Simulations of a BH-axion system

Hirotaka Yoshino
Hideo Kodama
(KEK)

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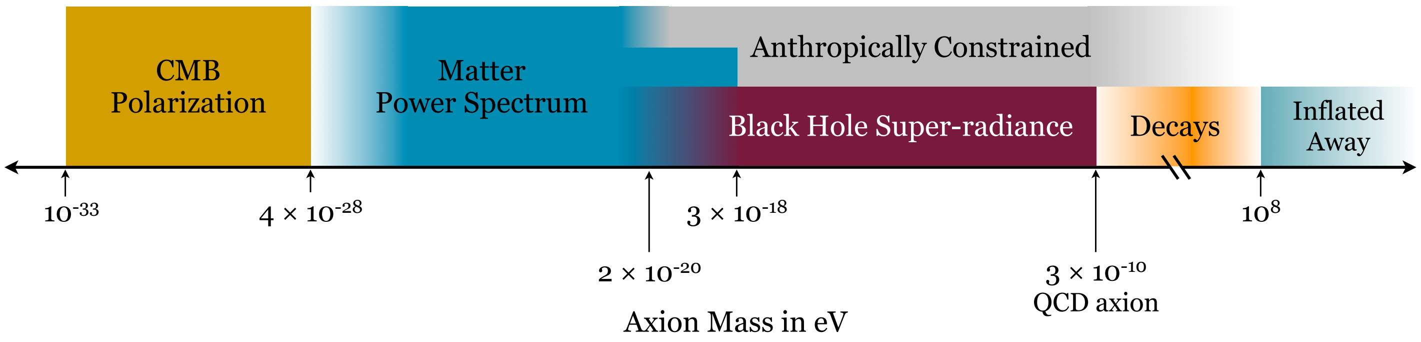
Axiverse

- QCD axion
  - QCD axion was introduced to solve the Strong CP problem.
  - It is one of the candidates of dark matter.

- String axions

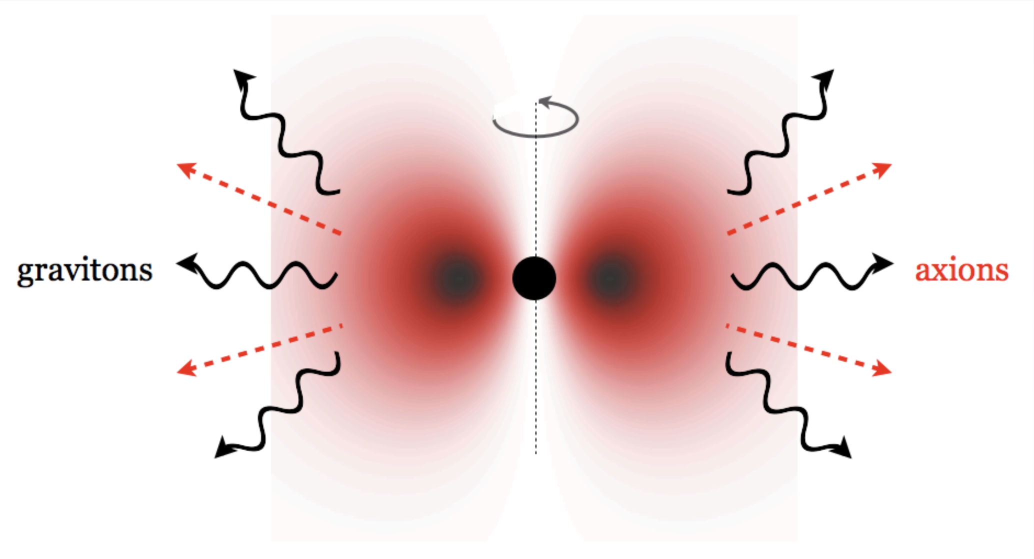
Arvanitaki, Dimopoulos, Dubvosky, Kaloper, March-Russel, PRD81 (2010), 123530.

  - String theory predicts the existence of 10-100 axion-like massive scalar fields.
  - There are various expected phenomena of string axions.



Axion field around a rotating black hole

- Axion field forms a cloud around a rotating BH and extract energy of the BH by "superradiant instability".



Arvanitaki, Dimopoulos, Dubvosky, Kaloper, March-Russel, PRD81 (2010), 123530.

Arvanitaki and Dubovsky, PRD83 (2011), 044026.

Kerr BH

- Metric

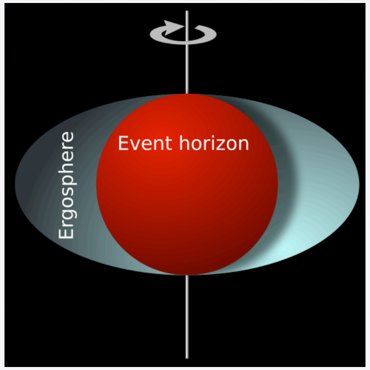
$$ds^2 = -\left(\frac{\Delta - a^2 \sin^2\theta}{\Sigma}\right) dt^2 - \frac{2a\sin^2\theta(r^2+a^2-\Delta)}{\Sigma} dt d\phi + \left[\frac{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}{\Sigma}\right] \sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2$$

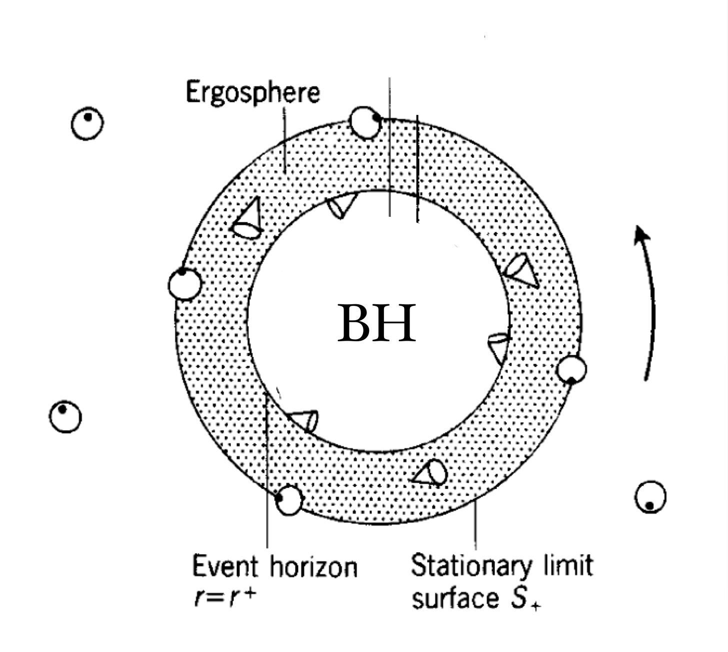
$$\Sigma = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 + a^2 - 2Mr.$$

$$J = Ma$$

- Ergo region

$$g_{tt} > 0$$



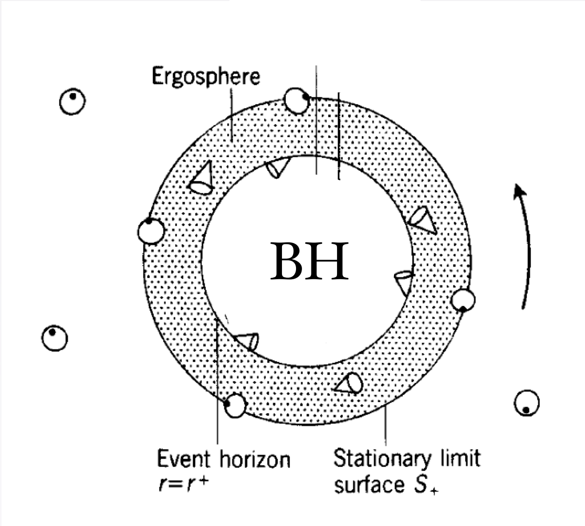


Energy extraction

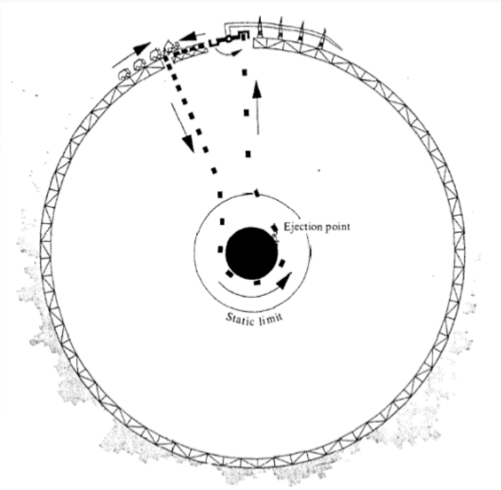
- BH's rotational energy

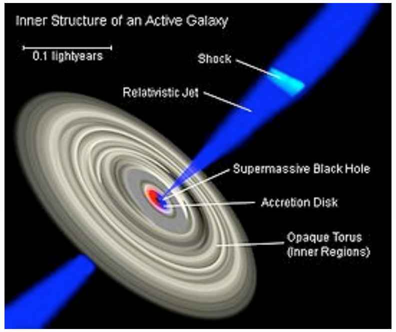
$$M_{\rm rot} = M - M_{\rm irr}$$

$$M_{\rm irr} = \sqrt{\frac{A_H}{16\pi}}$$



- Methods of energy extraction
  - Penrose process
  - Blandford-Znajek process
  - Superradiance





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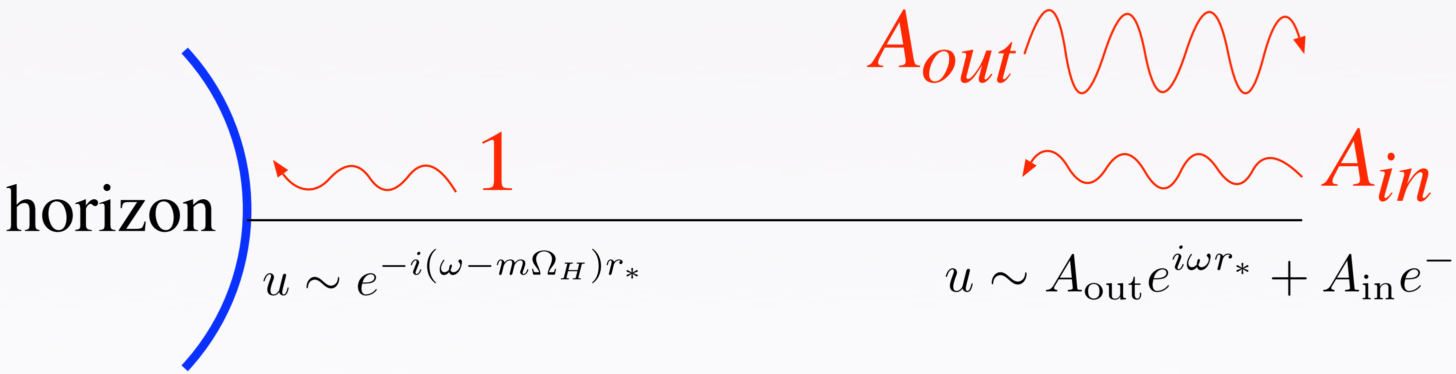
Superradiance

Zel'dovich (1971)

- Massless Klein-Gordon field $\nabla^2\Phi = 0$

$$\Phi = \mathrm{Re}[e^{-i\omega t}R(r)S(\theta)e^{im\phi}]$$

$$R = \frac{u}{\sqrt{r^2+a^2}} \Rightarrow \frac{d^2u}{dr_*^2} + \left[\omega^2 - V(\omega)\right]u = 0$$



$$u \sim e^{-i(\omega - m\Omega_H)r_*}$$

$$u \sim A_{\rm out}e^{i\omega r_*} + A_{\rm in}e^{-i\omega r_*}$$

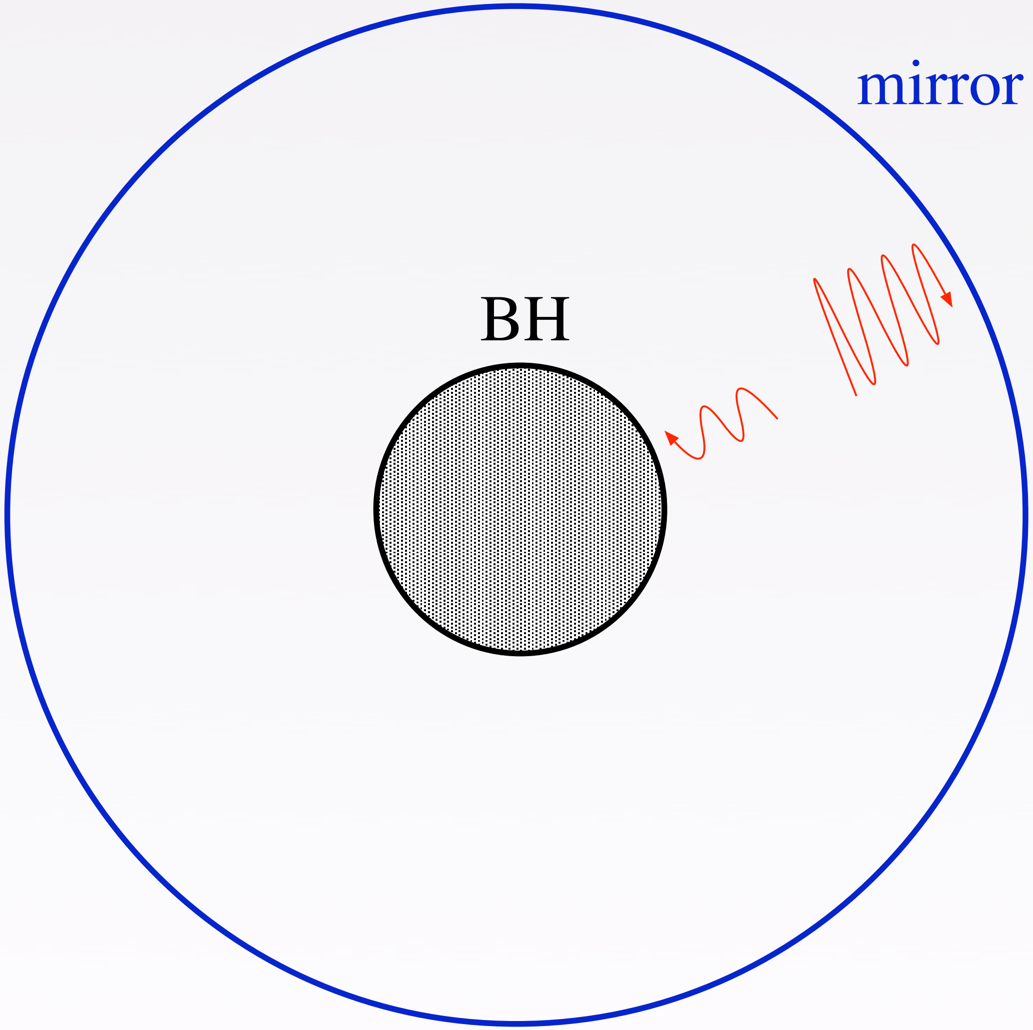
$$\left(1 - \frac{m\Omega_H}{\omega}\right)|T|^2 = 1 - |R|^2$$

Superradiant condition:

$$\omega < \Omega_H m$$

Black hole bomb

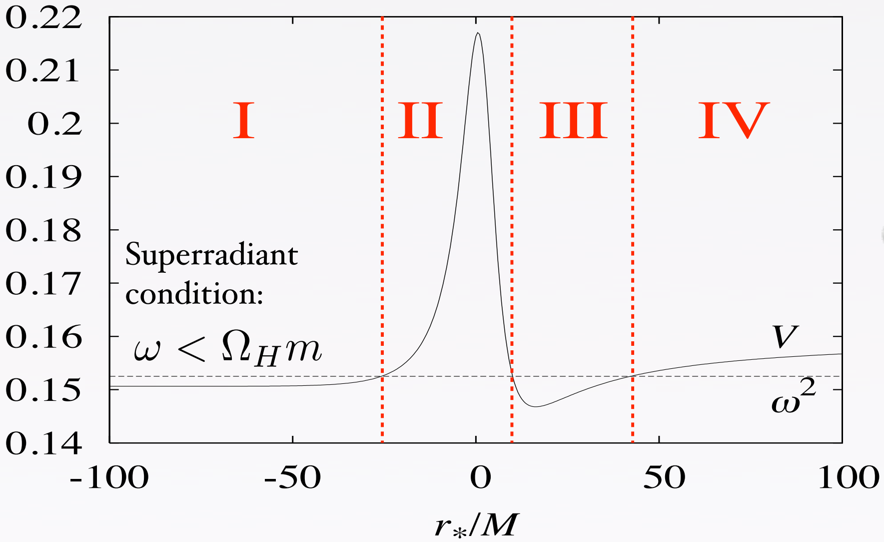
Press and Teukolsky (1972)



Bound state

Zouros and Eardley, Ann. Phys. 118 (1979), 139.
Detweiler, PRD22 (1980), 2323.

- Massive Klein-Gordon field $\nabla^2\Phi - \mu^2\Phi = 0$



- near horizon

$$u \sim e^{-i(\omega - m\Omega_H)r_*}$$

- distant region

$$u \sim e^{-\sqrt{\mu^2-\omega^2}\,r_*}$$

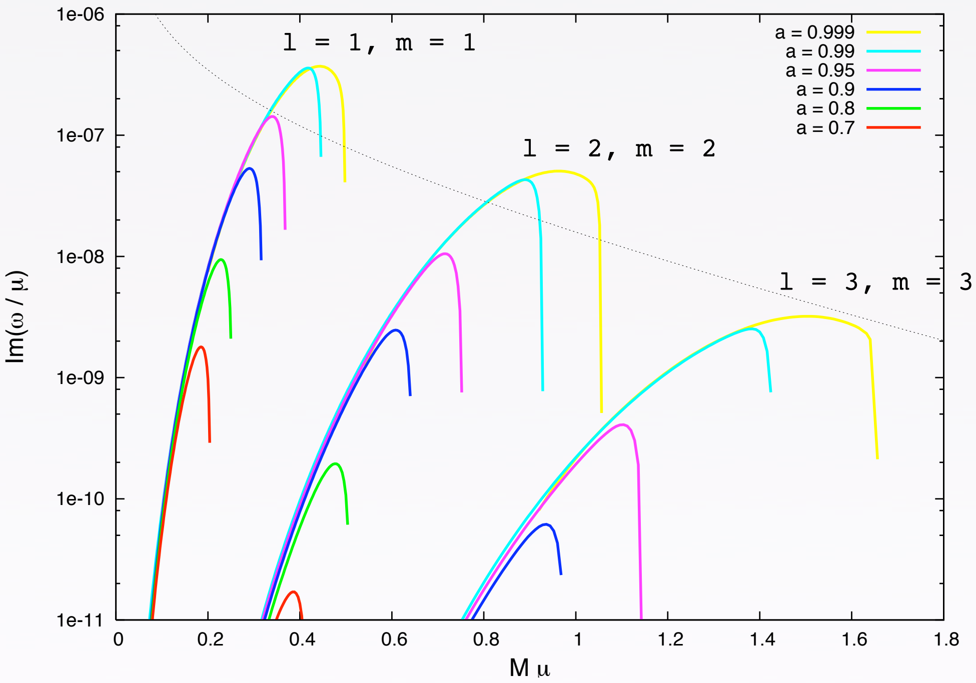
$$\Phi = \mathrm{Re}[e^{-i\omega t}R(r)S(\theta)e^{im\phi}]$$

$$R = \frac{u}{\sqrt{r^2+a^2}} \quad \Rightarrow \quad \frac{d^2u}{dr_*^2} + \left[\omega^2 - V(\omega)\right]u = 0$$

Growth rate

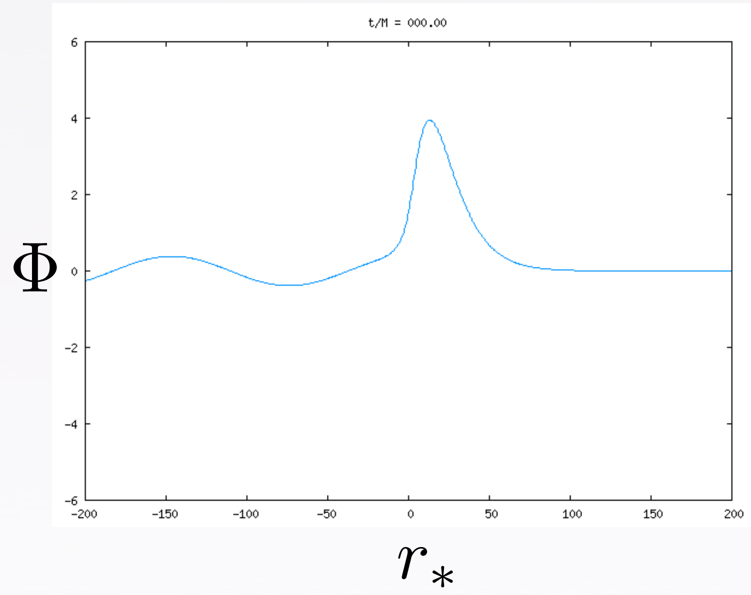
- Growth rate calculated by continued fraction method

Dolan, PRD76 (2007), 084001.



- Time evolution

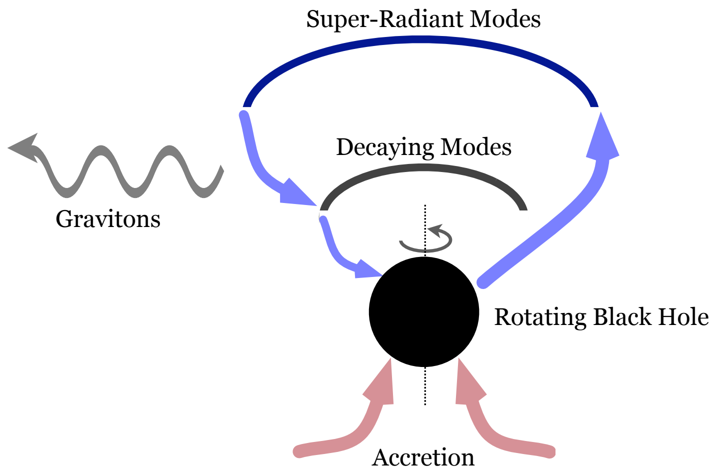
$$l = m = 1 \qquad M\mu = 0.4$$



(Near the horizon)

$$\Phi \sim e^{-i\omega t} e^{-i\tilde{\omega} r_*}$$
$$\tilde{\omega} = \omega - m\Omega_H$$

BH-axion system



Arvanitaki and Dubovsky, PRD83 (2011), 044026.

- Superradiant instability
  - Emission of gravitational waves
  - Pair annihilation of axions
- Effects of nonlinear self-interaction
  - Bosenova
  - Mode mixing

Nonlinear effect

- Typically, the potential of axion field becomes periodic

$$V = f_a^2 \mu^2 [1 - \cos(\Phi / f_a)]$$

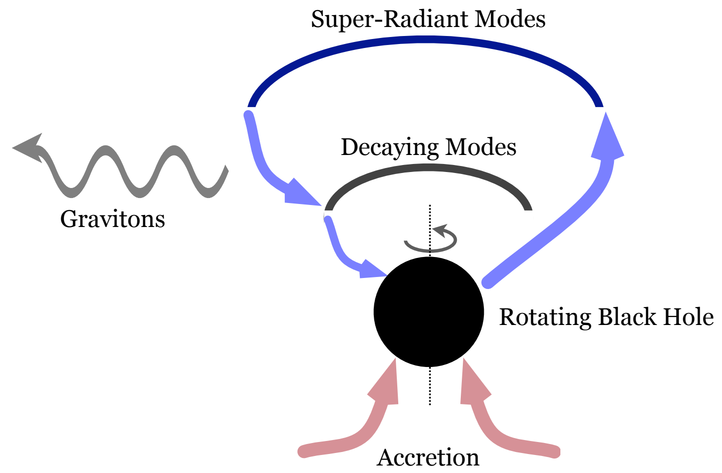
$$\Rightarrow \quad \boxed{\nabla^2 \varphi - \mu^2 \sin \varphi = 0} \qquad \varphi \equiv \frac{\Phi}{f_a}$$

- c.f., QCD axion

PQ phase transition — QCD phase transition

U(1)PQ symmetry $\Rightarrow$ potential becomes like a wine bottle $\Rightarrow$ Z(N) symmetry

BH-axion system



Arvanitaki and Dubovsky, PRD83 (2011), 044026.

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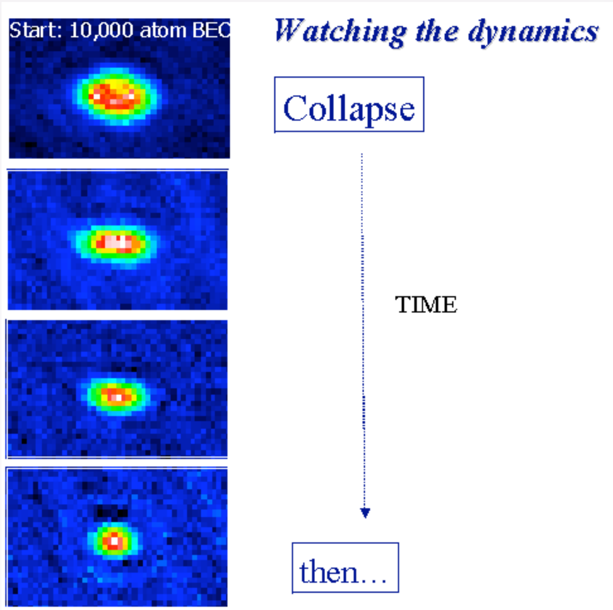
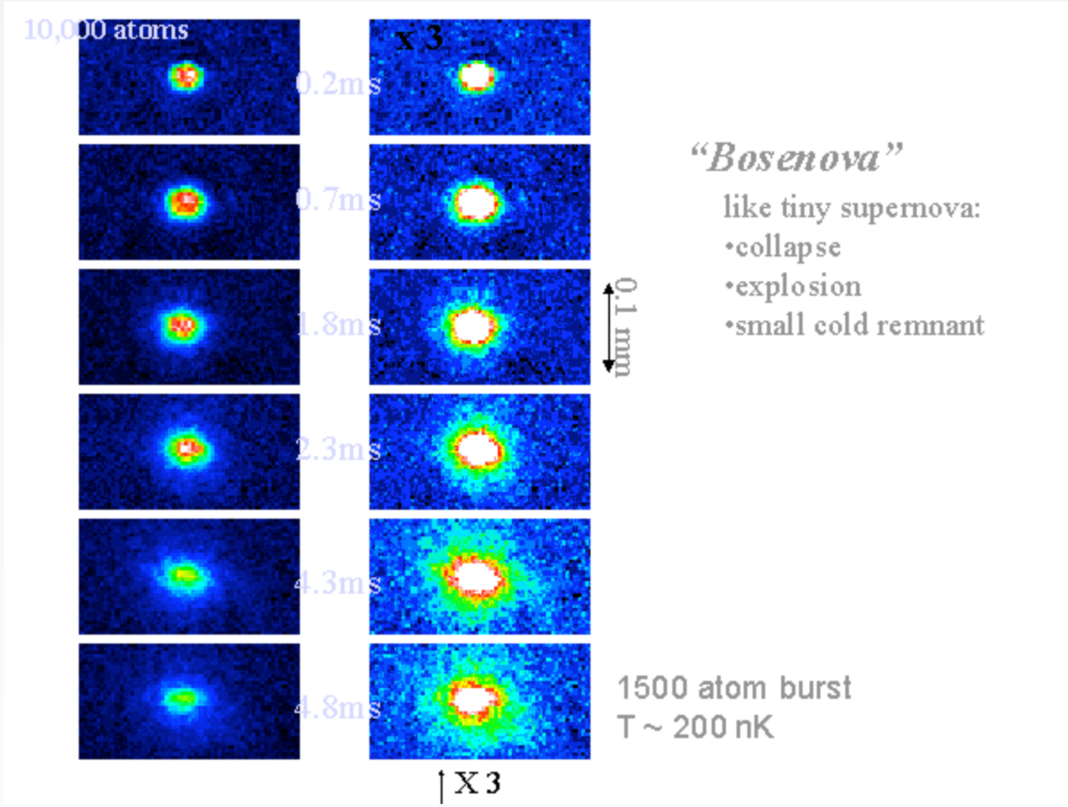
Bosenova in condensed matter physics

http://spot.colorado.edu/~cwieman/Bosenova.html





BEC state of Rb85 (interaction can be controlled)

Switch from repulsive interaction to attractive interaction

Wieman et al., Nature 412 (2001), 295

What we would like to do

- We would like to study the phenomena caused by axion cloud generated by the superradiant instability around a rotating black hole.

- In particular, we study numerically whether "Bosenova" happens when the nonlinear interaction becomes important.

- We adopt the background spacetime as the Kerr spacetime, and solve the axion field as a test field.

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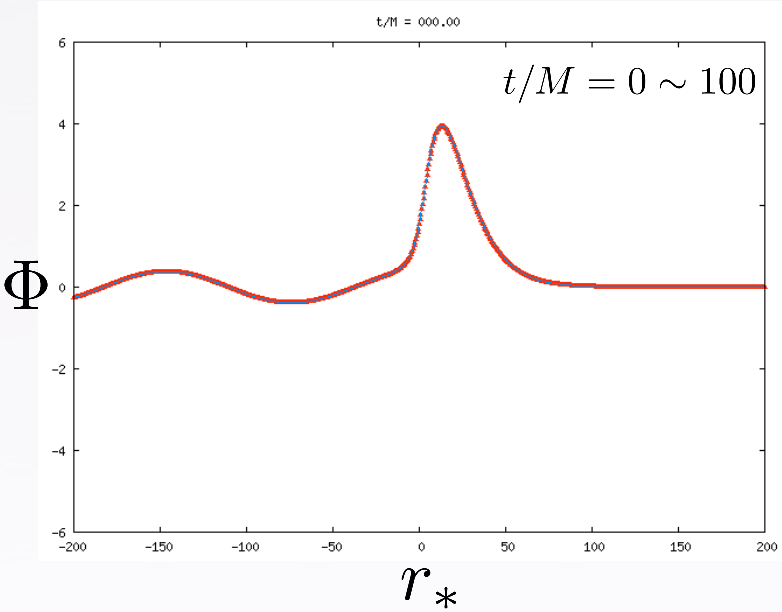
Our code

- 3D code of coordinates (r_*, θ, ϕ)
- Comparison with semianalytic solution of the Klein-Gordon case



$$\omega_I = \frac{\dot{E}}{2E} \simeq \frac{E(100M) - E(0)}{200ME(0)}$$

$$\omega_I^{(\mathrm{CF})}/\mu = 3.31 \times 10^{-7}$$

$$\omega_I^{(\mathrm{Numerical})}/\mu = 3.26 \times 10^{-7}$$

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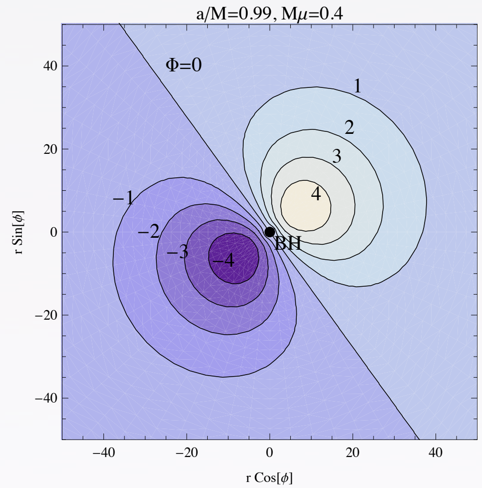
Numerical simulation

- Sine-Gordon equation

$$\nabla^2\varphi - \mu^2 \sin\varphi = 0$$

- Setup $\quad a/M = 0.99, \quad M\mu = 0.4$

As the initial condition, we choose the bound state of the Klein-Gordon field of the $l = m = 1$ mode.



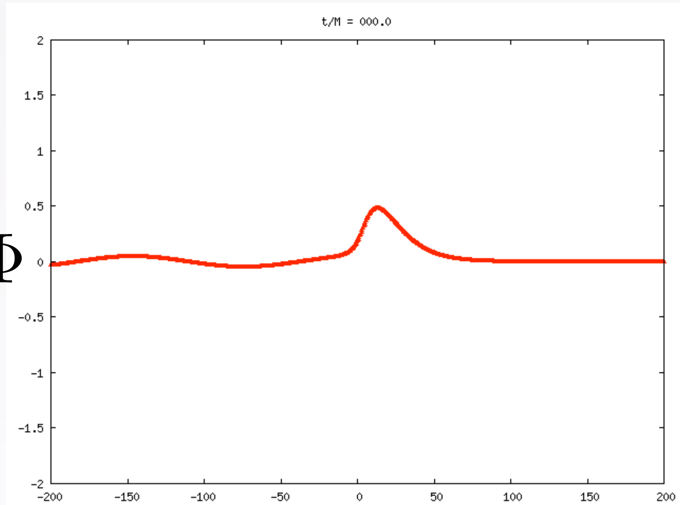
	Initial peak value	$E/[(f_a/M_p)^2 M]$
(A)	0.6	1370
(B)	0.7	1862

Simulation (A) $\quad \varphi_{\rm peak}(0) = 0.6$

- Axion field on the equatorial plane $\quad (\theta = \pi/2)$

$(\phi = 0)$

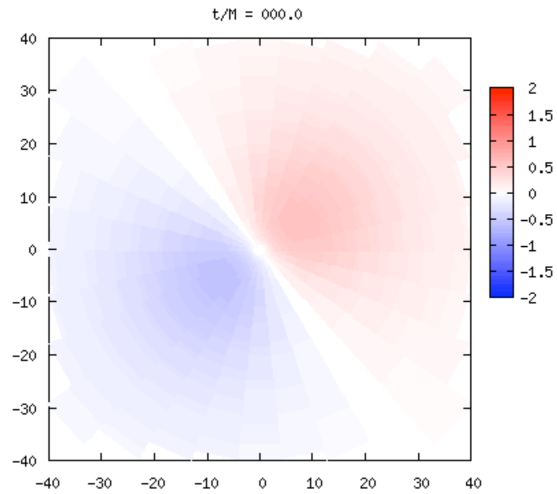
Φ



$$-200 \le r_*/M \le 200$$

(Equatorial plane)

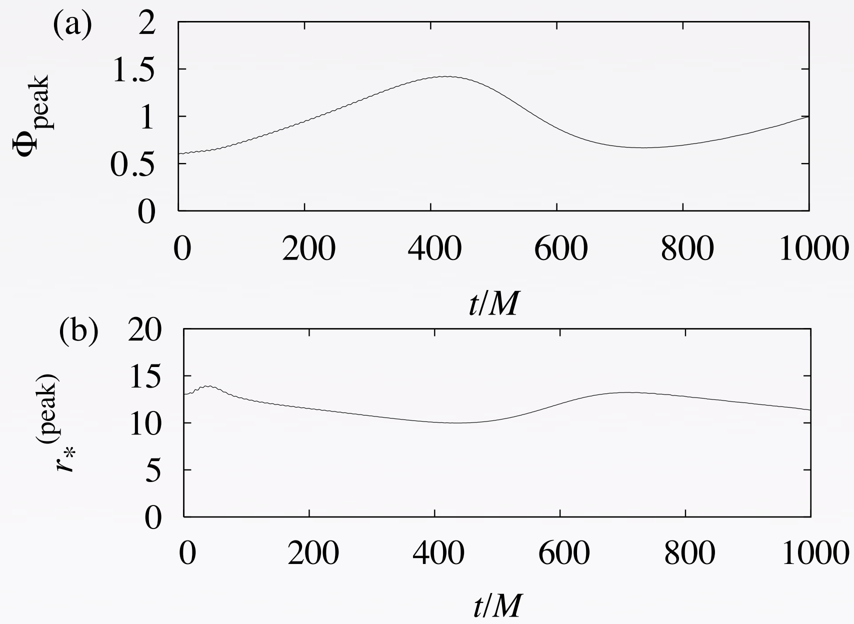
$r \sin\phi$



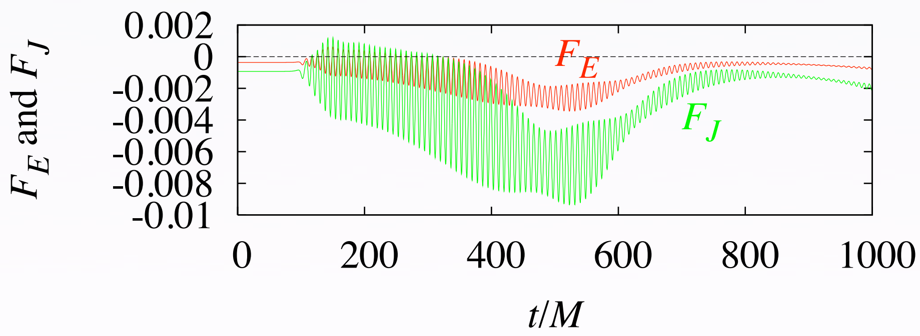
$r \cos\phi$

Simulation (A)

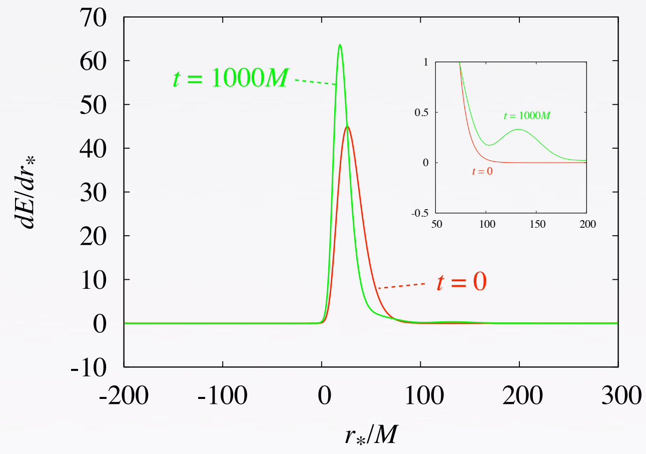
- Peak value and peak location

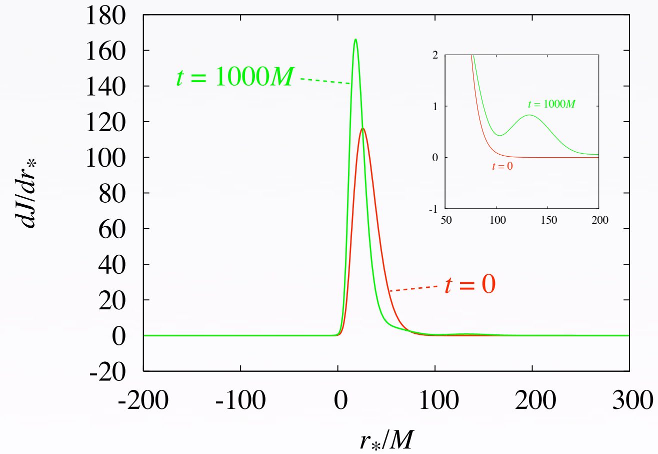


- Fluxes toward the horizon



- Energy and angular momentum distribution



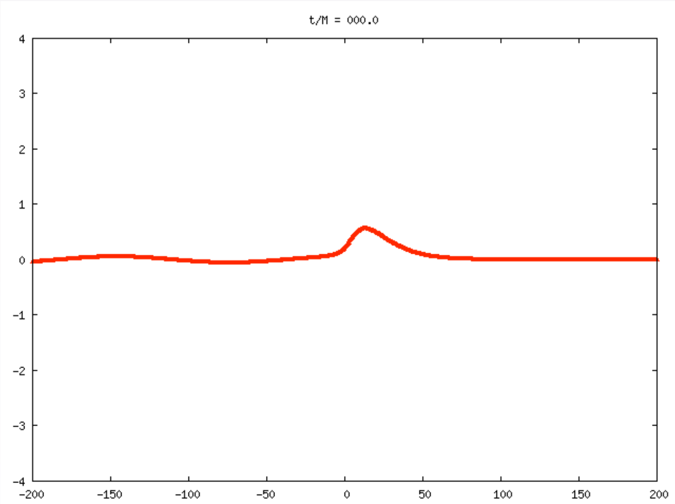


Simulation (B) $\varphi_{\rm peak}(0) = 0.7$

- Axion field on the equatorial plane $(\theta = \pi/2)$

$(\phi = 0)$

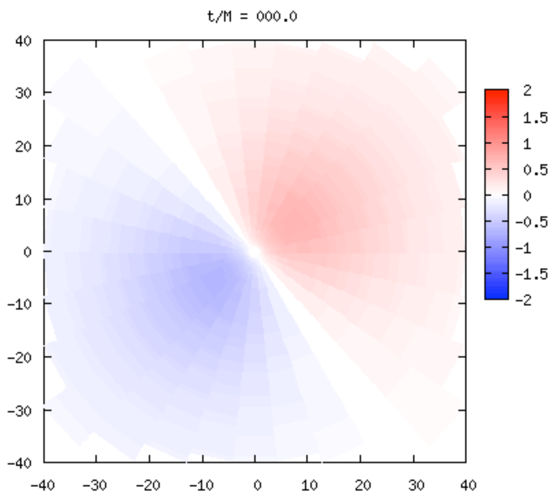
Φ



$-200 \leq r_*/M \leq 200$

(Equatorial plane)

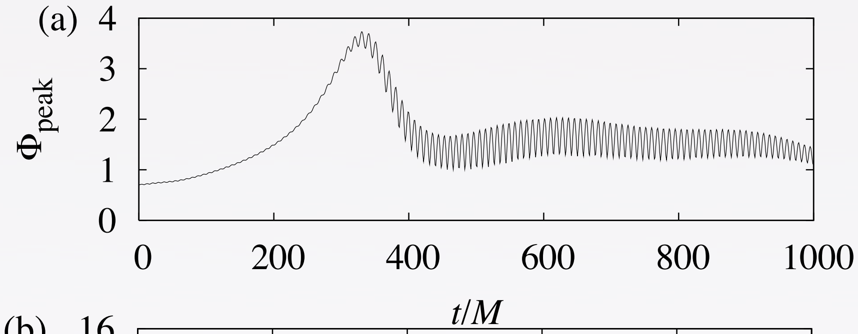
$r \sin\phi$

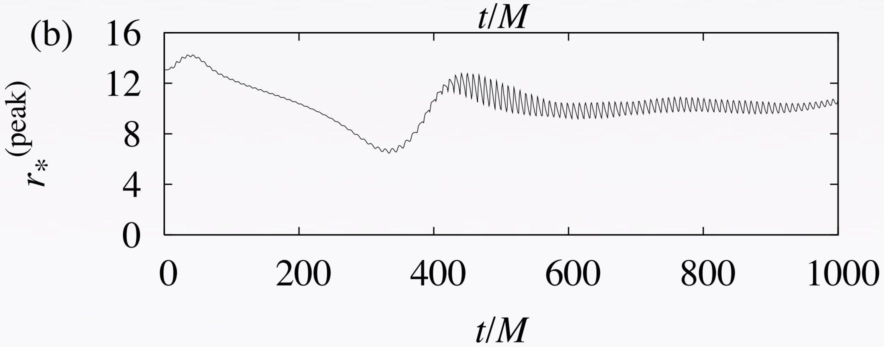


$r \cos\phi$

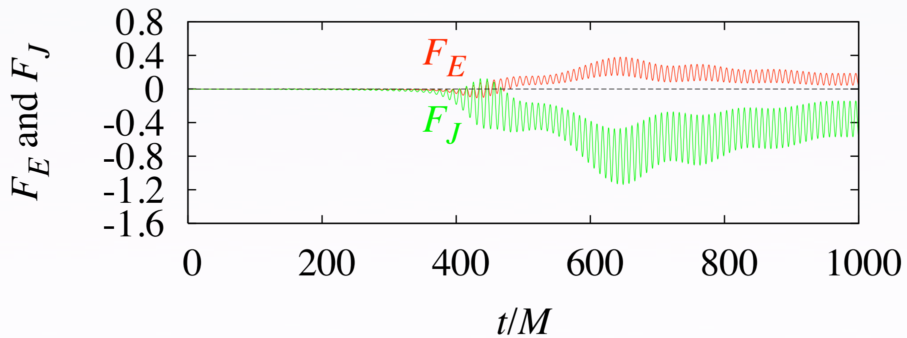
Simulation (B)

- Peak value and peak location

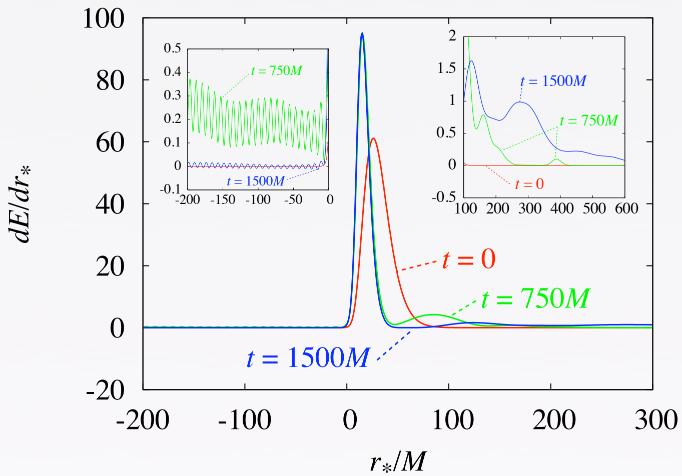


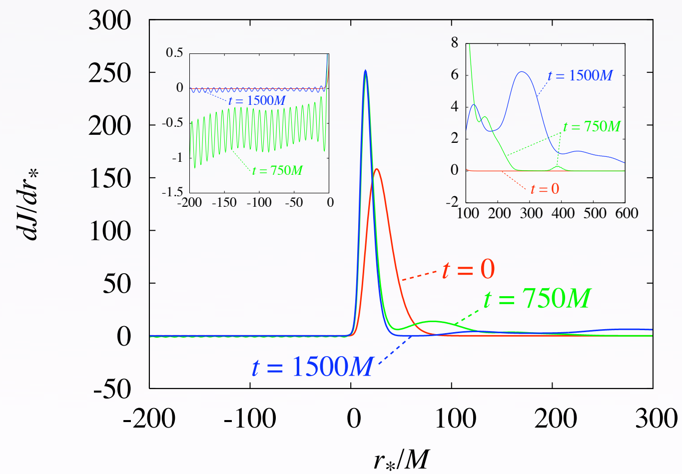


- Fluxes toward the horizon



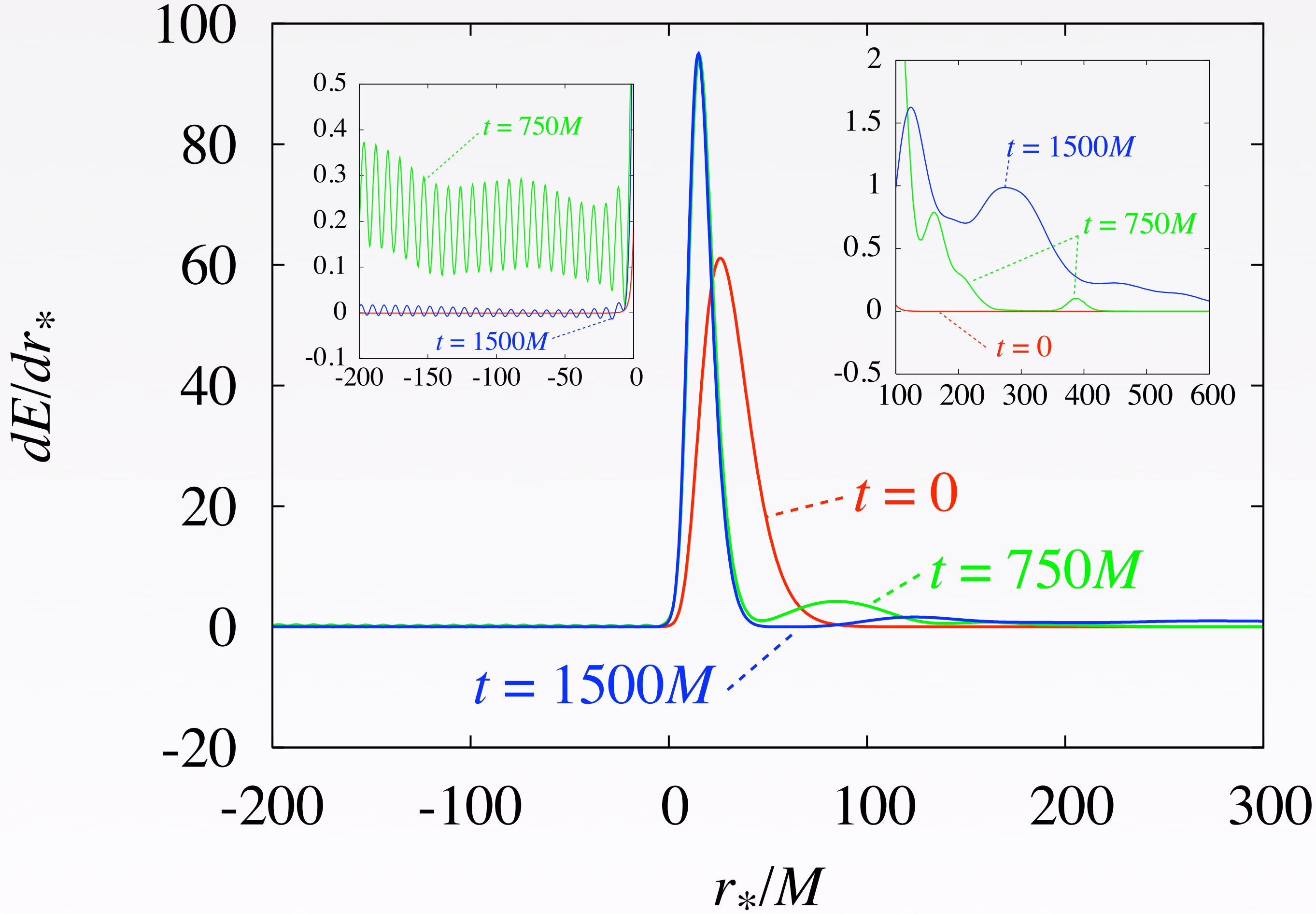
- Energy and angular momentum distribution





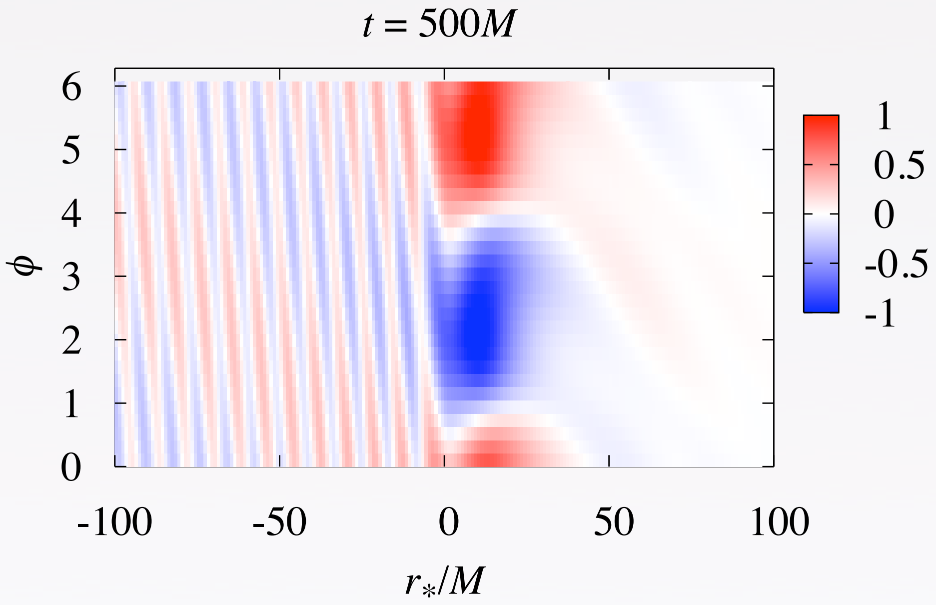
Simulation (B)

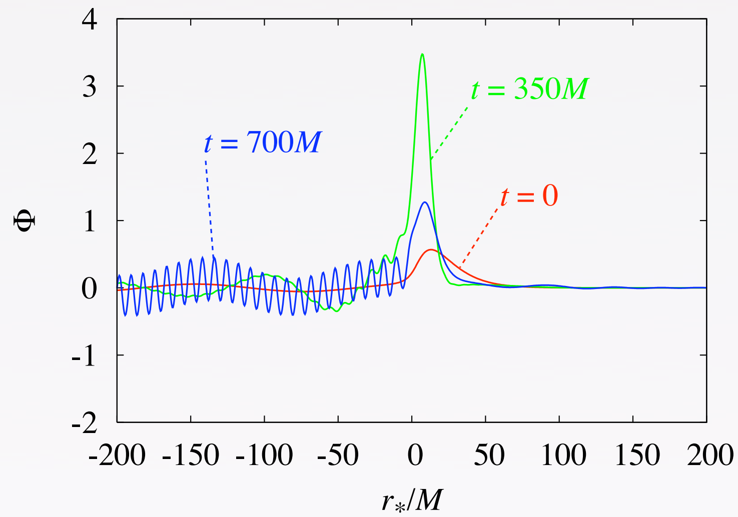
- Energy distribution



Simulation (B)

- Snapshots





m=-1 mode is generated!

(Near the horizon)

$$\Phi \sim e^{-i\omega t} e^{-i\tilde{\omega} r_*}$$
$$\tilde{\omega} = \omega - m\Omega_H$$

$$M\omega_{\rm KG} = 0.39 \qquad M\omega_{\rm NL} = 0.35$$
$$M\tilde{\omega}_{\rm KG} = -0.04 \qquad M\tilde{\omega}_{\rm NL} = 0.87$$

Summary of the simulations (A) and (B)

- (A)
  - When the peak value is not very large, the nonlinear term enhances the rate of superradiant instability.
  - The nonlinear effect makes energy distribute in the neighborhood of the black hole.
- (B)
  - When the peak value is sufficiently large, the bosenova collapse happens.
  - Once the bosenova happens, positive energy falls into the black hole, while the angular momentum continues to be extracted.

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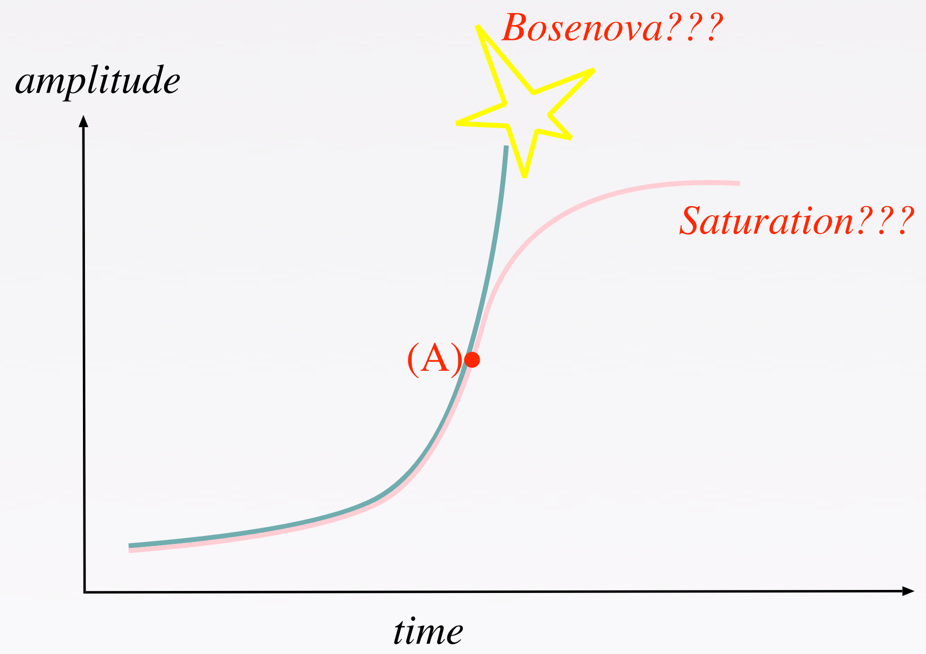
Does bosenova really happen?



- Additional simulation:

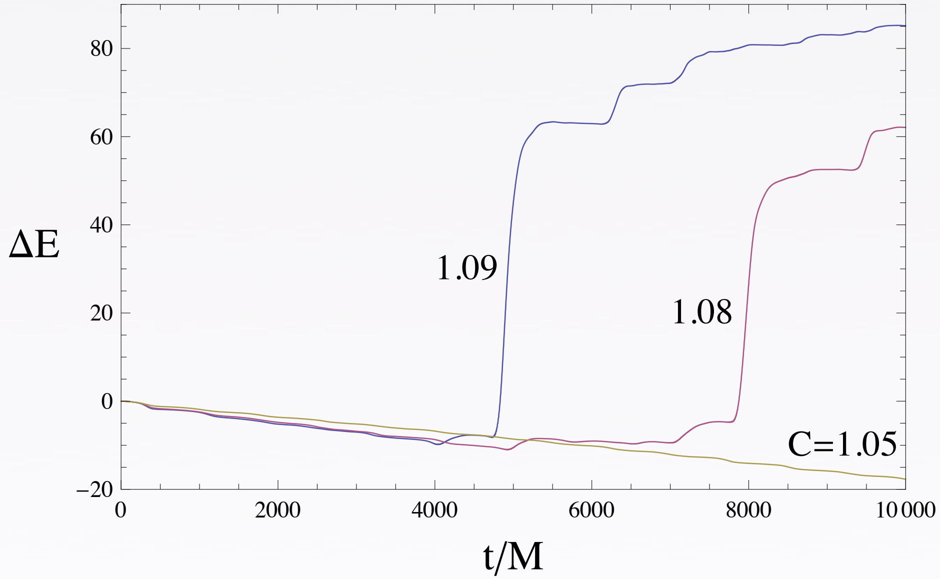
$$\varphi(0) = C\varphi^{(\mathrm{A})}(1000M)$$
$$\dot{\varphi}(0) = C\dot{\varphi}^{(\mathrm{A})}(1000M)$$

$$C = \begin{cases} 1.05 \\ 1.08 \\ 1.09 \end{cases}$$

Supplementary simulation

$$\varphi(0) = C\varphi^{(A)}(1000M)$$
$$\dot{\varphi}(0) = C\dot{\varphi}^{(A)}(1000M)$$

- Energy absorbed by the black hole $\Delta E := \int_0^t F_E dt$ $\quad C = \begin{cases} 1.05 \\ 1.08 \\ 1.09 \end{cases}$



The bosenova happens when $E \simeq 1600 \times (f_a/M_p)^2 M$

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Action

Saito and Ueda, PRA63 (2001), 043601

- BEC
  - Action

$$S = N\hbar \int d^3x dt \left[i\psi^* \dot{\psi} + \frac{1}{2}\psi^* \nabla^2 \psi - \frac{r^2}{2}\psi^*\psi - \frac{g}{2}(\psi^*\psi)^2 \right]$$

$$i\dot{\psi} = -\frac{1}{2}\nabla^2\psi + \frac{r^2}{2}\psi + g|\psi|^2\psi$$

Gross-Pitaevskii equation

- BH-axion
  - Action

$$\hat{S} = \int d^4x \sqrt{-g} \left[-\frac{1}{2}(\nabla\varphi)^2 - \mu^2 \left(\frac{\varphi^2}{2} + \hat{U}_{\rm NL}(\varphi) \right) \right],$$

  - Non-relativistic approximation

$$\varphi = \frac{1}{\sqrt{2\mu}} \left(e^{-i\mu t}\psi + e^{i\mu t}\psi^* \right)$$

$$\hat{S}_{\rm NR} = \int d^4x \left[\frac{i}{2} \left(\psi^*\dot{\psi} - \psi\dot{\psi}^* \right) - \frac{1}{2\mu}\partial_i\psi\partial_i\psi^* + \frac{\alpha_g}{r}\psi^*\psi - \mu^2 \tilde{U}_{\rm NL}(|\psi|^2/\mu) \right]$$

$$\tilde{U}_{\rm NL}(x) = -\sum_{n=2}^{\infty} \frac{(-1/2)^n}{(n!)^2} x^n.$$

Effective theory

Saito and Ueda, PRA63 (2001), 043601

$(\nu = \cos\theta)$

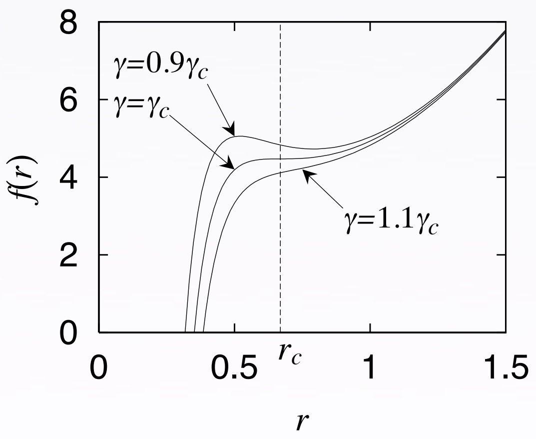
- BEC $\quad \psi = A(x,y,z,t)e^{i\phi(x,y,z,t)}$

$$A = \frac{\exp\left[-\left(\frac{x^2}{2d_x^2(t)} + \frac{y^2}{2d_y^2(t)} + \frac{z^2}{2d_z^2(t)}\right)\right]}{\sqrt{\pi^{3/2}d_x(t)d_y(t)d_z(t)}}$$

$$\phi = \frac{\dot{d}_x(t)}{2d_x(t)}x^2 + \frac{\dot{d}_y(t)}{2d_y(t)}y^2 + \frac{\dot{d}_z(t)}{2d_z(t)}z^2$$

- Spherical case $\quad d_x = d_y = d_z = r(t)$

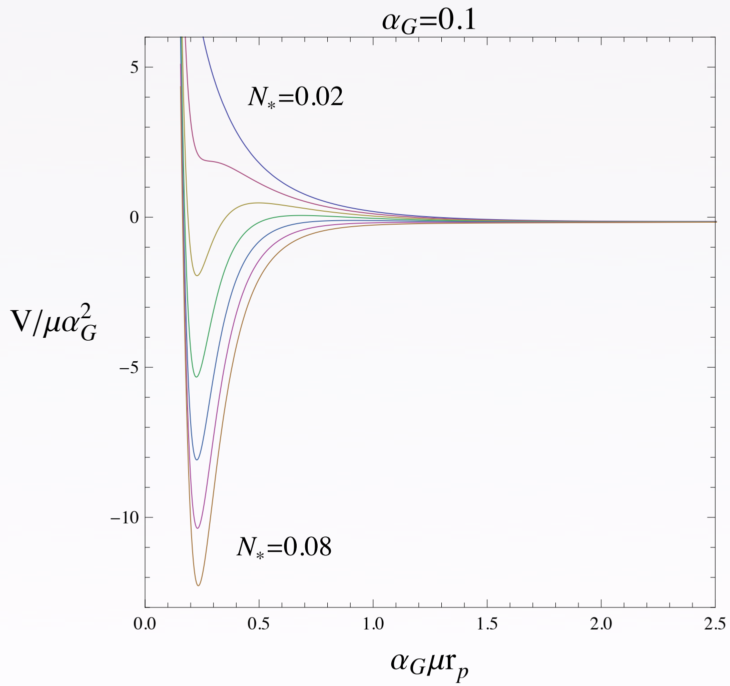
$$S = \frac{N\hbar}{4}\int dt\left[3\dot{r}^2 + 3\dot{r} - f(r)\right]$$



- BH-axion $\quad \psi = A(t,r,\nu)e^{iS(t,r,\nu)+im\phi}$

$$A(t,r,\nu) \approx A_0 \exp\left[-\frac{(r-r_p)^2}{4\delta_r r_p^2} - \frac{(\nu-\nu_p)^2}{4\delta_\nu}\right],$$

$$S(t,r,\nu) \approx S_0(t) + p(t)(r-r_p) + P(t)(r-r_p)^2 + \pi_\nu(t)(\nu-\nu_p)^2 + \cdots,$$

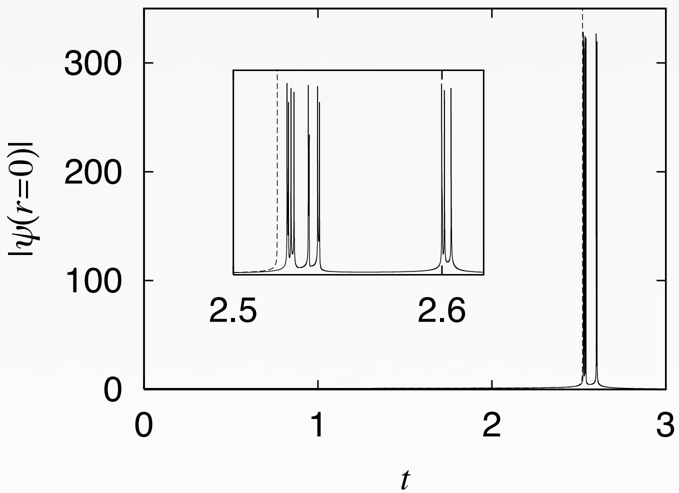


Simulation results

Saito and Ueda, PRA63 (2001), 043601

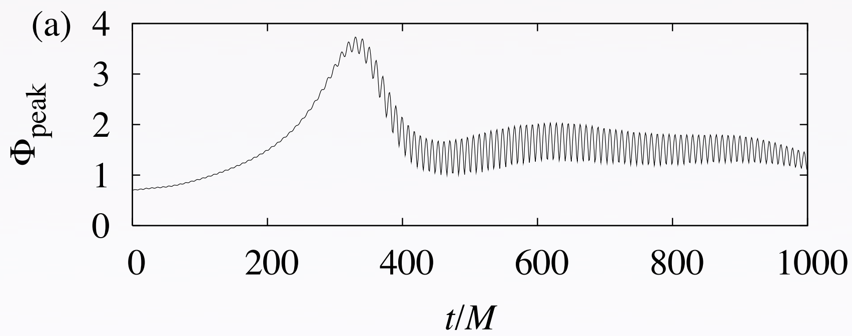
- BEC

$$i\dot{\psi} = -\frac{1}{2}\nabla^2\psi + \frac{r^2}{2}\psi + g|\psi|^2\psi - \frac{i}{2}\left(\frac{L_2}{2}|\psi|^2 + \frac{L_3}{6}|\psi|^4\right)\psi$$



- BH-axion

- Our simulation results



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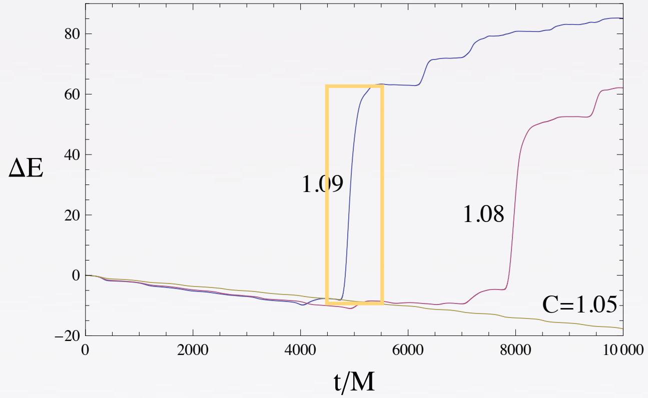
GWs emitted in the bosenova (rough estimate)

- Quadrupole moment
  - $$Q_{ij} \sim r_p^2 E$$

    $$r_p \sim 10M \qquad E_0 \sim 10^{-3}M$$



- About 5% of energy falls into the BH
  - $$E = E_0 + (\Delta E/2)\left[\cos(\pi t/\Delta t) - 1\right]$$

    $$\Delta E \sim 0.05E_0 \qquad \Delta t \sim 500M$$

- Amplitude of generated GWs

$$h \sim \frac{\ddot{Q}_{ij}}{r_{\rm obs}} \sim 10^{-7}\frac{M}{r_{\rm obs}}$$

Detectability

$$h \sim \frac{\ddot{Q}_{ij}}{r_{\rm obs}} \sim 10^{-7}\frac{M}{r_{\rm obs}}$$

- Supermassive BH of our galaxy (Sagittarius A*) $(10^{-4}\ \mathrm{Hz})$

$$h_{\rm rss} := \left[\int |h|^2 dt\right]^{1/2} \sim 10^{-16}(\mathrm{Hz})^{-1/2}$$

⇨ Detectable by the eLISA

Angular frequency of GW

- Solar-mass BH (e.g., Cygnus X-1) $(10^{2}\ \mathrm{Hz})$

$$h_{\rm rss} \sim 10^{-24}(\mathrm{Hz})^{-1/2}$$

⇨ below the sensitivity of the Advanced LIGO, KAGRA (LCGT), etc.

Contents

Summary

- We developed a reliable code and numerically studied the behaviour of axion field around a rotating black hole.
- The nonlinear effect enhances the rate of superradiant instability when the amplitude is not very large.
- The bosenova collapse would happen as a result of superradiant instability.

Issues for future

- Calculation of the gravitational waves emitted in bosenova.
- The case where axions couple to magnetic fields.