spin is carried by quarks. However, because the lepton reactions provide us only a limited piece of information on polarized partons [2], we should rely on other experimental methods such as those of the Relativistic Heavy Ion Collider (RHIC) -Spin project [3]. There, the approved activities are on various polarized proton-proton (pp) reactions.

Although the details of the pp reactions should be studied further [4], many pp processes have been investigated for a long time. On the other hand, there are few studies on the polarized deuteron reactions in connection with spin-dependent structure functions. We know that the deuteron has additional spin structure due to its spin-1 nature. The deuteron target is often used in the deep inelastic scattering; however, the major purpose is to extract the "neutron" structure functions in the deuteron. We think that we had better shed light on the deuteron spin structure itself rather than just using it for finding the neutron information. Within this context, there are some initial studies on the spin-1 structure functions. In the lepton scattering on the deuteron [5,6], there exist new tensor structure functions $b_1$, $b_2$, $b_3$, and $b_4$. Among them, the twist-two structure functions are $b_1$ and $b_2$, and they are related by the Callan-Gross type relation $2xb_1 = b_2$. A phenomenological sum rule was proposed for $b_1$ [7] in relation to the electric quadrupole structure. It could be important also to find the tensor polarization of sea quarks. Although the sum rule is valid for spin-1 hadrons within a quark model, there could have complexities in the deuteron because of nuclear shadowing effects [8].

There are some studies on the polarized lepton-deuteron reaction; however, the polarized deuteron has not been investigated in hadron-hadron reactions for finding the polarized structure functions. The general formalism of the polarized Drell-Yan process of spin-1/2 hadrons was studied in the pioneering paper of Ralston-Soper [9] and also the one by Donoghue and Gottlieb [10]. In these works, it was revealed that 48 structure functions can be studied in the reactions of spin-1/2 hadrons and the number becomes 11 after integrating over the virtual-photon transverse momentum $\vec{Q}_T$ or after taking the limit $Q_T \to 0$. The parton-model interpretation of these structure functions is also discussed in these works [9,10] as well as in the Tangerman-Mulders' paper [11].

The purpose of this paper is to investigate the general formalism of the polarized Drell-Yan processes with spin-1/2 and spin-1 hadrons in order to apply it to the polarized proton-deuteron (pd) reactions. In particular, we discuss what kind of new spin structure functions is measured and how they are related to the hadron polarizations. Another purpose is to facilitate future deuteron-spin projects such as the possible polarized deuteron reactions at RHIC [12].

In section II, we discuss general formalism of the polarized Drell-Yan process with spin-1/2 and spin-1 hadrons and obtain possible structure functions in the reaction. Then, we investigate how they are related to the Lorentz index structure of the hadron tensor in section III. Various spin asymmetries are discussed in section IV. The results are summarized in section V.

## II. FORMALISM FOR THE POLARIZED DRELL-YAN PROCESS WITH SPIN-1/2 AND SPIN-1 HADRONS

In the polarized Drell-Yan processes of spin-1/2 hadrons, possible structure functions were found by noting the Lorentz index structure in the hadron tensor [9]. Then, they were also discussed in a general framework [10] by using the Jacob-Wick helicity formalism [13]. In order to avoid missing some structure functions in the reactions of spin-1/2 and spin-1 hadrons, we first discuss the Donohue-Gottlieb type formalism [10] in this section. In section III, the Ralston-Soper type formalism is discussed.

We study the cross section of the Drell-Yan reaction

$$A\,(spin\,1/2) + B\,(spin\,1) \rightarrow \ell^+\ell^- + X \ , \qquad (2.1)$$

which is schematically shown in Fig. 2.1. The hadrons A and B are spin-1/2 and spin-1 particles, respectively. For example, they could be the proton and the deuteron; however, the following formalism can be applied to any other hadrons with spin-1/2 and spin-1.

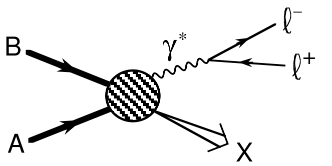


FIG. 2.1. Drell-Yan process.

In describing the cross section, a coordinate system has to be chosen. The center of momentum (c.m.) frame is easy to be visualized because it could be the same as or at least close to the laboratory frame of collider experiments. At this stage, the polarized deuteron acceleration is not planned at RHIC [12], so that its momentum is not known. The c.m. frame is shown in Fig. 2.2.

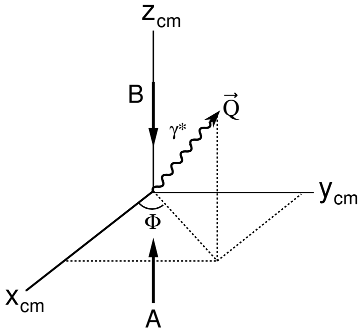


FIG. 2.2. Center of momentum frame.

The total dilepton momentum is denoted by $Q$, and it is expressed in the c.m. frame as

$$\begin{aligned} Q^\mu &\equiv k^\mu_{\ell+} + k^\mu_{\ell-} \\ &= (Q_0, Q_T cos\Phi, Q_T sin\Phi, Q_z)_{cm} \ . \end{aligned} \qquad (2.2)$$

The $z_{cm}$-axis is chosen in the direction of the hadron-A momentum $\vec{P}^{cm}_A$, so that the hadron-B momentum $\vec{P}^{cm}_B$ lies in the $-z_{cm}$ direction. The azimuthal angle of $\vec{Q}$ is denoted as $\Phi$.

The theoretical description becomes simple if the dilepton rest frame, rather than the c.m. frame, is chosen. It literally means that the total dilepton momentum vanishes ($\vec{Q}=0$) in the frame. In order to obtain this frame from the c.m. in Fig. 2.2, the frame has to be boosted first to the $z_{cm}$ direction so as to get $Q_z=0$, then to the $\vec{Q}_T$ direction so as to obtain $\vec{Q}=0$. The resulting frame is shown by $x$, $y$, and $z$ axes in Fig. 2.3. The momenta $\vec{P}_A$ and $\vec{P}_B$ are no longer along the $z$ axis. In the following formalism of this section, we take the dilepton rest frame as the Collins-Soper frame [14], which is shown by $x_0$, $y_0$, and $z_0$ axes in Fig. 2.3.

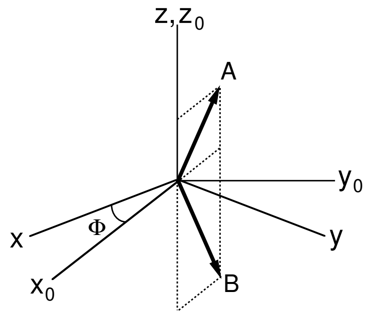


FIG. 2.3. Dilepton rest frame.

The $z_0$ axis is chosen so as to bisect the angle between the momenta $\vec{P}_A$ and $-\vec{P}_B$:

$$\hat{z}_0 = \frac{\vec{P}_A(P_B\cdot Q) - \vec{P}_B(P_A\cdot Q)}{|\vec{P}_A(P_B\cdot Q) - \vec{P}_B(P_A\cdot Q)|} \ . \qquad (2.3)$$

Throughout this paper, the $g_{\mu\nu}$ convention $g_{00}=-g_{11}=-g_{22}=-g_{33}=1$ is used. Then the $y_0$-axis is taken as

$$\hat{y}_0 = -\frac{\vec{P}_A\times\vec{P}_B}{|\vec{P}_A\times\vec{P}_B|} \ . \qquad (2.4)$$

The remaining $x_0$-axis is chosen so that it is orthogonal to $\hat{y}_0$ and $\hat{z}_0$. From the above definitions of $\hat{y}_0$ and $\hat{z}_0$, $\hat{x}_0$ is equal to the transverse unit vector $\hat{Q}_T$ which is in the direction away from $(\vec{P}_A+\vec{P}_B)_T$. Between the two dilepton rest frames, there is a difference of the azimuthal angle $\Phi$ as shown in Fig. 2.3. This difference becomes significant when the cross section is integrated over $\vec{Q}_T$ later.

The Drell-Yan cross section is given by

$$\begin{aligned} d\sigma = & \frac{1}{4\sqrt{(P_A\cdot P_B)^2 - M_A^2 M_B^2}} \sum_{s_{\ell+},s_{\ell-}} \sum_X \\ & \times (2\pi)^4 \delta^4(P_A+P_B-k_{\ell+}-k_{\ell-}-P_X) \\ \times & \ | < \ell^+\ell^- X|T|AB > |^2 \frac{d^3k_{\ell+}}{(2\pi)^3 2E_{\ell+}} \frac{d^3k_{\ell-}}{(2\pi)^3 2E_{\ell-}} \ , \end{aligned} \qquad (2.5)$$

where $M_A$ and $M_B$ are A and B masses, $s_{\ell^+}$ and $s_{\ell^-}$ indicate the $\ell^+$ and $\ell^-$ spins, $s$ is the center-of-mass energy squared $s = (P_A + P_B)^2$, and the matrix element is given by

$$< \ell^+ \ell^- X | T | AB > = \bar{u}(k_{\ell^-}, s_{\ell^-}) e \gamma_\mu v(k_{\ell^+}, s_{\ell^+}) \times \frac{g^{\mu\nu}}{(k_{\ell^+} + k_{\ell^-})^2} < X | e J_\nu(0) | AB > \ . \quad (2.6)$$

In the dilepton rest frame, the cross section becomes

$$\frac{d\sigma}{d^4Q d\Omega} = \frac{\alpha^2}{2\, s\, Q^4} L_{ij}\, W_{ij} \ , \quad (2.7)$$

where $\alpha = e^2/(4\pi)$ is the fine structure constant, the component $L_{00}$ vanishes in this frame, and the hadron and lepton masses are neglected by considering $M_A^2, M_B^2 \ll s$ and $m_\ell^2 \ll Q^2$. The lepton part is given by

$$\begin{aligned} L_{ij} &= \frac{1}{2} \sum_{s_{\ell^+}, s_{\ell^-}} [\, u(k_{\ell^-}, s_{\ell^-}) \gamma_i v(k_{\ell^+}, s_{\ell^+})\, ]^\dagger \\ &\qquad \times [\, u(k_{\ell^-}, s_{\ell^-}) \gamma_j v(k_{\ell^+}, s_{\ell^+})\, ] \\ &= 4\, \vec{k}^{\,2} \left( \delta_{ij} - k_i k_j / |\vec{k}|^2 \right) \ , \end{aligned} \quad (2.8)$$

where $\vec{k} \equiv \vec{k}_{\ell^+}$ and the lepton mass terms are neglected ($m_\ell^2 \ll \vec{k}^{\,2}$). The hadron part is given by

$$W_{ij} = \int \frac{d^4\xi}{(2\pi)^4} e^{iQ\cdot\xi} < AB \,|\, J_i(0)\, J_j(\xi) \,|\, AB > \ . \quad (2.9)$$

The polar and azimuthal angles $\theta$ and $\phi_0$ of the lepton $\ell^+$ are defined by

$$\hat{k} \cdot \hat{z}_0 = cos\theta, \quad \hat{k} \cdot \hat{y}_0 = sin\theta\, sin\phi_0 \ , \quad (2.10)$$

where $\hat{k} = \vec{k}/|\vec{k}|$. It is convenient to express the lepton tensor $L_{ij}$ as

$$L_{ij} = \frac{8\, \vec{k}^{\,2}}{3} \sum_{\lambda, \lambda', L, M} f(L) < 1\lambda : LM | 1\lambda' > \times D^{\,L}_{M0}(\phi_0, \theta, 0)\, \varepsilon_i(\lambda')\, \varepsilon_j^*(\lambda) \ , \quad (2.11)$$

where $< 1\lambda : LM | 1\lambda' >$ is a Clebsch-Gordan coefficient and $\varepsilon_i(\lambda)$ is the $i$ component of the spherical unit vector [15]. The coefficients $f(L)$ are defined by

$$f(0) = 1 \ , \quad f(1) = 0 \ , \quad f(2) = \frac{\sqrt{10}}{2} \ . \quad (2.12)$$

If the lepton mass cannot be neglected, $f(2)$ becomes the expression in Ref. [10]. We use the rotational matrix element $D^{\,j}_{mm'}(\alpha, \beta, \gamma)$ which is defined by [16]

$$\begin{aligned} D^{\,j}_{mm'}(\alpha, \beta, \gamma) &\equiv < jm \,|\, e^{-i\alpha J_z}\, e^{-i\beta J_y}\, e^{-i\gamma J_z} \,|\, jm' > \\ &= e^{-im\alpha}\, d^{\,j}_{mm'}(\beta)\, e^{-im'\gamma} \ . \end{aligned} \quad (2.13)$$

We should be careful about the definition of the $D$-matrix because there are different conventions. The definition of Eq. (2.13) is used throughout this paper; however, it is different, for example, from the one in Ref. [15].

Using these expressions and introducing spin density matrix, the cross section becomes

$$\begin{aligned} \frac{d\sigma}{d^4Q d\Omega_0} &= \frac{1}{4\,\pi} \sum_{L,M} f(L) D^{\,L}_{M0}(\phi_0, \theta, 0) \sum_{\lambda, \lambda'} \sum_X \\ &\quad \times \delta^4(P_A + P_B - Q - P_X) < 1\lambda : LM | 1\lambda' > \\ &\quad \times \sum_{\mu, \mu'} \sum_{\nu, \nu'} \rho(AB)_{\mu\nu:\mu'\nu'}\, F^\dagger_{\lambda'\mu'\nu'}\, F_{\lambda\mu\nu} \ . \end{aligned} \quad (2.14)$$

The helicity amplitude $F_{\lambda\mu\nu}$ is given by

$$F_{\lambda\mu\nu} = \sqrt{\frac{\pi\,\alpha^2}{3\, s\, \vec{k}^{\,2}}} < X \,|\, \vec{\varepsilon}^{\,*}(\lambda) \cdot \vec{J}(0) \,|\, A(\mu) B(\nu) > \ , \quad (2.15)$$

and the spin density matrix $\rho(AB)_{\mu\nu:\mu'\nu'}$ is

$$\rho(AB)_{\mu\nu:\mu'\nu'} = \rho(A)_{\mu\mu'}\, \rho(B)_{\nu\nu'} \ . \quad (2.16)$$

The density matrix for the hadron $A$ or $B$ is defined by

$$\rho = \frac{1}{2S+1} \sum_{K,N} < \tau_{KN}(S) > \tau^\dagger_{KN}(S) \ , \quad (2.17)$$

where the statistical tensor $\tau_{KN}(S)$ is expressed as [17]

$$\tau_{KN}(S) = \sqrt{2S+1} \sum_{\mu, \mu'} (-1)^{S-\mu} < S\mu' : S - \mu | KN > \times | S\mu' >< S\mu | \ . \quad (2.18)$$

Therefore, the density matrix becomes

$$\begin{aligned} \rho_{\mu\mu'} &= < S\mu \,|\, \rho \,|\, S\mu' > \\ &= \sum_{K,N} \frac{\sqrt{2K+1}}{2S+1} < S\mu' : KN | S\mu > < \tau^\dagger_{KN}(S) > \ . \end{aligned} \quad (2.19)$$

The quantities $< \tau^\dagger_{KN}(S) >$ for the spin 1/2 and 1 particles are

$$< \tau^\dagger_{00}(S) > = 1 \ , \quad (2.20)$$

$$< \tau^\dagger_{1N}(1/2) > = |\vec{S}|\, D^{\,1}_{N0}(\alpha, \beta, 0) \ , \quad (2.21)$$

$$< \tau^\dagger_{1N}(1) > = \sqrt{\frac{3}{2}}\, |\vec{S}|\, D^{\,1}_{N0}(\alpha, \beta, 0) \ , \quad (2.22)$$

where $\alpha$ and $\beta$ are azimuthal and polar angles of the polarization vector $\vec{S}$. In addition to these tensors, there exists another one for the spin-1 particle. As it is obvious from the definition of Eq. (2.18), the rank-two ($K = 2$) tensor is possible. We also express it in terms of the $D$-matrix:

$$< \tau^\dagger_{2N}(1) > = < \sqrt{2}\, \vec{S}^{\,2}\, D^{\,2}_{N0}(\alpha, \beta, 0) > \ . \quad (2.23)$$

However, special attention should be paid for $< \vec{S}^2 >= 2$ in calculating $< \tau^\dagger_{20}(1) >=< \vec{S}^2 (3\,cos^2\beta - 1)/\sqrt{2} >= < (3\,S_z^2 - \vec{S}^2)/\sqrt{2} >$. Furthermore, $< S_i\, S_j >$ should be replaced by $< S_i\, S_j + S_j\, S_i > /2$. For simplicity, the notation $<>$ of Eq. (2.23) is not written in the following equations. The polarization angles are denoted as $\alpha_{_A}$ and $\beta_{_A}$ for the hadron $A$ and as $\alpha_{_B}$ and $\beta_{_B}$ for $B$. The angles $\alpha_{_B}$ and $\beta_{_B}$ ($\alpha_{_A}$ and $\beta_{_A}$) are measured by taking the coordinates as $x_{_B} = x_{cm}$, $y_{_B} = -y_{cm}$, and $z_{_B} = -z_{cm}$ ($x_{_A} = x_{cm}$, $y_{_A} = y_{cm}$, and $z_{_A} = z_{cm}$). As it is pointed out in Ref. [10], the transverse axis could be chosen arbitrary. The azimuthal angles $\Phi$, $\alpha_{_A}$, and $\alpha_{_B}$ transform into $\Phi - \varepsilon$, $\alpha_{_A} - \varepsilon$, and $\alpha_{_B} + \varepsilon$ under the angle-$\varepsilon$ rotation about the $z$ axis, so that only the angles $\alpha_{_A} - \Phi$ and $\alpha_{_B} + \Phi$ have physical meaning. In this way, the angles $\alpha_{_A}$ and $\alpha_{_B}$ are replaced by $\alpha_{_A} - \Phi$ and $\alpha_{_B} + \Phi$, respectively in the expressions of the $D$-matrices. It is noteworthy to emphasize that new $\vec{S}^2$ terms exist in our Drell-Yan process whereas merely the linear terms of $\vec{S}$ are enough for the pp reaction. Therefore, the study of the new spin structure is in other words to investigate the rank-two $\vec{S}^2$ terms.

Substituting these expressions of $<\tau^\dagger_{_{KN}}(S)>$ into Eq. (2.19), we obtain the spin density matrix for the reaction in Eq. (2.16). Then, the cross section of Eq. (2.14) becomes

$$\begin{aligned}
\frac{d\sigma}{d^4Q d\Omega_0} = \frac{1}{4\pi} \sum_{L=0,2} f(L) \sum_{M=-L}^{L} D^L_{M0}(\phi_0,\theta,0) \Bigg[ & R^{M\,0\,0}_{L\;0\,0} + \sqrt{3}\,|\vec{S}_A| \sum_{N_1} R^{M\,N_1\,0}_{L\;\;1\;0}\, D^1_{N_1 0}(\alpha_A - \Phi, \beta_A, 0) \\
& + \frac{3}{\sqrt{2}}\,|\vec{S}_B| \sum_{N_2} R^{M\,0\,N_2}_{L\;0\;1}\, D^1_{N_2 0}(\alpha_B + \Phi, \beta_B, 0) + \sqrt{10}\,\vec{S}^2_B \sum_{N_2} R^{M\,0\,N_2}_{L\;0\;2}\, D^2_{N_2 0}(\alpha_B + \Phi, \beta_B, 0) \\
& + \frac{3\sqrt{3}}{\sqrt{2}}\,|\vec{S}_A|\,|\vec{S}_B| \sum_{N_1,N_2} R^{M\,N_1\,N_2}_{L\;\;1\;\;1}\, D^1_{N_1 0}(\alpha_A - \Phi, \beta_A, 0)\, D^1_{N_2 0}(\alpha_B + \Phi, \beta_B, 0) \\
& + \sqrt{30}\,|\vec{S}_A|\,\vec{S}^2_B \sum_{N_1,N_2} R^{M\,N_1\,N_2}_{L\;\;1\;\;2}\, D^1_{N_1 0}(\alpha_A - \Phi, \beta_A, 0)\, D^2_{N_2 0}(\alpha_B + \Phi, \beta_B, 0) \Bigg] ,
\end{aligned} \tag{2.24}$$

where the structure function $R^{MN_1N_2}_{LK_1K_2}$ is defined by the helicity amplitudes and the Clebsch-Gordan coefficients as

$$\begin{aligned}
R^{M\,N_1\,N_2}_{L\;K_1K_2} \equiv \frac{1}{6} \sum_{\lambda,\mu,\nu} \sum_X \delta^4(P_A + P_B - Q - P_X)\ & < 1\lambda : LM | 1\lambda' > \\
& \times F^\dagger_{\lambda'\mu'\nu'}\, F_{\lambda\mu\nu} < \frac{1}{2}\mu' : K_1 N_1 | \frac{1}{2}\mu >< 1\nu' : K_2 N_2 | 1\nu > \ .
\end{aligned} \tag{2.25}$$

The terms which do not exist in the pp reactions are those proportional to $\vec{S}^2_B$ and $|\vec{S}_A|\,\vec{S}^2_B$ in Eq. (2.24). These terms are, therefore, associated with the new spin structure for the spin-1 hadron. According to Eq. (2.24), the structure functions $R^{M\,0\,N_2}_{L\;0\;2}$ and $R^{M\,N_1\,N_2}_{L\;\;1\;\;2}$ appear as the additional ones.

We have shown that new structure functions exist in the polarized Drell-Yan reactions with spin-1/2 and spin-1 hadrons. However, it is not still clear how many structure functions could be studied in the reactions because the cross section is written in the summation form. Many of the possible functions $R^{M\,N_1\,N_2}_{L\;K_1K_2}$ are not independent, and the number can be reduced by imposing Hermiticity, parity conservation, and time-reversal invariance. The Hermiticity requires

$$R^{M\,N_1\,N_2}_{L\;K_1K_2} = (-1)^{M+N_1+N_2} \left( R^{-M\,-N_1\,-N_2}_{\;\;L\;\;\;\;K_1\;\;\;K_2} \right)^* \ . \tag{2.26}$$

The parity conservation and time-reversal invariance require

$$R^{M\,N_1\,N_2}_{L\;K_1K_2} = (-1)^{L+K_1+K_2+M+N_1+N_2}\, R^{-M\,-N_1\,-N_2}_{\;\;L\;\;\;\;K_1\;\;\;K_2} \ . \tag{2.27}$$

These equations indicate $\left( R^{M\,N_1\,N_2}_{L\;K_1K_2} \right)^* = (-1)^{L+K_1+K_2}\, R^{M\,N_1\,N_2}_{L\;K_1K_2}$. Therefore, the structure functions have the property

$$R^{M\,N_1\,N_2}_{L\;K_1K_2} = \begin{cases} \text{real} & \text{if } L + K_1 + K_2 = \text{even number}, \\ \text{imaginary} & \text{if } L + K_1 + K_2 = \text{odd number}, \\ 0 & \text{if } L + K_1 + K_2 = \text{odd number and } M + N_1 + N_2 = 0. \end{cases} \tag{2.28}$$

Imposing these conditions on the structure functions $R^{M\,N_1\,N_2}_{L\;K_1K_2}$, we find that 108 independent structure functions exist in our Drell-Yan processes. In the reactions of spin-1/2 hadrons, there are 48 structure functions. Therefore, there exist 60 additional ones in the case of spin-1/2 and spin-1 hadrons.

The polarized pp Drell-Yan processes will be investigated at RHIC; however, all of the 48 structure functions for the spin-1/2 hadrons would not be measured. Of course, all of them are not important at this stage. We know that even the leading-twist contributions are not well understood yet. The essential structure functions in our reactions can be extracted by integrating over the transverse momentum $\vec{Q}_T$. However, the integration over the azimuthal angle $\Phi$ of $\vec{Q}_T$ is sufficient in order to be compared with the result in section III. We have to be careful about the coordinates in calculating the integration because our present coordinate system is taken so that the $x_0$-axis is in the direction of $\vec{Q}_T$. The $x_0$ and $y_0$ axes should be rotated around $z$ when the integral of $\Phi$ is calculated. We fix the frame as the $(x, y, z)$ coordinates in Fig. 2.3, then the difference between the azimuthal angles is $\Phi$ as shown in the figure:

$$\frac{d\sigma}{d^4Q\, d\Omega} = \left. \frac{d\sigma}{d^4Q\, d\Omega_0} \right|_{\phi_0=\phi-\Phi} . \tag{2.29}$$

After the $\Phi$ integration, the only terms with $M = N_2 - N_1$ remain and the cross section becomes

$$\begin{aligned} 2\int d\Phi\, \frac{d\sigma}{d^4Q\, d\Omega} = \sum_{L=0,2} f(L) \sum_{M=-L}^{L} D^{\,L}_{M0}(\phi,\theta,0) \Big[ & \delta_{M,0}\, R^{M\,0\,0}_{L\;0\,0} + \sqrt{3}\, |\vec{S}_A|\, R^{M\,-M\,0}_{L\;\;1\;\;0}\, D^{\,1}_{-M0}(\alpha_A,\beta_A,0) \\ & + \frac{3}{\sqrt{2}}\, |\vec{S}_B|\, R^{M\,0\,M}_{L\;0\;1}\, D^{\,1}_{M0}(\alpha_B,\beta_B,0) + \sqrt{10}\, \vec{S}_B^{\,2}\, R^{M\,0\,M}_{L\;0\;2}\, D^{\,2}_{M0}(\alpha_B,\beta_B,0) \\ & + \frac{3\sqrt{3}}{\sqrt{2}}\, |\vec{S}_A|\, |\vec{S}_B| \sum_{N_1,N_2} \delta_{M,N_2-N_1}\, R^{M\,N_1\,N_2}_{L\;\;1\;\;1}\, D^{\,1}_{N_10}(\alpha_A,\beta_A,0)\, D^{\,1}_{N_20}(\alpha_B,\beta_B,0) \\ & + \sqrt{30}\, |\vec{S}_A|\, \vec{S}_B^{\,2} \sum_{N_1,N_2} \delta_{M,N_2-N_1}\, R^{M\,N_1\,N_2}_{L\;\;1\;\;2}\, D^{\,1}_{N_10}(\alpha_A,\beta_A,0)\, D^{\,2}_{N_20}(\alpha_B,\beta_B,0) \Big] . \end{aligned} \tag{2.30}$$

There exist 22 structure functions in this equation, and they are physically significant ones which could be investigated experimentally. In order to clarify the situation, we list these 22 structure functions in Table 2.1.

TABLE 2.1. List of the possible structure functions $R^{M\,N_1\,N_2}_{L\;K_1\,K_2}$ after the integration over $\Phi$. The asterisk * indicates a new structure function which does not exist in the Drell-Yan processes of spin-1/2 hadrons.

| $R^{M\,N_1\,N_2}_{L\;K_1\,K_2}$ | $L$ | $M$ | $K_1$ | $N_1$ | $K_2$ | $N_2$ |
|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 |
| * | 0 | 0 | 0 | 0 | 2 | 0 |
| | 0 | 0 | 1 | 0 | 1 | 0 |
| | 0 | 0 | 1 | 1 | 1 | 1 |
| * | 0 | 0 | 1 | 1 | 2 | 1 |
| | 2 | 0 | 0 | 0 | 0 | 0 |
| * | 2 | 0 | 0 | 0 | 2 | 0 |
| | 2 | 0 | 1 | 0 | 1 | 0 |
| | 2 | 0 | 1 | 1 | 1 | 1 |
| * | 2 | 0 | 1 | 1 | 2 | 1 |
| | 2 | 1 | 0 | 0 | 1 | 1 |
| * | 2 | 1 | 0 | 0 | 2 | 1 |
| | 2 | 1 | 1 | −1 | 0 | 0 |
| | 2 | 1 | 1 | −1 | 1 | 0 |
| * | 2 | 1 | 1 | −1 | 2 | 0 |
| | 2 | 1 | 1 | 0 | 1 | 1 |
| * | 2 | 1 | 1 | 0 | 2 | 1 |
| * | 2 | 1 | 1 | 1 | 2 | 2 |
| * | 2 | 2 | 0 | 0 | 2 | 2 |
| | 2 | 2 | 1 | −1 | 1 | 1 |
| * | 2 | 2 | 1 | −1 | 2 | 1 |
| * | 2 | 2 | 1 | 0 | 2 | 2 |

The 11 structure functions with the asterisk mark * in Table 2.1 are new ones which are associated with the spin-1 nature of the hadron $B$. The other 11 structure functions exist in the Drell-Yan processes of spin-1/2 hadrons, so that the interesting point of our reactions (e.g. the polarized proton-deuteron processes) is to investigate the details of the new 11 structure functions.

Substituting explicit expressions for the $D$-matrices and calculating the the summations over $L$, $M$, $N_1$, and $N_2$, we have the cross section with the 22 structure functions:

$$
\begin{aligned}
2 \int d\Phi \, \frac{d\sigma}{d^4Q \, d\Omega} = f(0) \Bigg[ & R^{000}_{000} + \frac{3\sqrt{3}}{\sqrt{2}} \, |\vec{S}_A| \, |\vec{S}_B| \left\{ cos\beta_A \, cos\beta_B \, R^{000}_{011} + sin\beta_A \, sin\beta_B \, cos(\alpha_A + \alpha_B) \, R^{011}_{011} \right\} \\
& + \frac{\sqrt{5}}{\sqrt{2}} \, \vec{S}_B^{\,2} \, (3 \, cos^2\beta_B - 1) \, R^{000}_{002} - 3\sqrt{10} \, |\vec{S}_A| \, \vec{S}_B^{\,2} \, sin\beta_A \, sin\beta_B \, cos\beta_B \, sin(\alpha_A + \alpha_B) \, i \, R^{011}_{012} \Bigg] \\
+ f(2) \, (3 \, cos^2\theta - 1) \Bigg[ & \frac{1}{2} \, R^{000}_{200} + \frac{3\sqrt{3}}{2\sqrt{2}} \, |\vec{S}_A| \, |\vec{S}_B| \left\{ cos\beta_A \, cos\beta_B \, R^{000}_{211} + sin\beta_A \, sin\beta_B \, cos(\alpha_A + \alpha_B) \, R^{011}_{211} \right\} \\
& + \frac{\sqrt{5}}{2\sqrt{2}} \, \vec{S}_B^{\,2} \, (3 \, cos^2\beta_B - 1) \, R^{000}_{202} - \frac{3\sqrt{5}}{\sqrt{2}} \, |\vec{S}_A| \, \vec{S}_B^{\,2} \, sin\beta_A \, sin\beta_B \, cos\beta_B \, sin(\alpha_A + \alpha_B) \, i \, R^{011}_{212} \Bigg] \\
+ f(2) \, sin\theta \, cos\theta \Bigg[ & sin(\phi - \alpha_A) \left\{ 3 \, |\vec{S}_A| \, sin\beta_A \, i \, R^{1\,-1\,0}_{2\;1\;0} + \frac{3\sqrt{5}}{\sqrt{2}} \, |\vec{S}_A| \, \vec{S}_B^{\,2} \, sin\beta_A \, (3 \, cos^2\beta_A - 1) \, i \, R^{1\,-1\,0}_{2\;1\;2} \right\} \\
& - sin(\phi + \alpha_B) \left\{ \frac{3\sqrt{3}}{\sqrt{2}} \, |\vec{S}_B| \, sin\beta_B \, i \, R^{101}_{201} + 3\sqrt{30} \, |\vec{S}_A| \, \vec{S}_B^{\,2} \, cos\beta_A \, sin\beta_B \, cos\beta_B \, i \, R^{101}_{212} \right\} \\
& - sin(\phi + \alpha_A + 2\alpha_B) \, \frac{3\sqrt{15}}{2} \, |\vec{S}_A| \, \vec{S}_B^{\,2} \, sin\beta_A \, sin^2\beta_B \, i \, R^{112}_{212} \\
& - cos(\phi - \alpha_A) \, \frac{9}{\sqrt{2}} \, |\vec{S}_A| \, |\vec{S}_B| \, sin\beta_A \, cos\beta_B \, R^{1\,-1\,0}_{2\;1\;1} \\
& + cos(\phi + \alpha_B) \left\{ 3\sqrt{10} \, \vec{S}_B^{\,2} \, sin\beta_B \, cos\beta_B \, R^{101}_{202} + \frac{9}{\sqrt{2}} \, |\vec{S}_A| \, |\vec{S}_B| \, cos\beta_A \, sin\beta_B \; R^{101}_{211} \right\} \Bigg] \\
+ f(2) \, sin^2\theta \Bigg[ & cos(2\phi + 2\alpha_B) \, \frac{3\sqrt{5}}{2\sqrt{2}} \, \vec{S}_B^{\,2} \, sin^2\beta_B \, R^{202}_{202} - cos(2\phi - \alpha_A + \alpha_B) \, \frac{9}{4} \, |\vec{S}_A| \, |\vec{S}_B| \, sin\beta_A \, sin\beta_B \, R^{2\,-1\,1}_{2\;1\;1} \\
& + sin(2\phi - \alpha_A + \alpha_B) \, \frac{3\sqrt{15}}{2} \, |\vec{S}_A| \, \vec{S}_B^{\,2} \, sin\beta_A \, sin\beta_B \, cos\beta_B \, i \, R^{2\,-1\,1}_{2\;1\;2} \\
& - sin(2\phi + 2\alpha_B) \, \frac{3\sqrt{15}}{2\sqrt{2}} \, |\vec{S}_A| \, \vec{S}_B^{\,2} \, cos\beta_A \, sin^2\beta_B \, i \, R^{202}_{212} \Bigg] \, . 
\end{aligned}
\tag{2.31}
$$

Here, the term $\vec{S}_B^{\,2}(3\,cos\beta_B^2 - 1)$ should be replaced by $3 < S_{Bz}^2 > - < \vec{S}_B^{\,2} >= 3 < S_{Bz}^2 > -2$ as it was explained after Eq. (2.23). It is interesting to find that the new structure functions with $K_2 = 2$ are multiplied by the polarization-angle factors, $3\,cos^2\beta_B - 1$, $sin^2\beta_B$, and $sin\beta_B \, cos\beta_B$. Because the factor $3\,cos^2\beta_B - 1 \sim Y_{20}$ is usually associated with the quadrupole structure of the spin-1 hadrons, it is easy to see that these structure functions are related to such tensor structure. In the same way, other factors are also related to the spherical harmonics as $sin^2\beta_B \sim Y_{22}$ and $sin\beta_B \, cos\beta_B \sim Y_{21}$, so that these structure functions are also related to the tensor structure. It is particularly interesting to find the factor $sin\beta_B \, cos\beta_B = sin(2\beta_B)/2$ in the cross section. It means that the structure functions with $sin\beta_B \, cos\beta_B$ *cannot* be found in the longitudinal ($\beta_B = 0$) or transverse ($\beta_B = \pi/2$) polarization experiments. In order to measure them, we should use the polarization condition $0 < \beta_B < \pi/2$ or $\pi/2 < \beta_B < \pi$. For example, $\beta_B = \pi/4$ is an appropriate choice. We call this polarization *intermediate polarization* in the sense that it is between the longitudinal and transverse polarization states. The longitudinal, transverse, and intermediate polarization states are denoted as $L$, $T$, and $I$, respectively in the following discussions.

In this section, we have shown that 108 structure functions exist in the Drell-Yan processes with spin-1/2 and spin-1 hadrons. After the integration over the azimuthal angle $\Phi$ (or over $\vec{Q}_T$), 22 structure functions can be investigated. Among them, there are 11 new structure functions which do not exist in the reactions of spin-1/2 hadrons.

## III. HADRON TENSOR AND STRUCTURE FUNCTIONS

Although the helicity couplings and polarization dependence are clearly shown in the formalism of the previous section, it is not straightforward to find the physics meaning of these new structure functions. It is also important to check whether the number 22 is the right one in an independent way. A popular method to describe the polarized pp Drell-Yan processes was developed by Ralston and Soper (RS) [9] by noting possible Lorentz index combinations in the hadron tensor $W^{\mu\nu}$. The RS type formalism is extended to the reactions of spin-1/2 and spin-1 hadrons in this section.

We expand the hadron tensor

$$W^{\mu\nu}=\int\frac{d^4\xi}{(2\,\pi)^4}\,e^{iQ\cdot\xi}\langle P_A S_A P_B S_B|J^\mu(0)J^\nu(\xi)|P_A S_A P_B S_B\rangle\,, \tag{3.1}$$

in terms of possible Lorentz index combinations including the hadron momenta and spins. We use the Lorentz vectors $X^\mu$, $Y^\mu$, and $Z^\mu$ in Ref. [9]:

$$\begin{aligned} X^\mu &= P_A^\mu\, Q^2\, Z\cdot P_B - P_B^\mu\, Q^2\, Z\cdot P_A \\ &\quad + Q^\mu\,(Q\cdot P_B\, Z\cdot P_A - Q\cdot P_A\, Z\cdot P_B)\ , \\ Y^\mu &= \epsilon^{\mu\alpha\beta\gamma}\ P_{A\alpha}\, P_{B\beta}\, Q_\gamma\ , \\ Z^\mu &= P_A^\mu\, Q\cdot P_B - P_B^\mu\, Q\cdot P_A\ , \end{aligned} \tag{3.2}$$

where the convention for the antisymmetric tensor is $\varepsilon_{0123}=1$. Then, we define the vector $T^\mu$ as [18]

$$T^\mu = \varepsilon^{\mu\alpha\beta\gamma} S_\alpha\, Z_\beta\, Q_\gamma\ . \tag{3.3}$$

In addition to the vectors in Eq. (3.2), $Q^\mu$, $S_A^\mu$, $S_B^\mu$, $T_A^\mu$, and $T_B^\mu$ are available for the analysis of $W^{\mu\nu}$. Instead of $S_A^\mu$ and $S_B^\mu$, it is more convenient to use the transverse vectors $S_{AT}^\mu$ and $S_{BT}^\mu$, which are defined by

$$S_T^\mu = \left( g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) S_\nu\ . \tag{3.4}$$

Even though $S^\mu$ is replaced by $S_T^\mu$ in Eq. (3.3), $T^\mu$ remains the same: $T^\mu=\varepsilon^{\mu\alpha\beta\gamma}S_\alpha Z_\beta Q_\gamma=\varepsilon^{\mu\alpha\beta\gamma}S_{T\alpha} Z_\beta Q_\gamma$, and it is a transverse vector $T^\mu=T_T^\mu$. We should obtain the 108 independent structure functions in the previous section by the combinations of these Lorentz vectors and pseudovectors. However, because it is too lengthy to write them down and all of them are not important in any case, we consider the limit $Q_T\to 0$. As we discuss in the end of this section, the same number of structure functions should be obtained as the one after integration over $\Phi$ in section II. The transverse momentum $Q_T$ is usually small because it is primarily due to the intrinsic transverse momenta of the partons. Therefore, it is roughly restricted by the hadron size $r$ as $Q_T \lesssim 1/r$.

In the case $Q_T\to 0$, the situation becomes simpler because $|\vec X|$ and $|\vec Y|$ are proportional to $Q_T$ and $X^0 = Y^0 = 0$ in the dilepton rest frame. It means that we do not have to take into account the vectors $X^\mu$ and $Y^\mu$ in analyzing $W^{\mu\nu}$. Furthermore, the hadron tensor has to satisfy the conditions

Hermiticity:

$$[W^{\nu\mu}(Q;P_AS_A;P_BS_B)]^* = W^{\mu\nu}(Q;P_AS_A;P_BS_B)\,,$$

parity conservation:

$$W^{\mu\nu}(Q;P_AS_A;P_BS_B) = W_{\mu\nu}(\overline{Q};\overline{P}_A-\overline{S}_A;\overline{P}_B-\overline{S}_B)\,,$$

time-reveral invariance:

$$[W^{\mu\nu}(Q;P_AS_A;P_BS_B)]^* = W_{\mu\nu}(\overline{Q};\overline{P}_A\overline{S}_A;\overline{P}_B\overline{S}_B)\,, \tag{3.5}$$

where $\overline{P}$ is defined by $\overline{P}^\mu=(P^0,-\vec{P})$. Expanding the hadron tensor in terms of all the possible combinations of the Lorentz vectors and pseudovectors so as to meet these requirements, we obtain

$$\begin{aligned} W^{\mu\nu} &= -g^{\mu\nu}A - \frac{Z^\mu Z^\nu}{Z^2}B' + Z^{\{\mu}T_A^{\nu\}}C + Z^{\{\mu}T_B^{\nu\}}D \\ &\quad + Z^{\{\mu}S_{AT}^{\nu\}}E + Z^{\{\mu}S_{BT}^{\nu\}}F - S_{BT}^\mu S_{BT}^\nu G' - S_{AT}^{\{\mu}S_{BT}^{\nu\}}H' \\ &\quad + T_A^{\{\mu}S_{BT}^{\nu\}}I' + S_{BT}^{\{\mu}T_B^{\nu\}}J + Q^\mu Q^\nu K + Q^{\{\mu}Z^{\nu\}}L \\ &\quad + Q^{\{\mu}S_{AT}^{\nu\}}M + Q^{\{\mu}S_{BT}^{\nu\}}N + Q^{\{\mu}T_A^{\nu\}}O + Q^{\{\mu}T_B^{\nu\}}P\ , \end{aligned} \tag{3.6}$$

where the notation $Q^{\{\mu}Z^{\nu\}}$ is, for example, defined by

$$Q^{\{\mu}Z^{\nu\}} \equiv Q^\mu Z^\nu + Q^\nu Z^\mu\ . \tag{3.7}$$

The coefficients $E$, $F$, $I'$, $J$, $M$, and $N$ are pseudoscalar quantities since $S_{AT}^\mu$ and $S_{BT}^\mu$ are pseudovectors. We should be careful that the $S_B^2$ type terms are allowed because the hadron $B$ is a spin-1 particle, whereas only the linear spin terms are allowed for the hadron $A$. As discussed in section II repeatedly, the rank-two tensors are possible in $W^{\mu\nu}$. Because the factors $T_B^\mu T_B^\nu$, $T_A^{\{\mu}T_B^{\nu\}}$, and $S_A^{T\{\mu}T_B^{\nu\}}$ are not independent from the others, these are not included in Eq. (3.6). This fact could be seen that the obtained cross sections from these terms have exactly the same polarization and angular dependence as the other ones. The hadron tensor $W^{\mu\nu}$ is expressed in terms of the factors $A$, $B'$, $\cdots$, and $P$ as the coefficients. However, in addition to the conditions in Eq. (3.5), it has to satisfy the current conservation

$$Q_\mu W^{\mu\nu} = 0\ . \tag{3.8}$$

Noting the relations $Q\cdot Z = Q\cdot S_{AT} = Q\cdot S_{BT} = Q\cdot T_A = Q\cdot T_B = 0$, we obtain $K=A/Q^2$ and $L=M=N=O=P=0$. Using these relations and adding the current conserving terms $g^{\mu\nu}-Q^\mu Q^\nu/Q^2$ and $g^{\mu\nu}-Q^\mu Q^\nu/Q^2 - Z^\mu Z^\nu/Z^2$ into the hadron tensor, we have

$$
\begin{aligned}
W^{\mu\nu} = & -\left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right) A - \left[\frac{Z^\mu Z^\nu}{Z^2} - \frac{1}{3}\left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right)\right] B + Z^{\{\mu}T_A^{\nu\}} C + Z^{\{\mu}T_B^{\nu\}} D + Z^{\{\mu}S_{AT}^{\nu\}} E + Z^{\{\mu}S_{BT}^{\nu\}} F \\
& - \left[S_{BT}^\mu S_{BT}^\nu - \frac{1}{2} S_{BT}\cdot S_{BT}\left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2}\right)\right] G - \left[S_{AT}^{\{\mu}S_{BT}^{\nu\}} - S_A^T\cdot S_B^T\left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2}\right)\right] H \\
& + \left[T_A^{\{\mu}S_{BT}^{\nu\}} - T_A\cdot S_{BT}\left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2}\right)\right] I + S_{BT}^{\{\mu}T_B^{\nu\}} J \ . \qquad (3.9)
\end{aligned}
$$

In this way, we expressed the hadron tensor $W^{\mu\nu}$ in terms of the ten coefficients $A$, $B$, $\cdots$, and $J$. However, these coefficients could still contain the spin factors in scalar or pseudoscalar forms. For example, the coefficient $A$ is obviously a scalar function. We consider possible scalar combinations among 1, $Z$, $S_A$, $S_B$, $T_A$, and $T_B$ up to the order of $S_A$ and $S_B^2$, then a structure function is assigned to each combination:

$$
\begin{aligned}
A &= A_1' + \frac{M_A M_B}{s\, Z^2} Z\cdot S_A\, Z\cdot S_B\, A_2 - S_{AT}\cdot S_{BT}\, A_3 + \frac{8\, M_B^2\, (Z\cdot S_B)^2}{s^2\, (Q\cdot P_B)^2} A_4' + \frac{M_B}{Z^2\, Q\cdot P_B} Z\cdot S_B\, T_A\cdot S_{BT}\, A_5 \\
&= A_1 + \frac{M_A M_B}{s\, Z^2} Z\cdot S_A\, Z\cdot S_B\, A_2 - S_{AT}\cdot S_{BT}\, A_3 - \left[\frac{8\, M_B^2\, (Z\cdot S_B)^2}{s^2\, (Q\cdot P_B)^2} + \frac{4}{3} S_B^2\right] A_4 + \frac{M_B}{Z^2\, Q\cdot P_B} Z\cdot S_B\, T_A\cdot S_{BT}\, A_5 \\
&= A_1 + \frac{1}{4}\lambda_A\, \lambda_B\, A_2 + |\vec{S}_{AT}|\, |\vec{S}_{BT}|\, cos(\phi_A - \phi_B)\, A_3 + \frac{2}{3}\, (2\, |\vec{S}_{BT}|^2 - \lambda_B^2)\, A_4 + \lambda_B\, |\vec{S}_{AT}|\, |\vec{S}_{BT}|\, sin(\phi_A - \phi_B)\, A_5 \ , \qquad (3.10)
\end{aligned}
$$

where the spin vectors are decomposed into the longitudinal and transverse components by

$$
\begin{aligned}
S_A^\mu &= \lambda_A P_A^\mu / M_A + S_{AT}^\mu - \delta_-^\mu (\lambda_A M_A / P_A^+) \ , \\
S_B^\mu &= \lambda_B P_B^\mu / M_B + S_{BT}^\mu - \delta_+^\mu (\lambda_B M_B / P_B^-) \ . \qquad (3.11)
\end{aligned}
$$

The $\lambda_A$ and $\lambda_B$ are the helicities of the hadrons $A$ and $B$, the momenta $P^+$ and $P^-$ are defined by $P^\pm = (P^0 \pm P^3)/\sqrt{2}$, and $\delta_\pm^\mu$ is defined by $\delta_+^\mu = [0, 1, \vec{0}_T]$ and $\delta_-^\mu = [1, 0, \vec{0}_T]$ in the expression of $a^\mu = [a_-, a_+, \vec{a}_T]$. The helicity and the transverse vector have the relation, $\lambda^2 + |\vec{S}_T|^2 = 1$. The $A_1'$ and $A_4'$ terms are combined so that the quantity $(Z\cdot S_B)^2$ and the constant $S_B^2 = -1$ become a typical quadrupole term $2(2 - 3\lambda_B^2)/3$. The $S_B^2$ term is not included in the first line of Eq. (3.10) because it is merely a constant. In obtaining the last line of Eq. (3.10), the relations

$$
\begin{aligned}
\vec{Z} &= (0, 0, |\vec{Z}|) \ , \\
\vec{S}_{AT} &= |\vec{S}_{AT}|\, (cos\phi_A, sin\phi_A, 0) \ , \\
\vec{S}_{BT} &= |\vec{S}_{BT}|\, (cos\phi_B, sin\phi_B, 0) \ , \\
\vec{T}_A &= Q\, |\vec{Z}|\, |\vec{S}_{AT}|\, (sin\phi_A, -cos\phi_A, 0) \ , \\
\vec{T}_B &= Q\, |\vec{Z}|\, |\vec{S}_{BT}|\, (sin\phi_B, -cos\phi_B, 0) \ , \qquad (3.12)
\end{aligned}
$$

are used. Because the coefficient $B$ is in $W^{\mu\nu}$ of Eq. (3.9) without multiplication of any spin factor, it is written in the same form as Eq. (3.10):

$$
\begin{aligned}
B &= B_1 + \frac{M_A M_B}{s\, Z^2} Z\cdot S_A\, Z\cdot S_B\, B_2 - S_{AT}\cdot S_{BT}\, B_3 - \left[\frac{8\, M_B^2\, (Z\cdot S_B)^2}{s^2\, (Q\cdot P_B)^2} + \frac{4}{3} S_B^2\right] B_4 + \frac{M_B}{Z^2\, Q\cdot P_B} Z\cdot S_B\, T_A\cdot S_{BT}\, B_5 \\
&= B_1 + \frac{1}{4}\lambda_A\, \lambda_B\, B_2 + |\vec{S}_{AT}|\, |\vec{S}_{BT}|\, cos(\phi_A - \phi_B)\, B_3 + \frac{2}{3}\, (2\, |\vec{S}_{BT}|^2 - \lambda_B^2)\, B_4 + \lambda_B\, |\vec{S}_{AT}|\, |\vec{S}_{BT}|\, sin(\phi_A - \phi_B)\, B_5 \ . \qquad (3.13)
\end{aligned}
$$

In the case of the coefficient $C$, the spin factor $S_A$ is already multiplied through $T_A$ in Eq. (3.9). Therefore, the hadron-$A$ spin factor cannot be used any longer and it becomes

$$
C = -\frac{1}{Q\, Z^2}\left[C_1 - \left\{\frac{8\, M_B^2\, (Z\cdot S_B)^2}{s^2\, (Q\cdot P_B)^2} + \frac{4}{3} S_B^2\right\} C_2\right] = +\frac{1}{Q\, |\vec{Z}^2|}\left[C_1 + \frac{2}{3}\, (2\, |\vec{S}_{BT}|^2 - \lambda_B^2)\, C_2\right] \ . \qquad (3.14)
$$

The factor $1/Q^2 Z^2$ is multiplied so as to cancel the kinematic factor which appears in calculating $(\delta_{ij} - \hat{k}_i \hat{k}_j)(Z_i T_{Aj} + Z_j T_{Ai})$ for obtaining the cross section.

Noting that the coefficient $D$ is multiplied by the spin factor $S_B$, we have

$$D = -\frac{1}{Q\,Z^2}\left[D_1 + \frac{M_A\,M_B}{s\,Z^2}\,Z\cdot S_A\,Z\cdot S_B\,D_2 - S_{AT}\cdot S_{BT}\,D_3\right]$$
$$= +\frac{1}{Q\,|\vec{Z}|^2}\left[D_1 + \frac{1}{4}\,\lambda_A\,\lambda_B\,D_2 + |\vec{S}_{AT}|\,|\vec{S}_{BT}|\,cos(\phi_A - \phi_B)\,D_3\right] \ . \qquad (3.15)$$

In the same way, we extract spin dependent factors from the other coefficients:

$$E = \frac{Q\,M_B}{Z^2\,Q\cdot P_B}\,Z\cdot S_B\,E_1 = -\frac{1}{|\vec{Z}|}\,\lambda_B\,E_1 \ ,$$
$$F = -\frac{Q\,M_A}{Z^2\,Q\cdot P_A}\,Z\cdot S_A\,F_1 + \frac{Q\,M_B}{Z^2\,Q\cdot P_B}\,Z\cdot S_B\,F_2 - \frac{1}{Z^2\,Q}\,T_A\cdot S_{BT}\,F_3$$
$$= -\frac{1}{|\vec{Z}|}\left[\lambda_A\,F_1 + \lambda_B\,F_2 + |\vec{S}_{AT}|\,|\vec{S}_{BT}|\,sin(\phi_A - \phi_B)\,F_3\right] \ ,$$
$$G = 2\,G_1 \ , \quad H = H_1 \ , \quad I = -\frac{M_B}{Z^2\,Q\cdot P_B}\,Z\cdot S_B\,I_1 = \frac{\lambda_B}{Q\,|\vec{Z}|}\,I_1 \ , \quad J = \frac{M_A}{Z^2\,Q\cdot P_A}\,Z\cdot S_A\,J_1 = \frac{\lambda_A}{Q\,|\vec{Z}|}\,J_1 \ , \qquad (3.16)$$

where the coefficients $E$, $F$, $I$, and $J$ are pseudoscalar quantities.

Now, we are ready to calculate the cross section by substituting the coefficients $A$, $B$, $\cdots$, and $J$ into Eq. (3.9) and then by using Eq. (2.7). The lepton momentum $\vec{k}$ is expressed in the polar coordinate as

$$\vec{k} = |\vec{k}|\,(sin\theta\,cos\phi, sin\theta\,sin\phi, cos\theta) \ . \qquad (3.17)$$

Calculating the cross section, we find redundancy in the structure functions. As the coefficient of the $2\,sin\theta\,cos\theta$ term in the cross section, we have the factor

$$sin(\phi - \phi_A)\,|\vec{S}_{AT}|\,[\,C_1 + 2\,(2\,|\vec{S}_{BT}|^2 - \lambda_B^2)\,C_2/3\,]$$
$$+\,|\vec{S}_{AT}|\,|\vec{S}_{BT}|^2\,[\,sin(\phi - \phi_B)\,cos(\phi_A - \phi_B)\,D_3 + cos(\phi - \phi_B)\,sin(\phi_A - \phi_B)\,F_3\,]$$
$$= sin(\phi - \phi_A)\,|\vec{S}_{AT}|\,[\,\{\,C_1 + (D_3 - F_3)/6\,\} + 2\,(2\,|\vec{S}_{BT}|^2 - \lambda_B^2)\,\{\,C_2 + (D_3 - F_3)/4\,\}/3\,]$$
$$+\,|\vec{S}_{AT}|\,|\vec{S}_{BT}|^2\,sin(\phi + \phi_A - 2\phi_B)\,(D_3 + F_3)/2 \ . \qquad (3.18)$$

This equation means that only three of the functions $C_1$, $C_2$, $D_3$, and $F_3$ are independent because we could redefine the functions as $C_1' = C_1 + (D_3 - F_3)/6$, $C_2' = C_2 + (D_3 - F_3)/4$, and $D_3' = (D_3 + F_3)/2$. This is the same as taking $F_3 = D_3$ without loosing generality, therefore we simply use $F_3 = D_3$ in our formalism. In this way, the hadron tensor is written in terms of the 22 structure functions: $A_1$, $A_2$, $\cdots$, and $J_1$. In order to avoid confusion, we mention that the functions $F_1$, $F_2$, $F_3$, and $G_1$ are nothing to do with those structure functions in the lepton-nucleon scattering.

The physics meaning is not clear in the notations $A_1$, $A_2$, $\cdots$, and $J_1$, so that they are expressed in terms of the notations similar to those in Refs. [9,11]:

$$\begin{array}{lllll}
A_1 = W_{0,0}\,, & A_2 = V_{0,0}^{LL}\,, & A_3 = V_{0,0}^{TT}\,, & A_4 = V_{0,0}^{UQ_0}\,, & A_5 = V_{0,0}^{TQ_1}\,, \\
B_1 = W_{2,0}\,, & B_2 = V_{2,0}^{LL}\,, & B_3 = V_{2,0}^{TT}\,, & B_4 = V_{2,0}^{UQ_0}\,, & B_5 = V_{2,0}^{TQ_1}\,, \\
C_1 = U_{2,1}^{TU}\,, & C_2 = U_{2,1}^{TQ_0}\,, & D_1 = U_{2,1}^{UT}\,, & D_2 = U_{2,1}^{LQ_1}\,, & D_3 = U_{2,1}^{TQ_2}\,, \\
E_1 = U_{2,1}^{TL}\,, & F_1 = U_{2,1}^{LT}\,, & F_2 = U_{2,1}^{UQ_1}\,, & & \\
G_1 = U_{2,2}^{UQ_2}\,, & H_1 = U_{2,2}^{TT}\,, & I_1 = U_{2,2}^{TQ_1}\,, & J_1 = U_{2,2}^{LQ_2}\,. &
\end{array} \qquad (3.19)$$

The functions $W$, $V$, and $U$ indicate an unpolarized structure function, a polarized one without the spin factors $S^\mu$ and $T^\mu$ in Eq. (3.9), and a polarized one with the spin factor. The subscripts of these structure functions indicate, for example, that $W_{L,M}$ is obtained by $\int d\Omega\,Y_{LM}\,d\sigma/(d^4Q\,d\Omega) \propto W_{L,M}$ in the unpolarized reaction. The superscripts indicate the polarization states of $A$ and $B$: e.g. $U_{L,M}^{pol_A\,pol_B}$. The superscripts $U$, $L$, and $T$ indicate unpolarized, longitudinally polarized, and transversely polarized states. The quadrupole polarizations $Q_0$, $Q_1$, and $Q_2$ are specific in the reactions with a spin-1 hadron, and they are associated with the quadrupole terms in section II:

$$\begin{array}{lll}
Q_0 & \text{for the term} & 3\,cos^2\beta_B - 1 \sim Y_{20} \\
Q_1 & \text{for} & sin\beta_B\,cos\beta_B \sim Y_{21} \\
Q_2 & \text{for} & sin^2\beta_B \sim Y_{22}\ .
\end{array} \qquad (3.20)$$

Using these structure-function expressions, we finally obtain the hadron tensor as

$$
\begin{aligned}
W^{\mu\nu} =& - \left[ g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right] \Bigg\{ W_{0,0} + \frac{M_A M_B}{s\, Z^2}\, Z\cdot S_A\, Z\cdot S_B\, V_{0,0}^{LL} - S_{AT}\cdot S_{BT}\, V_{0,0}^{TT} \\
& \qquad - \left( \frac{8\, M_B^2\, (Z\cdot S_B)^2}{s^2\, (Q\cdot P_B)^2} + \frac{4}{3}\, S_B^2 \right) V_{0,0}^{UQ_0} + \frac{M_B}{Z^2\, Q\cdot P_B}\, Z\cdot S_B\, T_A\cdot S_{BT}\, V_{0,0}^{TQ_1} \Bigg\} \\
& - \left[ \frac{Z^\mu Z^\nu}{Z^2} - \frac{1}{3} \left( g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) \right] \Bigg\{ W_{2,0} + \frac{M_A M_B}{s\, Z^2}\, Z\cdot S_A\, Z\cdot S_B\, V_{2,0}^{LL} - S_{AT}\cdot S_{BT}\, V_{2,0}^{TT} \\
& \qquad - \left( \frac{8\, M_B^2\, (Z\cdot S_B)^2}{s^2\, (Q\cdot P_B)^2} + \frac{4}{3}\, S_B^2 \right) V_{2,0}^{UQ_0} + \frac{M_B}{Z^2\, Q\cdot P_B}\, Z\cdot S_B\, T_A\cdot S_{BT}\, V_{2,0}^{TQ_1} \Bigg\} \\
& - Z^{\{\mu} T_A^{\nu\}}\, \frac{1}{\sqrt{Q^2}\, Z^2} \left\{ U_{2,1}^{TU} - \left( \frac{8\, M_B^2\, (Z\cdot S_B)^2}{s^2\, (Q\cdot P_B)^2} + \frac{4}{3}\, S_B^2 \right) U_{2,1}^{TQ_0} \right\} \\
& - Z^{\{\mu} T_B^{\nu\}}\, \frac{1}{\sqrt{Q^2}\, Z^2} \left\{ U_{2,1}^{UT} + \frac{M_A M_B}{s\, Z^2}\, Z\cdot S_A\, Z\cdot S_B\, U_{2,1}^{LQ_1} + S_{AT}\cdot S_{BT}\, U_{2,1}^{TQ_2} \right\} \\
& + Z^{\{\mu} S_{AT}^{\nu\}}\, \frac{\sqrt{Q^2}\, M_B}{Z^2\, Q\cdot P_B}\, Z\cdot S_B\, U_{2,1}^{TL} \\
& + Z^{\{\mu} S_{BT}^{\nu\}} \left\{ - \frac{\sqrt{Q^2}\, M_A}{Z^2\, Q\cdot P_A}\, Z\cdot S_A\, U_{2,1}^{LT} + \frac{\sqrt{Q^2}\, M_B}{Z^2\, Q\cdot P_B}\, Z\cdot S_B\, U_{2,1}^{UQ_1} - \frac{1}{\sqrt{Q^2}\, Z^2}\, T_A\cdot S_{BT}\, U_{2,1}^{TQ_2} \right\} \\
& - \left[ 2\, S_{BT}^\mu S_{BT}^\nu - S_{BT}^2 \left( g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) \right] U_{2,2}^{UQ_2} \\
& - \left[ S_{AT}^{\{\mu} S_{BT}^{\nu\}} - S_{AT}\cdot S_{BT} \left( g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) \right] U_{2,2}^{TT} \\
& - \left[ T_A^{\{\mu} S_{BT}^{\nu\}} - T_A\cdot S_{BT} \left( g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} - \frac{Z^\mu Z^\nu}{Z^2} \right) \right] \frac{M_B}{Z^2\, Q\cdot P_B}\, Z\cdot S_B\, U_{2,2}^{TQ_1} \\
& + S_{BT}^{\{\mu} T_B^{\nu\}}\, \frac{M_A}{Z^2\, Q\cdot P_A}\, Z\cdot S_A\, U_{2,2}^{LQ_2} \ . \qquad (3.21)
\end{aligned}
$$

Substituting this expression into Eq. (2.7), we obtain the Drell-Yan cross section

$$
\begin{aligned}
\frac{d\sigma}{d^4Q\, d\Omega} =& \frac{\alpha^2}{2\, s\, Q^2} \Bigg\{ 2 \Bigg[ W_{0,0} + \frac{1}{4}\, \lambda_A\, \lambda_B\, V_{0,0}^{LL} + |\vec{S}_{AT}|\, |\vec{S}_{BT}|\, \cos(\phi_A - \phi_B)\, V_{0,0}^{TT} \\
& \qquad + \frac{2}{3} \left( 2\, |\vec{S}_{BT}|^2 - \lambda_B^2 \right) V_{0,0}^{UQ_0} + |\vec{S}_{AT}|\, \lambda_B\, |\vec{S}_{BT}|\, \sin(\phi_A - \phi_B)\, V_{0,0}^{TQ_1} \Bigg] \\
& + \left( \frac{1}{3} - \cos^2\theta \right) \Bigg[ W_{2,0} + \frac{1}{4}\, \lambda_A\, \lambda_B\, V_{2,0}^{LL} + |\vec{S}_{AT}|\, |\vec{S}_{BT}|\, \cos(\phi_A - \phi_B)\, V_{2,0}^{TT} \\
& \qquad + \frac{2}{3} \left( 2\, |\vec{S}_{BT}|^2 - \lambda_B^2 \right) V_{2,0}^{UQ_0} + |\vec{S}_{AT}|\, \lambda_B\, |\vec{S}_{BT}|\, \sin(\phi_A - \phi_B)\, V_{2,0}^{TQ_1} \Bigg] \\
& + 2 \sin\theta \cos\theta \Bigg[ \sin(\phi - \phi_A)\, |\vec{S}_{AT}| \left( U_{2,1}^{TU} + \frac{2}{3} \left( 2\, |\vec{S}_{BT}|^2 - \lambda_B^2 \right) U_{2,1}^{TQ_0} \right) \\
& \qquad + \sin(\phi - \phi_B)\, |\vec{S}_{BT}| \left( U_{2,1}^{UT} + \frac{1}{4}\, \lambda_A\, \lambda_B\, U_{2,1}^{LQ_1} \right) + \sin(\phi + \phi_A - 2\phi_B)\, |\vec{S}_{AT}|\, |\vec{S}_{BT}|^2\, U_{2,1}^{TQ_2} \\
& \qquad + \cos(\phi - \phi_A)\, |\vec{S}_{AT}|\, \lambda_B\, U_{2,1}^{TL} + \cos(\phi - \phi_B)\, |\vec{S}_{BT}| \left( \lambda_A\, U_{2,1}^{LT} + \lambda_B\, U_{2,1}^{UQ_1} \right) \Bigg] \\
& + \sin^2\theta \Bigg[ \cos(2\phi - 2\phi_B)\, |\vec{S}_{BT}|^2\, U_{2,2}^{UQ_2} + \cos(2\phi - \phi_A - \phi_B)\, |\vec{S}_{AT}|\, |\vec{S}_{BT}|\, U_{2,2}^{TT} \\
& \qquad + \sin(2\phi - \phi_A - \phi_B)\, |\vec{S}_{AT}|\, \lambda_B\, |\vec{S}_{BT}|\, U_{2,2}^{TQ_1} + \sin(2\phi - 2\phi_B)\, \lambda_A\, |\vec{S}_{BT}|^2\, U_{2,2}^{LQ_2} \Bigg] \Bigg\} \ . \qquad (3.22)
\end{aligned}
$$

Using $\lambda_A = |\vec{S}_A| cos\theta_A$, $\lambda_B = |\vec{S}_B| cos\theta_B$, $|\vec{S}_{AT}| = |\vec{S}_A| sin\theta_A$, and $|\vec{S}_{BT}| = |\vec{S}_B| sin\theta_B$ in Eq. (3.22), and noting the relations: $\theta_A = \beta_A$, $\phi_A = \alpha_A$, $\theta_B = \pi - \beta_B$, and $\phi_B = -\alpha_B$, we obtain essentially the same equation as Eq. (2.31). In this way, we find that 22 structure functions exist in the Drell-Yan processes with spin-1/2 and spin-1 hadrons in the limit $Q_T \to 0$. We can repeat the analysis of this section by taking the integral over $\vec{Q}_T$ first. However, $X^\mu$ and $Y^\mu$ are averaged away by the $\vec{Q}_T$ integration. It means that exactly the same formalism can be applied without taking into account the vectors $X^\mu$ and $Y^\mu$ except for the replacements [9], $W_{00} \to \overline{W}_{00} = \int d\vec{Q}_T W_{00}$, $W_{20} \to \overline{W}_{20} = \int d\vec{Q}_T W_{20}$, and others. This is the reason why the same number of the structure functions are obtained after taking the limit $Q_T \to 0$ as discussed in this section and after the integration over $\vec{Q}_T$ (or $\Phi$) as discussed in section II.

Because there exist 11 structure functions after the $\vec{Q}_T$ integration in the Drell-Yan processes of spin-1/2 hadrons, there are 11 new structure functions in the reactions of spin-1/2 and spin-1 hadrons. In the pp Drell-Yan processes, the structure functions $U_{2,1}^{TL}$ and $U_{2,1}^{UT}$ are identified with $U_{2,1}^{LT}$ and $U_{2,1}^{TU}$. Therefore, there exist essentially 9 structure functions. In this sense, there are 13 additional structure functions in our Drell-Yan processes. As it is shown in Eq. (3.22), the 11 new structure functions are related to the quadrupole polarizations $Q_0$, $Q_1$, and $Q_2$. In order to measure these structure functions, various longitudinal, transverse, and intermediate polarization reactions should be combined. These are key structure functions to clarify the tensor structure of the spin-1 hadrons.

## IV. SPIN ASYMMETRIES

In the previous sections, we have derived the expressions for the Drell-Yan cross sections in Eqs. (2.31) and (3.22). However, it is not straightforward to see how each structure function can be measured. In this section, we discuss the relations between the structure functions and possible spin asymmetries. Because the Ralston-Soper type notations seem to be more popular, we use the cross-section expression in Eq. (3.22) for discussing the spin asymmetries.

In the proton-proton Drell-Yan cross sections, there are merely the following spin combinations [9,10,19]:

$$<\sigma>, \quad A_{LL}, \quad A_{TT}, \quad A_{LT}, \quad A_T \quad (\text{in pp}) \ , \qquad (4.1)$$

where $<\sigma>$ is the unpolarized cross section, $A_{LL}$ ($A_{TT}$) is the longitudinal (transverse) double spin asymmetry, $A_{LT}$ is the longitudinal-transverse spin asymmetry, and $A_T$ is the transverse single spin asymmetry. Here, the parity-violating asymmetries are not taken into account [20]. As it is obvious from the cross section in Eq. (3.22), there are many other spin combinations in the reactions with spin-1/2 and spin-1 hadrons (proton and deuteron).

First, the expression of our unpolarized cross section is the same as the pp one:

$$\left\langle \frac{d\sigma}{d^4Q \, d\Omega} \right\rangle = \frac{\alpha^2}{2\, s\, Q^2} \left[ 2\, W_{0,0} + \left( \frac{1}{3} - cos^2\theta \right) W_{2,0} \right] . \qquad (4.2)$$

The longitudinal double asymmetry may be defined in the similar way as the pp case: $A'_{LL} = [\sigma(\uparrow,-1) - \sigma(\uparrow,+1)]/[\sigma(\uparrow,-1) + \sigma(\uparrow,+1)]$. However, this is not an appropriate way to investigate the longitudinally polarized parton distributions because the tensor distribution $b_1$ contributes to the asymmetry $A_{LL}$. This asymmetry definition is used in analyzing the polarized lepton-deuteron scattering data for extracting $g_1^n$. Strictly speaking, this is not an appropriate way of handling the data. In order to avoid this kind of confusion, we use $2 <\sigma>$ in the denominator for defining the spin asymmetry. From the relation

$$\begin{aligned} 2 <\sigma> &= \sigma(\uparrow,+1) + \sigma(\uparrow,-1) \\ &\quad + \frac{1}{3} \left[ 2\, \sigma(\uparrow,0) - \sigma(\uparrow,+1) - \sigma(\uparrow,-1) \right] , \end{aligned} \qquad (4.3)$$

the term $2 <\sigma>$ agrees with the usual denominator $\sigma(\uparrow -1) + \sigma(\uparrow +1)$ if the quadrupole asymmetry can be ignored: $|\, 2\,\sigma(\uparrow,0) - \sigma(\uparrow,-1) - \sigma(\uparrow,+1)\,| \ll 6 <\sigma>$. In this way, we define a *modified* spin asymmetry by using the denominator $2<\sigma>$. Then, the longitudinal double (LL) spin asymmetry becomes

$$\begin{aligned} A_{LL} &= \frac{\sigma(\uparrow_L,-1_L) - \sigma(\uparrow_L,+1_L)}{2 <\sigma>} \\ &= - \frac{2\, V_{0,0}^{LL} + (\frac{1}{3} - cos^2\theta)\, V_{2,0}^{LL}}{4\, \left[ 2\, W_{0,0} + (\frac{1}{3} - cos^2\theta)\, W_{2,0} \right]} , \end{aligned} \qquad (4.4)$$

where the subscripts of $\uparrow_L$, $+1_L$, and $-1_L$ indicate the longitudinal polarization. Experimentally, it may be easier to obtain the usual asymmetry $A'_{LL}$ rather than the above one. Because the quadrupole effects are considered to be small, the result would not be changed significantly even if our $A_{LL}$ is simply replaced by $A'_{LL}$. However, the tensor structure is the major point in studying the polarized spin-1 hadron, so that its contributions to $A'_{LL}$ should be carefully taken into account. If the cross section is integrated over $\vec{Q}_T$, the denominator of Eq. (4.4) is expressed in terms of the unpolarized quark and antiquark distributions $f_1$ and $\overline{f}_1$ as $2\,\overline{W}_{0,0} + (\frac{1}{3} - cos^2\theta)\,\overline{W}_{2,0} = (1 + cos^2\theta)\,\overline{W}_T = (1 + cos^2\theta)\,(1/3) \sum_a e_a^2 f_1 \overline{f}_1$ in a parton model [11]. Our parton-model analysis of the reactions with spin-1/2 and spin-1 hadrons will be reported in Ref. [21].

In the LL asymmetry, the structure-function expression of Eq. (4.4) is not altered even if the spin asymmetry is taken in the hadron-$A$: $[\sigma(\downarrow_L,+1_L) - \sigma(\uparrow_L,+1_L)]$ $/(2 <\sigma>)$. However, the situation is different in the transverse double spin asymmetry. If the asymmetry is taken in the hadron-$B$ with a fixed transverse polarization of the hadron-$A$, the transverse double (TT) spin asymmetry becomes

$$
\begin{aligned}
A_{(T)T} = & \frac{\sigma(\phi_A=0,\phi_B=0)-\sigma(\phi_A=0,\phi_B=\pi)}{2<\sigma>} \\
= & \Big[ 2\,V_{0,0}^{TT} + (\frac{1}{3}-cos^2\theta)\,V_{2,0}^{TT} + sin^2\theta\, cos2\phi\, U_{2,2}^{TT} \\
& + 2\,sin\theta\, cos\theta\, sin\phi\, U_{2,1}^{UT} \Big] \Big/ \Big[ 2\,W_{0,0} + (\frac{1}{3}-cos^2\theta)\,W_{2,0} \Big] .
\end{aligned}
\tag{4.5}
$$

Hereafter, if $\phi_A$ or $\phi_B$ is indicated in the expression of $\sigma(pol_A, pol_B)$, it means that the hadron $A$ or $B$ is transversely polarized with the azimuthal angle $\phi_A$ or $\phi_B$. It should be noted that the transverse asymmetry is not equal to Eq. (4.5) if the spin asymmetry is taken in the hadron-$A$:

$$
\begin{aligned}
A_{T(T)} = & \frac{\sigma(\phi_A=0,\phi_B=0)-\sigma(\phi_A=\pi,\phi_B=0)}{2<\sigma>} \\
= & \Big[ 2\,V_{0,0}^{TT} + (\frac{1}{3}-cos^2\theta)\,V_{2,0}^{TT} + sin^2\theta\, cos2\phi\, U_{2,2}^{TT} \\
& + 2\,sin\theta\, cos\theta\, sin\phi \left( U_{2,1}^{TU} + \frac{4}{3}\,U_{2,1}^{TQ_0} + U_{2,1}^{TQ_2} \right) \Big] \\
& \Big/ \Big[ 2\,W_{0,0} + (\frac{1}{3}-cos^2\theta)\,W_{2,0} \Big] .
\end{aligned}
\tag{4.6}
$$

It is cumbersome to handle the above transverse-transverse asymmetries $A_{(T)T}$ and $A_{T(T)}$ because other structure functions mix with the TT-type ones. In order to separate the TT type, the following parallel transverse-transverse asymmetry should be studied:

$$
\begin{aligned}
A_{TT}^{\parallel} = & \frac{1}{2<\sigma>} \Big[ \frac{\sigma(\phi_A=0,\phi_B=0)+\sigma(\phi_A=\pi,\phi_B=\pi)}{2} \\
& - \frac{\sigma(\phi_A=\pi,\phi_B=0)+\sigma(\phi_A=0,\phi_B=\pi)}{2} \Big] \\
= & \frac{2\,V_{0,0}^{TT} + (\frac{1}{3}-cos^2\theta)\,V_{2,0}^{TT} + sin^2\theta\, cos2\phi\, U_{2,2}^{TT}}{2\,W_{0,0} + (\frac{1}{3}-cos^2\theta)\,W_{2,0}} .
\end{aligned}
\tag{4.7}
$$

Furthermore, the $U_{2,2}^{TT}$ part can be separated by the perpendicular transverse-transverse asymmetry

$$
\begin{aligned}
A_{TT}^{\perp} = & \Big[ \frac{\sigma(\phi_A=0,\phi_B=\pi/2)+\sigma(\phi_A=\pi,\phi_B=3\pi/2)}{2} \\
& - \frac{\sigma(\phi_A=\pi,\phi_B=\pi/2)+\sigma(\phi_A=0,\phi_B=3\pi/2)}{2} \Big] \\
& \Big/ (2<\sigma>) = \frac{sin^2\theta\, sin2\phi\, U_{2,2}^{TT}}{2\,W_{0,0} + (\frac{1}{3}-cos^2\theta)\,W_{2,0}} .
\end{aligned}
\tag{4.8}
$$

The situation is also complicated, for example, in the longitudinal-transverse (LT) asymmetry in the sense that different types of the structure functions could contribute if the asymmetry is taken either in the hadron-$A$ or in $B$. The optimum LT and TL asymmetries are given by

$$
\begin{aligned}
A_{LT} = & \frac{1}{2<\sigma>} \Big[ \frac{\sigma(\uparrow_L,\phi_B=0)+\sigma(\downarrow_L,\phi_B=\pi)}{2} \\
& - \frac{\sigma(\uparrow_L,\phi_B=\pi)+\sigma(\downarrow_L,\phi_B=0)}{2} \Big] \\
= & \frac{2\,sin\theta\, cos\theta\, cos\phi\, U_{2,1}^{LT}}{2\,W_{0,0} + (\frac{1}{3}-cos^2\theta)\,W_{2,0}} ,
\end{aligned}
\tag{4.9}
$$

$$
\begin{aligned}
A_{TL} = & \frac{1}{2<\sigma>} \Big[ \frac{\sigma(\phi_A=0,+1_L)+\sigma(\phi_A=\pi,-1_L)}{2} \\
& - \frac{\sigma(\phi_A=0,-1_L)+\sigma(\phi_A=\pi,+1_L)}{2} \Big] \\
= & \frac{2\,sin\theta\, cos\theta\, cos\phi\, U_{2,1}^{TL}}{2\,W_{0,0} + (\frac{1}{3}-cos^2\theta)\,W_{2,0}} ,
\end{aligned}
\tag{4.10}
$$

by combining both spin asymmetries in the hadrons $A$ and $B$. In the case of the asymmetry $A_{TL}$, we could have defined it in a simpler form as $A_{TL} = A_{(T)L} = [\sigma(\phi_A=0,+1_L)-\sigma(\phi_A=0,-1_L)]/(2<\sigma>)$, which becomes the same equation as Eq. (4.10) in terms of the structure functions $U_{2,1}^{TL}$, $W_{0,0}$, and $W_{2,0}$. However, we should be careful about the definitions of the LT and TL asymmetries. If the asymmetries are calculated by $A_{(L)T} = [\sigma(\uparrow_L,\phi_B=0)-\sigma(\uparrow_L,\phi_B=\pi)]/(2<\sigma>)$, $A_{L(T)} = [\sigma(\uparrow_L,\phi_B=0)-\sigma(\downarrow_L,\phi_B=0)]/(2<\sigma>)$, and $A_{T(L)} = [\sigma(\phi_A=0,+1_L)-\sigma(\phi_A=\pi,+1_L)]/(2<\sigma>)$, there are other contributions in addition to $U_{2,1}^{LT}$ or $U_{2,1}^{TL}$. Therefore, the asymmetries $A_{(L)T}$, $A_{L(T)}$, and $A_{T(L)}$ are not appropriate quantities for studying exclusively the LT and TL structure functions. In the following discussions of this section on various asymmetries, the details are no longer discussed about a number of different possibilities. We simply show the optimum asymmetries.

In the single spin asymmetries, we express the unpolarized state explicitly, for example, as $A_{UT}$ which is the transverse single spin asymmetry for the hadron $B$. It is because $A_{UT}$ and $A_{TU}$ are in general different in our case although it does not matter in the pp. Then, the single spin asymmetries become

$$
\begin{aligned}
A_{UT} = & \frac{\sigma(\bullet,\phi_B=0)-\sigma(\bullet,\phi_B=\pi)}{2<\sigma>} \\
= & \frac{2\,sin\theta\, cos\theta\, sin\phi\, U_{2,1}^{UT}}{2\,W_{0,0} + (\frac{1}{3}-cos^2\theta)\,W_{2,0}} ,
\end{aligned}
\tag{4.11}
$$

$$
\begin{aligned}
A_{TU} = & \frac{\sigma(\phi_A=0,\bullet)-\sigma(\phi_A=\pi,\bullet)}{2<\sigma>} \\
= & \frac{2\,sin\theta\, cos\theta\, sin\phi\, U_{2,1}^{TU}}{2\,W_{0,0} + (\frac{1}{3}-cos^2\theta)\,W_{2,0}} ,
\end{aligned}
\tag{4.12}
$$

where $\bullet$ indicates the unpolarized case.

In defining the quadrupole spin asymmetry $Q_0$, we consider the parton-model definition of the $b_1$ structure function with the tensor distribution [5]: $\delta q = [q_0 - (q_{+1} + q_{-1})/2]/2$ where the subscripts denote $z$ components of the hadron spin. The quadrupole asymmetry $Q_0$ is, therefore, defined by

$$A_{UQ_0} = \frac{1}{2 <\sigma>} \left[ \sigma(\bullet, 0_L) - \frac{\sigma(\bullet, +1_L) + \sigma(\bullet, -1_L)}{2} \right]$$
$$= \frac{2\, V_{0,0}^{UQ_0} + (\frac{1}{3} - cos^2\theta)\, V_{2,0}^{UQ_0}}{2\, W_{0,0} + (\frac{1}{3} - cos^2\theta)\, W_{2,0}} \; , \qquad (4.13)$$

where the average is taken over the angle $\phi_B$ in the polarization state $0_L$. If we wish to specify the azimuthal angle in defining the asymmetry, it could be written as $A_{UQ_0} = [\, \{\sigma(\bullet, \phi_B = 0) + \sigma(\bullet, \phi_B = \pi)\}/2 - \{\sigma(\bullet, +1_L) + \sigma(\bullet, -1_L)\}/2\,]/(2 <\sigma>)$. The situation is illustrated in Fig. 4.4. The $Q_0$ polarization is related to the difference between the longitudinal and transverse cross sections.

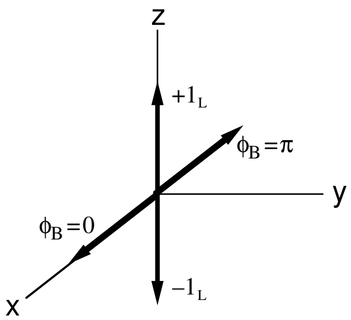


FIG. 4.4. Quadrupole asymmetry $Q_0$. The arrows indicate polarization directions of the hadron $B$.

This is a new asymmetry which does not exist in the pp reactions. We will show in Ref. [21] that this asymmetry is related to the $b_1$ distribution. This quadrupole asymmetry is one of the interesting quantities to be investigated in our reactions. The $b_1$ structure function will be studied in the polarized lepton-deuteron scattering by the HERMES collaboration [22]; however, the experimental accuracy may not be good enough to find the small quantity. Therefore, the idea is to use an accelerator with enough beam intensity. A realistic possibility is to measure it in the ELFE (Electron Laboratory for Europe) project [23] or in a similar one [24]. However, these projects are not approved yet. It would take several years to start investigating $b_1$ experimentally by considering the present situation. The polarized pd reactions are the timely and pioneering works if they are studied in the next generation RHIC-Spin project.

In the same way, transverse-quadrupole $Q_0$ ($TQ_0$) asymmetry is given by

$$A_{TQ_0} = \frac{1}{2 <\sigma>} \Big[ \{\, \sigma(\phi_A = 0, 0_L) - \sigma(\phi_A = \pi, 0_L)\, \}/2$$
$$- \{\, \sigma(\phi_A = 0, +1_L) + \sigma(\phi_A = 0, -1_L)$$
$$- \sigma(\phi_A = \pi, +1_L) - \sigma(\phi_A = \pi, -1_L)\, \}/4 \Big]$$
$$= \frac{2\, sin\theta\, cos\theta\, sin\phi\, U_{2,1}^{TQ_0}}{2\, W_{0,0} + (\frac{1}{3} - cos^2\theta)\, W_{2,0}} \; . \qquad (4.14)$$

Here, the $\sigma(\phi_A, 0_L)$ term cannot be replaced by $[\sigma(\phi_A, \phi_B = 0) + \sigma(\phi_A, \phi_B = \pi)]/2$ as it was discussed just after Eq. (4.13). If we would like to specify $\phi_B$, it should be replaced by $[\sigma(\phi_A, \phi_B = 0) + \sigma(\phi_A, \phi_B = \pi/2) + \sigma(\phi_A, \phi_B = \pi) + \sigma(\phi_A, \phi_B = 3\pi/2)]/4$ in order to cancel out the $Q_2$-type structure functions.

We find other peculiar asymmetries and structure functions. There are structure functions which do not exist in the transverse and longitudinal polarization reactions; however, they can exist in the intermediate polarization reactions in Fig. 4.5.

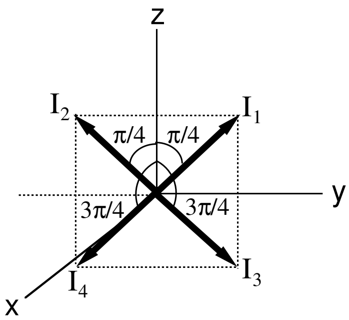


FIG. 4.5. Quadrupole asymmetry $Q_1$. The polarization vectors are in the $yz$ plane.

The single spin asymmetry for the intermediate polarization is:

$$A_{UQ_1} = \frac{\sigma(\bullet, I_1) - \sigma(\bullet, I_3)}{2 <\sigma>}$$
$$= \frac{sin\theta\, cos\theta\, sin\phi\, U_{2,1}^{UQ_1}}{2\, W_{0,0} + (\frac{1}{3} - cos^2\theta)\, W_{2,0}} \; , \qquad (4.15)$$

where the intermediate polarizations are denoted as $I_1$ and $I_3$ and they are shown in Fig. 4.5. The longitudinal-quadrupole $Q_1$ (or intermediate) asymmetry is

$$A_{LQ_1} = \frac{1}{8 <\sigma>} \Big[ \sigma(\downarrow_L, I_1) - \sigma(\downarrow_L, I_2) - \sigma(\downarrow_L, I_3)$$
$$+ \sigma(\downarrow_L, I_4) - \sigma(\uparrow_L, I_1) + \sigma(\uparrow_L, I_2) + \sigma(\uparrow_L, I_3) - \sigma(\uparrow_L, I_4) \Big]$$
$$= \frac{sin\theta\, cos\theta\, cos\phi\, U_{2,1}^{LQ_1}}{4\, \left[\, 2\, W_{0,0} + (\frac{1}{3} - cos^2\theta)\, W_{2,0}\, \right]} \; , \qquad (4.16)$$

where the additional polarizations $I_2$ and $I_4$ are also shown in Fig. 4.5. The asymmetry definition is more complicated than the one in Eq. (4.15) in order to cancel out other contributions. In the same way, these intermediate polarizations should be combined with the transverse polarization states of the hadron-$A$ for obtaining the transverse-quadrupole $Q_1$ (or intermediate) asymmetry:

$$\begin{aligned}A_{TQ_1} = & \frac{1}{8 <\sigma>} \Big[ -\sigma(\phi_A=0,I_1)+\sigma(\phi_A=0,I_2) \\ & +\sigma(\phi_A=\pi,I_1)-\sigma(\phi_A=\pi,I_2)+\sigma(\phi_A=0,I_3) \\ & -\sigma(\phi_A=0,I_4)-\sigma(\phi_A=\pi,I_3)+\sigma(\phi_A=\pi,I_4)\Big] \\ = & \frac{2\,V_{0,0}^{TQ_1}+(\frac{1}{3}-cos^2\theta)\,V_{2,0}^{TQ_1}+sin^2\theta\, cos2\phi\, U_{2,2}^{TQ_1}}{2\,\left[\,2\,W_{0,0}+(\frac{1}{3}-cos^2\theta)\,W_{2,0}\,\right]} \; . \end{aligned} \tag{4.17}$$

There are other interesting asymmetries which are related to the quadrupole polarization in the transverse plane. There are structure functions with the $sin(2\phi_B)$ or $cos(2\phi_B)$ factor in Eq. (3.22). These terms vanish if the cross-section difference is taken between those of the opposite transverse polarizations, $\phi_B=0$ and $\phi_B=\pi$. They are associated with the difference between $\phi_B=0$ and $\phi_B=\pi/2$ cross sections as illustrated in Fig. 4.6.

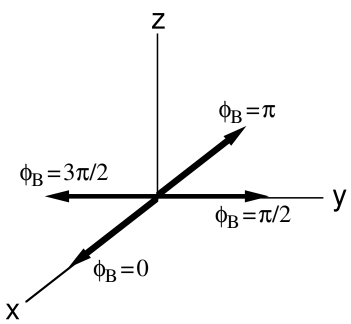


FIG. 4.6. Quadrupole asymmetry $Q_2$.

The single quadrupole asymmetry $Q_2$ is defined and it is expressed in terms of our structure functions as

$$\begin{aligned}A_{UQ_2} = & \frac{1}{2<\sigma>}\Big[\frac{\sigma(\bullet,\phi_B=0)+\sigma(\bullet,\phi_B=\pi)}{2} \\ & -\frac{\sigma(\bullet,\phi_B=\pi/2)+\sigma(\bullet,\phi_B=3\pi/2)}{2}\Big] \\ = & \frac{sin^2\theta\, cos2\phi\, U_{2,2}^{UQ_2}}{2\,W_{0,0}+(\frac{1}{3}-cos^2\theta)\,W_{2,0}} \; . \end{aligned} \tag{4.18}$$

In the same way, the other $Q_2$ asymmetries are given as

$$\begin{aligned}A_{LQ_2} = & \frac{1}{8<\sigma>}\Big[\sigma(\uparrow_L,\phi_B=0)+\sigma(\uparrow_L,\phi_B=\pi) \\ & -\sigma(\uparrow_L,\phi_B=\pi/2)-\sigma(\uparrow_L,\phi_B=3\pi/2)-\sigma(\downarrow_L,\phi_B=0) \\ & -\sigma(\downarrow_L,\phi_B=\pi)+\sigma(\downarrow_L,\phi_B=\pi/2)+\sigma(\downarrow_L,\phi_B=3\pi/2)\Big] \\ = & \frac{sin^2\theta\, sin2\phi\, U_{2,2}^{LQ_2}}{2\,W_{0,0}+(\frac{1}{3}-cos^2\theta)\,W_{2,0}} \; , \end{aligned} \tag{4.19}$$

$$\begin{aligned}A_{TQ_2} = & \frac{1}{8<\sigma>}\Big[\sigma(\phi_A=0,\phi_B=0)+\sigma(\phi_A=0,\phi_B=\pi) \\ & -\sigma(\phi_A=0,\phi_B=\pi/2)-\sigma(\phi_A=0,\phi_B=3\pi/2) \\ & -\sigma(\phi_A=\pi,\phi_B=0)-\sigma(\phi_A=\pi,\phi_B=\pi) \\ & +\sigma(\phi_A=\pi,\phi_B=\pi/2)+\sigma(\phi_A=\pi,\phi_B=3\pi/2)\Big] \\ = & \frac{2\, sin\theta\, cos\theta\, sin\phi\, U_{2,1}^{TQ_2}}{2\,W_{0,0}+(\frac{1}{3}-cos^2\theta)\,W_{2,0}} \; . \end{aligned} \tag{4.20}$$

We have found that there are a variety of polarization asymmetries in our Drell-Yan processes in comparison with those in the pp reactions. We propose that the following asymmetries be measured for finding the structure functions:

$$\begin{aligned} & <\sigma>, \; A_{LL}, \; A_{TT}, \; A_{LT}, \; A_{TL}, \\ & A_{UT}, \; A_{TU}, \; A_{UQ_0}, \; A_{TQ_0}, \; A_{UQ_1}, \\ & A_{LQ_1}, \; A_{TQ_1}, \; A_{UQ_2}, \; A_{LQ_2}, \; A_{TQ_2}. \end{aligned} \tag{4.21}$$

The new structure functions with tensor structure could be found by the quadrupole polarizations $Q_0$, $Q_1$, and $Q_2$. For example, the $Q_0$ structure functions could be found by the asymmetry $A_{UQ_0}$ in Eq. (4.13). It should be associated with the unobserved structure function $b_1$, so that it is important to measure it in the hadron-hadron reaction. In addition, there are interesting quadrupole polarizations. There exist structure functions for the intermediate polarization $Q_1$, and one of them could be investigated, for example, by the asymmetry $A_{UQ_1}$ in Eq. (4.15). The $Q_2$ structure functions are related to the quadrupole polarization in the transverse plane, and one of them could be measured by the asymmetry $A_{UQ_2}$ in Eq. (4.18). In this way, it was revealed that there are many unexplored topics in the Drell-Yan processes with a spin-1 hadron. A parton-model analysis of the structure functions is discussed in Ref. [21]. However, further investigations are necessary on the structure of spin-1 hadrons and relations to various reactions with them.

A bonus of our formalism is the possibility of discussing the $\bar{u}/\bar{d}$ asymmetry [25] in the longitudinally polarized and transversity distributions by combining the pp and pd Drell-Yan data. It is well known that the proton-deuteron Drell-Yan asymmetry is sensitive to the ratio $\bar{u}/\bar{d}$ in the unpolarized case. In fact, the recent E866 experimental data [26] clearly showed the ratio $\bar{u}/\bar{d}$ as a function of the momentum fraction $x$. In the same way, it is in principle possible to study the polarized antiquark asymmetries $\Delta\bar{u}/\Delta\bar{d}$ and $\Delta_{_T}\bar{u}/\Delta_{_T}\bar{d}$ by the pp and pd Drell-Yan processes in addition to the $W$ production studies (only for $\Delta\bar{u}/\Delta\bar{d}$ [27]) at RHIC. However, the polarized pd formalism had not been available until our studies. It is now possible to discuss the relation between the polarized p-d Drell-Yan asymmetry and the polarized flavor asymmetry.

Because the structure functions of the spin-1 hadrons have not been well studied, it is important to investigate more about their spin structure theoretically. Furthermore, the experimental possibility of the polarized pd reactions should be discussed seriously, especially at RHIC [12].

## V. SUMMARY

We have shown that 108 structure functions can be studied in the polarized Drell-Yan processes with spin-1/2 and spin-1 hadrons. Because the number is 48 in the reactions of spin-1/2 hadrons, there are 60 new structure functions. After integrating over $\vec{Q}_T$ or after taking the limit $Q_T \to 0$, we have shown that 22 structure functions exist by two independent methods with the requirements of Hermiticity, parity conservation, and time-reversal invariance. Because the number is 11 in the processes of spin-1/2 hadrons, there are 11 new structure functions in the reactions of spin-1/2 and spin-1 hadrons. We have shown that these are related to the tensor structure of the spin-1 hadron and they can be measured by the quadrupole polarization experiments. A number of spin asymmetries were shown for extracting the new structure functions. The quadrupole spin asymmetries with the $Q_0$, $Q_1$, and $Q_2$ polarizations are valuable for measuring the tensor structure functions. It is also interesting to find the structure functions for the intermediate polarizations in the sense that they do not contribute to the longitudinally and transversely polarized cross sections. In this paper, we have discussed generally what kind of structure functions is investigated in the polarized processes with spin-1/2 and spin-1 hadrons. We hope that our studies will be materialized experimentally as the polarized proton-deuteron reactions in the RHIC-Spin project and also other future projects.

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