Searching for de Sitter String Vacua

Gary Shiu

University of Wisconsin & HKUST

Sister cities?

Hong Kong

22.3000° N, 114.1667° E

Madison

43.0731° N, 89.4011° W

Obihiro, Hokkaidō 42.9167° N, 143.2000° E

From Wikipedia, the free encyclopedia

Obihiro (帯広市 *Obihiro-shi*?) is a city located in Tokachi, Hokkaidō, Japan. Obihiro is the only designated city in the Tokachi area. The next most populous municipality in Tokachi is the adjacent town of Otofuke, with less than a third of Obihiro's population. The city had approximately 500 foreign residents in 2008.[1] The city contains the headquarters of the fifth division of the northern army of the Japan Ground Self-Defense Force. It also hosts the Rally Japan World Rally Championship-event. In 2008, Obihiro was designated as a 'model environmental city' in Japan.[2]

International sister cities

Obihiro has three international sister-cities:

- Seward, Alaska, United States - (1968)

While on a business trip in Alaska, a (former) teacher at Obihiro's Agricultural High School, Yasuhiko Ohzono, was asked by the mayor of Seward to create some sort of cultural exchange between the two cities. On March 21, 1967, the mayor of Obihiro sent a picture album and other materials to introduce the city to the mayor of Seward. The mayor of Seward sent a message, a coat of arms, and a medal; all of which were personally delivered by a member of the entourage of the U.S.-Japan Fishing Industry Negotiation Team in Japan at the time. Obihiro sends the Mayor of Seward a wooden carving of a bear. On January 31, 1968 the resolution made by the Seward City Council arrives. The City of Obihiro also created a resolution on March 27, 1968, the sister city agreement was signed by both sides, and exchange between the two cities began. Since the Obihiro Economic Observation Group visited Seward in September, 1971, there have been various exchanges between Seward and Obihiro. Both mayors and many citizens of both cities have participated in exchanges, and the high school student exchange program has been put on every year since the summer of 1973.

- Chaoyang, Liaoning, People's Republic of China - (2000)

Interaction between the two cities began with Chaoyang's Economic Observation Group Visit to Obihiro on May 30, 1985. In September that same year, Obihiro sent the 15 member Northeast China Friendship and Observation Group to Chaoyang. Since then various groups have made exchange visits, agricultural trainees have been received, and there has even been exchanges of craft projects between elementary students. Since 1987, administrative and agricultural trainees have made 13 visits. In addition, JICA (Japan International Cooperation Agency) has been sending agricultural specialists to Chaoyang. At the end of October in 1999, the mayor of Obihiro at the time, Toshifumi Sunagawa, lead the Official Friendship Visit Group to Chaoyang, and he exchanged memos regarding the signing of a Friendship City Agreement. On November 17, 2000, the mayor of Chaoyang at the time, Daicao Wang, lead a delegation to Obihiro where a Friendship City Agreement was signed with the purpose of deepening interaction between the two cities across a wide range of fields, and to promote further friendship and peace between the two cities; not to mention China and Japan. The two cities have run a high school student exchange program since 2002.

- Madison, Wisconsin, United States - (2006)

Obihiro became sister cities with Madison in October 2006. The two cities have almost the same latitude, and have similar climates. The content of the sister-city relationship has been mainly various visits to Madison regarding the field of mental health, but since the official start of the relationship there have been various fact-finding missions to and from Madison. There was even a short visit to Obihiro by two Madison area students, in August 2007. Obihiro hopes to learn more about Madison agriculture, mental health systems and facilities, and about how the University of Wisconsin–Madison runs various programs and organizations that have helped make it the university it is today. For example, the Obihiro University of Agriculture and Veterinary Medicine has shown interest in marketing ice cream and other dairy products as the Babcock Dairy does at UW–Madison.

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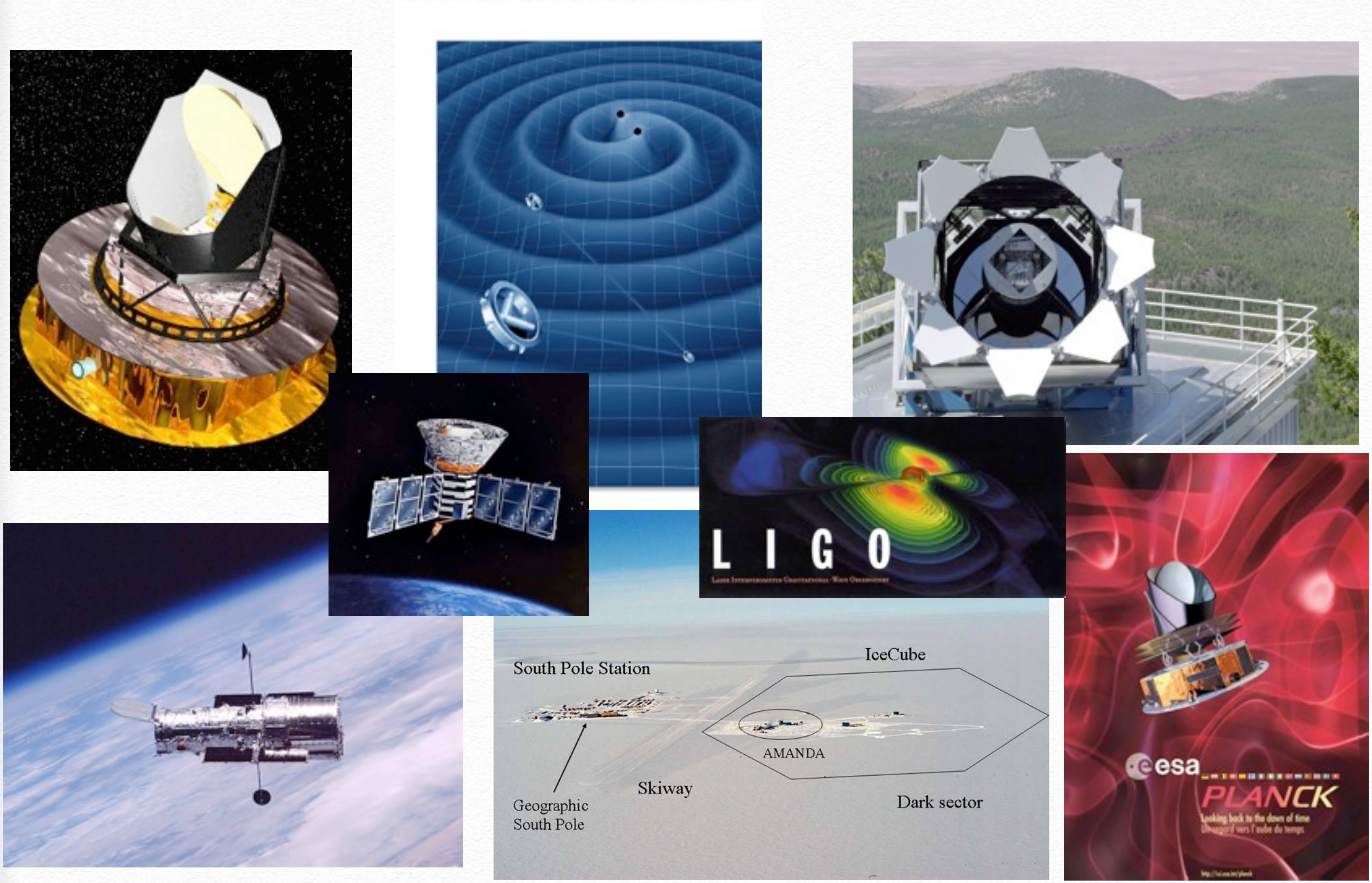
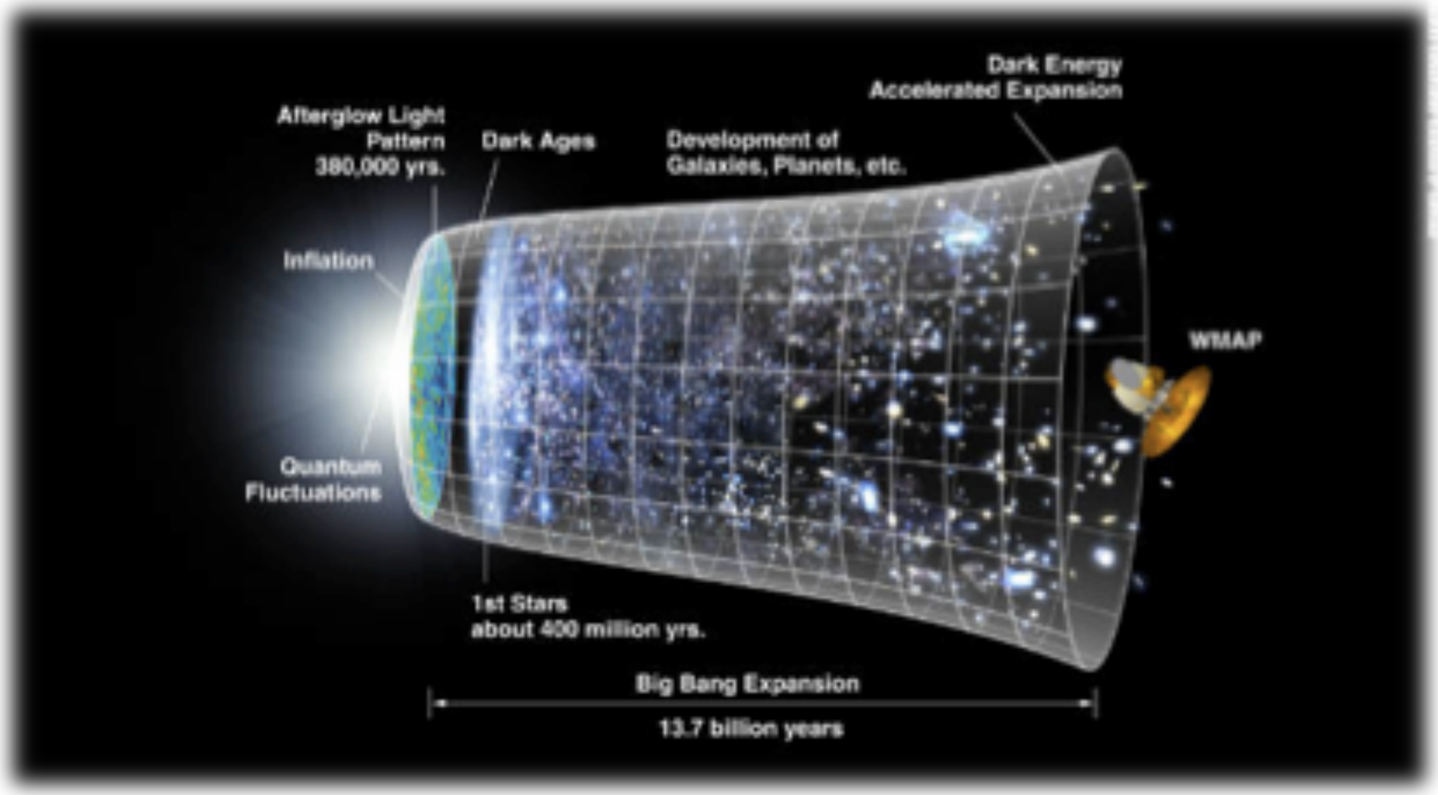
Outline of these Lectures

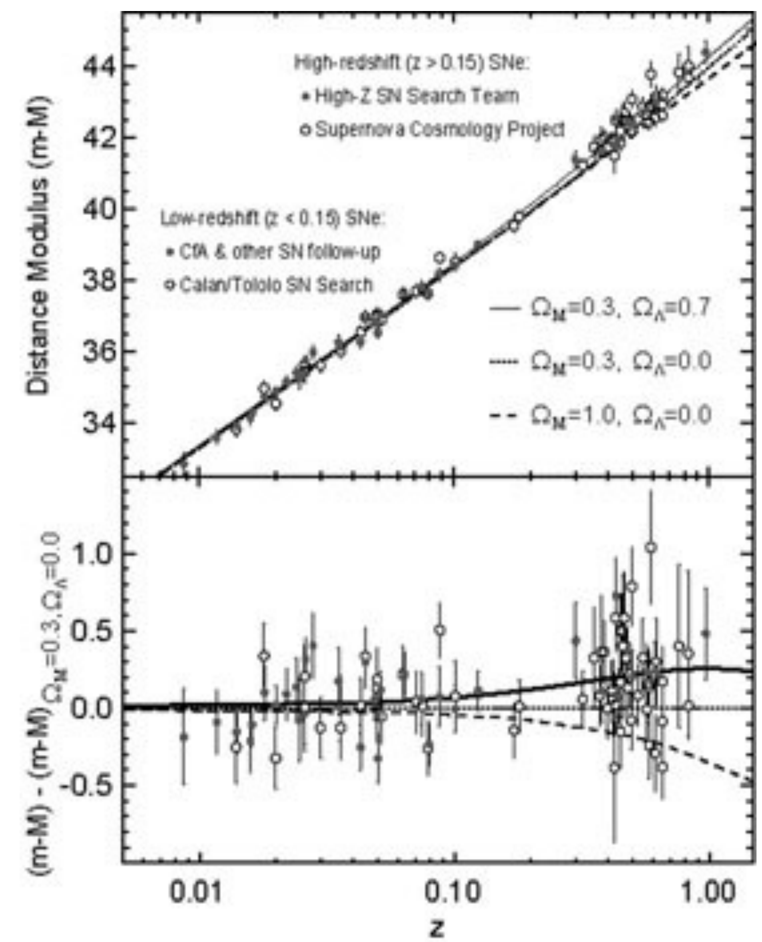
- Lecture 1: No-go theorems for dS and explicit model building
  - S.S. Haque, GS, B. Underwood, T. Van Riet, Phys. Rev. D79, 086005 (2009).
  - U.H. Danielsson, S.S. Haque, GS, T. Van Riet, JHEP 0909, 114 (2009).
  - U.H. Danielsson, S.S. Haque, P. Koerber, GS, T. Van Riet, T. Wrase, Fortsch. Phys. 59, 897 (2011).
  - GS, Y. Sumitomo, JHEP 1109, 052 (2011).
- Lecture 2: Two roles of tachyons in String Cosmology
  - Random (Super)gravities & Implications to the Landscape
    - X. Chen, GS, Y. Sumitomo, H. Tye, JHEP 1204, 026 (2012)+work in progress
  - Detectable Primordial Gravity Waves in Small Field Inflation
    - N. Barnaby, J. Moxon, R. Namba, M. Peloso, GS, P. Zhou, arXiv:1206.6117.

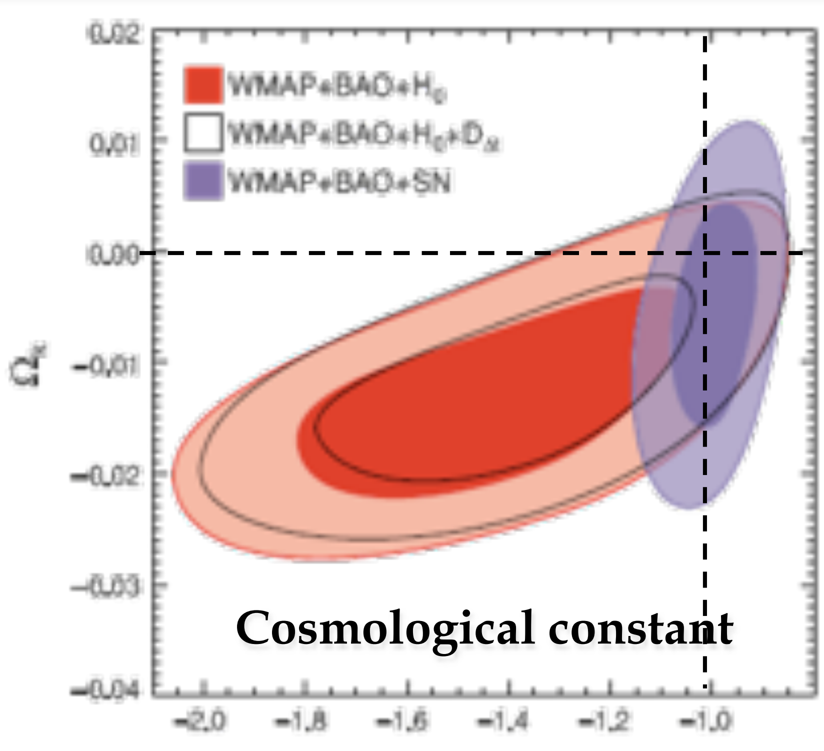
Golden Age of Cosmology



Dark Energy








Saul Perlmutter


Brian P. Schmidt


Adam G. Riess

Photo: Roy Kaltschmidt. Courtesy: Lawrence Berkeley National Laboratory

Saul Perlmutter

Photo: Belinda Pratten, Australian National University

Brian P. Schmidt

Photo: Homewood Photography

Adam G. Riess

A challenge:

Still, while Riess and his team made a striking discovery, the findings also revealed a new mystery. The universe's acceleration is thought to be driven by an immensely powerful force that since has been labeled "dark energy" — but precisely what that is remains an enigma, "perhaps the greatest in physics today," according to the academy that annually awards Nobel Prizes.

Riess called dark energy the "leading candidate" to explain the acceleration of the universe's expansion, but said he and others in his field have plenty of work to do before they determine how it works.

"You'll win a Nobel Prize if you figure it out," Riess said. "In fact, I'll give you mine."

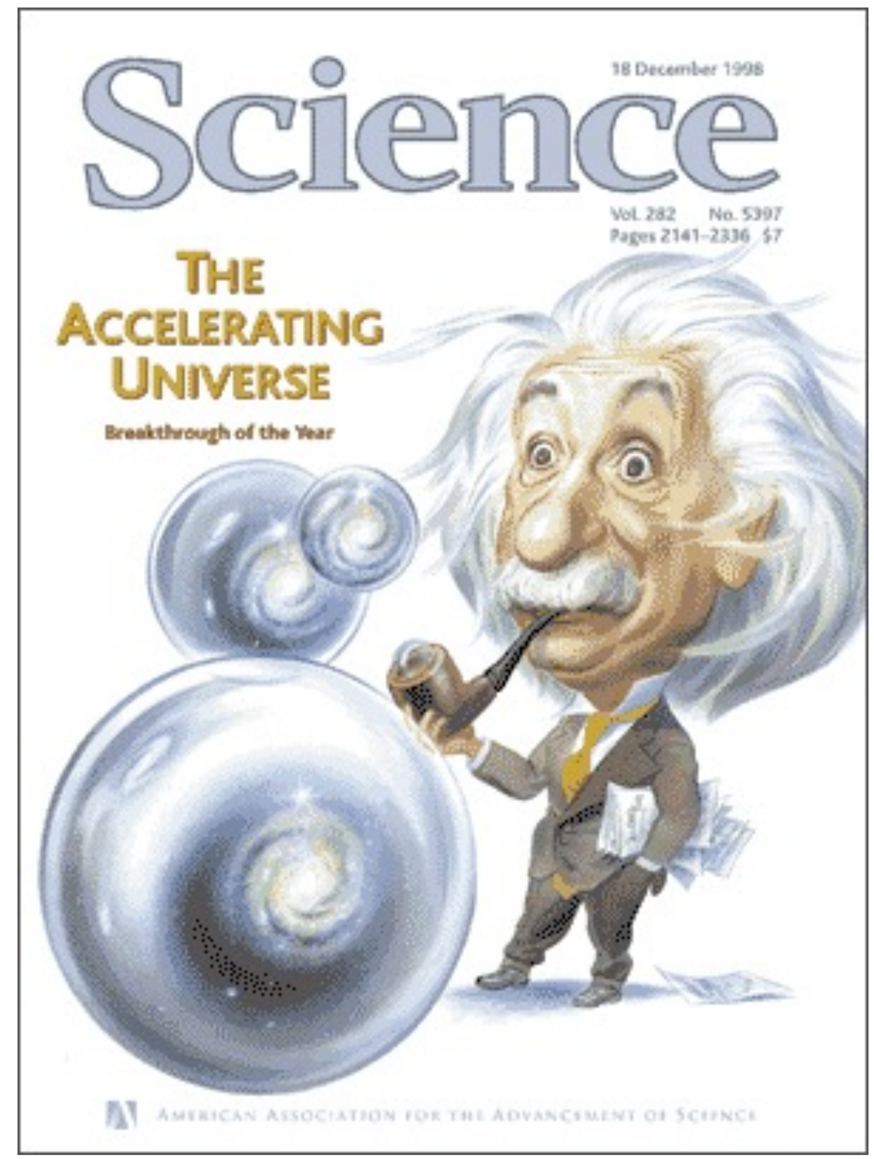
Cosmic Acceleration & String Theory

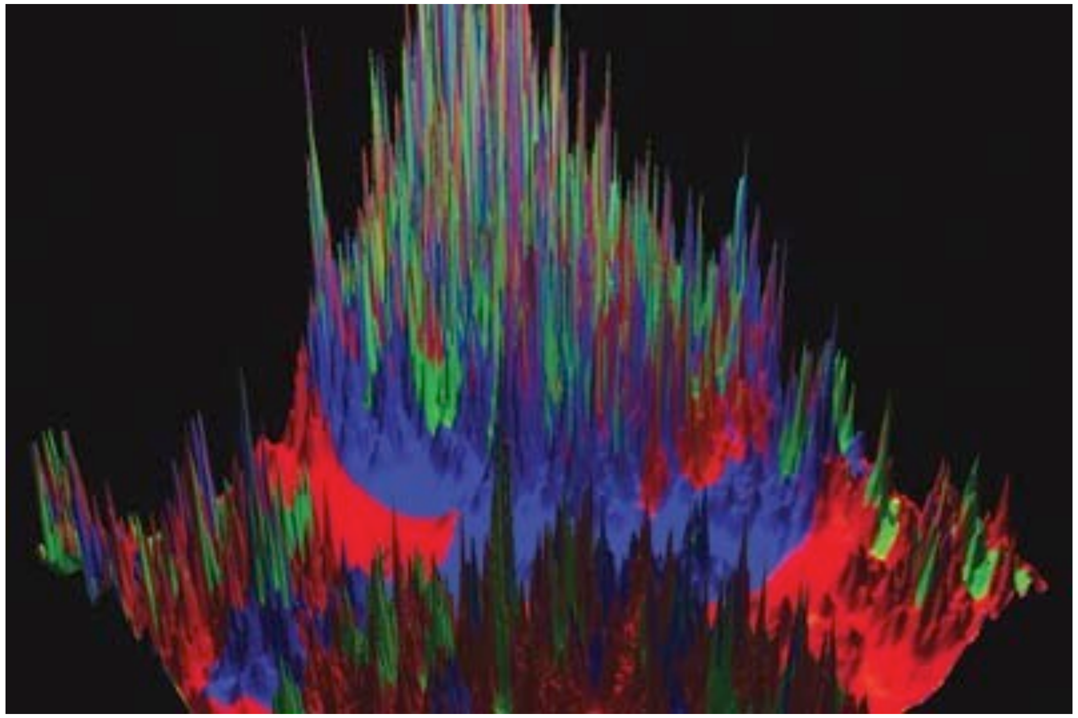
The zero of the vacuum energy:

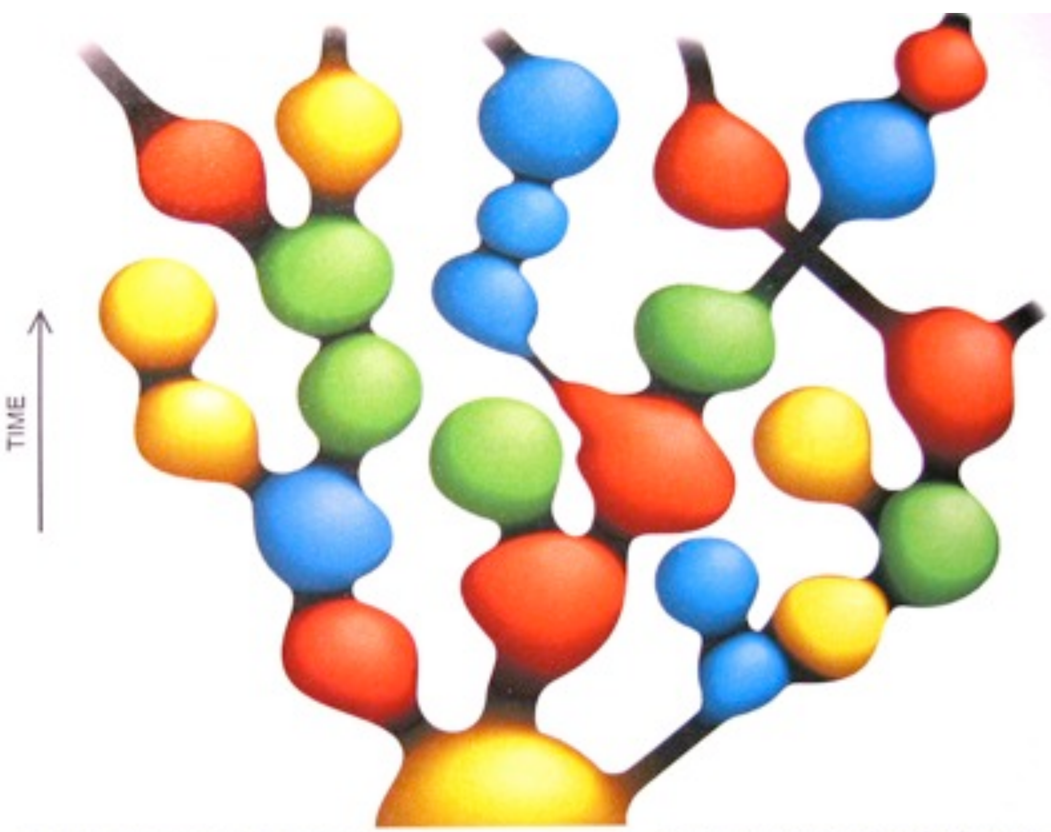
✤ is immaterial in the absence of gravity,

✤ can be tuned at will classically.

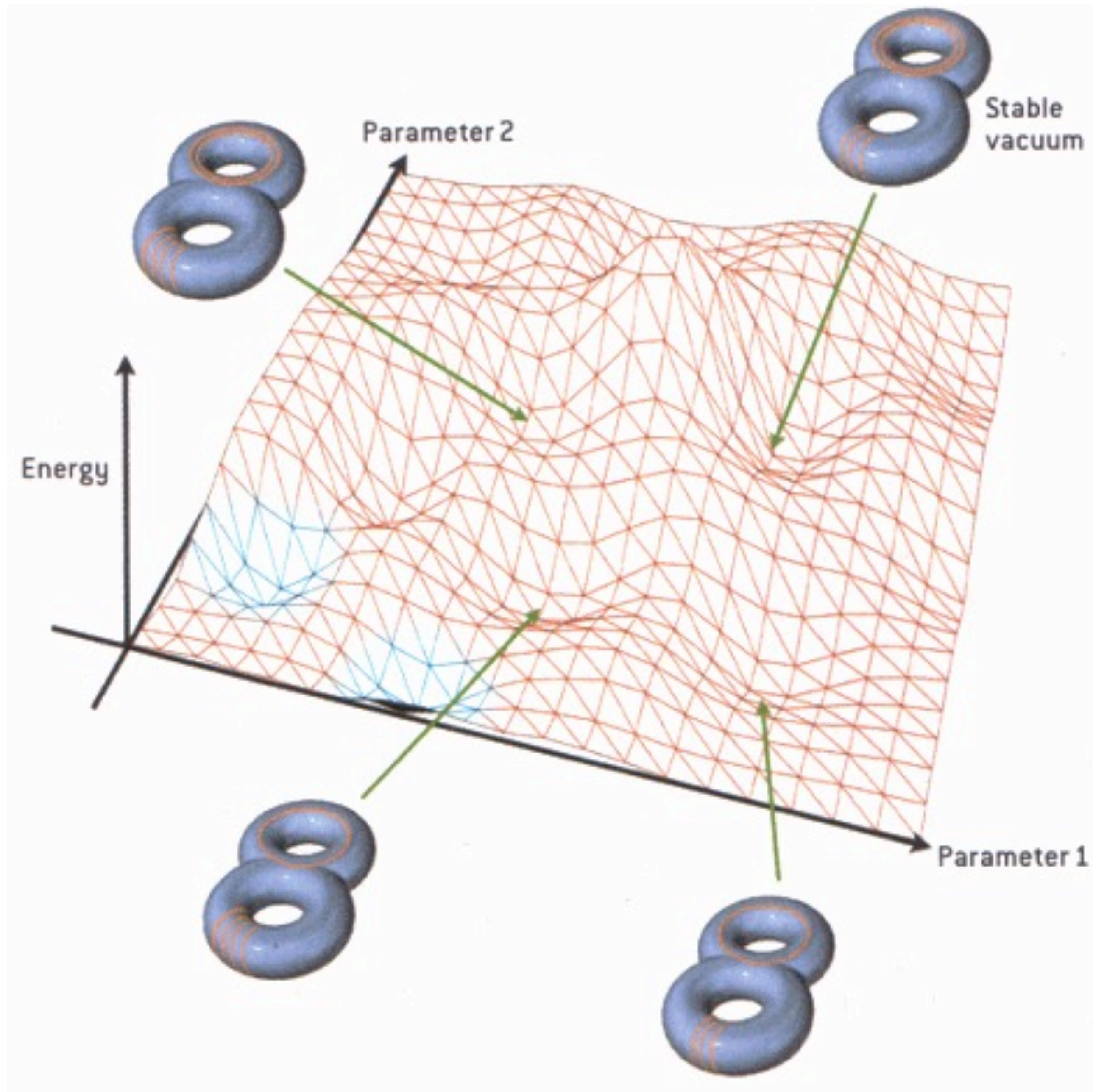
Solution to the dark energy problem likely requires quantum gravity!



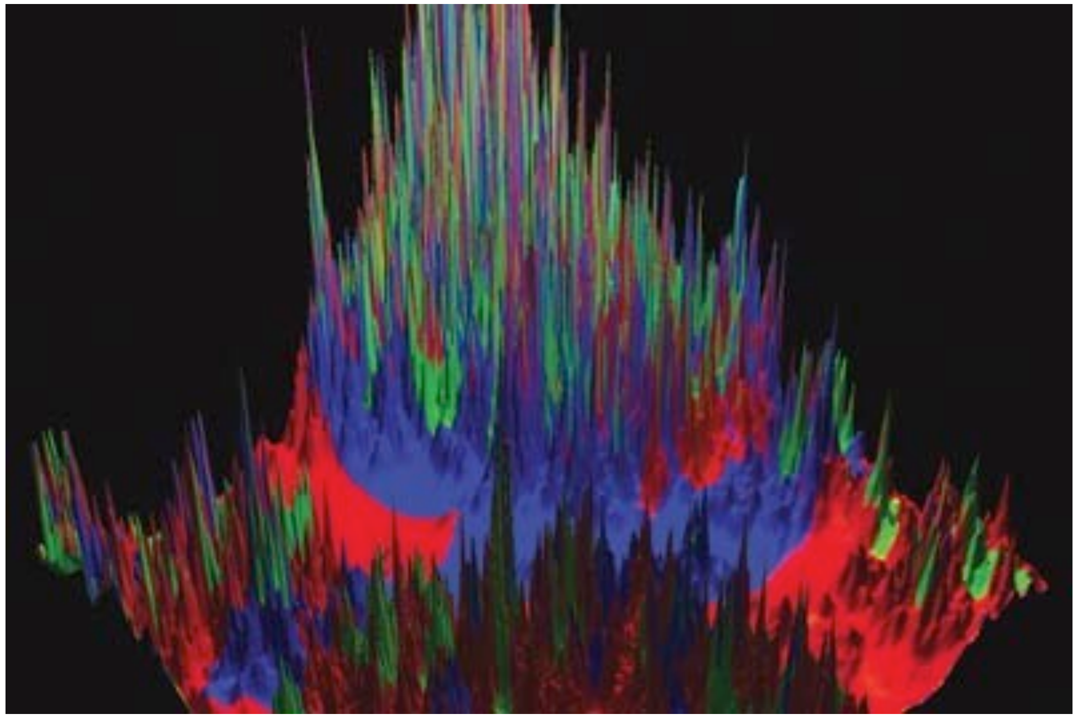


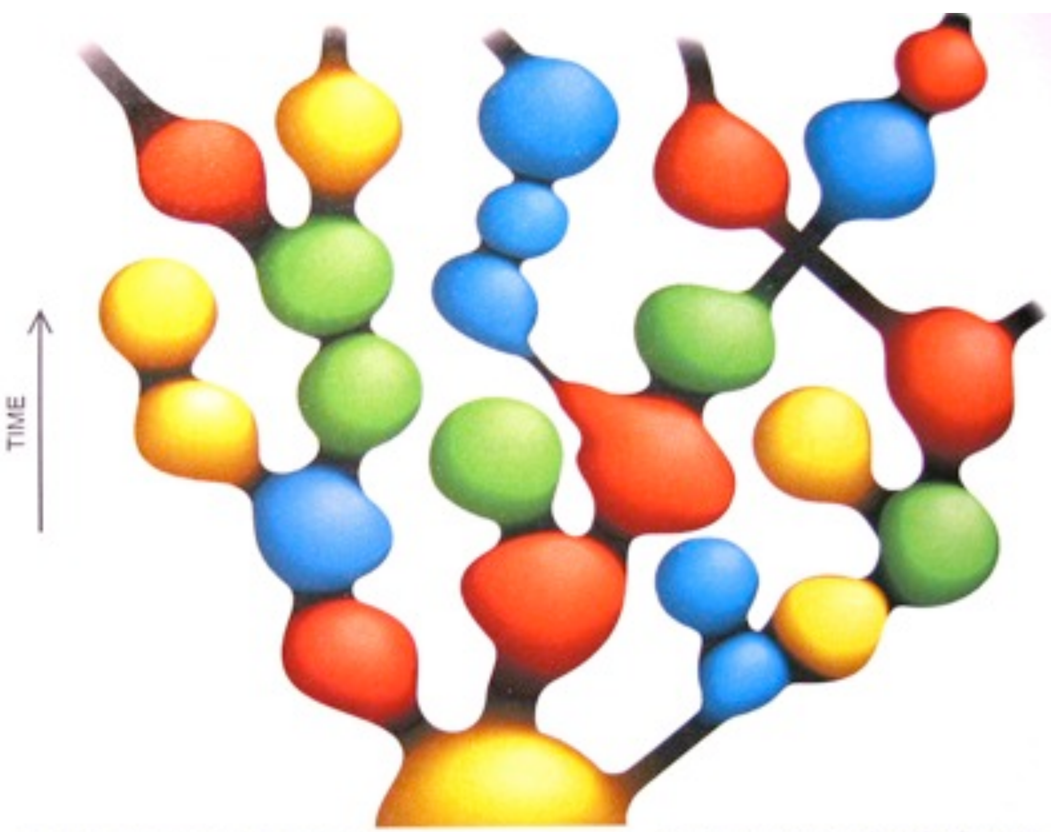


SELF-REPRODUCING COSMOS appears as an extended branching of inflationary bubbles. Changes in color represent "mutations" in the laws of physics from parent universes. The properties of space in each bubble do not depend on the time when the bubble formed. In this sense, the universe as a whole may be stationary, even though the interior of each bubble is described by the big bang theory.



"The Landscape" (Picture from *Scientific American*)





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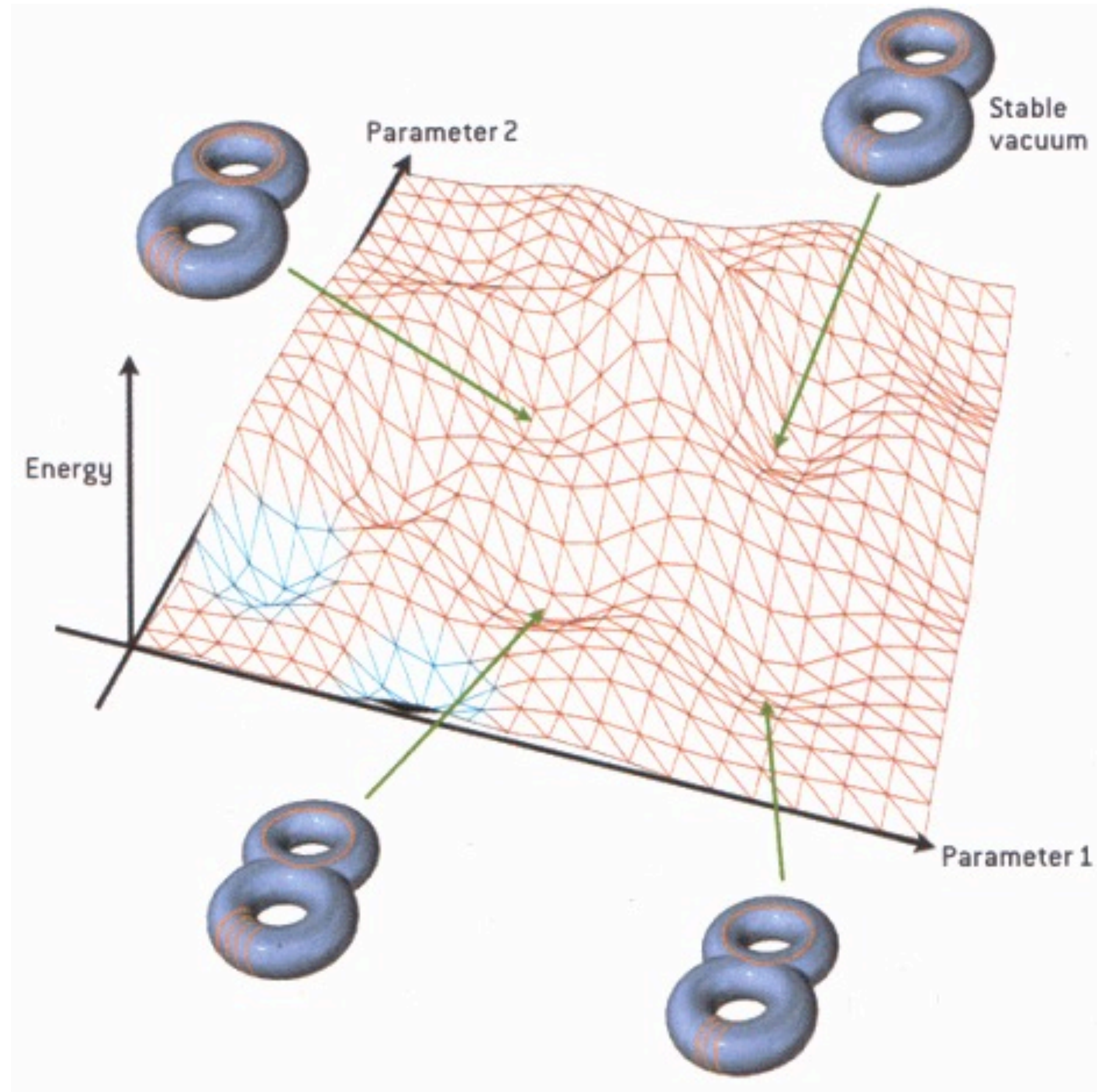


"The Landscape" (Picture from *Scientific American*)

A landscape of string vacua?

The de Sitter Menu

Antipasti

Fluxes stabilize complex structure moduli; Kahler moduli remain unfixed.

Entree

Non-perturbative effects (D7 gauge instantons or ED3 instantons)

stabilize the Kahler moduli.

Desserts

Anti-branes to "uplift" vacuum energy.

Kachru, Kallosh, Linde, Trivedi;
Balasubramanian, Berglund, Conlon, Quevedo;
.....

In fine print ....

- **Non-perturbative effects:** difficult to compute *explicitly*. Most work aims to illustrate their existence, rather than to compute the actual contributions:

$$W_{\rm np} = Ae^{-a\rho} \quad \Longrightarrow \quad W_{\rm np} = A(\zeta_i)e^{-a\rho}$$

Moreover, the full moduli dependence is suppressed.

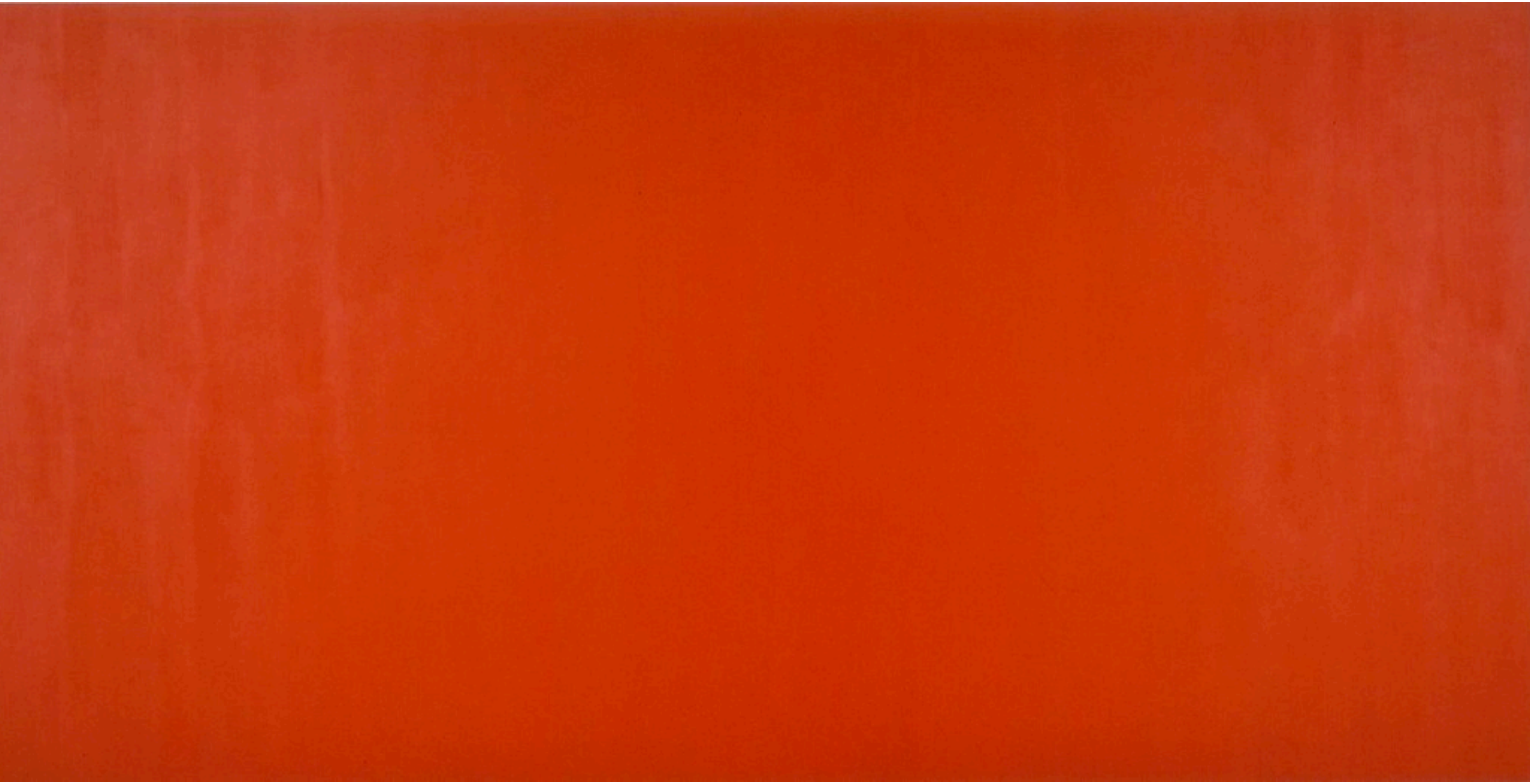
- **Anti D3-branes:** backreaction on the 10D SUGRA proves to be very challenging.

[DeWolfe, Kachru, Mulligan];[McGuirk, GS, Sumitomo];[Bena, Grana, Halmagyi], [Dymarsky], ...

WIKIPEDIA
The Free Encyclopedia

Minimalism describes movements in various forms of art and design, especially visual art and music, where the work is stripped down to its most fundamental features. As a specific movement in the arts it is identified with developments in post-World War II Western Art, most strongly with American visual arts in the late 1960s and early 1970s. Prominent artists associated with this movement include Donald Judd, Agnes Martin and Frank Stella. It is rooted in the reductive aspects of Modernism, and is often interpreted as a reaction against Abstract Expressionism and a bridge to Postmodern art practices.

Richard Pousette-Dart, *Symphony No. 1, The Transcendental*, oil on canvas, 1941-42, Metropolitan Museum of Art



Barnett Newman, *Anna's light,* 1968

Barnett Newman, *Onement 1*, 1948. Museum of Modern Art, New York. The first example of Newman using the so-called "zip" to define the spatial structure of his paintings.

Towards simple de Sitter vacua

- Explicitly computable within classical SUGRA.
- Absence of np effects, and explicit SUSY breaking localized sources, e.g., anti-branes.
- Solve 10D equations of motion (c.f., 4D EFT).
- (For now) content with simple dS solutions w/o requiring a realistic cc & SUSY breaking scale: explicit models help address conceptual issues.
- Readily amenable to statistical studies (later).

Our Ingredients

- Fluxes: contribute *positively* to energy and tend to make the internal space *expands:*

$$S = -\frac{1}{2p!}\int_6 \sqrt{g_6}\, F_{\mu_1\ldots\mu_{p+1}} F^{\mu_1\ldots\mu_{p+1}}$$

- Branes: contribute *positively* to energy and tend to *shrink* the internal space (reverse for O-plane which has negative tension):

$$S = -T_{\rm brane}\int_{\rm brane} \sqrt{g_{\rm brane}}$$

- Curvature: Positively (negatively) curved spaces tend to shrink (expand) and contribute a negative (positive) energy:

$$\int_{10} \sqrt{|g_{10}|}\mathcal{R}_{10} = \int_4 \sqrt{g_4}\Big(\big(\int_6 \sqrt{g_6}\big)\mathcal{R}_4 + \int_6 \sqrt{g_6}\mathcal{R}_6\Big)$$

Universal Moduli

✤ Consider metric in 10D string frame and 4d Einstein frame:

$$\mathrm{d}s_{10}^2 = \tau^{-2}\mathrm{d}s_4^2 + \rho\,\mathrm{d}s_6^2\,, \qquad\qquad \tau \equiv \rho^{3/2}\mathrm{e}^{-\phi}\,,$$

ρ, τ are the *universal moduli*.

✤ The various ingredients contribute to V in some specific way:

$$V_R = U_R\rho^{-1}\tau^{-2}, \quad U_R(\varphi) \sim \int \sqrt{g_6}\;(-R_6),$$

$$V_H = U_H\rho^{-3}\tau^{-2}, \quad U_H(\varphi) \sim \int \sqrt{g_6}\;H^2,$$

$$V_q = U_q\rho^{3-q}\tau^{-4}, \quad U_q(\varphi) \sim \int \sqrt{g_6}F_q^2 > 0$$

$$V_p = U_p\rho^{\frac{p-6}{2}}\tau^{-3}, \quad U_p(\varphi) = \mu_p\ \mathrm{Vol}(M_{p-3}).$$

✤ The full 4D potential $V(\rho,\tau,\phi_i) = V_R + V_H + V_q + V_p$.

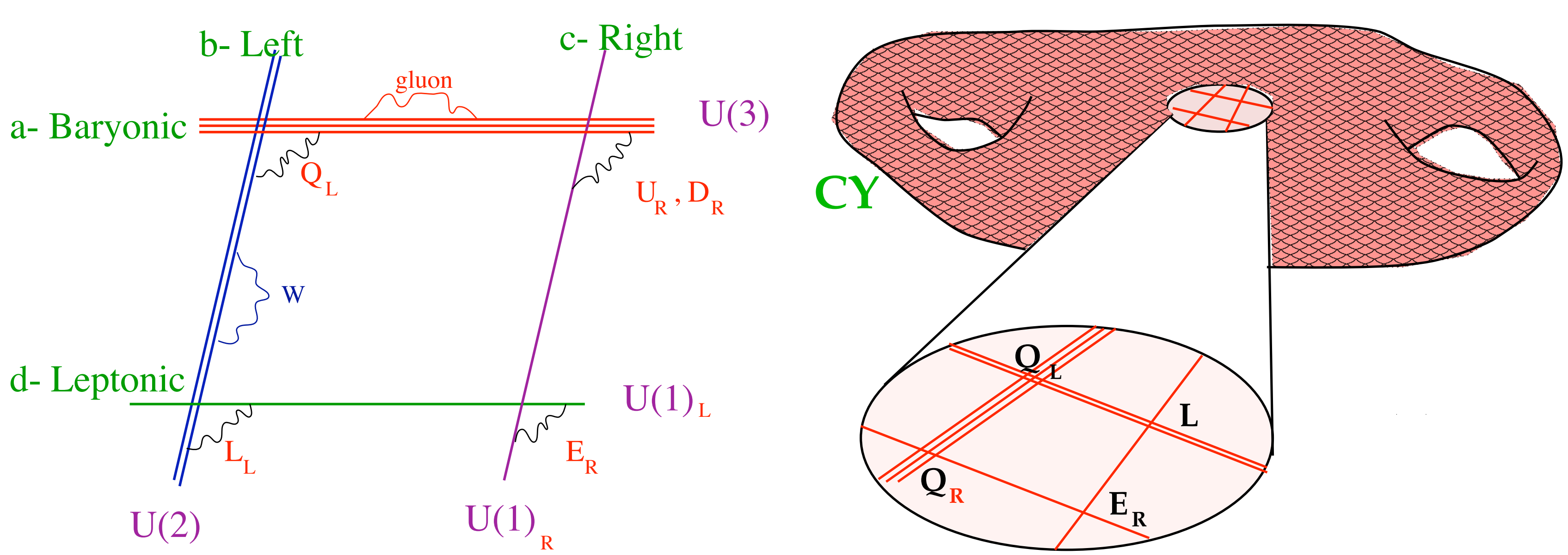
No-go Theorem(s)

- From the scalings of V, can prove no-goes for dS by finding:

$$-a\tau\partial_\tau V - b\rho\partial_\rho V \geq cV \qquad\qquad c > 0$$

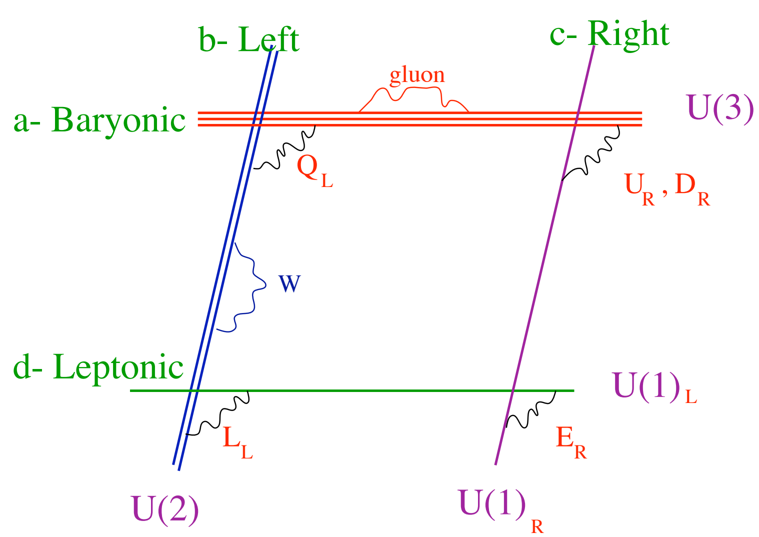
  in various Type II settings with D-branes/O-planes [Haque, GS, Underwood, Van Riet]; [Danielsson, Haque, GS, van Riet];[Wrase, Zagermann].

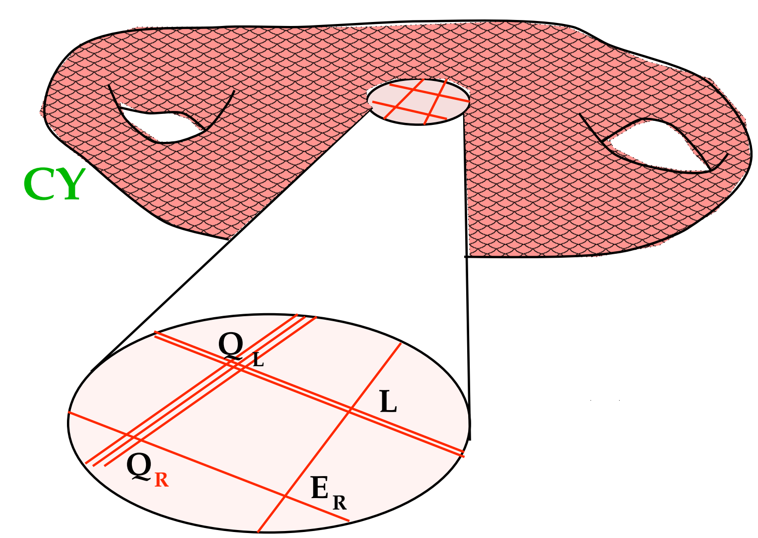
- This *excludes* classical **dS vacua** in Type IIA CY orientifolds with intersecting D6-branes [Hertzberg, Kachru, Taylor, Tegmark].



Intersecting Brane Models

✤ Popular framework for building particle physics models from string theory. See e.g., [Blumenhagen, Cvetic, Langacker, GS] for reviews.





✤ For Calabi-Yau, V_R =0, we have: $$V = V_H + \sum_q V_q + V_{D6} + V_{O6}$$

✤ The universal moduli dependence leads to an inequality:

$$-\rho \frac{\partial V}{\partial \rho} - 3\tau \frac{\partial V}{\partial \tau} = 9V + \sum_q q V_q \geq 9V$$

excludes dS vacuum $\frac{\partial V}{\partial \rho} = \frac{\partial V}{\partial \tau} = 0$ and $V > 0$ and inflation!

Stability Constraints

[GS, Sumitomo]

- Goal: seek for a systematic way to check whether ∃ unstable modes (or flat directions) in the full moduli mass matrix.
- Diagonalize full mass matrix: time-consuming, case by case, ...
- Useful necessary conditions for stability can be stated by restricting the analysis to the universal moduli subspace.
- Sylvester's Criterion:

  *An N x N Hermitian matrix is positive definite iff the determinants of the upper-left n x n submatrices (n≤N) are **all** positive.*

- The 1x1 and 2x2 universal moduli subspace of the full mass matrix must both have positive determinants.

Stability Constraints

[GS, Sumitomo]

- The mass matrix M of the 2D universal moduli subspace must satisfy:

$$\det M > 0, \quad \mathrm{tr} M > 0$$

- The *minimal* ingredients for classical dS extrema tabulated in [Danielsson, Haque, GS, van Riet, 09];[Wrase, Zagermann,10]:

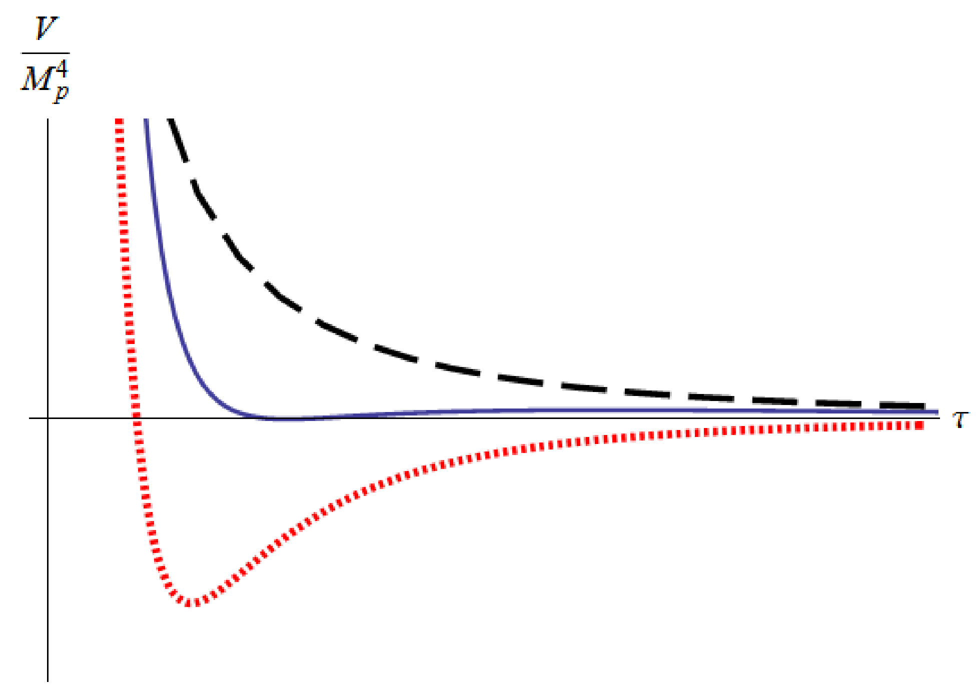
Curvature	No-go, if	No no-go in IIA with	No no-go in IIB with
$V_{R_6} \sim -R_6 \leq 0$	$q+p-6 \geq 0, \forall p, q,$ $\epsilon \geq \frac{(3+q)^2}{3+q^2} \geq \frac{12}{7}$	O4-planes and H, F_0-flux	O3-planes and H, F_1-flux
$V_{R_6} \sim -R_6 > 0$	$q+p-8 \geq 0, \forall p, q,$ (except $q=3, p=5$) $\epsilon \geq \frac{(q-3)^2}{q^2-8q+19} \geq \frac{1}{3}$	O4-planes and F_0-flux O4-planes and F_2-flux O6-planes and F_0-flux	O3-planes and F_1-flux O3-planes and F_3-flux O3-planes and F_5-flux O5-planes and F_1-flux

all turn out to have an unstable mode (or flat direction)!

- Similar analyses likely applicable to other dS constructions.

No-go Theorem(s)

- Evading these no-goes: O-planes [introduced in any case because of [Gibbons; de Wit, Smit, Hari Dass; Maldacena,Nunez]], fluxes, often also *negative* curvature. [Silverstein + above cited papers]



Heuristically: negative internal scalar curvature acts as an uplifting term.

- *Classical AdS vacua* from IIA flux compactifications were known [Behrndt & Cvetic, Derendinger et al; Villadoro et al; De Wolfe et al; Camara et al].

- *Minimal ingredients* needed for dS [Haque, GS, Underwood, Van Riet]:

  1) O6-planes 2) Romans mass 3) H-flux 4) Negatively curved internal space.

Generalized Complex Geometry

- Interestingly, such extensions were considered before in the context of generalized complex geometry (GCG).
- Among these GCG, many are negatively curved (e.g., twisted tori), at least in some region of the moduli space [Lust et al; Grana et al; Kachru et al; ...].
- Attempts to construct explicit dS models were made soon after no-goes [Haque,GS,Underwood,Van Riet];[Flauger,Paban,Robbins, Wrase]; [Caviezel,Koerber,Lust,Wrase,Zagermann];[Danielsson,Haque,GS,van Riet]; [de Carlos,Guarino,Moreno];[Caviezel,Wrase,Zagermann];[Danielsson, Koerber,Van Riet]; ....
- We report on the result of a systematic search within a broad class of such manifolds [Danielsson, Haque, Koerber, GS, van Riet, Wrase].

Two Approaches

SUSY broken @ or above KK scale	SUSY broken below KK scale

Do not lead to an effective SUGRA in dim. reduced theory

[Silverstein, 07];
[Andriot, Goi, Minasian, Petrini, 10];
[Dong, Horn, Silverstein, Torroba, 10];
...

Lead to a 4d SUGRA (N=1):

[This talk]

➡ Spontaneous ~~SUSY~~ state

➡ Potentially lower ~~SUSY~~ scale

➡ Much more control on the EFT

➡ c.f. dS searches within SUGRA

10d vs 4d

✤ We advocate 10d point of view, so why consider 4d $V(\rho,\tau)$?

It can be shown [Danielsson, Haque, GS, Van Riet]:

$$\Box\phi = 0 = \sum_n \frac{a_n}{2n!} \mathrm{e}^{a_n\phi} F_n^2 \pm \frac{p-3}{4} \mathrm{e}^{(p-3)\phi/4} |\mu_p|\, \delta(\Sigma)\,,$$

and trace of

$$R_{ab} = \sum_n \Big(-\frac{n-1}{16n!} g_{ab} \mathrm{e}^{a_n\phi} F_n^2 + \frac{1}{2(n-1)!} \mathrm{e}^{a_n\phi} (F_n)^2_{ab}\Big) + \tfrac{1}{2}(T^{loc}_{ab} - \tfrac{1}{8} g_{ab} T^{loc})\,,$$

(upon smearing of sources) are equivalent to $\partial_\rho V = \partial_\tau V = 0$ & trace of $R_{\mu\nu}$ equation just gives def. of V; a useful first pass.

✤ When backreaction of localized sources cannot be ignored (more later), 10d eoms are harder to solve, a warped 4D EFT is needed. [Kodama,Uzawa];[Giddings,Maharana];[Koerber,Martucci]; [GS,Torroba, Underwood, Douglas]; ...

Search Strategy

- GCG: natural framework for N=1 SUSY compactifications when backreaction from fluxes are taken into account.

- Type IIA SUSY AdS vacua arise from specific SU(3) structure manifolds [Behrndt,Cvetic];[Lust,Tsimpis];[Caviezel et al];[Koerber, Lust, Tsimpsis]; ...

- Modify the AdS ansatz for the fluxes (which solves the flux eoms from the outset) and search for dS solutions.

- Spontaneously SUSY breaking state in a 4D SUGRA: powerful results & tools from SUSY, GCG.

SU(3) Structure

- SUSY implies the existence of a nowhere vanishing internal 6d spinor η_+ (and complex conjugate η_-).

- Characterized by a real 2-form J and a complex 3-form Ω:

$$J = \frac{i}{2||\eta||^2}\eta_+^\dagger \gamma_{i_1 i_2}\eta_+ \mathrm{d}x^{i_1} \wedge \mathrm{d}x^{i_2}$$
$$\Omega = \frac{1}{3!||\eta||^2}\eta_-^\dagger \gamma_{i_1 i_2 i_3}\eta_+ \mathrm{d}x^{i_1} \wedge \mathrm{d}x^{i_2} \wedge \mathrm{d}x^{i_3}$$

satisfying $\Omega \wedge J = 0\,, \qquad \Omega \wedge \Omega^* = (4i/3)\, J \wedge J \wedge J = 8i\, \mathrm{vol}_6\,.$

- J, Ω define SU(3) structure, not SU(3) holonomy: generically dJ≠0 and dΩ≠0.

SU(3) Structure

✤ Build an almost complex structure:

$$I^l{}_k = c\,\varepsilon^{m_1 m_2 \ldots m_5 l} (\Omega_R)_{k m_1 m_2} (\Omega_R)_{m_3 m_4 m_5} \qquad I^l_k I^k_m = -\delta^l_m$$

for which J is of (1,1) and Ω is of (3,0) type.

✤ The metric then follows:

$$g_{mn} = -I^l{}_m J_{ln}$$

✤ The global existence of these forms implies the structure group of the frame bundle to be SU(3).

SU(3) Torsion Classes

The non-closure of the exterior derivatives characterized by:

$$\mathrm{d}J = \frac{3}{2}\mathrm{Im}\left(\mathcal{W}_1\Omega^*\right) + \mathcal{W}_4 \wedge J + \mathcal{W}_3$$

$$\mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega$$

Torsion classes	Name
$\mathcal{W}_1 = \mathcal{W}_2 = 0$	Complex
$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$	Symplectic
$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Nearly Kähler
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$	Kähler
$\mathrm{Im}\,\mathcal{W}_1 = \mathrm{Im}\,\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Half-flat
$\mathcal{W}_1 = \mathrm{Im}\,\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Nearly Calabi-Yau
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Calabi-Yau
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 0, (1/2)\mathcal{W}_4 = (1/3)\mathcal{W}_5 = -\mathrm{d}A$	Conformal Calabi-Yau

SU(3) Torsion Classes

The non-closure of the exterior derivatives characterized by:

$$\mathrm{d}J = \frac{3}{2}\mathrm{Im}\left(\mathcal{W}_1\Omega^*\right) + \mathcal{W}_4 \wedge J + \mathcal{W}_3$$

$$\mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega$$

Torsion classes	Name
$\mathcal{W}_1 = \mathcal{W}_2 = 0$	Complex
$\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$	Symplectic
$\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Nearly Kähler
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$	Kähler
$\mathrm{Im}\,\mathcal{W}_1 = \mathrm{Im}\,\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Half-flat
$\mathcal{W}_1 = \mathrm{Im}\,\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Nearly Calabi-Yau
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$	Calabi-Yau
$\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 0, (1/2)\mathcal{W}_4 = (1/3)\mathcal{W}_5 = -\mathrm{d}A$	Conformal Calabi-Yau

Half Flat Manifolds

- For the SU(3) structure manifold to be compatible with the orbifold/orientifold symmetries we consider (more later):

$$\mathrm{d}J = \frac{3}{2} W_1 \Omega_R + W_3 \,,$$
$$\mathrm{d}\Omega_R = 0 \,,$$
$$\mathrm{d}\Omega_I = W_1 J \wedge J + W_2 \wedge J \,,$$

$W_{1,2,3}$ are real

- $W_{1,2,3}$ is a scalar, a (1,1) form, & a (1,2) +(2,1) form satisfying:

$$W_2 \wedge J \wedge J = 0 \,, \qquad W_3 \wedge J = 0 \,;$$
$$W_2 \wedge \Omega = 0 \,, \qquad W_3 \wedge \Omega = 0$$

- Ricci tensor can be expressed explicitly in terms of J, Ω and the torsion forms [Bedulli, Vezzoni].

Universal Ansatz

- In terms of the universal forms: $\Big\{J, \Omega, W_1, W_2, W_3\Big\}$

  one finds a natural ansatz for the fluxes:

$$
\begin{aligned}
e^{\Phi}\hat{F}_0 &= f_1\,, \\
e^{\Phi}\hat{F}_2 &= f_2 J + f_3 \hat{W}_2\,, \\
e^{\Phi}\hat{F}_4 &= f_4 J \wedge J + f_5 \hat{W}_2 \wedge J\,, \\
e^{\Phi}\hat{F}_6 &= f_6 \mathrm{vol}_6\,, \\
H &= f_7 \Omega_R + f_8 \hat{W}_3\,, \\
j &= j_1 \Omega_R + j_2 \hat{W}_3\,.
\end{aligned}
$$

- Universal ansatz: forms appear in *all* SU(3) structure (in this case, half flat) manifolds.

- Also the ansatz for the SUSY AdS vacua in [Lust, Tsimpis]

O-planes

- To simplify, we take the *smeared* approximation:

$$\delta \to \text{constant}$$

  i.e., we solve the eoms in an “average sense”. If backreaction is ignored, eoms are not satisfied pointwise [Douglas, Kallosh].

- Finding backeacted solutions with localized sources proves to be challenging (more later) [Blaback, Danielsson, Junghans, Van Riet, Wrase, Zagermann].

- The Bianchi identity becomes:

$$\mathrm{d}\hat{F}_2 + H\hat{F}_0 = -j , \qquad e^{\Phi} j = j_1 \Omega_R + j_2 \hat{W}_3 \,.$$

- The source terms of smeared O-planes in dilaton/Einstein eoms can be found in [Koerber, Tsimpis, 07].

Finding Solutions

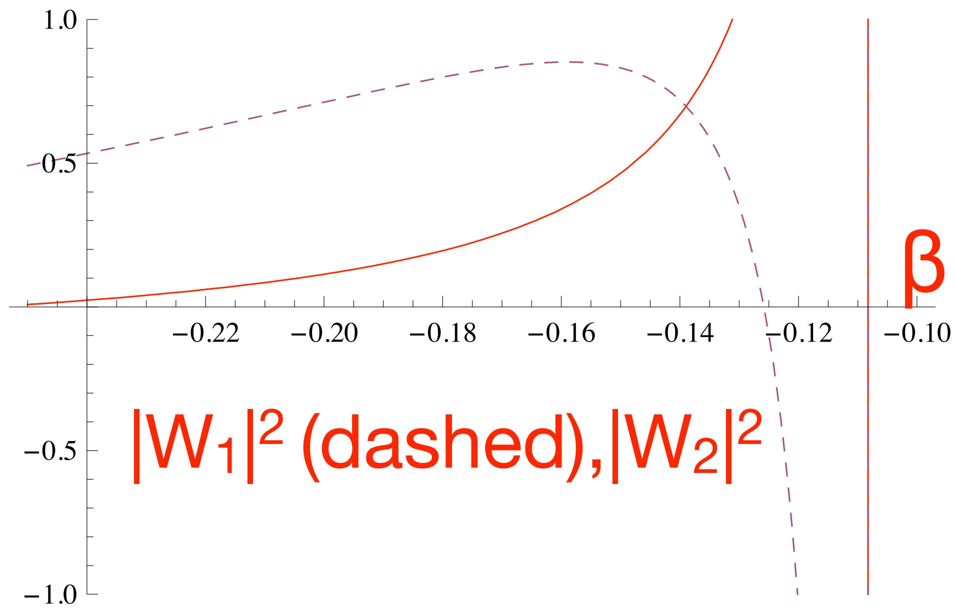
- The dilaton/Einstein/flux eoms and Bianchi identities can be expressed as algebraic equations (skip details).
- To find solutions other than the SUSY AdS, impose constraints:

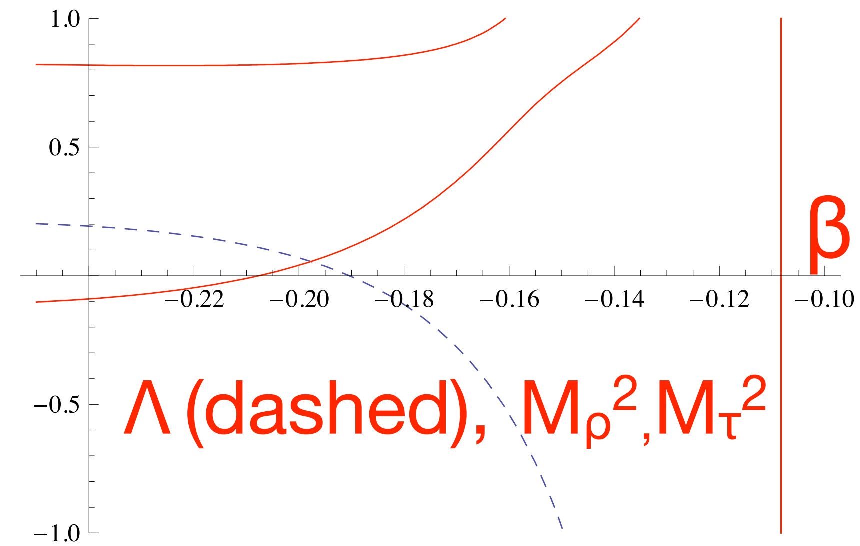
$$
\begin{aligned}
\mathrm{d}\hat{W}_2 &= c_1 \Omega_R + d_1 \hat{W}_3\,,\\
\hat{W}_2 \wedge \hat{W}_2 &= c_2 J \wedge J + d_2 \hat{W}_2 \wedge J\,,\\
\mathrm{d} \star_6 \hat{W}_3 &= c_5 J \wedge J + c_3 \hat{W}_2 \wedge J\,,\\
\frac{1}{2}(\hat{W}_{3\,ikl} \hat{W}_{3\,j}{}^{kl})^+ &= d_4 J_{ik} \hat{W}_2{}^k{}_j\,.
\end{aligned}
$$

for some c's and d's.

Finding Solutions

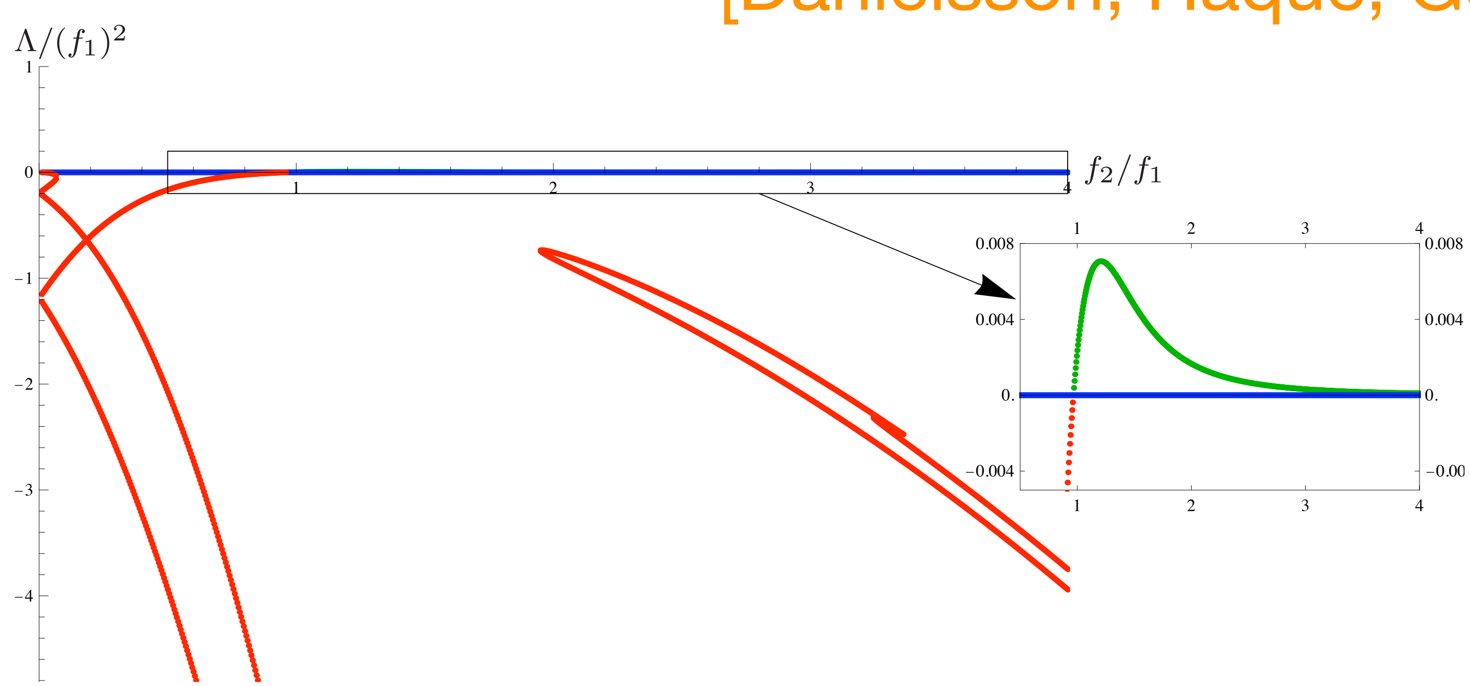
$W_3 = 0$





[Danielsson, Haque, GS, Van Riet]

$W_2 = 0$



[Danielsson, Koerber, Van Riet]

Universal de Sitter

- Bottom-up approach: we found *necessary* constraints on fluxes & torsion classes for *universal* dS solutions, a useful first step.

- Next: explicit geometries, stabilization of model-dependent moduli, flux quantization, unsmeared sources, etc.

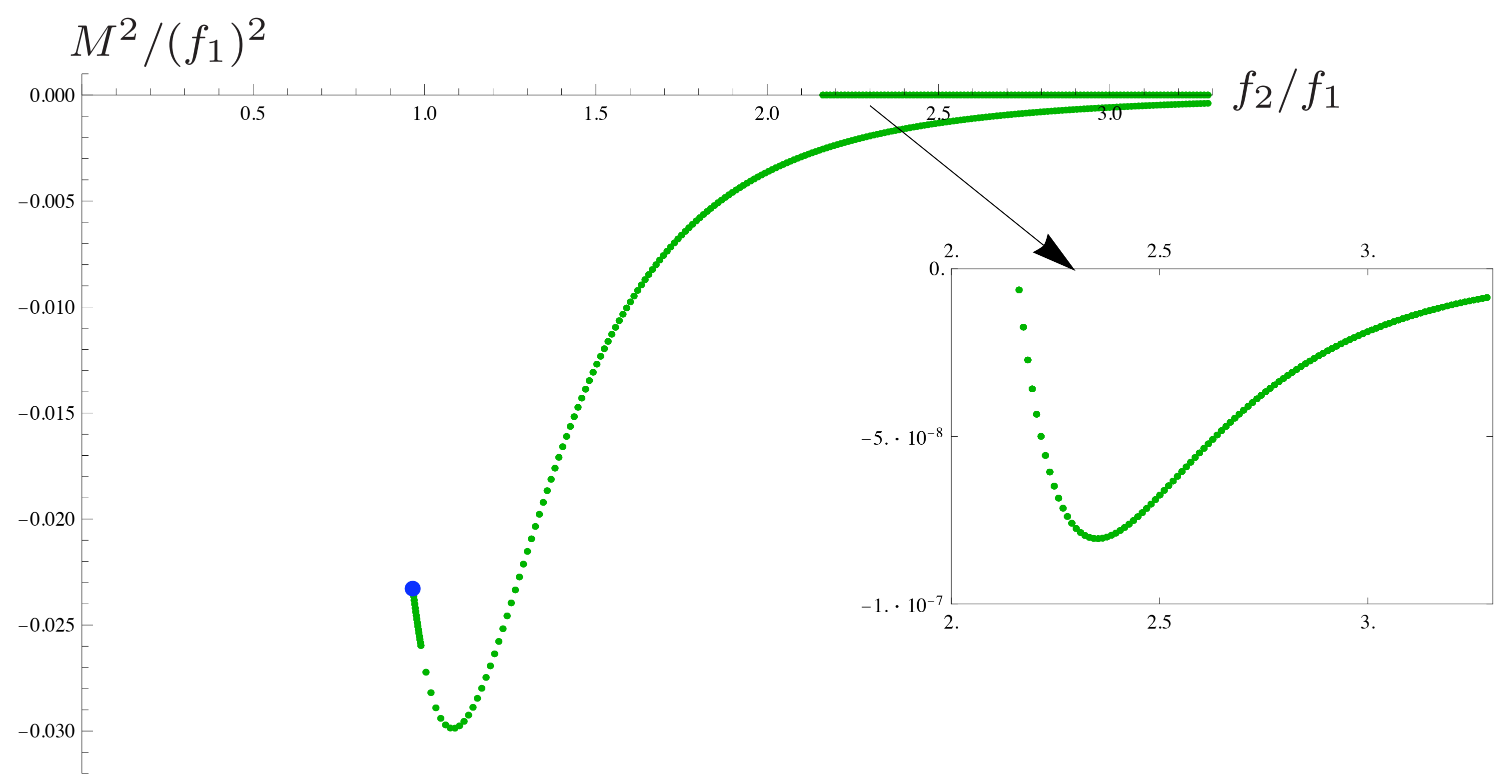
- Homogenous spaces (group/coset spaces) seem a promising first trial: can explicitly construct SU(3) structure.

Example

Bottom-up constraints (with W_2=0) can be satisfied with an explicit model: an SU(2) x SU(2) group manifold.

This realizes a solution obtained by 4d SUGRA approach[Caviezel, Koerber, Kors, Lust, Wrase, Zagermann]

Unfortunately, out of 14 scalars, one is tachyonic !

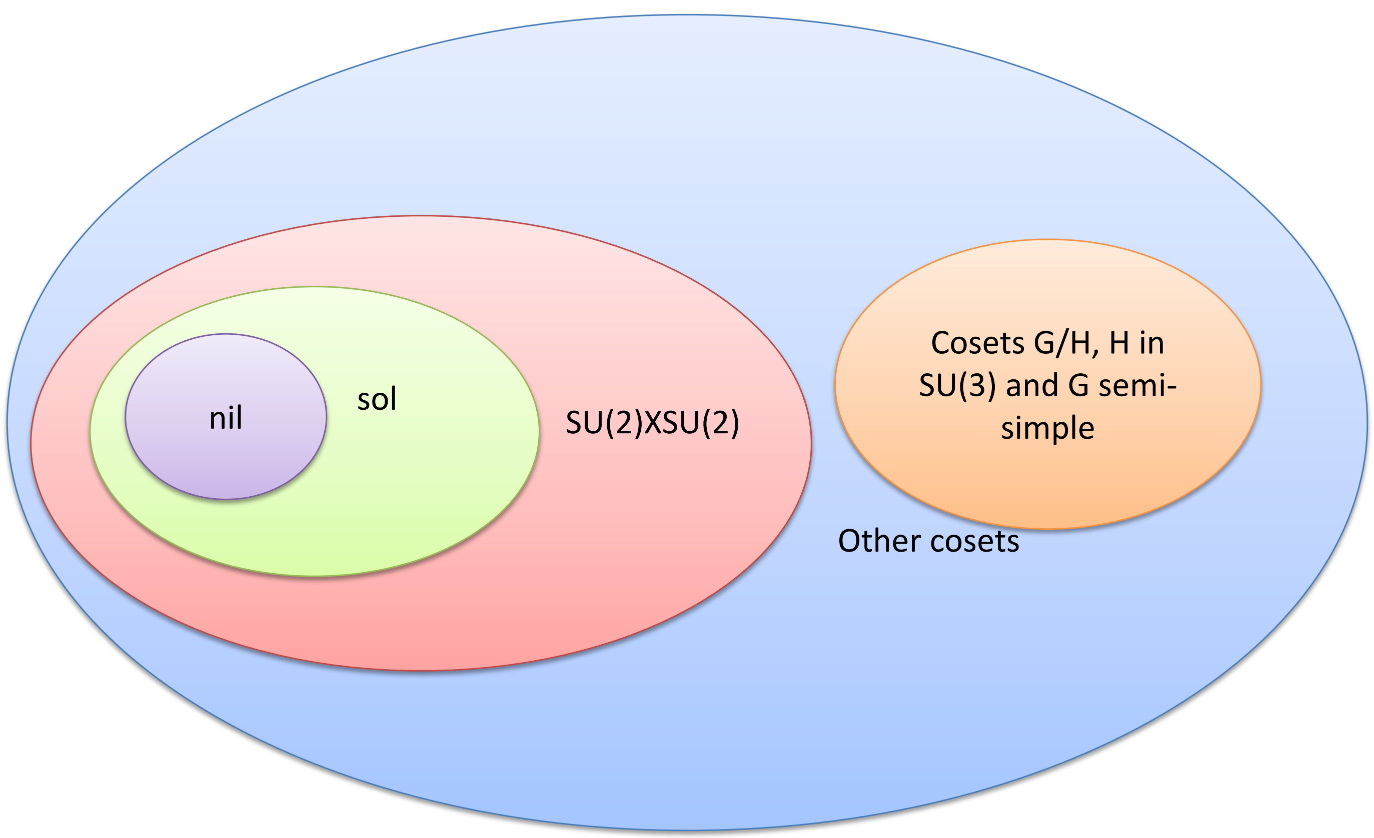


[Danielsson, Koerber, Van Riet]

A Systematic Search

[Danielsson, Haque, Koerber, GS, Van Riet, Wrase]

- Focus on homogenous spaces (G/H, H ⊆ SU(3)) where we can explicitly construct the SU(3) structure.



G=Semi-simple
[Caviezel,Koerber,Lust,Tsimpis, Zagermann]; ...

Nilmanifold

Solmanifold
[Grana, Minasian, Petrini, Tomasiello]; [Andriot, Goi, Minasian, Petrini]; ...

Unexplored!

➡ We cover ***all*** group manifolds, by classifying 6d groups.

Group Manifolds

- A coframe of left-invariant forms: $g^{-1}\mathrm{d}g = e^a T_a$

  that obeys the Maurer-Cartan relations: $\mathrm{d}e^a = -\frac{1}{2} f^a{}_{bc} e^b \wedge e^c$

- From these MC forms, we can construct J, Ω, and the metric:

$$\mathrm{d}s^2 = \mathcal{M}_{ab} e^a \otimes e^b$$

- Levi's theorem: $\mathfrak{g} = \mathfrak{s} \ltimes \mathfrak{r}$

  semi-simple $\mathfrak{s}$; radical $\mathfrak{r}$ = largest solvable ideal

  Ideal: $[\mathfrak{g}, \mathfrak{i}] \subseteq \mathfrak{i}$.

  Solvable: $\mathfrak{g}^n = [\mathfrak{g}^{n-1}, \mathfrak{g}^{n-1}]$ vanishes at some point

Group Manifolds

- Semi-simple:

Case
$\mathfrak{so}(3) \times \mathfrak{so}(3)$
$\mathfrak{so}(3) \times \mathfrak{so}(2,1)$
$\mathfrak{so}(2,1) \times \mathfrak{so}(2,1)$
$\mathfrak{so}(3,1)$

- Semi-direct product of semi-simple algebra & radical: $\mathfrak{g} = \mathfrak{s} \ltimes \mathfrak{r}$

Unimodular algebra:

$$f^a{}_{ab} = 0\,, \quad \text{for all } b$$

necessary condition for non-compact group space to be made compact.

Case	Representations
$\mathfrak{so}(3) \ltimes_\rho \mathfrak{u}(1)^3$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$ and $\rho = \mathbf{3}$
$\mathfrak{so}(3) \ltimes_\rho \mathrm{Heis}_3$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$
$\mathfrak{so}(3) \ltimes_\rho \mathfrak{iso}(2)$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$
$\mathfrak{so}(3) \ltimes_\rho \mathfrak{iso}(1,1)$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$
$\mathfrak{so}(2,1) \ltimes_\rho \mathfrak{u}(1)^3$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$, $\rho = \mathbf{1} \oplus \mathbf{2}$ and $\rho = \mathbf{3}$
$\mathfrak{so}(2,1) \ltimes_\rho \mathrm{Heis}_3$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$ and $\rho = \mathbf{1} \oplus \mathbf{2}$
$\mathfrak{so}(2,1) \ltimes_\rho \mathfrak{iso}(2)$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$
$\mathfrak{so}(2,1) \ltimes_\rho \mathfrak{iso}(1,1)$	$\rho = \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$

[Danielsson, Haque, Koerber, GS, Van Riet, Wrase]

Group Manifolds

- Solvable groups:

Name	Algebra	O5	O6	Sp
$\mathfrak{g}_{3.4}^{-1} \oplus \mathbb{R}^3$	$(q_1 23, q_2 13, 0, 0, 0, 0) \quad q_1, q_2 > 0$	14, 15, 16, 24, 25, 26, 34, 35, 36	123, 145, 146, 156, 245, 246, 256, 345, 346, 356	✓
$\mathfrak{g}_{3.5}^{0} \oplus \mathbb{R}^3$	$(-23, 13, 0, 0, 0, 0)$	14, 15, 16, 24, 25, 26, 34, 35, 36	123, 145, 146, 156, 245, 246, 256, 345, 346, 356	✓
$\mathfrak{g}_{3.1} \oplus \mathfrak{g}_{3.4}^{-1}$	$(-23, 0, 0, q_1 56, q_2 46, 0) \quad q_1, q_2 > 0$	14, 15, 16, 24, 25, 26, 34, 35, 36	-	✓
$\mathfrak{g}_{3.1} \oplus \mathfrak{g}_{3.5}^{0}$	$(-23, 0, 0, -56, 46, 0)$	14, 15, 16, 24, 25, 26, 34, 35, 36	-	✓
$\mathfrak{g}_{3.4}^{-1} \oplus \mathfrak{g}_{3.5}^{0}$	$(q_1 23, q_2 13, 0, -56, 46, 0) \quad q_1, q_2 > 0$	14, 15, 16, 24, 25, 26, 34, 35, 36	-	✓
$\mathfrak{g}_{3.4}^{-1} \oplus \mathfrak{g}_{3.4}^{-1}$	$(q_1 23, q_2 13, 0, q_3 56, q_4 46, 0) \quad q_1, q_2, q_3, q_4 > 0$	14, 15, 16, 24, 25, 26, 34, 35, 36	-	✓
$\mathfrak{g}_{3.5}^{0} \oplus \mathfrak{g}_{3.5}^{0}$	$(-23, 13, 0, -56, 46, 0)$	14, 15, 16, 24, 25, 26, 34, 35, 36	-	✓
$\mathfrak{g}_{4.5}^{p,-p-1} \oplus \mathbb{R}^2$	?			-
$\mathfrak{g}_{4.6}^{-2p,p} \oplus \mathbb{R}^2$	?			-
$\mathfrak{g}_{4.8}^{-1} \oplus \mathbb{R}^2$	$(-23, q_1 34, q_2 24, 0, 0, 0) \quad q_1, q_2 > 0$	14, 25, 26, 35, 36	145, 146, 256, 356	-
$\mathfrak{g}_{4.9}^{0} \oplus \mathbb{R}^2$	$(-23, -34, 24, 0, 0, 0)$	14, 25, 26, 35, 36	145, 146, 256, 356	-
$\mathfrak{g}_{5.7}^{1,-1,-1} \oplus \mathbb{R}$	$(q_1 25, q_2 15, q_2 45, q_1 35, 0, 0) \quad q_1, q_2 > 0$	13, 14, 23, 24, 56	125, 136, 146, 236, 246, 345	✓
$\mathfrak{g}_{5.8}^{-1} \oplus \mathbb{R}$	$(25, 0, q_1 45, q_2 35, 0, 0) \quad q_1, q_2 > 0$	13, 14, 23, 24, 56	125, 136, 146, 236, 246, 345	✓
$\mathfrak{g}_{5.13}^{-1,0,r} \oplus \mathbb{R}$	$(q_1 25, q_2 15, -q_2 r 45, q_1 r 35, 0, 0)\ r \neq 0,\ q_1, q_2 > 0$	13, 14, 23, 24, 56	125, 136, 146, 236, 246, 345	✓
$\mathfrak{g}_{5.14}^{0} \oplus \mathbb{R}$	$(-25, 0, -45, 35, 0, 0)$	13, 14, 23, 24, 56	125, 136, 146, 236, 246, 345	✓
$\mathfrak{g}_{5.15}^{-1} \oplus \mathbb{R}$	$(q_1(25-35), q_2(15-45), q_2 45, q_1 35, 0, 0) \quad q_1, q_2 > 0$	14, 23, 56	146, 236	✓
$\mathfrak{g}_{5.17}^{p,-p,r} \oplus \mathbb{R}$	$(q_1(p25+35), q_2(p15+45), q_2(p45-15), q_1(p35-25), 0, 0)$ $r^2 = 1,\ q_1, q_2 > 0$	14, 23, 56 $p = 0$: 12, 34	146, 236 $p = 0$: 126, 135, 245, 346	✓
$\mathfrak{g}_{5.18}^{0} \oplus \mathbb{R}$	$(-25-35, 15-45, -45, 35, 0, 0)$	14, 23, 56	146, 236	✓
$\mathfrak{g}_{6.3}^{0,-1}$	$(-26, -36, 0, q_1 56, q_2 46, 0) \quad q_1, q_2 > 0$	24, 25	134, 135, 456	✓
$\mathfrak{g}_{6.10}^{0,0}$	$(-26, -36, 0, -56, 46, 0)$	24, 25	134, 135, 456	✓

[Turkowski];[Andriot,Goi,Petrini,Minasian];
[Grana,Minasian,Petrini,Tomasiello]

Orientifolding

- dS critical point of effective N=1 SUGRA from group manifolds.
- Orbifolding further by discrete $\Gamma \subset SU(3)$.
- Among the Abelian orbifolds of (twisted) T^6, only two $Z_2 \times Z_2$ orientifolds can evade $\varepsilon \geq O(1)$ [Flauger, Paban, Robbins, Wrase]
- Consider $Z_2 \times Z_2$ orientifolds of the group spaces we classified.

$$\theta_1 : \begin{cases} e^1 & \to & -e^1 \\ e^2 & \to & -e^2 \\ e^3 & \to & e^3 \\ e^4 & \to & -e^4 \\ e^5 & \to & e^5 \\ e^6 & \to & -e^6 \end{cases} , \qquad \theta_2 : \begin{cases} e^1 & \to & -e^1 \\ e^2 & \to & e^2 \\ e^3 & \to & -e^3 \\ e^4 & \to & e^4 \\ e^5 & \to & -e^5 \\ e^6 & \to & -e^6 \end{cases} \qquad \sigma : \begin{cases} e^1 & \to & e^1 \\ e^2 & \to & e^2 \\ e^3 & \to & e^3 \\ e^4 & \to & -e^4 \\ e^5 & \to & -e^5 \\ e^6 & \to & -e^6 \end{cases}$$

[Other $Z_2 \times Z_2$ orientifold has a different σ]

Constructing SU(3) Structure

e^1	e^2	e^3	e^4	e^5	e^6
$\otimes$	$\otimes$	$\otimes$	$-$	$-$	$-$
$\otimes$	$-$	$-$	$\otimes$	$\otimes$	$-$
$-$	$\otimes$	$-$	$-$	$\otimes$	$\otimes$
$-$	$-$	$\otimes$	$\otimes$	$-$	$\otimes$

- O-planes: $j_6 = j_A e^{456} + j_B e^{236} + j_C e^{134} + j_D e^{125}$
- J and Ω_R are odd under orientifolding:

$$J = ae^{16} + be^{24} + ce^{35}\,,$$
$$\Omega_R = v_1 e^{456} + v_2 e^{236} + v_3 e^{134} + v_4 e^{125}\,,$$

- The metric fluxes are even:

$$\mathrm{d}e^1 = f^1{}_{23}e^{23} + f^1{}_{45}e^{45}\,, \qquad \mathrm{d}e^2 = f^2{}_{13}e^{13} + f^2{}_{56}e^{56}\,,$$
$$\mathrm{d}e^3 = f^3{}_{12}e^{12} + f^3{}_{46}e^{46}\,, \qquad \mathrm{d}e^4 = f^4{}_{36}e^{36} + f^4{}_{15}e^{15}\,,$$
$$\mathrm{d}e^5 = f^5{}_{14}e^{14} + f^5{}_{26}e^{26}\,, \qquad \mathrm{d}e^6 = f^6{}_{34}e^{34} + f^6{}_{25}e^{25}\,.$$

- Metric g and Ω_I can be expressed in terms of the “moduli”:

$$g = \frac{1}{\sqrt{v_1 v_2 v_3 v_4}}\Big(av_3v_4\,,\; -bv_2v_4\,,\; cv_2v_3\,,\; -bv_1v_3\,,\; cv_1v_4\,,\; av_1v_2\Big) \qquad \sqrt{v_1v_2v_3v_4} = -abc$$

$$\Omega_I = \sqrt{v_1v_2v_3v_4}\Big(v_1^{-1}\,e^{123} + v_2^{-1}\,e^{145} - v_3^{-1}\,e^{256} - v_4^{-1}e^{346}\Big)$$

Constructing SU(3) Structure

- Parity under orientifolding implies Im W_1= Im W_2= W_4 = W_5=0

  ➡ Half-flat SU(3) Structure Manifold

- Construct the remaining torsion classes:

$$
\begin{aligned}
W_1 &= -\tfrac{1}{6} \star_6 (\mathrm{d}J \wedge \Omega_I)\,, \\
W_2 &= - \star \mathrm{d}\Omega_I + 2W_1 J\,, \\
W_3 &= \mathrm{d}J - \tfrac{3}{2} W_1 \Omega_R\,.
\end{aligned}
$$

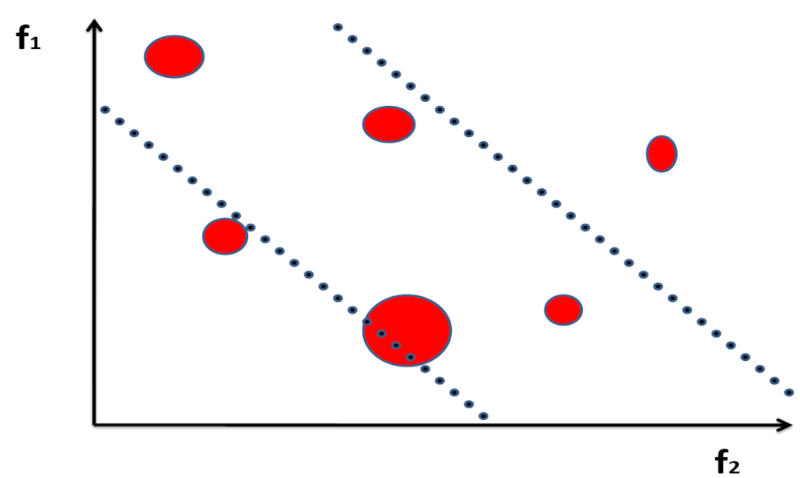
- Search for dS solutions satisfying constraints obtained earlier.

Challenges

- In all models, there are at least one tachyon among the left-invariant modes! (generic? c.f. [Gomez-Reino, Louis, Scrucca], ...)

- Flux quantization:

Pictorially



For SU(2)xSU(2) examples, can explicitly check flux quantization demands solutions outside SUGRA.

- Backreaction of localized sources: $\boxed{R_6 = +\frac{1}{4}T_6 - \frac{3}{4}T_4}$ Douglas,Kallosh

constant negative curvature

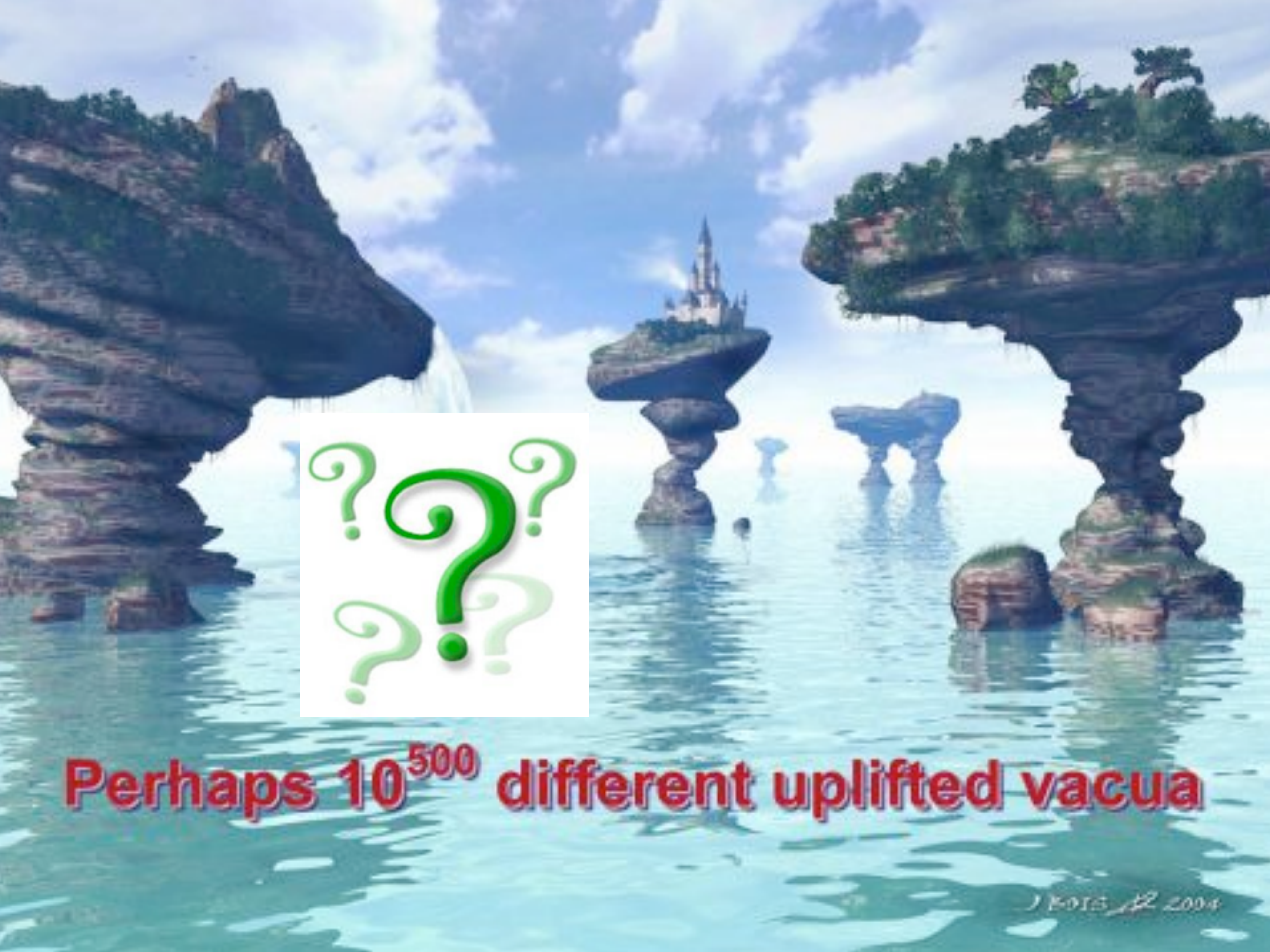
localized negative tension

See [Blaback, Danielsson, Junghans, Van Riet, Wrase, Zagermann]


de Sitter solutions are hard to find

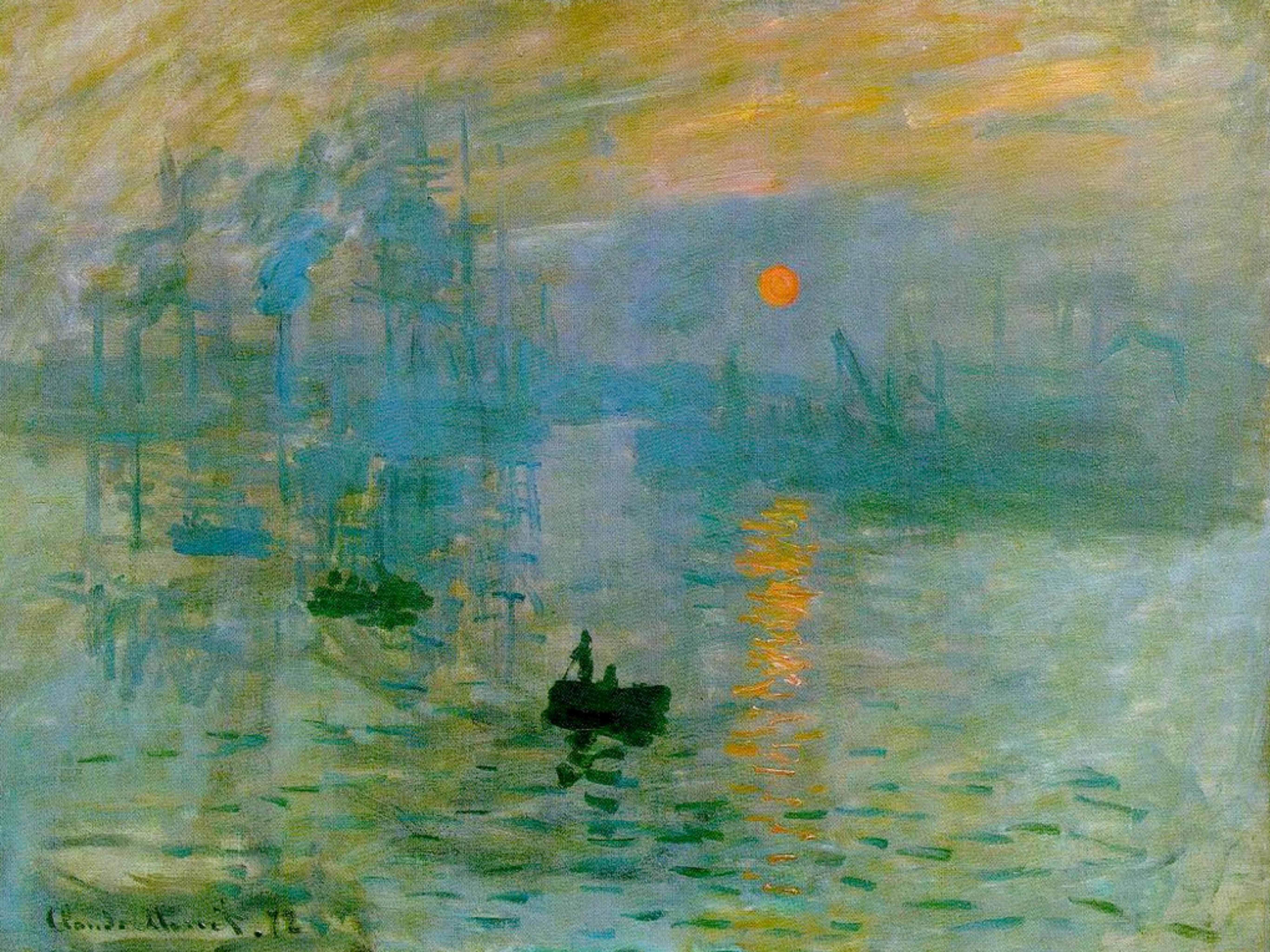
de Sitter solutions are hard to find

In candidate vacua, tachyons seem ubiquitious


Perhaps 10^{500} different uplifted vacua

Summary

- No-go theorems for de Sitter vacua/inflation from string theory, and the *minimal* ingredients to evade them.
- Motivate dS construction from SU(3) structure manifolds.
- Bottom-up approach: de Sitter ansatz.
- A systematic search for dS vacua within a broad class of group manifolds that admit an explicit construction of SU(3) structure.
- dS solutions hard to come by; even for solutions found, tachyons seem ubiquitous. Other issues: backreaction, flux quantization.
- In the next lecture, we will try to understand this difficulty in finding de Sitter vacua statistically, using random matrix theory.



ありがとう

THANKS