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Q^2 evolution of structure functions in the nucleon and nuclei

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Abstract

Q^2 evolution of structure functions in the nucleon and nuclei is investigated by using usual DGLAP equations and parton-recombination equations. Calculated results for proton's F_2 and for the ratio F_2^{Ca}/F_2^D are compared with various experimental data. Furthermore, we study nuclear dependence of Q^2 evolution in tin and carbon nuclei: $\partial[F_2^{Sn}/F_2^C]/\partial[\ln Q^2] \neq 0$, which was found by NMC.

1. Introduction

Internal structure of the nucleon can be investigated in high-energy lepton-nucleon scattering. Measured structure functions depend on two variables, $Q^2 = -q^2$ and $x = Q^2/2P \cdot q$, where q is the four-momentum transfer and P is the nucleon momentum. Their Q^2 dependence can be calculated within perturbative QCD. An intuitive way of describing the Q^2 variation is to use the DGLAP equations [1]. They are often used in experimental analysis and also theoretical calculations, so it is important to investigate the numerical solution of these equations.

In addition, Q^2 dependence of nuclear structure functions becomes increasingly interesting. It is because the NMC (New Muon Collaboration) showed Q^2 variations of the ratio F_2^A/F_2^D with reasonably good accuracy [2]. Furthermore, it is found recently that there exist significant differences between tin and carbon Q^2 variations, $\partial[F_2^{Sn}/F_2^C]/\partial[\ln Q^2] \neq 0$. It is the first indication of nuclear effects in the Q^2 evolution of F_2 and is worth investigating theoretically. However, it is not obvious whether DGLAP could be applied to the nuclear case. In particular, the longitudinal localization size of a parton with momentum fraction x could exceed an average nucleon separation in a nucleus if x is small ($x < 0.1$). In this case, partons in different nucleons could interact and the interactions are called parton recombinations (PR). It is considered that their contributions enter into the evolution, and modified Q^2 evolution equations are proposed in Ref. [3].

The purpose of our study is to investigate the numerical solution of these Q^2 evolution equations. We calculate the Q^2 variation of proton's F_2 and the ratio F_2^{Ca}/F_2^D. Nuclear dependence of Q^2 evolution $\partial[F_2^{Sn}/F_2^C]/\partial[\ln Q^2]$ is also calculated in order to understand the NMC measurements.

2. Numerical solution

The DGLAP and PR evolution equations are given by following integrodifferential equations:

$$\frac{\partial}{\partial t}\, q_i\,(x,t) \;=\; \int_x^1 \frac{dy}{y}\, \left[\, \sum_j P_{q_i q_j}\left(\frac{x}{y}\right)\; q_j\,(y,t) \;+\; P_{qg}\left(\frac{x}{y}\right)\; g\,(y,t)\,\right]$$

$$+\;\left(recombination\ terms\ \propto\ \frac{\alpha_s A^{1/3}}{Q^2}\right)\;, \qquad (1a)$$

$$\frac{\partial}{\partial t}\, g\,(x,t) \;=\; \int_x^1 \frac{dy}{y}\, \left[\, \sum_j P_{g q_j}\left(\frac{x}{y}\right)\; q_j\,(y,t) \;+\; P_{gg}\left(\frac{x}{y}\right)\; g\,(y,t)\,\right]$$

$$+\;\left(recombination\ terms\ \propto\ \frac{\alpha_s A^{1/3}}{Q^2}\right)\;, \qquad (1b)$$

where the variable t is defined by $t = -(2/\beta_0)\ln[\alpha_s(Q^2)/\alpha_s(Q_0^2)]$. The functions $q_i(x,t)$ and $g(x,t)$ are flavor-i quark and gluon distributions, respectively. $P_{p_i p_j}(z)$ are called splitting functions which determine the probability that a parton p_j with the nucleon's momentum fraction y splits into a parton p_i with the momentum fraction x and another parton. In the PR evolution case, there are additional higher-twist terms which are proportional to $1/Q^2$. These terms are also proportional to $A^{1/3}$ because recombination effect is proportional to the magnitude of parton overlap in the longitudinal direction. Furthermore, there is an extra evolution equation for a higher-dimensional gluon distribution G_{HT}. Explicit expressions of recombination contributions are found in Ref. [3, 4].

As a numerical method for solving the evolution equations, we have been studying a Laguerre-polynomial method [5]. It is very efficient by considering computing time and numerical accuracy. However, this method has some difficulties, e.g. in handling the non-linear recombination terms. Therefore, we decide to employ a brute-force method [4]. In this method, the variables x and t are divided into N_x and N_t steps and integration and differentiation are defined by $df(x)/dx = [f(x_{m+1}) - f(x_m)]/\Delta x_m$ and $\int dx f(x) = \sum_{m=1}^{N_x} \Delta x_m\ f(x_m)$. If initial distributions are given, we can solve the Q^2 evolution equations step by step. This is the simplest method for solving the integrodifferential equations, but the number of steps N_x and N_t have to be large enough to get accurate results. Furthermore, the small x region becomes increasingly important with the development of high-energy accelerators such as HERA. So it is necessary to have a good accuracy at small x as small as 10^{-5}. In order to satisfy this condition, the logarithmic-x step $\Delta(log_{10}x) = |log_{10}x_{min}|/N_x$ is taken in our analysis.

It is useful to have a computer program for solving the evolution equations accurately because they are frequently used in theoretical and experimental studies. Therefore, we provide a FORTRAN program BF1 [4] which is available at the following homepage

http://www.cc.saga-u.ac.jp/saga-u/riko/physics /quantum1/structure.html,

File name : bf1.fort77.gz.

This is a very useful program for studying spin-independent structure functions in the nucleon and nuclei.

3. Results

Using the program BF1, we calculate various Q^2 evolution. First, we calculate Q^2 evolution of proton's F_2 structure function by using leading-order (LO) DGLAP, next-to-leading-order (NLO) DGLAP, and parton-recombination (PR) evolution equations. As initial distributions, we choose MRS(G) distributions which are given at 4 GeV^2. The results are shown with various experimental data in Fig. 1. In this figure, our results agree with the data very well except for the data at small x and at small Q^2 where perturbative QCD would not work. However, we can not test the recombinations because their effects in the nucleon are very small.

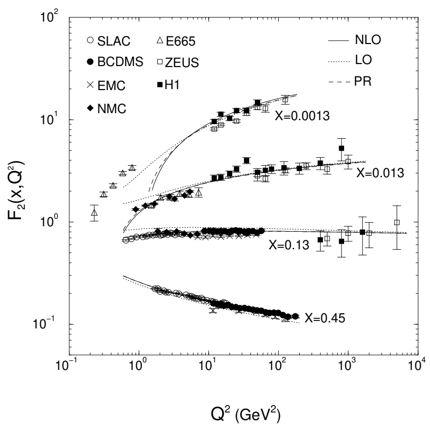

Figure 1: Q^2 dependence of proton's F_2.

Next, we investigate nuclear cases. In order to calculate Q^2 evolution of nuclear F_2 structure functions, we need to have input parton distributions in nuclei at certain Q^2. At this stage, there are various models [6] which can explain x dependence of measured ratios F_2^A/F_2^D. In such models, we have a hybrid model with parton-recombination and Q^2 rescaling mechanisms [7, 8]. According to this model, nuclear parton distributions are calculated at Q_0^2=0.8 GeV^2. We evolve these initial distributions to those at larger Q^2

In Fig. 2, we show the results of F_2^{Ca}/F_2^D at x=0.0085 with NMC experimental data [2]. The solid, dashed, and dot-dashed curves are obtained by LO-DGLAP, NLO-DGLAP, and PR evolution equations respectively with Λ=0.2 GeV and N_f=3. As shown by the figure, NLO and recombination contributions to the ratio are conspicuous at small x. If we evolve F_2 from Q_0^2=0.8 GeV^2, the

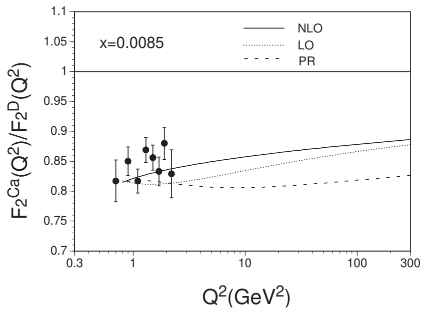

Figure 2: Q^2 evolution of F_2^{Ca}/F_2^D.

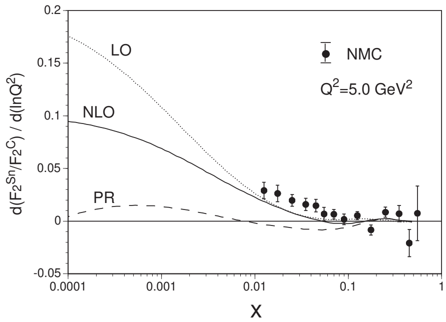

Figure 3: Nuclear dependence of Q^2 evolution.

recombination effect is larger than the NLO one. It is interesting to find such large recombination contributions. However, it is obvious that the recombination cannot be tested at this stage because we do not have data at large Q^2 with small x. In order to investigate the details of recombination, we need data in the wide Q^2 region at small x.

We also investigate the nuclear dependence of Q^2 evolution in tin and carbon nuclei [9]. Calculated results for $\partial[F_2^{Sn}/F_2^C]/\partial[\ln Q^2]$ are shown at Q^2=5 GeV^2 together with preliminary NMC data [10] in Fig. 3. The dotted, solid, and dashed curves correspond to LO-DGLAP, NLO-DGLAP, and PR evolution results respectively. The DGLAP evolution curves agree roughly with the experimental tendency. It is interesting to find that the PR results disagree with experimental data. Because of the significant discrepancy from the data, large parton-recombination contributions could be ruled out. However, it does not mean that the parton-recombination mechanism itself is in danger. Actually, there is an essential parameter K_{HT}, which determines how large the higher-dimensional gluon distribution is ($xG_{HT}(x,Q_0^2) = K_{HT}[xg(x,Q_0^2)]^2$). The magnitude of K_{HT} is unknown at this stage and we choose $K_{HT} = 1.68$ from Ref. [3]. In order to discuss the validity of the PR evolution, the constant K_{HT} must be evaluated theoretically. It is encouraging to study the details of the recombination mechanism in comparison with the NMC data. On the other hand, proposed HERA nuclear program should be able to clarify this issue by taking small x ($\approx 10^{-4}$) data in Fig. 3.

Finally, we comment on a spin-dependent case. Because NLO spin-dependent splitting functions are recently evaluated, we investigate Q^2 evolution of spin-dependent structure functions with the NLO contributions. This study is still in progress, and it is partly discussed in Ref. [11].

4. Conclusions

We investigate numerical solution of DGLAP and PR equations. We provide a FORTRAN program BF1 for solving these equations in a brute-force method. Using this program, we calculate the Q^2 evolution of proton's F_2 and the ratio F_2^{Ca}/F_2^{D}. Our results are consistent with experimental data. However, parton recombinations in the nucleon cannot be found because their effects are very small. Their contributions cannot be tested at this stage even in the nuclear case. In order to investigate the details of the recombination in nuclei, we need data in the wide Q^2 region at small x. We also find the nuclear dependence of Q^2 evolution $\partial[F_2^{Sn}/F_2^{C}]/\partial[\ln Q^2]$ provides an important clue to the recombination mechanism.

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