

Non-perturbative Effects in Type II/F-Theory

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Outline:

Non-perturbative physics w/ D-branes:

- I. Type II (w/ D-branes at small string coupling)
 - Standard Model & GUT's (local and global)
 - D-instantons → new hierarchy for couplings
- II. F-theory (string theory w/ D-branes at finite coupling)
 - primarily GUT's
 - instantons

I. Particle Physics implications (Type II):

Recent focus: landscape of realistic D-brane quivers w/ D-instantons

String Consistent MSSM Quivers w/ realistic fermion textures

M.C., J. Halverson, R. Richter; & P. Langacker '09-'10

only highlights

Most MSSM Quivers-string inconsistent

What are the simplest extensions?

With no additional nodes:

landscape analysis all MSSM quivers & additional matter

(compatible with string constraints) → stringy inputs on exotic matter

M.C., J. Halverson & P. Langacker, 1108.5187

With additional $U(1)$'s or $U(N)$ node:

implications for SUSY breaking, dark matter, Z'

M.C., J. Halverson & H. Piragua, UPR-2041-T, to appear

II. D-instantons – formal developments

Focus on F-theory

Theory at finite string coupling g_s w/ no fundamental formulation

→ multi-pronged approaches

Recent Past:

i) zero mode structure

neutral (3-3) zero modes → monodromies in F-theory; anomaly inflow

[M.C., I. Garcia-Etxebarria, R. Richter, 0911.0012],

[M.C., I. Garcia-Etxebarria, J. Halverson, 1107.2388]

charged (3-7) zero modes → string junctions

[M.C., I. Garcia-Etxebarria, J. Halverson, 1107.2388],...

Recent/Current:

ii) Superpotential via dualities & directly in F-theory

→ ii) F-theory instanton superpotential

Focus on Pfaffians (7-brane moduli dependent prefactors):

i) Via Heterotic Duality → Geometric interpretation of zero loci
(including E_8 symmetric point)

[M.C., I. Garcia-Etxebarria & J. Halverson, 1107.2388]

ii) Inclusion of fluxes & direct F-theory results

[M.C., R. Donagi, J. Halverson & J. Marsano, UPR-1040-T, to appear]

iii) Effective Superpotential via N=2 D=3 M-theory

[study of anomaly cancellation as a prerequisite]

[M.C., T. Grimm, J. Halverson & D. Klevers, work in progress]

Not much time

I. Type II: Model Building with D-branes →

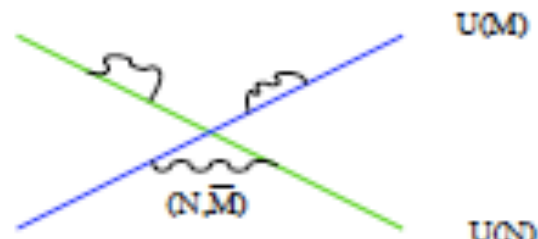
fertile ground for particle physics model building

[Pedagogical review: TASI'10 lectures, M.C., Halverson arXiv:1101.2907]

Illustrate: Type IIA w/intersecting D-branes →

key features of SM & SU(5) GUT spectrum →

non-Abelian gauge symmetry, chirality & family replication Geometric



The diagram shows two intersecting lines, one green and one blue, representing D-branes. The intersection is labeled with (N, \bar{M}) . To the right of the green line is the label $U(M)$, and to the right of the blue line is the label $U(N)$.

Representation	Multiplicity
(\bar{a}, b)	$\pi_a \circ \pi_b$
(\bar{a}, \bar{b})	$\pi_a \circ \pi'_b$
$\begin{array}{ c } \hline a \\ \hline \end{array}$	$\frac{1}{2} (\pi'_a \circ \pi_a + \pi_{O6} \circ \pi_a)$
$\begin{array}{ c c } \hline & a \\ \hline \end{array}$	$\frac{1}{2} (\pi'_a \circ \pi_a - \pi_{O6} \circ \pi_a)$

Large classes (order of 100's) of supersymmetric,
globally consistent (Gauss's law for D-brane charge)

[Aldazabal et al.'00-'01]; [Blumenhagen et al.'00-'01]

SM-like & GUT constructions; also coupling calculations

(primarily toroidal orbifolds)

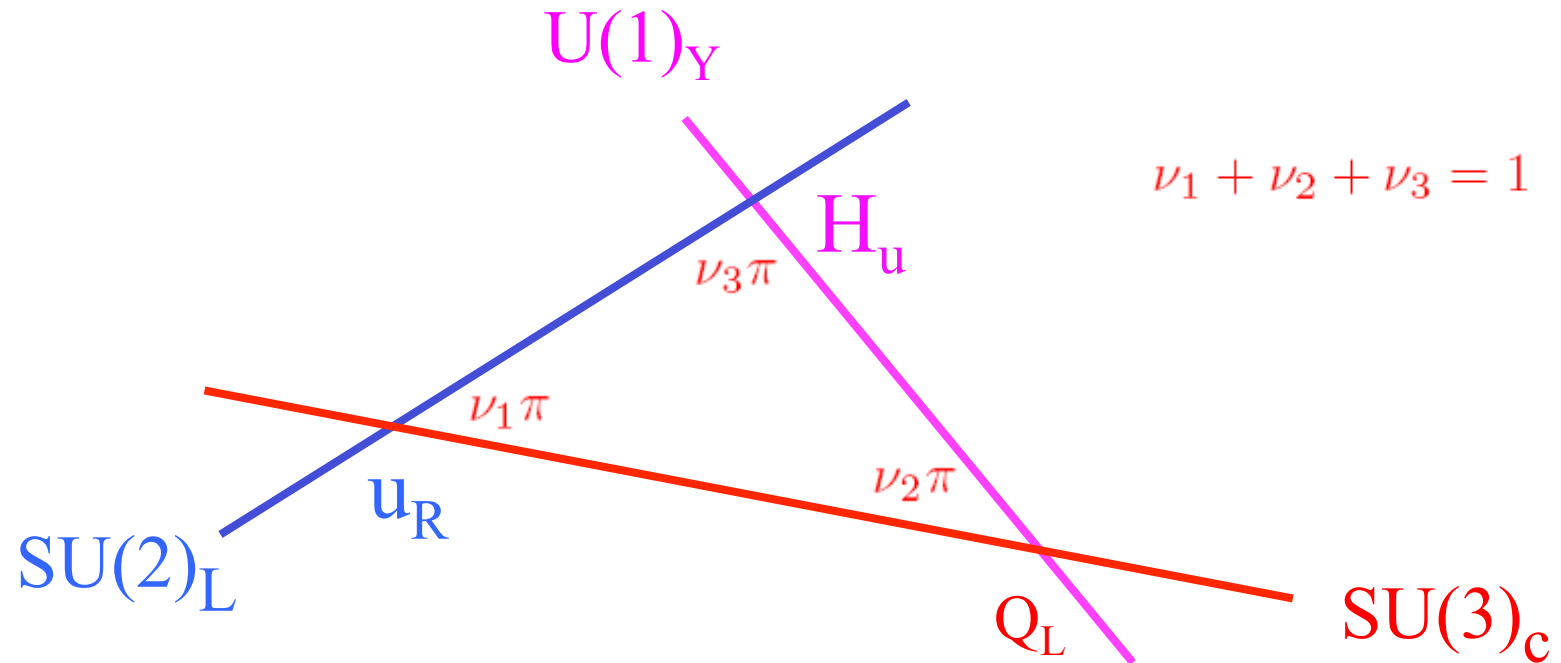
[M.C. ,Shiu, Uranga'01]...

[M.C. Papadimitriou '03],

[Cremades, Ibáñez, Marchesano'03]...

Yukawa Couplings

(schematic) Intersections in internal space (schematic on i^{th} -two-torus of an orbifold)



[M.C., Papadimitriou'03]

$$Y = (2\pi)^{\frac{3}{2}} g_{st} \prod_{i=1}^3 \left[\frac{\Gamma(1 - \nu_1^i) \Gamma(1 - \nu_2^i) \Gamma(1 - \nu_3^i)}{\Gamma(\nu_1^i) \Gamma(\nu_2^i) \Gamma(\nu_3^i)} \right]^{\frac{1}{4}} \sum_I \exp\left(-\frac{A_I^1 + A_I^2 + A_I^3}{2\pi\alpha'}\right)$$

quantum part

classical part A_I^i -triangle areas on i^{th} two-torus lattice

Non-perturbative effects → D-instantons

Motivation:

i) Important role in moduli stabilization

... [Kachru, Kallosh, Linde, Trivedi'03],...

[Balasubramanian, Berglund, Conlon, Quevedo'05],...

ii) New types of D-instantons: generate certain perturbatively absent couplings for charged sector matter

[Blumenhagen, M.C., Weigand, hep-th/0609191],

[Ibañez, Uranga, hep-th/0609213],

- charges matter coupling corrections

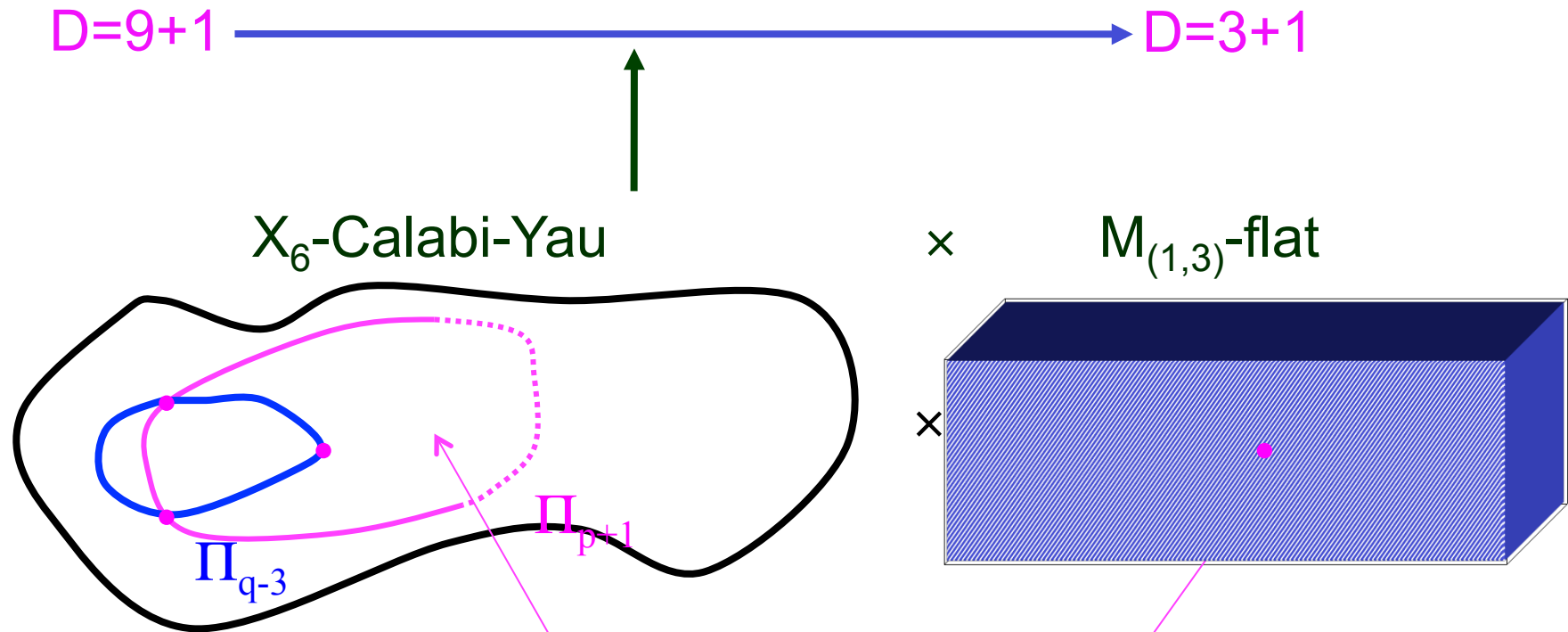
[Floreza, Kachru, McGreevy, Saulina, hep-th/0610003]

-supersymmetry breaking

Review: [Blumenhagen, M.C., Kachru, Weigand, 0902.3251]

Encoded in non-perturbative violation of ``anomalous'' $U(1)$'s

Illustrate: Type II A D-Instantons (geometric)- Euclidean D-brane



Wraps cycle Π_{p+1} cycles of X_6 point-in 3+1 space-time

New geometric hierarchies for couplings:

$$\mathcal{R}e(e^{-S_{E2}}) = e^{-\frac{2\pi}{\ell_s^3 g_s} \text{Vol}_{E2}} = e^{-\frac{2\pi}{\alpha_{\text{GUT}}} \frac{\text{Vol}_{E2}}{\text{Vol}_{D6}}} \quad \text{stringy!}$$

Instanton can intersect with D-brane (charged - zero modes)

→ generate non-perturbative couplings of charged matter

Rigid $O(1)$ instantons \rightarrow direct contribution to superpotential

I. Wrap rigid cycles homologically related to orientifold cycles-

Neutral zero modes $\bar{\tau}^{\dot{\alpha}}$ projected out

[Argurio et al.0704.0262]

\rightarrow 4 bosonic modes x_E^μ & only 2 fermionic modes θ_α

yield directly superpotential measure: $\int d^4 x_E d^2 \theta$

$$W \sim e^{-S_{E2}^{cl}} \prod_i \Phi_i,$$

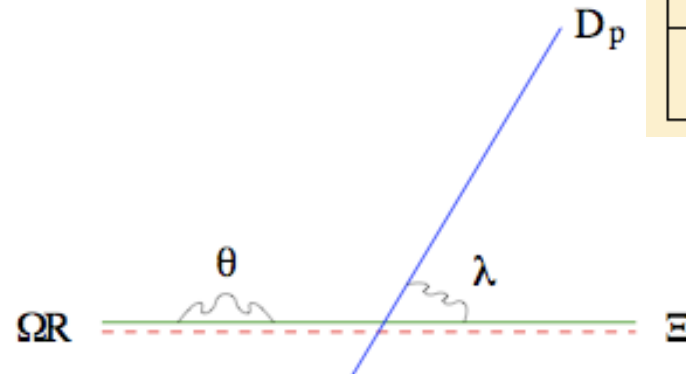
II. Charged Zero modes from strings between $E2$ and $D6_a$:

\rightarrow Localized at each intersection of $E2$ and $D6_a$:

One fermionic zero mode λ_a per intersection

Stringy & Geometric!

Zero modes	Reps	Number
$\lambda_{a,I}$	$(-1_E, \square_a)$	$I = 1, \dots, [\Xi \cap \Pi_a]^+$
$\bar{\lambda}_{a,I}$	$(1_E, \bar{\square}_a)$	$I = 1, \dots, [\Xi \cap \Pi_a]^-$
$\lambda_{a',I}$	$(-1_E, \bar{\square}_a)$	$I = 1, \dots, [\Xi \cap \Pi'_a]^+$
$\bar{\lambda}_{a',I}$	$(1_E, \square_a)$	$I = 1, \dots, [\Xi \cap \Pi'_a]^-$



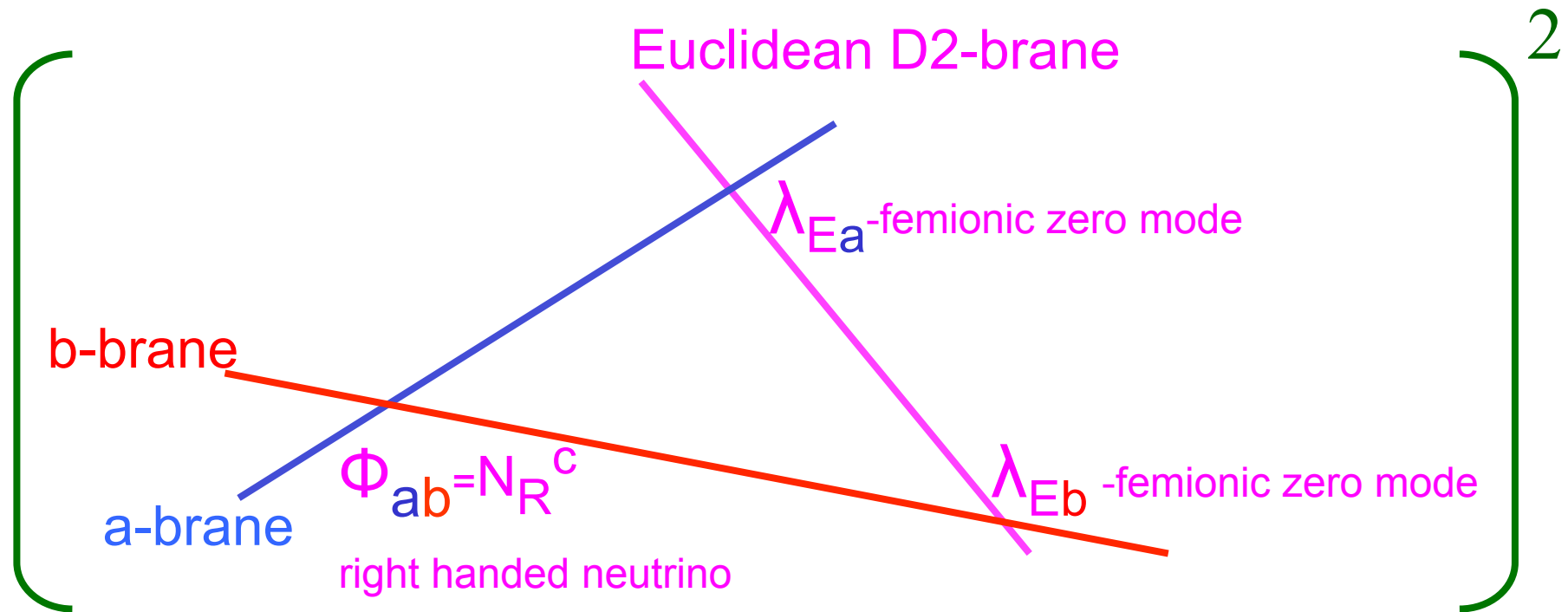
III. Develop conformal field theory instanton calculus

[Blumenhagen, M. C., Weigand, hep-th/0609191, ...]

Building blocks: disc-level couplings of two λ modes to

matter Φ_{ab} : $S = \int_{\Xi} \lambda_a \Phi_{ab} \bar{\lambda}_b$

Illustrate: Type IIA Euclidean D2-brane O(1) Instanton
 & Majorana neutrino mass: wraps 3-cycle $[\Pi_{E3}]$ in internal space



There is non-zero non-perturbative coupling: $M_m N_R^c N_R^c$

for Euclidean D2-instanton w/ $[\Pi_a]^\circ[\Pi_{E3}] = 2$ & $[\Pi_b]^\circ[\Pi_{E3}] = -2 \rightarrow$
 λ zero modes appear precisely ONCE and thus M_m non-zero
 (CFT calculation for M_m on an orbifold [M.C., Richter, Weigand'07])
 Geometric!

Specific examples of instanton induced charged matter couplings:

i) Majorana neutrino masses original papers...

ii) Nonpert. Dirac neutrino masses [M.C., Langacker, 0803.2876]

iii) $10 \ 10 \ 5$ GUT coupling in $SU(5)$ GUT's

[Blumenhagen, M.C. Lüst, Richter, Weigand, 0707.1871]

iv) Polonyi-type couplings

[Aharony, Kachru, Silverstein, 0708.0493], [M.C. Weigand, 0711.0209, 0807.3953],
[Heckman, Marsano, Sauline, Schäfer-Nameki, Vafa, 0808.1286]...

Original examples primarily for
Local Type IIA toroidal orbifolds SU(5) GUT's



Global models → Type I/IIB/F-theory (algebraic geometry)

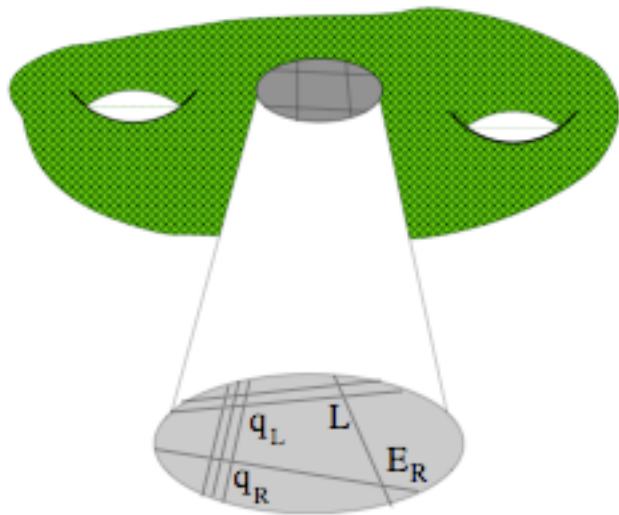
- i. Type I GUT's on compact elliptically fibered Calabi-Yau
First global chiral (four-family) SU(5) GUT's w/ D-instanton
generated Polonyi & Majorana neutrino masses
[M.C., T.Weigand, 0711.0209, 0807.3953]
- ii. Global Type IIB GUT's : $10^{10} 5_H$ non-perturbative coupling
(two family) SU(5) GUT on CY as hypersurface in toric variety
[Blumenhagen, Grimm, Jurke, Weigand, 0811.2938]
- iii. Global F-theory lift [M.C., I. Garcia-Etxebarria, J. Halverson, 003.5337]
[Develop a code to calculate zero modes/spectrum in Type IIB and F-theory on
toric varieties; code w/ new efficient technique →
[Blumenhagen, Jurke, Rahn & Roschy, 1003.5217]]

Most examples instantons addressed SU(5) GUT's
How about directly Standard Model?

Local Madrid quiver [Ibañez,Richter, 0811.1583]



Systematic Analysis of D-Instanton effects for MSSM's quivers
(compatible with global/stringy constraints)



Landscape analysis of MSSM w/
realistic fermion textures

[M.C., J. Halverson, R. Richter, 0905.3379;
0909.4292; 0910.2239]

Stringy Weinberg operator neutrino masses
(examples of low string scale)

[M.C., J. Halverson, P. Langacker, R. Richter, 1001.3148]

Singlet-extended MSSM landscape

[M.C. J. Halverson, P. Langacker, 1006.3341]

Bottom-up approach initiated [Aldazabal,Ibanez,Quevedo,Uranga'00]..

Related recent works: Specific 3-stack [Leontaris, 0903.3691]

Madrid quiver [Anastasopoulos, Kiritsis, Lionetto, 0905.3044]

SU(5) GUT's [Kiritsis, Lennek, Schellekens, 0909.0271]...

MSSM at toric singularities: [Krippendorff, Dolan,Maharana,Quevedo,1002.1790, 1106.6039]...

Approach: Bottom-up quivers

Spectrum and couplings **geometric** 

efficient **classification** of key physics

[compatible w/ global constraints → **stringy**, but without delving into specifics of globally defined string compactifications]

Quiver data: massless **spectrum** &

examination of **couplings**

[both **perturbative** & **non-perturbative-instantons**]



Probe “quiver landscape”

to identify **realistic quivers** in the landscape of string vacua

I. Spectrum: exact MSSM w/ 3 right-handed neutrinos
compatible with RR tadpole cancellation & global constraint for massless $U(1)_Y$
[fixes specific reps., e.g., bi-fund., (anti-)symmetric; different reps. for diff. fams.]



of order 10^4 quivers (3&4 stacks); of order 10^6 quivers (5-stacks)

à la [Anastasopoulos, Dijkstra, Kiritsis, Schellekens, hep-th/0605226]

Couplings:

- i. top Yukawa coupling perturbative
- ii. charged fermion textures (pert. and/or non-pert.) & μ -parameter (non-pert.) – in the desired regime
- iii. Neutrino masses (non-pert.): seesaw or non-pert. Dirac
- iv. Fermion texture instantons do not generate:
 μ -term & R-parity violating and dim-5 proton decay ops.

[i.-iv. fix $O(1)$ -instanton intersection numbers & size of its S_{cl}]

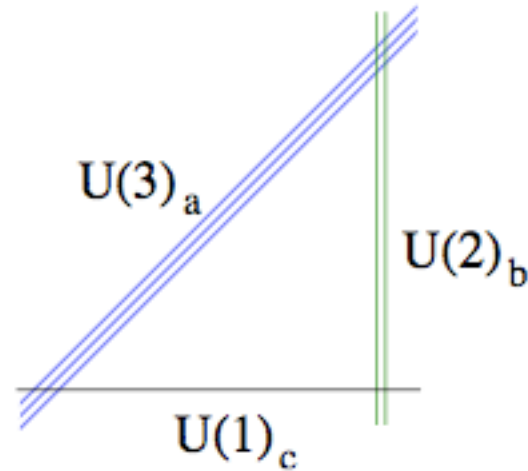


of order 30 MSSM quivers w/potentially realistic textures

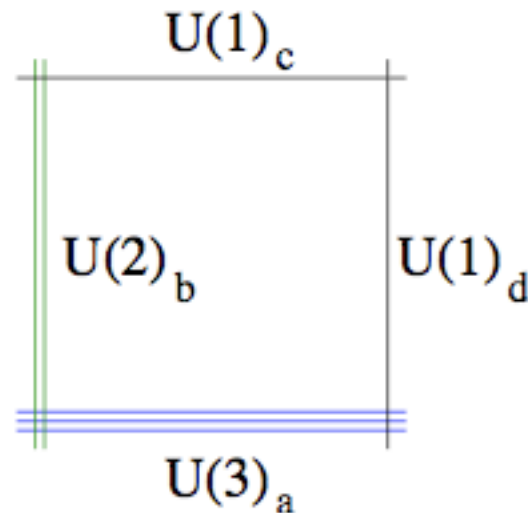
[M.C., J. Halverson, R. Richter, 0905.3379]

Multi-stack MSSM quivers

Employ three-stack MSSM $U(3)_a \times U(2)_b \times U(1)_c$



& four-stack MSSM $U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$



five-stack....

Four-stack set of MSSM models w/ 3 N_R & potentially viable fermion textures

[M.C., J. Halverson, R. Richter, 0905.3379]

Solution #	q_L		d_R			u_R		L			E_R		N_R			H_u				H_d		
	(a, b)	(a, \bar{b})	(\bar{a}, c)	(\bar{a}, \bar{d})	Γ_a	(\bar{a}, \bar{c})	(\bar{a}, d)	(b, \bar{c})	(b, d)	(\bar{b}, d)	(c, \bar{d})	\perp_c	\perp_d	\top_b	\top_c	(c, d)	(\bar{c}, \bar{d})	(b, c)	(\bar{b}, c)	(b, \bar{d})	(\bar{b}, \bar{d})	(b, \bar{c})
1	3	0	3	0	0	0	3	0	0	3	0	0	3	2	0	0	1	0	0	0	1	1
2	3	0	2	0	1	0	3	0	0	3	0	0	3	2	0	0	1	0	0	0	1	1
3	3	0	1	0	2	0	3	0	0	3	0	0	3	2	0	0	1	0	0	0	1	1
4	3	0	0	0	3	0	3	0	0	3	0	0	3	2	0	0	1	0	0	0	1	1
5	3	0	0	0	3	0	3	0	0	3	0	0	3	3	0	0	0	1	0	0	0	1
6	3	0	3	0	0	2	1	0	0	3	0	2	1	2	0	1	0	0	0	0	1	1
7	3	0	3	0	0	3	0	0	0	3	2	1	0	2	0	0	1	0	1	0	0	1
8	3	0	3	0	0	3	0	0	0	3	0	2	1	2	0	0	1	0	1	0	0	1
9	3	0	3	0	0	3	0	0	0	3	1	2	0	2	0	1	0	0	1	0	0	1
10	2	1	3	0	0	1	2	0	0	3	0	0	3	0	0	0	3	1	0	0	0	1
11	2	1	3	0	0	1	2	0	0	3	3	0	0	0	0	3	0	1	0	0	0	1
12	2	1	3	0	0	3	0	0	0	3	3	0	0	0	0	0	3	1	0	0	0	1
13	2	1	3	0	0	3	0	0	1	2	3	0	0	0	0	0	3	0	1	0	0	1
14	1	2	3	0	0	3	0	0	3	0	3	0	0	0	0	0	3	1	0	0	0	1
15	0	3	0	3	0	0	3	3	0	0	0	1	2	0	3	0	0	0	0	1	0	1
16	0	3	0	0	3	0	3	0	3	0	0	0	3	0	3	0	0	1	0	0	0	1
17	0	3	0	0	3	0	3	1	2	0	1	0	2	0	3	0	0	0	0	1	0	1
18	0	3	0	0	3	0	3	3	0	0	2	0	1	0	3	0	0	0	0	1	0	1
19	0	3	0	0	3	0	3	3	0	0	0	1	2	0	3	0	0	0	0	1	0	1
20	0	3	0	0	3	1	2	1	2	0	2	0	1	0	3	0	0	1	0	0	0	1
21	0	3	0	0	3	1	2	1	2	0	0	1	2	0	3	0	0	1	0	0	0	1
22	0	3	0	0	3	1	2	3	0	0	3	0	0	0	3	0	0	1	0	0	0	1
23	0	3	0	0	3	1	2	3	0	0	1	1	1	0	3	0	0	1	0	0	0	1
24	0	3	0	0	3	2	1	0	3	0	3	0	0	0	3	0	0	1	0	0	0	1
25	0	3	0	0	3	2	1	0	3	0	1	1	1	0	3	0	0	1	0	0	0	1
26	0	3	0	0	3	2	1	2	1	0	2	1	0	0	3	0	0	1	0	0	0	1
27	0	3	0	0	3	2	1	2	1	0	0	2	1	0	3	0	0	1	0	0	0	1
28	0	3	0	3	0	3	0	3	0	0	0	3	0	0	3	0	0	1	0	0	0	1
29	0	3	0	2	1	3	0	3	0	0	0	3	0	0	3	0	0	1	0	0	0	1
30	0	3	0	1	2	3	0	3	0	0	0	3	0	0	3	0	0	1	0	0	0	1
31	0	3	0	0	3	3	0	1	2	0	1	2	0	0	3	0	0	1	0	0	0	1

Solutions for hypercharge $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c - \frac{1}{2}U(1)_d$ - Madrid embedding

Concrete 5-stack model (benchmark)

(w/ three mass scales in top, bottom in charged lepton sector)

Sector	Matter Fields	Transformation	Multiplicity	Hypercharge
ab	q_L	(a, \bar{b})	1	$\frac{1}{6}$
ab'	q_L	(a, b)	2	$\frac{1}{6}$
ac'	u_R	(\bar{a}, \bar{c})	2	$-\frac{2}{3}$
ad'	u_R	(\bar{a}, \bar{d})	1	$-\frac{2}{3}$
aa'	d_R	$\begin{array}{ c } \hline \square \\ \hline \end{array}_a$	3	$\frac{1}{3}$
bc'	H_u	(b, c)	1	$\frac{1}{2}$
bd'	L	(\bar{b}, \bar{d})	3	$-\frac{1}{2}$
be'	H_d	(\bar{b}, \bar{e})	1	$\frac{1}{2}$
ce'	E_R	(c, e)	2	1
ce	N_R	(\bar{c}, e)	1	0
dd'	E_R	$\begin{array}{ c c } \hline \square & \square \\ \hline \end{array}_d$	1	1
de	N_R	(\bar{d}, e)	2	0

Allows for full (inter- & intra-) family mass hierarchy via “factorization of Yukawa matrices” due to vector-pairs of zero fermion modes-stringy (technical, no time)

Recent: String constraints & matter beyond the MSSM

[M.C., J. Halverson, P. Langacker, 1108.5387]

I. Classify ALL possible MSSM quivers (three & four stacks)

irrespective of global conditions → most quivers inconsistent

What is additional matter to be compatible w/ global constraints?

→ stringy inputs on exotic matter

3-stack analysis: global conditions ($T_{a,b,c}=0$) constraining, e.g., MSSM w/

$$U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c \quad T_a = 0 \quad T_b = \pm 2n \quad T_c = 0 \bmod 3 \quad \text{with } n \in \{0, \dots, 7\},$$

w/ preferred additions: quasi-chiral Higgs pairs, MSSM singlets

hypercharge-less SU(2) triplets, &

various quark anti-quark pairs, all w/ integer el. ch.;

one (massless) Z' quiver

4-stack analysis: richer structure

sizable number of quivers w/ Z', including leptophobic (tuned);

additional structures: possible $SH_{\underline{u}}H_{\underline{d},}$ v-masses;

exotics w/ fractional el. ch. ...

● 105

3-node quivers (≤ 5 additions)

Multiplicity	Matter Additions				
4	$\boxtimes_b, (1, 3)_0$	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$	$(a, \bar{b}), (3, 2)_{\frac{1}{6}}$	$(\bar{a}, \bar{b}), (\bar{3}, 2)_{-\frac{1}{6}}$
4	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$			
4	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$			
4	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$
4	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$
4	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	$(a, \bar{b}), (3, 2)_{\frac{1}{6}}$	$(\bar{a}, \bar{b}), (\bar{3}, 2)_{-\frac{1}{6}}$
4	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$			
4	$\boxplus_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$		
4	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	
4	$(a, \bar{b}), (3, 2)_{\frac{1}{6}}$	$\boxplus_a, (\bar{3}, 1)_{\frac{1}{3}}$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(\bar{a}, \bar{c}), (\bar{3}, 1)_{-\frac{2}{3}}$	$\boxtimes_c, (1, 1)_1$
4	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$
4	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$
4	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$		
4	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	
4	$\boxtimes_b, (1, 3)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$
4	$\boxplus_b, (1, 1)_0$				
4	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	
4	$\boxtimes_b, (1, 3)_0$	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	
4	$\boxtimes_b, (1, 3)_0$	$\boxtimes_b, (1, 3)_0$	$\boxplus_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$
4	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	$\boxplus_b, (1, 1)_0$	

Multiplicity	Matter Additions				
4	$\overline{\square}b, (1, 3)_0$	$\overline{\square}b, (1, 3)_0$	$\overline{\square}b, (1, 3)_0$	$\overline{\square}b, (1, 1)_0$	$\overline{\square}b, (1, 1)_0$
4	$\overline{\square}b, (1, 3)_0$	$\overline{\square}b, (1, 3)_0$	$\square b, (1, 1)_0$		
1	$\square a, (\overline{3}, 1)_{\frac{1}{3}}$	$\square b, (1, 3)_0$	$\square b, (1, 1)_0$	$(a, \overline{c}), (3, 1)_{-\frac{1}{3}}$	
1	$\overline{\square}a, (3, 1)_{-\frac{1}{3}}$	$\square b, (1, 3)_0$	$\square b, (1, 1)_0$	$(\overline{a}, c), (\overline{3}, 1)_{\frac{1}{3}}$	
1	$\square a, (\overline{3}, 1)_{\frac{1}{3}}$	$\overline{\square}b, (1, 3)_0$	$\square b, (1, 1)_0$	$(a, \overline{c}), (3, 1)_{-\frac{1}{3}}$	
1	$\overline{\square}a, (3, 1)_{-\frac{1}{3}}$	$\overline{\square}b, (1, 3)_0$	$\square b, (1, 1)_0$	$(\overline{a}, c), (\overline{3}, 1)_{\frac{1}{3}}$	
1	$\square a, (\overline{3}, 1)_{\frac{1}{3}}$	$\overline{\square}b, (1, 1)_0$	$\overline{\square}b, (1, 1)_0$	$(a, \overline{c}), (3, 1)_{-\frac{1}{3}}$	
1	$\overline{\square}a, (3, 1)_{-\frac{1}{3}}$	$\overline{\square}b, (1, 1)_0$	$\overline{\square}b, (1, 1)_0$	$(\overline{a}, c), (\overline{3}, 1)_{\frac{1}{3}}$	
1	$\square a, (\overline{3}, 1)_{\frac{1}{3}}$	$\overline{\square}b, (1, 1)_0$	$(b, \overline{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	$(a, \overline{c}), (3, 1)_{-\frac{1}{3}}$
1	$\overline{\square}a, (3, 1)_{-\frac{1}{3}}$	$\overline{\square}b, (1, 1)_0$	$(b, \overline{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	$(\overline{a}, c), (\overline{3}, 1)_{\frac{1}{3}}$
1	$(a, \overline{b}), (3, 2)_{\frac{1}{6}}$	$(b, \overline{c}), (1, 2)_{-\frac{1}{2}}$	$(\overline{a}, c), (\overline{3}, 1)_{\frac{1}{3}}$	$(\overline{a}, \overline{c}), (\overline{3}, 1)_{-\frac{2}{3}}$	$\square c, (1, 1)_1$
1	$\square a, (\overline{3}, 1)_{\frac{1}{3}}$	$\overline{\square}b, (1, 3)_0$	$\overline{\square}b, (1, 1)_0$	$\overline{\square}b, (1, 1)_0$	$(a, \overline{c}), (3, 1)_{-\frac{1}{3}}$
1	$\overline{\square}a, (3, 1)_{-\frac{1}{3}}$	$\overline{\square}b, (1, 3)_0$	$\overline{\square}b, (1, 1)_0$	$\overline{\square}b, (1, 1)_0$	$(\overline{a}, c), (\overline{3}, 1)_{\frac{1}{3}}$
1	$\square a, (\overline{3}, 1)_{\frac{1}{3}}$	$\square a, (\overline{3}, 1)_{\frac{1}{3}}$	$\square b, (1, 1)_0$	$(a, \overline{c}), (3, 1)_{-\frac{1}{3}}$	$(a, \overline{c}), (3, 1)_{-\frac{1}{3}}$
1	$\overline{\square}a, (3, 1)_{-\frac{1}{3}}$	$\overline{\square}a, (3, 1)_{-\frac{1}{3}}$	$\square b, (1, 1)_0$	$(\overline{a}, c), (\overline{3}, 1)_{\frac{1}{3}}$	$(\overline{a}, c), (\overline{3}, 1)_{\frac{1}{3}}$
1	$\square a, (\overline{3}, 1)_{\frac{1}{3}}$	$\square b, (1, 1)_0$	$(a, \overline{c}), (3, 1)_{-\frac{1}{3}}$		
1	$\overline{\square}a, (3, 1)_{-\frac{1}{3}}$	$\square b, (1, 1)_0$	$(\overline{a}, c), (\overline{3}, 1)_{\frac{1}{3}}$		
1	$\square a, (\overline{3}, 1)_{\frac{1}{3}}$	$\overline{\square}b, (1, 3)_0$	$\overline{\square}b, (1, 3)_0$	$\square b, (1, 1)_0$	$(a, \overline{c}), (3, 1)_{-\frac{1}{3}}$
1	$\overline{\square}a, (3, 1)_{-\frac{1}{3}}$	$\overline{\square}b, (1, 3)_0$	$\overline{\square}b, (1, 3)_0$	$\square b, (1, 1)_0$	$(\overline{a}, c), (\overline{3}, 1)_{\frac{1}{3}}$

M.C., J. Halverson & H. Piragua, UPR-1041-T, to appear

II. MSSM's with additional Hidden Sector nodes

Up-to n-additional $U(1)$'s or one $U(N)$

Systematic search (w/implement global consistency conditions)

i) SM singlets by far the most common fields

&(light anomalous) $U(1)$ '-monochromatic gamma ray line dark matter scenario
à la Dudas, Mambrini, Pokorski, Romagnoni 1205.1520

w/ coupling to SM automatically forbidden by anomalous $U(1)$

decay to $Z \gamma$ possible (via $Z' - B_Y - B_Y$ "Chern-Simons" vertex).

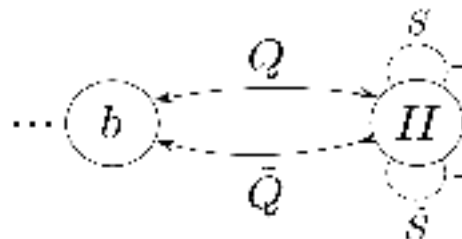
ii) E.g., Stringy dynamical SUSY breaking scenario (à la Fayet)

Aharony, Kachru, Silverstein '07

Quasi-chiral matter \rightarrow messenger masses instanton suppressed;

lifetime of metastable vacuum set by instanton superpotential

$$W \supset \lambda_{(0,0)} M_* S \tilde{S} + \lambda_{(-2,0)} M_* Q \tilde{Q} + \lambda_{(-2,0)} \frac{1}{M_*} S \tilde{S} Q \tilde{Q}$$



II. Model building in F-theory

Vafa'96..

Revival: geometric features of particle physics w/ intersecting branes
& exceptional gauge symmetries common in the heterotic string
-- at finite string coupling g_s

Geometry of F-theory: Elliptically fibered Calabi-Yau fourfold Y_4 ;
complexified g_s encoded in T^2 fibration over the base B_3

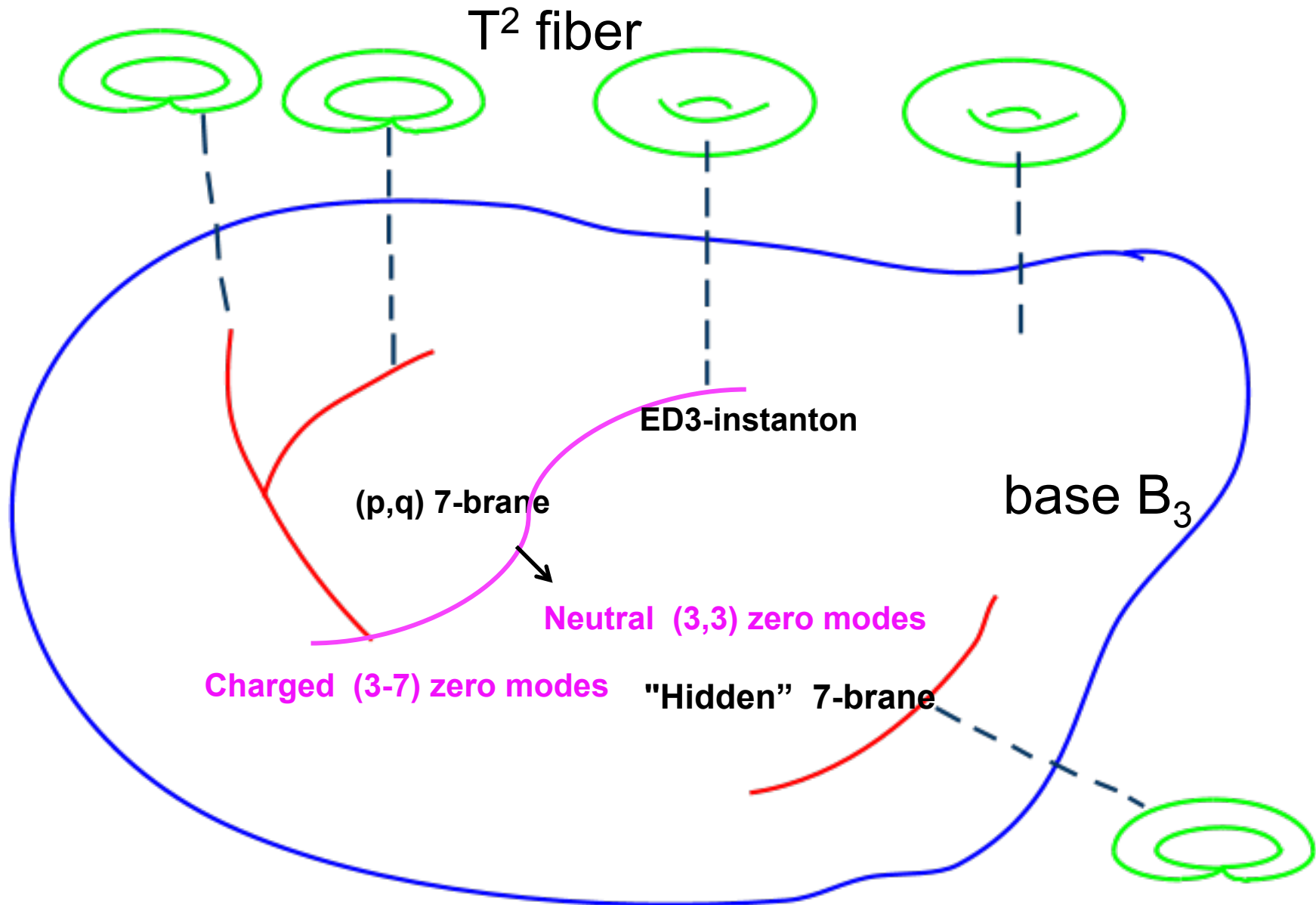
Gauge Symmetry: where fiber degenerates (say for T^2 $pA+qB$ cycle) a
co-dim 1 singularity signified a location (p,q) 7-branes in the base B_3

Matter: Intersecting 7-branes at co-dim 2 singularities G_4 -flux needed (for chirality)

(Semi-) local & (limited) global SU(5) GUT's: chiral matter &
Yukawa couplings (co-dim two (and three) singularities on the GUT 7-brane)...
[Donagi, Wijnholt'08'11'12], [Beasley, Heckman, Vafa'08], ...
[Marsano, Schäfer-Nameki, Saulina'08'10'11], [Marsano Schäfer-Nameki'11],
[Blumehagen, Grimm, Jurke, Weigand'09],
[M.C., Garcia-Etxebarria, Halverson, 1003.533], ...
[Grimm, Weigand'10], [Grimm, Hayashi'11]; [Krause, Mayrhofer, Weigand'11'12], ...
[Esole, Yau'11], ... [Cecotti, Cordova, Heckman, Vafa'10], ...

Cartoon of **F-theory compactification** (Y_4 as T^2 over B_3)

Instanton: Euclidean D3 brane (ED3) wrapping divisor in B_3



Instantons in F-theory

Past Work:

[Witten'96], [Donagi, Grassi, Witten'96], [Katz, Vafa'96],
[Ganor'96], ..., [Diaconescu, Gukov'98], ...

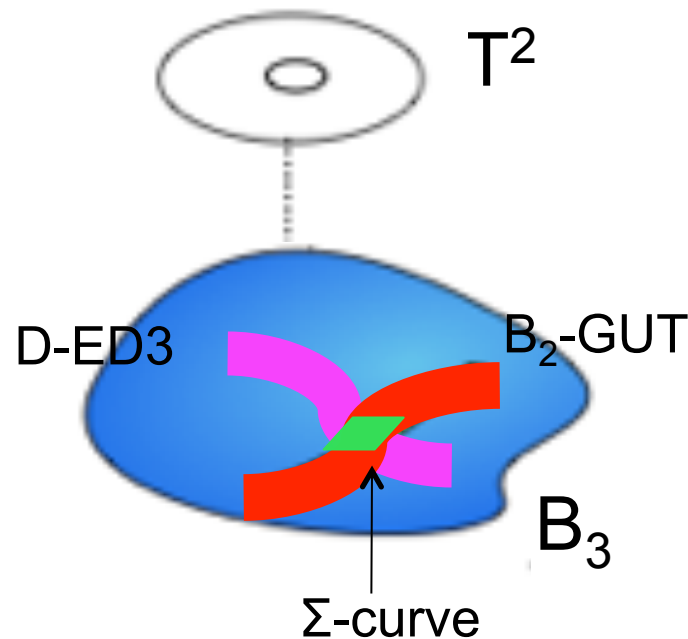
Recent Work:

[Blumenhagen, Collinucci, Jurke'10],
[M.C., García-Etxebarria, Halverson'10,'11], [Donagi, Wijnholt'11],
[Grimm, Kerstan, Palti, Weigand'11],
[Marsano, Saulina, Schäfer-Nameki'11],
[Bianchi, Collinucci, Martucci'11], [Kerstan, Weigand'12]



Related recent works focus on G_4 -fluxes and $U(1)$'s

Non-pert. Superpotential for moduli stabiliz. $W \sim Ae^{-T}$
 due to ED3 wrapping divisor D in $B_{3,1}$
 in the presence of (E_6) GUT 7-brane wrapping B_2
 w/local structure captured by intersection curve Σ & flux G_4 there



Key upshots:

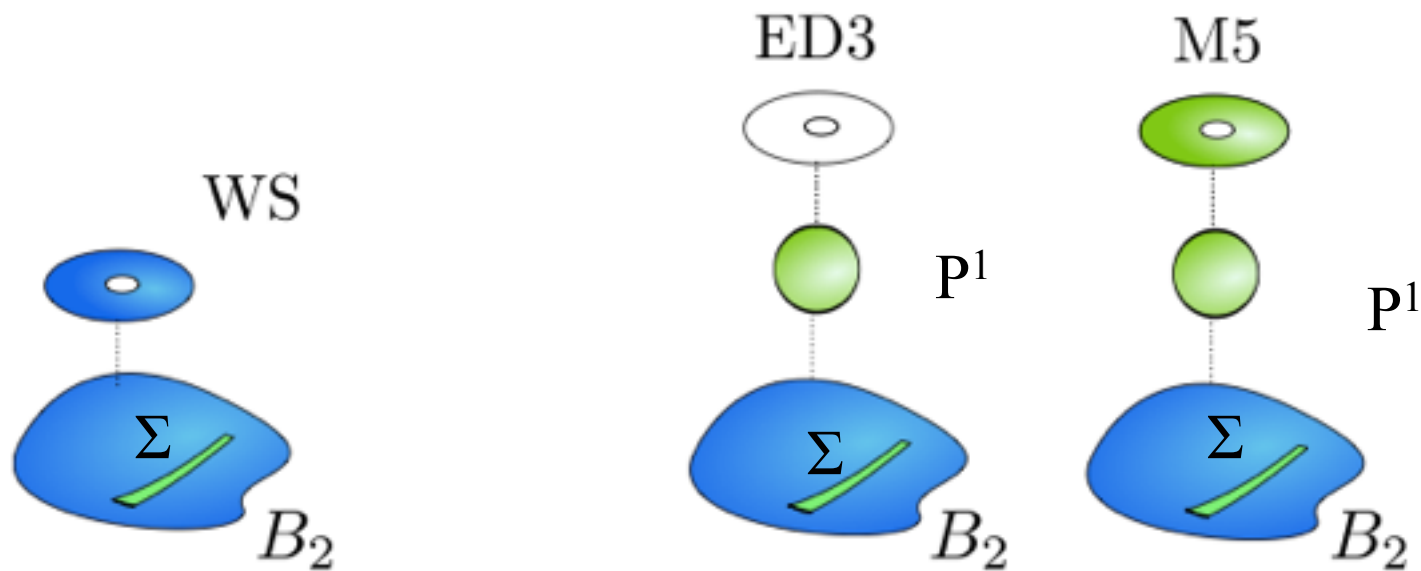
- i) Conjecture how to compute Pfaffian A
 (7-brane moduli dependent prefactor)
- ii) Explicit F-theory examples;
 analyse substructure, such as points of E_8 enhancement

F-theory ED3-instanton via duality (brief):

Heterotic \longleftrightarrow F-theory \longleftrightarrow M-theory ^{*}Digression

Shrink elliptic fiber
w/ fixed compl. str.

M5 with a leg in the fiber
(vertical divisor)



*Digression: F-theory via D=3, N=2 M-theory compactification

[Grimm, Hayashi'11], [Grimm,Klevers'12]

Analyze 4D F-theory in D=3, N=2 supergravity on Coulomb branch

$$\text{F-theory on } X_4 \times S^1 = \text{M-theory on } \hat{X}_4$$

Matching of two effective theories possible only at 1-loop



1-loop in F-theory (by integrating out massive matter) =
classical supergravity terms in M-theory

Rich and controllable example: 3D Chern-Simons terms $\Theta_{IJ} A^I \wedge F^J$

⇒ 3D Chern-Simons terms Θ_{IJ} encode

[Aharony, Hanany, Intriligator,
Seiberg, Strassler'97]

1) 4D chiral index $\chi(R)$: $\Theta_{IJ}^M = \int_{X_4} G_4 \wedge \omega_I \wedge \omega_J \iff \Theta_{IJ}^{loop} \sim \chi(R)$
(M-theory/supergravity) (F-theory)

2) 4D anomaly cancellation implies relation among different CS-terms Θ_{IJ}

[M.C., Grimm, Klevers, to appear]

3) Index of charged instanton zero modes encoded in certain CS-terms.

[MC, Grimm, Halverson, Klevers, in preparation]

F-theory ED3-instanton via duality:

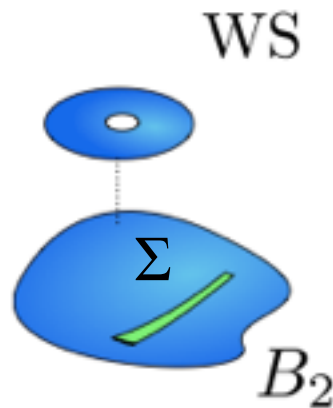
Heterotic \longleftrightarrow F-theory \longleftrightarrow M-theory

X_3 ellipt. fibered over B_2

Vector bundle $V \leftrightarrow$
 (C_{Het}, L) -spectral cover data

Worldsheet inst. wraps Σ in B_2
 Fermionic left-moving zero modes

$$h^0(\Sigma, V|_{\Sigma} \otimes \mathcal{O}(-1)) \\ \cong h^0(c, \mathcal{L} \otimes K_c^{\frac{1}{2}})$$

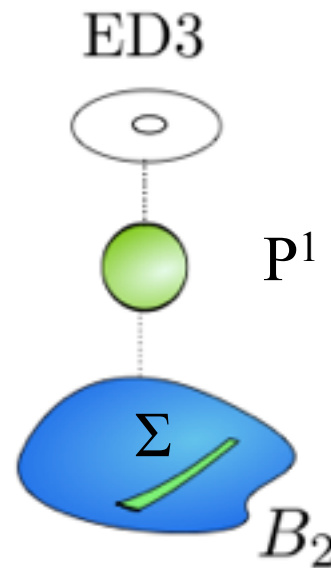


Y_4 ellipt. fibered over B_3
 with $B_3: P^1$ over B_2

Flux $G_4 \leftrightarrow$
 (C_F, N) -spectral cover data

ED3 wraps P^1 over Σ

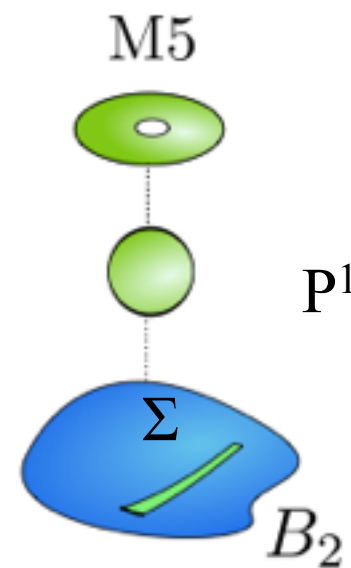
Fermionic “ λ ” (3-7) modes



$$\Sigma = \text{inst} \cdot_{B_3} S_{GUT}$$

Shrink elliptic fiber
 w/ fixed compl. str.

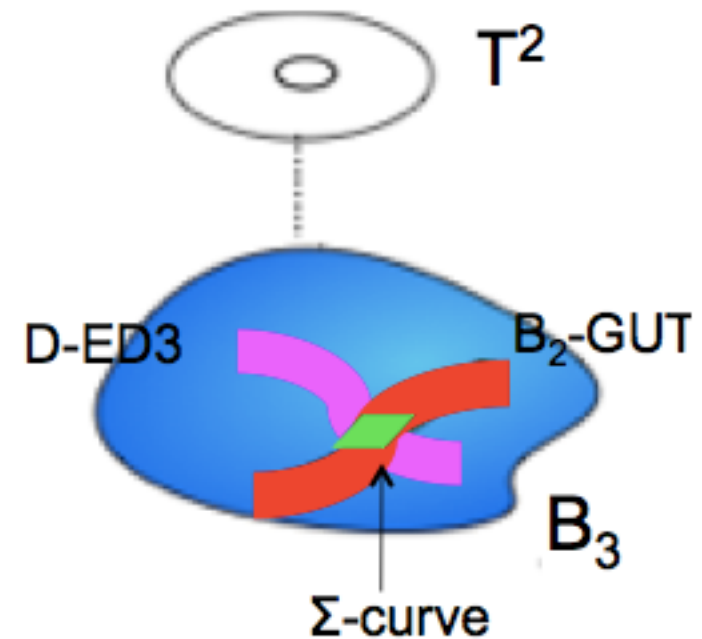
M5 with a leg in the fiber
 (vertical divisor)



Instanton data in F-theory:

ED3 on divisor D in the presence
 of (E_6) GUT divisor $B_2 \subset B_3$
 by gauge theory on $R^{(3,1)} \times B_2$ data
 (G_4 info) specified by Higgs bundle
 \leftrightarrow spectral cover data $(\mathcal{C}_{loc}, \mathcal{N}_{loc})$

Spectral surface, line bundle (G_4 info)



zero modes localize on $\Sigma \equiv D \cap B_2 \subset B_3$

\rightarrow study vector bundle cohomology on the intersection curve Σ

\rightarrow line bundle cohomology on a spectral curve $\mathcal{C}_{loc} = \mathcal{C}_{loc} \cap \pi^* \Sigma$

Defining equation of \mathcal{C}_{loc} specified by moduli \rightarrow
 7-brane moduli in the instanton world-volume

Computing Pfaffian prefactor:

Class of curve c_{loc} : $[c_{loc}] = 3s + rF$

E_6 GUT (n=3)

$\mathcal{E} \sim \pi^* \Sigma$

↑ section of \mathcal{E}
↑ elliptic fiber class

w/ further algebraic data:

$$c_1(\mathcal{N}_{c,loc}|\mathcal{E}) = \frac{1}{2}(ns + (r + \chi)F) + \lambda(ns - (r - n\chi)F) \quad \chi \equiv c_{1,B_2} \cdot_{B_2} \Sigma \quad r \equiv \eta \cdot_{B_2} \Sigma \in \mathbb{Z}$$

$$\mathcal{L}_A \equiv (\mathcal{N}_{c,loc} \otimes \mathcal{O}_{\mathcal{E}}(-F))|_{c_{loc}}$$

Pfaffian can be determined via moduli dependence of cohomology
 [w/short exact Koszul sequence $0 \rightarrow \mathcal{L}_A \otimes \mathcal{O}_{\mathcal{E}}(-c_{loc}) \rightarrow \mathcal{L}_A \rightarrow \mathcal{L}_A|_{c_{loc}} \rightarrow 0 \rightarrow$
 long exact sequence in cohomologies
 (determine moduli dependent matrix whose det is a Pfaffian)]

Analogous to heterotic computation

[Buchbinder,Donagi,Ovrut'02,...,Curio'08,09,10]

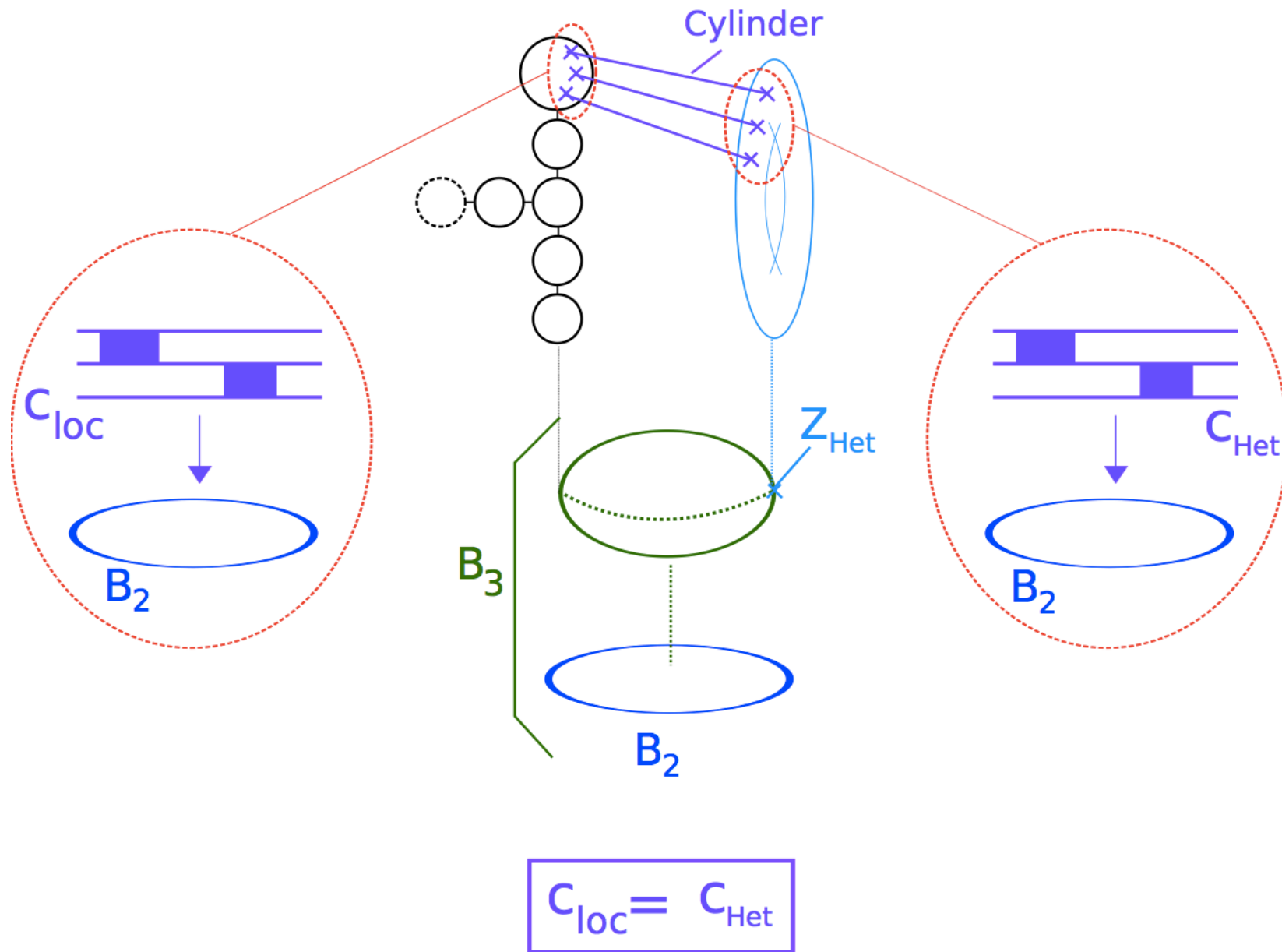
Non-trivial checks via duality:

Heterotic: cohomology isomorphism via cylinder map*
when a dual exists

Type IIB: gauge dependent data localized at instanton
and 7-brane intersection →
natural interpretation as (3-7) charged “ λ ” modes

M-theory: when a heterotic dual exists,
 $Jac(C_{loc})$ & $IJac(M5)$ are deeply related
[Further study...]

* cohomology on C_{loc} isomorphic to cohomology on C_{Het}
[under the cylinder map [Curio, Donagi'98], the curves are the same]



W/ heterotic dual \rightarrow cohomology on C_{loc} isomorphic to cohomology on C_{Het}
 [under the cylinder map [Curio, Donagi'98], the curves are the same]

Setting up the computation in F-theory:

- given B_3 , find an ED3 divisor D and GUT divisor B_2 which intersect at a curve Σ (P^1)

- compute spectral cover data:

$$\chi \equiv c_{1,B_2} \cdot_{B_2} \Sigma \quad r \equiv \eta \cdot_{B_2} \Sigma \in \mathbb{Z}.$$

and λ which satisfies D3 tadpole

→ Class of the spectral curve c_{loc} in \mathcal{E} & line bundle \mathcal{L}_A determined

→ compute Pfaffian via Koszul exact sequence

$$0 \rightarrow \mathcal{L}_A \otimes \mathcal{O}_{\mathcal{E}}(-c_{loc}) \rightarrow \mathcal{L}_A \rightarrow \mathcal{L}_A|_{c_{loc}} \rightarrow 0$$

Typical Pfaffian prefactor structure: $Pfaff \sim \prod_i f_i^{k_i}$

f_i - polynomials in complex structure of
7-brane moduli restricted to instanton world-volume

→ depend on local subset of full moduli data
[the same correction could arise in different compactifications]

→ interesting physics can determine the
substructure of each f_i

Example: Pfaffian calculation directly in F-theory (without a dual)

B_3 in terms of toric data (generalization of weighted projective spaces):

Holomorphic coord.

	x_0	x_1	x_2	x_3	z	x_5	class
GLSM charges (scaling weights) \rightarrow	1	0	0	1	1	1	X
	1	1	0	0	1	0	E_1
	1	0	1	1	0	0	E_2

$\langle x_2x_3, x_3x_5, zx_5, x_0x_1x_2, x_0x_1z \rangle$
Stanley-Reisner ideal

Divisor classes

- E_6 GUT on $B_2 = \{z = 0\}$ and ED3 instanton at $D = \{x_1 = 0\}$
- Y_4 defining equation: $y^2 = x^3 + f x z^4 v^4 + g z^6 v^6 + z^2 v^4 (b_0 z^3 v^3 + b_2 z v x + b_3 y)$
with sections $b_{(0,2,3)}$ in terms of $\mathcal{O}(a, b, c) \equiv \mathcal{O}(aX + bE_1 + cE_2)$
- compute: $r = 5, \chi = 1$ & $\lambda = \frac{3}{2} \rightarrow [c_{loc}] = 3s + 5F \quad \mathcal{L}_A = \mathcal{O}_{\mathcal{E}}(6s - F)$

- only subset of moduli b_m in Pfaffian: $\tilde{b}_m \equiv b_m|_{GUT \cap D3} = b_m|_{z=x_1=0}$

$$\tilde{b}_0 = \psi_1 x_3^5 + \psi_2 x_3^4 x_2^1 + \psi_3 x_3^3 x_2^2 + \psi_4 x_3^2 x_2^3 + \psi_5 x_3^1 x_2^4 + \psi_6 x_2^5$$

$$\tilde{b}_2 = \phi_1 x_3^3 + \phi_2 x_3^2 x_2^1 + \phi_3 x_3^1 x_2^2 + \phi_4 x_2^3$$

$$\tilde{b}_3 = \chi_1 x_3^2 + \chi_2 x_3^1 x_2^1 + \chi_3 x_2^2$$

- using defining eq. for c_{loc} $f_{c_{loc}} = \tilde{b}_0 W + \tilde{b}_2 u X + \tilde{b}_3 q$
to compute via Koszul exact sequence

- result:

$$pfaff \sim f_{E8}^4 = (\chi_1^2 \chi_3 \phi_3^2 - \chi_1^2 \chi_2 \phi_3 \phi_4 - 2\chi_1 \chi_3^2 \phi_3 \phi_1 - \chi_1 \chi_2 \chi_3 \phi_3 \phi_2 + \chi_2^2 \chi_3 \phi_1 \phi_3 + \phi_4^2 \chi_1^3 - 2\phi_2 \phi_4 \chi_3 \chi_1^2 + \chi_1 \chi_3^2 \phi_2^2 + 3\phi_1 \phi_4 \chi_1 \chi_2 \chi_3 + \phi_2 \chi_1 \phi_4 \chi_2^2 + \phi_1^2 \chi_3^3 - \phi_2 \chi_2 \phi_1 \chi_3^2 - \phi_4 \phi_1 \chi_2^3)^4 ,$$

Comments:

- beautiful factorization $pfaff \sim f_{E8}^4$
other examples (c.f., later) w/ substructure ubiquitous & w/ E_8 enhancement often
- the physics governing the substructure $\rightarrow f_{E8} = \det(M) = Res(\tilde{b}_2, \tilde{b}_3)$

E_8 enhanced point in instanton world-volume!

$$Pfaff = 0 \quad \Leftrightarrow \quad \exists \text{ pt of } E_8 \text{ in instanton WV}$$

$$M \equiv \begin{pmatrix} \phi_4 & \phi_3 & \phi_2 & \phi_1 & 0 \\ 0 & \phi_4 & \phi_3 & \phi_2 & \phi_1 \\ \chi_3 & \chi_2 & \chi_1 & 0 & 0 \\ 0 & \chi_3 & \chi_2 & \chi_1 & 0 \\ 0 & 0 & \chi_3 & \chi_2 & \chi_1 \end{pmatrix}$$

Sylvester matrix

- Is this relation more general? \rightarrow quantified further (no time)
[E_8 points can cause the Pfaffian to vanish even for SU(5) GUTs as a sublocus within the vanishing locus of the Pfaffian]
- Phenomenological implications: in SU(5) GUTs, points of E_8 enhancement can give natural flavor structure, minimal gauge mediated supersymmetry breaking... [Heckman, Tavanfar, Vafa'10]

Calculation well-defined → Scanning across B_3 bases:
 Toric B_3 -from triangulations of 4308 d=3 polytopes (99%) of
 Kreuzer-Skarke d=3 list

r	χ	M	N	Multiplicity	Comments
1	0	6	-2	2454	$pfaff = 0$ example with het. dual
5	1	6	-1	13163	pts of E_8 , example without het. dual
6	1	6	-2	15034	$pfaff = 0$ F-theory
7	1	6	-3	2897	transition matrix
8	1	6	-4	55	not computed
11	2	6	-2	13070	not computed
12	2	6	-3	5356	not computed
13	2	6	-4	168	not computed
16	3	6	-2	2200	not computed
17	3	6	-3	2507	not computed
18	3	6	-4	155	not computed
19	3	6	-5	7	not computed
23	4	6	-4	33	not computed

Spectral data:

$$[c_{loc}] = 3s + rF \quad \nearrow E_6$$

$$\mathcal{L}_A \equiv \mathcal{O}(Ms + NF)|_{c_{loc}}$$

$$\lambda = \frac{3}{2}$$

Comments:

- Many examples are identically zero → implic. for moduli stabil.
- Many examples are the points of E_8 Pfaffian
- Only 13 unique functions; high Pfaffian degeneracy

Transition (32x32) matrix M for B₃ with
(r=7 χ =1, M=6, N=-3) spectral data

```
sage: print m.str()
[a1 a2 a3 a4 a5 a6 a7 a8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 a1 a2 a3 a4 a5 a6 a7 a8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]
[b1 b2 b3 b4 b5 b6 0 0 0 a1 a2 a3 a4 a5 a6 a7 a8 0 0 0 0 0 0 0 0 0 0 0]
[0 b1 b2 b3 b4 b5 b6 0 0 0 a1 a2 a3 a4 a5 a6 a7 a8 0 0 0 0 0 0 0 0 0]
[0 0 b1 b2 b3 b4 b5 b6 0 0 0 a1 a2 a3 a4 a5 a6 a7 a8 0 0 0 0 0 0 0]
[0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 a1 a2 a3 a4 a5 a6 a7 a8 0 0 0 0 0 0]
c1 c2 c3 c4 c5 0 0 0 0 0 0 0 0 0 0 0 0 0 a1 a2 a3 a4 a5 a6 a7 a8 0 0 0]
[0 c1 c2 c3 c4 c5 0 0 0 0 0 0 0 0 0 0 0 0 0 a1 a2 a3 a4 a5 a6 a7 a8 0]
[0 0 c1 c2 c3 c4 c5 0 0 0 0 0 0 0 0 0 0 0 a1 a2 a3 a4 a5 a6 a7 a8 0]
[0 0 0 c1 c2 c3 c4 c5 0 0 0 0 0 0 0 0 0 0 a1 a2 a3 a4 a5 a6 a7 a8 0]
[0 0 0 0 c1 c2 c3 c4 c5 0 0 0 0 0 0 0 0 0 0 a1 a2 a3 a4 a5 a6 a7 a8]
[0 0 0 0 0 c1 0 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5 b6]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4 b5]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2 b3 b4]
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[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1 b2]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0 b1]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1 0]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 c1]
sage:
```

Pfaff=Det(M)

Conclusions:

- I) Type II model building with D-branes & D-Instanton effects
- II) F-theory and D-instantons

Most recent results: Moduli dependent instanton Pfaffian prefactors

- i) Pfaffian can be computed in F-theory GUT's via line bundle cohomology on the spectral curve over the instanton-7 brane intersection

Checks: when heterotic dual exists, in Type IIB limit

- ii) Pfaffian has a rich structure
 - typically factorizes into non-trivial powers of moduli polynomials
 - points of E_8 enhancement can cause Pfaffian to vanish;
quantified conditions for when this occurs
 - physics implication