CMB imprints of a pre-inflationary climbing phase

Noriaki Kitazawa
Tokyo Metropolitan University

E.Dudas, NK, S.P.Patil and A.Sagnotti, JCAP 05 (2012) 012,
E.Dudas, NK, A.Sagnotti, Phys. Lett. B694 (2010) 80

1. Some motivations to string models and "Climbing Phase"

The Standard Model gives precise description of particle physics in laboratories.

Lack of the origin of Yukawa couplings
-> no understanding on the flavor structure

Lack of the gravitational interaction
-> impossible to describe cosmological phenomena
(possible at classical level by including gravity and additional fields (dark matter and inflaton ...)

Quantum field theory --> “String Theory”

Geometrical understandings on Yukawa couplings
-> the complicated flavor structure may be come from rather simple geometry (including D-brane configurations)

Natural inclusion of the gravitational interaction and the matter beyond the Standard Model
-> natural descriptions or understandings of the cosmology may be possible

Here, we think “String Theory” as a tool to describe physics,
not “Theory of Everything”.
(Surely, the quantum field theory has been a good tool.)

Unfortunately, “String Theory” is not well-constructed as that for the quantum field theory.

We do not have a complete model which include the Standard Model and satisfactory describes particle physics in laboratories.

How can we have predictions of “String Theory” to the cosmology, especially to the cosmological phenomena on which physics at very high energies plays important role?

What only we can do at present is listing up possible predictions under some reasonable assumptions, and check them by observations.

“Climbing phase” is a possible prediction of “String Theory” under the following assumptions.

- Non-SUSY string models with so called “brane SUSY breaking”, or KKLT scenario with SUSY breaking uplifts.
  -> exponential-type potentials for moduli fields

- Tadpoles (NS-NS) can be considered perturbatively.
  (exponential-type potentials give linear terms.)

- String loop corrections are small.
  (g_s corrections are small.)

- The effects of higher derivative terms are small.
  (α' corrections are small.)

2. “Brane supersymmetry breaking” and “Climbing Phase”

Charges of D-branes and O-planes (Orientifold fixed planes)

	tension (mass)	RR charge
D-branes	positive	positive
anti-D-branes	positive	negative
O_+-plane	negative	negative
O_--plane	positive	positive

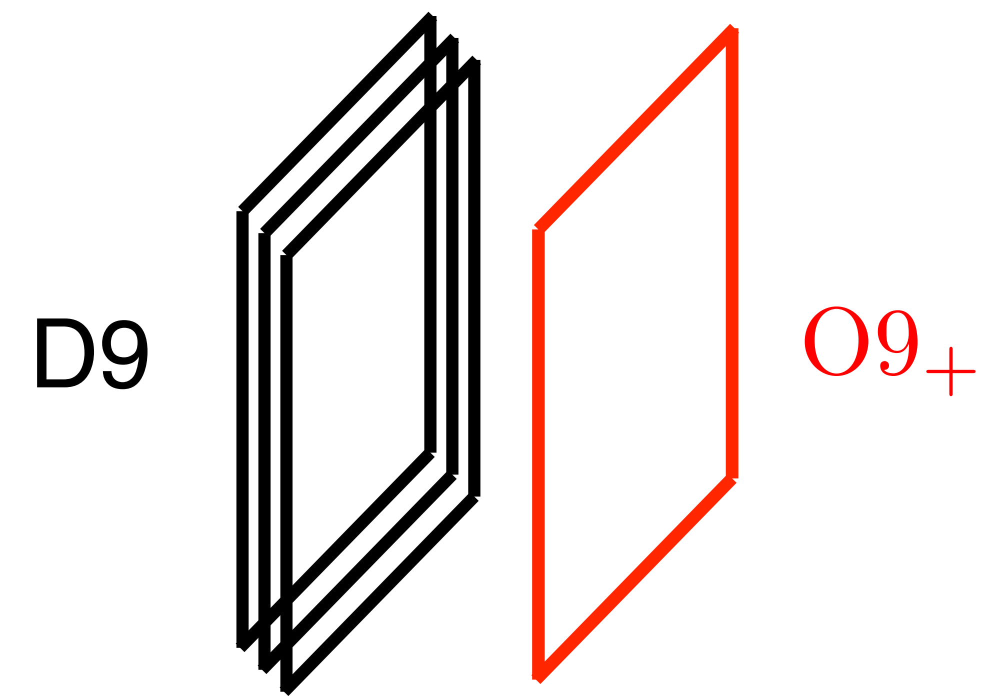
Angelantonj-Sagnotti (2002)

Cancelation of RR charges -> anomaly free

Cancelation of tension -> supersymmetry

Example 1 : type I superstring theory

type I = type IIB + orientifold projection + 32 D9-branes,
namely,
a system of 32 D9-brane and an $\mathrm{O9}_{+}$-plane.



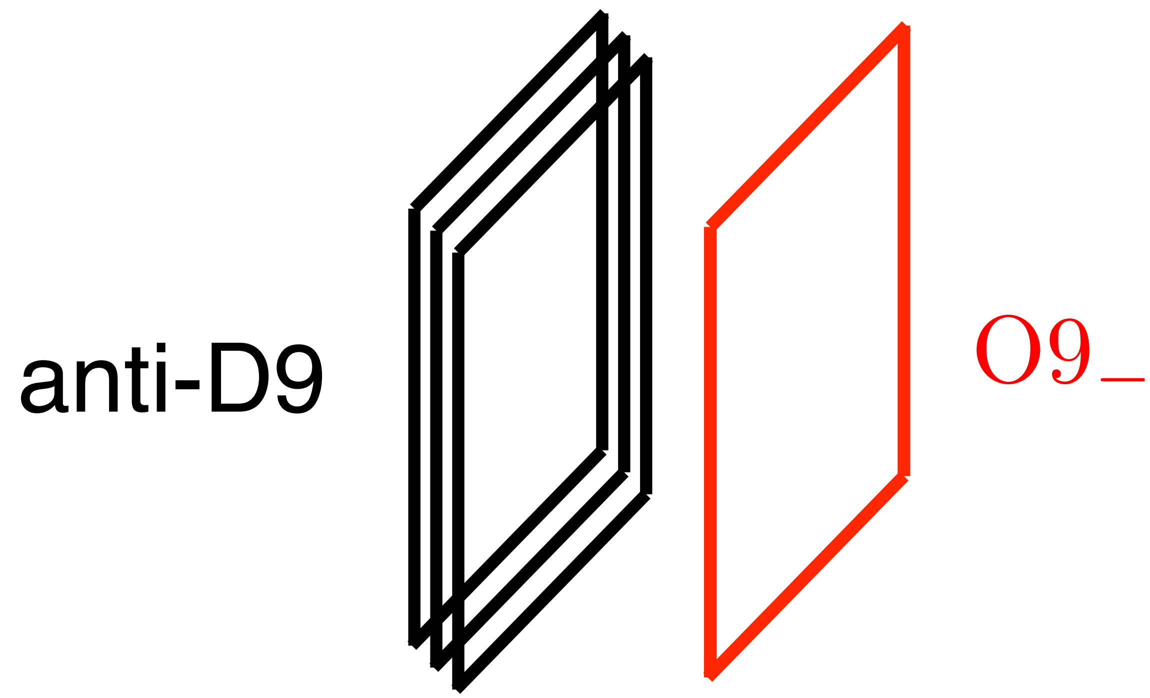
Both tension and RR charges are canceled out.

A consistent theory:
10D N=1 SUSY SO(32) gauge theory with gravity.

Example 2 : Sugimoto model

Sugimoto (1999)

a system of 32 anti-D9-brane and an $\mathrm{O9}_{-}$-plane.



RR charges are canceled out, but not for tensions.

A consistent theory:
10D non-SUSY Sp(32) gauge theory with gravity
(no tachyonic states).

"Brane SUSY Breaking"

Antoniadis-Dudas-Sagnotti (1999)

There is a tadpole for dilaton ϕ in this model.

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-g} \left\{ \left[R - \frac{1}{2}(\partial\phi)^2 - 2\kappa_{10}^2 \cdot 32T_9\, e^{\frac{3\phi}{2}} \right] - \frac{1}{12}\, e^{\phi} H^2 \right\}$$

$$\boxed{T_p \equiv \frac{\sqrt{\pi}}{\kappa_{10}} \left(4\pi^2\alpha'\right)^{\frac{3-p}{2}}}$$

This exponential potential includes a tadpole term $-3\kappa_{10}^2 \cdot 32T_9\phi$, which indicates that the present vacuum is not a solution of “String Theory”.

Fischler-Susskind (1986)

We simply assume that tadpole can be considered perturbatively.

Make a compactification (dimensional reduction) keeping only overall breathing mode:

$$g_{IJ}^{(10)} = e^{\sigma}\delta_{IJ}, \quad g_{\mu\nu}^{(10)} = e^{-3\sigma} g_{\mu\nu}^{(4)}$$

Effective action in four-dimensional space-time:

$$S_4 \;=\; \frac{1}{2}\int d^4x\,\sqrt{-g}\,\Big[R \,-\, (\partial\Phi_s)^2 \,-\, (\partial\Phi_t)^2 \,-\, 2\,\alpha\; e^{-\sqrt{6}\Phi_t} + \cdots\Big]$$

$$e^{3\sigma}e^{\frac{\phi}{2}} \;=\; e^{\sqrt{2}\Phi_s}, \qquad e^{\sigma}e^{-\frac{\phi}{2}} \;=\; e^{\sqrt{\frac{2}{3}}\;\Phi_t}$$

$$\kappa_4^2 = \kappa_{10}^2/V_6 = 1, \qquad \alpha \propto 32T_9$$

We have an exponential potential for a moduli field.

Remember the value of its logarithmic slope $\sqrt{6}$.

“Climbing Phase”

Start with a simple action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

with metric

$$ds^2 = -e^{2B(\xi)}d\xi^2 + e^{2A(\xi)}d\mathbf{x}\cdot d\mathbf{x}.$$

Take a convenient gauge to solve equations of motion:

$$V(\phi)e^{2B} = \frac{M^2}{2}$$

M is some scale in the potential

With the redefinitions

$$\tau = M\sqrt{\frac{3}{2}}\xi, \quad \varphi = \sqrt{\frac{3}{2}}\phi, \quad \mathcal{A} = 3A,$$

we have equations of motion

$$\frac{d\mathcal{A}}{d\tau} = \sqrt{1 + \left(\frac{d\varphi}{d\tau}\right)^2}\ ,$$

$$\frac{d^2\varphi}{d\tau^2} + \frac{d\varphi}{d\tau}\sqrt{1 + \left(\frac{d\varphi}{d\tau}\right)^2} + \left[1 + \left(\frac{d\varphi}{d\tau}\right)^2\right]\frac{1}{2V}\frac{\partial V}{\partial\varphi} = 0\ .$$

Consider the solutions of the second equation with

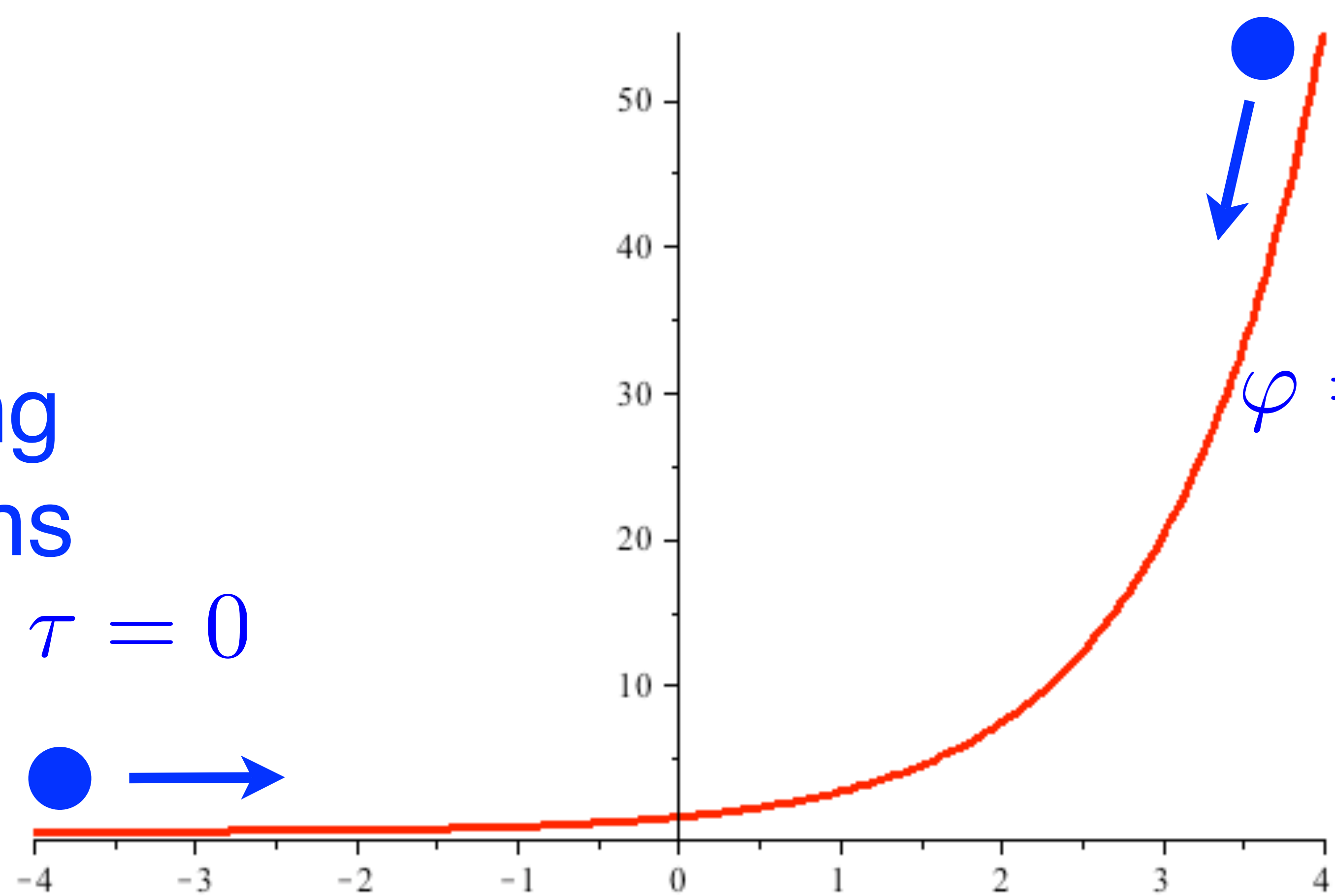
$$V = \frac{M^2}{2} e^{2\gamma\varphi} \ .$$

The value of $\gamma = 1$ corresponds to the logarithmic slope $\sqrt{6}$.

We can expect two types of solutions from "Big-Bang singularity" $a = e^{\mathcal{A}/3} = 0 \text{ at } \tau = 0$.

climbing solutions

$\varphi = -\infty \text{ at } \tau = 0$



descending solutions

$\varphi = +\infty \text{ at } \tau = 0$

For $0 < \gamma < 1$, both two types of the solutions are allowed.

$$\varphi = \varphi_0 + \frac{1}{1+\gamma} \ln \sinh(\frac{\tau}{2}\sqrt{1-\gamma^2}) - \frac{1}{1-\gamma} \ln \cosh(\frac{\tau}{2}\sqrt{1-\gamma^2})$$

"climbing solutions"

$$\varphi = \varphi_0 + \frac{1}{1+\gamma} \ln \cosh(\frac{\tau}{2}\sqrt{1-\gamma^2}) - \frac{1}{1-\gamma} \ln \sinh(\frac{\tau}{2}\sqrt{1-\gamma^2})$$

"descending solutions"

For $\gamma \geq 1$, only climbing solutions are allowed.

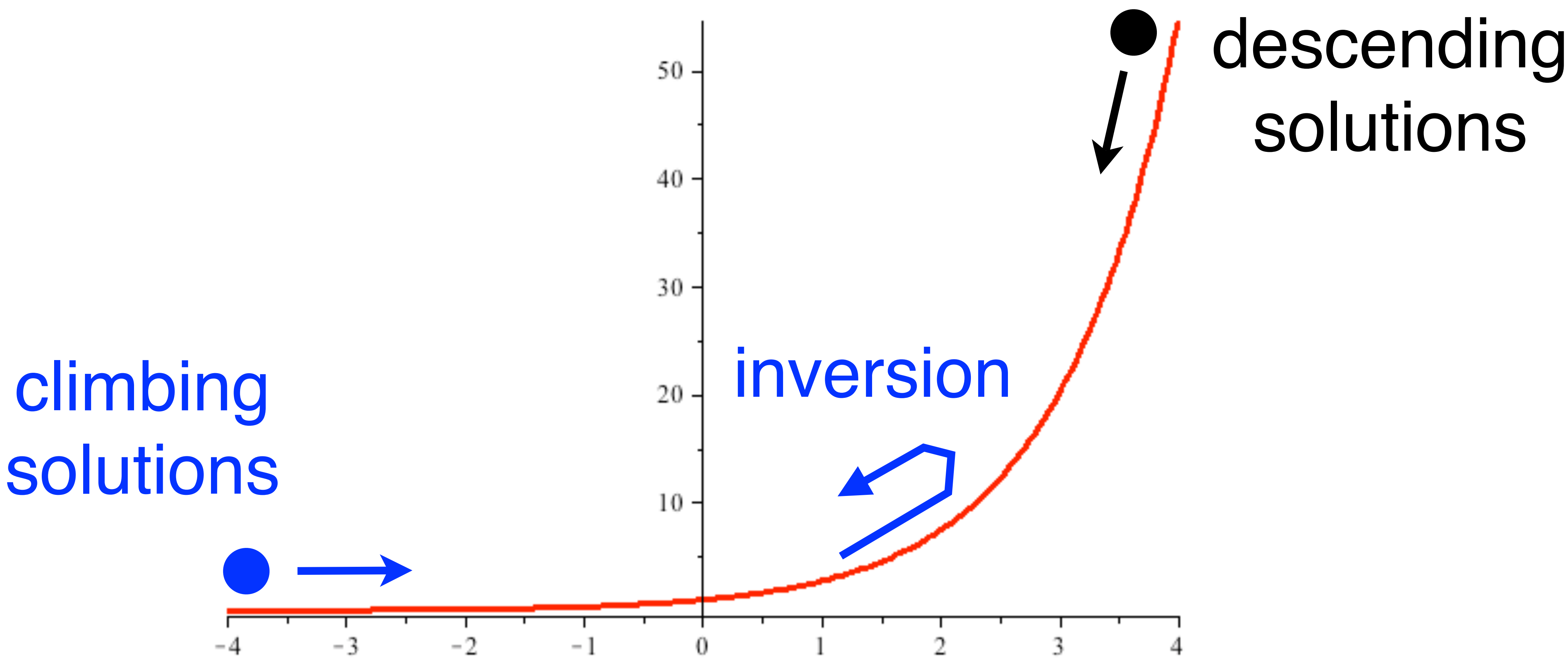
"Climbing Phenomena"

Russo (2004)
Dudas-NK-Sagnotti (2010)

$$\varphi = \varphi_0 + \frac{1}{2} \ln \tau - \frac{\tau^2}{4}, \quad \text{for } \gamma = 1,$$

$$\varphi = \varphi_0 + \frac{1}{\gamma+1} \ln \sin(\frac{\tau}{2}\sqrt{\gamma^2-1}) + \frac{1}{\gamma-1} \ln \cos(\frac{\tau}{2}\sqrt{\gamma^2-1}), \text{ for } \gamma > 1.$$

The reason of "climbing phenomena"



There is an asymptotic attractor solution with "velocity"

$$\frac{d\varphi}{d\tau} = -\frac{\gamma}{\sqrt{1-\gamma}} \ .$$

Lucchin-Matarrese (1985)

Since the "velocities" of descending solutions approach to this "limiting velocity" from the above, they must disappear for $\gamma \geq 1$.

Remember that a 4D compactification of Sugimoto model gives the system with $\gamma = 1$.

Although there is no guarantee that "climbing phenomena" is always realized in the present class of string models, we can imagine that climbing solutions are realized, or the system is in "climbing phase".

Many kinds of corrections (g_s corrections, α' corrections, for example) might destroy "climbing phenomena", but there are possibilities that some solutions with climbing up the potential survive and realize in Nature.

"Climbing phase" gives some imprints in CMB?

3. KKLT scenario and "climbing phenomena"

Consider typeIIB theory with flux compactification.

Assume that all the complex structure moduli and dilaton are stabilized by the effect of the flux, and assume that only an overall Kähler moduli remains to be stabilized.

Superpotential and Kähler potential for the moduli field:

$$W = W_0 + ae^{-bT}, \qquad K = -3\ln(T+\bar{T})$$

"non-SUSY flux" non-perturbative effect three complex space dimensions of the compact space.

The real part of $T = e^{\sqrt{\frac{2}{3}}\Phi_t} + i\sqrt{\frac{2}{3}}\theta$ is stabilized with anti-de Sitter vacuum.

Consider F-term uplift potential with SUSY breaking to obtain small cosmological constant

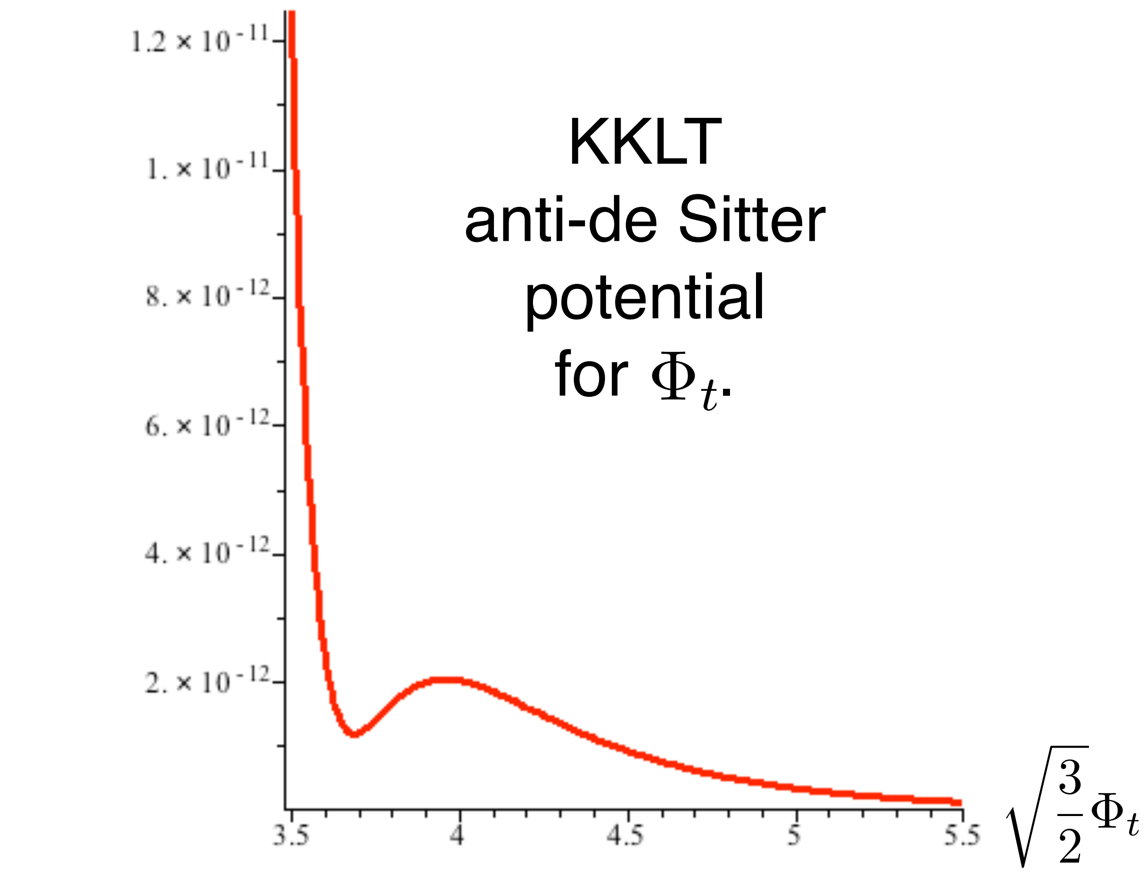
$$V = V_F + \frac{c}{(T+\bar{T})^3} = V_F + \frac{c}{8} e^{-\sqrt{6}\Phi_t}$$

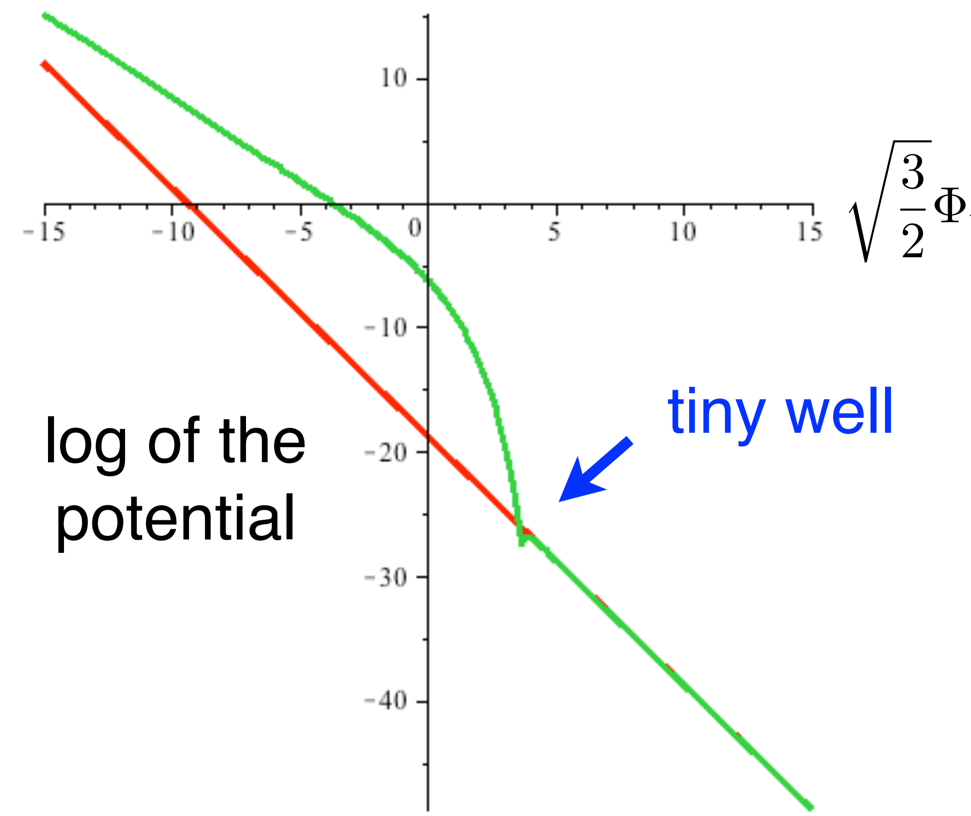
$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} (\partial \Phi_t)^2 - \frac{1}{2} e^{\frac{2\sqrt{2}}{\sqrt{3}} \Phi_t} (\partial \theta)^2 - V(\Phi_t, \theta) \right]$$

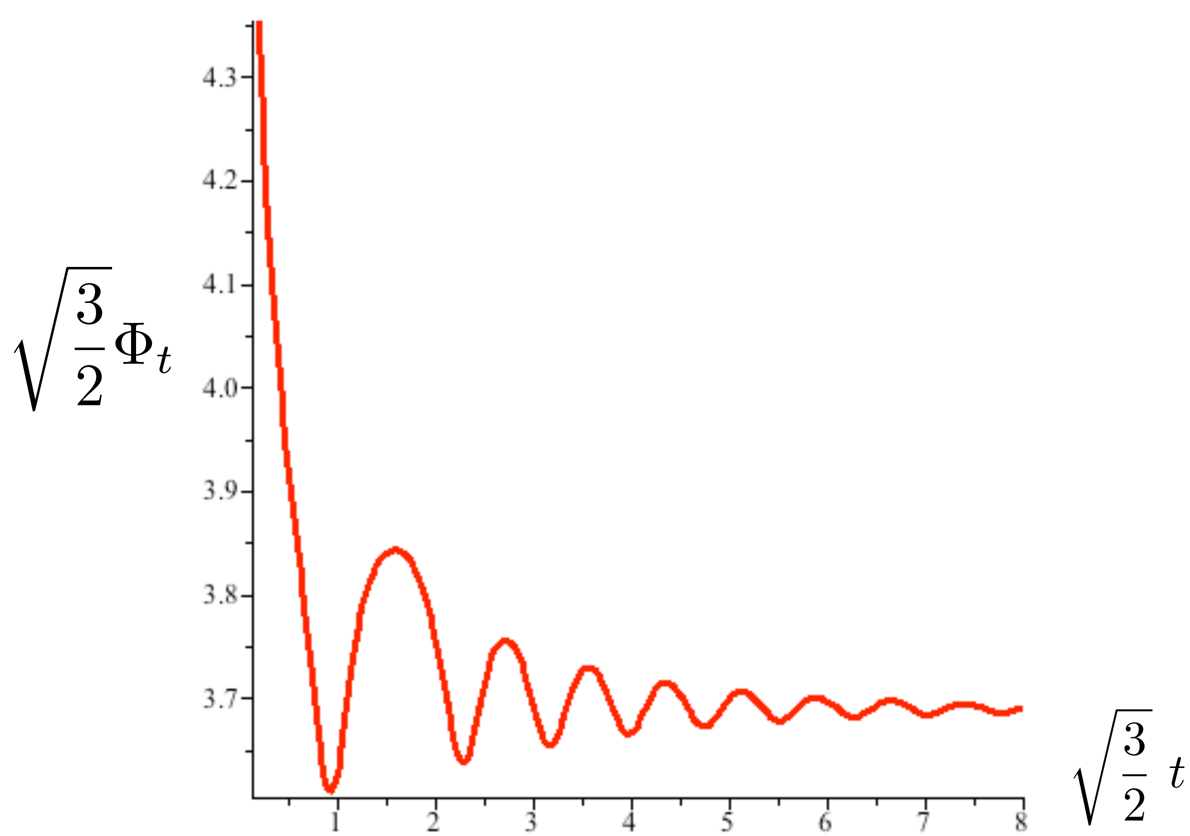
Again, we obtain an exponential potential with critical logarithmic slope.

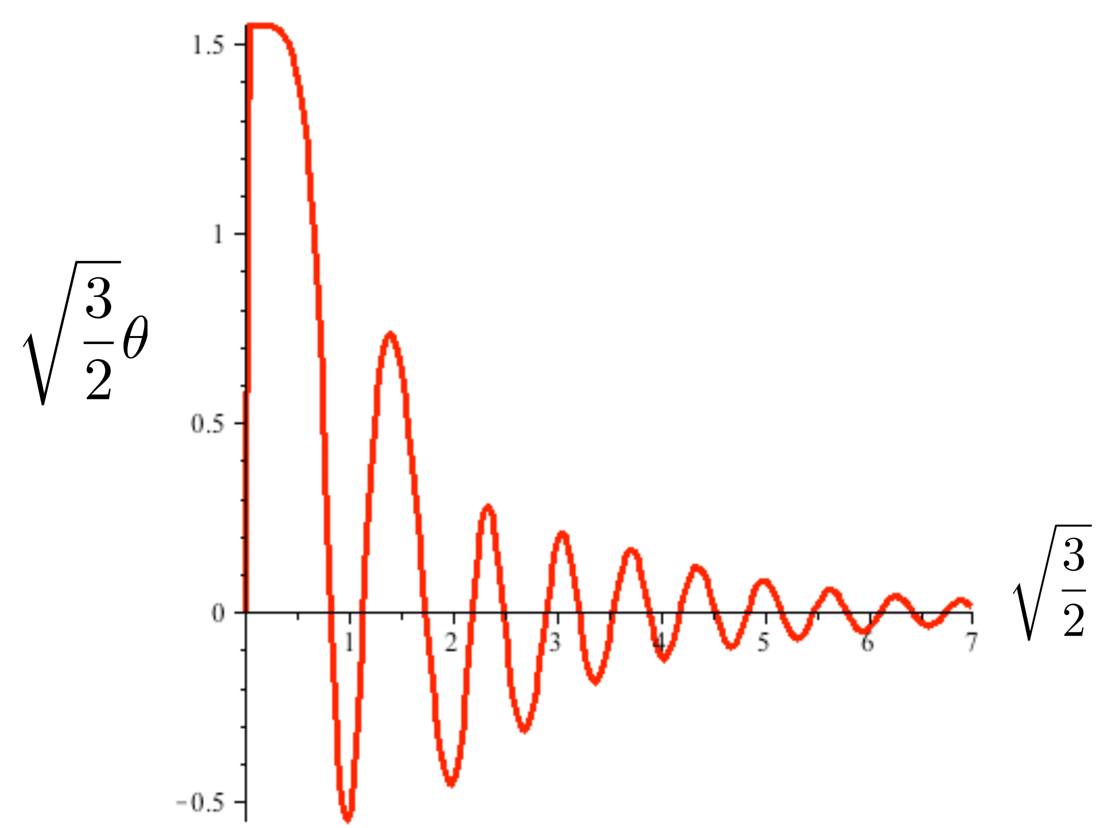
It is difficult to rigorously proof "climbing phenomena" in this complicated non-linear system, but we can find climbing solutions, namely, "climbing phase" is possible in this system.

“dynamical moduli stabilization?”









4. “Climbing phase” and CMB

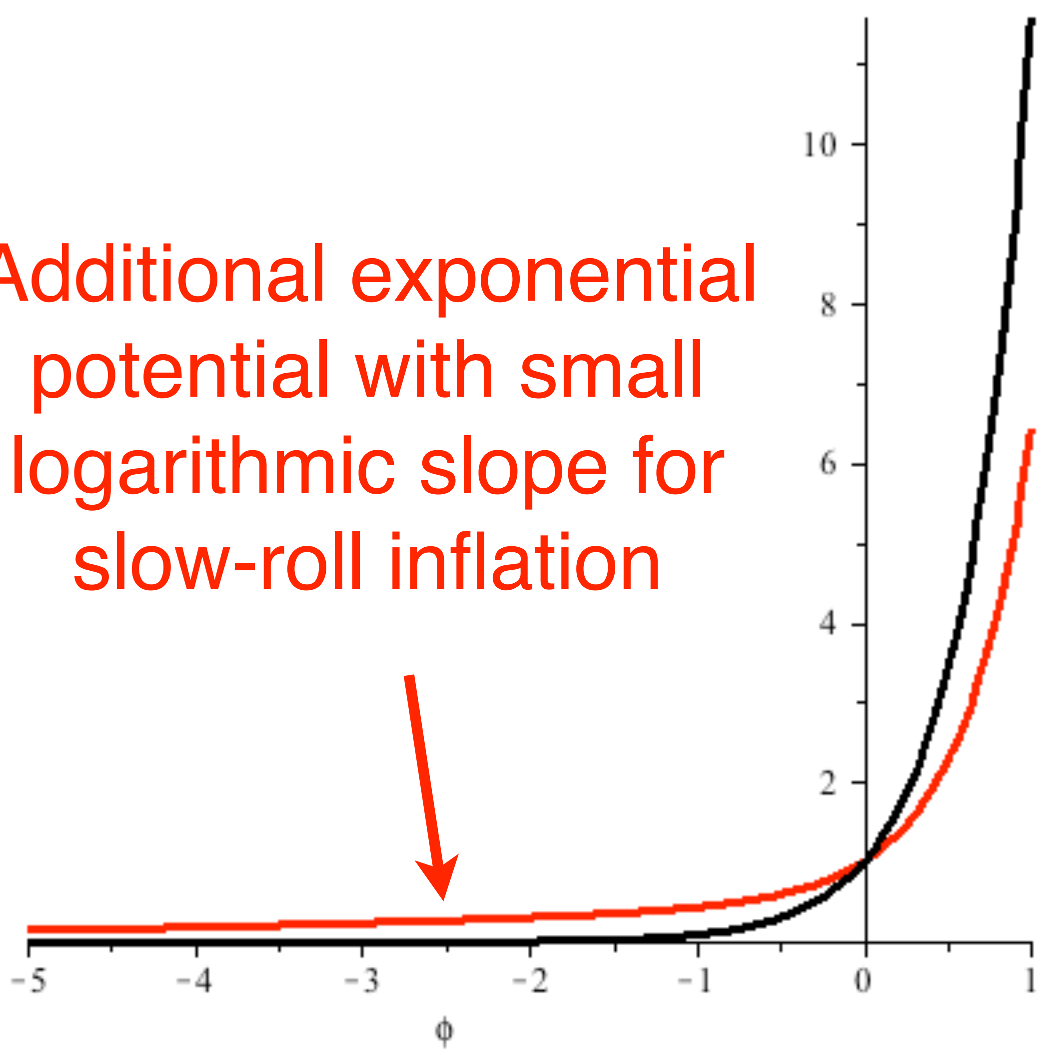
$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right],$$

$$V = M^2 e^{\sqrt{6}\phi}$$

$$V = \frac{M^2}{2}\left(e^{\sqrt{6}\phi} + e^{\sqrt{6}\gamma\phi}\right)$$

$$\gamma = \frac{1}{4\sqrt{6}}$$

Additional exponential potential with small logarithmic slope for slow-roll inflation



The inversion point in climbing solutions do not give enough number of e-folds.

“Climbing phenomena” are not ruined by additional potential (descending solutions are forbidden).

Numerically solve background field equations
(equations of motion for ϕ and scale factor a)
as well as the equation for fluctuations:
Mukhanov-Sasaki equation

$$\left(z \equiv \frac{a}{H}\frac{d\phi}{dt}\right)$$

$$\left[\frac{d^2}{dt^2} + H\frac{d}{dt} + \frac{k^2}{a^2} - \frac{1}{z}\left(\frac{d^2 z}{dt^2} + H\frac{dz}{dt}\right)\right] S_{\mathcal{R}}(k;t) = 0.$$

The initial conditions for background fields and fluctuations are set at enough early time with kinetic energy dominance.

- Use approximate analytic solutions of background fields: "climbing solution" for smaller logarithmic slopes
  (an integration constant: ϕ_0)
- Use an approximate analytic solution for $S_{\mathcal{R}}$ in case of the above background.

The primordial power spectrum is given by

$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\frac{|S_{\mathcal{R}}(k;\eta_{\rm LS})|^2}{z(\eta_{\rm LS})^2}$$

η : conformal time $(ad\eta = dt)$

η_{LS} : comformal time at the last scattering

The CMB multipole coefficient for $l < 30$ is given by

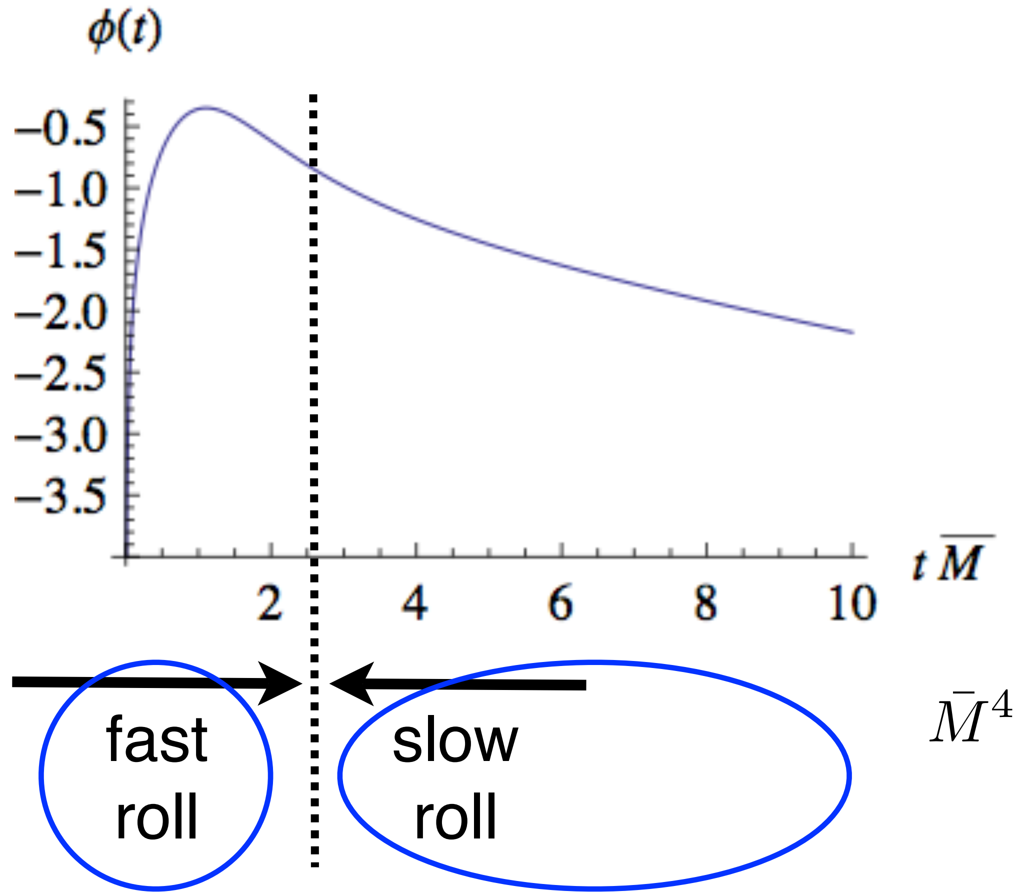
$$C_l = \frac{4\pi}{9}\int_0^\infty \frac{dk}{k} P_{\mathcal{R}}(k)\Big\{j_l(k(\eta_0 - \eta_{\rm LS}))\Big\}^2.$$

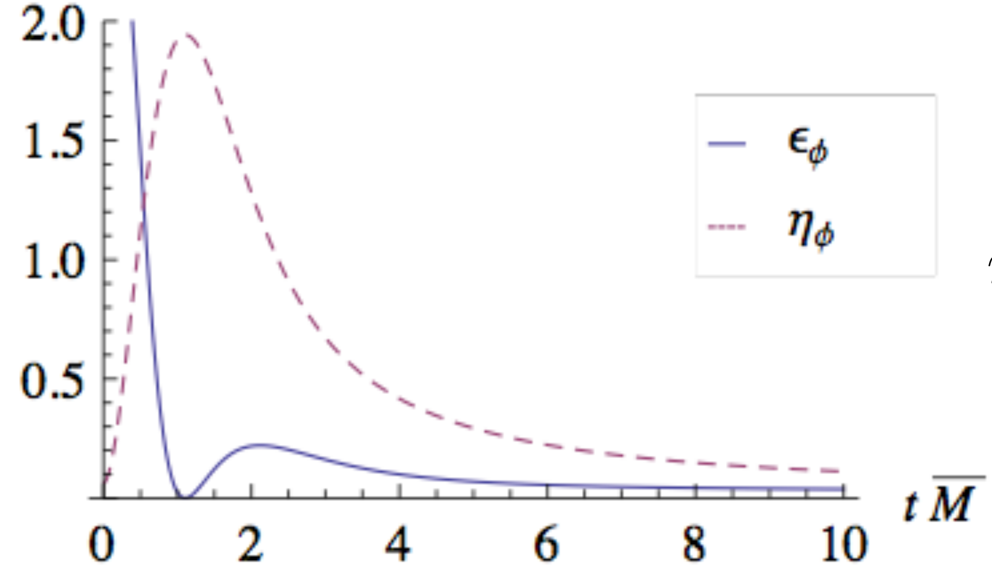
Mukhanov (2004)

η_0 : comformal time at present

$(\eta_0 - \eta_{\rm LS} = 3.296/H_0)$

Background fields for $\gamma = \frac{1}{4\sqrt{6}}, \quad \phi_0 = -4$

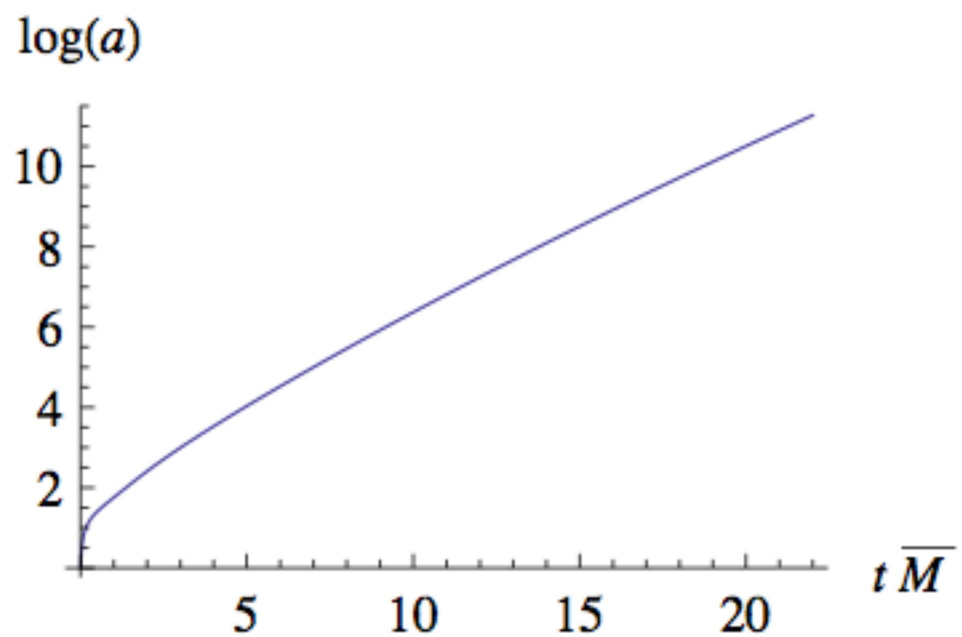


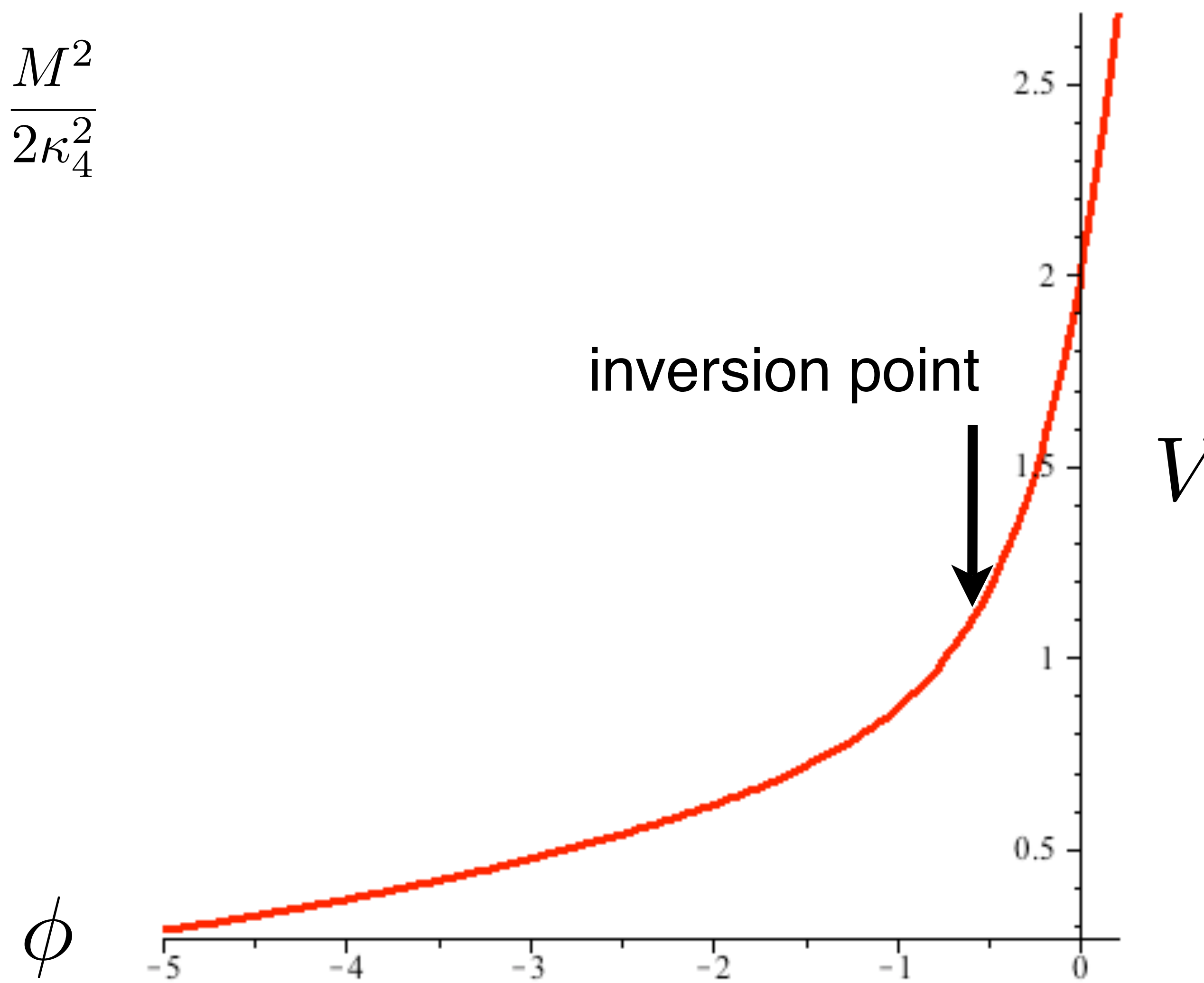


$$\epsilon_\phi = -\frac{\dot{H}}{H^2}$$

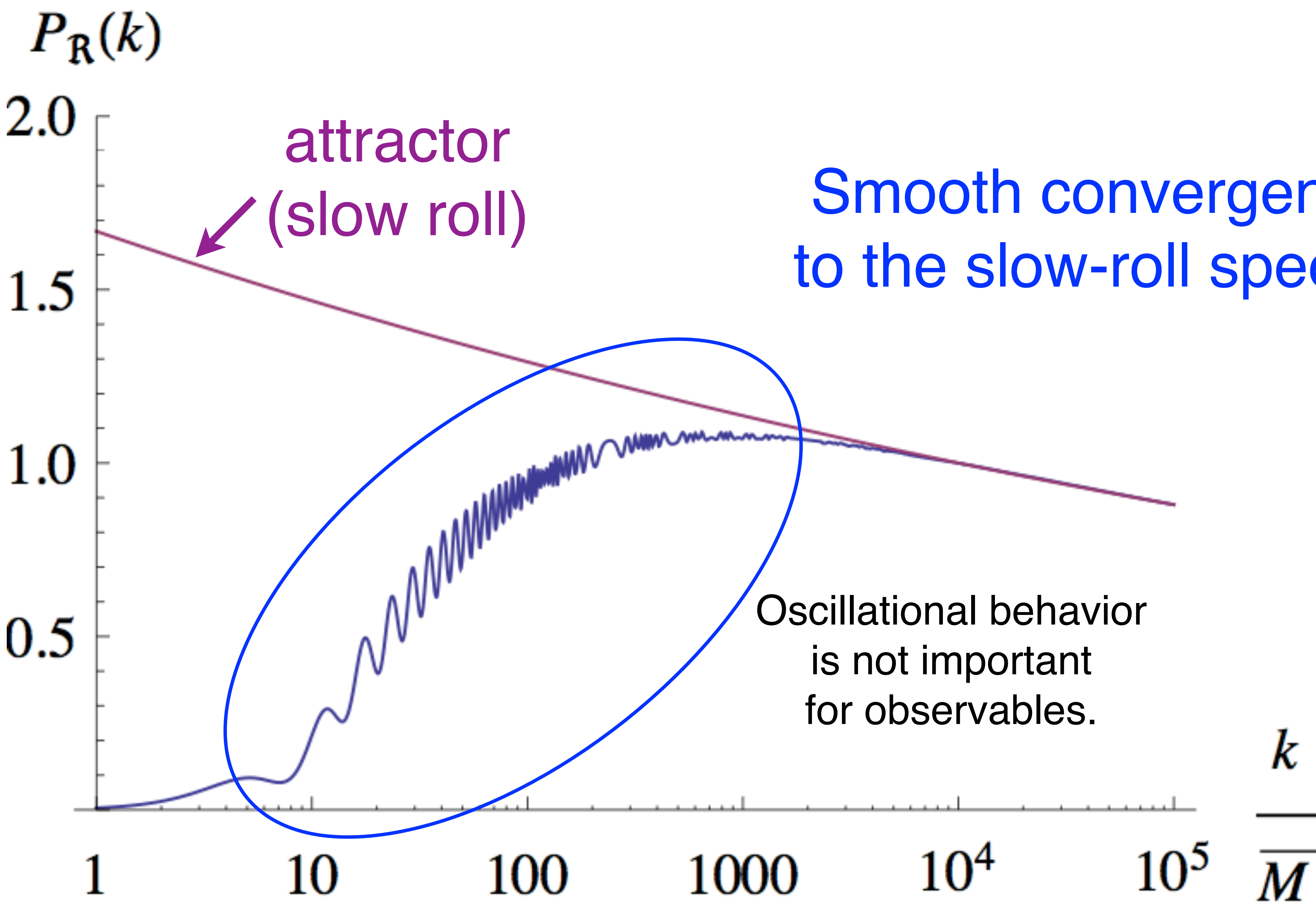
$$\eta_\phi = \frac{1}{V}\frac{\partial^2 V}{\partial \phi^2}$$

$$\bar{M}^4 = \frac{M^2}{2\kappa_4^2}$$



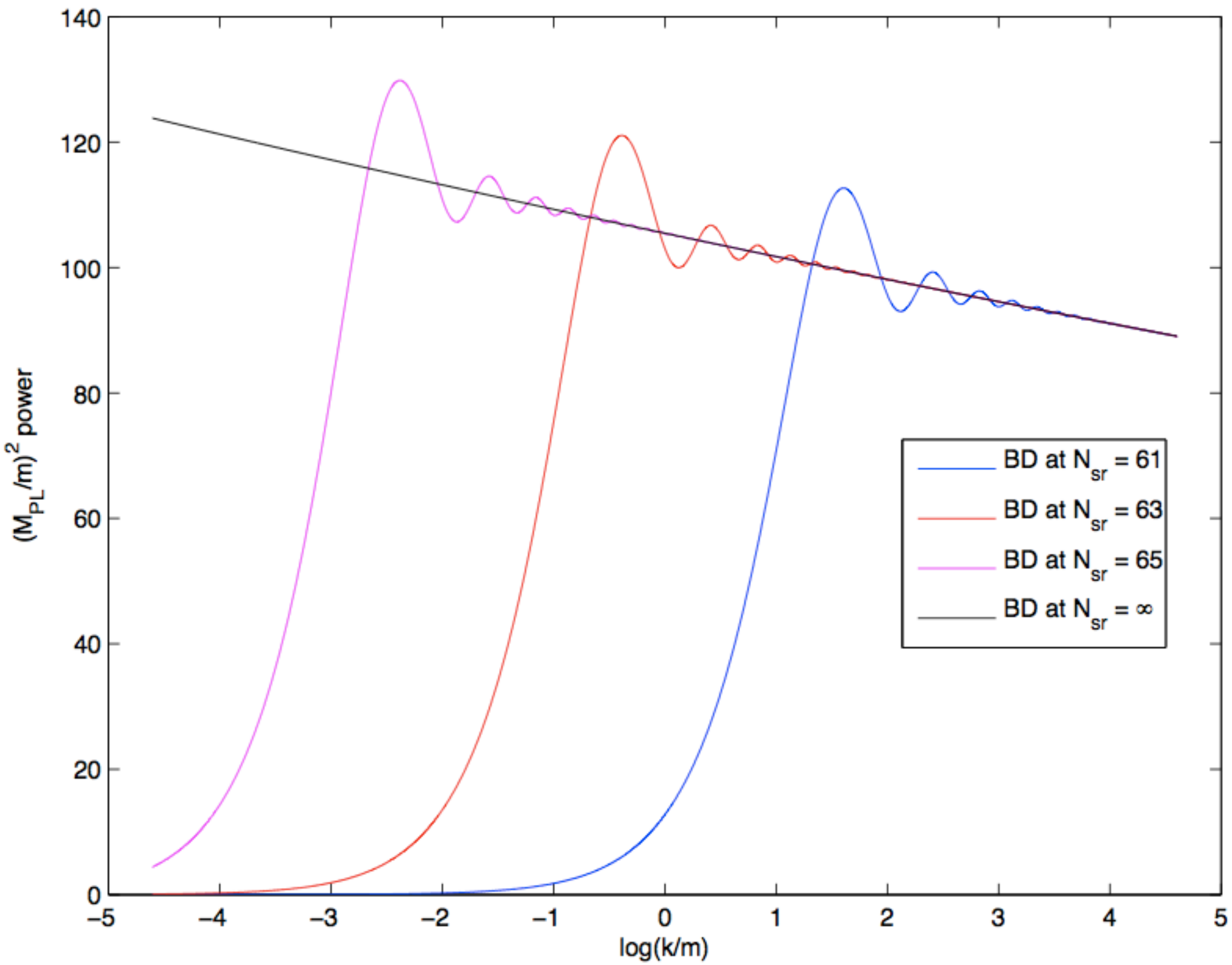


The power spectrum for scalar perturbations



Suppression of large-scale perturbations

A typical behavior from fast-roll to slow-roll (Destri-deVega-Sanches (2010))



$$V \sim M^4 \left\{ -\frac{1}{2}\phi^2 + \frac{\lambda}{4}\left(\frac{M_{\rm pl}}{m}\right)^4 \phi^4 \right\}$$

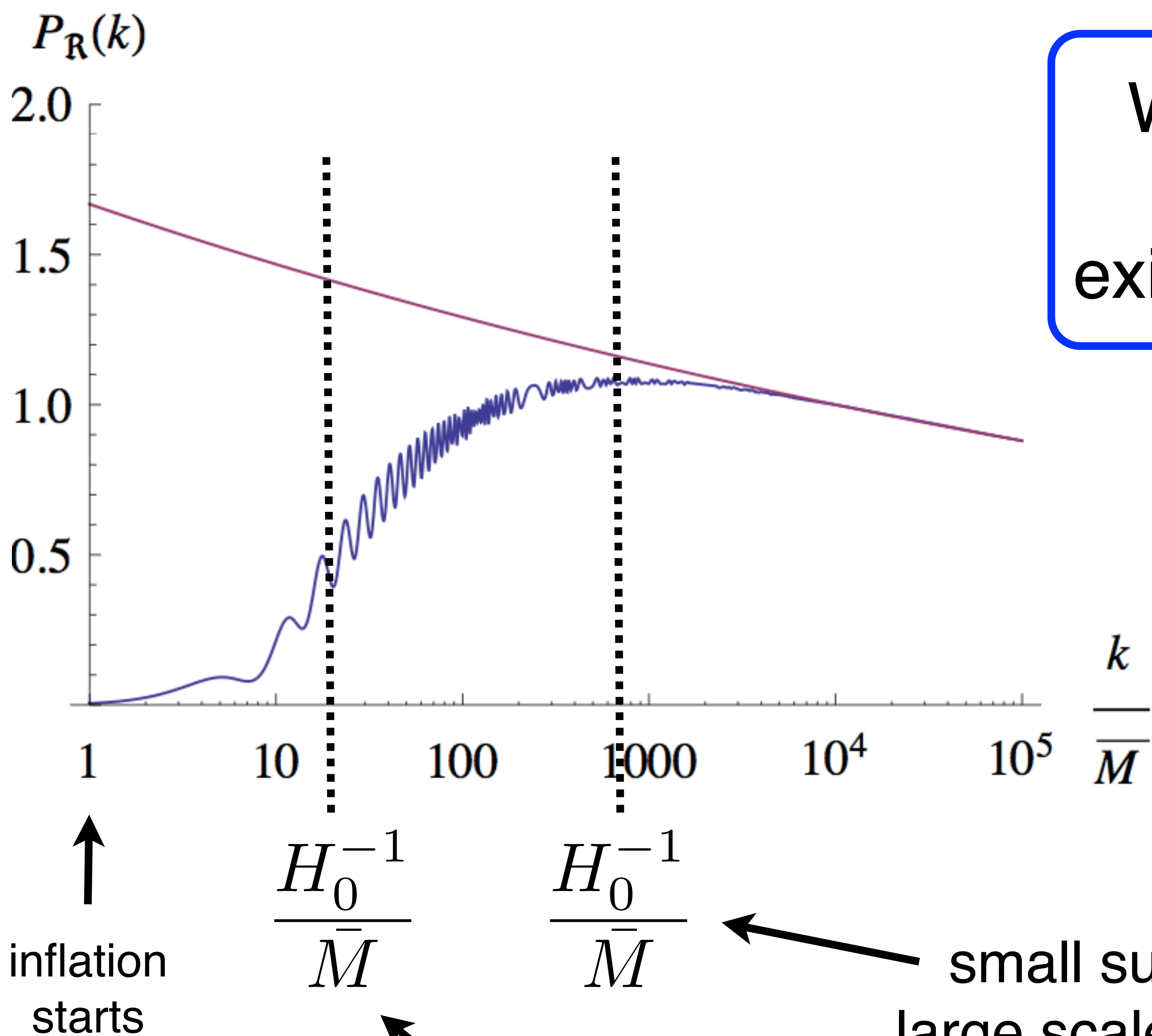
Descending solutions from Big-Bang singularity

Rather quick convergence to the slow-roll spectra with overshoots

Smooth convergence to the slow-roll spectra is a characteristic feature of ''climbing phase''.

Suppression of large scale perturbations is observable?

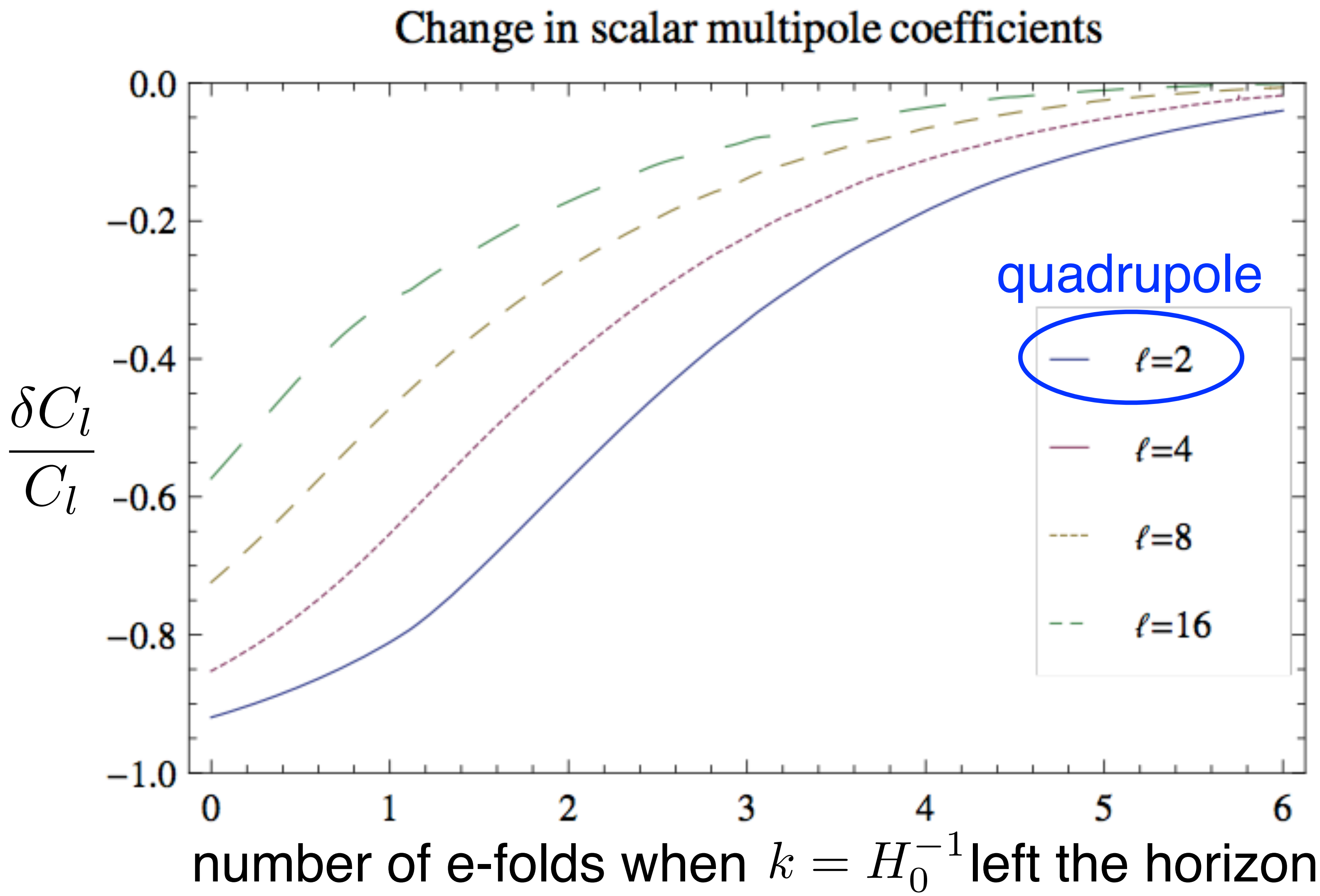
When the scale H_0^{-1} exited the horizon?



small suppression of large scale perturbations

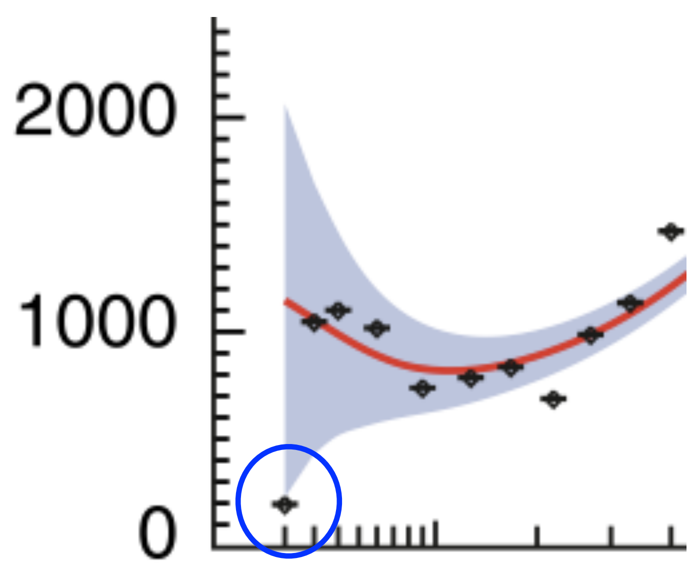
large suppression of large scale perturbations

Suppression of CMB multipole coefficients for small l

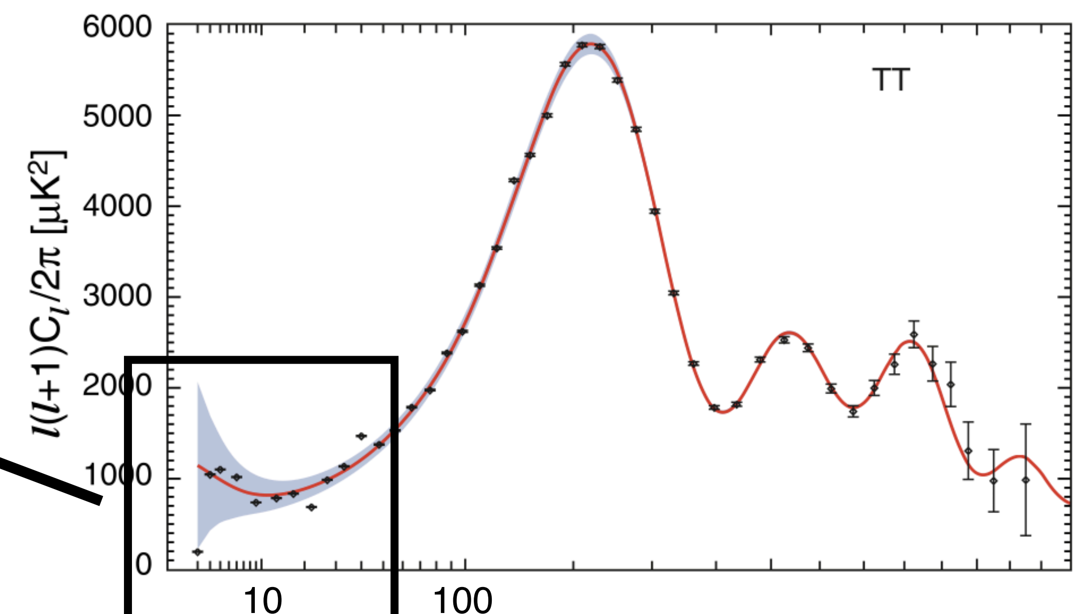


Quadrupole suppression in the observation may be explained.

Limitation of Cosmic variance... Only quadrupole???



WMAP7 (temperature)



5. Summary

- Some comments on String Cosmology
  (We need to impose reasonable assumptions.)

- Appearances of exponential potentials in
  the systems with “brane supersymmetry breaking”
  (also in KKLT scenario)

- “Climbing Phenomena” and “Climbing Phase”
  ((inevitable) climbing up the potential from Big-Bang)

- A characteristic shape of the power spectra in
  “climbing phase” and suppressions of large-scale
  perturbations (a prediction of String Theory?)