SUPERFIELDS
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Superstring Cosmophysics
Obihiro
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String Theory and
Fundamental Interactions
MIUR Grant - RBFR10QS5J
FUTURO IN RICERCA

New facts on *classical* and *quantum N=8* supergravity

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together with:

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G. Inverso and M. Trigiante hep-th/1208.xxxx

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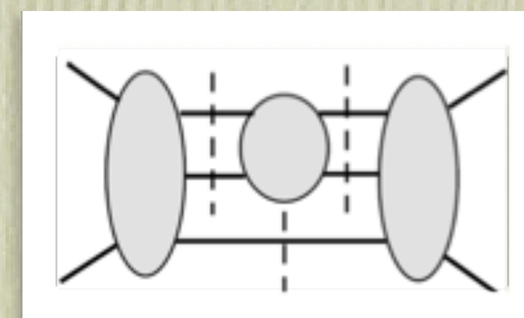
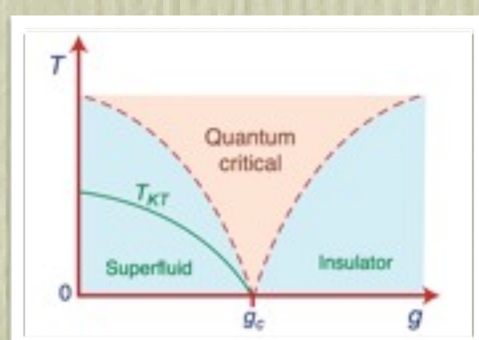
Why N=8 Supergravity (again)?

- *N=8 sugra as a unified framework for gauge interactions failed but:*

Finiteness (?)

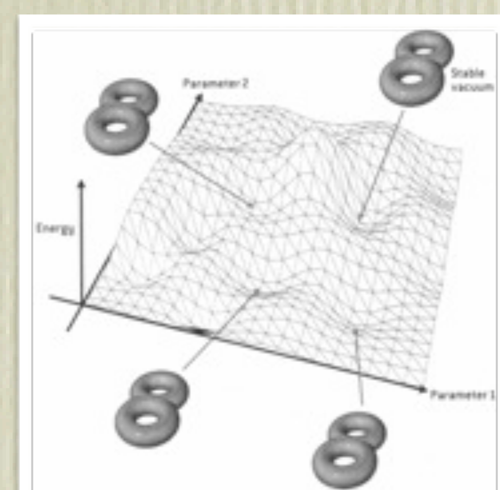


Other Minkowski vacua?



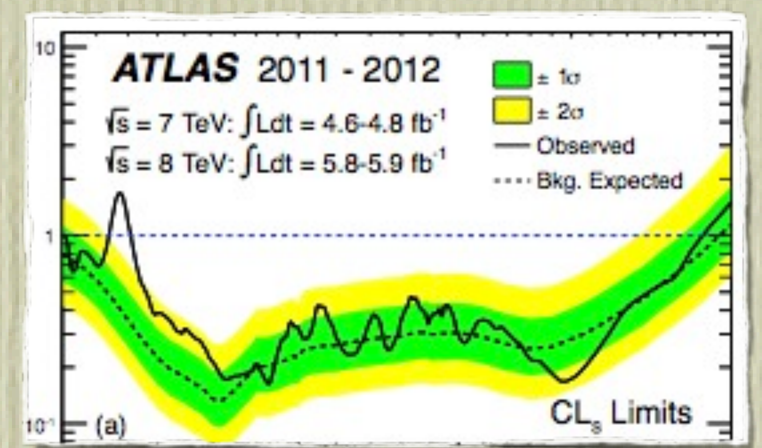
AdS/CMT applications

dS vacua in N>1
(and eventually string theory)



Why N=8 Supergravity (again)?

2012: Higgs discovery.



What about New TeV Physics?

ICHEP2012
Melbourne

Raman SUNDRUM

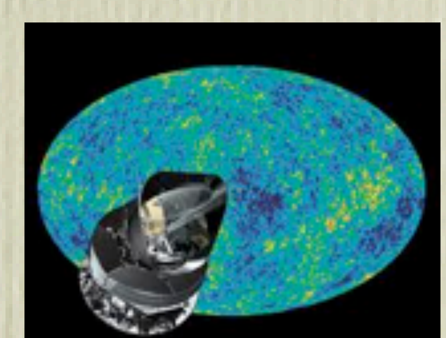
SUSY - What's left?

Andy PARKER

SUSY Searches (ATLAS/CMS): the Lady Vanishes

Next scale we are sure of: ***Quantum Gravity***

Cosmological observations may also help and *guide* us



Why N=8 Supergravity (again)?

Which (Quantum) Gravity Theories?

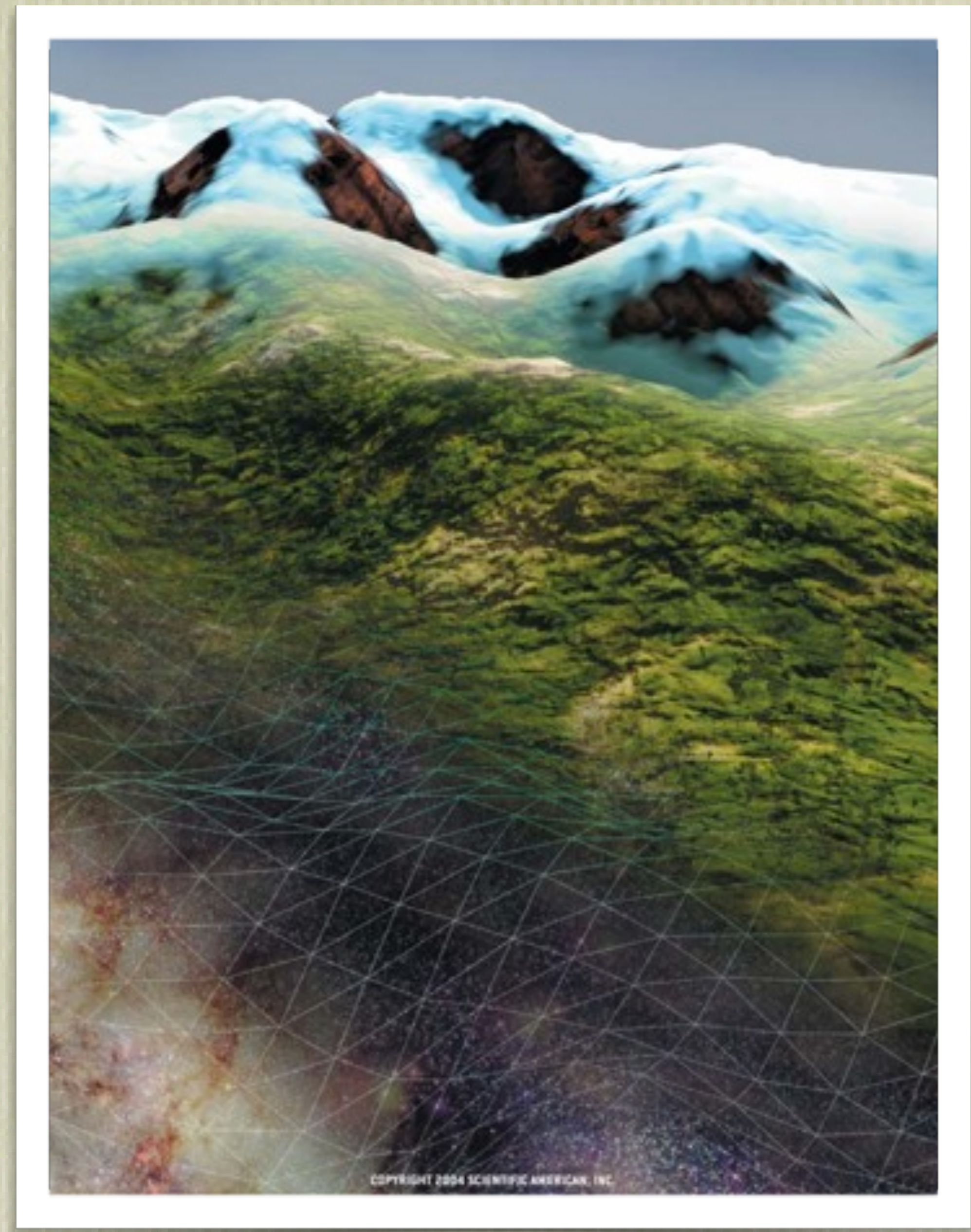
- Highly constrained
- Where well-defined calculations can be carried out

"Supersymmetric theories have so many theoretical properties that you can really make wonderfully significant progress studying the dynamics of quantum field theories. And you do it by studying them in their ***most supersymmetric*** *aspect first."*

NIMA ARKANI-HAMED

The String Theory Landscape

- *What is landscape and what is swampland?*
- 10^{500} *vacua with positive cc?* **(or none !?!)**
- *What is the right SUSY breaking mechanism?*



Λ *from quantum corrections*

Quantum corrections to flat classical potential (string moduli) may lead to slow-roll conditions

At 1 loop:

$$\Lambda_{eff} \simeq \frac{1}{64\pi^2} \sum_i (-1)^F \int^{M_{\mathrm{cut}}} d^4p \log(p^2 + m_i^2)$$

Better grasp on **Quantum Stability** of N=8 *Minkowski* and *de Sitter vacua*

The N=8 Landscape of Minkowski vacua

- [1979-2011] **Unique** way to get spontaneously broken N=8 Minkowski vacua: Scherk–Schwarz mechanism

$$\mathrm{U}(1) \ltimes T^{27} \ \textit{gauging} \qquad \begin{cases} [X_0, X^I] & = \quad Q^I{}_J\, X^J \\ [X^I, X^J] & = \quad 0 \end{cases}$$

- Masses classified by $\mathrm{U}(1) \subset \mathrm{Sp}(8,\mathbb{R})$ charge matrices
[4 mass parameters]

- Quantum properties:

$$\mathrm{Str}\,\mathcal{M}^2 = \mathrm{Str}\,\mathcal{M}^4 = \mathrm{Str}\,\mathcal{M}^6 = 0$$

  - 1-loop finite

Sezgin-Van Nieuwenhuizen

Ferrara-Zumino

$$\mathrm{Str}\,\mathcal{M}^8 \neq 0$$

  - $V_{\text{1-loop}} < 0$

Here:

i) New classical properties

a. New *classes of gaugings* G.D., Inverso, Trigiante

b. *New method* for new *vacua* of N=8 gauged supergravity G.D., Inverso

c. New susy-breaking patterns (CSO* models) G.D., Inverso, Zwirner

ii) New quantum properties G.D., Zwirner

a. **1-loop finiteness** (New relations for the Supertraces)

b. *Instability* of *1-loop potential* for all known examples

N=8 (Gauged) Supergravity

N=8 Supergravity: Field Content

Unique multiplet: **2^8 = 256 = 128_B + 128_F d.o.f.**

$\|+2\rangle$	*1 Graviton*	$g_{\mu\nu}$
$\|+3/2, i\rangle = Q_i\|+2\rangle$	*8 Gravitini*	ψ_μ^i
$\|+1, [ij]\rangle = Q_iQ_j\|+2\rangle$	*28 Vectors*	A_μ^{ij}
$\|+1/2, [ijk]\rangle = Q_iQ_jQ_k\|+2\rangle$	*56 fermions*	χ^{ijk}
$\|0, [ijkl]\rangle = Q_iQ_jQ_kQ_l\|+2\rangle$	*70 scalars*	ϕ^{ijkl}

and similarly for negative helicity up to -2

N=8 (GAUGED) SUPERGRAVITY

- The scalar manifold is a *coset space* $\mathcal{M}_{scal} = E_{7(7)}/SU(8)$

- Scalars defined by *coset representatives*

$$L(\phi) = \exp\begin{pmatrix} 0 & \phi_{ijkl} \\ \phi^{ijkl} & 0 \end{pmatrix} \qquad \delta L(\phi) = \Lambda L(\phi) - L(\phi) h(\phi)$$

- SU(8)-invariant parameterization via $\mathcal{M} = L L^T$

- The E7 action is **linear** on

$$\delta\mathcal{M} = \Lambda\,\mathcal{M} + \mathcal{M}\,\Lambda^T$$

$E_{7(7)}$ is the ***U-duality group*** in 4 dimensions

U-duality = generalized *electric-magnetic* duality

In 4d we have
electric & *magnetic*
vector fields
Hodge-dual to each other

eoms + BI

$$\begin{cases} dF^{\Lambda} = 0 \\ dG_{\Lambda} = d\left(\dfrac{\partial \mathcal{L}}{\partial F^{\Lambda}}\right) = 0 \end{cases}$$

The sum of eoms+BI is *invariant under duality transformations*

The sum of eoms+BI is *invariant under duality transformations*

Consistency of the dual field-strength definition constrains S to be a *Symplectic Transformation*

GAILLARD-ZUMINO

invariance

$$\delta \begin{pmatrix} F' \\ G' \end{pmatrix} = S \begin{pmatrix} F \\ G \end{pmatrix}$$

$$S \in U \subset Sp(2n_V, \mathbb{Z})$$

The Lagrangian is ***not invariant*** but ***eoms+BI*** *are invariant*

When scalars are present the duality group is reduced to the isometry group of the scalar manifold

In N=8 Supergravity

$$n_V = 28$$

and

$$U = E_{7(7)} \subset Sp(56, \mathbb{R})$$

invariance

$$\delta \begin{pmatrix} F' \\ G' \end{pmatrix} = S \begin{pmatrix} F \\ G \end{pmatrix}$$

$$S \in U \subset Sp(2n_V, \mathbb{Z})$$

In Supergravity one can still change the starting Lagrangian by changing the symplectic frame

Inequivalent theories for any $Sp(56, \mathbb{R})/E_{7(7)}$ transformation

(non-minimal couplings change)

The ungauged theory has:

- *$U(1)^{28}$ gauge group*
- *no scalar potential (only 4d vacua = Minkowski)*
- *Quantum theory finite? (consensus: up to 7 loops)*

We would like:

- *Non abelian gauge group*
- *scalar potential (Spontaneous SUSY breaking!)*

This leads to ***N=8 Gauged supergravites:***

~we make local some of the isometries

The 133 $E_{7(7)}$ generators t_α act on the 28+28 vectors A_μ^M

The group of global symmetries of the Lagrangian is called Electric Group

$$(t_\alpha)_P{}^Q = \begin{pmatrix} * & * \\ 0 & * \end{pmatrix}$$

Dual gauge fields do **not** appear in the lagrangian

One can gauge also **magnetic generators** at the expense of introducing *tensor fields*

N=8 Gauged Supergravity Theories:

Fix the Symplectic Frame

(choose electric and magnetic vectors)

Make local a subgroup of the duality group

Modern approach to the gauging of supergravity theories:

THE EMBEDDING TENSOR FORMALISM

Isometries of the scalar manifolds act on the coset representatives

$$\delta L = \epsilon^{\alpha} t_{\alpha} L$$

and on the vector fields as *global symmetries*:

$$\delta A_{\mu}^{M} = -\epsilon^{\alpha} (t_{\alpha})_{N}{}^{M} A_{\mu}^{N}$$

Gauging* = *make local $X_M \in G \subset E_{7(7)}$

Simple derivatives replaced by covariant ones:

$$\partial_\mu \to D_\mu \equiv \partial_\mu - g\, A_\mu^M\, X_M$$

Vector fields promoted to gauge connections

$$\delta A_\mu^M = \partial_\mu \epsilon^M + g A_\mu^N X_{NP}{}^M \epsilon^P$$

Susy compatibility constrains allowed G!

New couplings:

- *O(g)* modification in *susy rules for fermions*
- *O(g)* mass term for fermions
- *O(g²)* scalar potential

- The gauge group is determined by the ***Embedding Tensor***

gauge generators

$$\boxed{X_M = \Theta_M{}^\alpha t_\alpha}$$

E_7 generators

- Consistency imposes quadratic constraints:

$$[X_M, X_N] = -X_{MN}{}^P X_P \qquad \Theta_M{}^\alpha \Theta_N{}^\beta \Omega^{MN} = 0$$

- Supersymmetry imposes a linear constraint:

$$t_{\alpha M}{}^N \Theta_N{}^\alpha = 0, \qquad (t_\beta t^\alpha)_M{}^N \Theta_N{}^\beta = -\frac{1}{2}\Theta_M{}^\alpha$$

- *Altogether:*

$$\mathbf{56 \times 133 = \cancel{56} + 912 + \cancel{6480}}$$

- In string compactifications electric vectors usually split:

$$A_\mu^M \to \begin{cases} A_\mu^m & \textit{in the Adjoint of G} \\ A_\mu^i & \textit{in some representation of G} \end{cases}$$

*The vector fields that are **not** in the adjoint have to be **dualized***

$$A_\mu^i \longrightarrow \mathcal{H}^i_{\mu\nu} = F^i_{\mu\nu} + g\, X_{PQ}{}^i B^{PQ}_{\mu\nu}$$

These are the magnetic gaugings $\quad (t_\alpha)_P{}^Q = \begin{pmatrix} * & * \\ * & * \end{pmatrix}$

(magnetic) Free-Differential-Algebra vs. Lie Algebra (electric)

- One can always perform a *field redefinition* to a symplectic frame where the gauging is *purely electric*

- Our main interest is in classifying the vacua and understanding if **stable dS** and/or **Minkowski** is possible.

- Relevant part of the action:

$$\mathcal{L}_{scalars} = \frac{1}{8}\,\mathrm{Tr}(\partial_\mu \mathcal{M} \partial^\mu \mathcal{M}^{-1}) - V(\phi)$$

- The scalar potential depends also on the **gauging**

$$V(\phi) = (X_{MN}{}^{R} X_{PQ}{}^{S} \mathcal{M}^{MP} \mathcal{M}^{NQ} \mathcal{M}_{RS} + 7\, X_{MN}{}^{Q} X_{PQ}{}^{N} \mathcal{M}^{MP})$$

- Note that the potential depends on special combinations of coset representatives and embedding tensor:

$$V(\phi) = V\left(L(\phi), \Theta\right) = V(L^{-1}\Theta)$$

- The *action* of $U \in E_{7(7)}$ is *linear* on both

$$\Theta \to \Theta' = U\Theta \qquad U L(\phi) = L(\phi')h(\phi,\phi'), \quad \text{with } h \in \mathrm{SU}(8)$$

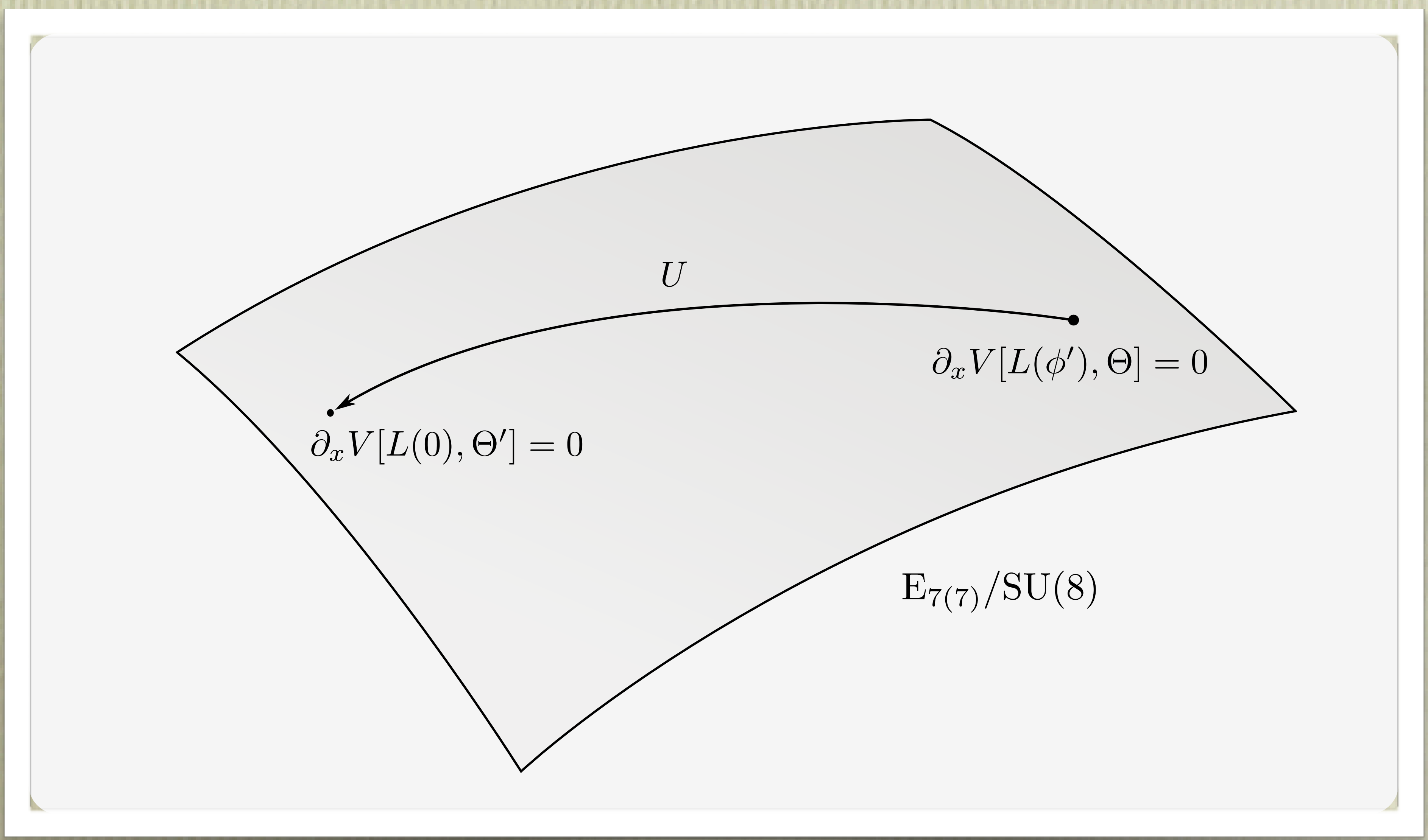
- This means that we can **undo** a *change of parameterization* by an *inverse change on the embedding tensor*

$$\boxed{V(L(\phi), \Theta') = V(L(\phi'), \Theta)}$$

G.D.-Inverso
Dibitetto-
Guarino-Roest

- The result is that we can **simplify drastically** the *extremization* of the scalar potential

- *Trade 70 nonlinear conditions on the scalars for a set of* ***quadratic*** *and* ***linear*** *conditions on the embedding tensor!*



Compute everything for vanishing scalars

- **Example:** *N=1 vacua of STU model*
- The STU model is $\left[\dfrac{SU(1,1)}{U(1)}\right]^3$
- Parameterized by the representatives $L = \begin{pmatrix} e^{\phi} & e^{-\phi}C \\ 0 & e^{-\phi} \end{pmatrix}$

$$\mathcal{M} = \frac{2i}{\tau - \bar{\tau}} \begin{pmatrix} |\tau|^2 & \frac{1}{2}(\tau + \bar{\tau}) \\ \frac{1}{2}(\tau + \bar{\tau}) & 1 \end{pmatrix} \qquad \tau = C + i\, e^{2\phi}$$

- Model: *3 copies of* $\mathcal{L}_{scalars} = \dfrac{1}{(\tau - \bar{\tau})^2} \partial_\mu \tau \partial^\mu \bar{\tau}$

- Scalar potential from superpotential

$$W = \alpha + \beta\, S + \gamma\, T + \delta\, ST$$

- Various vacua. One example is:

$$\partial_I V = 0 \Leftarrow st = \frac{\beta\gamma - \alpha\delta}{\delta^2} \quad \sigma = \frac{\gamma}{\delta} \quad \tau = \frac{\beta}{\delta}$$

- **At origin** $S = \sigma - is = -i\,,\, T = \tau - it = -i\,:$

$$\beta = \gamma = 0, \qquad \alpha = -\delta$$

- These are the vacua of $W = \alpha\,(1 + ST)$

- We find more general vacua using isometries

$$\tau \to \frac{a\tau + b}{c\tau + d} \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SU(1,1)$$

- *For each modulus:*

$$D : \tau \to \lambda\tau \qquad (a = 1/d = \sqrt{\lambda})$$

$$S : \tau \to \tau + 1 \qquad (a = d = b = 1)$$

- For instance, from $W = \alpha\,(1 + ST)$, by using

$$T \to T + \frac{\beta}{\alpha}$$

we generate

$$W = \alpha + \beta S + \alpha\, ST$$

Summarizing:

N=8 supergravity:

matter content fixed

couplings change (many different N=8 supergravities!)

Theories connected by

Symplectic transformations (change theory)

U-duality transformations (change Lagrangian, not eoms+ BI)

Summarizing:

Ungauged *models depend on*

Symplectic embedding

Gauged *models depend*

$$G \subset E_{7(7)} \subset Sp(56, \mathbb{R})$$

(Embedding Tensor)

Coset manifolds allow for easy classification of vacua at the origin

New CLASSICAL Properties

Vacua of N=8 gauged supergravity

- Since the '80s attempts at constructing gaugings and finding vacua

- *Few vacua in the SO(8) model* found soon: Warner, Hull, Nicolai...

Symmetry	Cosm. const.	SUSY
SO(8)	AdS	N=8
$SO(7)_-$	AdS	N=0
$SO(7)_+$	AdS	N=0
G_2	AdS	N=1
SU(4)	AdS	N=0
SU(3) x U(1)	AdS	N=2
SO(3) x SO(3)	AdS	N=0

- Since the '80s attempts at constructing gaugings and finding vacua
- *Few vacua in the SO(8) model* found soon — Warner, Hull, Nicolai...
- Few vacua in other models:

Gauging	Cosm. const.	SUSY
SO(3,5)	dS	N=0
SO(4,4)	dS	N=0
CSO(2,0,6)	Mink	N=0

Vacua of N=8 gauged supergravity

- Since the '80s attempts at constructing gaugings and finding vacua
- *Few vacua in the SO(8) model* found soon — Warner, Hull, Nicolai...
- Few vacua in other models
- New (approximate) **vacua with numerics** — Fischbacher

This part of the collection describes 41 critical points, 7 of which have been known for more than two decades, 8 of which were discovered recently, and 26 are novel. The residual gauge symmetries of these 41 critical points (likely) are SO(8) with N=8 SUSY (1x), SO(7) (2x), SU(4) (1x), G2 with N=1 SUSY (1x), SU(3)xU(1) with N=2 SUSY (1x), SO(3)xSO(3) (2x), SO(3)xU(1)xU(1) (1x), SO(3)xU(1) (3x), SO(3) (3x), U(1)xU(1) with N=1 SUSY (1x), U(1)xU(1) without SUSY (4x), U(1) (11x), and None (10x). Analytic conjectures (not yet proven but overwhelmingly likely correct) are given for the locations and cosmological constants of some critical points.

Vacua of N=8 gauged supergravity

- Our **new method** allows for *easy identification of vacua* and computation of the mass spectrum *(mainly missing also for known vacua)*

Stable!

- Some interesting examples:

Gauging	Cosm. const.	SUSY
SO(6,2)=SO*(8)	Mink	N=0
	dS	N=0
[illegible]	Mink	N=0
CSO*(8-2n,0,2n)	Mink	N=2n

New class of vacua
New SS-type mechanism

Beyond "exhaustive classifications"?!

- Full mass spectrum.

Stable!

#	G_{gauge}	G_{res}	Λ	m^2
vi	SO(2,6)	SO(2) × SO(6)	Mink	$2^{(2)}, 1/2^{(20)}, 0^{(48)}$
	CSO(2,0,6)	SO(2)		
xii	SO(4)× SO(2,2) ⋉ T^{16}	SO(2)^2× SO(4)	Mink	$4^{(4)}, 2^{(12)}, 1^{(16)}, 0^{(38)}$
xiii	SO(2)^2 ⋉ T^{20}	SO(2)^2		
i	SO(8)	SO(7)	AdS	$2^{(1)}, -4/5^{(27)}, -2/5^{(35)}, 0^{(7)}$
iii	SO(7,1)			
viii	SO(7) ⋉ T^7			
ii	SO(8)	SO(6)	AdS	$2^{(2)}, -1^{(20)}, -1/4^{(20)}, 0^{(28)}$
iv	SO(7,1)			
ix	SO(7) ⋉ T^7			
x	SO(6)× SO(1,1) ⋉ T^{12}			
v	SO(7,1)	SO(5)	AdS	$2^{(3)}, -4/3^{(14)}, 2/3^{(5)}, 0^{(48)}$
	SO(6)× SO(1,1) ⋉ T^{12}			
vii	SO(3,5)	SO(3) × SO(5)	dS	$-2^{(1)}, 4^{(5)}, 2^{(30)}, 4/3^{(14)}, -2/3^{(5)}, 0^{(15)}$

Unstable

- So far no stable dS, but stable N=0 Minkowski

- Interesting pattern emerging: *masses completely fixed by (large) symmetries*

Interesting pattern emerging: *masses completely fixed by (large) symmetries*

General pattern *(not yet explained)*

Original gauge group:

compact: ***AdS vacua***

non-compact:

AdS* , *dS* or *Mink

Residual gauge group

$$G = U(1) \times G'$$

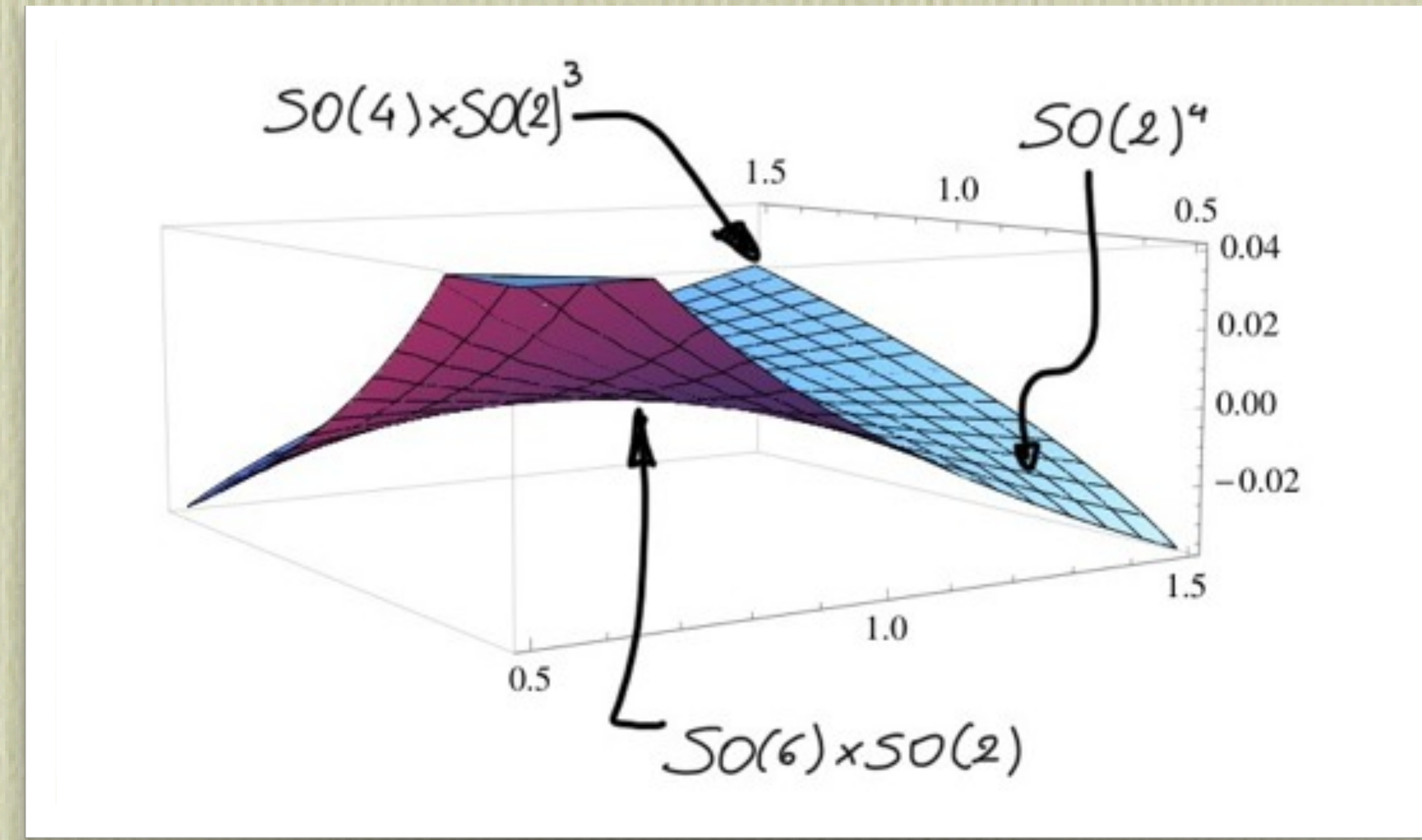
Mink vacua

Dirac masses

semisimple

AdS*, *dS vacua

- **Non-trivial Moduli space**



Mass value
for 2 moduli of the
SO(6,2) gauging

- *At the boundary of the moduli space SO(6,2) becomes CSO(2,0,6)*

FLOP TRANSITION??

- What about the *string theory uplift?*
- *SO(8) gauged supergravity* = *M-theory on S*[7]
- Other vacua from *geometric* **and** *non-geometric* fluxes

$\mathbf{1}_{+\mathbf{7}}$	g_7	$(\mathbf{140}+\mathbf{7})_{+\mathbf{3}}$	$\tau^i_{jk}+\delta^i_j\tau_k$	$\mathbf{28}_{-\mathbf{1}}$	$\theta_{(ij)}$
$\mathbf{1}_{-\mathbf{7}}$	$\tilde{g}_7$	$(\mathbf{140}'+\mathbf{7}')_{-3}$	$Q^{jk}_i+\delta^j_i Q^k$	$\mathbf{28}'_{+1}$	$\xi^{(ij)}$
$\mathbf{35}_{-\mathbf{5}}$	h^{ijkl}	$\mathbf{224}_{-1}$	f^i_{jkl}	$\mathbf{21}_{-\mathbf{1}}$	$\theta_{[ij]}$
$\mathbf{35}'_{+5}$	g_{ijkl}	$\mathbf{224}'_{+1}$	R^{jkl}_i	$\mathbf{21}'_{+1}$	$\xi^{[ij]}$

Intermezzo:

N=1 and Flux Compactifications

- We start with a *G2-holonomy* manifold which is a 7-torus:

$$X_7 = \mathbb{T}^7/(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2)$$

JOYCE

- $\mathbb{Z}_2$ action:

$$\begin{aligned}
\mathbb{Z}_2(x_i) &= \{-x_5, -x_6, -x_7, -x_8, x_9, x_{10}, x_{11}\} \\
\mathbb{Z}_2'(x_i) &= \{a_1 - x_5, a_2 - x_6, x_7, x_8, -x_9, -x_{10}, x_{11}\} \\
\mathbb{Z}_2''(x_i) &= \{a_3 - x_5, x_6, a_4 - x_7, x_8, a_5 - x_9, x_{10}, -x_{11}\}
\end{aligned}$$

- The topology gives $b_1=b_2=0$, $b_3=7$, so the effective theory is described by 1 graviton, 7 chiral multiplets and (for $a_1=a_5=0$, $a_2=a_3=a_4=1/2$) one can turn on 7 4-form fluxes and 6 geometrical deformations

G.D.-PREZAS

The ***twist*** makes X_7 a *G2-structure* manifold

$$d\Phi = W_1 \star \Phi + W_{27}, \quad d \star \Phi = 0$$

The ***Kæhler potential*** and ***Superpotential*** read

$$K = -3 \log \left(\frac{1}{7} \int_{X_7} \Phi \wedge \star \Phi \right)$$

$$W = \int_{X_7} [d\Phi + i\,(dC + 2g)] \wedge (\Phi + iC)$$

Torsion contribution *Defines the T-moduli*

- In the original N=8 the *U-dual fluxes* in $GL(7,\mathbb{R})$ basis

$\mathbf{1}_{+7}$	g_7	$(\mathbf{140}+\mathbf{7})_{+3}$	$\tau^i_{jk}+\delta^i_j\tau_k$	$\mathbf{28}_{-1}$	$\theta_{(ij)}$
$\mathbf{1}_{-7}$	$\tilde{g}_7$	$(\mathbf{140}'+\mathbf{7}')_{-3}$	$Q^{jk}_i+\delta^j_iQ^k$	$\mathbf{28}'_{+1}$	$\xi^{(ij)}$
$\mathbf{35}_{-5}$	h^{ijkl}	$\mathbf{224}_{-1}$	f^i_{jkl}	$\mathbf{21}_{-1}$	$\theta_{[ij]}$
$\mathbf{35}'_{+5}$	g_{ijkl}	$\mathbf{224}'_{+1}$	R^{jkl}_i	$\mathbf{21}'_{+1}$	$\xi^{[ij]}$

- ***Geometric fluxes***
- ***Locally geometric fluxes***
- ***Also geometric in sphere reductions!***
- Fluxes classified by O(1,1) weight

- We have a ***Generalized geometric description*** for the superpotential

$$\boxed{W = \sum_{fluxes} \int_{X_7} (C + i\,\Phi) \wedge [(weight\ \mathbf{n}\ flux) \cdot (C + i\,\Phi)^{\frac{5-n}{2}}]}$$

where the dot product means

$$\phi^I \cdot \xi \cdot \phi^J = dy^a \wedge dy^b \wedge dy^c \wedge dy^d \left(\phi^I_{abi} \xi^{ij} \phi^J_{jcd}\right)$$

- W is $SL(2,\mathbb{R})^7$ ***U-duality invariant***

$$T_I \to \frac{a_I T_I + b_I}{c_I T_I + d_I} \qquad \Theta \to \Theta' = U\Theta$$

Combined

...Back to Classical N=8

- ***The landscape of such models is much larger than expected!!***
- Massive models depend on the *gauge group* **and** on its *symplectic embedding*
- First known example: *SO(8) gauged supergravity* (*set* θ *to 1*)

  CREMMER-JULIA
  DE WIT-NICOLAI
- We have an **infinite class of SO(8) models.**
- We will show that **most of them are *NEW***

SO(8) gauged supergravity

$$X_M = \Theta_M{}^\alpha t_\alpha$$

where $t_\alpha \in SO(8) \subset E_{7(7)} \subset Sp(56,\mathbb{R})$

But which SO(8)??

Decompose $E_{7(7)}$ in SO(8) irrepses

$$\mathbf{912} \to 2 \times (\mathbf{1} + \mathbf{35}_s + \mathbf{35}_c + \mathbf{35}_v + \mathbf{350})$$

Two singlets:

$$\theta_{AB} = \delta_{AB} \qquad \xi^{AB} = \delta^{AB}$$

In SL(8,R) basis the $E_{7(7)}$ generators are

$$t_\alpha = \{t_A{}^B, t_{ABCD}, t^{ABCD}\}$$

The embeddings are

$$\Theta_{AB}{}^C{}_D = \delta^C_{[A}\theta_{B]D} \qquad \Theta^{ABC}{}_D = c\,\delta^{[A}_D\xi^{B]C}$$

For *c=0* we find the "Old SO(8) theory", but what about c non-zero?

Are these theories equivalent to the known one or not?

New N=8 gauged supergravity models

- Dual models should have the same *duality invariants* *(see e.g. black holes)* Ferrara et al.

$$\boxed{I_4(Q) = d_{MNPQ}\, Q^M\, Q^N\, Q^P\, Q^Q}$$

- Gaugings should be classified by *duality invariant* combinations of the *embedding tensor*.

- **No scalar invariant**, but we can use ***tensor classifiers*** Trigiante et al.

- For $\xi = c\theta^{-1}$ we find **inequivalent models** $0<c<1$

- We also found **different vacua** for different values of c!

- The *eigenvalues of*

$$\boxed{B_{\alpha\beta}^{\gamma\delta} = \Theta_\alpha^M\,\Theta_\beta^N\,\Theta^{P\gamma}\,\Theta^{Q\delta}\,d_{MNPQ}}$$

are *duality invariant*

$$B \to U B U^{-1}, \qquad U \in E_7$$

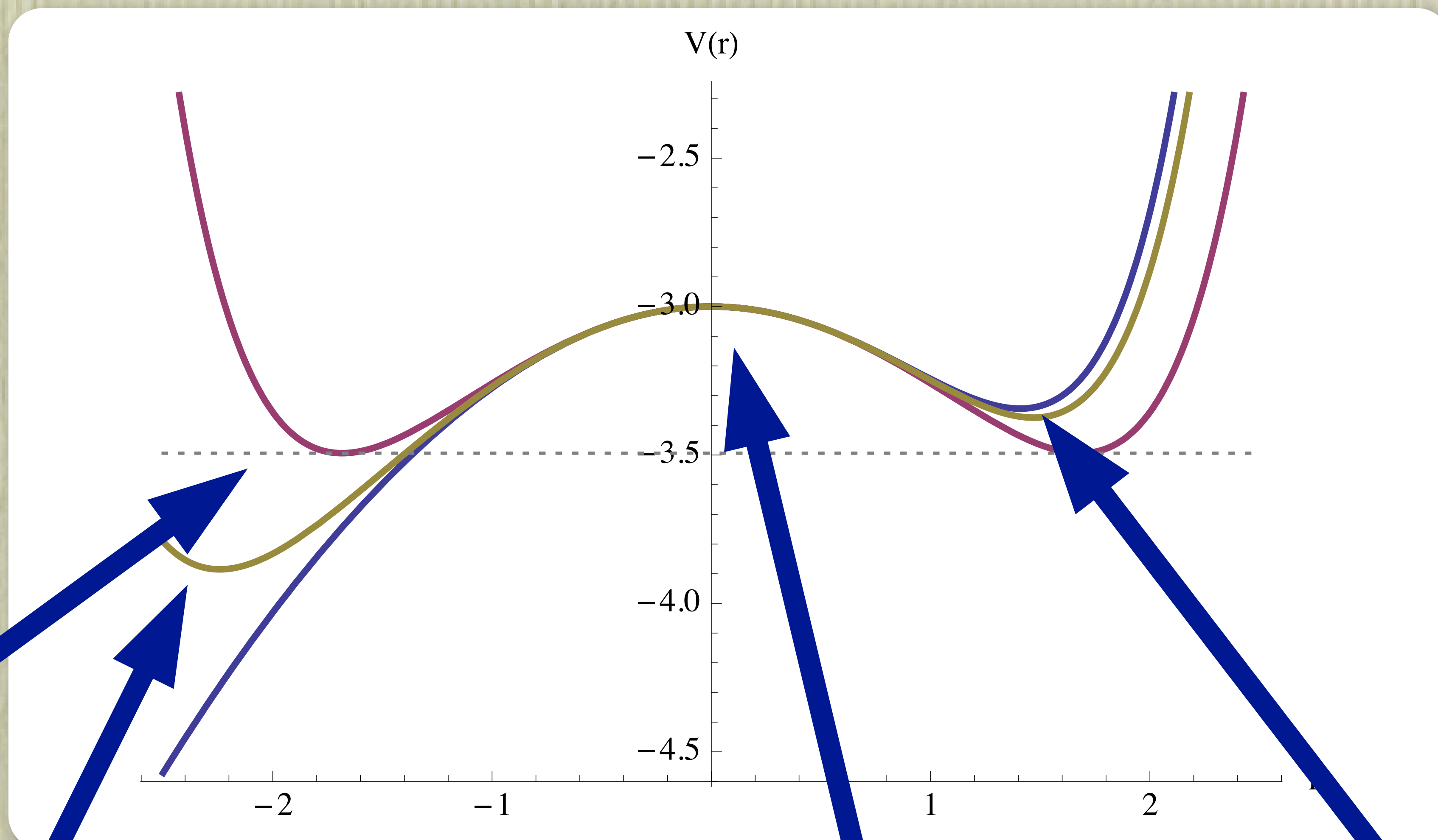
- For $\xi = c\theta^{-1}$ we find **inequivalent models** for c in the range [0,1]

- We also found **different vacua** for different values of c!

- Still, *spectrum* classified by *residual G*

New N=8 gauged supergravity models

The scalar potential for an SO(7) truncation, for



c=0

c=1

c=0.41

New AdS vacua

N=8 AdS vacuum

SO(7)⁺ vacuum

Other couplings also change.

Gravitino masses: $c = \tan\omega$

$$m_\psi = e^{i\omega}(1+\phi^2/4) + e^{-i\omega}\phi^3/6 + \mathcal{O}(\phi^4)$$

Gauge kinetic functions:

$$e^{-1}\mathcal{L}_{vect} = \frac{1}{4}\mathcal{I}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma\,\mu\nu} + \frac{1}{4}\mathcal{R}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}\widetilde{F}^{\Sigma\,\mu\nu}$$

$$\mathcal{N} = \mathcal{R} + i\,\mathcal{I} = -i\frac{\cos\omega\, e^{\phi\Lambda} - i\sin\omega\, e^{-\phi\Lambda}}{\cos\omega\, e^{-\phi\Lambda} - i\sin\omega\, e^{\phi\Lambda}}$$

where Λ *is the generator of the scalar*

Summarizing:

New method for finding vacua of N=8 gauged supergravity

***Many new vacua** (dS, AdS and Minkowski) with full mass spectrum*

*Relation **masses - residual gauge group***

Interesting moduli space

New (infinite) classes of gaugings

New QUANTUM Properties

- ***Ungauged*** N=8 supergravity is *finite* up to 4 loops and possibly more
- ***Gauging* = *new couplings*** *(and masses)*
- **Arguments from ungauged theory do not apply**
- *One loop* divergencies controlled by ***supertraces***

$$Str\left(\mathcal{M}^{2k}\right) = \sum_J (-1)^{2J}(2J+1)tr(\mathcal{M}_J)^{2k}$$

- Example: *One loop effective potential*

$$\begin{aligned} V_{eff} &= \tfrac{1}{64\pi^2}\mathrm{Str}\mathcal{M}^0\Lambda^4\log\tfrac{\Lambda^2}{\mu^2} + \tfrac{1}{32\pi^2}\mathrm{Str}\mathcal{M}^2\Lambda^2 - \tfrac{1}{64\pi^2}\mathrm{Str}\mathcal{M}^4\log\Lambda^2 \\ &+ \tfrac{1}{64\pi^2}\mathrm{Str}\left(\mathcal{M}^4\log\mathcal{M}^2\right) \end{aligned}$$

- We computed the ***quadratic*** and ***quartic*** supertrace mass formulae for a *generic* N=8 gauged supergravity
- Using:

1. *Critical point condition;*
2. *Vanishing cosmological constant;*
3. *Quadratic constraints*

we find that they **vanish**

G.D., Zwirner

$$\boxed{Str\ \left(\mathcal{M}^2\right) = \ Str\ \left(\mathcal{M}^4\right) = 0}$$

The computation is easy for a generic embedding tensor using the fermi susy shift matrices:

$$\begin{aligned} \delta_g \psi^i_\mu &= \delta_0 \psi^i_\mu + \sqrt{2}\, A^{ij}\, \gamma_\mu \epsilon_j\,, \\ \delta_g \chi^{ijk} &= \delta_0 \chi^{ijk} - 2\, A_l{}^{ijk}\, \epsilon^l. \end{aligned}$$

where

$$\mathcal{V}^{-1\,kl\,M} \mathcal{V}^{-1\,N}_{mi} X_{MN}{}^P \mathcal{V}_P{}^{mj} = \frac{3}{2} \delta_i^{[k} A_1^{l]j} - \frac{3}{4} A_{2i}{}^{jkl}$$

Example: gravitino

$$\mathcal{M}_{(3/2)}{}^{ij} = \sqrt{2}\, A^{ij}\,, \qquad \mathrm{Tr}\, \mathcal{M}^2_{(3/2)} = 2\, A_{ij} A^{ij}$$

For the other traces, we used the ***Minkowski*** vacuum conditions

Example: vectors

$$\mathcal{M}^2_{(1)} = \begin{pmatrix} \mathcal{M}_{ij}{}^{kl} & \mathcal{M}_{ijkl} \\ \mathcal{M}^{ijkl} & \mathcal{M}^{ij}{}_{kl} \end{pmatrix}$$

$$\mathcal{M}_{ij}{}^{kl} = (\mathcal{M}^{ij}{}_{kl})^* = -\frac{1}{6} A_{[i}{}^{npq} \delta^{[k}_{j]} A^{l]}{}_{npq} + \frac{1}{2} A_{[i}{}^{pq[k} A^{l]}{}_{j]pq}$$

$$\mathcal{M}_{ijkl} = (\mathcal{M}^{ijkl})^* = \frac{1}{36} A_{[i}{}^{pqr} \epsilon_{j]pqrmns[k} A_{l]}{}^{mns} .$$

For the other traces, we used the ***Minkowski*** vacuum conditions

Example: vectors

$$\mathrm{Tr}\,\mathcal{M}^2_{(1)} = \mathcal{M}_{ij}{}^{ij} + \mathcal{M}^{ij}{}_{ij} = \frac{2}{3}\,A_i{}^{jkl}A^i{}_{jkl} \overset{\Lambda=0}{=} 12\,A_{ij}A^{ij}$$

$$V_0(\phi) = \frac{1}{24}A_n{}^{jkl}A^n{}_{jkl} - \frac{3}{4}A_{kl}A^{kl}$$

For (A)dS vacua we get other constraints!

$$\mathrm{Tr}\,\mathcal{M}^2_{(0)} = \mathcal{M}_{ijkl}{}^{ijkl} = \frac{7}{2}\,A_i{}^{jkl}A^i{}_{jkl} - 35\,A_{ij}A^{ij} \overset{\Lambda=0}{=} 28\,A_{ij}A^{ij}$$

We can say more for the 4 classes of models we have now:

Scherk-Schwarz $U(1) \ltimes T^{27}$	$SO(6,2)$
$SO(2,2) \times SO(4) \ltimes T^{16}$	$U(1)^2 \ltimes T^{20}$

Gauge symmetry is broken on the vacuum

$$SO(6,2) \to SO(6) \times SO(2) \to \dots$$

All masses are fixed by the charges wrt unbroken U(1) factors

$$\boxed{m^2_{ij} = \vec{q}_i \cdot \vec{q}_j = \sum_A q_i^A \, q_j^A \, \mu^2_A \, .}$$

Assign U(1) charges to the 8 supersymmetry operators $\mathcal{Q}_i$

Other states follow:

$$\begin{aligned}
|2\rangle &: \quad M^2 = 0 \,, \quad \vec{0} \,, \\
|3/2, i\rangle &: \quad M^2 = (\vec{q}_i)^2 \,, \quad \vec{q}_i \,, \\
|1, [ij]\rangle &: \quad M^2 = (\vec{q}_i + \vec{q}_j)^2 \,, \quad \vec{q}_i + \vec{q}_j \,, \\
|1/2, [ijk]\rangle &: \quad M^2 = (\vec{q}_i + \vec{q}_j + \vec{q}_k)^2 \,, \quad \vec{q}_i + \vec{q}_j + \vec{q}_k \,, \\
|0, [ijkl]\rangle &: \quad M^2 = (\vec{q}_i + \vec{q}_j + \vec{q}_k + \vec{q}_l)^2 \,, \quad \vec{q}_i + \vec{q}_j + \vec{q}_k + \vec{q}_l \,,
\end{aligned}$$

$$\begin{aligned}
&|+2\rangle \,, \\
|+3/2, i\rangle &= Q_i\, |+2\rangle \,, \\
|+1, [ij]\rangle &= Q_i\, Q_j\, |+2\rangle \,, \\
|+1/2, [ijk]\rangle &= Q_i\, Q_j\, Q_k\, |+2\rangle \,, \\
|0, [ijkl]\rangle &= Q_i\, Q_j\, Q_k\, Q_l\, |+2\rangle \,,
\end{aligned}$$

Higgs, Super-Higgs and consistency force a maximum of 4 independent U(1) factors

$$M^2_{ijkl} = \sum_A \left(\vec{q}_i^{\,A} + \vec{q}_j^{\,A} + \vec{q}_k^{\,A} + \vec{q}_l^{\,A}\right)^2 \ \mu_A^2$$

Example 1: CSS gauging *(single unbroken U(1), 4 parameters)*

$$q_1 = -q_2 = e_1, q_3 = -q_4 = e_2, q_5 = -q_6 = e_3, q_7 = -q_8 = e_4$$

Universal modulus

(No-scale model)

$$\mu^2 = \phi^2$$

$$m^2 = \begin{cases} \phi^2 | \pm e_1 \pm e_2 \pm e_3 \pm e_4 |^2 \\ \phi^2 | \pm e_i \pm e_j |^2 \quad i \neq j \\ 0 \end{cases}$$

$$M^2_{ijkl} = \sum_A \left(\vec{q}_i^{\,A} + \vec{q}_j^{\,A} + \vec{q}_k^{\,A} + \vec{q}_l^{\,A}\right)^2 \;\; \mu_A^2$$

Example 2: SO(6,2) *(unbroken U(1)4)*

$$\vec{q}_1 = -\vec{q}_2 = (+1,+1,+1,+1), \;\; \vec{q}_3 = -\vec{q}_4 = (+1,+1,-1,-1)$$

$$\vec{q}_5 = -\vec{q}_6 = (+1,-1,+1,-1), \;\; \vec{q}_7 = -\vec{q}_8 = (+1,-1,-1,+1)$$

$$\mu_1^2 = \frac{(x-y)^2(1+x^2y^2)}{8x^2y^2}\,, \;\; \mu_2^2 = \mu_3^2 = 0\,, \;\; \mu_4^2 = \frac{(1+x^2y^2)(1+xy^3)^2}{8x^2y^4}$$

Compute

$$\mathrm{Str}\,\mathcal{M}^{2k} = \mathrm{Tr}\,[\mathcal{M}^2_{(0)}]^k - 2\ \mathrm{Tr}\,[\mathcal{M}^2_{(1/2)}]^k + 2\ \mathrm{Tr}\,[\mathcal{M}^2_{(1)}]^k - 2\ \mathrm{Tr}\,[\mathcal{M}^2_{(3/2)}]^k$$

we obtain G.D., Zwirner

$$\mathrm{Str}\,\mathcal{M}^2 = \mathrm{Str}\,\mathcal{M}^4 = \mathrm{Str}\,\mathcal{M}^6 = 0\,,$$

$$\mathrm{Str}\,\mathcal{M}^8 = 40320 \sum_{A=1}^{n} \left(\prod_{i=1}^{8} q_i^A \right) \mu_A^8\,.$$

Performing a numerical computation we also get that

$$V_{1\text{-loop}} < 0$$

Summarizing:

*Quadratic and quartic supertraces vanish for **any** gauging*

1-loop potential is then finite and calculable

Masses of known examples determined by charges wrt residual U(1) factors

1-loop effective potential is negative definite

A New Susy-breaking mechanism?

Scherk–Schwarz from the d=4 perspective:

Gravitino masses depend on 4 parameters m_i

Each combination corresponds to a gauging – *superpositions* of the basic ones

$m_2 = m_3 = m_4 = 0$	$N = 6$	$\mathrm{U}(1) \ltimes T^{12}$
$m_3 = m_4 = 0$	$N = 4$	$\mathrm{U}(1) \ltimes T^{18}$
$m_4 = 0$	$N = 2$	$\mathrm{U}(1) \ltimes T^{24}$
	$N = 0$	$\mathrm{U}(1) \ltimes T^{24}$

$$m_1 = m_2 = m_3 = m_4 : CSO(2,0,6)$$

In gauged supergravity one can also gauge $SO^*(p)$ groups

$$U^T U = 1 \qquad \text{and} \qquad U^T J U = J \qquad J^2 = -1$$

Pattern just like SS:

$$CSO^*(2,0,6) \to U(1) \qquad N = 6$$

$$CSO^*(4,0,4) \to U(1) \times SU(2) \qquad N = 4$$

$$CSO^*(6,0,2) \to U(1) \times SU(3) \qquad N = 2$$

$$CSO^*(8) \to U(1) \times SU(4) \qquad N = 0$$

superpositions of CSO(2,0,6)*

$$SO^*(8) \simeq SO(6,2)$$

Outlook

- *More vacua? dS vacua?*
- *General proof of mass formula and/or of 1-loop instability? (or counterexamples)*
- *Stringy uplift?*
- *Nature of the obstructions to positive (or zero) cc?*
- *Dual interpretation of new SO(8) models?*