

QCD Axion, Moduli Stabilization and SUSY Breaking in String Theory

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Tomorrow:

TeV scale SUSY with the recent LHC results

Outline of Today's Lecture

- QCD axion in string theory
 - * Axion solution to the strong CP problem and its realization in string theory
 - * Cosmological constraints on GUT scale QCD axion
 - * Intermediate scale QCD axion with anomalous $U(1)$ gauge symmetry
- Moduli stabilization and SUSY breaking with intermediate scale QCD axion
 - * Effective theory for SUSY breaking by anti-brane in KKLT moduli stabilization
 - * KKLT axiverse

Strong CP problem:

Low energy QCD involves a CP-violating interaction

$$\frac{1}{32\pi^2} \bar{\theta} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$

which gives rise to the neutron EDM

$$d_n \sim 10^{-16} \bar{\theta} \text{ e} \cdot \text{cm}$$

and therefore is constrained as

$$|\bar{\theta}| \lesssim 10^{-9} \quad (|d_n| \lesssim 10^{-25} \text{ e} \cdot \text{cm}).$$

On the other hand,

$$\text{SM : } \bar{\theta} = \theta_{\text{QCD}} + \text{ArgDet}(y_u y_d),$$

$$\text{MSSM : } \bar{\theta} = \theta_{\text{QCD}} + \text{ArgDet}(y_u y_d) + 3\text{Arg}(M_{\tilde{g}}) + 3\text{Arg}(B\mu),$$

so it is quite unnatural that $|\bar{\theta}| \lesssim 10^{-9}$ within the CKM paradigm which explains CP violations in the weak interactions through the complex Yukawa couplings $y_{u,d}$.

Axion solution: Peccei and Quinn

At scales below an appropriate energy scale f_a , the theory is assumed to possess a non-linearly realized global $U(1)$ symmetry:

Axionic shift symmetry $U(1)_{\text{PQ}} : a \rightarrow a + \text{constant}$,

which is explicitly broken dominantly by the QCD anomaly.

$$\Rightarrow \mathcal{L}_{\text{axion}} = \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{f_a}\partial_\mu a J^\mu + \frac{1}{32\pi^2} \left(\frac{a}{f_a} + \bar{\theta} \right) G\tilde{G} + \Delta\mathcal{L}$$

$(\Delta\mathcal{L} = \text{nonderivative couplings of } a \text{ other than } aG\tilde{G})$

$$\begin{aligned} \partial_\mu J_{\text{PQ}}^\mu &= \frac{1}{32\pi^2} G\tilde{G} + f_a \frac{\partial}{\partial a} \Delta\mathcal{L} \quad (J_{\text{PQ}}^\mu = f_a \partial^\mu a + J^\mu) \\ &= \text{QCD anomaly} + \text{other explicit } U(1)_{\text{PQ}} \text{ breakings} \end{aligned}$$

$$\begin{aligned} V(a) &= V_{\text{QCD}}(a) + V_{\text{UV}}(a) \\ &= -f_\pi^2 m_\pi^2 \sqrt{\frac{m_u^2 + m_d^2 + 2m_u m_d \cos(a/f_a + \bar{\theta})}{(m_u + m_d)^2}} \\ &\quad + \epsilon M_{\text{UV}}^4 \cos(a/f_a + \alpha) \quad (\alpha \neq \bar{\theta}) \end{aligned}$$

If explicit $U(1)_{\text{PQ}}$ breakings other than the QCD anomaly are highly suppressed, so that

$$V_{\text{UV}}(a) \lesssim 10^{-9} f_\pi^2 m_\pi^2 \sim 10^{-78} M_{\text{GUT}}^4,$$

then V_{QCD} drives the axion VEV to cancel $\bar{\theta}$ with an accuracy of $\mathcal{O}(10^{-9})$, regardless of the values of the low energy parameter $\bar{\theta}$ and the UV parameter α :

$$\left| \left\langle \frac{a}{f_a} \right\rangle + \bar{\theta} \right| \lesssim 10^{-9}.$$

This is an elegant mechanism, but raises a question:

Q1: What is the origin of such global symmetry which is explicitly broken in a quite peculiar way?

Simply assuming such a global symmetry is not likely to be sensible as quantum gravity effects generically break global symmetries, so can generate $V_{\text{UV}}(a) \gg f_\pi^2 m_\pi^2$.

Astrophysical and cosmological considerations lead to various constraints on the axion scale f_a .

* Axion emission from red giants, neutron stars, SN1987A:

$$\Rightarrow f_a \gtrsim 10^9 \text{ GeV}.$$

* Relic axions produced by an initial misalignment $\delta a \equiv f_a \theta_i$:

$$\Omega_a h^2 \simeq 2 \times 10^4 \left(\frac{f_a}{10^{16} \text{ GeV}} \right)^{7/6} \langle \theta_i^2 \rangle \lesssim 0.12$$

(assuming no entropy production after the QCD phase transition)

$$\Rightarrow f_a \lesssim 3 \times 10^{11} \langle \theta_i^2 \rangle^{-6/7} \text{ GeV}.$$

So, if the misalignment angle takes a value in the natural range, i.e. $\theta_i \sim 1$, the axion scale is required to be

$$10^9 \text{ GeV} \lesssim f_a \lesssim 3 \times 10^{11} \text{ GeV}.$$

This raises another question:

Q2: What is the dynamics to generate such an intermediate axion scale?

In SUSY models, the axion scale f_a is in fact a dynamical field, the saxion or modulus partner of axion, and then the axion scale is determined by the mechanism to stabilize saxion or moduli.

Higher-dim gauge symmetry as the origin of $U(1)_{\text{PQ}}$

Higher-dim theory with a p -form gauge field ($p = 1, 2, \dots$), compactified on internal space involving a p -cycle S_p :

* Higher-dim gauge symmetry:

$$G_C : C_p \rightarrow C_p + d\Lambda_{p-1} \quad (\Lambda_{p-1} = \text{globally well-defined } (p-1)\text{-form})$$

* Axion fluctuation:

$$C_{[m_1 m_2 \dots m_p]}(x, y) = a(x) \omega_{[m_1 m_2 \dots m_p]}(y) = a(x) \partial_{[m_1} \tilde{\Lambda}_{m_2 \dots m_p]} \\ (x^\mu, y^m) = (4\text{d Minkowski coordinates, internal coordinates})$$

Here ω_p is a harmonic p -form with $\int_{S_p} \omega_p = 1$, so $\tilde{\Lambda}_{p-1}$ is only locally well-defined.

Then the axionic shift symmetry $U(1)_{\text{PQ}} : a \rightarrow a + \text{constant}$ is locally equivalent to the higher-dim gauge symmetry G_C , but not globally:

$$G_C : C_p \rightarrow C_p + d\Lambda_{p-1} \quad \text{for} \quad \int_{S_p} d\Lambda_{p-1} = 0,$$

$$U(1)_{\text{PQ}} : C_p \rightarrow C_p + \text{constant} \times \omega_p \quad \text{for} \quad \int_{S_p} \omega_p \neq 0$$

$U(1)_{\text{PQ}}$ can be explicitly broken, but only through the effects associated with non-trivial global topology of the p -cycle S_p , in particular associated with

$$\int_{S_p} \omega_p \neq 0$$

* QCD anomaly:

$$G_C\text{-invariant } \int C_p \wedge G \wedge G \rightarrow U(1)_{\text{PQ}}\text{-breaking } \int_{4\text{D}} a G \tilde{G} \int_{S_p} \omega_p$$

* UV instantons wrapping S_p :

$$V_{\text{UV}}(a) = \epsilon_0 e^{-S_{\text{ins}}} M_{\text{UV}}^4 \cos(a/f_a + \alpha) \quad \left(M_{\text{UV}} \sim M_{\text{GUT}} \text{ or } M_{\text{string}} \right)$$

$$\left(\epsilon_0 = \text{model-dependent zero-mode factors possibly involving } \left(\frac{m_{3/2}}{M_{\text{Pl}}} \right)^n \right)$$

This suggests that if S_p has a relatively large volume to have the instanton action $S_{\text{ins}} \gtrsim \mathcal{O}(100)$, a good $U(1)_{\text{PQ}}$ can appear as a low energy remnant of higher-dim gauge symmetry.

Obviously string theory is the best place to realize this scenario.

Axion scale (= axion decay constant):

Canonically normalized stringy axion typically has Planck-scale suppressed interactions, so a decay constant $f_a \sim M_{\text{GUT}}$:

$$\frac{\hat{a}}{M_{\text{Pl}}} G\tilde{G} \equiv \frac{1}{32\pi^2} \frac{\hat{a}}{f_a} G\tilde{G} \quad \rightarrow \quad f_a \sim \frac{M_{\text{Pl}}}{32\pi^2} \sim M_{\text{GUT}}$$

This can be easily seen in supersymmetric compactification:

Axion Superfield: $T = t + ia + \sqrt{2}\theta\tilde{a} + \theta^2 F^T$

$$\left(t \propto \text{Vol}(S_p), \text{ normalized as } \langle t \rangle \sim \frac{1}{g_{\text{GUT}}^2} \right)$$

$$\begin{aligned} \mathcal{L}_{\text{axion}} &= M_{\text{Pl}}^2 \frac{\partial^2 K}{\partial t^2} \partial_\mu a \partial^\mu a + \frac{1}{4} a G\tilde{G} + \dots \\ \Rightarrow \quad f_a &= \sqrt{\frac{\partial^2 K}{\partial t^2} \frac{M_{\text{Pl}}}{8\pi^2}} \sim 3 \sqrt{\frac{\partial^2 K}{\partial t^2}} \times 10^{16} \text{ GeV}, \end{aligned}$$

For

$$K \simeq -n \ln(T + T^*) \quad \rightarrow \quad \frac{\partial^2 K}{\partial t^2} \simeq \frac{n}{t^2} \sim g_{\text{GUT}}^4,$$

so the axion scale is indeed around 10^{16} GeV. [KC, Kim\(1985\)](#); [Svrcek, Witten\(2006\)](#)

It is in principle possible to have $\sqrt{\frac{\partial^2 K}{\partial t^2}} \ll 1$, and therefore $f_a \ll M_{\text{GUT}}$.

For instance, S_p might be a relatively small cycle embedded in a much larger bulk volume (Large Volume Scenario), or it might be located at a highly warped region in the internal space (Warped Compactification):

$$\frac{\partial^2 K}{\partial t^2} \sim \frac{1}{\text{large bulk volume}} \quad \text{or} \quad \text{small warp factor} \sim 10^{-10},$$

which would give

$$f_a \sim 3 \times 10^{11} \text{ GeV}$$

and

$$\Omega_a h^2 \sim 0.1 \left(\frac{f_a}{3 \times 10^{11} \text{ GeV}} \right)^{7/6} \theta_i^2 \sim 0.1 \quad \text{for } \theta_i \sim 1.$$

However in such scheme, the cutoff scale of 4D visible sector physics is red-shifted also, making it difficult to accommodate the unification scale $M_{\text{GUT}} \sim 2 \times 10^{16} \text{ GeV}$ within the scheme.

More on cosmology of GUT scale QCD axion:

Fox,Pierce,Thomas(2004); Mack(2009)

Even for GUT scale QCD axion, the relic axion mass density can have an acceptable value if

- (i) the misalignment angle θ_i is small enough and/or
- (ii) there is a dilution of axions by late entropy production after the QCD phase transition:

$$\Omega_a h^2 \simeq 2 \times 10^4 \left(\frac{f_a}{10^{16} \text{ GeV}} \right)^{7/6} \langle \theta_i^2 \rangle \quad (\text{No dilution})$$

$$\Omega_a h^2 \simeq 40 \left(\frac{T_{RH}}{6 \text{ MeV}} \right) \left(\frac{f_a}{10^{16} \text{ GeV}} \right)^2 \langle \theta_i^2 \rangle$$

(Entropy production by late decaying massive particles with $6 \text{ MeV} \lesssim T_{RH} \lesssim \Lambda_{\text{QCD}}$)

In order for a GUT scale QCD axion to have $\Omega_a h^2 \lesssim 0.1$,

$$\langle \theta_i^2 \rangle \lesssim 5 \times 10^{-6} \quad \text{or} \quad 2.5 \times 10^{-3} \left(\frac{6 \text{ MeV}}{T_{RH}} \right)$$

It is often argued that in inflationary cosmology with $U(1)_{PQ}$ non-linearly realized during inflation, such small $\langle \theta_i^2 \rangle$ can be explained by anthropic selection rule since galaxies in a Universe with $\Omega_a h^2 > \mathcal{O}(1)$ will have too large mass densities to accommodate life. [Linde\(1986\)](#); [Tegmark,Rees,Wilczek\(2009\)](#)

On the other hand, in such scenario, the axion misalignment square receives an irreducible contribution from the axion fluctuation produced during inflation:

$$\begin{aligned}\langle \theta_i^2 \rangle &= \langle \theta_i \rangle^2 + \langle (\theta_i - \langle \theta_i \rangle)^2 \rangle = \langle \theta_i \rangle^2 + \sigma_\theta^2, \\ f_a \sigma_\theta &\sim \frac{H_I}{2\pi} \quad (= \text{RMS axion fluctuation during inflation}),\end{aligned}$$

There is no dynamical mechanism to determine $\langle \theta_i \rangle$, so in principle it can have an arbitrarily small value, while for given inflation model, σ_θ is predicted to be of the order of $H_I/2\pi$.

If anthropic selection is the correct explanation for small $\langle \theta_i^2 \rangle$ of GUT scale QCD axion, it implies that the actual value of $\Omega_a h^2$ should not be far below the anthropic upper bound $\sim \mathcal{O}(1)$, so it has a high probability to be around 0.1.

Axion fluctuation produced during inflation is of isocurvature type, so leads to an isocurvature CMB fluctuation: [Fox,Pierce,Thomas\(2004\)](#)

$$\begin{aligned}\alpha &\equiv \frac{\langle(\delta T/T)_{\text{iso}}^2\rangle}{\langle(\delta T/T)_{\text{tot}}^2\rangle} \simeq \frac{8}{25} \frac{(\Omega_a/\Omega_M)^2}{\langle(\delta T/T)_{\text{tot}}^2\rangle} \frac{\sigma_\theta^2(2\langle\theta_i\rangle^2 + \sigma_\theta^2)}{(\langle\theta_i^2\rangle)^2} \\ &= 1.5 \times 10^{11} \left(\frac{\Omega_a h^2}{\langle\theta_i^2\rangle} \right)^2 \sigma_\theta^2 (2\langle\theta_i\rangle^2 + \sigma_\theta^2).\end{aligned}$$

Imposing the observational bound on this isocurvature CMB fluctuation

$$\alpha \sim 1.5 \times 10^{11} \left(\frac{\Omega_a h^2}{\langle\theta_i^2\rangle} \right)^2 \left(\frac{H_I}{2\pi f_a} \right)^2 \left(2\langle\theta_i\rangle^2 + \left(\frac{H_I}{2\pi f_a} \right)^2 \right) \lesssim 0.072,$$

one finds a rather strong constraint on the inflation scale:

For $f_a \sim 10^{16}$ GeV and $\Omega_a h^2 \sim 0.1$:

$$\begin{aligned}H_I &\lesssim 6 \times 10^8 \text{ GeV} \quad (\text{no dilution}) \\ &\lesssim 1.6 \times 10^{10} \text{ GeV} / \sqrt{T_{\text{RH}}/6 \text{ MeV}} \quad (\text{late entropy production})\end{aligned}$$

In slow roll inflation scenario,

$$\epsilon \equiv \frac{1}{2} M_{\text{Pl}}^2 \left(\frac{V'}{V} \right)^2 \sim \frac{1}{8\pi^2 (\delta T/T)^2} \left(\frac{H_I}{M_{\text{Pl}}} \right)^2 \sim 10^8 \left(\frac{H_I}{M_{\text{Pl}}} \right)^2$$
$$n_s - 1 = -6\epsilon + 2\eta \quad \left(\eta \equiv M_{\text{Pl}}^2 \frac{V''}{V} \right),$$

so the above upper bound on H_I from the isocurvature fluctuation of GUT scale QCD axion implies

$$\epsilon \lesssim 10^{-11} \text{ (no dilution)}$$
$$\left(\lesssim \frac{7 \times 10^{-9}}{(T_{\text{RH}}/6 \text{ MeV})} \text{ (late entropy production)} \right)$$
$$\eta = -\mathcal{O}(10^{-2}).$$

This implies that the tensor mode in CMB is too small to be observed (but note Gary's talk), and may require a fine tuning of either the initial condition or the model parameters in the underlying inflation model.

This motivates us to explore an alternative possibility for

i) Natural $U(1)_{\text{PQ}}$ being a low energy remnant of higher-dim gauge symmetries,

ii) Intermediate axion scale $f_a \sim 10^{9-11}$ GeV, while $M_{\text{GUT}} \sim 10^{16}$ GeV.

Lowering the axion scale with anomalous $U(1)$ gauge symmetry

KC, Jeong, Okumura, Yamaguchi (2011)

Symmetries:

* Global axionic shift symmetry from higher-dim gauge symmetry:

$$a_C \rightarrow a_C + \text{constant} \quad (C_p(x, y) = a_C(x)\omega_p(y))$$

* Anomalous $U(1)_A$ gauge symmetry:

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x), \quad a_C \rightarrow a_C + \delta_{\text{GS}} \alpha(x)$$

$$X_i \rightarrow e^{iq_i \alpha(x)} X_i \quad \left(\delta_{\text{GS}} = \frac{1}{8\pi^2} \sum_i q_i \text{Tr}(T_a^2(X_i)) \right)$$

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & M_{\text{Pl}}^2 \frac{\partial^2 K}{\partial t^2} (\partial_\mu a_C - \delta_{\text{GS}} A_\mu)^2 + \frac{1}{4} a_C G \tilde{G} \\ & + D_\mu X_i^* D^\mu X_i - \frac{g_A^2}{2} \left(\delta_{\text{GS}} M_{\text{Pl}}^2 \frac{\partial K}{\partial t} - \sum_i q_i |X_i|^2 \right)^2 + \dots \\ & (t = \text{modulus partner of } a_C) \end{aligned}$$

If $U(1)_A$ has a mixed anomaly with the SM gauge group, which is cancelled by the GS mechanism, some $U(1)_A$ charged scalar field X should have a $\text{VEV} \gtrsim 10^9 \text{ GeV}$ in order for the model to be phenomenologically viable.

Then there are two axion-like fields, a_C and $\text{Arg}(X)$, and the physical axion is given by the $U(1)_A$ invariant combination:

$$a \propto a_C + \frac{q_X}{\delta_{\text{GS}}} \text{Arg}(X),$$

while the other combination becomes the longitudinal component of A_μ .

There are two key mass scales in this type of model:

$$\text{FI-term : } \xi_{FI} = \delta_{\text{GS}} M_{\text{Pl}}^2 \frac{\partial K}{\partial t},$$

$$\text{Stückelberg mass : } M_{ST}^2 = \delta_{\text{GS}}^2 M_{\text{Pl}}^2 \frac{\partial^2 K}{\partial t^2},$$

$$\text{* D-flat condition : } D_A = \xi_{FI} - q_X |X|^2 = 0,$$

$$\text{* } U(1)_A \text{ gauge boson mass : } M_A^2 = M_{ST}^2 + q_X^2 |X|^2$$

* Decay constant of a_C originating from p -form gauge field :

$$f_{a_C} \sim M_{ST} \sim 10^{16} \text{ GeV}$$

$$\text{* Decay constant of physical axion : } f_a \sim \frac{|X| M_{ST}}{\sqrt{M_{ST}^2 + |X|^2}}$$

We can now consider two possibilities:

A. Anomalous $U(1)$ gauge symmetry not affecting the moduli sector:

For a (metastable) vacuum configuration with

$$\xi_{FI} = \delta_{\text{GS}} M_{\text{Pl}}^2 \frac{\partial K}{\partial t} = q_X |X|^2 \gg M_{ST}^2 = \delta_{\text{GS}}^2 M_{\text{Pl}}^2 \frac{\partial^2 K}{\partial t^2},$$

the anomalous $U(1)_A$ gauge boson gets a mass $M_A \sim \sqrt{\xi_{FI}}$ by the VEV of $U(1)_A$ -charged matter field X , and is decoupled without affecting the moduli sector.

Then the low energy theory has a physical axion which is mostly a_C and has a decay constant

$$f_a \sim M_{ST} \sim 10^{16} \text{ GeV}.$$

Example:

$$\begin{aligned} a_C &= \text{model-independent axion in heterotic string} \\ t &= \text{heterotic string dilaton} \end{aligned}$$

$$\Rightarrow K = -\ln t, \quad \langle t \rangle = \frac{1}{g_{\text{GUT}}^2} \quad \Rightarrow \quad \xi_{FI} \sim \frac{8\pi^2}{g_{\text{GUT}}^2} M_{ST}^2$$

B. Anomalous $U(1)$ gauge symmetry eliminating a stringy axion a_C and its modulus partner t from low energy spectrum, while leaving a field theoretic axion with intermediate scale decay constant:

For another type of vacuum configuration with

$$\xi_{FI} \ll M_{ST}^2,$$

$U(1)_A$ gauge boson absorbs mostly a_C to get a mass $M_A \sim M_{ST}$ through the Stückelberg mechanism, leaving an anomalous global $U(1)_{PQ}$ in low energy theory, which is essentially the global part of $U(1)_A$ and spontaneously broken by

$$\langle X \rangle \sim \sqrt{\xi_{FI}} \ll M_{ST}$$

In this case, the physical axion is mostly $\text{Arg}(X)$, and therefore the axion scale f_a is determined by the dynamics to fix the vacuum value of X .

A simple and attractive possibility is that X is stabilized by an interplay between SUSY breaking effects and Planck-scale-suppressed effects, naturally giving an intermediate QCD axion scale:

[Kim, Nilles\(1984\); KC, Chun, Kim\(1997\)](#)

$$f_a \sim \langle X \rangle \sim \sqrt{m_{\text{soft}} M_{\text{Pl}}} \sim 10^{10-11} \text{ GeV}.$$

In fact, vacuum configuration with $\xi_{FI} \ll M_{ST}^2$ is quite common in D -brane models realized in type IIB or IIA string theory (and some heterotic string compactification also), which allows a supersymmetric solution with vanishing ξ_{FI} in the limit that all $U(1)_A$ -charged matter fields have vanishing VEVs.

Moduli stabilization and SUSY breaking with intermediate scale QCD axion

Thanks to the progress in moduli stabilization during the last decade, we now have several interesting scenarios of moduli stabilization, which have a good potential to stabilize all moduli at phenomenologically viable vacuum with nearly vanishing cosmological constant.

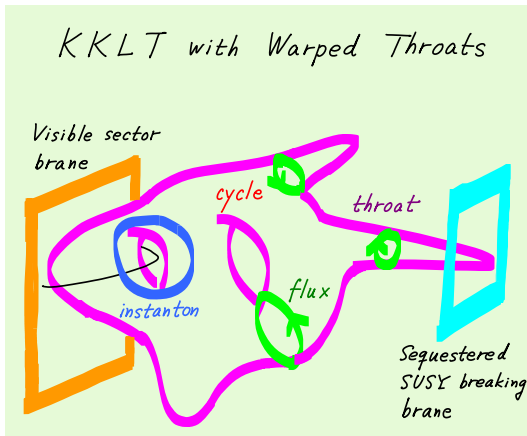
Here I will discuss a **KKLT Axiverse Scenario** which can easily accommodate an intermediate scale QCD axion as well as other (ultralight) axion-like fields with decay constants $\sim M_{\text{GUT}}$.

(It should be noticed that **Large Volume Scenario (LVS)** can also accommodate an intermediate scale QCD axion in a similar manner.

[KC, Nilles, Shin, Trappetti \(2010\)](#))

KKLT moduli stabilization

- Flux stabilization of all moduli except for the Kähler moduli $\{T_I\}$
- Non-perturbative stabilization of (some of) $\{T_I\}$ by instanton-induced (or gaugino-condensation-induced) superpotential
- Sequestered SUSY-breaking by anti-brane at the tip of throat



Pattern of moduli masses:

- * Dilaton or complex structure moduli stabilized by flux in bulk CY:

$$m_{S,U} \sim \frac{1}{M_{\text{string}} R^3} \sim 10^{15} \text{ GeV} \quad (R = \text{bulk CY radius})$$

- * Complex structure modulus describing the throat:

$$m_{U'} \sim e^A M_{\text{string}} \sim 10^{10} \text{ GeV}$$

(e^{2A} = warp factor at the end of throat $\sim 10^{-14}$)

- * (Some of) Kähler moduli stabilized by instantons or gaugino condensation:

$$m_T \sim m_{3/2} \ln(M_{\text{Pl}}/m_{3/2}) \sim 10^6 \text{ GeV}$$

Before introducing anti- D_3 brane, these moduli were stabilized at SUSY AdS solution, which is lifted to a dS vacuum with nearly vanishing cosmological constant after anti- D_3 brane is introduced:

$$V \simeq -3m_{3/2}^2 M_{\text{Pl}}^2 + V_{\text{lift}} \simeq 0 \quad \left(V_{\text{lift}} \sim M_{D_3}^3 \sim e^{4A} M_{\text{Pl}}^4 \right)$$
$$\Rightarrow m_{3/2} \sim e^{2A} M_{\text{Pl}} \sim 10^4 \text{ GeV}$$
$$m_{\text{soft}} \sim \frac{m_{3/2}}{\ln(M_{\text{Planck}}/m_{3/2})} \sim 10^3 \text{ GeV}$$

If all Kähler moduli are stabilized by nonperturbative superpotential as was proposed in the original KKLT, there would not be any axionic shift symmetry which is required for $U(1)_{PQ}$ in low energy theory.

So, to implement the axion solution to the strong CP problem, we need some Kähler moduli not stabilized by non-perturbative superpotential, but by other effects (uplifting potential or D -term potential) preserving the axionic shift symmetries.

In fact, this can be considered to be more likely and more desirable than the original KKLT scenario, since

- i) it is somewhat difficult that the Kähler modulus of a cycle supporting charged chiral fermions, e.g. the visible sector Kähler modulus, gets a sizable instanton-induced superpotential due to the suppression by charged chiral zero modes,
- ii) an axionic shift symmetry unbroken by the moduli potential is required for the axion solution to the strong CP problem.

4D effective theory for SUSY breaking by anti- D_3 brane

KC,Nilles,Falkowski,Olechowski(2005)

To proceed, let us discuss the 4D effective action describing SUSY breaking by anti-brane stabilized at the end of warped throat.

For simplicity, we consider first the case of single Kähler modulus, and integrate out the heavy moduli S , U and U' to write down the effective action of the Kähler modulus and visible gauge and matter fields:

$$\mathcal{L}_{\text{eff}} = \int d^4\theta \Omega + \left[\int d^2\theta \left(\frac{1}{4} f_a \mathcal{W}^{a\alpha} \mathcal{W}_\alpha^a + C^3 W \right) + \text{c.c} \right]$$

$$\Omega = \Omega_{\text{bulk}} + \Omega_{\bar{D}_3}$$

$$\Omega_{\text{bulk}} = -3CC^* e^{-K_{\text{bulk}}/3} = \left[-3e^{-K_0(T,T^*)/3} + Y_I(T,T^*) \Phi_I^* \Phi_I + \dots \right]$$

$$(C = C_0 + F^C \theta^2 = \text{SUGRA chiral compensator})$$

$$f_a = T$$

$$W = W_0 + A e^{-aT} + \frac{\lambda_{IJK}}{6} \Phi_I \Phi_J \Phi_K + \dots$$

In $\Omega_{\bar{D}_3}$, SUSY is non-linearly realized with the Goldstino superfield

$$\Lambda^\alpha = \theta^\alpha + \frac{1}{M_{\bar{D}_3}^2} \xi^\alpha + \dots \quad (M_{\bar{D}_3} \sim e^A M_{\text{string}}).$$

$$\begin{aligned} \Rightarrow \quad \Omega_{\bar{D}_3} &= -C^2 C^{2*} e^{4A} \Lambda^2 \bar{\Lambda}^2 P + (e^{3A} C^3 \bar{\Lambda}^2 \Gamma + \text{c.c.}) \\ &= -C^2 C^{2*} e^{4A} \theta^2 \bar{\theta}^2 P + (e^{3A} C^3 \bar{\theta}^2 \Gamma + \text{c.c.}) \\ &\quad + \text{Goldstino-dependent terms} \end{aligned}$$

(SUSY appear to be explicitly broken.)

The compensator dependence in $\Omega_{\bar{D}_3}$ can be fixed by the Weyl-invariance under

$$\eta_{\mu\nu} \rightarrow e^{2(\tau_0 + \tau_0^*)} \eta_{\mu\nu}, \quad C \rightarrow e^{-2\tau_0} C, \quad \theta^\alpha \rightarrow e^{-\tau_0 + 2\tau_0^*} \theta^\alpha,$$

and the simple dimensional analysis determines the dependence on the warp factor e^A .

From the above superspace action, the equations of motion for the auxiliary F -components of the chiral superfields C and T can be derived, which would determine the on-shell expressions of $F^{C,T}$.

The SUSY breaking term $e^{3A}\theta^2\Gamma^*$ in $\Omega_{\bar{D}_3}$ modifies the on-shell expressions of $F^{C,T}$ (compared to the standard $N = 1$ SUGRA expression obtained in the absence of anti-brane), however the modification is suppressed by e^{3A} :

$$\begin{aligned}\frac{F^C}{C_0} &= m_{3/2}^* + \frac{1}{3}F^T\partial_T K + \mathcal{O}(e^{3A}M_{\text{Pl}}) \\ F^T &= -e^{K/2}K^{T^*T}(D_TW)^* + \mathcal{O}(e^{3A}M_{\text{Pl}})\end{aligned}$$

Also the on-shell expression of the moduli potential is modified by $e^{3A}\theta^2\Gamma$ and $e^{4A}\theta^2\bar{\theta}^2P$ as:

$$V = e^K \left(K^{T^*T} |D_TW|^2 - 3|W|^2 \right) + e^{4A}e^{2K/3}P + \mathcal{O}(e^{3A}m_{3/2}M_{\text{Pl}}^3)$$

To make the cosmological constant to be nearly vanishing, the warp factor should have a size

$$e^A \sim \sqrt{m_{3/2}/M_{\text{Pl}}}.$$

Then the modification of on-shell expressions of $F^{C,T}$ can be safely ignored, and one can use the standard $N = 1$ SUGRA expressions for the auxiliary components:

$$\begin{aligned}\frac{F^C}{C_0} &= m_{3/2}^* + \frac{1}{3} F^I \partial_I K, \\ F^I &= -e^{K/2} K^{I\bar{J}} (D_{\bar{J}} K)^*,\end{aligned}$$

while the scalar potential is well approximated by

$$V = e^K \left(F^{I\bar{J}} D_I W (D_{\bar{J}} W)^* - 3|W|^2 \right) + e^{4A} e^{2K/3} P.$$

It turns out that SUSY breaking by anti-D3 at the tip of KS throat is sequestered well from the light degrees of freedom in bulk CY.

KC,Jeong; Kachru,McAllister,Sundrum

As a consequence, $e^{4A}P$ is (nearly) independent of the Kähler modulus T as well as of the visible matter and gauge fields which are presumed to be localized at D -branes in bulk CY:

$$e^{4A}P = e^{4A_0}P_0 = \text{Constant of } \mathcal{O}(m_{3/2}^2 M_{\text{Pl}}^2)$$

$$\Rightarrow V_{\text{lift}} = e^{4A_0}P_0 e^{2K(T,T^*,\Phi,\Phi^*)/3}$$

Now the moduli VEV can be determined by minimizing

$$V = V_{\text{SUGRA}} + V_{\text{lift}} = V_{\text{SUGRA}} + e^{4A_0} P_0 e^{2K_0(T+T^*)/3} \\ \left(V_{\text{lift}} \propto \frac{1}{(T+T^*)^2} \text{ for } K_0 = -3 \ln(T+T^*) \right)$$

under the condition of nearly vanishing cosmological constant.

Normally it is technically difficult to find a SUSY-breaking (local) minimum of generic SUGRA potential.

However, in KKLT scenario, the (meta-stable) vacuum is near the supersymmetric configuration, which makes it possible to compute the moduli VEVs, F -components and masses in the perturbative expansion in

$$\frac{1}{\ln(M_{\text{Planck}}/m_{3/2})} \sim \frac{1}{4\pi^2}$$

In this simple example, regardless of the form of $K_0(T + T^*)$,

$$m_T \sim \frac{\partial_T^2 W}{\partial_T \partial_{T^*} K} \sim m_{3/2} \ln(M_{Pl}/m_{3/2})$$

$$\langle T \rangle = T_0 + \delta T, \quad \frac{\delta T}{T_0} = \mathcal{O}\left(\frac{m_{3/2}^2}{m_T^2}\right) \quad (D_T W|_{T=T_0} = 0)$$

$$\frac{F^T}{T + T^*} \sim m_T \frac{\delta T}{T} \sim \frac{m_{3/2}^2}{m_T} \sim \frac{m_{3/2}}{\ln(M_{Pl}/m_{3/2})} \sim \frac{1}{4\pi^2} \frac{F^C}{C_0}$$

$$\frac{F^C}{C_0} = m_{3/2}^* + \frac{1}{3} F^T \partial_T K_0 \simeq m_{3/2}$$

$$\Rightarrow \quad \textbf{modulus mediation of } \mathcal{O}\left(\frac{F^T}{T}\right) \sim \textbf{anomaly mediation of } \mathcal{O}\left(\frac{m_{3/2}}{8\pi^2}\right)$$

This pattern of SUSY breaking is quite robust and persists even when the scheme is generalized in various different directions, including the KKLT axiverse scenario incorporating an intermediate scale QCD axion.

KKLT Axiverse KC, Jeong

* Start with a generic compactification with multiple Kähler moduli $\{T_I\}$, and also multiple anomalous $U(1)_A$ gauge symmetries under which some Kähler moduli have non-linear transformation to implement the GS anomaly cancellation mechanism:

$$U(1)_A : V_A \rightarrow V_A - \Lambda - \Lambda^*, \quad T_I \rightarrow T_I + \delta_{GS,I} \Lambda, \quad X_i \rightarrow e^{q_i \Lambda} X_i$$

* At leading order, the Kähler potential of Kähler moduli takes the no-scale form, obeying

$$K^{I\bar{J}} K_I K_{\bar{J}} = 3, \quad (T_I + T_I^*) K_I = -3, \quad K^{I\bar{J}} K_{\bar{J}} = -(T_I + T_I^*).$$

* V_A for $U(1)_A$ with $\xi_{FI} \gg M_{ST}^2$ gets a mass $M_A \sim \sqrt{\xi_{FI}} \gg M_{ST}$ by the VEV of some $U(1)_A$ charged matter fields, and can be integrated out without affecting the moduli sector.

* For other type of $U(1)_A$ with $\xi_{FI} \ll M_{ST}^2$, V_A gets a mass $M_A \sim M_{ST}$ by absorbing a Kähler modulus superfield T_A , and can be integrated out while obeying

$$\frac{\partial K}{\partial V_A} = \delta_{GS,A} \frac{\partial K}{\partial T_A} = 0.$$

* In low energy effective theory, the global part of such $U(1)_A$ appears as an anomalous global symmetry which is spontaneously broken by the VEV of $U(1)_A$ charged matter field at scales far below M_{ST} :

$$U(1)_{PQ} : X_i \rightarrow e^{iq_i \alpha} X_i \text{ with } \langle X_i \rangle \ll M_{ST} \sim M_{GUT}$$

and the physical axion scale is determined by the dynamics to fix the matter field VEVs $\langle X_i \rangle$.

* The Kähler potential of the remained light Kähler moduli $\{T_M\}$ still obeys the no-scale condition: $\{T_I\} = \{T_A, T_M\}$

$$K^{M\bar{N}} K_M K_{\bar{N}} = 4, \quad (T_M + T_M^*) K_M = -3, \quad K^{M\bar{N}} K_{\bar{N}} = -(T_M + T_M^*).$$

* Some Kähler moduli $\{T_m\}$ among $\{T_M\} = \{T_m, T_u\}$ are stabilized by non-perturbative superpotential:

$$W_{NP} = \sum_m A_m e^{-a_m T_m},$$

while the other Kähler moduli $\{T_u\}$ are stabilized by the uplifting potential invariant under the axionic shift symmetries.

* As for axions, an intermediate scale QCD axion can be obtained from $\text{Arg}(X_i)$ if the biggest VEV of X_i is determined by an interplay between SUSY breaking terms and Planck-scale-suppressed terms as

$$\text{Max}(\langle X_i \rangle) \sim \sqrt{m_{\text{soft}} M_{\text{Pl}}}.$$

There can be also ultralight axions $\text{Im}(T_u)$ having a decay constant $\sim M_{\text{GUT}}$, which would be harmless if they are light enough and may lead to the axiverse phenomenology discussed in [Arvanitaki et. al, arXiv:0905.4720](#).

Moduli stabilization and SUSY breaking in KKLT axiverse

After integrating out all massive $U(1)_A$ vector superfields as well as the complex structure moduli and the dilaton stabilized by flux, the effective theory contains three sectors:

- i) Kähler moduli sector,
- ii) PQ sector which would break spontaneously the anomalous global $U(1)$ symmetries which are the low energy remnant of $U(1)_A$ with $\xi_{\text{FI}} \ll M_{\text{ST}}^2$,
- iii) the MSSM sector.

$$\mathcal{L}_{\text{eff}} = \int d^4\theta [CC^* \Omega_{\text{bulk}} + C^2 C^{*2} e^{4A_0} P_0] + \int d^2\theta C^3 W + \dots,$$

$$\Omega_{\text{bulk}} = -3e^{-K_0(T_M + T_M^*)/3} + Y_i(T_M + T_M^*)X_i^* X_i + Y_\alpha(T_M + T_M^*)\Phi_\alpha^* \Phi_\alpha$$

$$W = W_0 + \sum_m A_m e^{-a_m T_m} + W_{\text{PQ}}(X_i) + W_{\text{MSSM}}(\Phi_\alpha)$$

$$(\{T_M\} = \{T_m, T_u\}, \quad \{X_i\} = \text{PQ sector matter}, \quad \{\Phi_\alpha\} = \text{MSSM matter})$$

A particularly attractive feature of this setup is that much of low energy physics is independent of the detailed form of K_0 and Y_I ($I = i, \alpha$) as long as

* K_0 is of no-scale form:

$$K^{M\bar{N}} K_M K_{\bar{N}} = 4, \quad (T_M + T_M^*) K_M = -3, \quad K^{M\bar{N}} K_{\bar{N}} = -(T_M + T_M^*),$$

* $Y_I(T_M + T_M^*)$ obey the simple scaling law:

$$Y_I(\lambda(T_M + T_M^*)) = \lambda^{n_I} Y_I(T_M + T_M^*),$$

which are true at leading order in α' and string-loop expansions.

Grimm, Louis (2004); Conlon, Cremades, Quevedo (2007)

Moduli stabilization

T_m are stabilized by the NP superpotential,

$$\Delta W_{\text{NP}} = \sum_m A_m e^{-a_m T_m},$$

while $t_u = T_u + T_u^*$ are stabilized by the uplifting potential

$$V_{\text{lift}} = e^{4A_0} P_0 e^{2K_0(T_M + T_M^*)/3}.$$

* Moduli masses:

$$m_{T_m} \simeq 2m_{3/2} \ln(M_{\text{Pl}}/m_{3/2}), \quad m_{t_u} \simeq \sqrt{2}m_{3/2}, \quad m_{a_u} = 0$$

* Modulino masses:

$$m_{\tilde{T}_m} \simeq 2m_{3/2} \ln(M_{\text{Pl}}/m_{3/2}), \quad m_{\tilde{T}_u} \simeq m_{3/2}$$

* **Universal moduli F -terms** (up to small corrections suppressed by $1/\ln(M_{\text{Pl}}/m_{3/2})$):

$$\frac{F^m}{T_m + T_m^*} = \frac{F^\alpha}{T_\alpha + T_\alpha^*} = \frac{m_{3/2}}{\ln(M_{\text{Pl}}/m_{3/2})}$$

Of course, we also have the SUGRA compensator F -component

$$\frac{F^C}{C_0} \simeq m_{3/2}$$

which is responsible for the anomaly mediated SUSY breaking, and also plays a key role in generating intermediate axion scale in KKLT axiverse scenario.

Generation of an intermediate QCD axion scale

For the PQ sector, we consider a simple example to generate an intermediate axion scale:

$$U(1)_{\text{PQ}} : X_1 \rightarrow e^{i\alpha} X_1, \quad X_2 \rightarrow e^{-3i\alpha} X_2$$

$$W_{\text{PQ}} = \frac{\lambda}{M_{\text{Pl}}} X_1^3 X_2^2$$

X_i get model-dependent moduli and anomaly mediated soft masses of $\mathcal{O}(m_{3/2}/8\pi^2)$, however X_i are stabilized mostly by an interplay between the model-independent compensator-mediated A term with $A \simeq m_{3/2}$ and the Planck-scale suppressed F -term potentials:

$$\begin{aligned} V_{\text{PQ}} &\simeq \sum_i \left| \frac{\partial W_{\text{PQ}}}{\partial X_i} \right|^2 + \left(\frac{\lambda m_{3/2}}{M_{\text{Pl}}} X_1^3 X_2^2 + \text{c.c.} \right) \\ \Rightarrow |X_1|^2 &\simeq 3|X_2|^2 \simeq \frac{1}{3\sqrt{3}\lambda} m_{3/2} M_{\text{Pl}}, \\ \frac{F^{X_1}}{X_1} &\simeq \frac{F^{X_2}}{X_2} \simeq -\frac{2}{3} m_{3/2}. \end{aligned}$$

$$\Rightarrow \text{Intermediate scale QCD axion with } f_{a_{\text{QCD}}} \sim \sqrt{m_{3/2} M_{\text{Pl}}} \sim 10^{11} \text{ GeV.}$$

MSSM soft terms:

* Anomaly mediation:

$$m_{\text{soft}}^{\text{anomaly}} \sim \frac{1}{8\pi^2} \frac{F^C}{C_0} \sim \frac{m_{3/2}}{8\pi^2}$$

* Moduli mediation:

$$m_{\text{soft}}^{\text{moduli}} \sim \frac{F^{T_M}}{T_M} \sim \frac{m_{3/2}}{\ln(M_{\text{Pl}}/m_{3/2})}$$

* Gauge mediation if some of the PQ breaking fields X_i couple to the gauge charged messengers $\Phi + \Phi^c$:

$$\Delta W = \kappa X_i \Phi \Phi^c \quad \rightarrow \quad m_{\text{soft}}^{\text{gauge}} \sim \frac{1}{8\pi^2} \frac{F^{X_i}}{X_i} \sim \frac{m_{3/2}}{8\pi^2}$$

These three mediations give comparable contribution to soft masses, so the KKLT axiverse scenario gives rise to mixed-moduli-anomaly-gauge mediation (= deflected mirage mediation) yielding a quite distinctive pattern of sparticle masses. [KC,Jeong,Okumura,Nakamura,Yamaguchi\(2009\)](#)

As the moduli F -components are universal,

$$\frac{F^{T_M}}{T_M + T_M^*} = \frac{m_{3/2}}{\ln(M_{\text{Pl}}/m_{3/2})} \equiv M_0,$$

we don't need to know the explicit moduli-dependence of f_a and Y_α to determine the moduli-mediated soft masses, but the information on the scaling weights is enough:

$$\begin{aligned} M_a^{\text{moduli}} &= F^{T_M} \partial_{T_M} \ln \text{Re}(f_a) = M_0 \\ A_{\alpha\beta\gamma}^{\text{moduli}} &= -F^{T_M} \partial_{T_M} \ln \left(\frac{\lambda_{\alpha\beta\gamma}}{Y_\alpha Y_\beta Y_\gamma} \right) = (n_\alpha + n_\beta + n_\gamma) M_0 \\ m_\alpha^{2\text{moduli}} &= -F^{T_M} F^{T_N^*} \partial_{T_M} \partial_{T_N^*} \ln Y_\alpha = n_\alpha M_0^2 \end{aligned}$$

$$\begin{aligned} K &= K_0(T_M + T_M^*) + Z_\alpha(T_M + T_M^*) \Phi_\alpha^* \Phi_\alpha + \dots \\ \Omega &= -3e^{-K/3} = -3e^{-K_0/3} + Y_\alpha \Phi_\alpha^* \Phi_\alpha + \dots \\ W &= W_0(T_m) + \frac{1}{6} \lambda_{\alpha\beta\gamma} \Phi_\alpha \Phi_\beta \Phi_\gamma + \dots \end{aligned}$$

$\lambda_{\alpha\beta\gamma}$ = moduli-independent holomorphic Yukawa couplings

$$Y_\alpha(T_M + T_M^*) = e^{-K_0(T_M + T_M^*)/3} Z_\alpha(T_M + T_M^*)$$

Since the scaling weights n_α are typically flavor-blind, we can assure that soft masses are flavor-universal, even without knowing the explicit form of the matter Kähler metric $Z_\alpha(T_M + T_M^*)$

Conclusions

- * String theory provides the best theoretical framework to realize the axion solution to the strong CP problem.
- * Typical string theory axion has a decay constant \sim GUT scale, so we may need a mechanism to lower the QCD axion scale down to the intermediate scale $\sim 10^{9-11}$ GeV, while keeping $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV.
- * Compactification with anomalous $U(1)$ gauge symmetry can provide such a mechanism, generating an intermediate QCD axion scale $f_a \sim \sqrt{m_{3/2} M_{\text{Pl}}}$ by an interplay between SUSY breaking effects and Planck-scale-suppressed effects.
- * A simple and plausible generalization of KKLT setup is proposed to accommodate an intermediate scale QCD axion as well as ultralight GUT scale axions for axiverse.
- * Such KKLT axiverse scenario gives rise to the deflected mirage mediation of SUSY breaking, yielding a quite distinctive pattern of soft masses.