Renormalizable 4D Quantum Gravity and Cosmological Constant Problem

K. Hamada

http://research.kek.jp/people/hamada/

Reference

Renormalizable 4D Quantum Gravity as a Perturbed Theory from CFT, arXiv:0907.3969[hep-th].
Recent developments for $m > M_P$

- Resolve the problem of space-time singularities
  - At $E > M_P$, particle excitations become black holes
    - give up graviton picture
  - I proposed non-perturbative CFT approach

- Understand space-time dynamics of the early universe
  - I proposed Inflation scenario without a scalar field: $a = e^{M_P \tau}$
    - New scale $\Lambda_{QG} \simeq 10^{17}$ GeV $\Rightarrow$ space-time transition
    - Number of e-foldings $\Rightarrow N_e \simeq M_P / \Lambda_{QG}$
    - Initial condition of the universe $\Rightarrow$ CFT spectra
    - Amplitude reduces during inflation $\Rightarrow \delta R / R \simeq (\Lambda_{QG} / M_P)^2$

Physical quantities given by ratios of two gravitational scales
Evolution of fluctuation

K.H., Horata, Yukawa, arXiv:0908.0192

Planck phenomena (CFT) ➔ space-time transition (big bang) ➔ today (CMB)

Scale-inv. spectrum at Planck time

Inflation era

From Planck length to cosmological distance

\[ 10^{59} = 10^{30} + 10^{29} \]

inflation  Friedmann

Spectrum at transition point = almost HZ spectrum

Amplitude decreases during inflation ➔ Resolve the flatness problem

\[ \Lambda_{QG} \simeq 10^{17} \text{ GeV} \]
The Aim of This Talk

- Explain small cosmological constant in the context of quantum gravity dynamics
Renormalizable 4D Quantum Gravity as a Perturbed Theory from CFT
Renormalizable Quantum Gravity

The Action (Weyl + Euler + Einstein)

\[ I = \int d^4x \sqrt{-g} \left\{ -\frac{1}{t^2} C_{\mu\nu\lambda\sigma}^2 - b G_4 + \frac{1}{\hbar} \left( \frac{1}{16\pi G} R - \Lambda + \mathcal{L}_M \right) \right\} \]

- conformally invariant (no \( R^2 \))
- Planck constant

“t” is a **unique** dimensionless gravitational coupling constant indicating asymptotic freedom

At high energies \( t \to 0 \), \( C_{\mu\nu\lambda\sigma} \to 0 \) conformally flat

(Perturbation is defined about this config.)

“b” is not independent coupling, which is expanded by t

At \( \hbar \to 0 \) the Einstein action dominates
Conformal Symmetry is “Gauge” Symmetry in Quantum Gravity

Metric field is expanded about \( C_{\mu\nu\lambda\sigma} = 0 \)

\[
g_{\mu\nu} = e^{2\phi}(\mathring{g}_{\mu\nu} + t h_{\mu\nu} + \cdots)
\]

No coupling const.

Diffeomorphism includes conformal transformation at UV limit (\( t \to 0 \))

\[
\delta_{\zeta} g_{\mu\nu} = g_{\mu\lambda} \nabla_{\nu} \zeta^\lambda + g_{\nu\lambda} \nabla_{\mu} \zeta^\lambda
\]

with

\[
\nabla_{\mu} \zeta_{\nu} + \nabla_{\nu} \zeta_{\mu} - \frac{1}{2} \mathring{g}_{\mu\nu} \nabla_{\lambda} \zeta^\lambda = 0
\]

\[
\delta_{\zeta} \phi = \zeta^{\lambda} \mathring{\nabla}_{\lambda} \phi + \frac{1}{4} \mathring{\nabla}_{\lambda} \zeta^{\lambda}
\]

\[
\delta_{\zeta} h_{\mu\nu} = \zeta^{\lambda} \mathring{\nabla}_{\lambda} h_{\mu\nu} + \frac{1}{2} h_{\mu\lambda} \left( \mathring{\nabla}_{\nu} \zeta^{\lambda} - \mathring{\nabla}^{\lambda} \zeta_{\nu} \right) + \frac{1}{2} h_{\nu\lambda} \left( \mathring{\nabla}_{\mu} \zeta^{\lambda} - \mathring{\nabla}^{\lambda} \zeta_{\mu} \right)
\]

(\# other gauge d.o.f. are gauge-fixed in usual way)

\[
\delta_{\kappa} h_{\mu\nu} = \mathring{\nabla}_{\mu} \kappa_{\nu} + \mathring{\nabla}_{\nu} \kappa_{\mu} - \frac{1}{2} \mathring{g}_{\mu\nu} \mathring{\nabla}_{\lambda} \kappa^{\lambda}
\]

(cf. Field indep.)
Induced Riegert-Wess-Zumino Action

\[ Z = \int [dg \cdots]_g \exp (iI) \]
\[ = \int [d\phi dh \cdots]_g \exp (iS(\phi) + iI) \]

Practical measure defined on the background

Jacobian to preserve diff. inv.

= WZ action for conformal anomaly

Lowest term of \( S (= \text{Riegert action}) \) is coupling-independent

\[ S_1(\phi, \bar{g}) = -\frac{b_1}{(4\pi)^2} \int d^4x \int_0^\phi d\phi \sqrt{-g} E_4 \]
\[ = -\frac{b_1}{(4\pi)^2} \int d^4x \sqrt{-\bar{g}} \left( 2\phi \Delta_4 \phi + \bar{E}_4 \phi \right) \]

\[ E_4 = G_4 - \frac{2}{3} \nabla^2 R \]

Dynamics of conformal mode is induced from the measure

\[ b_1 = \frac{(N_X + 11N_D + 62N_A)}{360} + \frac{769}{180} \]
The Perturbation about CFT

This model:

- CFT + perturbations (by single coupling “t”)
  - Riegert + Weyl
  - Non-perturbative (conformal mode is treated exactly)
  - No graviton

cf. Early 4-derivative models in 1970’s

- Free + perturbations (by two couplings)
  - R^2 + Weyl
  - Perturbative (all modes are treated in perturbation)
  - Graviton picture
Dimensional Regularization

Bare action $\leftrightarrow$ D-dimensional WZ integrability coupled with QED

$$I = \int d^D x \sqrt{-g} \left\{ \frac{1}{t^2} C^2_{\mu\nu\lambda\sigma} + b G_D + \frac{1}{4} F^2_{\mu\nu} + \sum_{j=1}^{n_F} i \bar{\psi}_j D \psi_j - \frac{M_P^2}{2} R + \Lambda \right\}$$

$$C^2_{\mu\nu\lambda\sigma} = R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} - \frac{4}{D - 2} R_{\mu\nu} R^{\mu\nu} + \frac{2}{(D - 1)(D - 2)} R^2$$

$$G_D = G_4 + \frac{(D - 3)^2(D - 4)}{(D - 1)^2(D - 2)} R^2$$

Renormalization factors

$Z_\phi = 1$  
conformal mode is not renormalized 
because this mode has no its own coupling constant

$A_\mu = Z_3^{1/2} A_\mu^r$, $\psi_j = Z_2^{1/2} \psi_j^r$, $h_{\mu\nu} = Z_h^{1/2} h_{\mu\nu}^r$

$e = Z_e e_r$, $t = Z_t t_r$  
($Z_e = Z_3^{-1/2}$)

Ward-Takahashi identity

Ambiguity is fixed!
**Induced Interactions**

\[ Z_\phi = 1 \quad \text{Residues are functions} \]

\[ Z_3 = 1 + \frac{x_1}{D-4} + \frac{x_2}{(D-4)^2} + \cdots \]

Bare action \( \Rightarrow \) vertices and counterterms

\[
\frac{1}{4} \int d^D x \sqrt{g} F_{\mu \nu} F^{\mu \nu} \\
= \frac{1}{4} Z_3 \int d^D x e^{(D-4)\phi} F_{\mu \nu}^r F_{\lambda \sigma}^r g^{\mu \lambda} g^{\nu \sigma} \\
= \frac{1}{4} \int d^D x \left\{ \left( \frac{x_1}{D-4} + \frac{x_2}{(D-4)^2} + \cdots \right) F_{\mu \nu}^r F_{\lambda \sigma}^r g^{\mu \lambda} g^{\nu \sigma} \\
+ \left( D-4 + \frac{x_1}{D-4} + \frac{x_2}{D-4} + \cdots \right) \phi F_{\mu \nu}^r F_{\lambda \sigma}^r g^{\mu \lambda} g^{\nu \sigma} \\
+ \frac{1}{2} \left( (D-4)^2 + (D-4)x_1 + x_2 + \cdots \right) \phi^2 F_{\mu \nu}^r F_{\lambda \sigma}^r g^{\mu \lambda} g^{\nu \sigma} \\
+ \cdots \right\}
\]

Bare Weyl action

\[
-\frac{1}{t^2} \int d^D x \sqrt{g} C_{\mu \nu \lambda \sigma}^2 = \frac{1}{t^2} \int d^D x \sqrt{\bar{g}} e^{(D-4)\phi} \bar{C}_{\mu \nu \lambda \sigma}^2
\]

Wess-Zumino action for conformal anomaly
Laurent expansion of $b$

$$b = \frac{1}{(4\pi)^2} \sum_{n=1}^{\infty} \frac{b_n}{(D - 4)^n}$$

Euler term

$$b \int d^D x \sqrt{g} G_D$$

$$= \frac{1}{(4\pi)^2} \int d^D x \left\{ \left( \frac{b_1}{D - 4} + \frac{b_2}{(D - 4)^2} + \cdots \right) \bar{G}_4 + \left( b_1 + \frac{b_2}{D - 4} + \cdots \right) \left( 2\phi \Delta_4 \phi + \bar{E}_4 \phi + \frac{1}{18} \bar{R}^2 \right) + \frac{1}{2} \left( (D - 4)b_1 + b_2 + \cdots \right) \left( 2\phi^2 \Delta_4 \phi + \bar{E}_4 \phi^2 + \cdots \right) + \cdots \right\}$$

Kinetic term (Riegert action) is induced

$$b_1(t_r, e_r) = b_1 + b'_1(t_r, e_r)$$

Positive constant

$$b_1 = \frac{11N_F}{360} + \frac{40}{9}$$

$\left\{ \text{counterterms} \right\}$

$\left\{ \text{new WZ actions and new counterterms} \right\}$

Dynamics of conformal mode is induced
Beta functions

\[ \beta_t = -\left( \frac{n_F}{40} + \frac{10}{3} \right) \frac{t_r^3}{(4\pi)^2} - \frac{7n_F}{72} \frac{e_r^2 t_r^3}{(4\pi)^4} + o(t_r^5) \]

\[ \beta_e = \frac{4n_F}{3} \frac{e_r^3}{(4\pi)^2} + \left( 4n_F - \frac{8n_F^2}{9b_1} \right) \frac{e_r^5}{(4\pi)^4} + o(e_r^3 t_r^2) \]

Negative for \( n_F > 24 \)

Residues \( b_n \) [ \( b_1(t_r, e_r) = b_1 + b'_1(t_r, e_r) \) ]

\[ b_1 = \frac{11n_F}{360} + \frac{40}{9}, \quad b'_1 = -\frac{n_F^2}{6} \frac{e_r^4}{(4\pi)^4} + o(t_r^2), \]

\[ b_2 = \frac{2n_F^3}{9} \frac{e_r^6}{(4\pi)^6} + o(t_r^4) \]


( corrections from diagrams with internal gravitational lines)

Non-renormalization of Conformal Mode \( Z_\phi = 1 \)

\[
\frac{2b_1}{(4\pi)^2} k^4 \left[ -3 \frac{t_r^2}{(4\pi)^2} \left( \frac{1}{\tilde{\epsilon}} - \log \frac{z^2}{\mu^2} + \frac{7}{6} \right) \right] + \frac{2b_1}{(4\pi)^2} k^4 \left[ 3 \frac{t_r^2}{(4\pi)^2} \left( \frac{1}{\tilde{\epsilon}} - \log \frac{z^2}{\mu^2} + \frac{7}{12} \right) \right]
\]

\( z \): infinitesimal fictitious mass (IR regularization)  
Not gauge invariant \( \Rightarrow \) cancel out!

Propagator \( 1/k^4 \rightarrow 1/(k^2 + z^2)^2 \)

[Remark: Einstein action cannot be considered as the mass term due to the existence of exponential factor of conformal mode]
The $e^4$ order correction

\[ \epsilon = \frac{(4 - D)}{2} \]

= UV finite ( $Z_\phi = 1$ )

Two-point functions at $e^6$ and $e^6/b_1$ have been checked.

It was also checked that vertex functions ( $\phi F_{\mu\nu}^2$ ) at $e^6$ and $e^6/b_1$ are renormalized by the condition $Z_\phi = 1$.

Running Coupling Constant

$$\Gamma_W = \left\{ \frac{1}{t^2_r} - 2\beta_0 \phi + \beta_0 \log \left( \frac{k^2}{\mu^2} \right) \right\} \tilde{C}^{r_2}_{\mu \nu \lambda \sigma}$$

$$\beta_t = - \beta_0 t_r^3$$

where

$$\tilde{t}_r^2(p) = \frac{1}{\beta_0 \log(p^2/\Lambda_{QG}^2)}$$

$k$: comoving momentum defined on the flat background

Asymptotic Freedom

Physical momentum: $p^2 = k^2/a^2$ with $a = e^\phi$

(# Conf. anomaly is necessary to preserve diff. inv.)

New IR scale breaking conformal invariance:

$$\Lambda_{QG} = \mu \exp(-1/2\beta_0 t_r^2)$$
Renormalization of Cosmological Constants
Renormalization of Cosmological Constant at Large $b_1$

Lower derivative gravitational actions

$$I_{\text{low}} = \int d^D x \left\{ Z_\Lambda \Lambda_r e^{D\phi} - \frac{1}{2} Z_{\text{EH}} M_P^2 e^{(D-2)\phi} \left( \partial^2 \phi + \cdots \right) \right\}$$

(expand in $\epsilon = (4 - D)/2$ and $Z_\phi = 1$)

\[ e^{A\phi} = \sum_n \frac{4^n}{n!} \phi^n \]

Anomalous dim. \[ \gamma_\Lambda = \frac{4}{b_1} + \frac{8}{b_1^2} - \frac{18\pi^2 M_P^4}{b_1^2 \Lambda_r} + o(b_1^{-3} , t_r^2) \]

(and also $\gamma_{\text{EH}} = \frac{1}{b_1} + \frac{1}{b_1^2} + o(1/b_1^3 , t_r^2)$)

See reference
Effective Cosmological Constant

Introduce constant background by the shift: \( \phi \rightarrow \phi + \sigma \)

Large b\_1 approx. \( \Rightarrow \) one-loop approximation becomes good

\[
V_{\text{loop}} = \sum \Lambda_r e^{4\sigma} e^{2\sigma} = \frac{1}{2} \int \frac{d^D k}{(2\pi)^D} \log \left\{ 1 - \frac{24\pi^2}{b_1} M_P^2 e^{2\sigma} \frac{1}{k^2} + \frac{64\pi^2}{b_1} \Lambda_r e^{4\sigma} \frac{1}{k^4} \right\}
\]

Effective potential

\[
V = e^{4\sigma} \left\{ \Lambda_r + 4\sigma \left( \frac{\Lambda_r}{b_1} - \frac{9\pi^2}{2} \frac{M_P^4}{b_1^2} \right) + \left( \frac{\Lambda_r}{b_1} - \frac{9\pi^2}{2} \frac{M_P^4}{b_1^2} \right) 3 - \log \left( \frac{\Lambda_r}{b_1} e^{4\sigma} \right) \right\}
\]

\[
-6\pi \frac{M_P^2}{b_1} \sqrt{\frac{\Lambda_r}{b_1} - \frac{9\pi^2}{4} \frac{M_P^4}{b_1^2}} \arccos \left( \frac{3\pi}{2} \frac{M_P^2 / b_1}{\sqrt{\Lambda_r / b_1}} \right) \}
\]

(# IR divergences cancel out)
Running Mass Constants

Renormalization group equation (RGE)

\[-\frac{d}{d\sigma} \tilde{t}_r = \beta_t (\tilde{t}_r) \]

\[-\frac{d}{d\sigma} \Lambda = - \left[ 4 + \gamma_\Lambda (\tilde{t}_r, \Lambda, \overline{M}_P^2) \right] \Lambda \]

\[-\frac{d}{d\sigma} \overline{M}_P^2 = - \left[ 2 + \gamma_{EH} (\tilde{t}_r) \right] \overline{M}_P^2 \]

Solutions of RGE

\[\overline{\Lambda}(\sigma = 0) = \Lambda_r, \quad \overline{M}_P(\sigma = 0) = M_P^0\]

\[\gamma_\Lambda = \frac{4}{b_1} - \frac{18\pi^2}{b_1^2} \frac{M_P^4}{\Lambda_r}, \quad \gamma_{EH} = \frac{1}{b_1}\]

\[\overline{\Lambda}(\sigma) = e^{4\sigma} \left\{ \Lambda_r + 4 \left( \frac{\Lambda_r}{b_1} - \frac{9\pi^2}{2} \frac{M_P^4}{b_1^2} \right) \sigma + \cdots \right\} \]

\[\overline{M}_P^2(\sigma) = M_P^2 e^{2\sigma} \left( 1 + \frac{1}{b_1} \sigma + \cdots \right)\]
Renormalization Group

UV limit: \( \sigma \to -\infty \)  
IR limit: \( \sigma \to \infty \)

If \( \frac{\Lambda_r}{b_1} - \frac{9\pi^2}{2} \frac{M_P^4}{b_1^2} < 0 \)

Small cosmological constant \( e^{-4\sigma} \Lambda(\sigma) \to 0 \)

\( \Lambda(0) = M_P^2(0) = 1 \)
\( b_1 = 10 \)

GUTs
Renormalization Group Improvement

Solution to ’t Hooft-Weinberg equation

\[ V \left( \Lambda(\sigma), \overline{M}_P^2(\sigma), \mu \right) = e^{4\sigma} V \left( \Lambda_r, \overline{M}_P^2, \mu \right) \]

RG improved effective cosmological constant

\[
V = \overline{\Lambda}(\sigma) + \left( \frac{\overline{\Lambda}(\sigma)}{b_1} - \frac{9\pi^2}{2} \frac{\overline{M}_P^4(\sigma)}{b_1^2} \right) \left\{ 3 - \log \left( \frac{(8\pi)^2 \overline{\Lambda}(\sigma)}{\mu^4} \right) \right\} \\
-6\pi \frac{\overline{M}_P^2(\sigma)}{b_1} \sqrt{\frac{\overline{\Lambda}(\sigma)}{b_1} - \frac{9\pi^2}{4} \frac{\overline{M}_P^4(\sigma)}{b_1^2}} \arccos \left( \frac{3\pi \overline{M}_P^2(\sigma)/b_1}{2\sqrt{\overline{\Lambda}(\sigma)/b_1}} \right)
\]

Can analytically continue to >1 using \( \arccos \omega = i \log(\omega + \sqrt{\omega^2 - 1}) \)
Plot of Effective Potential

\[ b_1 = 10, \quad \mu = 3 \]

\[ \overline{\Lambda}(\sigma) = 0 \]

\[ V = \frac{9\pi^2}{2b_1^2} \overline{M}_P^4 \left[ -3 + \log \left( \frac{24\pi^2 \overline{M}_P^2}{b_1 \mu^2} \right)^2 \right] \]

\( V \) is real \( \Rightarrow \overline{\Lambda}(\sigma), \overline{M}_P^2(\sigma) \geq 0 \)
Conclusion and Discussion
I investigated the cosmological constant problem in the context of renormalizable 4D quantum gravity formulated as a perturbed theory from CFT.

Renormalization of the cosmological constant was carried out.

I studied the RG equation and found a solution that running cosmological constant becomes small in IR limit.

RG improved effective cosmological constant was calculated.
Non-perturbative effects by gravitational instantons

- study of self-dual Weyl configurations

Gravitational solitons

- If unstable (=graviballs) ➔ dynamics of space-time transition, baryogenesis, …

- If stable ➔ candidate for cold dark matter

Supersymmetrization of Riegert action