Begun to write since June 15th.　Reformed on June 16th. Revised on July 17th, 2009.

※注：20090717 時点においては、未だ個人見解レベルの検討書です。

Particle　Accelerator　Development　Note

パルス伝送方程式の解
～　No.45: (C//R)(L//R)マッチング　～

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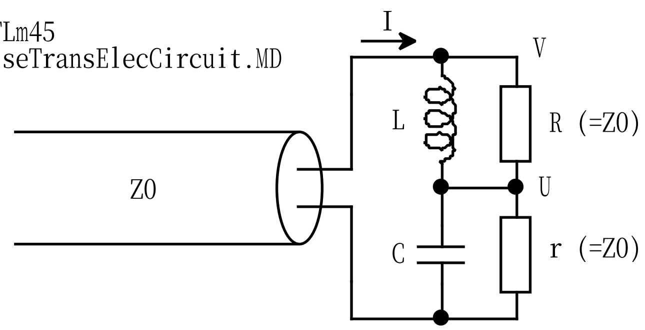
要約

負荷に対するパルス伝送方程式の解をケース別にまとめる作業を実施している。
今回はマッチングを重視した下記回路。完全整合であるが、立上り時間は遅い。

ENTLm45
PulseTransElecCircuit.MD



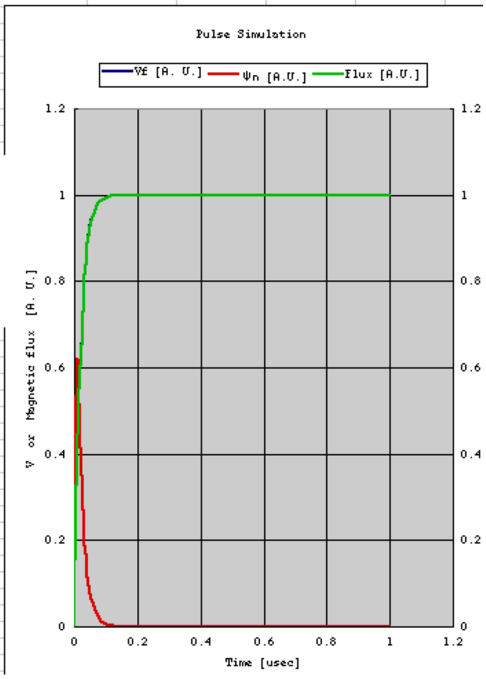
PADN445CRLRv1.xls					
			ramda	0.02	[us]
dt	0.01		Z0	100	[ohm]
t0	0		L	0.01	[uH]
			tau	0.0001	[us]
			ramda tau	0.005	
	T [us]	Vf [A. U.]	Ψn [A.U.]	Flux [A.U.]	
	0	0	0	0	
	0.01	0.39346934	0.60957855	0.390421448	
	0.02	0.63212056	0.36972808	0.630271918	

$$\Psi \equiv V - U = 2\,V_0\,(1 - e^{-\lambda t}) - Z_0\,I - U = \frac{V_0}{1-\tau\lambda}\,(e^{-\lambda t} - e^{-\frac{t}{\tau}})$$

$$\Phi = \int_0^t \Psi\,\mathrm{d}t = \frac{V_0}{1-\tau\lambda}\,\{\frac{1-e^{-\lambda t}}{\lambda} - \tau\,(1 - e^{-\frac{t}{\tau}})\}$$

$$= \frac{V_0}{\lambda} + \frac{V_0}{1-\tau\lambda}\,(\frac{-e^{-\lambda t}}{\lambda} + \tau\,e^{-\frac{t}{\tau}})$$

	T [us]	Vf [A. U.]	Ψn [A.U.]	Flux [A.U.]	
	0.13	0.99849656	0.00151099	0.998489006	
	0.14	0.99908812	0.00091646	0.999083536	
	0.15	0.99944692	0.00055586	0.999444136	
	0.16	0.99966454	0.00033715	0.999662852	
	0.17	0.99979653	0.00020449	0.999795509	
	0.18	0.99987659	0.00012403	0.99987597	
	0.19	0.99992515	7.5228E-05	0.999924772	
	0.2	0.9999546	4.5628E-05	0.999954372	
	0.21	0.99997246	2.7675E-05	0.999972325	
	0.22	0.9999833	1.6786E-05	0.999983214	
	0.23	0.99998987	1.0181E-05	0.999989819	
	0.24	0.99999386	6.1751E-06	0.999993825	
	0.25	0.99999627	3.7454E-06	0.999996255	
	0.26	0.99999774	2.2717E-06	0.999997728	
	0.27	0.99999863	1.3778E-06	0.999998622	
	0.28	0.99999917	8.3571E-07	0.999999164	
	0.29	0.9999995	5.0688E-07	0.999999493	
	0.3	0.99999969	3.0744E-07	0.999999693	
	0.31	0.99999981	1.8647E-07	0.999999814	
	0.32	0.99999989	1.131E-07	0.999999887	
	0.33	0.99999993	6.8599E-08	0.999999931	



(45) (L//R) (C//R) $\left(R \equiv \sqrt{\frac{L}{C}} \ ; \ C = \frac{L}{R^2} \right)$

$$Z = \frac{R\ j\omega L}{R + j\omega L} + \frac{R \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R\ j\omega L\,(R - j\omega L)}{(R + j\omega L)(R - j\omega L)} + \frac{R}{1 + j\omega C R}$$

$$\frac{Z}{R} = \frac{(\omega L)^2 + j\omega L R}{R^2 + (\omega L)^2} + \frac{1 - j\omega C R}{1 + (\omega C R)^2} = \frac{(\omega L)^2 + j\omega L R}{R^2 + (\omega L)^2} + \frac{1 - j\omega \frac{L}{R^2} R}{1 + (\omega \frac{L}{R^2} R)^2}$$

$$= \frac{(\omega L)^2 + j\omega L R}{R^2 + (\omega L)^2} + \frac{R^2 - j\omega L R}{R^2 + (\omega L)^2} = \frac{R^2 + (\omega L)^2}{R^2 + (\omega L)^2} = 1$$

$$2 V_f = V + Z_0 I \ ; \ V_f \equiv V_0 (1 - e^{-\lambda t})$$

$$2 V_0 (1 - e^{-\lambda t}) = V + Z_0 I$$
$$V - U = L \left(I - \frac{V - U}{R} \right)',$$
$$I = C U' + \frac{U}{R} = C \{ (V - U) e^{\frac{t}{CR}} \}' e^{-\frac{t}{CR}}$$

$$2 V_0 (1 - e^{-\lambda t}) = V + Z_0 \left(C U' + \frac{U}{R} \right)$$
$$V - U = L \left(C U' + \frac{U}{R} - \frac{V - U}{R} \right)',$$

$$\tau \equiv \frac{L}{R} \ ; \ Z_0 \equiv R \ ; \ \eta \equiv C R \ ; \ C \equiv \frac{L}{R^2} \ ; \ \eta = \frac{L}{R^2} R = \frac{L}{R} = \tau \ ; \ C L = \tau^2$$
$$2 V_0 (1 - e^{-\lambda t}) = V + \tau U' + U$$
$$V - U = \tau^2 U'' - \tau V' + 2 \tau U'$$

$$2 V_0 \lambda e^{-\lambda t} = V' + \tau U'' + U'$$
$$2 V_0 (1 - e^{-\lambda t}) - \tau U' - U - U = \tau^2 U'' - \tau (2 V_0 \lambda e^{-\lambda t} - \tau U'' - U') + 2 \tau U'$$
$$2 V_0 (1 - e^{-\lambda t}) - \tau U' - 2 U = 2 \tau^2 U'' - 2 V_0 \tau \lambda e^{-\lambda t} + 3 \tau U'$$
$$\tau^2 U'' + 2 \tau U' + U = V_0 (1 - e^{-\lambda t}) + V_0 \lambda \tau e^{-\lambda t} = V_0 \{ 1 + (\lambda \tau - 1) e^{-\lambda t} \}$$

$$\phi \equiv \tau U' + U$$
$$\tau \phi' + \phi = V_0 \{ 1 + (\lambda \tau - 1) e^{-\lambda t} \} = \tau (\phi e^{\frac{t}{\tau}})' e^{-\frac{t}{\tau}}$$

$$\phi e^{\frac{t}{\tau}} - \phi_0 = \int_0^t \frac{V_0}{\tau} \{ 1 + (\lambda \tau - 1) e^{-\lambda t} \} e^{\frac{t}{\tau}} d t$$

$$\frac{\tau}{V_0} \phi e^{\frac{t}{\tau}} = \tau (e^{\frac{t}{\tau}} - 1) + \frac{\lambda \tau - 1}{\frac{1}{\tau} - \lambda} (e^{\frac{t}{\tau} - \lambda t} - 1) = \tau (e^{\frac{t}{\tau}} - 1) + \tau \frac{\lambda \tau - 1}{1 - \lambda \tau} (e^{\frac{t}{\tau} - \lambda t} - 1) = \tau (e^{\frac{t}{\tau}} - 1)$$
$$- \tau (e^{\frac{t}{\tau} - \lambda t} - 1) = \tau (e^{\frac{t}{\tau}} - e^{\frac{t}{\tau} - \lambda t})$$

$$\frac{\phi}{V_0} = 1 - e^{-\lambda t}$$
$$\frac{\tau U' + U}{V_0} = 1 - e^{-\lambda t} = \frac{\tau (U e^{\frac{t}{\tau}})' e^{-\frac{t}{\tau}}}{V_0}$$
$$U e^{\frac{t}{\tau}} - U_0 = \int_0^t \frac{V_0}{\tau} (1 - e^{-\lambda t}) e^{\frac{t}{\tau}} d t$$
$$\frac{\tau U}{V_0} e^{\frac{t}{\tau}} = \tau (e^{\frac{t}{\tau}} - 1) - \frac{1}{\frac{1}{\tau} - \lambda} (e^{\frac{t}{\tau} - \lambda t} - 1)$$
$$\frac{U}{V_0} = (1 - e^{-\frac{t}{\tau}}) - \frac{1}{1 - \tau \lambda} (e^{-\lambda t} - e^{-\frac{t}{\tau}})$$

$$\frac{R I}{V_0} = \tau U' + U = \phi = 1 - e^{-\lambda t}$$

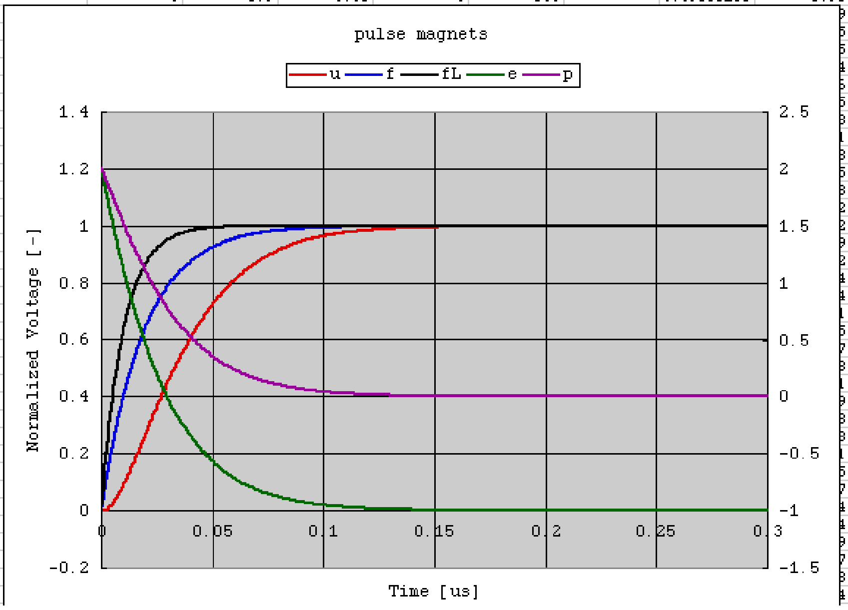
$$\Psi \equiv V - U = 2 V_0 (1 - e^{-\lambda t}) - Z_0 I - U = \frac{V_0}{1 - \tau \lambda} (e^{-\lambda t} - e^{-\frac{t}{\tau}})$$
$$\Phi = \int_0^t \Psi \, d t = \frac{V_0}{1 - \tau \lambda} \left\{ \frac{1 - e^{-\lambda t}}{\lambda} - \tau (1 - e^{-\frac{t}{\tau}}) \right\}$$
$$= \frac{V_0}{\lambda} + \frac{V_0}{1 - \tau \lambda} \left(\frac{- e^{-\lambda t}}{\lambda} + \tau e^{-\frac{t}{\tau}} \right)$$
$$\frac{\lambda \Phi}{V_0} = 1 + \frac{\tau \lambda e^{-\frac{t}{\tau}} - e^{-\lambda t}}{1 - \tau \lambda}$$

$$\lim_{\lambda \to 0} \frac{\lambda \Phi}{V_0} = 1 + \frac{\tau\ 0\ e^{-\frac{t}{\tau}} - 1}{1 - 0} = 0 \quad (O.K.)$$
$$\lim_{\lambda \to \infty} \frac{\lambda \Phi}{V_0} = 1 - e^{-\frac{t}{\tau}} \quad (O.K.)$$

電流負荷時（１頁の図と同等）

LRCRsim.xls

		L [uH]	0.5							
		Z [ohm]	25							
		tau [us]	0.02							
		C [pF]	800							
c0	0		0	2	0					
c1	0.001									
N	t [us]		u	e	f	u'	e'		fL	p
0	0		0	2	0				0	2
1	0.001		0	1.9	0.05	0	-100		0.09516258	1.95
2	0.002									
3	0.003									
4	0.004									
5	0.005									
6	0.006									
7	0.007									
8	0.008									
9	0.009									
10	0.01									
11	0.011									
12	0.012									
13	0.013									
14	0.014									
15	0.015									
16	0.016									
17	0.017									
18	0.018									
19	0.019									
20	0.02									
21	0.021									
22	0.022									
23	0.023									
24	0.024									
25	0.025									
26	0.026									
27	0.027									
28	0.028									
29	0.029									
30	0.03									
31	0.031									
32	0.032									
33	0.033									
34	0.034									
35	0.035									



電圧負荷時（Droop は殆ど無く完全整合）

LRCRsim.xls

		L [uH]	1000							
		Z [ohm]	130							
		tau [us]	7.69230769							
		C [pF]	59171.5976							
c0	0		0	2	0					
c1	0.001									
N	t [us]		u	e	f	u'	e'		fL	p
0	0		0	2	0				0	2
1	0.001		0	1.99974	0.00013	0	-0.26		0.00025997	1.99987
2	0.002									
3	0.003									
4	0.004									
5	0.005									
6	0.006									
7	0.007									
8	0.008									
9	0.009									
10	0.01									
11	0.011									
12	0.012									
13	0.013									
14	0.014									
15	0.015									
16	0.016									
17	0.017									
18	0.018									
19	0.019									
20	0.02									
21	0.021									
22	0.022									
23	0.023									
24	0.024									
25	0.025									
26	0.026									
27	0.027									
28	0.028									
29	0.029									
30	0.03									
31	0.031									
32	0.032									
33	0.033									
34	0.034									
35	0.035