Supersymmetry at TeV scales with the recent LHC results

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* Supersymmetric Standard Model
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  Higgs boson mass in the MSSM

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Motivations for TeV scale SUSY

The standard model (SM) of particle physics has been enormously successful to explain most of the observed particle physics phenomena at energy scales below TeV.

However still there are numerous fundamental questions not answered by the SM:

* Origin of the electroweak symmetry breaking

* Dark matter, Matter-antimatter asymmetry in the Universe

* Origin of the flavor structure

* Strong CP problem, Cosmic inflation, Grand unification, Quantum gravity, ...

So we have many compelling reasons to anticipate new physics beyond the SM at high energy scales.

Then what is the energy scale where new physics appears first?

Standard Model:

Effective field theory for the strong, weak and electromagnetic forces with a priori unknown cutoff scale Λ_{SM} which can be identified as the scale where new physics beyond the SM appears first.

In principle, Λ_{SM} can be anywhere between TeV and $M_{\text{Planck}} \sim 10^{18}$ GeV.

Light degrees of freedom in the model:

- spin = 1 gauge bosons for $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry:

$$G_\mu = (8,1)_0, \;\; W_\mu = (1,3)_0, \;\; B_\mu = (1,1)_0$$

- 3-generations of spin = 1/2 quarks and leptons:

$$q_i = (3,2)_{\frac{1}{6}}, \;\; u_i^c = (\bar{3},1)_{-\frac{2}{3}}, \;\; d_i^c = (\bar{3},1)_{\frac{1}{3}},$$

$$\ell_i = (1,2)_{-\frac{1}{2}}, \;\; e_i^c = (1,1)_1 \qquad (i = 1,2,3)$$

- spin = 0 Higgs boson: $\;H = (1,2)_{\frac{1}{2}}$

Attractive features of the SM:

* Local masses of gauge bosons, quarks and leptons are forbidden by $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry, so can be generated only through a spontaneous symmetry breaking, which allows them naturally light compared to the cutoff scale $\Lambda_{\rm SM}$.

  (A light point-like degree of freedom is in fact something special in QFT as its mass can receive a potentially large self-energy contribution from UV physics around the cutoff scale. The only known way to make it natural is to have a symmetry which forbids non-zero mass in the symmetric limit.)

* Baryon and lepton numbers (B & L) are good accidental symmetries of the renormalizable part of the model, which nicely explains why protons are long-lived and neutrinos are light.

* Flavor violation occurs only through the charged-current weak interactions described by the CKM matrix, which nicely explains the suppression of FCNC effects in the light meson system .

Incomplete or unattractive features of the SM:

* There are many fundamental questions not answered by the SM:

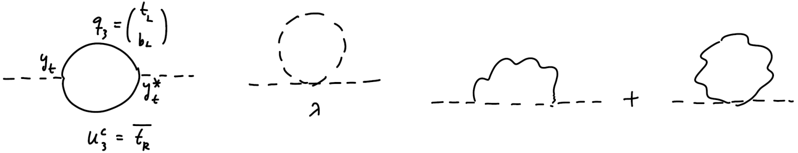
Dark matter, Matter-antimatter asymmetry, Flavor, Strong CP problem, Cosmic inflation, Grand unification, Quantum gravity, ...

* **Hierarchy problem:**

Higgs boson mass is not protected by any symmetry, so it receives a large quantum correction from UV physics around $\Lambda_{\rm SM}$:

$$\mathcal{L}_{\rm higgs} = D_\mu H^\dagger D^\mu H - m_H^2|H|^2 - \frac{1}{4}\lambda|H|^4 + y_t H q_3 u_3^c + ...$$

$$\Rightarrow \quad \delta m_H^2 = \left[-3y_t^2 + 3\lambda + \frac{9g_2^2+3g_1^2}{8} + ...\right]\frac{\Lambda_{\rm SM}^2}{16\pi^2}$$



On the other hand, we know that

$$|m_H^2| = |m_{\rm bare}^2 + \delta m_H^2| \sim (100\ \text{GeV})^2$$

$$\left(\delta m_H^2 = \left[-3y_t^2 + 3\lambda + \frac{9g_2^2 + 3g_1^2}{8} + ...\right]\frac{\Lambda_{\rm SM}^2}{16\pi^2}\right)$$

So if $\Lambda_{\rm SM} \gg 1$ TeV, we need a fine tuning of $\mathcal{O}\left((\text{TeV}/\Lambda_{\rm SM})^2\right)$ to have the correct electroweak symmetry breaking at the weak scale.

To avoid this fine tuning, SM should be modified at scales around TeV in a way to regulate the quadratically divergent Higgs boson mass.

If it is indeed true, the first new physics that appear at the lowest scale is likely to be the one to regulate the top-quark-loop contribution to the Higgs boson mass.

So the hierarchy problem implies that new physics beyond the SM (BSM physics) is likely to be around the TeV scale, and **SUSY** is the prime candidate for such new physics regulating the quadratically divergent Higgs boson mass.

In fact, SUSY does not only solve the hierarchy problem, but also provide an attractive theoretical framework to address many other fundamental questions such as dark matter, baryogenesis, grand unification and quantum gravity:

* Lightest SUSY particle is a good dark matter candidate.

* Some squark or slepton fields can have a nontrivial cosmological evolution which would generate baryon or lepton asymmetry in the early Universe.

* With SUSY around the TeV scale, the three gauge couplings of $SU(3)_c \times SU(2)_L \times U(1)_Y$ are successfully unified at $M_{\rm GUT} \sim 10^{16}$ GeV.

* SUSY is an essential component of string/M theory.

Minimal Supersymmetric Standard Model (MSSM)

Field contents:

- $SU(3)_c \times SU(2)_W \times U(1)_Y$ gauge multiplets:

$$V_a = -\theta\sigma^\mu\bar{\theta}A^a_\mu + i\theta\theta\bar{\theta}\bar{\lambda}^a - i\bar{\theta}\bar{\theta}\lambda^a + \frac{1}{2}\theta^2\bar{\theta}^2 D^a$$

$$(A^a_\mu, \lambda_a) = (G_\mu, \tilde{g}),\ (W_\mu, \tilde{W}),\ (B_\mu, \tilde{B})$$

- 3 generations of quark and lepton multiplets:

$$\Phi^I = \phi^I + \sqrt{2}\theta\psi^I + \theta^2 F^I \equiv (\phi^I, \psi^I)$$

$$Q_i = (\tilde{q}_i, q_i), \quad U^c_i = (\tilde{u}^c_i, u^c_i), \quad D^c_i = (\tilde{d}^c_i, d^c_i),$$
$$L_i = (\tilde{\ell}_i, \ell_i), \quad E^c_i = (\tilde{e}^c_i, e^c_i) = (1,1)_1$$

- Higgs multiplets:

$$H_u = (H_u, \tilde{H}_u) = (1,2)_{\frac{1}{2}}, \quad H_d = (H_d, \tilde{H}_d) = (1,2)_{-\frac{1}{2}}$$

Lagrangian:

$$\int d^4\theta\, Z_{\bar{I}J}\Phi^{I*}e^V\Phi^J + \left[\int d^2\theta\, \left(\frac{1}{4}f_a\mathcal{W}^{a\alpha}\mathcal{W}^a_\alpha + W\right) + \text{c.c}\right]$$

$$Z_{\bar{I}J} \;=\; \delta_{\bar{I}J} - m^2_{\bar{I}J}\theta^2\bar{\theta}^2, \qquad f_a \;=\; \frac{1}{g_a^2}\left(1 - M_a\theta^2\right)$$

$$W \;=\; W_{\text{MSSM}} + \Delta W$$

$$\begin{aligned} W_{\text{MSSM}} \;=\; & \mu(1 - B\theta^2)H_uH_d + y^u_{ij}(1 - A^u_{ij}\theta^2)H_uQ_iU^c_j \\ & + y^d_{ij}(1 - A^d_{ij}\theta^2)H_dQ_iD^c_j + y^\ell_{ij}(1 - A^\ell_{ij}\theta^2)H_dL_iE^c_j \end{aligned}$$

$$\begin{aligned} \Delta W \;=\; & \mu'_iL_iH_u + \lambda_{ijk}L_iL_jE^c_k + \lambda'_{ijk}L_iQ_jD^c_k + \lambda''_{ijk}U^c_iD^c_jD^c_k \\ & + \frac{\gamma_{ijkl}}{M_{\text{Planck}}}Q_iQ_jQ_kL_l + \frac{\gamma'_{ijkl}}{M_{\text{Planck}}}U^c_iU^c_jD^c_kE^c_l + \ldots \end{aligned}$$

$W_{\text{MSSM}} =$ B and L conserving renormalizable superpotential including the associated soft SUSY breaking terms

$\Delta W =$ Potentially dangerous B and/or L violating superpotential

Compared to the SM, the MSSM includes bunch of new interactions which can induce dangerous **flavor, CP, B or L violating processes**, which are severely constrained by low energy data.

$$
\begin{aligned}
\mathcal{L}_{\rm soft} \;=\;& -\frac{1}{2}\left(M_{\tilde g}\tilde g\tilde g + M_{\tilde W}\tilde W\tilde W + M_{\tilde B}\tilde B\tilde B + \text{c.c.}\right) \\
& - (B\mu H_u H_d + \text{c.c.}) - m_{H_u}^2|H_u|^2 - m_{H_d}^2|H_d|^2 \\
& - (m_{\tilde q}^2)_{ij}\tilde q_i^*\tilde q_j - (m_{\tilde u}^2)_{ij}\tilde u_i^{c*}\tilde u_j^c - (m_{\tilde d}^2)_{ij}\tilde d_i^{c*}\tilde d_j^c \\
& - (m_{\tilde\ell}^2)_{ij}\tilde\ell_i^*\tilde\ell_j - (m_{\tilde e}^2)_{ij}\tilde e_i^{c*}\tilde e_j^c \\
& - \left(A_{ij}^u y_{ij}^u H_u\tilde q_i\tilde u_j^c + A_{ij}^d y_{ij}^d H_d\tilde q_i\tilde d_j^c + A_{ij}^\ell y_{ij}^\ell H_d\tilde\ell_i\tilde e_j^c + \text{c.c.}\right)
\end{aligned}
$$

$$
\begin{aligned}
\Delta W \;=\;& \mu_i' L_i H_u + \lambda_{ijk}L_iL_jE_k^c + \lambda_{ijk}'L_iQ_jD_k^c + \lambda_{ijk}''U_i^cD_j^cD_k^c \\
& + \frac{\gamma_{ijkl}}{M_{\rm Planck}}Q_iQ_jQ_kL_l + \frac{\gamma_{ijkl}'}{M_{\rm Planck}}U_i^cU_j^cD_k^cE_l^c + \ldots
\end{aligned}
$$

The price for solving the hierarchy problem with SUSY is not cheap!

We lose the two nice features of the SM: (i) automatic B/L conservation at renormalizable level, and (ii) GIM suppression of flavor violation.

This might not be a problem, but provides an opportunity to understand the underlying UV theory of TeV scale SUSY model.

Constraints from proton decay:

$$\begin{aligned}\Delta W \;=\; & \mu_i' L_i H_u + \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c \\ & + \frac{\gamma_{ijkl}}{M_{\text{Planck}}} Q_i Q_j Q_k L_l + \frac{\gamma'_{ijkl}}{M_{\text{Planck}}} U_i^c U_j^c D_k^c E_l^c + \ldots\end{aligned}$$

$$\begin{aligned}
&\frac{\mu_i'}{\mu}\lambda''_{112} \lesssim 10^{-21}\left(\frac{m_{\text{soft}}}{\text{TeV}}\right)^2 \;\; (i=1,2),\\
&\lambda'_{i1k}\lambda''_{11k} \lesssim 10^{-24}\left(\frac{m_{\text{soft}}}{\text{TeV}}\right)^2 \;\; (i=1,2),\\
&\lambda_{33i}\lambda''_{112} \lesssim (10^{-16}-10^{-21})\left(\frac{m_{\text{soft}}}{\text{TeV}}\right)^2 \;\; (i=1,2,3)\\
&\gamma_{112i} \lesssim 10^{-8}\left(\frac{m_{\text{soft}}}{\text{TeV}}\right) \;\; (i=1,2,3),\\
&\gamma'_{12ij} \lesssim 10^{-7}\left(\frac{m_{\text{soft}}}{\text{TeV}}\right) \;\; (i,j=1,2),
\end{aligned}$$

so we need some symmetry to suppress these B/L violating couplings!

Symmetries to suppress the B/L violating couplings:

A. Symmetry involving an exact R-parity $= (-1)^{3(B-L)} e^{2\pi i J_z}$:

A-1) Matter parity $Z_2 = (-1)^{3(B-L)}$

$$\Rightarrow \quad \mu' = \lambda = \lambda' = \lambda'' = 0$$

(But matter parity alone does not explain why γ and γ' are so small.)

A-2) Proton hexality $Z_6 = (-1)^{2B}(-1)^{3(B-L)}$

$$\Rightarrow \quad \mu' = \lambda = \lambda' = \lambda'' = \gamma = \gamma' = 0$$

B. Symmetry not involving an exact R-parity:

B-1) Baryon triality $Z_3 = (-1)^{2B}$

$$\Rightarrow \quad \lambda'' = \gamma = \gamma' = 0$$

(Still need to explain why μ'/μ, λ and λ' are small.)

B-2) Spontaneoulsy broken discrete R-symmetry

$$\Rightarrow \quad B \text{ or } L \text{ violating couplings } \propto (m_{3/2}/M_{\rm Planck})^{\Delta}$$

($\Delta = R$-charge dependent rational numbers)

B-3) Other symmetries such as $U(1)_{PQ}$, ...

Why R-parity ($\equiv$ matter parity) is special?

* All known ordinary particles $\ni$ quarks, leptons, gauge bosons, Higgs bosons, graviton (also axion if exists): Even under R-parity

* All superpartners $\ni$ squarks, sleptons, gauginos, Higgsinos, gravitino, (axino): Odd under R-parity

So, if R-parity is an exact symmetry, the lightest superpartner (LSP) is stable, which has important implications for SUSY signatures at colliders and cosmology:

i) Missing transverse energy (MET) carried away by invisible LSPs in CMS and ATLAS detectors

ii) LSP as dark matter.

With exact R-parity,

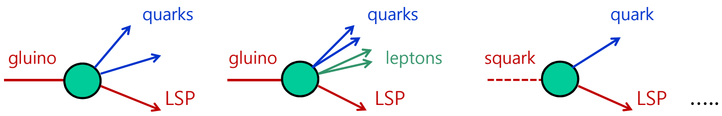
* At the LHC, SUSY particles are produced always in pair:

proton + proton → pair of SUSY particles (mostly gluino or squark)

* Each of the produced SUSY particles decays as

heavier SUSY particle → lighter SUSY particle + SM particles

→ lightest SUSY particle (LSP) + more SM particles



LSP is stable, so it should not have electromagnetic and strong interactions in order to be cosmologically viable, which means that

LSP is invisible in the particle detector!

(In many cases, LSP is a good dark matter candidate.)

Typical SUSY events at the LHC:

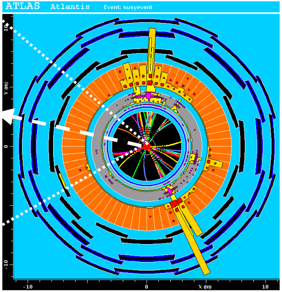
proton + proton ➔ gluino (or squark)+ gluino (or squark)

➔ Visible SM particles + Invisible LSP pair

LSP

missing momentum

LSP



Key feature:

Momentum imbalance in the transverse direction due to the invisible LSP pair

= Missing ET

There can be many different types of missing ET events with different set of visible SM particles, which depend on the pattern of SUSY particle masses, so provide hint for SUSY particle mass spectrum.

Constraints from flavor or CP violations

$$
\begin{aligned}
\mathcal{L}_{\rm soft} \quad = \quad & -\frac{1}{2}\left(M_{\tilde{g}}\tilde{g}\tilde{g} + M_{\tilde{W}}\tilde{W}\tilde{W} + M_{\tilde{B}}\tilde{B}\tilde{B} + \text{c.c.}\right) \\
& - \Big(B\mu H_u H_d + \text{c.c.} \Big) - m_{H_u}^2 |H_u|^2 - m_{H_d}^2 |H_d|^2 \\
& - (m_{\tilde{q}}^2)_{ij}\tilde{q}_i^* \tilde{q}_j - (m_{\tilde{u}}^2)_{ij}\tilde{u}_i^{c*}\tilde{u}_j^c - (m_{\tilde{d}}^2)_{ij}\tilde{d}_i^{c*}\tilde{d}_j^c \\
& - (m_{\tilde{\ell}}^2)_{ij}\tilde{\ell}_i^* \tilde{\ell}_j - (m_{\tilde{e}}^2)_{ij}\tilde{e}_i^{c*}\tilde{e}_j^c \\
& - \Big(A_{ij}^u y_{ij}^u H_u \tilde{q}_i \tilde{u}_j^c + A_{ij}^d y_{ij}^d H_d \tilde{q}_i \tilde{d}_j^c + A_{ij}^\ell y_{ij}^\ell H_d \tilde{\ell}_i \tilde{e}_j^c + \text{c.c.} \Big)
\end{aligned}
$$

Since the present data implies that flavor violation beyond the SM should be quite suppressed, it is convenient to decompose flavor-violating soft masses into two parts:

$$
\text{soft mass} \;=\; \text{flavor-universal part} \;+\; \text{flavor-non-universal part}
$$

$$
\begin{aligned}
\left(m_\phi^2\right)_{ij} \quad &= \quad m_\phi^2 \delta_{ij} + \left(\Delta m_\phi^2\right)_{ij} \quad \left(\phi = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{\ell}, \tilde{e}\right) \\
A_{ij}^x \quad &= \quad A_x + \Delta A_{ij}^x \quad \left(x = u, d, \ell\right)
\end{aligned}
$$

Note: This decomposition is not unique, and is just for an order of magnitude estimate of the flavor constraints.

Some bounds on soft masses at TeV scale from FCNC and CPV:

- K-$\bar{K}$ mass difference and ϵ_K :

$$\sqrt{\left(\mathrm{Re}, \mathrm{Im}\right)\left(\frac{(\Delta m^2_{\tilde{q}})_{12}(\Delta m^2_{\tilde{d}})_{12}}{m^2_{\tilde{q}} m^2_{\tilde{d}}}\right)} \leq \left(5 \times 10^{-3},\, 4 \times 10^{-4}\right)\left(\frac{m_{\tilde{q},\tilde{d}}}{1\ \mathrm{TeV}}\right)$$

- $\mu \to e\gamma$:

$$\frac{(\Delta A^{\ell})_{12}}{m_{\tilde{\ell}}} \leq 2 \times 10^{-2}\left(\frac{m_{\tilde{\ell}}}{100\ \mathrm{GeV}}\right)\left(\frac{M_{\tilde{W}}}{100\ \mathrm{GeV}}\right)$$

- EDMs:

$$\mathrm{Arg}\left(\frac{M_a}{M_b}, \frac{M_a}{A_x}, \frac{M_a}{B}\right) \leq \left(10^{-2} - 10^{-3}\right) \times \left(\frac{m_{\tilde{q},\tilde{\ell}}}{100\ \mathrm{GeV}}\right)^2$$

$\left(m_{\tilde{q},\tilde{d},\tilde{\ell}} = \text{1st or 2nd generation squark and slepton masses}\right)$

Flavor and CP constraints imply

A. Soft masses are nearly flavor-universal (at least for the 1st and 2nd generations) and CP conserving:

$$\frac{\Delta m_\phi^2}{m_\phi^2},\ \frac{\Delta A^x}{m_\phi},\ \mathrm{Arg}\left(\frac{M_a}{M_b},\frac{M_a}{A_x},\frac{M_a}{B}\right)\ \text{ are small enough}$$

or

B. Sfermion masses (at least of the 1st and 2nd generations) are heavy enough:

$$m_{\tilde{q},\tilde{d}} \gtrsim \mathcal{O}(10-100)\ \text{TeV}, \quad m_{\tilde{\ell}} \gtrsim \text{few TeV}$$

As we will see, stop masses as light as possible are more favoured for more natural (less unnatural) electroweak symmetry breaking.

As a viable UV completion of MSSM, we may then search for a SUSY breaking scheme yielding either

A. flavor-universal and CP-conserving soft masses

or

B. inverted sfermions mass hierarchy with

$$m_{\tilde{q}} \gtrsim \mathcal{O}(10-100)\ \text{TeV}, \quad m_{\tilde{t}} \lesssim 1\ \text{TeV},$$

$$\big(m_{\tilde{q}} = \text{1st and 2nd generation squark masses}\big)$$

Conditions for natural EWSB:

In fact, some of the sparticle masses, in particular the Higgsino, stop and gluino masses, are closely linked to the electroweak symmetry breaking (EWSB), so constrained if we wish to have an EWSB without any severe fine tuning.

To examine this issue, we first recall the fine tuning problem of the EWSB in the SM.

$$V_{\rm SM} = m_H^2|H|^2 + \frac{\lambda}{4}|H|^4$$

$$\begin{aligned}
\Rightarrow \quad \frac{M_Z^2}{2} &= \frac{g_1^2+g_2^2}{4}\langle|H|^2\rangle = -\frac{g_1^2+g_2^2}{4}\left(\frac{2m_H^2}{\lambda}\right) \\
&= -\left(\frac{g_1^2+g_2^2}{2\lambda}\right)\left(m_{H,\rm bare}^2 + \delta m_H^2\right) \\
&= -\left(\frac{g_1^2+g_2^2}{2\lambda}\right)\left[m_{H,\rm bare}^2 - \frac{\Lambda_{\rm SM}^2}{16\pi^2}\left(3y_t^2 - 3\lambda - \frac{3g_1^2+8g_2^2}{8} + \dots\right)\right]
\end{aligned}$$

$$\Rightarrow \quad \Lambda_{\rm SM} \lesssim \mathcal{O}(1)\ \text{TeV to avoid a severe fine tuning}$$

Electroweak symmetry breaking in MSSM:

$$\begin{aligned} V_{\text{MSSM}} &= (m_{H_u}^2+|\mu|^2)|H_u|^2+(m_{H_d}^2+|\mu|^2)|H_d|^2-(B\mu H_u H_d+\text{c.c}) \\ &+ \left(\frac{g_1^2+g_2^2}{8}\left(|H_u|^2-|H_d|^2\right)^2+\frac{g_2^2}{2}|H_u^\dagger H_d|^2\right) \end{aligned}$$

$$\frac{\partial V}{\partial H_u}=\frac{\partial V}{\partial H_d}=0$$

$$\begin{aligned} \Rightarrow \quad \text{i)} \quad \frac{M_Z^2}{2} &= \frac{g_1^2+g_2^2}{2}\langle\, |H_u|^2+|H_d|^2\rangle = \frac{m_{H_d}^2-m_{H_u}^2\tan^2\beta}{\tan^2\beta-1}-|\mu|^2 \\ &\simeq -m_{H_u}^2-|\mu|^2+\frac{m_{H_d}^2}{\tan^2\beta} \qquad \left(\tan\beta=\frac{\langle|H_u|\rangle}{\langle|H_d|\rangle}\right) \end{aligned}$$

$$\text{ii)} \quad \frac{2|B\mu|}{\sin 2\beta} = m_{H_u}^2+m_{H_d}^2+2|\mu|^2$$

RG evolutions which can significantly affect the value of $m_{H_u}^2$ in the EWSB condition:

$$16\pi^2 \frac{d}{d\ln\Lambda} m_{H_u}^2 = 6y_t^2\left(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2\right) + \ldots$$
$$16\pi^2 \frac{d}{d\ln\Lambda} m_{\tilde{t}_{L,R}}^2 = -\frac{32}{3} g_3^2 M_{\tilde{g}}^2 + \frac{8}{3\pi^2} g_3^4 m_{\tilde{q}}^2 + \ldots$$

$$y_t = y_{33}^u, \quad m_{\tilde{t}_L}^2 = (m_{\tilde{q}}^2)_{33}, \quad m_{\tilde{t}_R}^2 = (m_{\tilde{u}}^2)_{33},$$

$m_{\tilde{q}}$ = 1st and 2nd generation squark masses which are presumed to be comparable to each other

$$\Rightarrow \quad m_{H_u}^2 = m_{H_u,\mathrm{bare}}^2 - \frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln\left(\frac{M_{\mathrm{mess}}}{m_{\tilde{t}}}\right) - \frac{2y_t^2}{\pi^2}\frac{g_3^2}{4\pi^2} M_{\tilde{g}}^2 \left(\ln\left(\frac{M_{\mathrm{mess}}}{M_{\tilde{g}}}\right)\right)^2 + \ldots$$
$$m_{\tilde{t}}^2 = m_{\tilde{t},\mathrm{bare}}^2 + \frac{2g_3^2}{3\pi^2} M_{\tilde{g}}^2 \ln\left(\frac{M_{\mathrm{mess}}}{M_{\tilde{g}}}\right) - \frac{1}{6}\left(\frac{g_3^2}{\pi^2}\right)^2 m_{\tilde{q}}^2 \ln\left(\frac{M_{\mathrm{mess}}}{m_{\tilde{q}}}\right) + \ldots$$

M_{mess} = messenger scale where soft terms are generated as local operator

$$m_{\tilde{t}}^2 = \frac{m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2}{2}, \quad m_{\phi,\mathrm{bare}} = m_\phi(M_{\mathrm{mess}}) \ \text{for}$$

Potential fine tuning problem in the MSSM:

$$
\begin{aligned}
\frac{M_Z^2}{2} &\simeq -m_{H_u}^2 - |\mu|^2 + \frac{m_{H_d}^2}{\tan^2\beta} \\
&= \left[-m_{H_u,\mathrm{bare}}^2 + \frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln\left(\frac{M_{\mathrm{mess}}}{m_{\tilde{t}}}\right) + \frac{2y_t^2}{\pi^2}\frac{g_3^2}{4\pi^2} M_{\tilde{g}}^2 \left(\ln\left(\frac{M_{\mathrm{mess}}}{M_{\tilde{g}}}\right)\right)^2 + \dots \right] \\
&\quad - |\mu|^2 + \frac{m_{H_d}^2}{\tan^2\beta}
\end{aligned}
$$

* EWSB is only logarithmically sensitive to M_{mess}, so the messenger scale (= UV cutoff scale for soft masses) can be close to M_{Planck} without causing a severe fine tuning problem.

(Except for gauge mediation, most of the known mediation schemes have M_{mess} close to the GUT scale or the Planck scale.)

* However EWSB in the MSSM is quadratically sensitive to m_{soft}, in fact most sensitive to $m_{\tilde{t}}$ and $M_{\tilde{g}}$ due to the large top-quark Yukawa coupling and QCD coupling.

As a result, if $m_{\tilde{t}}$ or $M_{\tilde{g}}$ is far above M_Z, EWSB requires a fine tuning of $\mathcal{O}(M_Z^2/m_{\tilde{t}}^2)$ or of $\mathcal{O}(M_Z^2/M_{\tilde{g}}^2)$.

A more quantitative estimate of the naturalness condition

$$
\begin{aligned}
m_{\tilde{t}} &\lesssim 0.5 \left(\frac{10\,\%}{\epsilon_{\text{tuning}}}\right)^{1/2} \left(\frac{3}{\ln(M_{\text{mess}}/m_{\tilde{t}})}\right)^{1/2} \text{ TeV} \\
M_{\tilde{g}} &\lesssim 1.3 \left(\frac{10\,\%}{\epsilon_{\text{tuning}}}\right)^{1/2} \left(\frac{3}{\ln(M_{\text{mess}}/M_{\tilde{g}})}\right) \text{ TeV} \\
\mu &\lesssim 0.2 \left(\frac{10\,\%}{\epsilon_{\text{tuning}}}\right)^{1/2} \text{ TeV}
\end{aligned}
$$

* **Fully natural** ($\epsilon_{\text{tuning}} > 10\%$) **EWSB (for** $M_{\text{mess}} \sim M_{\text{GUT}}$**) if**

$$m_{\tilde{t}} \sim M_{\tilde{g}} \sim \mu \sim m_{H_u} \sim M_Z.$$

However it turns out that Nature does not take this natural option.

*** Not-so-unnatural EWSB with acceptable fine tuning:**

We may accept $\epsilon_{\text{tuning}} = \mathcal{O}(1)\%$ for $M_{\text{mess}} \sim M_{\text{GUT}}$.

$$\big(\text{or } \epsilon_{\text{tuning}} = \mathcal{O}(10)\% \text{ for } M_{\text{mess}} \sim 10 \text{ TeV}\big)$$

$$\Rightarrow \quad m_{\tilde{t}} \lesssim 0.5 \text{ TeV}, \quad M_{\tilde{g}} \lesssim 1.3 \text{ TeV}, \quad \mu \lesssim 0.2 - 0.7 \text{ TeV}$$

In these days, people call this "Natural SUSY".

Note:

* It should be noted that this is not a real constraint, but just a favoured range of soft masses in view of the naturalness.

* In many cases, such a light stop favored by natural EWSB is in conflict with the Higgs boson mass $m_h \simeq 125$ GeV.

$$m_{\tilde{t}}^2 = m_{\tilde{t},\mathrm{bare}}^2 + \frac{2g_3^2}{3\pi^2} M_{\tilde{g}}^2 \ln\left(\frac{M_{\mathrm{mess}}}{M_{\tilde{g}}}\right) - \frac{1}{6}\left(\frac{g_3^2}{\pi^2}\right)^2 m_{\tilde{q}}^2 \ln\left(\frac{M_{\mathrm{mess}}}{m_{\tilde{q}}}\right) + \ldots$$

$$\left(m_{\tilde{q}} = \text{1st and 2nd generation sfermion masses}\right)$$

Again, if we wish to avoid a fine tuning worse than $\mathcal{O}(1-10)$ %,

$$m_{\tilde{q}} \lesssim \mathcal{O}(10\, M_{\tilde{g}}) \text{ or } \mathcal{O}(10\, m_{\tilde{t}}),$$

so there is a limitation on $m_{\tilde{q}}$ in **the inverted sfermion mass scenario** which has been proposed as a possible option to satisfy flavor and CP constraints.

This implies also that for any mediation scheme yielding such an inverted sfermion mass spectrum, one needs a careful examination of the low energy stop mass to make sure that it has a phenomenologically viable value.

Higgs boson masses in the MSSM:

MSSM Higgs sector: $H_u = (H_u^+, H_u^0)$, $H_d = (H_d^0, H_d^-)$

* 3 Goldstone bosons for the longitudinal components of $W^\pm, Z$
* 2 CP-even neutral Higgs bosons
* 1 CP-odd neutral Higgs boson
* 1 charged Higgs boson

As the recent experimental hint of SM-like Higgs boson with $m_h \simeq 125$ GeV is a hot issue, here we focus on the lightest CP-even Higgs boson which behaves like the SM Higgs boson in most cases.

For simplicity, we take the limit that all Higgs bosons other than the lightest CP-even Higgs are heavy enough, and consider the effective theory of the light Higgs boson after the heavy Higgs bosons are integrated out.

$$
\begin{aligned}
V_{\text{MSSM}} &= (m_{H_u}^2 + |\mu|^2)|H_u|^2 + (m_{H_d}^2 + |\mu|^2)|H_d|^2 - (B\mu H_u H_d + \text{c.c}) \\
&+ \left(\frac{g_1^2 + g_2^2}{8} \left(|H_u|^2 - |H_d|^2\right)^2 + \frac{g_2^2}{2} |H_u^\dagger H_d|^2 \right)
\end{aligned}
$$

$$
H_u^0 = \frac{h \sin\beta}{\sqrt{2}}, \quad H_d^0 = \frac{h \cos\beta}{\sqrt{2}} \quad \big(h = \text{light CP-even neutral Higgs boson}\big)
$$

$$
\begin{aligned}
\Rightarrow \quad V_{\text{higgs}} &= -m^2 h^2 + \frac{\lambda}{16} h^4 \qquad \left(\lambda = \frac{g_1^2 + g_2^2}{2} \cos^2 2\beta\right) \\
m_h^2 &= \left.\frac{\partial^2 V}{\partial h^2}\right|_{h=\langle h \rangle} = M_Z^2 \cos^2 2\beta
\end{aligned}
$$

$$
\left(M_Z^2 = \frac{g_1^2 + g_2^2}{2} \langle\, |H_u|^2 + |H_d|^2 \rangle = \frac{g_1^2 + g_2^2}{4} \langle h^2 \rangle \right)
$$

In generic case, this value of m_h corresponds to the upper bound on the tree level mass of the lightest CP-even Higgs boson in the MSSM.

So, at tree level, the MSSM predicts a Higgs boson lighter than M_Z, which has been excluded a long time ago.

But there are important radiative corrections which saves the life of the MSSM Higgs boson.

$$\begin{aligned} \Delta V &= \frac{1}{64\pi^2} \mathrm{Str}\,\mathcal{M}^4 \ln\left(\frac{\mathcal{M}^2}{Q^2}\right) \qquad (Q = \text{renormalization point}) \\ &= \frac{3}{32\pi^2}\left[\sum_{i=1,2} m_{\tilde{t}_i}^4 \ln\left(\frac{m_{\tilde{t}_i}^2}{Q^2}\right) - 2m_t^4 \ln\left(\frac{m_t^2}{Q^2}\right) + \ldots\right] \end{aligned}$$

For simplicity, let us consider the limit

$$\tan\beta \gg 1 \quad \left(\Rightarrow \; H_u^0 \simeq \frac{h}{\sqrt{2}}\right), \qquad \frac{m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2}{2} \gg y_t A_t h \gg \frac{|m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2|}{2}$$

in which a sizable $A_t \equiv A_{33}^u$ is helpful for raising up the Higgs boson mass:

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + y_t^2 h^2/2 & y_t A_t h/\sqrt{2} \\ y_t A_t h/\sqrt{2} & m_{\tilde{t}_R}^2 + y_t^2 h^2/2 \end{pmatrix}, \qquad m_t = y_t h/\sqrt{2}$$

$$\Rightarrow \quad m_{\tilde{t}_i}^2 \simeq m_{\tilde{t}}^2 + \frac{y_t^2 h^2}{2} \pm \frac{y_t A_t h}{\sqrt{2}} \qquad \left(m_{\tilde{t}}^2 \equiv \frac{m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2}{2}\right) \qquad (i = 1, 2)$$

$$\Rightarrow \qquad \Delta V = \frac{\Delta\lambda}{16} h^4 + \ldots \qquad \left(\Delta\lambda \simeq \frac{3y_t^4}{4\pi^2} \left[\ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \frac{A_t^2}{m_{\tilde{t}}^2} - \frac{1}{12}\frac{A_t^4}{m_{\tilde{t}}^4} \right] \right)$$

$$\Rightarrow \qquad \Delta m_h^2 \simeq \frac{3y_t^2 m_t^2}{4\pi^2} \left[\ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \frac{A_t^2}{m_{\tilde{t}}^2} - \frac{1}{12}\frac{A_t^4}{m_{\tilde{t}}^4} \right]$$

In more general situation including the case with small $\tan\beta$, we have

$$\left(m_h^2\right)_{\mathrm{MSSM}} \simeq M_Z^2 \cos^2 2\beta + \frac{3y_t^2 m_t^2}{4\pi^2} \left[\ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right) + \frac{|X_t|^2}{m_{\tilde{t}}^2} - \frac{1}{12}\frac{|X_t|^4}{m_{\tilde{t}}^4} \right]$$

$$\left(X_t = A_t - \mu \cot\beta\right)$$

Implications of the recent LHC results for SUSY

Relatively simple realizations of SUSY predict

* LHC events with missing transverse momentum (=Missing ET events)

* Light Higgs boson near the Z-boson mass $M_Z \sim 91$ GeV

which can play a key role for the experimental verification of SUSY.

However recent results in the LHC experiments indicate

* No appreciable missing ET events beyond the SM backgrounds

* Higgs boson is a bit heavier than what the minimal SUSY model favors

→ * SUSY might be heavier than what we have hoped,

and/or

* Some sort of extension or modification of the minimal SUSY scenario might be necessary.

Search for missing ET events

LHC has been searching for the events

proton + proton ➔ gluino (or squark) + gluino (or squark)

➔ light quarks + invisible LSP pair

(and many other types of missing ET events also)

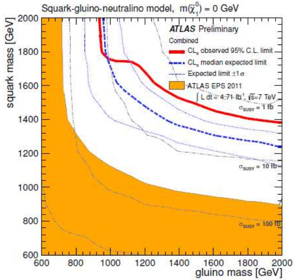
but so far could not find any appreciable amount of missing ET events.

This implies that either

* **SUSY is heavy:**

Gluinos and squarks are heavier than what we have hoped, so not yet copiously produced at the LHC:

$$m_{gluino} \sim m_{squark} > 1.5 \text{ TeV}$$



or

* **Missing ET events are mostly top-rich, so more difficult to be identified:**

Inverted sfermion mass spectrum:

Light quarks couple to the Higgs boson very weakly, so their superpartners can have a mass well above 1 TeV without making the Higgs boson self energy too large.

m_{squark} ~ multi-TeV for the superpartners of light quarks

m_{stop} & m_{gluino} ~ sub-TeV

→ Gluinos decay mostly into $t + \bar{t}$ + LSP, yielding much more complicate final states:

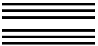
proton + proton → gluino + gluino → $t + \bar{t}$ + LSP + $t + \bar{t}$ + LSP

→ $b\ \bar{b}\ b\ \bar{b}$ W^+ W^- W^+ W^- ($W \to q\bar{q}'$ or $\ell\nu$)

or

*** Missing ET is softer than what we have expected**:

Compressed SUSY spectrum:

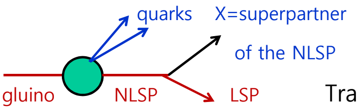


In the compressed limit

$m_{gluino} - m_{LSP} \rightarrow$ small,

Missing ET = total transverse momentum of the LSP pair $\rightarrow$ small

Stealth SUSY:



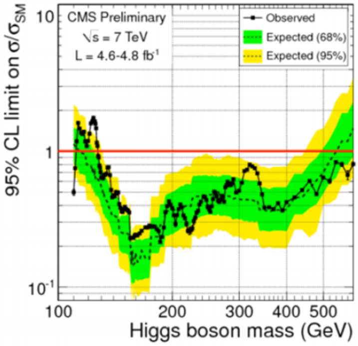
In the stealth limit

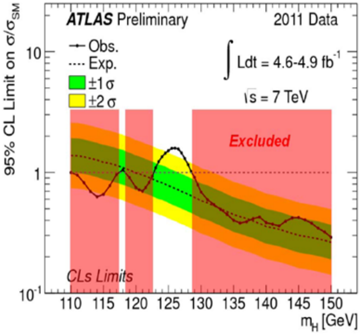
$m_{NLSP} - m_X \rightarrow$ small

$m_{LSP} \rightarrow$ small

Transverse momentum of the LSP $\rightarrow$ small

Implications of SM-like Higgs boson with $m_h \simeq 125$ GeV:





125 GeV Higgs in the MSSM:

$$\begin{aligned}(m_h^2)_{\rm MSSM} &\simeq M_Z^2\cos^2 2\beta + \frac{3y_t^2 m_t^2}{4\pi^2}\left[\ln\left(\frac{m_{\tilde t}^2}{m_t^2}\right) + \frac{|X_t|^2}{m_{\tilde t}^2} - \frac{1}{12}\frac{|X_t|^4}{m_{\tilde t}^4}\right] \\ &= (91\ \mathrm{GeV})^2 + (89\ \mathrm{GeV})^2 = (125\ \mathrm{GeV})^2 \qquad (\tan\beta \gg 1) \\ &\quad (X_t = A_t - \mu\cot\beta)\end{aligned}$$

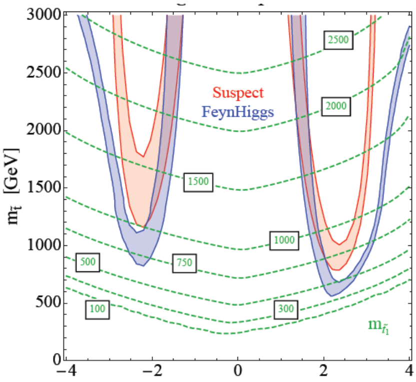
To have such large radiative correction, we need heavy stop and/or a large stop mixing $X_t = A_t - \mu\cot\beta \simeq \sqrt{6}m_{\tilde t}$!

But this is precisely what EWSB does not like: more fine tuning!
(**Little hierarchy problem**)

$$16\pi^2\frac{dm_{H_u}^2}{d\ln\Lambda} = 12y_t^2\left(m_{\tilde t}^2 + \frac{|A_t|^2}{2}\right) + \ldots, \qquad \frac{M_Z^2}{2} \simeq -m_{H_u}^2 - |\mu|^2 + \frac{m_{H_d}^2}{\tan^2\beta}$$

$$\Rightarrow \quad \sqrt{m_{\tilde t}^2 + \frac{|A_t|^2}{2}} \lesssim 0.5\left(\frac{10\,\%}{\epsilon_{\rm tuning}}\right)^{1/2}\left(\frac{3}{\ln(M_{\rm mess}/m_{\tilde t})}\right)^{1/2}\ \mathrm{TeV}$$

125 GeV Higgs in the MSSM:



$X_t/m_{\tilde{t}}$

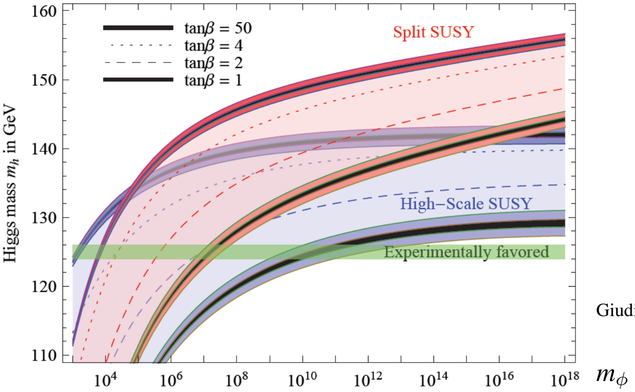
$\tan\beta = 20$

Hall,Pinner,Ruderman (2012)

* Unless $|X_t/m_{\tilde{t}}| \simeq \sqrt{6}$, we need $m_{\tilde{t}} = \text{few} - \mathcal{O}(10)$ TeV.

* For $M_{\rm mess} \sim M_{\rm GUT}$, the required degree of fine tuning is at least $\epsilon_{\rm tuning} = \mathcal{O}(0.1)$ even for nearly maximal mixing $|X_t/m_{\tilde{t}}| \simeq \sqrt{6}$, and becomes significantly worse for other values of $X_t/m_{\tilde{t}}$.

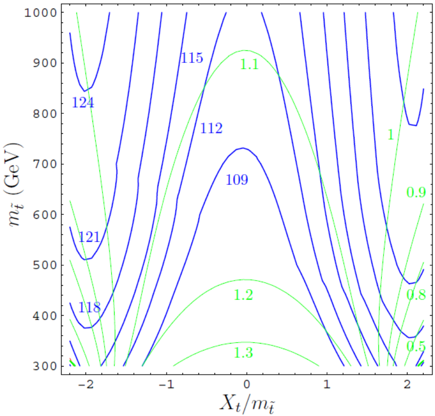
Although a bit heavier than what we have hoped, still $m_h \simeq 125$ GeV is not a bad news for low energy SUSY.



Giudice,Strumia (2011)

Unless $\tan\beta$ is quite small, $m_h \simeq 125$ GeV implies that the sfermion mass m_ϕ is rather close to the lower end ($\sim$ TeV) which is favored by the naturalness argument.

However within the MSSM, a relatively light stop favored by natural EWSB, e.g. $m_{\tilde{t}} \sim 0.5$ TeV, indicates a higgs boson mass somewhat lighter than 125 GeV:



Dermisek,Low (2007)

This motivates an extension of the MSSM in the direction to have additional contribution to the Higgs boson mass other than the top-stop loops.

*** Next to Minimal Supersymmetric Standard Model (NMSSM):**

Perhaps the simplest extension of the MSSM providing additional Higgs boson mass is the NMSSM including a singlet S with

$$\Delta W = \kappa S H_u H_d$$

$$\Rightarrow \quad \Delta V = \left|\frac{\partial W}{\partial S}\right|^2 = \kappa^2 |H_u H_d|^2 = \frac{\kappa^2 \sin^2 2\beta}{16} h^4$$

$$\left(H_u^0 = \frac{h \sin\beta}{\sqrt{2}}, \;\; H_d^0 = \frac{h \cos\beta}{\sqrt{2}} \right)$$

$$\Rightarrow \quad \text{Higgs quartic coupling :} \quad \lambda_{\text{NMSSM}} = \lambda_{\text{MSSM}} + \kappa^2 \sin^2 2\beta$$

$$\Rightarrow \quad \left(m_h^2\right)_{\text{NMSSM}} \simeq \left(m_h^2\right)_{\text{MSSM}} + \frac{2\kappa^2 \sin^2 2\beta}{g_1^2 + g_2^2} M_Z^2$$

Additional Higgs boson mass can be sizable in the small $\tan\beta$ limit.

* Models with extra $U(1)$:

Models with extra $U(1)$ gauge symmetry under which the Higgs bosons are charged can give rise to additional Higgs quartic coupling through the D-term potential.

$$\begin{aligned}\Delta V &= \frac{g'^2 m_{\rm soft}^2}{2M_{Z'}^2}\left(|H_u|^2 - |H_d|^2\right)^2 = \frac{g'^2 m_{\rm soft}^2}{M_{Z'}^2}\frac{\cos^2 2\beta}{8}h^4 \\ \Rightarrow \quad \Delta m_h^2 &= \left(\frac{4g'^2 m_{\rm soft}^2}{g_1^2+g_2^2}M_{Z'}^2\right)M_Z^2\cos^2 2\beta\end{aligned}$$

Can be sizable in the large $\tan\beta$ limit and $M_{Z'}$ is near its lower bound.

Conclusions

- SUSY at the TeV scale has been proposed to avoid the extreme fine tuning for electroweak symmetry breaking (EWSB) in the SM, so for many years natural EWSB has been an important guideline for SUSY model building.
- There is now a tension between natural EWSB and the LHC results such as the SM-like 125 GeV Higgs boson and the non-observation of missing ET events, so the simplest version of SUSY is now in trouble.
- This may enforce us to abandon the idea of naturalness, or suggest a modification of the simplest version in various different directions:

  i) Compressed (or stealth) SUSY spectrum?

  ii) Inverted sfermion mass hierarchy?

  iii) Loop split SUSY?

  iv) Broken *R*-parity?

  v) Extended Higgs sector or extra U(1) ?

  ...

  Hopefully LHC will provide further guidelines for the right direction.