Searching for de Sitter String Vacua

Gary Shiu

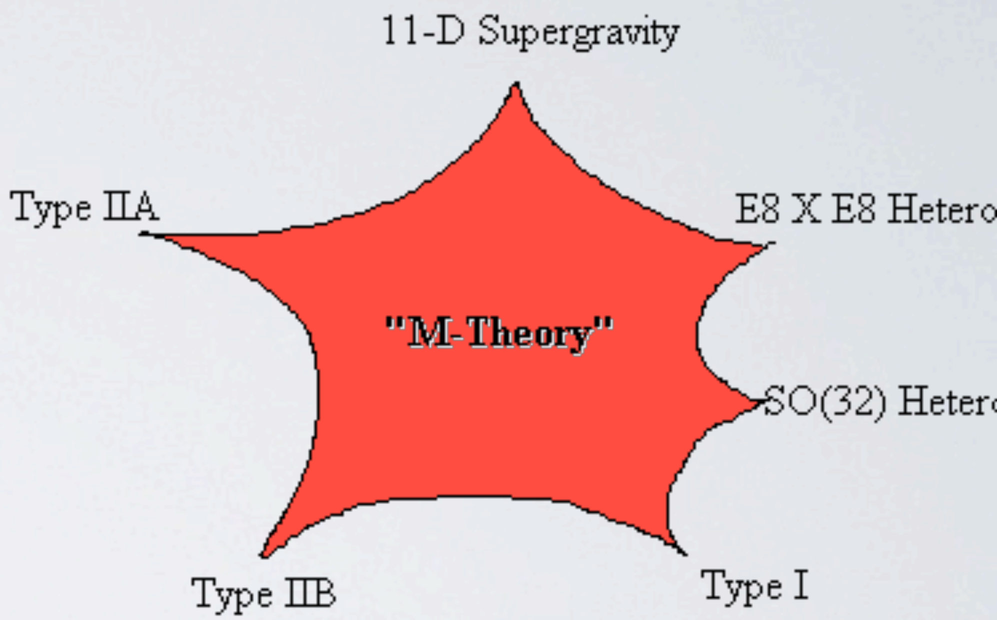
University of Wisconsin & HKUST

Outline of these Lectures

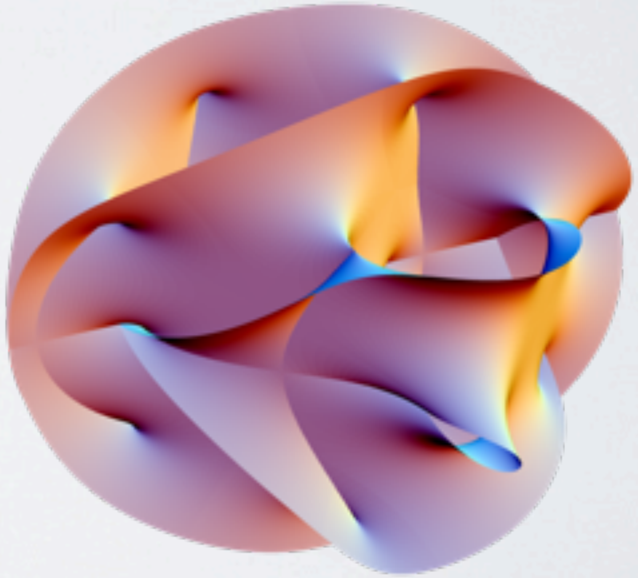
- Lecture 1: No-go theorems for dS and explicit model building
  - S.S. Haque, GS, B. Underwood, T. Van Riet, Phys. Rev. D79, 086005 (2009).
  - U.H. Danielsson, S.S. Haque, GS, T. Van Riet, JHEP 0909, 114 (2009).
  - U.H. Danielsson, S.S. Haque, P. Koerber, GS, T. Van Riet, T. Wrase, Fortsch. Phys. 59, 897 (2011).
  - GS, Y. Sumitomo, JHEP 1109, 052 (2011).
- Lecture 2: Two roles of tachyons in String Cosmology
  - Random (Super)gravities & Implications to the Landscape
    - X. Chen, GS, Y. Sumitomo, H. Tye, JHEP 1204, 026 (2012)+work in progress
  - Detectable Primordial Gravity Waves in Small Field Inflation
    - N. Barnaby, J. Moxon, R. Namba, M. Peloso, GS, P. Zhou, arXiv:1206.6117.

STRING THEORY LANDSCAPE

- Many perturbative formulations:



- In each perturbative limit, many topologies:



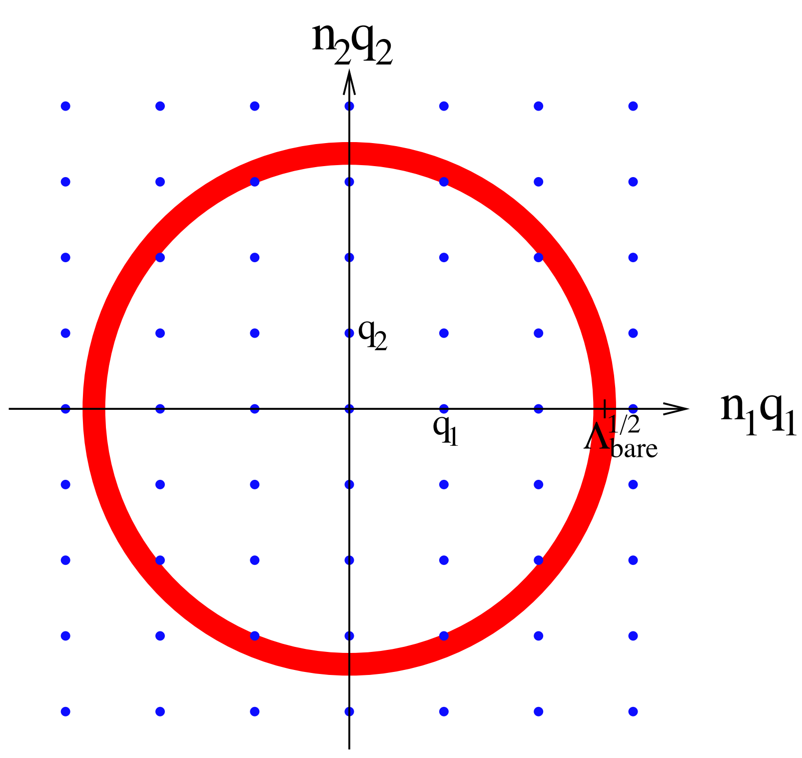
- For a fixed topology, many choices of fluxes.

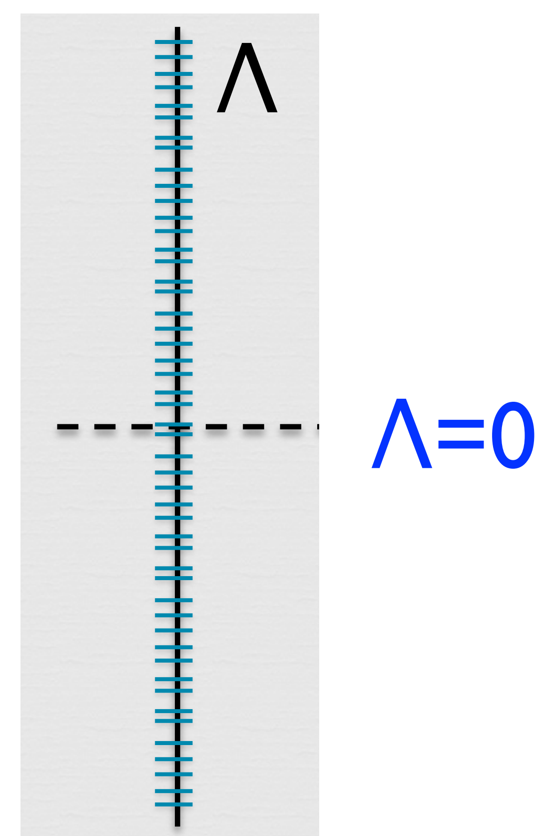
A Flux Landscape

- Quantized fluxes contribute to vacuum energy:

$$\int_\Sigma F \in \mathbb{Z} \qquad\qquad \Lambda = \Lambda_{\rm bare} + \frac{1}{2}\sum_i n_i^2 q_i^2$$

- A finely spaced discretuum [Bousso, Polchinski]:



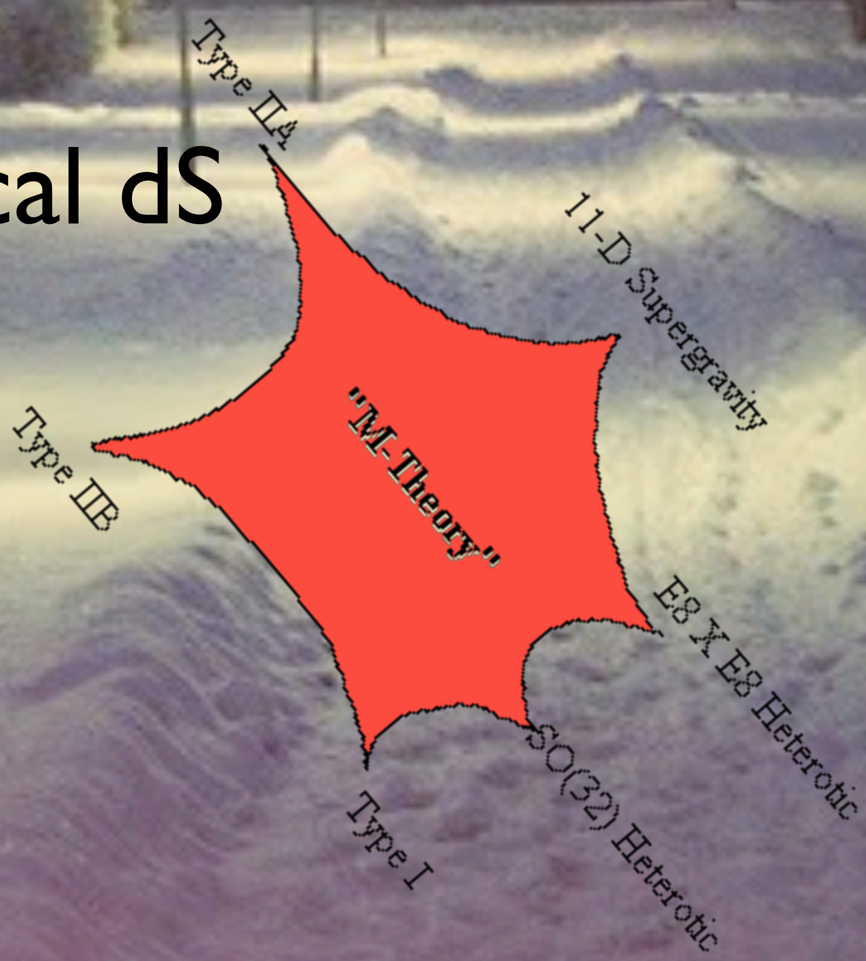


- # solutions ~ (# flux quanta)$^{\#\text{moduli}}$ ~m^N~10^{500}

Explicit Constructions

Classical dS

KKLT, LVS, ...



A Mini Landscape

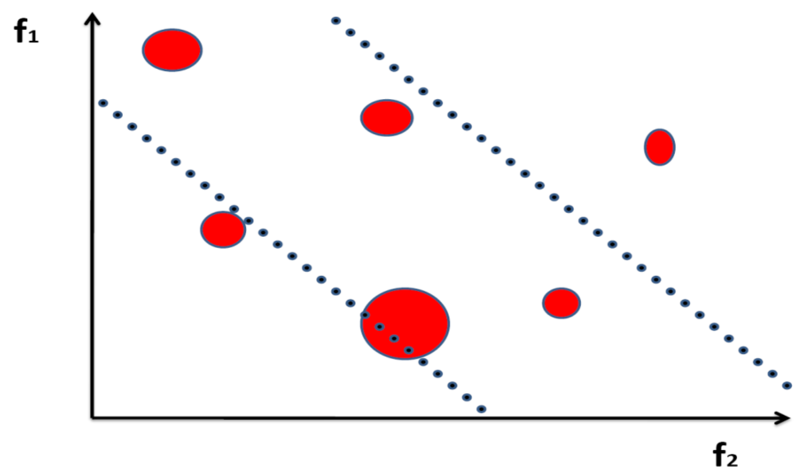
✤ # of unipotent 6D group spaces ~ *O*(50). Among them, only a handful have de Sitter critical points that are compatible with orbifold/orientifold symmetries.

✤ Each of these group spaces has *O*(10) left-invariant modes. Tadpole constraints restrict flux quanta on each cycle ≤ O(10).

✤ A sample space of O(10^{10}) solutions, no dS that is tachyon free.

✤ Flux quantization:

Pictorially



For SU(2)xSU(2) examples, can explicitly check flux quantization demands solutions outside SUGRA.

Probability Estimate

- Consider $V(\phi) = \sum_{j=1}^{N} V_j(\phi_j)$
- Then $V_{min}(\phi) = \sum_j V_{j,min}(\phi_j)$. If V_j has n_j minima, then there are $\prod n_j$ classical minima. For $n_j \sim n$, # minima = n^N [Susskind]. This is implicit in BP.
- Say V_j has $2n_j$ extrema, roughly half of which are minima.
- Probability for an extremum to be a minimum is

$$\mathcal{P} = 1/2^N = e^{-N \ln 2}$$

- Still, there are $\mathcal{P}$ x (# extrema) = $e^{N \ln n}$ minima.

Probability for de Sitter Vacua

- We are interested in dS vacua from string theory.
- The various Φ_j interact with each other. It is difficult to estimate how many minima there are.
- Explicit form of V is typically very complicated, e.g., in IIA:

$$V = e^K\left(K^{ij}D_{t^i}W\overline{D_{t^j}W} + K^{K\overline{L}}D_{N^K}W\overline{D_{N^L}W} - 3|W|^2\right) + \frac{1}{2}\left(\mathrm{Re}f\right)^{-1^{\alpha\beta}} D_\alpha D_\beta$$

$$K = -\,2\,\ln\left(-i\int e^{-2\phi}\Omega\wedge\Omega^*\right) - \ln\left(\frac{4}{3}\int J\wedge J\wedge J\right) \qquad J = k^i Y_i^{(2-)}$$

$$\Omega = \mathcal{F}_K Y_K^{(3-)} + i\mathcal{Z}^K Y_K^{(3+)}$$

$$\sqrt{2}W = \int\left(\Omega_c\wedge(-iH + dJ_c) + e^{iJ_c}\wedge\hat{F}\right)$$

$$f_{\alpha\beta} = -\,\hat{\kappa}_{i\alpha\beta}t^i, \qquad \hat{\kappa}_{i\alpha\beta} = \int Y_i^{(2-)}\wedge Y_\alpha^{(2+)}\wedge Y_\beta^{(2+)}, \qquad J_c = J - iB = t^i Y_i^{(2-)}$$

$$D_\alpha = -\,\frac{e^{\phi_4}}{\sqrt{2\mathrm{vol}_6}}\hat{r}_\alpha^K\mathcal{F}_K, \qquad dY_\alpha^{(2+)} = \hat{r}_\alpha{}^K Y_K^{(3+)}. \qquad \Omega_c = e^{-\phi}\mathrm{Im}(\Omega) + iC_3 = N^K Y_K^{(3+)}$$

Stability of Extrema

- The Hessian mass matrix ***H***$= V_{ij}$ at an extremum $V_i = 0$ must be positive definite for (meta)stability.
- We can use Sylvester's criterion to check whether there are tachyons, but time-consuming for a large Hessian ***H*** (c.f. last lecture).
- If the Hessian is large and complicated, how do we estimate the probability of an extremum to be a min.?


Random MATRIX

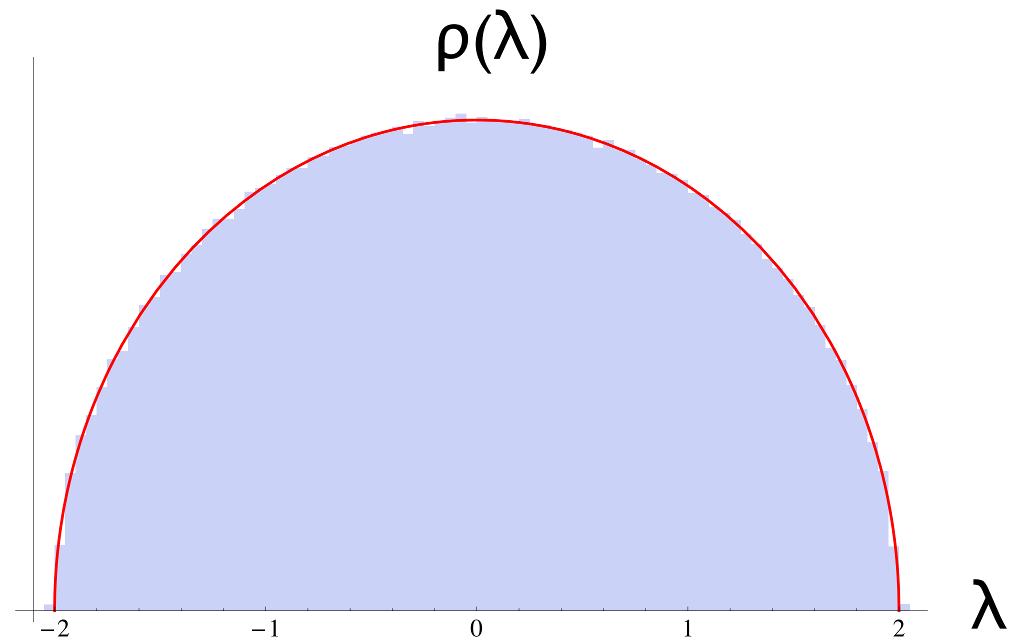
Random Matrix Theory

- A tool to study a large complicated matrix statistically [Wigner, Tracy-Widom, ...]
- Given a random ***H***, the theory of fluctuation of extreme eigenvalues allows one to compute the probability of drawing a *positive definite* matrix from the ensemble.
- Eigenvalue repulsion: probability for ***H*** to have no negative eigenvalue is *Gaussianly suppressed*.
- Some initial foray in applying these RMT results to cosmology was made [Aazami, Easther (2005)].

Wigner Ensemble

$$M = A + A^{\dagger}$$

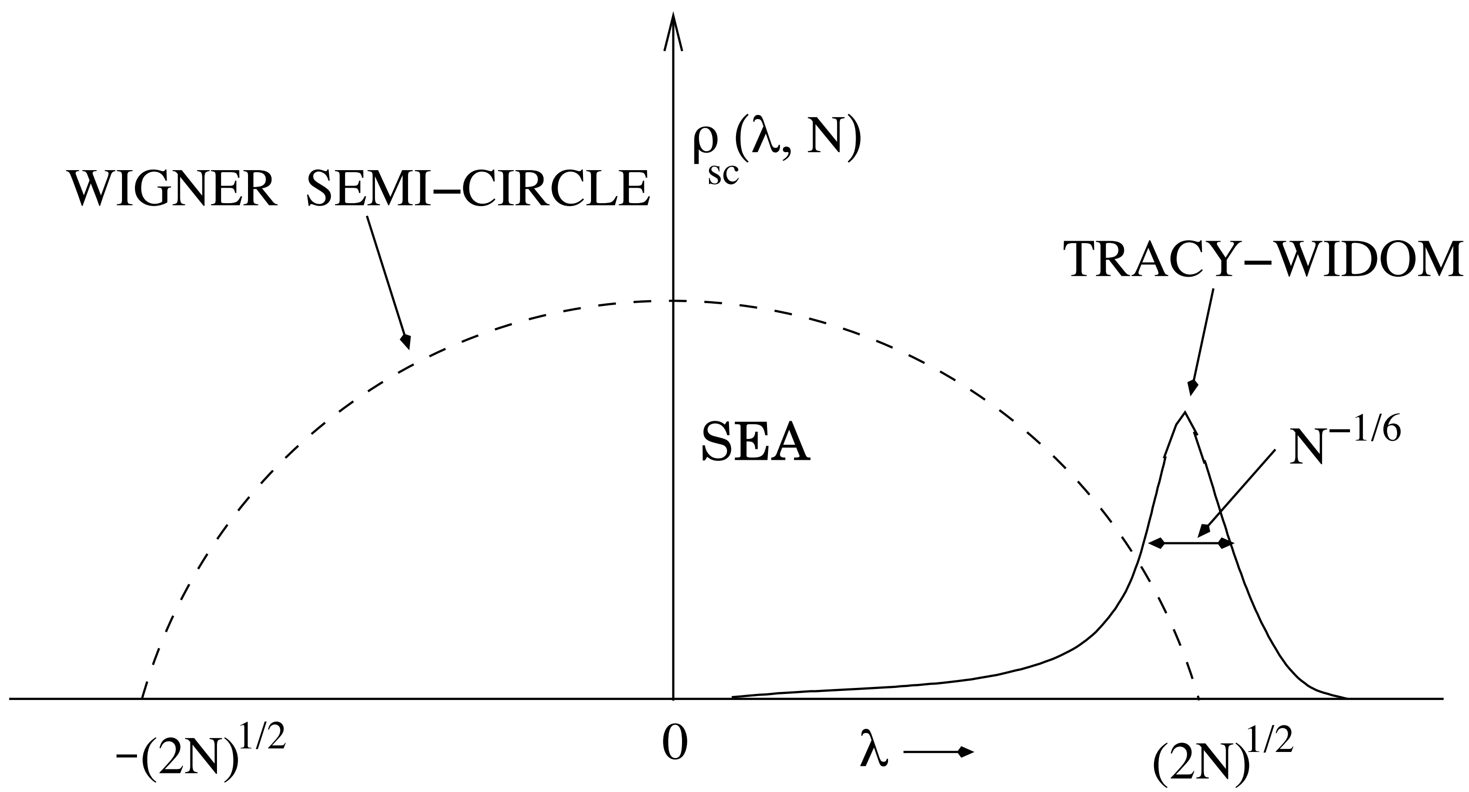
Dyson



Wigner's semi-circle

Elements of A are independent identically distributed variables drawn from some statistical distribution.

Tracy-Widom & Beyond



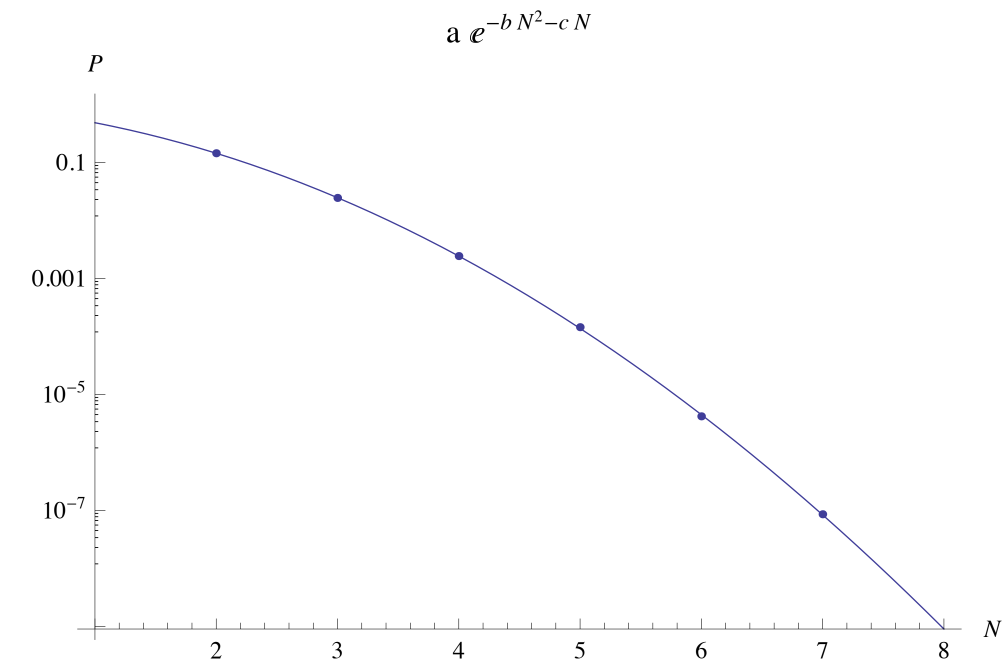
Study of the fluctuations of the smallest (largest) eigenvalue was initiated by Tracy-Widom, and generalized to large fluctuations by Dean and Majumdar (cond-mat/0609651).

Probability of Stability

Consider a Gaussian orthogonal ensemble

Probability of the form:

$$\mathcal{P} = a\, e^{-bN^2 - cN}$$ [Chen, GS, Sumitomo, Tye]



seems to work well, and agrees with:

The large N analytic result of Dean & Mujumdar $\mathcal{P} \approx e^{-\frac{\ln 3}{4} N^2}$
and further refinement by Borot et al:

$$\mathcal{P} = \exp\left[-\frac{\ln 3}{4} N^2 + \frac{\ln(2\sqrt{3} - 3)}{2} N - \frac{1}{24} \ln N - 0.0172 \right]$$

If the probability is Gaussianly suppressed, while # extrema goes like e^{cN} (recall 10^{500}), unlikely to find metastable vacua.

Random Supergravities

- Consider the SUGRA potential:

$$V = e^K \left(D^A W D_A W - 3|W|^2 \right)$$

  and its Hessian, which is a function of $D_A W$, $D_A D_B W$, and $D_A D_B D_C W$, as well as W.

- Instead of randomizing elements of ***H***, one can randomize K, W, and its covariant derivatives [Denef, Douglas];[Marsh, McAllister, Wrase]

- This approach is applicable to F-term breaking, but not to D-term breaking, and models with explicit SUSY breaking.

- Also a different ansatz $\mathcal{P} = ae^{-bN^c}$ was used. Quantitative details differ, but $\mathcal{P}$ less likely than exponential also found.

Random Supergravities

The Hessian is well approximated by a sum of a Wigner matrix and two Wishart matrices.

$$M = A + A^\dagger \qquad\qquad M = AA^\dagger$$

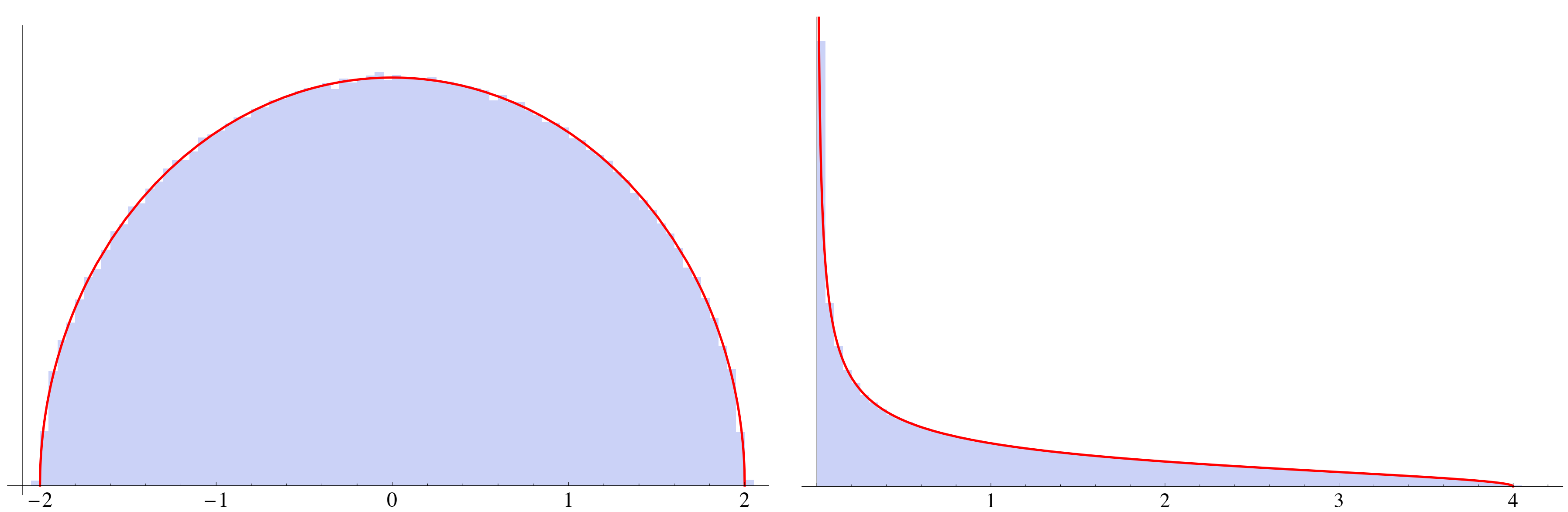


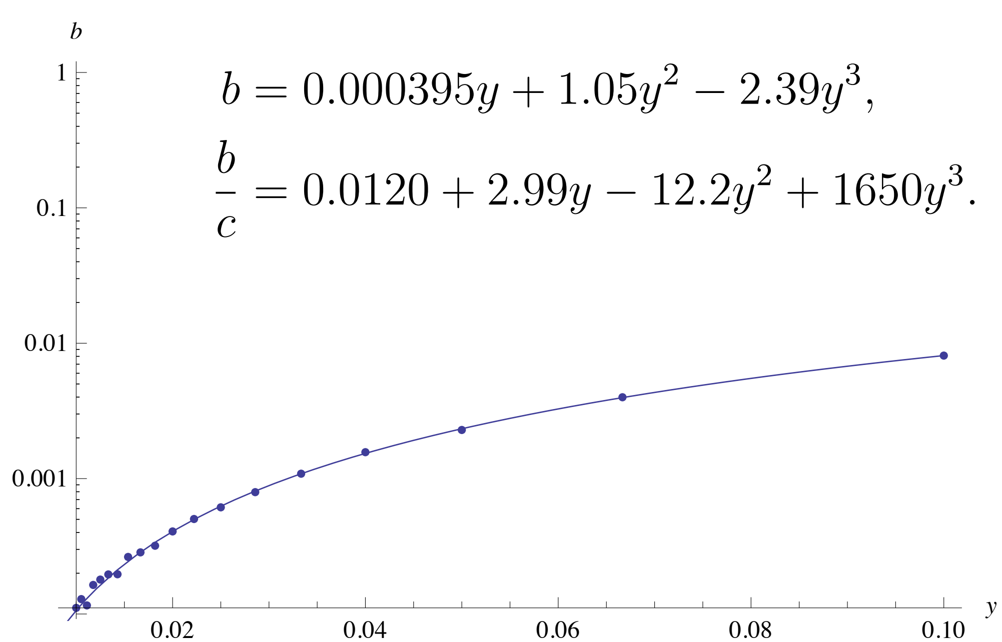
Figure 1: The eigenvalue spectra for the Wigner ensemble (left panel), and the Wishart ensemble with $N = Q$ (right panel), from 10^3 trials with $N = 200$.

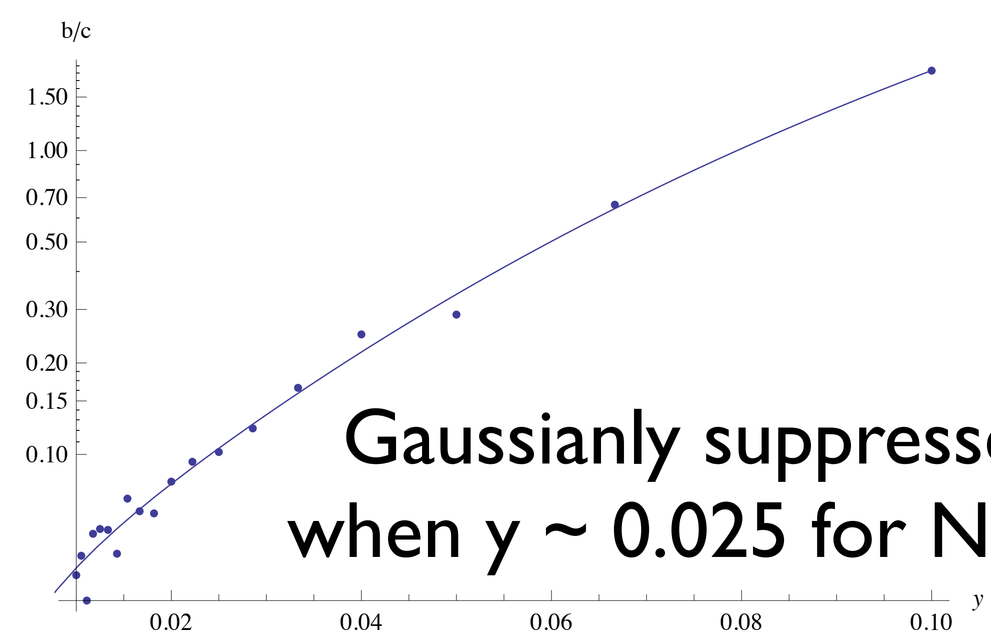
IIA Flux Vacua

- An infinite family of AdS vacua are known to arise from flux compactifications of IIA SUGRA [Derendinger et al; Villadoro et al; De Wolfe et al; Camara et al].
- Attempts to construct IIA dS flux vacua often start with similar setups as SUSY AdS ones and then introduce new ingredients to uplift (e.g., negative curvature of internal space).
- We can model the Hessian as ***H*** = A + B where A= diagonal mass matrix at AdS min., B is uplift contribution.
- A does not have to be positive definite for stability, as long as the BF bound is satisfied. To play it safe, we start with a SUSY AdS vacuum with A=*positive definite* diagonal matrix.

IIA Flux Vacua

- We take B to be a randomized real symmetric matrix.
- A and B have variances σ_A and σ_B. The relative ratio $y = \sigma_B/\sigma_A$ determines the amount of uplift.
- The ansatz $\mathcal{P} = a\, e^{-bN^2 - cN}$ works well when the mass matrix is not completely random, but has a hierarchy:





Gaussianly suppressed when y ~ 0.025 for N=10

Chen, GS, Sumitomo, Tye

A Type IIA Example

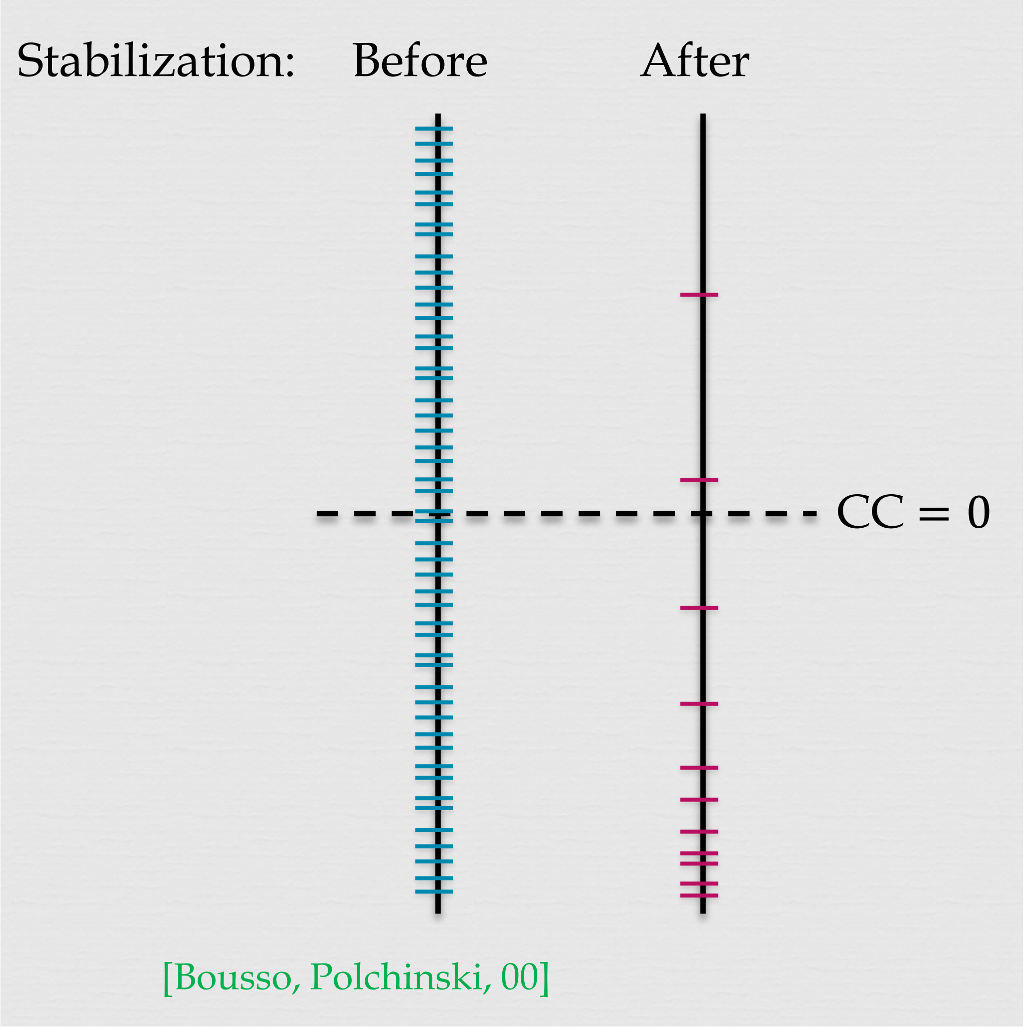
- Return to the SU(2)xSU(2) group manifold studied earlier in the systematic search of [Danielsson, Haque, Koerber, GS, Van Riet, Wrase]
- This model evades the no-goes for dS extrema and stability in the universal moduli subspace. There are 14 moduli.
- Evaluating the variance: $y \sim \left(\frac{\frac{1}{14\times 13/2} \sum_{A<B} M_{AB}^2}{\frac{1}{14} \sum_{A=1}^{14} M_{AA}^2} \right)^{1/2} = 0.274.$ >> 0.025.
- There is no surprise that tachyon appears.
- Tachyon appears in a 3x3 sub-Hessian. Chen, GS, Sumitomo, Tye
- In this model, η = V''/V ≲ -2.4 at the extremum, so the tachyon becomes more tachyonic as the CC increases.

CC and Stability

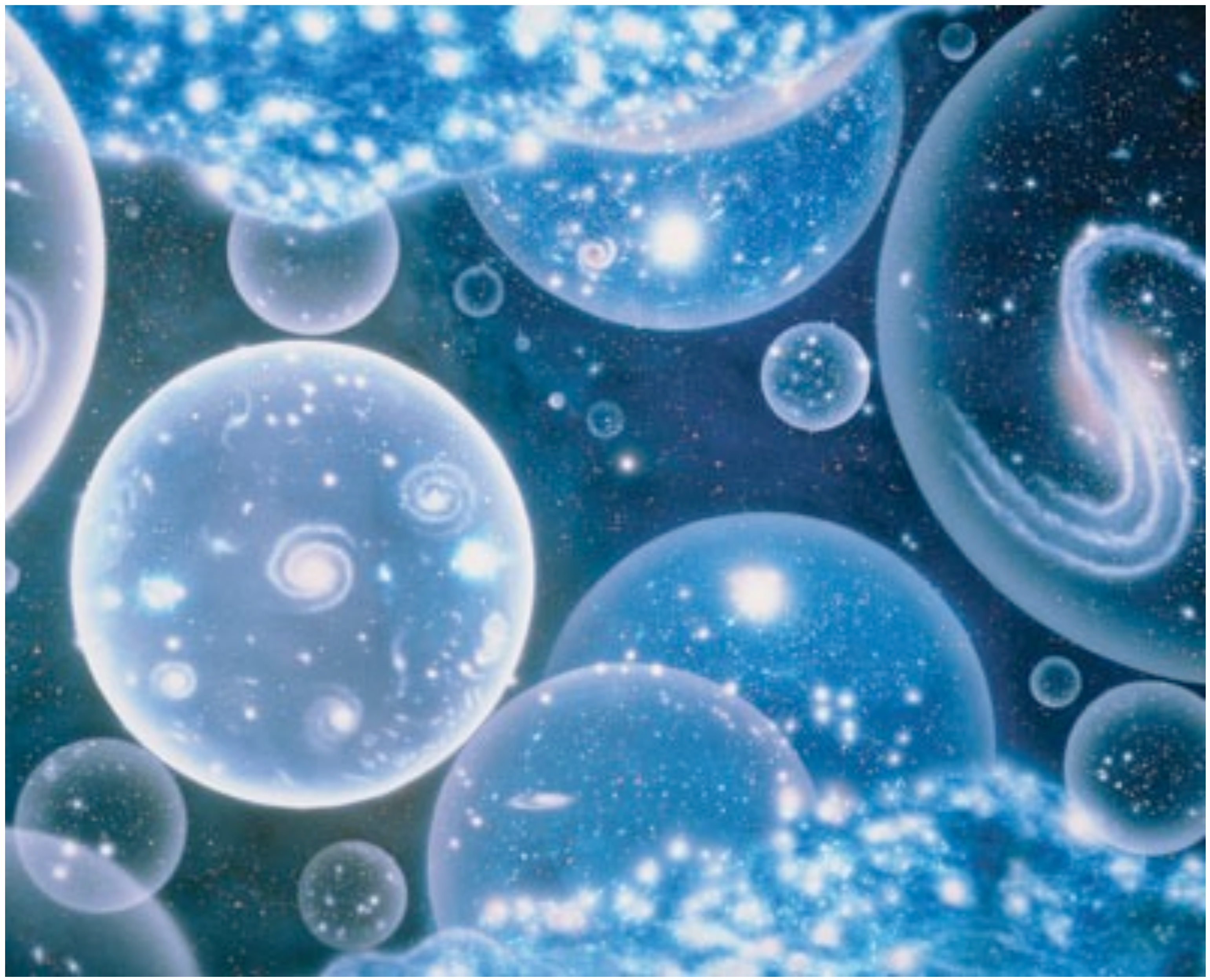
- As we lift the CC, the off-diagonal terms become bigger and the extremum becomes unstable.
- In general, we expect some moduli to be very heavy and essentially decouple from the light sector, so $N = N_H + N_L$.
- The # of extrema is controlled by N, while the fraction of stable critical points is controlled by N_L.
- Example: a 2-sector SUGRA where some moduli have very large SUSY masses while SUSY is broken in a decoupled sector involving only the light moduli.
- As we go to higher energies, more moduli come into play (larger eff. N) ⇨ probability more Gaussianly suppressed.

Less Democratic Landscape

Raising the CC destabilizes the classically stable vacua.



Implications to the Landscape?



Detectable Primordial Gravity Waves without Large Field Inflation

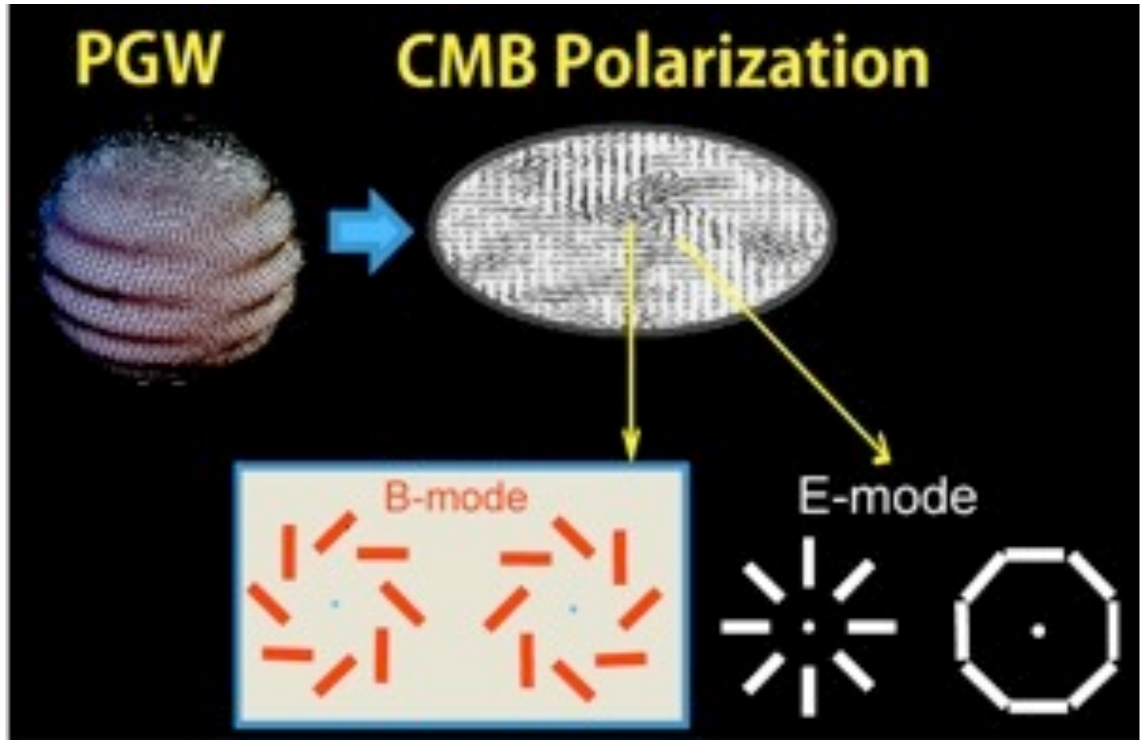
[when having tachyons is a good thing]

Gravitational Waves

- A consequence of General Relativity, predicted by Einstein in 1916.
- Remains a Holy Grail of Observational Cosmology.
- Indirect evidence: e.g., Hulse-Taylor binary.
- Direct detection has so far come up empty. Present and future interferometers include LIGO, VIRGO, Einstein Telescope, LISA, TAMA, KAGRA, Decigo, ...

Primordial Gravity Waves

- Besides astrophysical sources of GW, even more interesting are perhaps "echoes" from the Big Bang.
- Spacetime metric undergoes quantum fluctuations during inflation leading to production of GW ➪ B-mode polarization of CMB.



Tensor Modes & Large Field Inflation

- Tensor to scale ratio: $r=P_T/P_S$

$$P_T \sim \frac{H^2}{M_P^2}\,, \quad P_S \sim H^2 \left(\frac{H}{\dot{\phi}}\right)^2 \qquad \Rightarrow r = 16\epsilon$$

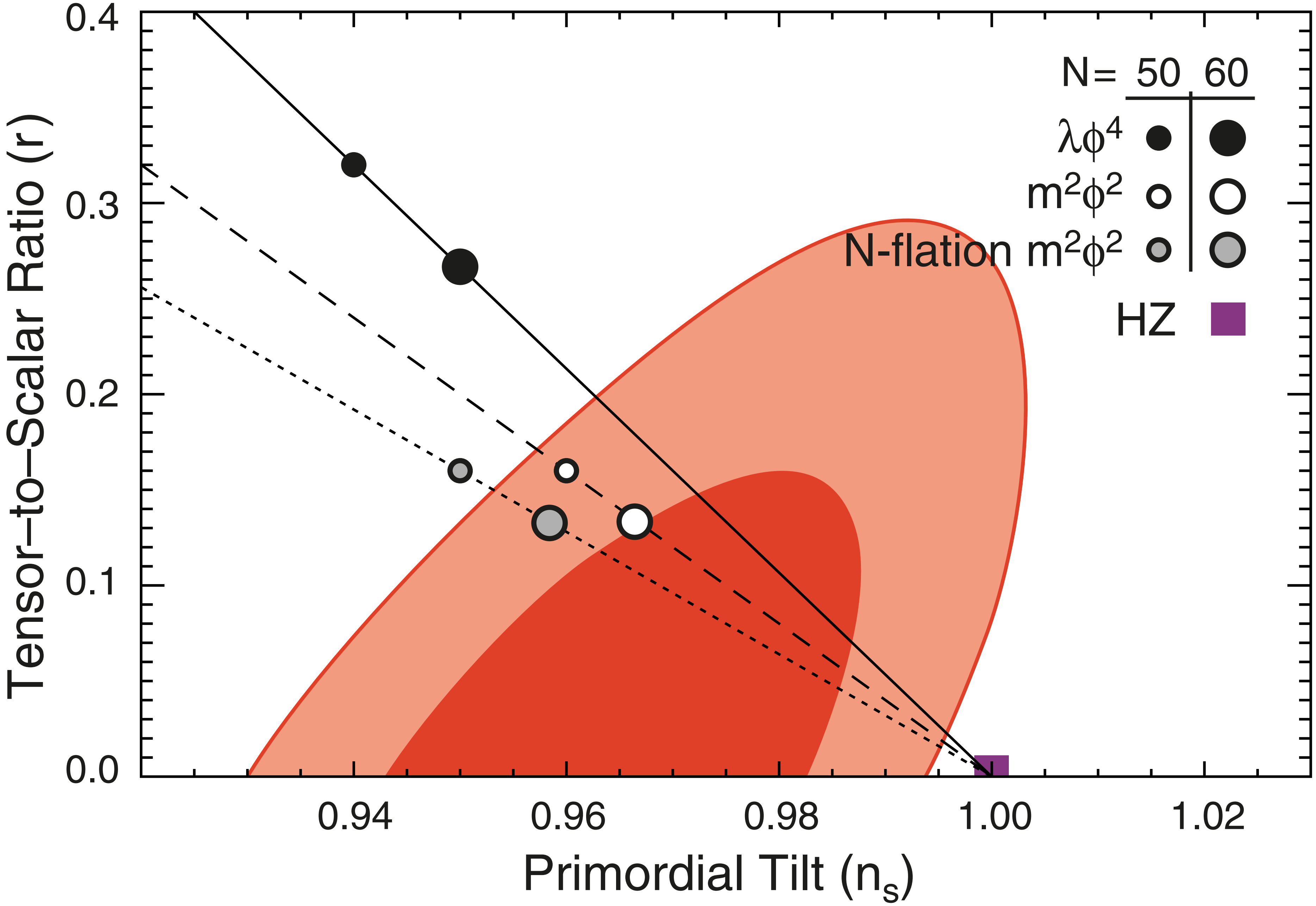
- Current bound (WMAP+SPT+H_0+BAO): r<0.17, while r≃0.01 may be detectable in future missions.

- Lyth Bound: $$\frac{\Delta\phi}{M_P} = \frac{1}{\sqrt{8}}\int_0^{\mathcal{N}_{\rm end}} d\mathcal{N}\; r^{1/2} \gtrsim 1.06 \times \left(\frac{r_*}{0.01}\right)^{1/2}$$

- Detectable tensors ⇔ *Large* field inflation

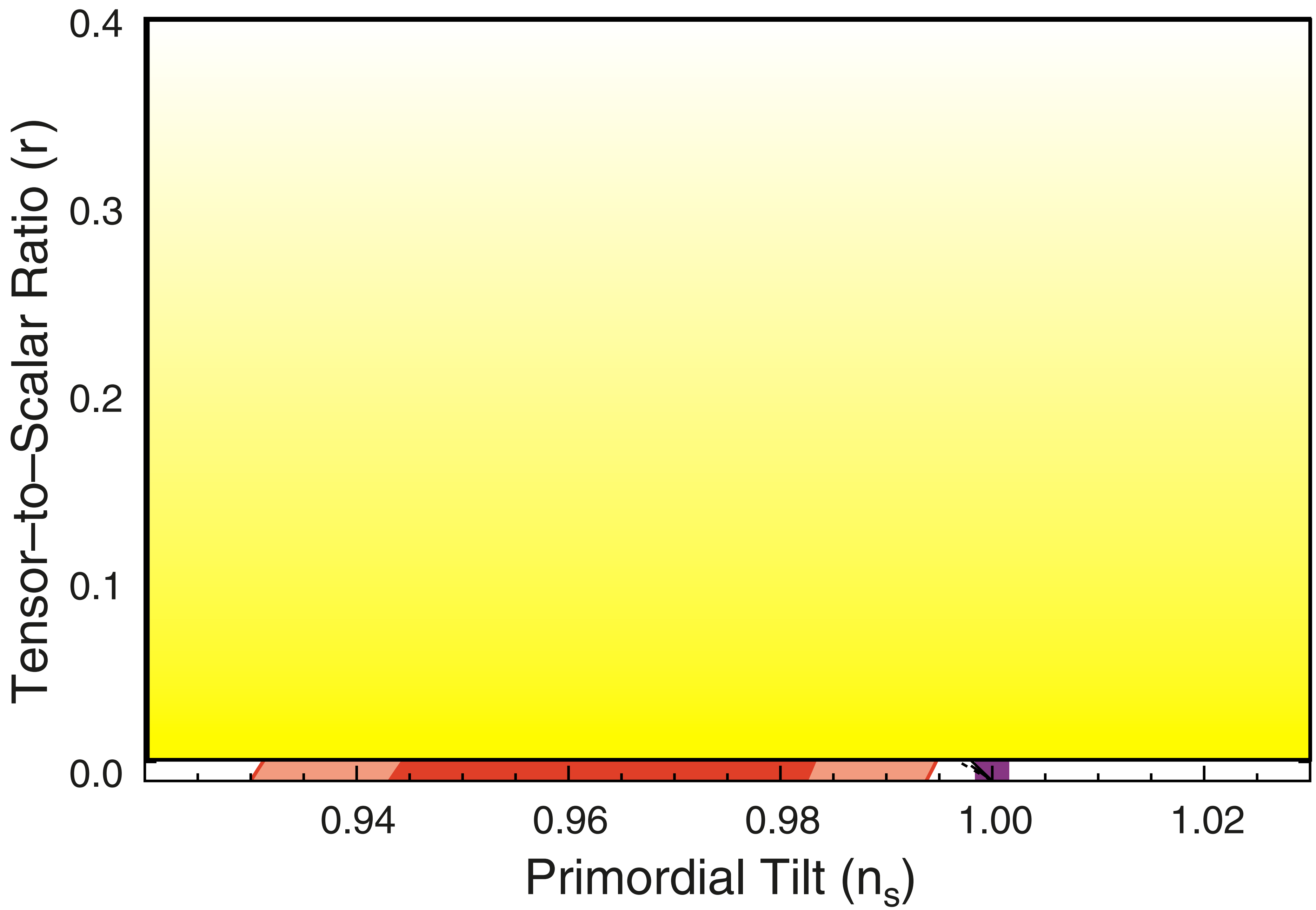
- Need to control an *infinite* set of operators in the EFT of inflation; a UV completion is necessary.

Tensor Modes



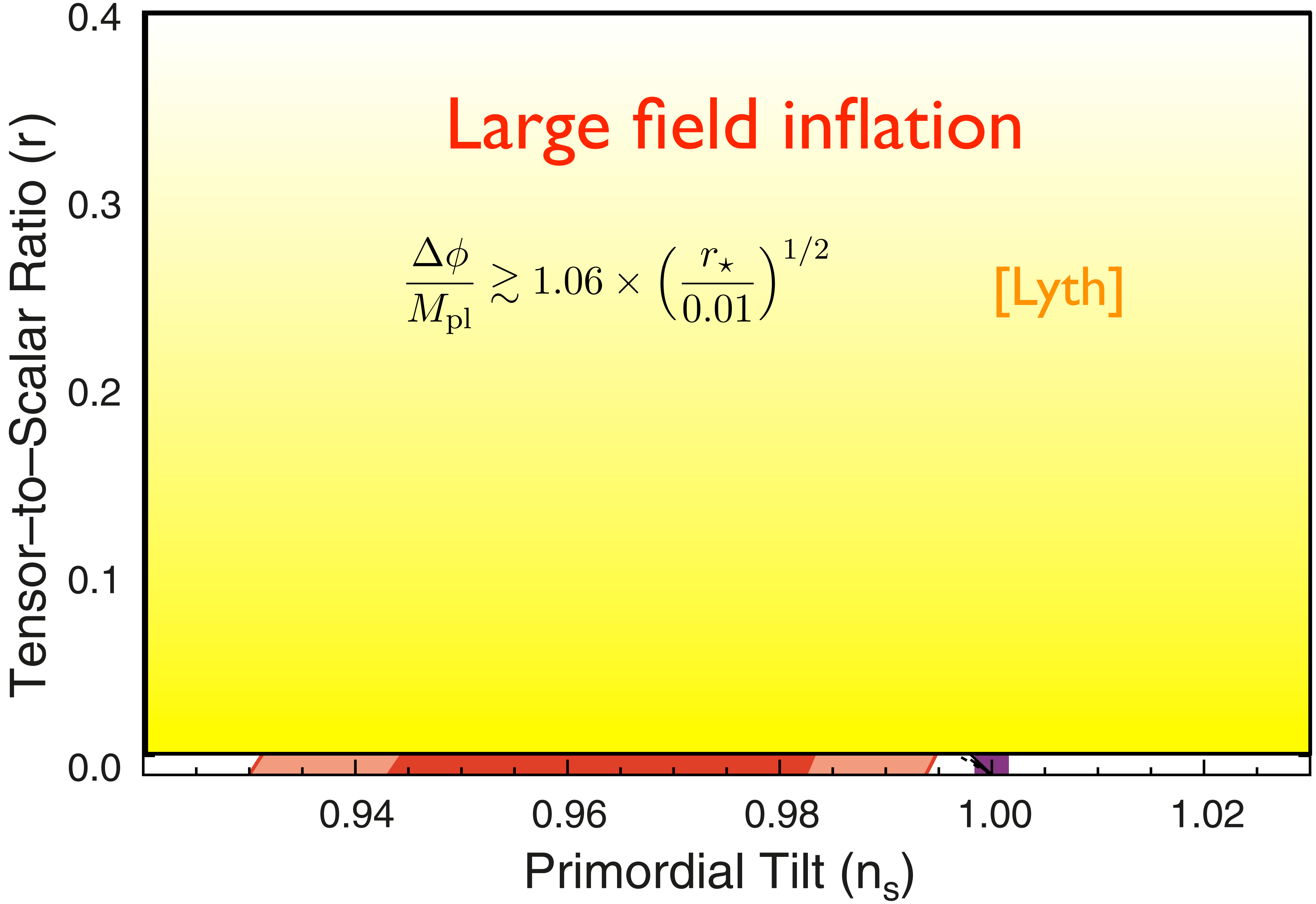
WMAP7

Tensor Modes



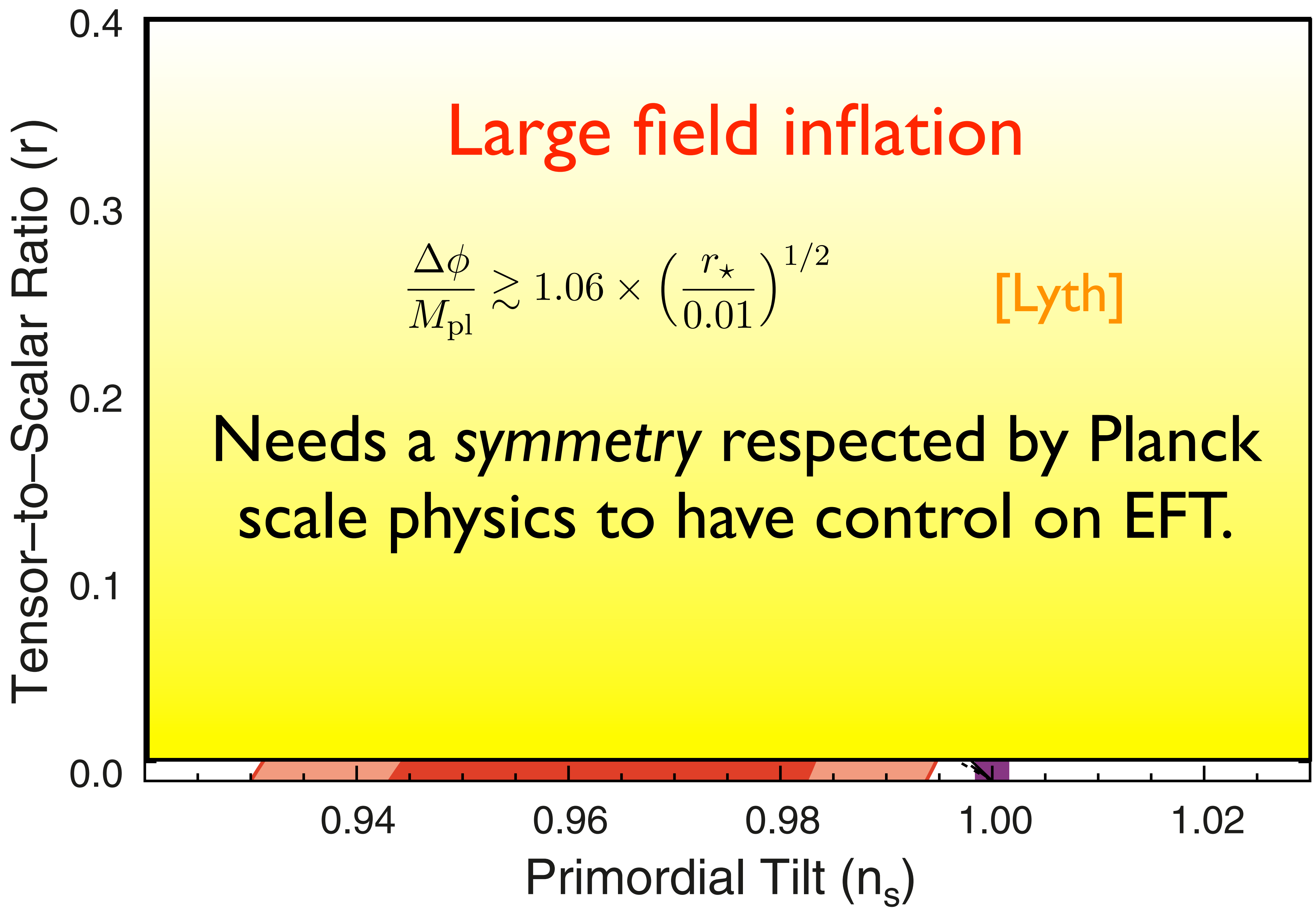
WMAP7

Tensor Modes



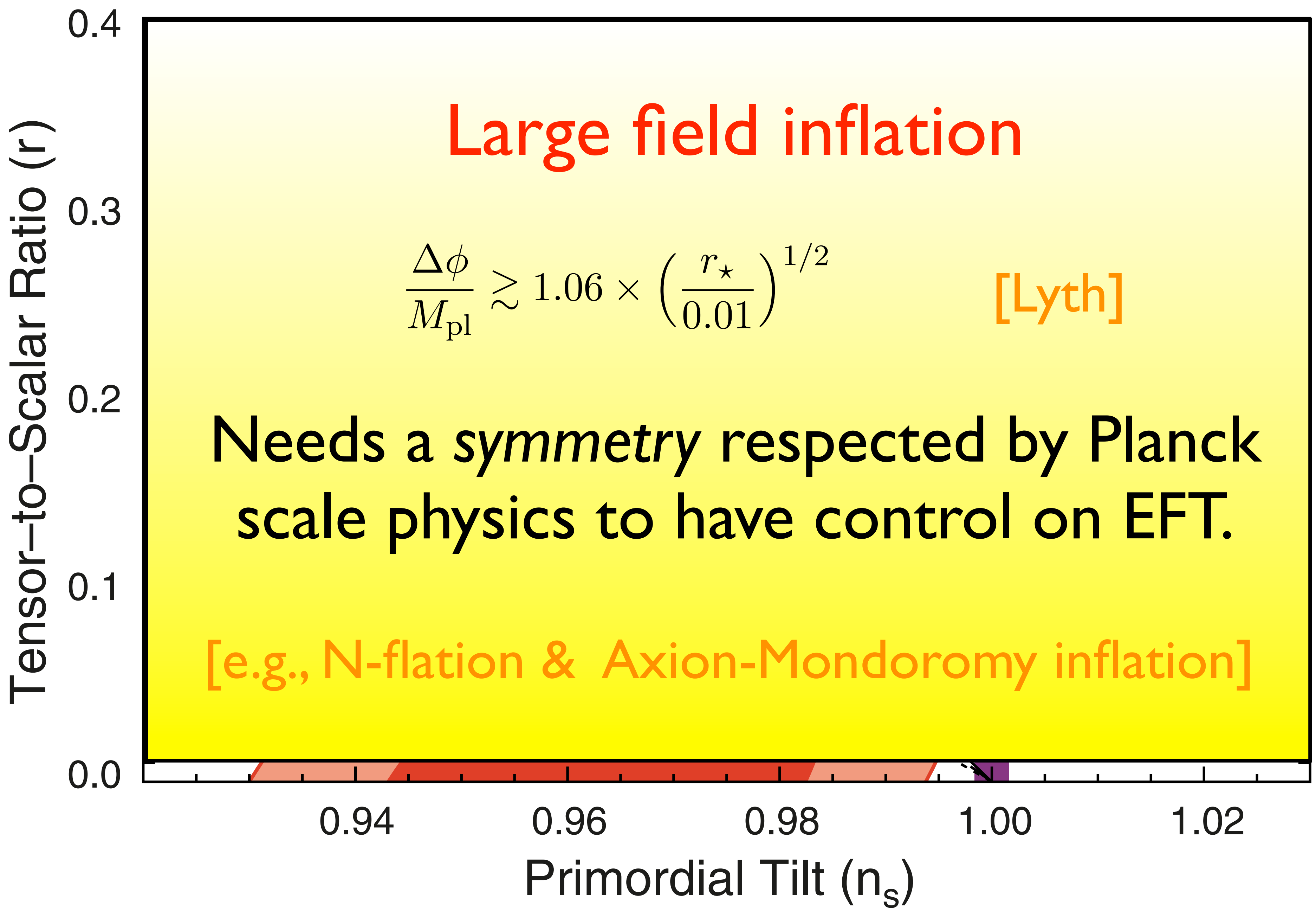
WMAP7

Tensor Modes



WMAP7

Tensor Modes



WMAP7

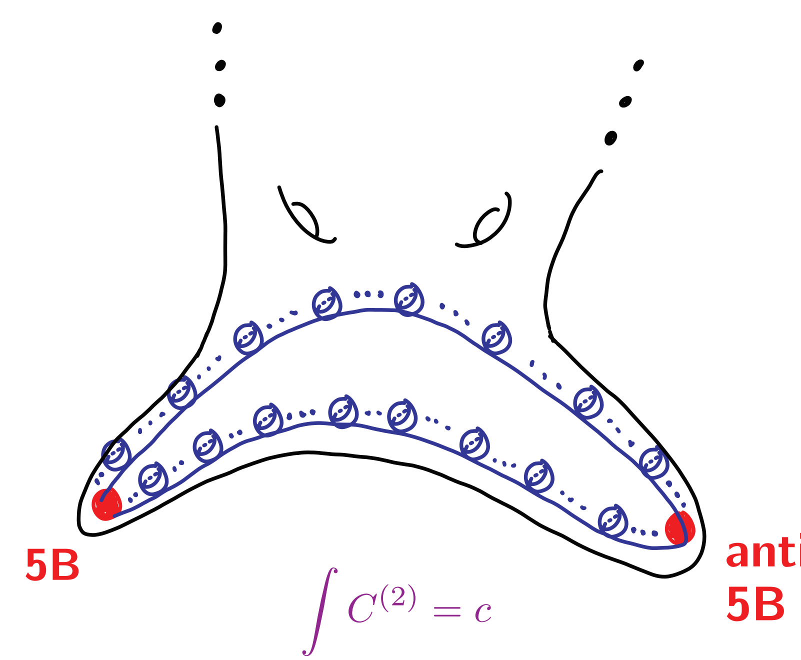
Challenges for Large Field Inflation

- N-flation [Dimopoulos, Kachru, McGreevy, Wacker]

$$V(\{\phi_n\}) = \sum_{n=1}^{N} V_n(\phi_n) = \sum_{n=1}^{N} \Lambda_n^4 \cos\left(\frac{2\pi\phi_n}{f_n}\right)$$

assisted inflation with N~O(500) axions; backreaction on M_P.

- Axion-monodromy inflation [McAllister, Silverstein, Westphal]



- Issues on backreaction and entropy bound [Conlon]
- Seems to be statistically disfavored [Westphal]

Summary of Our Work

- Tachyonic production of gauge fields induced by axion couplings during (even small field) inflation can lead to detectable tensors [Barnaby, Moxon, Namba, Peloso, GS, Zhou]
- Our scenario may find a natural home in string theory.
- Studied also scalar, fermion, vector particle production during inflation due to a non-adiabatic change of mass.
- Particle production sources not only GW but scalar spectrum; only axion model leads to detectable tensors while consistent with constraints on scalar spectrum.

Vector Production by Axion

- A simple model (φ=inflaton, ψ=axion):

$$S=\int d^4x\sqrt{-g}\left[\,\frac{M_p^2}{2}R-\underbrace{\frac{1}{2}(\partial\varphi)^2-V(\varphi)}_{\text{inflaton sector}}-\underbrace{\frac{1}{2}(\partial\psi)^2-U(\psi)-\frac{1}{4}F^2-\frac{\psi}{4f}F\tilde{F}}_{\text{hidden sector}}\,\right]$$

- Time dependence of axion leads to particle production:

$$\left[\partial_\tau^2+k^2\pm\frac{2k\xi}{\tau}\right]A_\pm(\tau,k)=0\,,\qquad \xi\equiv\frac{\dot{\psi}^{(0)}}{2Hf}\,.$$

- One helicity (with tachyonic mass) gets copiously produced:

$$A_+(\tau,k)\approx\left(\frac{-\tau}{8\xi k}\right)^{1/4}e^{\pi\xi-\sqrt{-2\xi k\tau}}\,,\qquad A_+'(\tau,k)\approx\left(\frac{2\xi k}{-\tau}\right)^{1/2}A_+(\tau,k)$$

- Vector particles produced source GW & scalar spectrum.

Background Dynamics

- Energy density of gauge fields & axion must be subdominant:

$$\frac{1}{2}\langle \vec{E}^2 + \vec{B}^2 \rangle \ll \frac{\dot{\psi}^{(0)2}}{2}$$

$$\frac{1}{2}\left(\dot{\psi}^{(0)}\right)^2 + U\left(\psi^{(0)}\right) \ll 3H^2 M_p^2$$

which can be satisfied if the axion decay constant:

$$0.074\frac{\sqrt{\epsilon \mathcal{P}} e^{\pi\xi}}{\xi^{5/2}} \ll \frac{f}{M_p} \ll \frac{1.2}{\xi}\sqrt{1 - \frac{U\left(\psi\right)}{V\left(\varphi\right)}}$$

- The parameter ξ is adiabatically evolving if

$$\frac{\ddot{\psi}^{(0)}}{H\,\dot{\psi}^{(0)}} \ll 1 \quad \Leftrightarrow \quad m_\psi \ll \frac{3H}{2}$$

Scalar Spectrum

- Particle production contribute additionally to power spectrum:

$$P_{\zeta,s}(k) = \frac{\xi e^{4\pi\xi} H^4}{128\pi^2 M_p^4} \int \frac{d^3q}{(2\pi)^3} q^{1/2} |\hat{k} - \vec{q}|^{1/2} \left[1 - (q - |\hat{k} - \vec{q}|)^2\right]^2 \left[1 - \frac{\vec{q} \cdot (\hat{k} - \vec{q})}{q|\hat{k} - \vec{q}|}\right]^2 \mathcal{I}^2 \left[q, |\hat{k} - \vec{q}|\right] ,$$

where $\vec{q} \equiv \vec{p}/k, \hat{k} \equiv \vec{k}/k$ and

$$\mathcal{I}[a,b] \equiv \int_{-k\tau}^{\infty} dz \, \frac{\sin z - z \cos z}{z^{1/2}} \, e^{-2\sqrt{2\xi z}\left[\sqrt{a}+\sqrt{b}\right]} \qquad z \equiv -k\tau'$$

- Super-horizon regime: -kτ<<1, and for ξ≳O(1),

$$\mathcal{I}[a,b] \approx \int_0^{\infty} dz \, \frac{z^{5/2}}{3} \, e^{-2\sqrt{2\xi z}\left[\sqrt{a}+\sqrt{b}\right]} = \frac{15}{32\sqrt{2}\left(\sqrt{a}+\sqrt{b}\right)^7 \xi^{7/2}}$$

- Numerically evaluating integral gives: $P_{\zeta,s}(k) \approx 4 \cdot 10^{-10} \frac{H^4}{M_p^4} \frac{e^{4\pi\xi}}{\xi^6}$

GW and Scalar Spectrum

- Power Spectrum: $$P_\zeta \approx \mathcal{P}\left[1+2.5\cdot 10^{-6}\,\epsilon^2\mathcal{P}\frac{e^{4\pi\xi}}{\xi^6}\right]\,, \qquad \mathcal{P}\equiv\frac{H^2}{8\pi^2\epsilon M_p^2}$$

  ☞ Particle production contribution is subdominant.

- Gravity Wave: $$\begin{aligned} P_+ &= P_{+,\mathrm{v}}+P_{+,\mathrm{s}} \simeq \frac{H^2}{\pi^2 M_p^2}\left[1+8.6\cdot 10^{-7}\,\frac{H^2}{M_p^2}\frac{e^{4\pi\xi}}{\xi^6}\right] \\ P_- &= P_{-,\mathrm{v}}+P_{-,\mathrm{s}} \simeq \frac{H^2}{\pi^2 M_p^2}\left[1+1.8\cdot 10^{-9}\,\frac{H^2}{M_p^2}\frac{e^{4\pi\xi}}{\xi^6}\right] \end{aligned}$$

- Standard "consistency condition" is violated

$$r\equiv\frac{\sum_\lambda P_\lambda}{P_\zeta}\approx 16\epsilon\frac{1+3.4\cdot 10^{-5}\epsilon\mathcal{P}\frac{e^{4\pi\xi}}{\xi^6}}{1+2.5\cdot 10^{-6}\epsilon^2\mathcal{P}\frac{e^{4\pi\xi}}{\xi^6}}$$

- r interpolates between 16ε and 218, observable signal can be obtained for any ε in this model.

Non-Gaussianity

- Non-Gaussianity (bispectrum) is peaked at equilateral limit

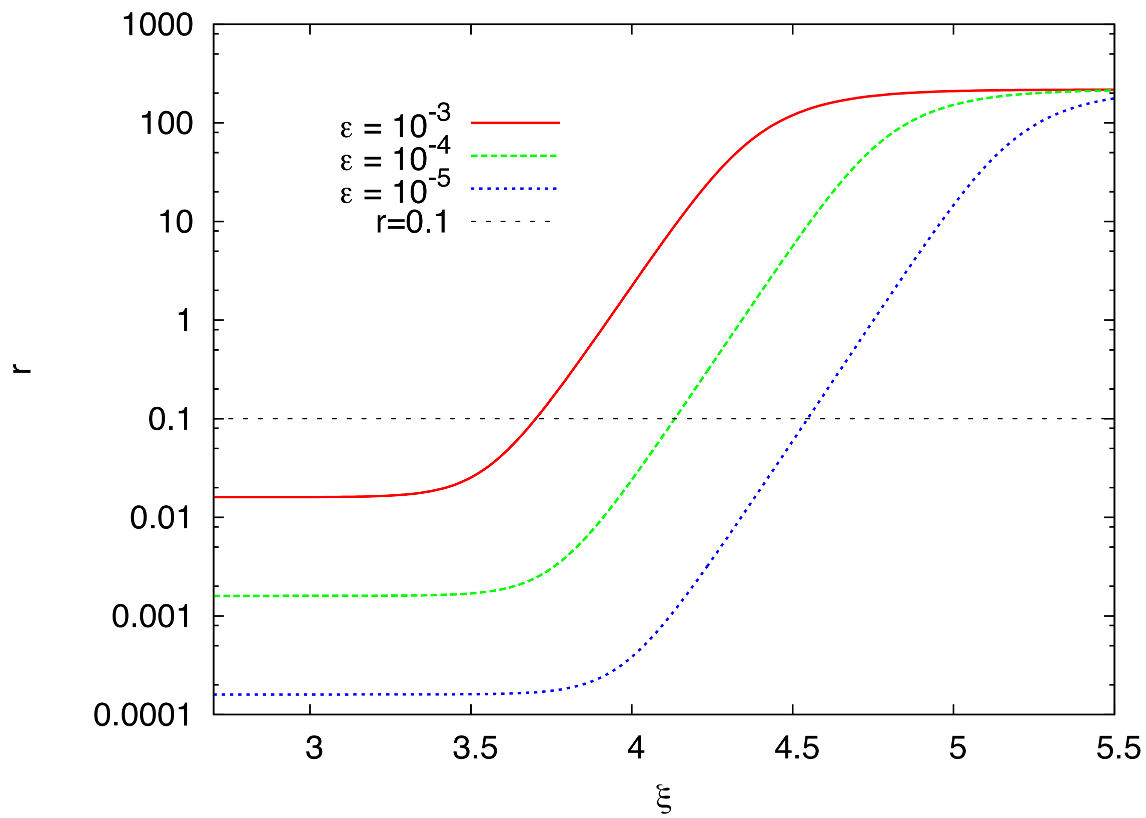
$$\left\langle \zeta\left(\vec{k}_1\right)\zeta\left(\vec{k}_2\right)\zeta\left(\vec{k}_3\right)\right\rangle = B_\zeta\left(\vec{k}_i\right)\,\delta^{(3)}\left(\vec{k}_1+\vec{k}_2+\vec{k}_3\right) \qquad B_\zeta\left(k_1=k_2=k_3\equiv k\right)\approx 2.6\cdot 10^{-13}\,\frac{H^6}{M_p^6}\,\frac{e^{6\pi\xi}}{\xi^9}\,\frac{1}{k^6}$$

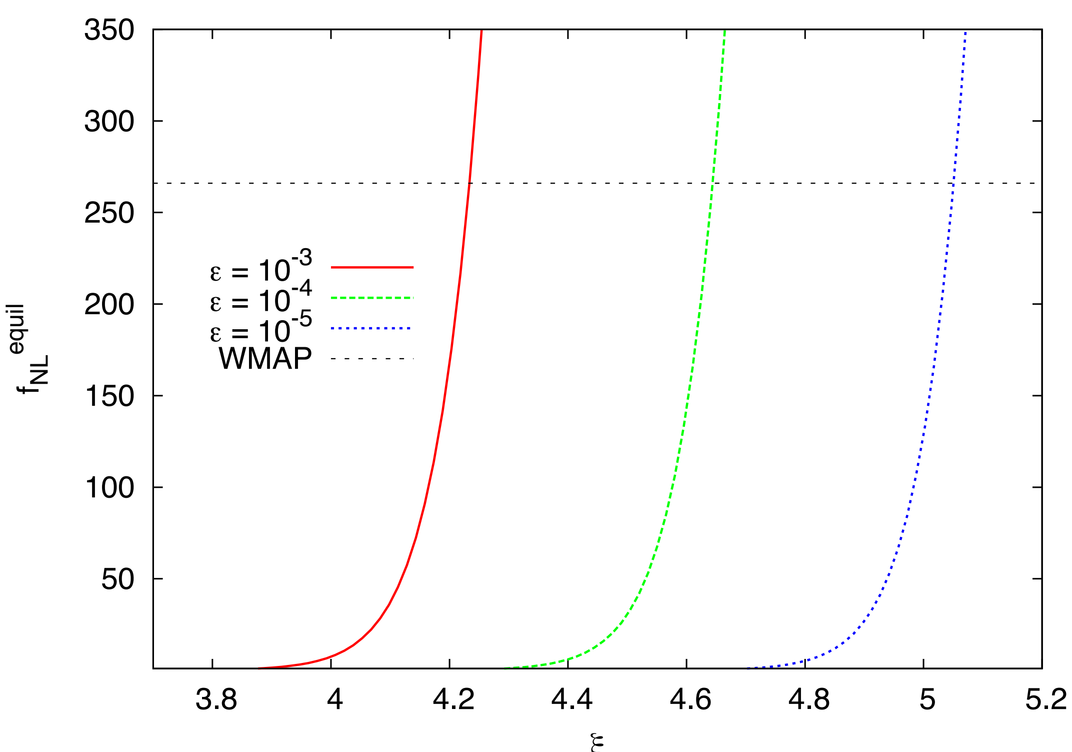
because the source at any moment is dominated by modes with wavelength comparable to horizon at that moment.

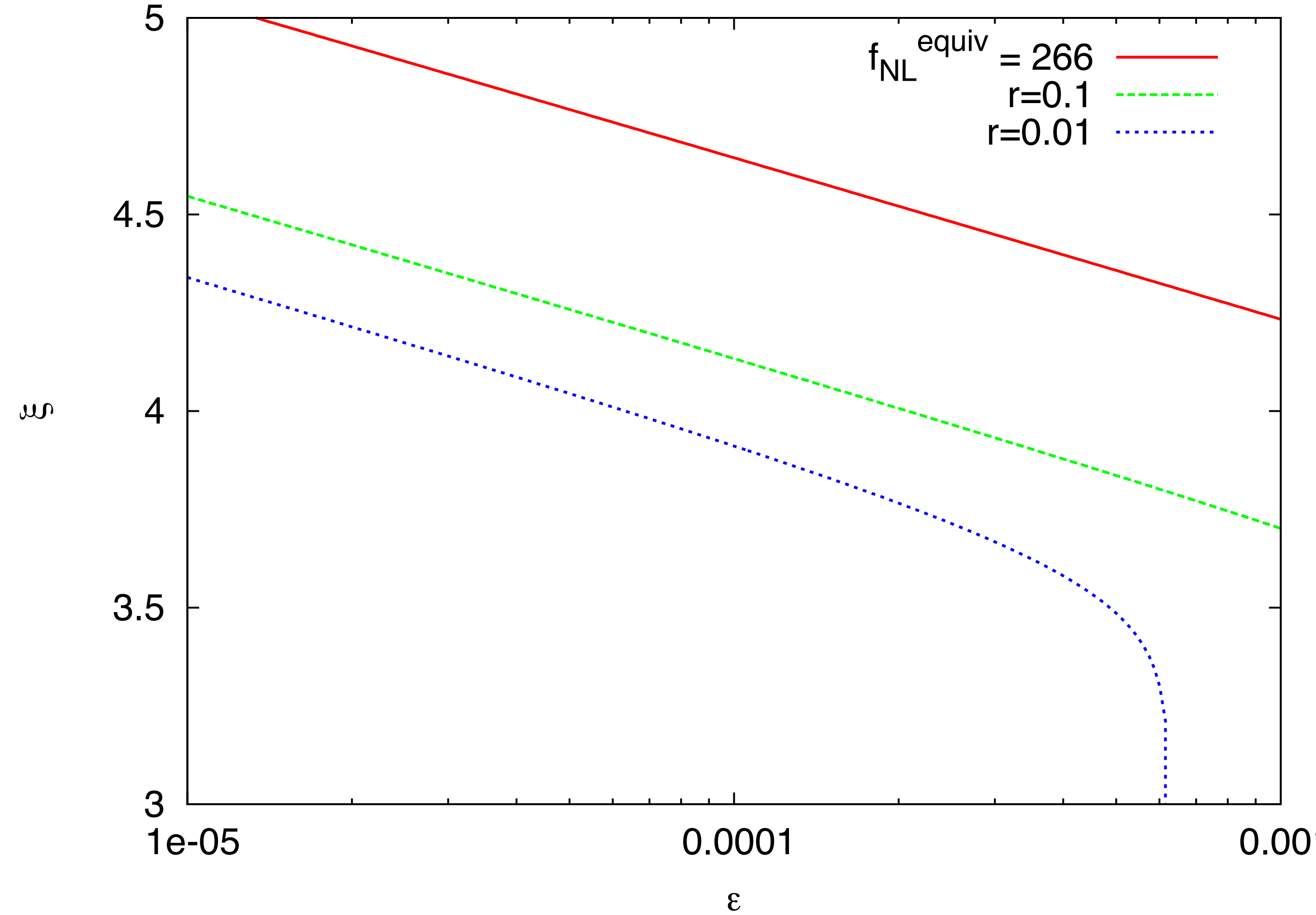
- To quantify the size of non-Gaussianity: $f_{NL}^{\mathrm{equil.eff}} \approx 1.5\cdot 10^{-9}\epsilon^3\,\frac{\mathcal{P}^3}{P_\zeta^2}\,\frac{e^{6\pi\xi}}{\xi^9}$

- Current bound: $-214 < f_{NL}^{\mathrm{equil}} < 266$

Signatures & Constraints





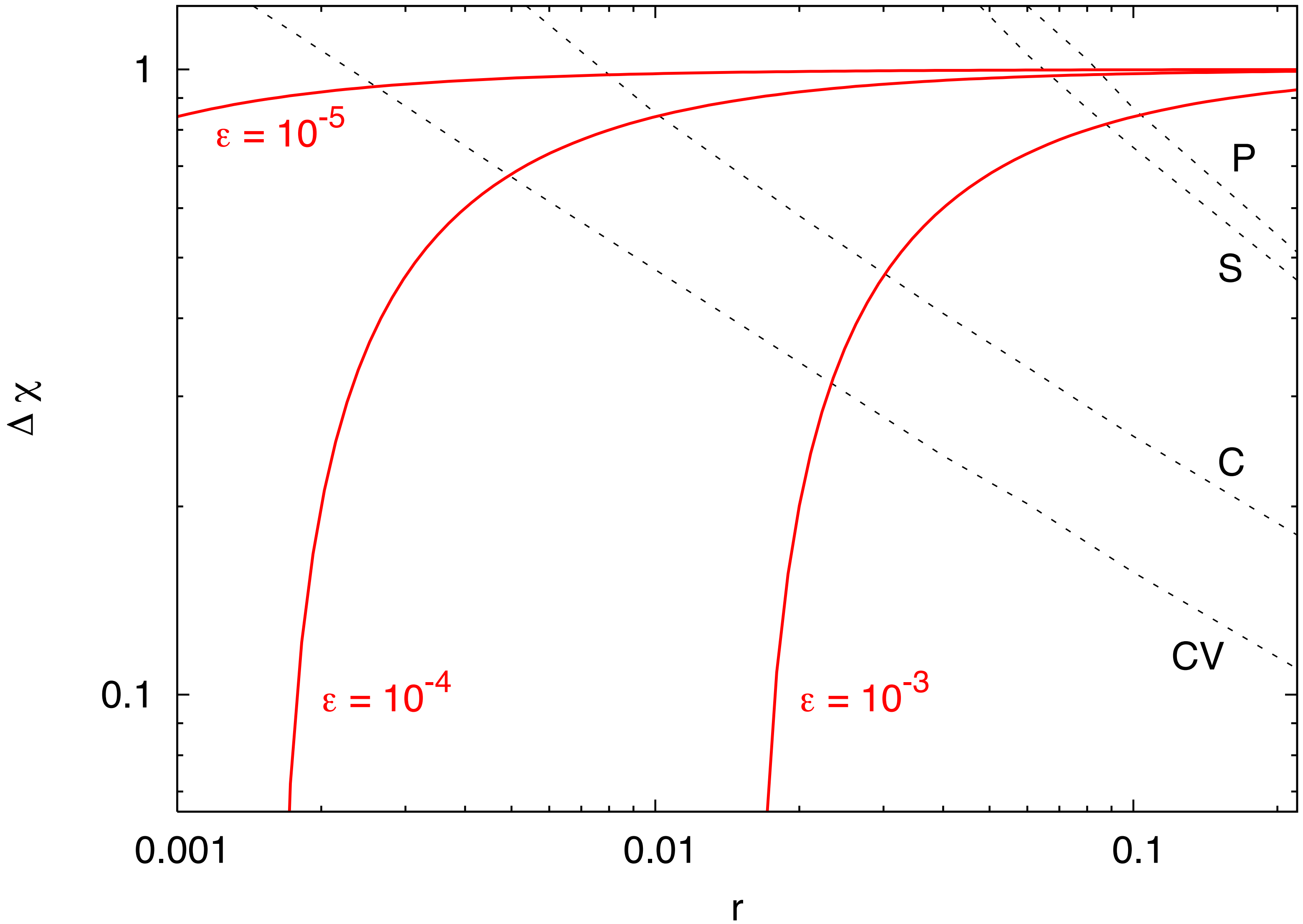


For detectable tensor, consistency of model requires:

$$10^{-4} << \frac{f}{M_P} \lesssim 10^{-2}$$

Parity Violating Effects in GW

$$|\Delta\chi| \equiv \left|\frac{P_+ - P_-}{P_+ + P_-}\right| = \frac{3.4 \cdot 10^{-5} \epsilon \mathcal{P} \frac{e^{4\pi\xi}}{\xi^6}}{1 + 3.4 \cdot 10^{-5} \epsilon \mathcal{P} \frac{e^{4\pi\xi}}{\xi^6}} \simeq 1 - \frac{16\,\epsilon}{r}$$



P=Planck, S=SPIDER, C=CMB-Pol, CV=cosmic-variance limited expt.

Comments

- Many axion-like fields in string theory, e.g.,

$$a = C_0, \quad a_i = \int_{\Sigma_2^i} C_2, \quad a_I = \int_{\Sigma_4^I} C_4$$

- Typically f ~ M_{GUT}, sits comfortably in our allowed window.
- Pseudoscalar coupling comes from, e.g., $\int_{D5} C_2 \wedge F \wedge \tilde{F}$
- ∃ inflation candidates that couple only gravitationally to *hidden sector*, e.g., another axion with:

$$\int \omega_a \wedge * \omega_b = 0 \ , \quad \int \omega_a \wedge \tilde{\omega_b} = 0$$

$\omega_{a,b}$= p-form associated with each axion

or D-brane moduli that do not couple to the axion and the D-brane on which the U(1) lives.

Comments

- Detection of GW probes not the scale of inflation, but the nature and dynamics of axions.
- GW sourced by scalar particle production during inflation has recently been explored ([Cook, Sorbo];[Senatore, Silverstein, Zaldarriaga]), though constraints from backreaction on scalar spectrum and bispectrum were not known. Thus, effects on LIGO scales were instead considered.
- We computed the backreaction due to particle production on the power spectrum and bispectrum of such models.

Comments

- We derived a *universal formula* for GW sourced by scalar, fermion, vector production due to non-adiabatic change of mass:

$$P_{\lambda,\mathrm{s}} \simeq \frac{2\, g_s\, k^3}{15\pi^4 a^2 M_p^4} \tilde{\mathcal{T}}_k^2 \int dp\, p^6 |\beta_p|^2 \left(|\alpha_p|^2 + (-1)^{2s}\; |\beta_p|^2 \right)$$

where s=spin of source, g_s= # of dof of a spin-s field

$$m(\tau) = \dot{m}_*\,(t - t_*) = \frac{\dot{m}_*}{H} \ln\left(\frac{-\tau_*}{-\tau}\right) = -\frac{\dot{m}_*}{H} \ln(-H\tau)$$

$$A_\lambda(k) = \alpha_k(\tau)\, f_k(\tau) + \beta_k(\tau)\, f_k^*(\tau) \quad , \quad f_k \equiv \frac{\mathrm{e}^{-i\int^\tau d\tau'\,\omega(\tau')}}{\sqrt{2\omega(\tau)}} \qquad \alpha_k(t \gg t_*) \simeq \sqrt{1 + \mathrm{e}^{-\frac{\pi k^2}{\dot{m}_*}}} \quad , \quad \beta_k(t \gg t_*) \simeq -\mathrm{e}^{-\frac{\pi k^2}{2\,\dot{m}_*}}$$

$$\mathcal{T}_k \simeq \frac{a(\tau)\,\dot{m}_*}{27\,H^3}\; {}_2F_3\left(\frac{3}{2}, \frac{3}{2}; \frac{5}{2}, \frac{5}{2}, \frac{5}{2}; \frac{-k^2}{4H^2}\right)$$

- We found the GW signal too small on CMB scales.

御清聴有難う御座います

THANKS