PADN- 446 (PADN446laddersimV1.doc)
Begun to write since Oct. 26th. (Reformed on . Revised on), 2009.
※注：20091026 時点においては、未だ個人見解レベルの検討書です。
Particle Accelerator Development Note

パルス伝送方程式の解
～ No.46: ラダー回路の計算例（式の簡素化） ～

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要約
分布定数型負荷の場合、ステージ数が多くなると解析的に解きにくく、シミュレーターに頼りがちになる。しかしながら、式の構築は実際のところさほど難解ではなく、ある程度整った形で記述でき、簡単な差分方程式で解くことができる。ここでは、その一例を紹介する。

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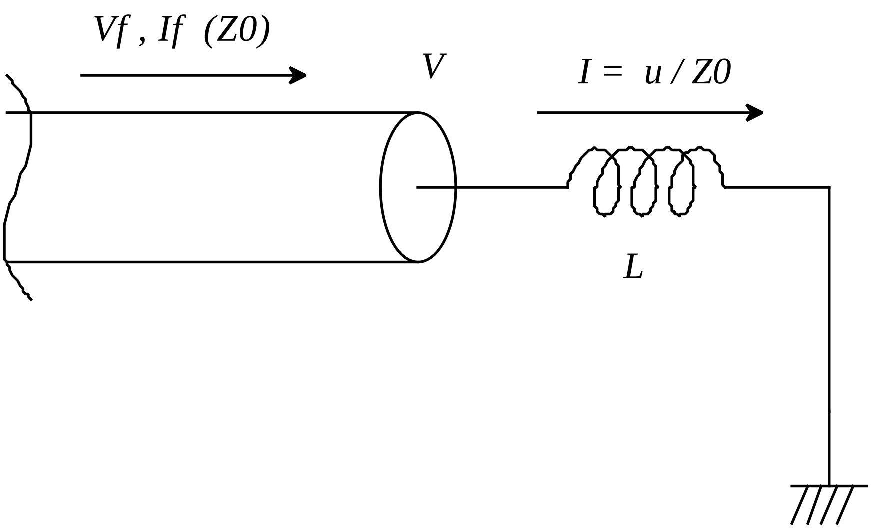
(1) Simple Excitation

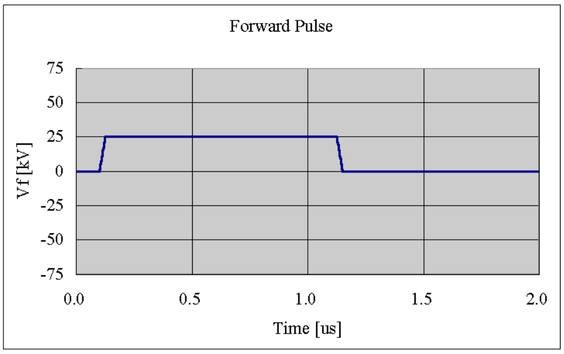


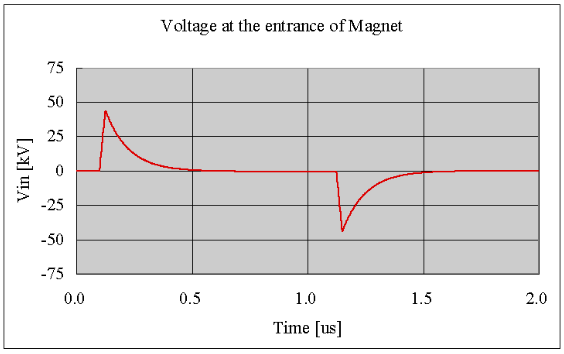
$$2\ V_f = V + Z_0\ I$$

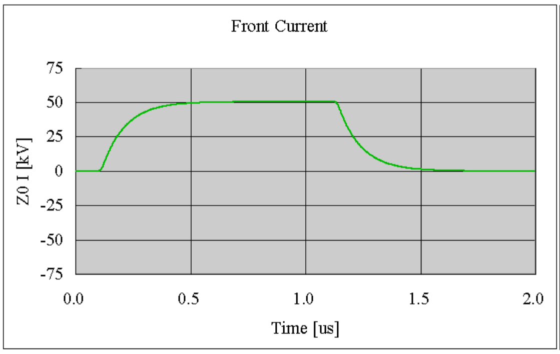
$$(1)\quad V = \frac{\partial \Phi}{\partial t} = L\ \frac{\partial I}{\partial t} \equiv L\ I'\quad ;\quad u \equiv Z_0\ I\quad and\quad \tau \equiv \frac{L}{Z_0}$$

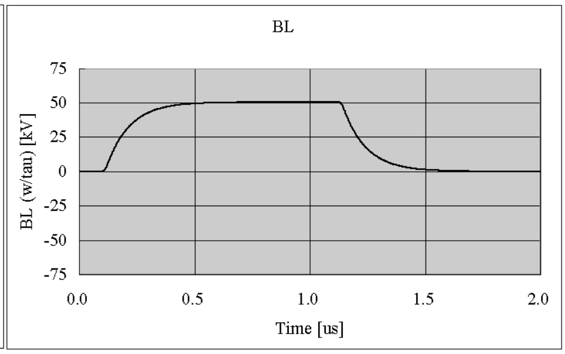
$$2\ V_f = L\ I' + Z_0\ I = L\ I' + Z_0\ I = \tau\ u' + u = \tau\ (u\ e^{\frac{t}{\tau}})'\ e^{-\frac{t}{\tau}}$$

$$\Rightarrow\quad u = \frac{2}{\tau}\ e^{-\frac{t}{\tau}} \int_0^t V_f\ e^{\frac{t}{\tau}}\ dt\quad and\quad V = \tau\ u' = 2\ V_f - u$$

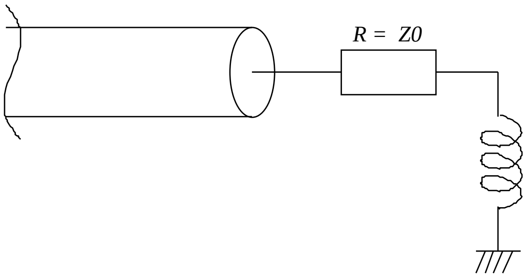

Forward Pulse
Vf [kV]
75
50
25
0
-25
-50
-75
0.0
0.5
1.0
1.5
2.0
Time [us]


Voltage at the entrance of Magnet
Vin [kV]
75
50
25
0
-25
-50
-75
0.0
0.5
1.0
1.5
2.0
Time [us]

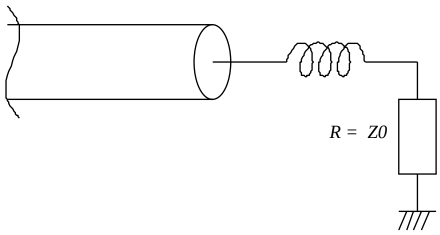

Front Current
Z0 I [kV]
75
50
25
0
-25
-50
-75
0.0
0.5
1.0
1.5
2.0
Time [us]


BL
BL (w/tau) [kV]
75
50
25
0
-25
-50
-75
0.0
0.5
1.0
1.5
2.0
Time [us]

(2) L-R case



or

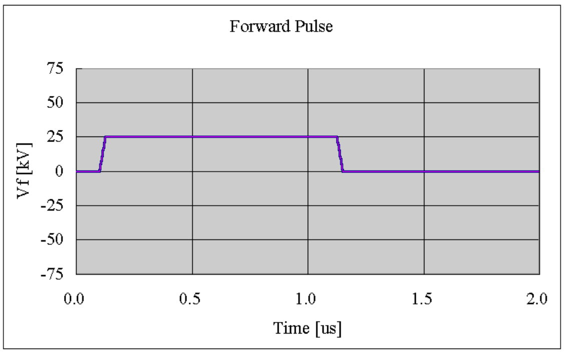


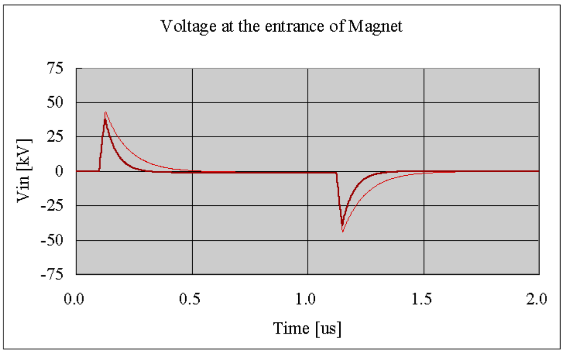
$$2\ V_f = V + Z_0\ I$$

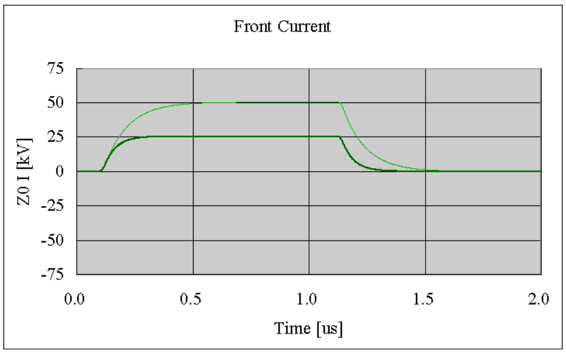
$$(2)\quad V = \frac{\partial \Phi}{\partial t} + R\ I = L\ I' + Z_0\ I \quad ; \quad u \equiv Z_0\ I \quad and \quad \tau \equiv \frac{L}{Z_0}$$

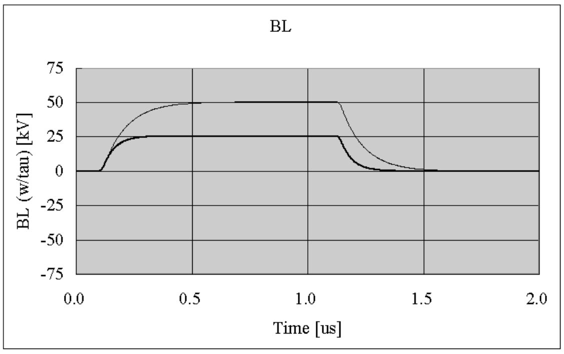
$$2\ V_f = \tau\ u' + 2\ u = \tau\ (u\ e^{\frac{2t}{\tau}})'\ e^{-\frac{2t}{\tau}}$$

$$\Rightarrow \quad u = \frac{2}{\tau}\ e^{-\frac{2t}{\tau}} \int_0^t V_f\ e^{\frac{2t}{\tau}}\ dt \quad and \quad V = \tau\ u' = 2\ V_f - 2\ u$$

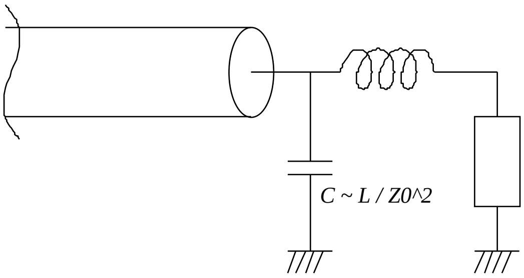








(3) One-stage L//C-R case

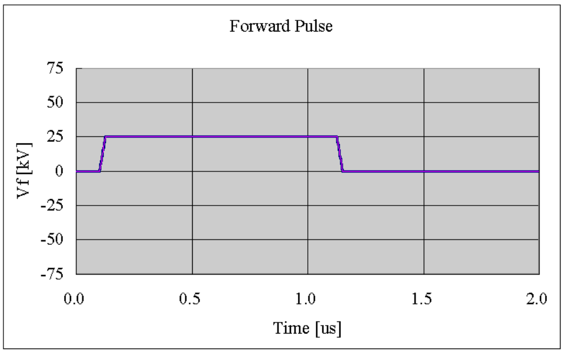


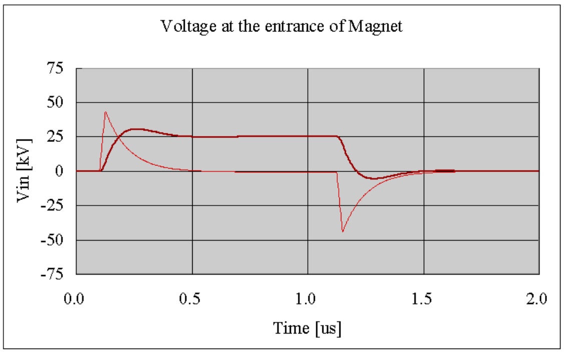
$$(3)\quad V = L\,J' + Z_0\,J = \frac{Q}{C} \quad and \quad I = J + Q' \;;$$

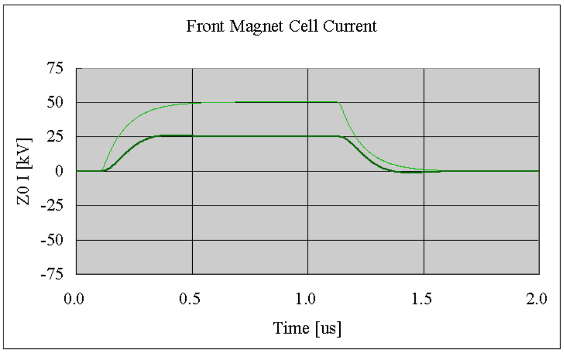
$$u \equiv Z_0\,I\,,\; \omega \equiv \frac{1}{\sqrt{C\,L}} \quad and \quad \tau \equiv \frac{L}{Z_0} \;;$$

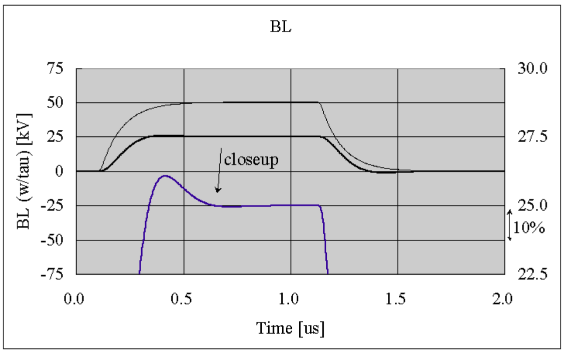
$$Z_0 \equiv \sqrt{\frac{L}{C}} \qquad \omega = \frac{Z_0}{L} = \frac{1}{\tau} \qquad C\,Z_0 = \frac{L}{Z_0} = \tau$$

$$\begin{aligned} 2\,V_f &= L\,J' + Z_0\,J + Z_0\,(J + Q') \\ &= L\,J' + 2\,Z_0\,J + Z_0\,(C\,L\,J' + C\,Z_0\,J)' \\ &= Z_0\,C\,L\,J'' + (L + C\,Z_0^2)\,J' + 2\,Z_0\,J \\ &= C\,L\,u'' + (\frac{L}{Z_0} + C\,Z_0)\,u' + 2\,u \\ &= \tau^2\,u'' + 2\,\tau\,u' + 2\,u \end{aligned}$$

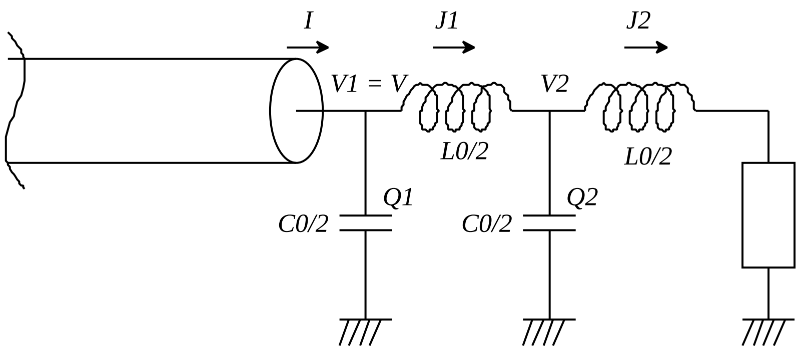








(4) Two-stage (L//C)-(L//C)-R case



$$(4)\quad \begin{cases} 2\ V_f = V + Z_0\ I \\ V = V_1 = L\ J_1{}' + V_2 = \dfrac{Q_1}{C} \\ V_2 = L\ J_2{}' + Z_0\ J_2 = \dfrac{Q_2}{C} \\ I = J_1 + Q_1{}' \\ J_1 = J_2 + Q_2{}' \end{cases}$$

$$u \equiv Z_0\ I,\quad u_1 \equiv Z_0\ J_1,\quad u_2 \equiv Z_0\ J_2,\quad \omega \equiv \frac{1}{\sqrt{C\ L}}\ \text{and}\ \ \tau \equiv \frac{L}{Z_0}\ ;$$

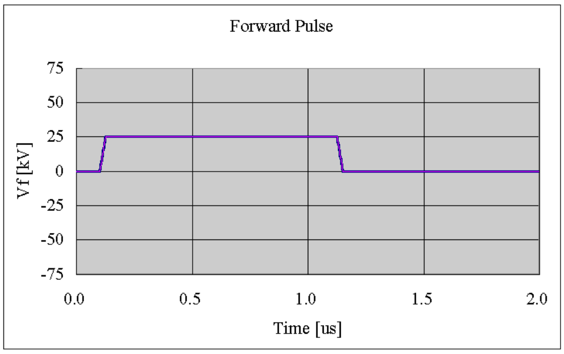
$$Z_0 \equiv \sqrt{\frac{L}{C}}\quad \omega = \frac{Z_0}{L} = \frac{1}{\tau}\quad C\ Z_0 = \frac{L}{Z_0} = \tau$$

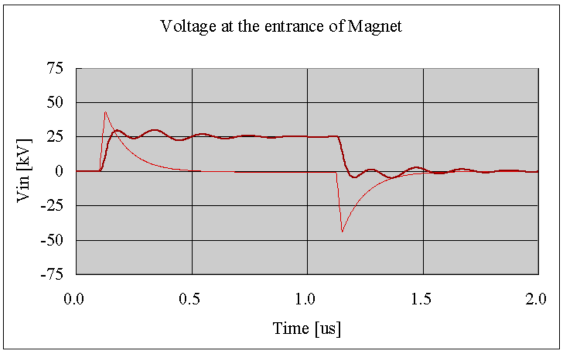
$$\begin{cases} 2\ V_f = V + u \\ V = V_1 = \tau\ u_1{}' + V_2 = \dfrac{Q_1}{C} \\ V_2 = \tau\ u_2{}' + u_2 = \dfrac{Q_2}{C} \\ u = u_1 + \tau\ \dfrac{Q_1}{C}{}' \\ u_1 = u_2 + \tau\ \dfrac{Q_2}{C}{}' \end{cases}$$

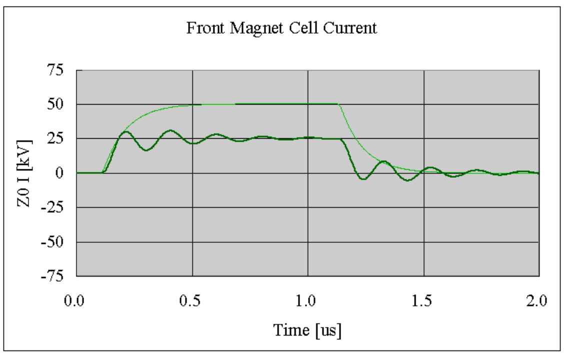
$$\begin{cases} 2\ V_f = V + u \\ V = V_1 = \tau\ u_1{}' + \tau\ u_2{}' + u_2 = \dfrac{Q_1}{C} \\ \tau\ u_2{}' + u_2 = \dfrac{Q_2}{C} \\ u = u_1 + \tau\ \dfrac{Q_1}{C}{}' \\ u_1 = u_2 + \tau\ \dfrac{Q_2}{C}{}' \end{cases}$$

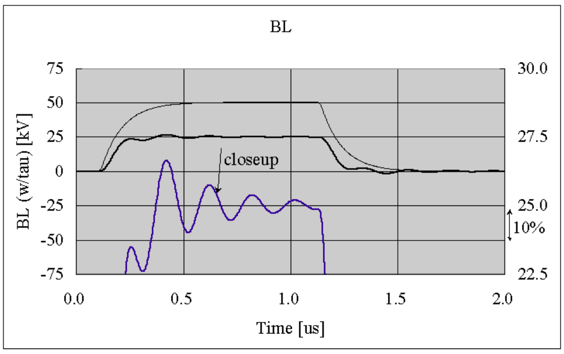
$$\begin{cases} 2\ V_f = \tau\ u_1{}' + \tau\ u_2{}' + u_2 + u \\ u = u_1 + \tau\ (\tau\ u_1{}' + \tau\ u_2{}' + u_2)' = u_1 + \tau^2\ u_1{}'' + \tau\ u_2{}' + \tau^2\ u_2{}'' \\ u_1 = u_2 + \tau\ (\tau\ u_2{}' + u_2)' = u_2 + \tau\ u_2{}' + \tau^2\ u_2{}'' \end{cases}$$

$$\begin{cases} 2\ V_f = \tau\ u_1{}' + \tau\ u_2{}' + u_2 + u \\ u = u_1 + \tau^2\ u_1{}'' + \tau\ u_2{}' + \tau^2\ u_2{}'' \\ u_1 = u_2 + \tau\ u_2{}' + \tau^2\ u_2{}'' \end{cases}$$

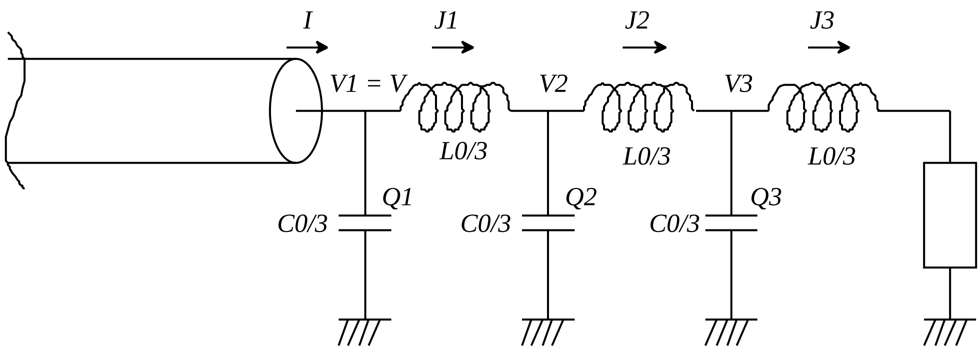








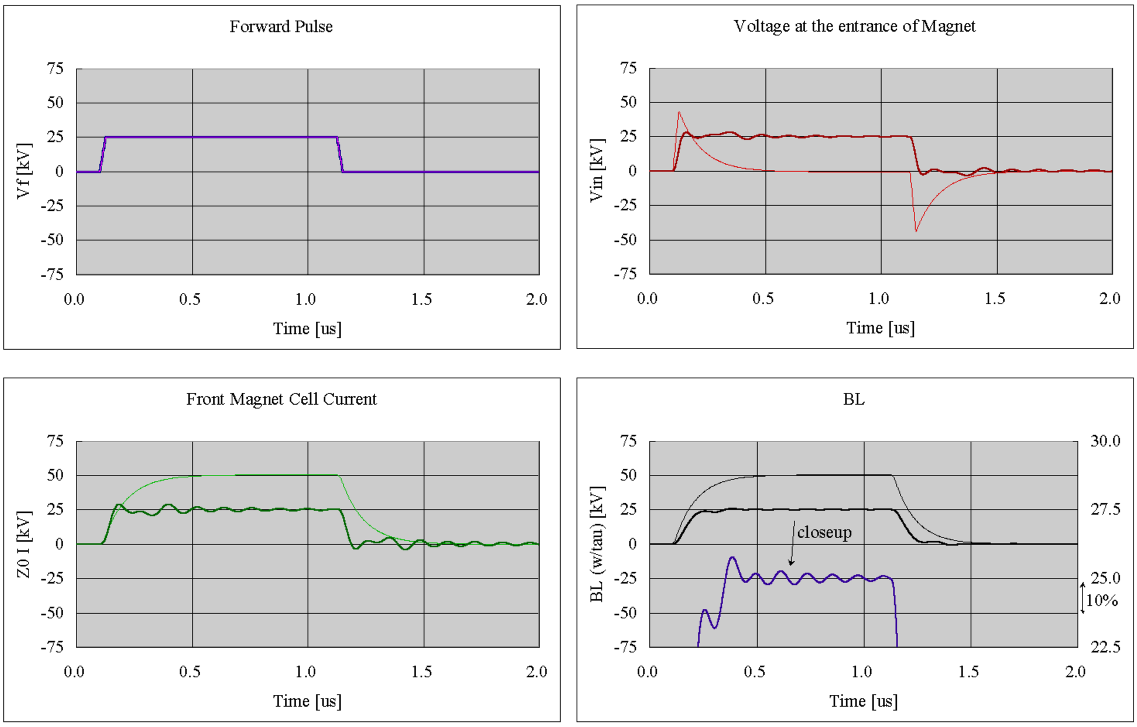
(5) Three-stage (L//C)-(L//C)-(L//C)-R case



$$2\,V_f = 2\,u_1 - u_2 + u_3 + \tau\,(u_1 + u_2 + u_3)' + \tau^2\,u_1''$$

$$\left\{\begin{array}{l} u_1 = 2\,u_2 - u_3 + \tau^2\,u_2'' \\ u_2 = u_3 + \tau\,u_3' + \tau^2\,u_3'' \end{array}\right.$$

$$V_1 = \tau\,(u_1 + u_2 + u_3)' + u_3$$

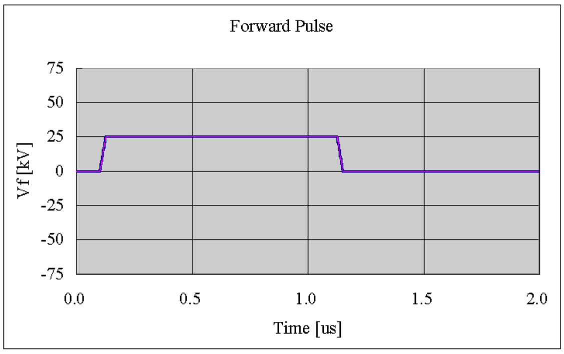


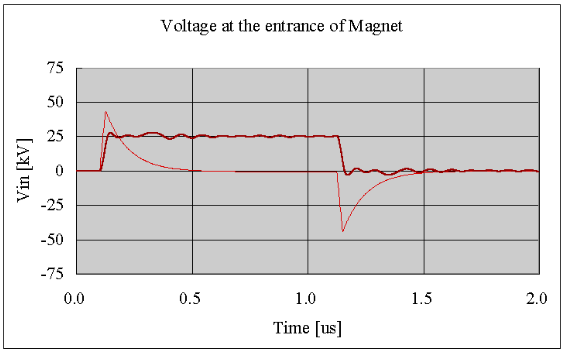
(6) Four-stage (L//C)-(L//C)-(L//C)-(L//C)-R case

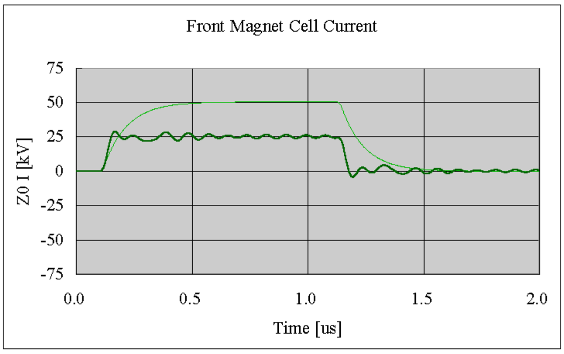
$$2V_f = \tau u_1' + \tau u_2' + \tau u_3' + \tau u_4' + u_4 + 2u_1 - u_2 + \tau^2 u_1''$$

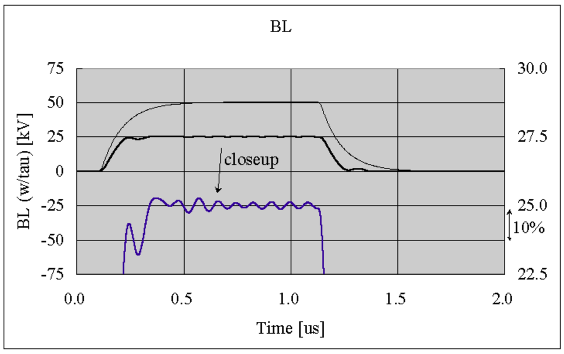
$$\begin{cases} u_1 = 2u_2 - u_3 + \tau^2 u_2'' \\ u_2 = 2u_3 - u_4 + \tau^2 u_3'' \\ u_3 = u_4 + \tau u_4' + \tau^2 u_4'' \end{cases}$$

$$V_1 = \tau u_1' + \tau u_2' + \tau u_3' + \tau u_4' + u_4$$







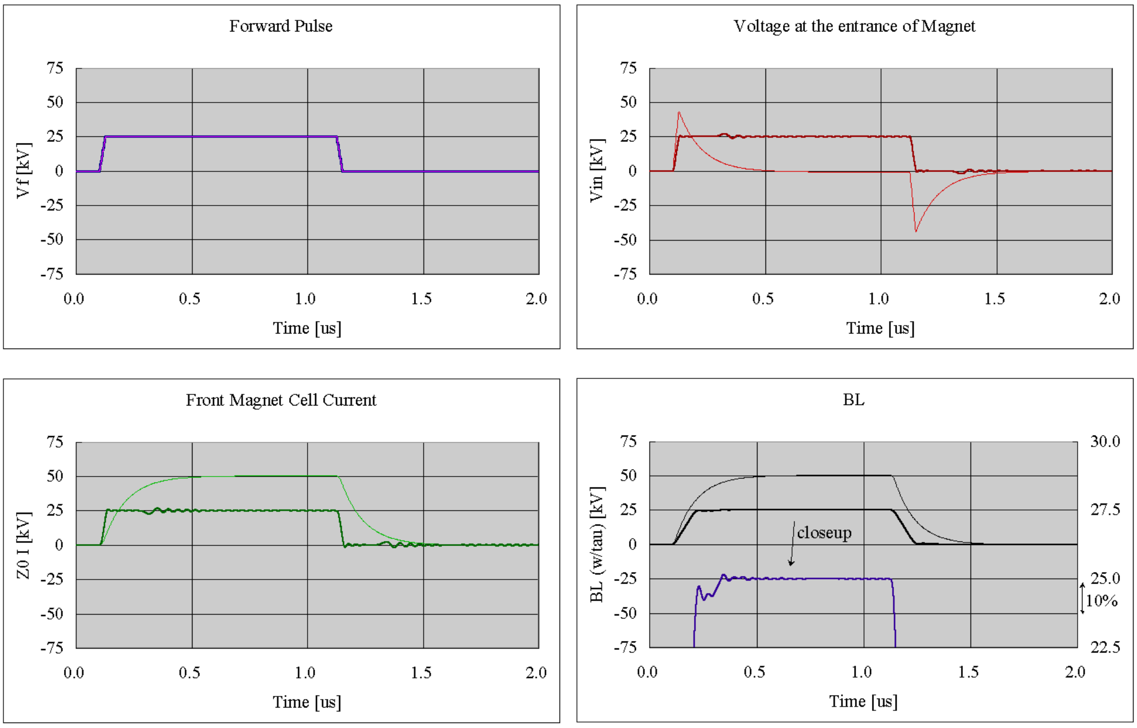


(*)　Multi-stage equations

$$V_1 = \tau \sum_{n=1}^{N} u_n' + u_N$$

$$\begin{cases} \tau^2 u_1'' = 2 V_f - \tau \sum_{n=1}^{N} u_n' - 2 u_1 + u_2 - u_N \\ \tau^2 u_n'' = u_{n-1} - 2 u_n + u_{n+1} \quad \textit{for} \quad 2 \le n \le N-1 \\ \tau^2 u_N'' = u_{N-1} - u_N - \tau u_N' \end{cases}$$

<< N = 10 case >>



(*) 10 stage (C/2)-L-(C/2) ladder case

<< N = 10 case >>