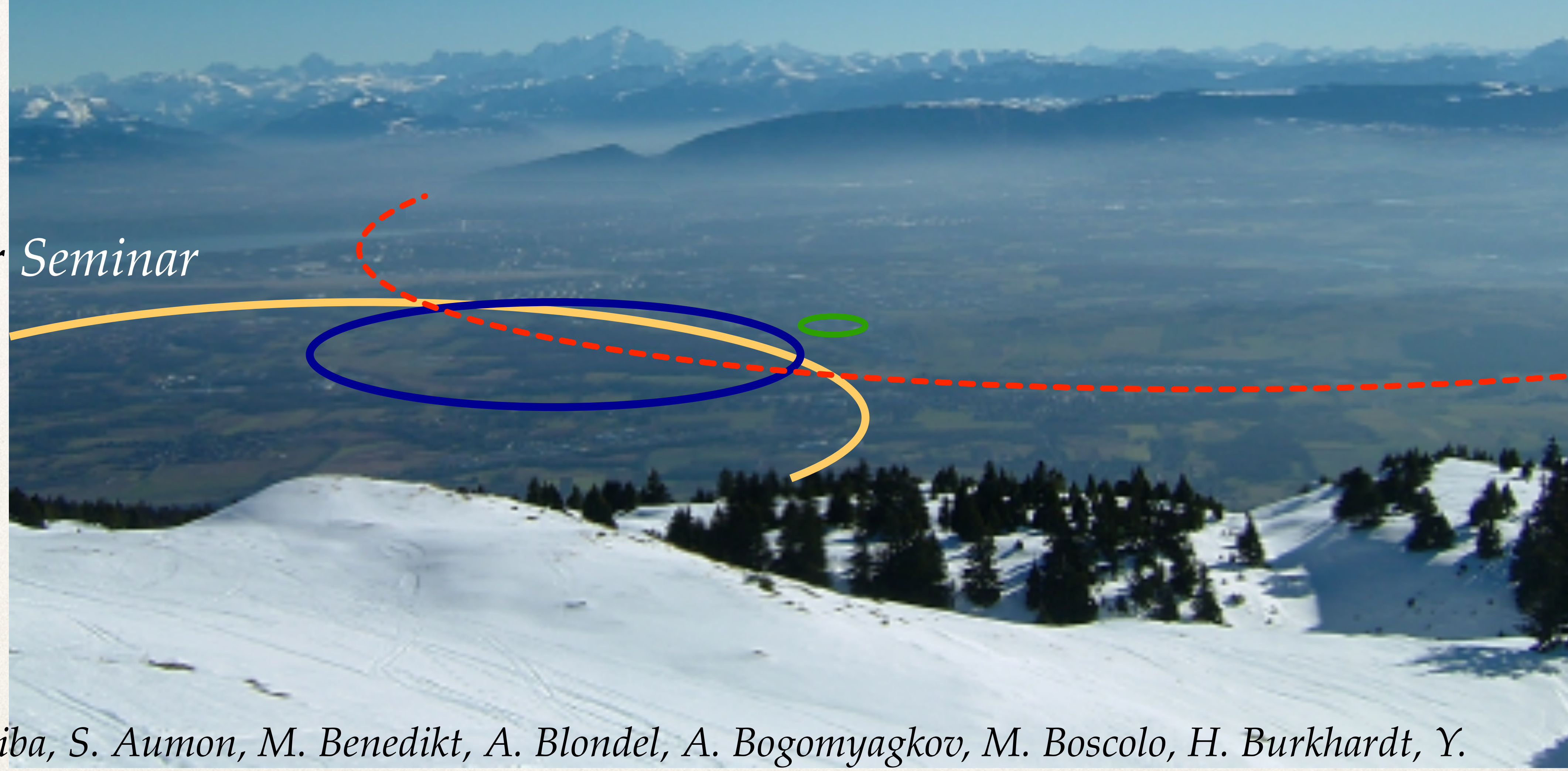
FCC-ee Machine Layout and Beam Optics + Matching with synchrotron motion



KEK Accelerator Seminar

27 Apr. 2016

KEK, Tsukuba

K. Oide

Contributed by M. Aiba, S. Aumon, M. Benedikt, A. Blondel, A. Bogomyagkov, M. Boscolo, H. Burkhardt, Y. Cai, A. Doblehammer, B. Haerer, B. Holzer, J. Jowett, I. Koop, M. Koratzinos, E. Levitchev, L. Medrano, K. Ohmi, Y. Papaphilippou, P. Piminov, D. Shatilov, S. Sinyatkin, M. Sullivan, J. Wenninger, D. Zhou, F. Zimmermann.

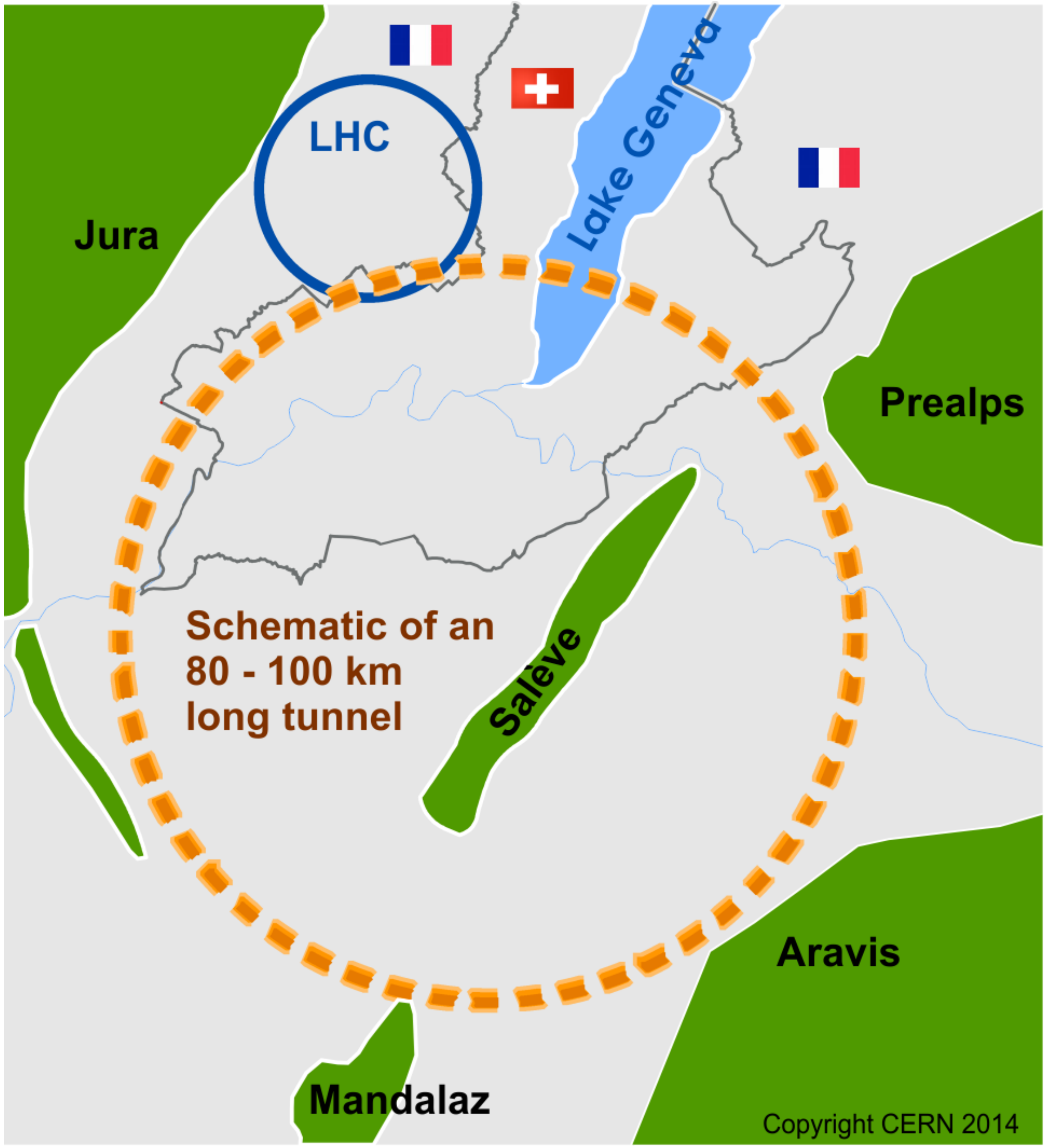
Future Circular Collider Study
GOAL: CDR and cost review for the next ESU (2019)

International FCC collaboration (CERN as host lab) to study:

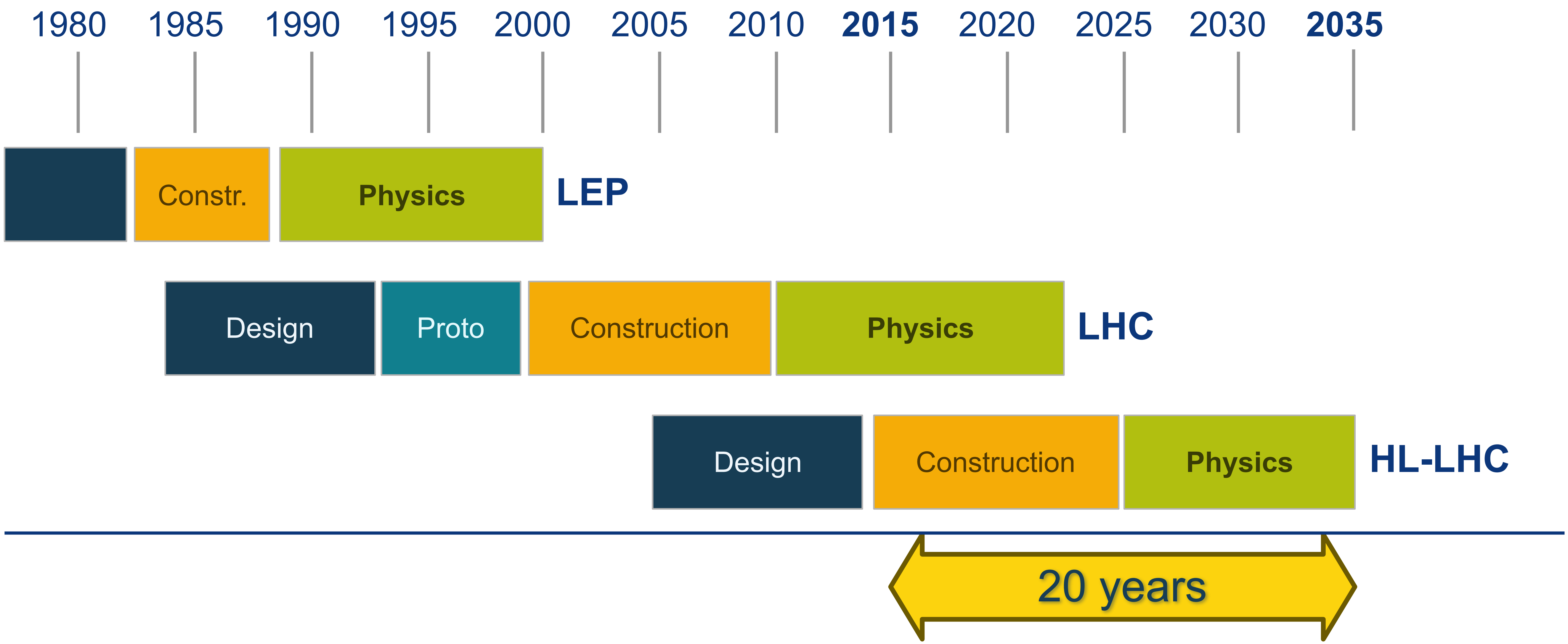
- ***pp*-collider (*FCC-hh*)**
  → main emphasis, defining infrastructure requirements

~16 T ⇒ 100 TeV *pp* in 100 km

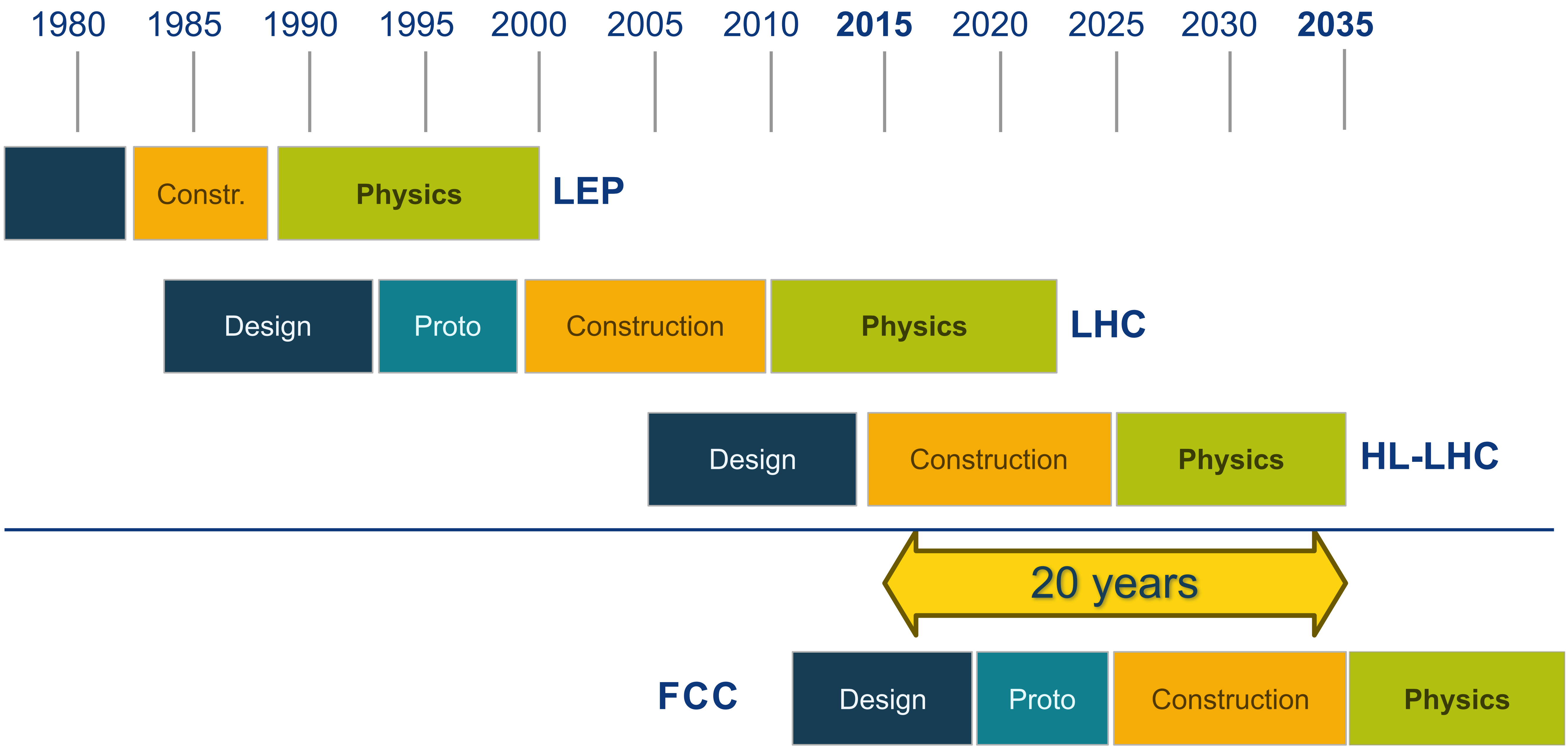
- **80-100 km tunnel infrastructure** in Geneva area
- ***e⁺e⁻* collider (*FCC-ee*)** as potential first step
- ***p-e* (*FCC-he*) option**
- **HE-LHC** with *FCC-hh* technology



CERN Circular Colliders & FCC

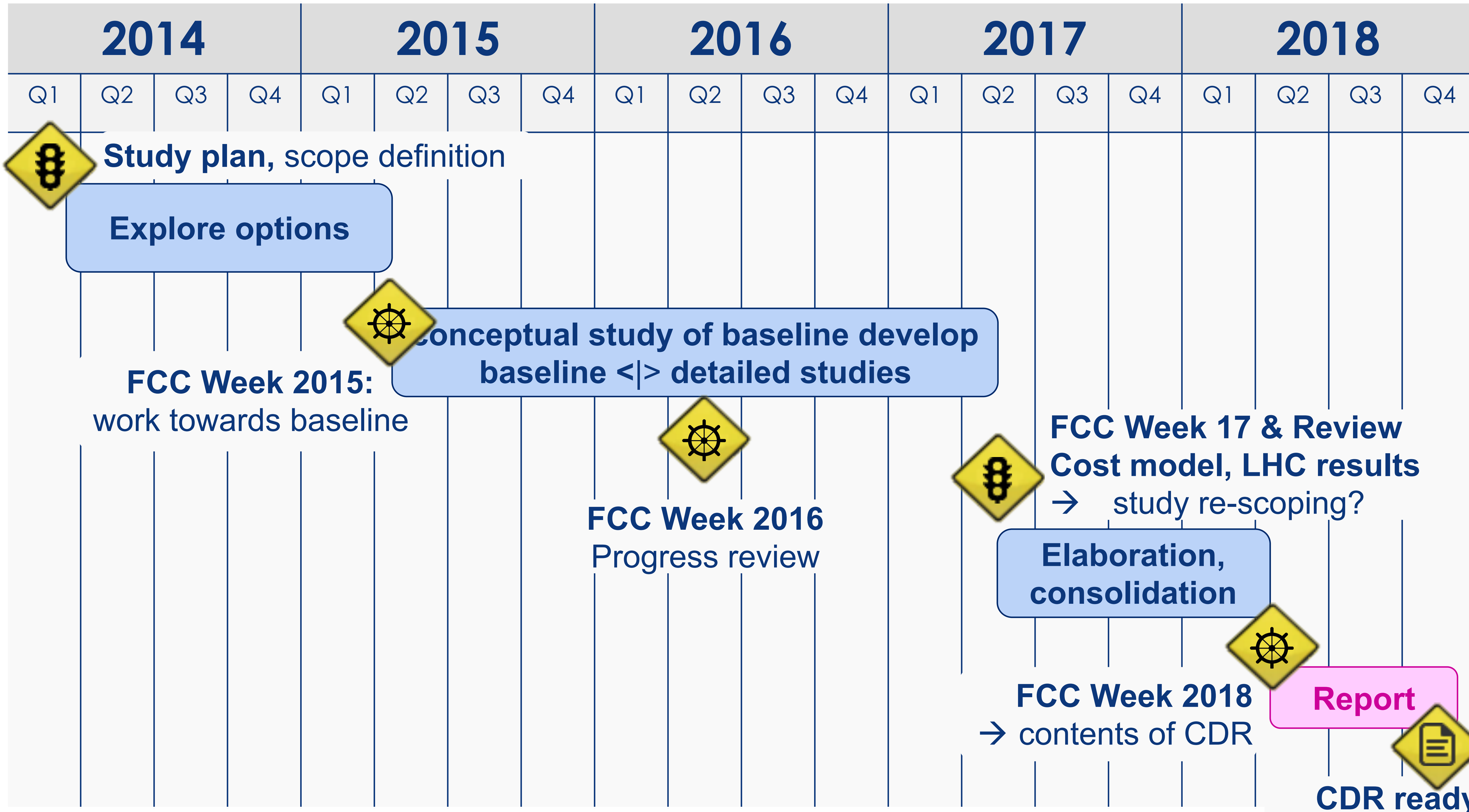


CERN Circular Colliders & FCC

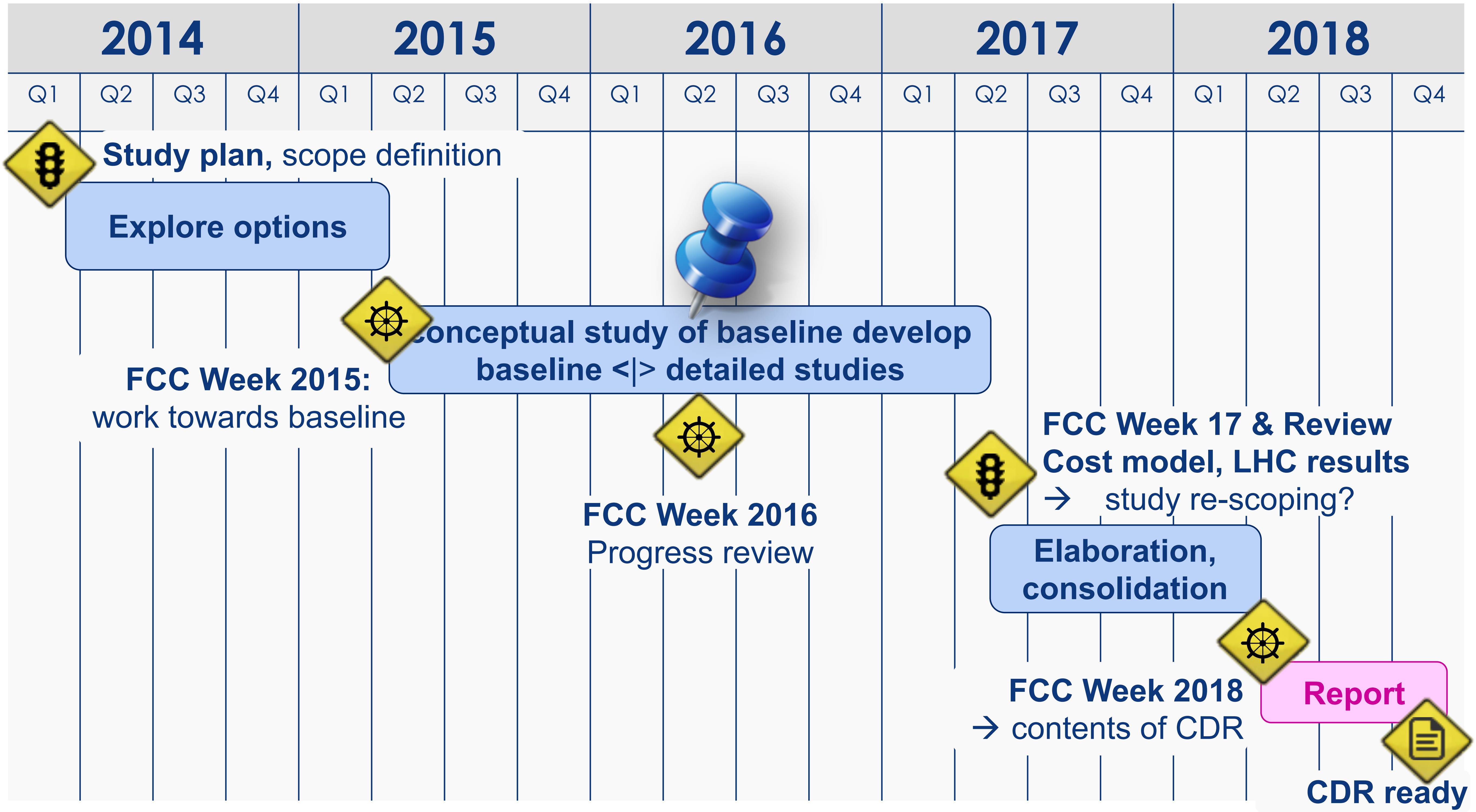


Now is the time to plan for the period 2035 – 2040

CDR Study Time Line



CDR Study Time Line



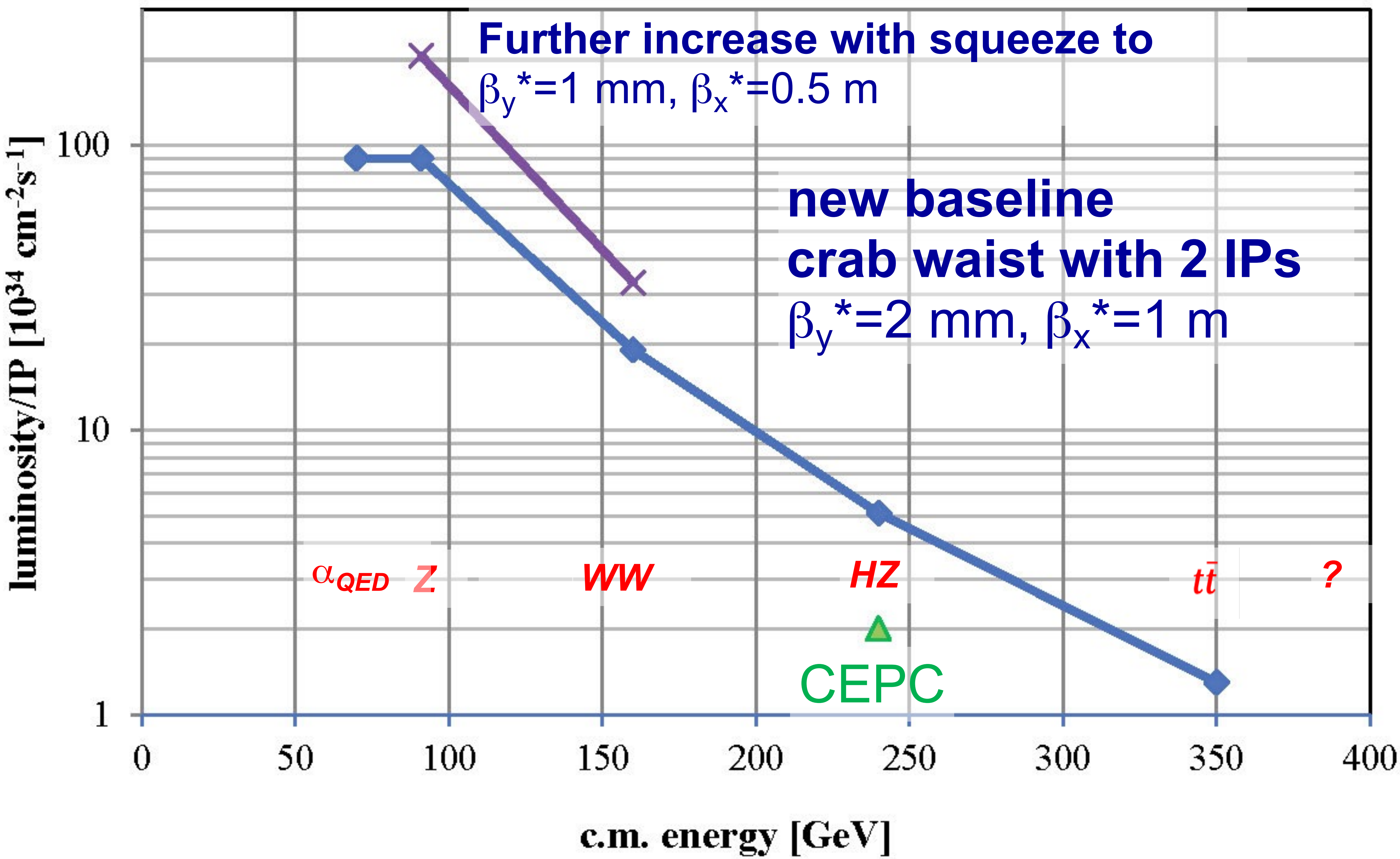
lepton collider parameters

parameter	FCC-ee (400 MHz)					CEPC	LEP2
Physics working point	**Z**		**WW**	**ZH**	**tt_{bar}**	H	
energy/beam [GeV]	**45.6**		**80**	**120**	**175**	120	105
bunches/beam	30180	**91500**	**5260**	**780**	**81**	50	4
bunch spacing [ns]	7.5	**2.5**	**50**	**400**	**4000**	3600	22000
bunch population [10^{11}]	1.0	**0.33**	**0.6**	**0.8**	**1.7**	3.8	4.2
beam current [mA]	1450	**1450**	**152**	**30**	**6.6**	16.6	3
luminosity/IP x $10^{34}cm^{-2}s^{-1}$	210	**90**	**19**	**5.1**	**1.3**	2.0	0.0012
energy loss/turn [GeV]	0.03	**0.03**	**0.33**	**1.67**	**7.55**	3.1	3.34
synchrotron power [MW]	**100**					103	22
RF voltage [GV]	0.4	**0.2**	**0.8**	**3.0**	**10**	6.9	3.5

identical FCC-ee baseline optics for all energies

FCC-ee: 2 separate rings CEPC, LEP: single beam pipe

FCC-ee luminosity per IP



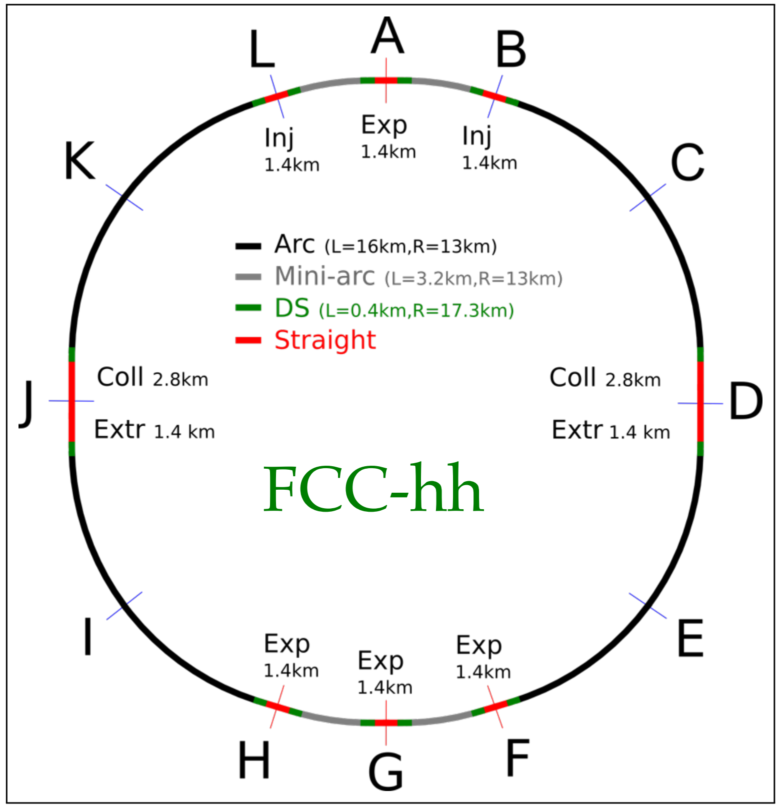
Design constraints & assumptions

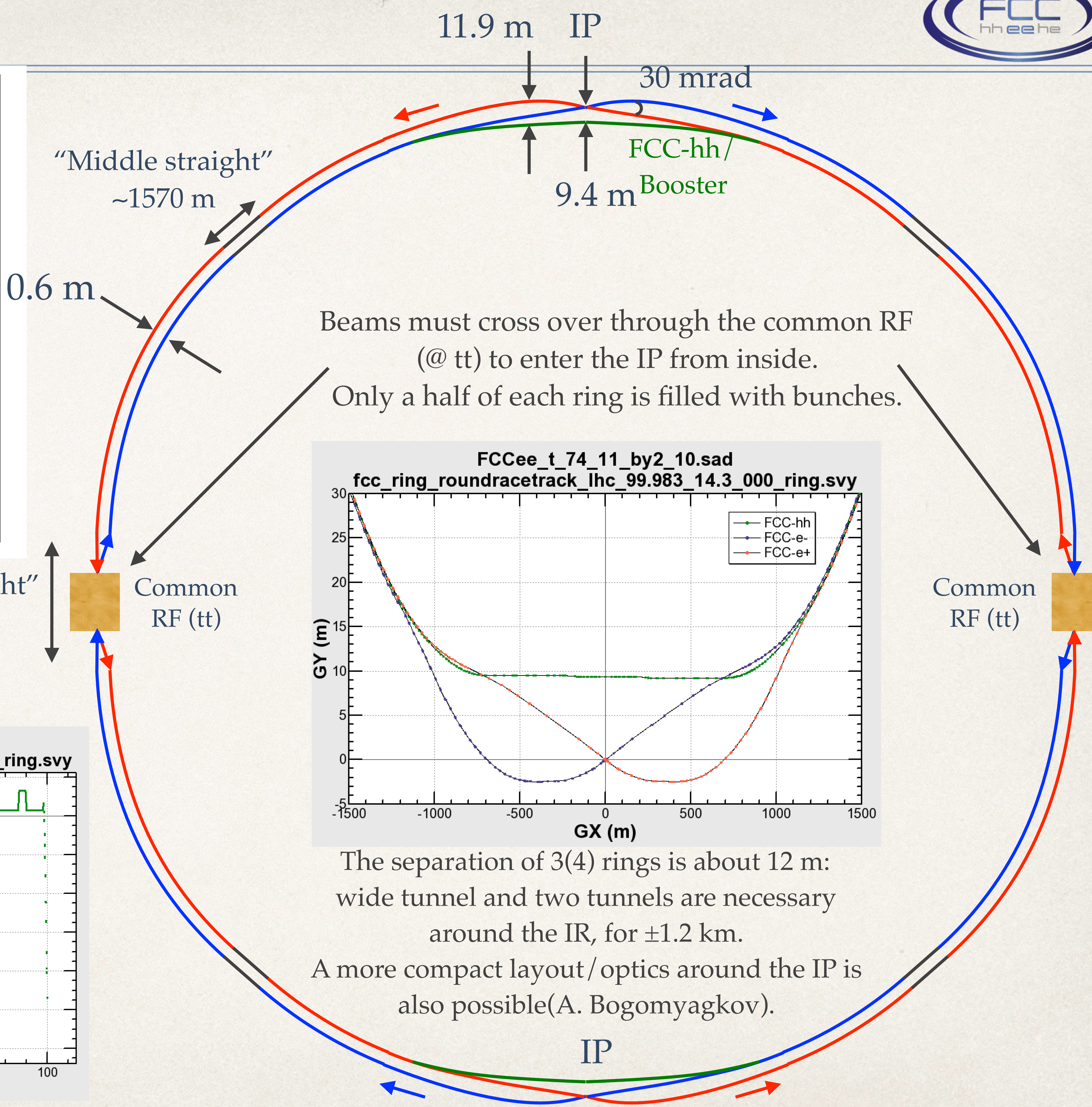
- C = 100 km, fits to the FCC-hh tunnel as much as possible.
- 2 IPs / ring.
- 30 mrad crossing angle at the IP with crab waist.
- Common lattice for all energies.
- $\varepsilon_x \leqq 1.3$ nm @ 175 GeV.
- ±2% momentum acceptance at 175 GeV.
- Vertical emittance less than 1 pm at 175 GeV.
- $\beta_{x,y}^* =$ (1 m, 2 mm) at 175 GeV, (0.5 m, 1 mm) at 45.6 GeV.
- Suppress the synchrotron radiation to the IP below 100 keV, up to 500 m upstream (as suggested by H. Burkhardt).
- "tapering" to cure the sawtooth at high energy.

Parameters

Circumference [km]	99983.76	
Number of IPs / ring	2	
Crossing angle at IP [mrad]	30	
Solenoid with compensation at IP	±2 T × 1 m	
ℓ^* [m] (asymmetric version)	2.2 / 2.9	
Critical energy of photons to IP	< 100 keV @ 175 GeV, up to 510 m upstream	
IR Optics	asymmetric	
Local chromaticity correction	Y	
Crab sexts	integrated with LCCS	
Arc cell	FODO, 90°/90°	
Arc sextuple families	292 (paired)	
mom. comp. [10^{-5}]	0.70	
Tunes (x/y)	387.08 / 387.14	
Ebeam [GeV]	**45.6**	**175**
SR energy loss per turn [GeV]	0.0346	7.47
Current / beam [mA]	1450	6.6
$P_{SR,tot}$ [MW]	100.3	98.6
ε_x [nm]	0.86	1.26
β^*_x [m]	0.5 (1)	1 (0.5)
β^*_y [mm]	1 (2)	2 (1)
RF frequency [MHz]	400	
$\sigma_{\delta,SR}$ [%]	0.038	0.141
$\sigma_{z,SR}$ [mm]	2.8 @ V_c = 78 MV	2.4 @ V_c = 9.04 GV
Synchrotron tune	-0.0158 @ V_c = 78 MV	-0.0657 @ V_c = 9.04 GV

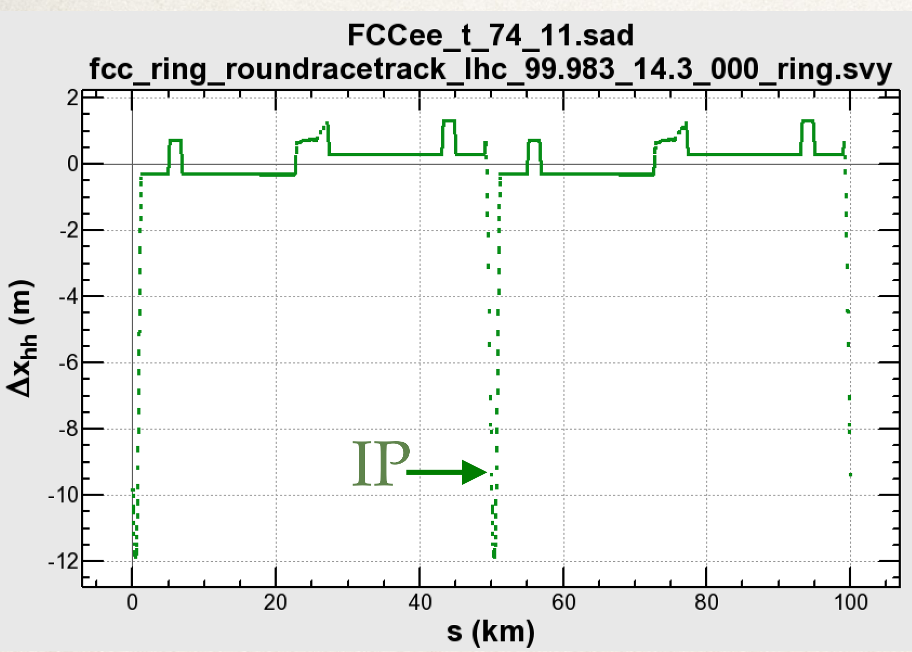
Layout of FCC-ee





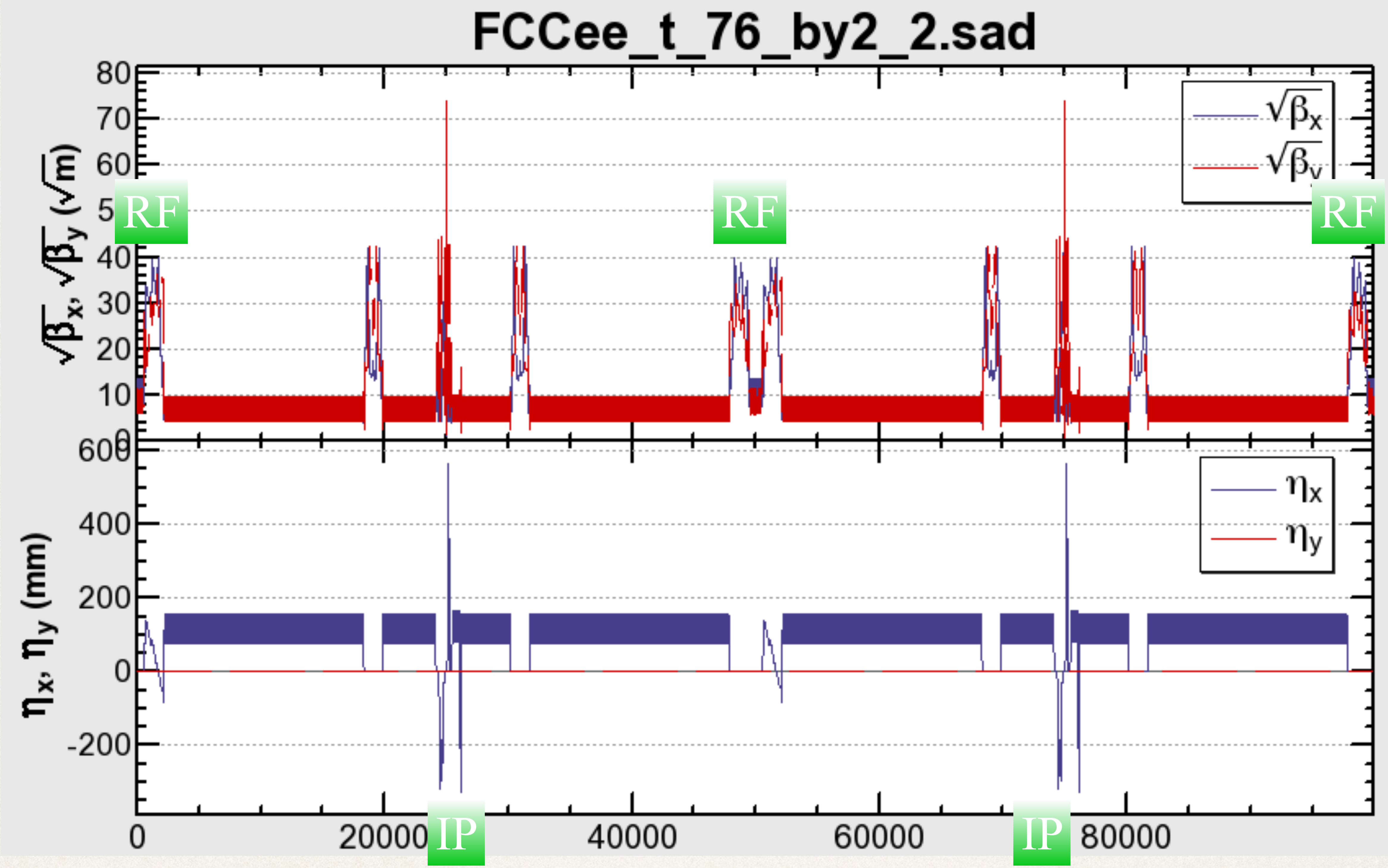
Beams must cross over through the common RF (@ tt) to enter the IP from inside.
Only a half of each ring is filled with bunches.

The separation of 3(4) rings is about 12 m: wide tunnel and two tunnels are necessary around the IR, for ±1.2 km.
A more compact layout/optics around the IP is also possible(A. Bogomyagkov).



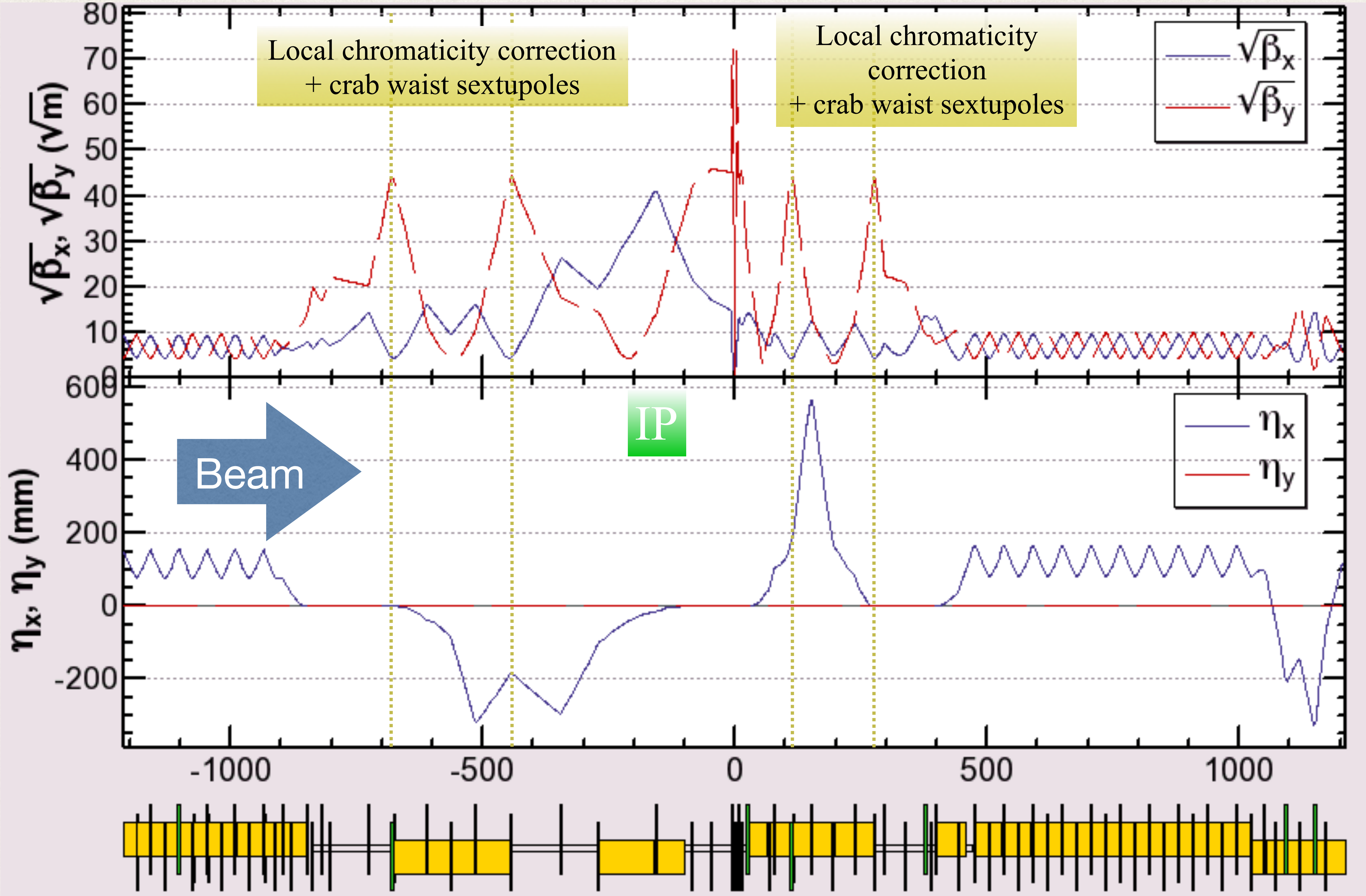
Relative distance to FCC-hh

Ring Optics



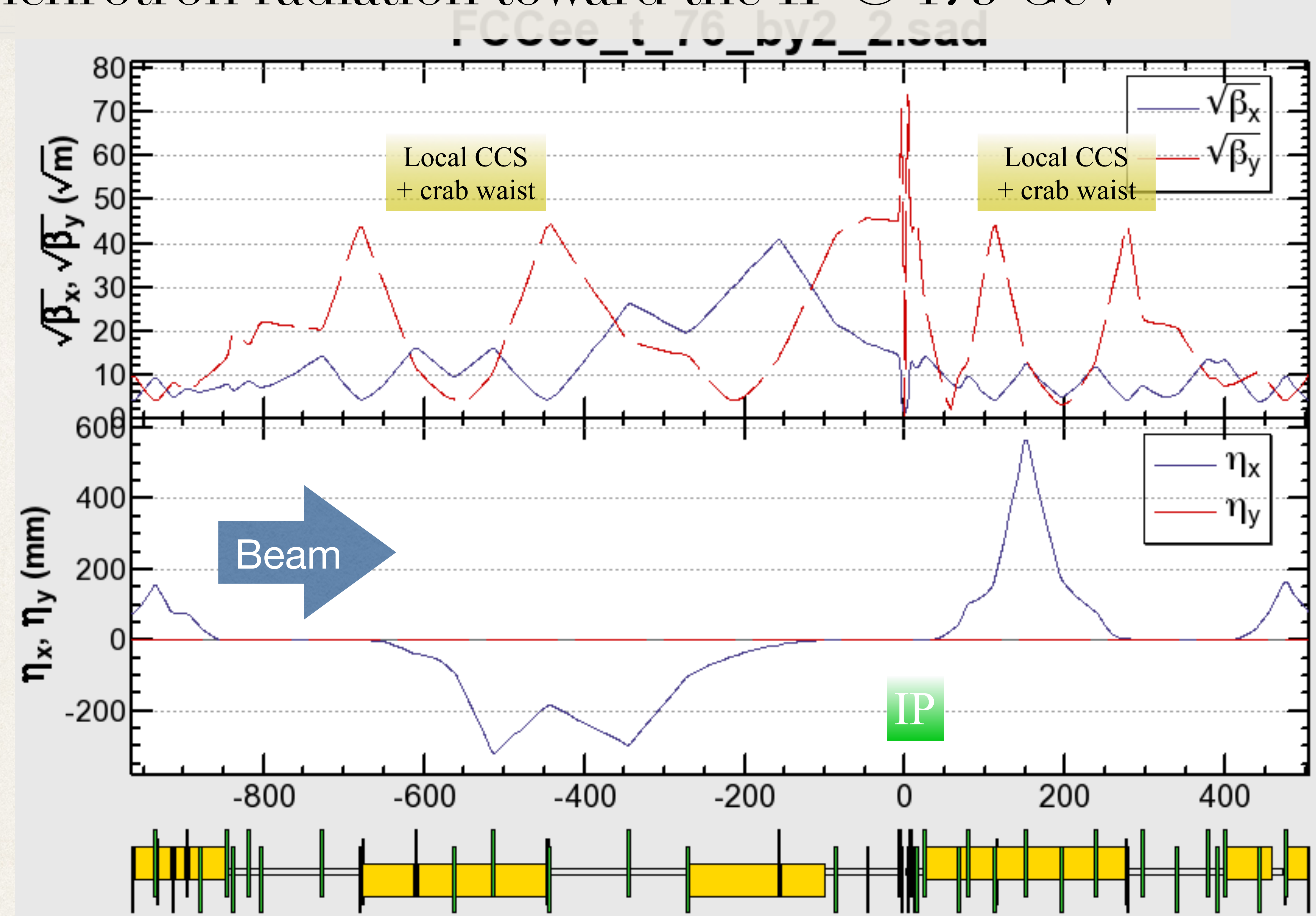
- Above are the optics for tt, $\beta^*_{x/y}$ = 1 m / 2 mm.
- 2 IP/ring.
- The optics for straight sections except for the IR are tentative, customizable for infection/extraction/collimation, etc.

Interaction Region



- The optics in the interaction region are asymmetric.
- The synchrotron radiation from the upstream dipoles are suppressed below 100 keV up to 450 m from the IP.
- The crab sextuples are integrated in the local chromaticity correction in the vertical plane.

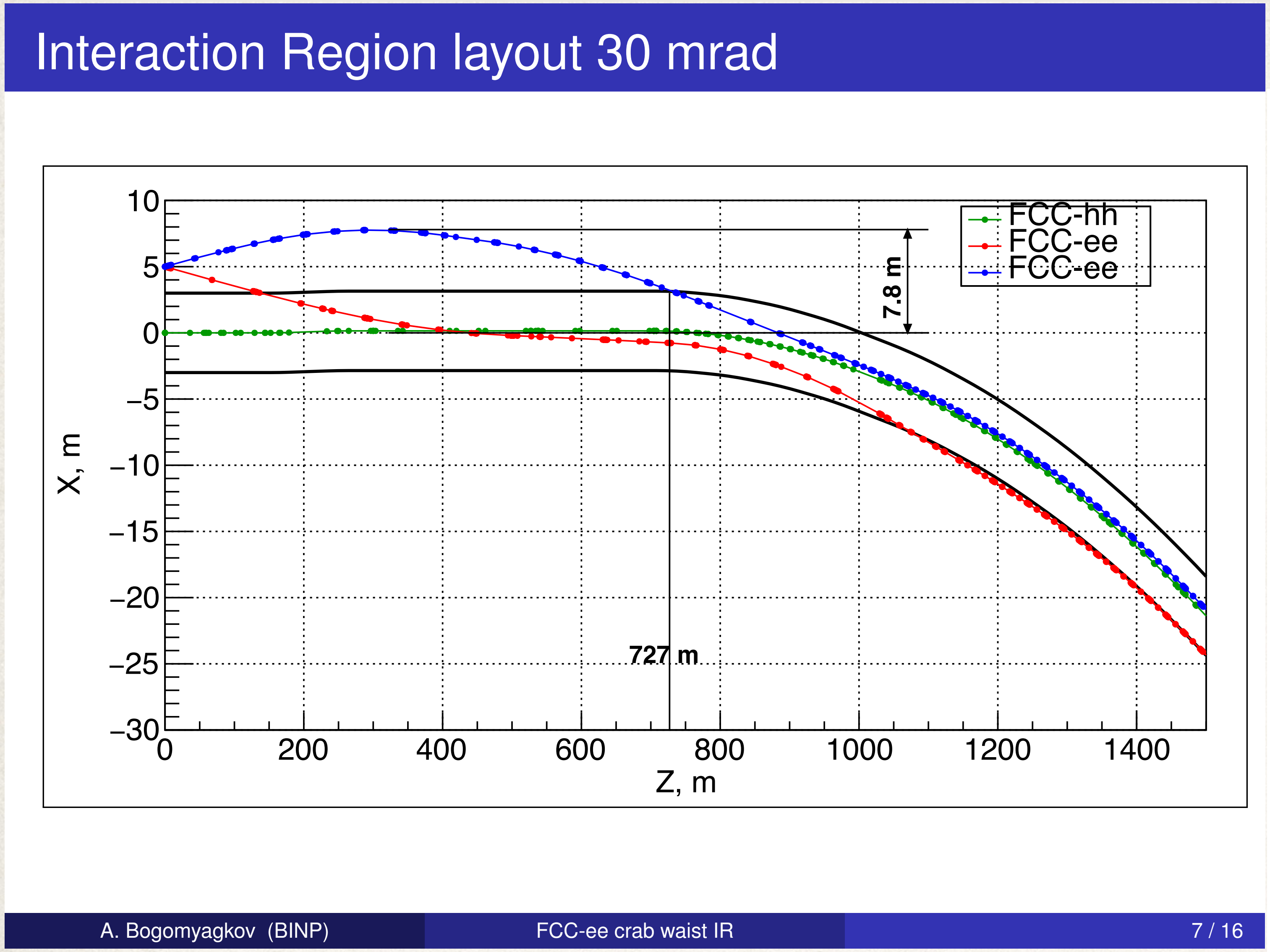
Synchrotron radiation toward the IP @ 175 GeV

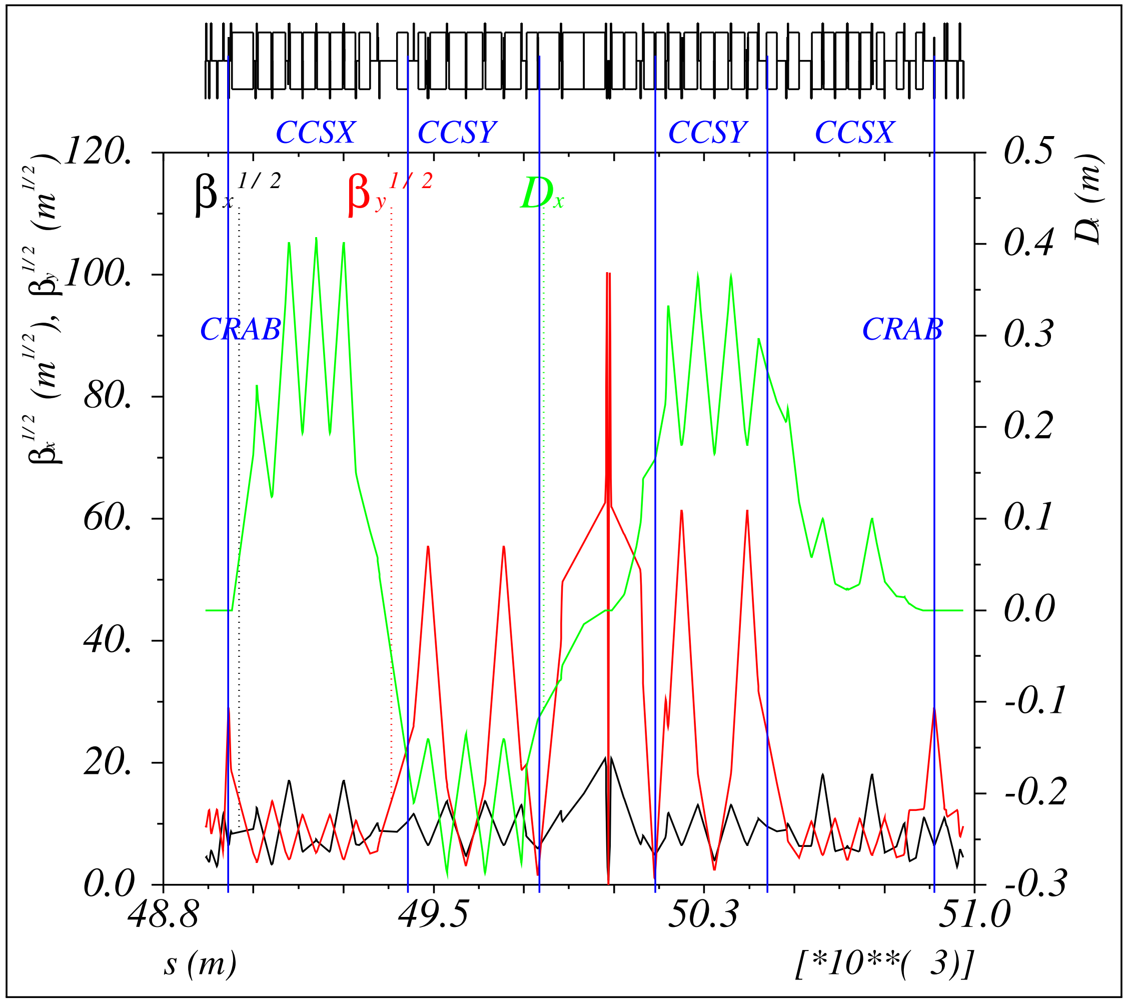


u_c (keV)	1062 930	204 449 292 9.1	100 100	691 1472 296 533
P_{SR} (kW)	16.6 15.3	1.7 5.4 2.3 0.003	0.67 0.34	11.2 38.7 1.9 6.9

$u_c < 100$ keV up to 510 m from the IP.

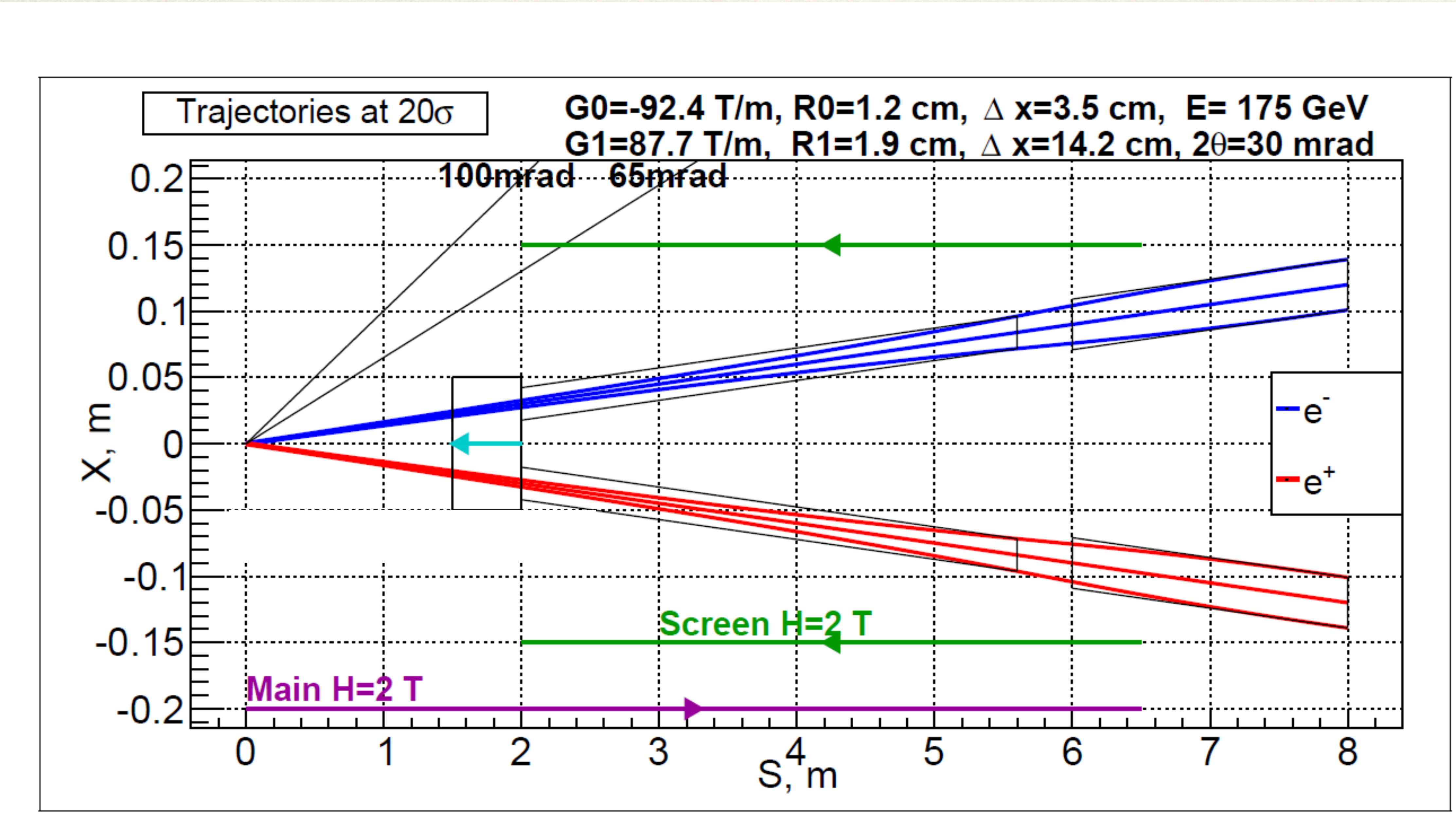
More compact IR (A. Bogomyagkov)





- A more compact layout / optics (AB Lattice) has been developed by A, Bogomyagkov.
- The deviation from FCC-hh is reduced to 5 m (9.5 m), the maximum excursion 7.8 m (11.9 m), the wide tunnel region ±730 m (1,200 m).
- Local chromaticity correction for both X and Y can be installed.
- A stronger dipoles are necessary for upstream of the IP (100 keV up to ~200 m, 200 keV up to ~300 m).

Solenoid compensation / shielding at the IR

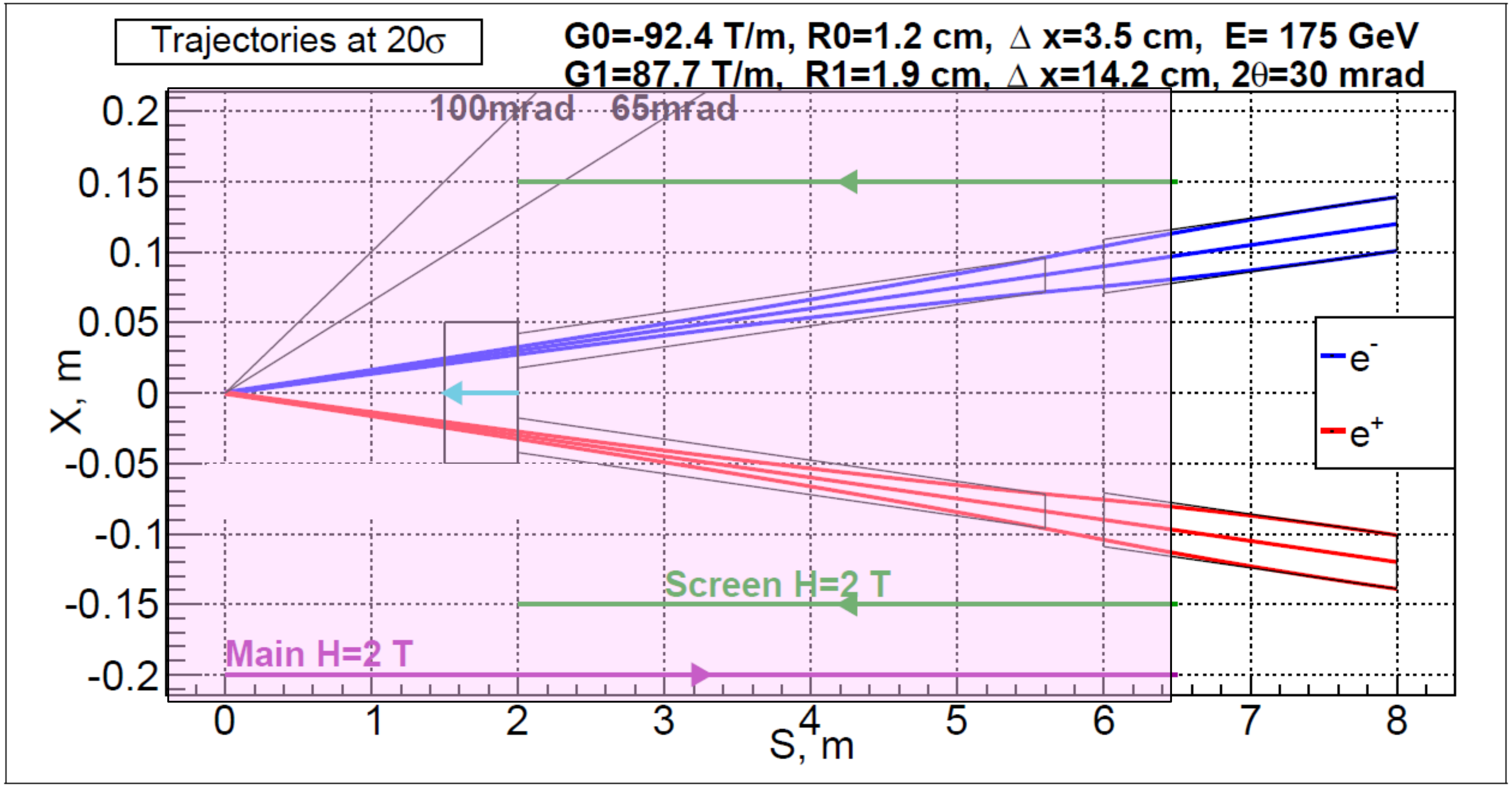


Favoured design at the moment. (it is not clear that a luminometer can fit inside the compensating solenoid)

- The effect of the solenoids are locally compensated within ±2 m around the IP.
- The final quads are shielded.

M. Koratzinos

Solenoid compensation / shielding at the IR



Main detector solenoid

Favoured design at the moment. (it is not clear that a luminometer can fit inside the compensating solenoid)

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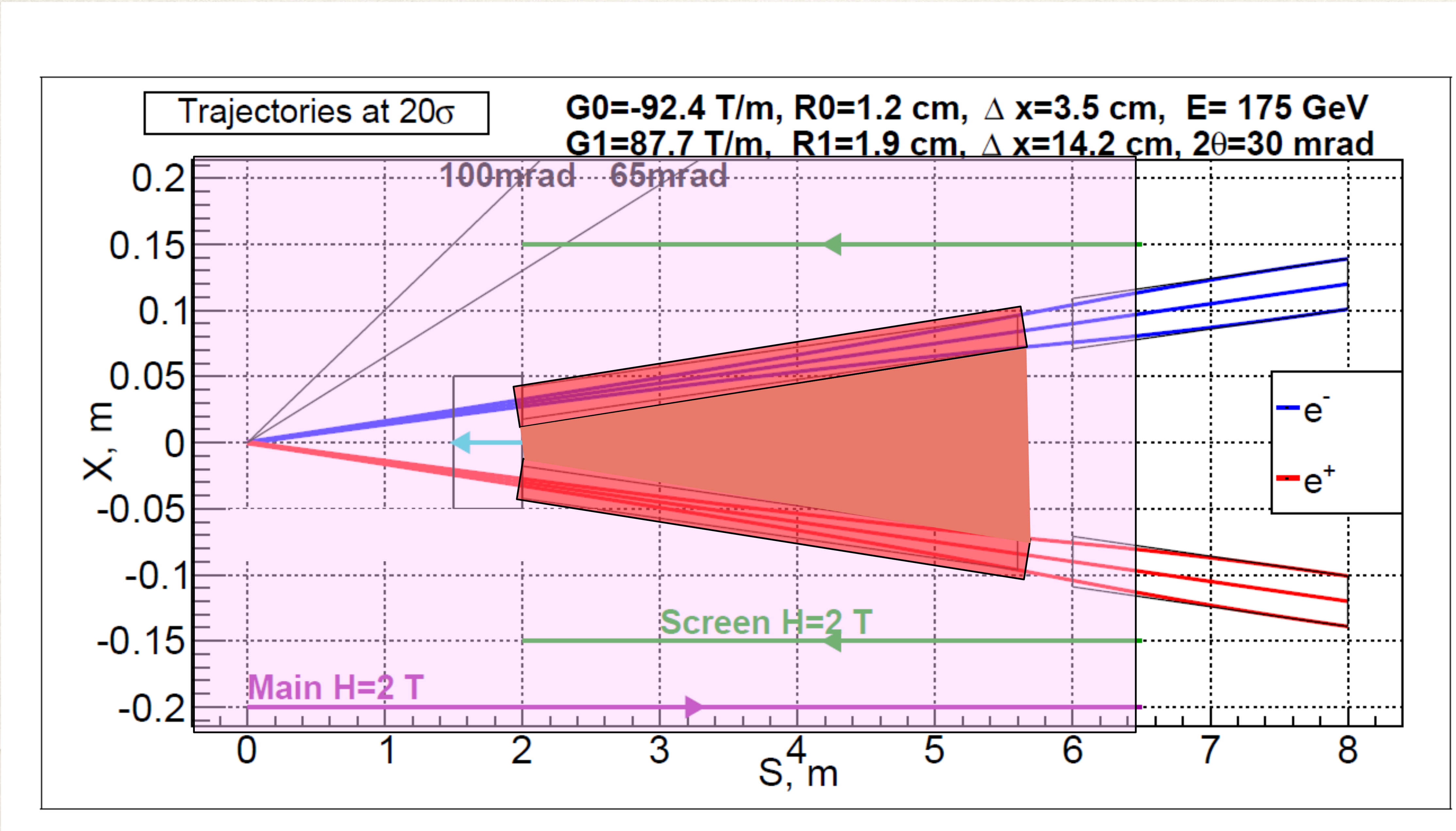

Final quads

Main detector solenoid

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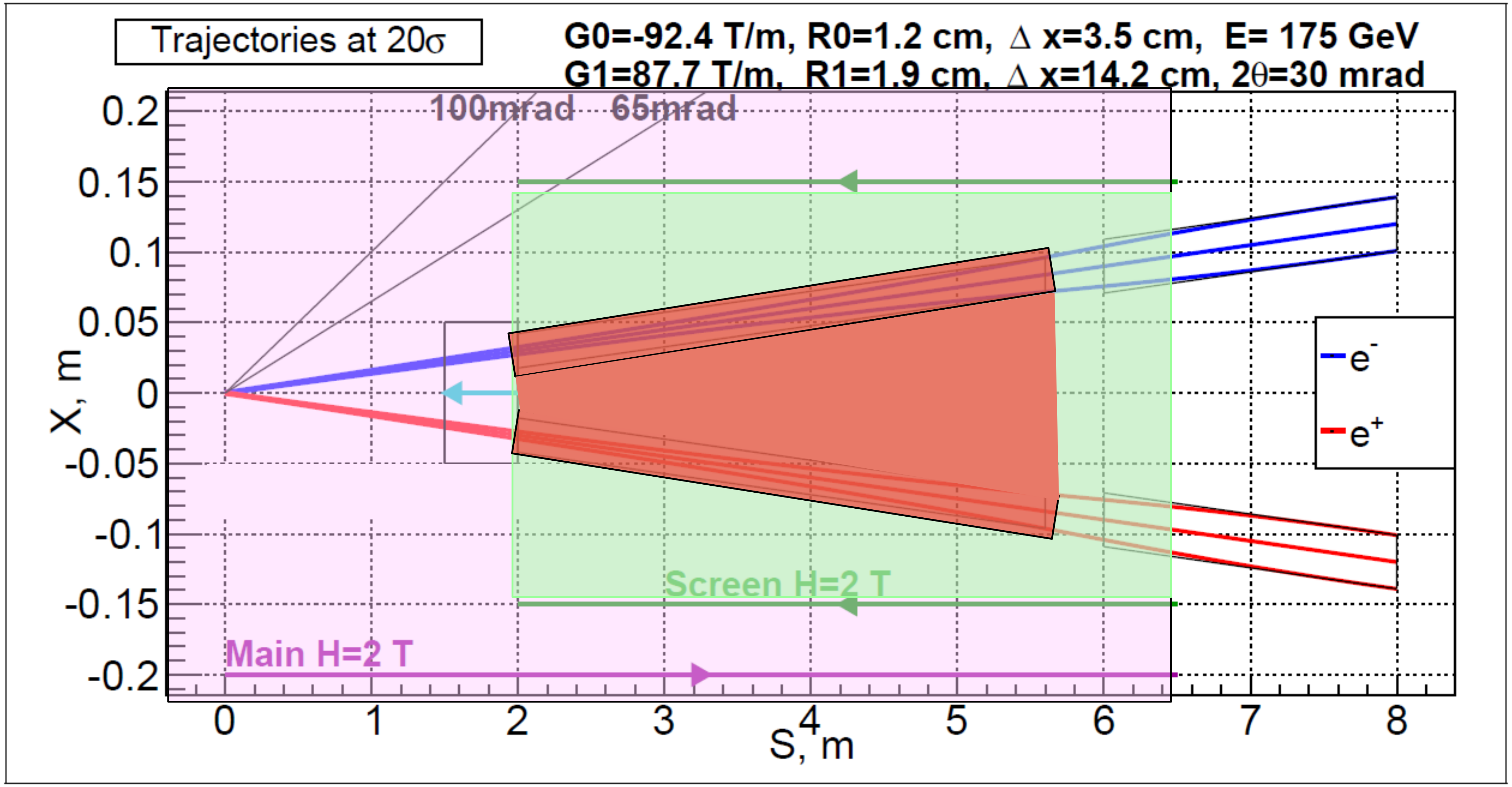

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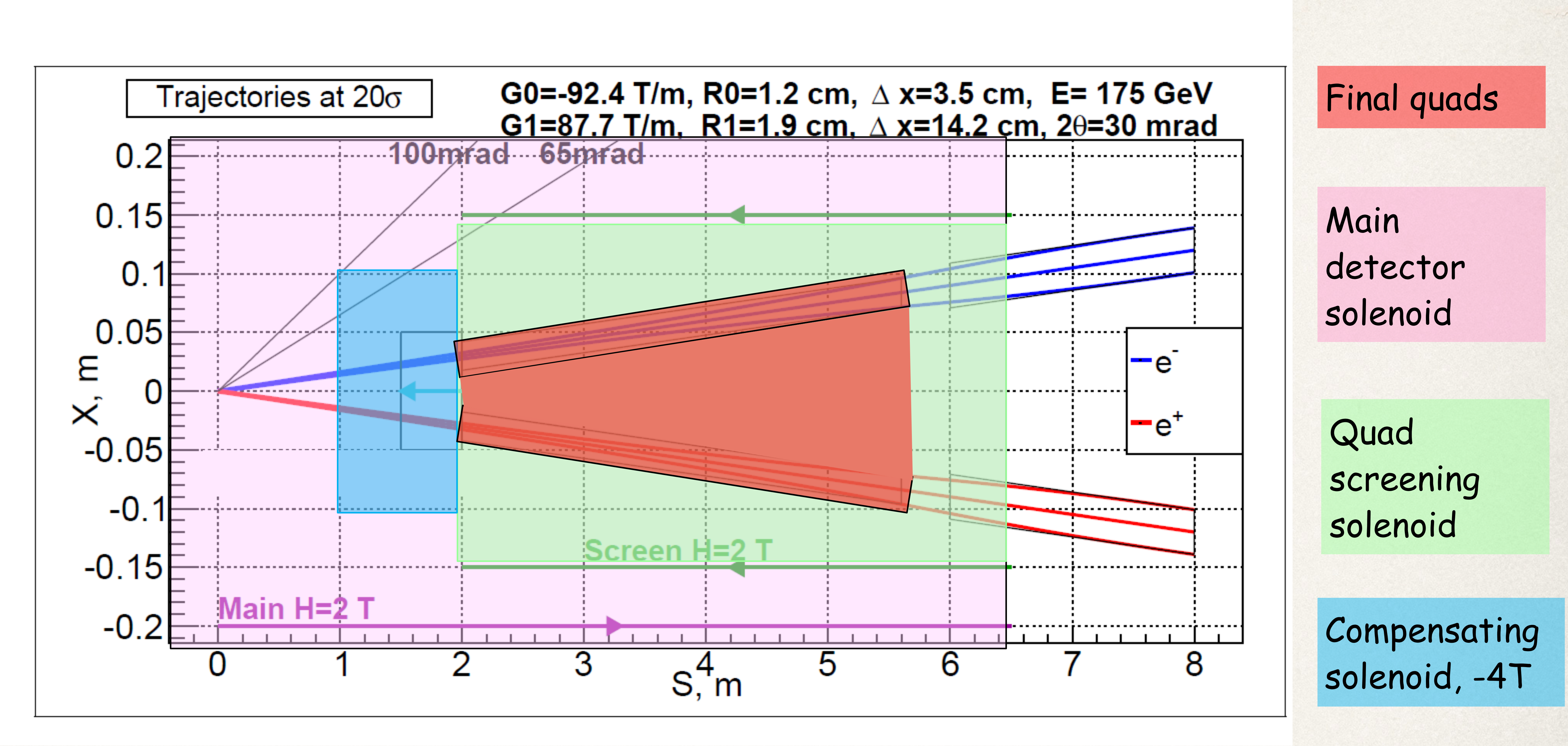

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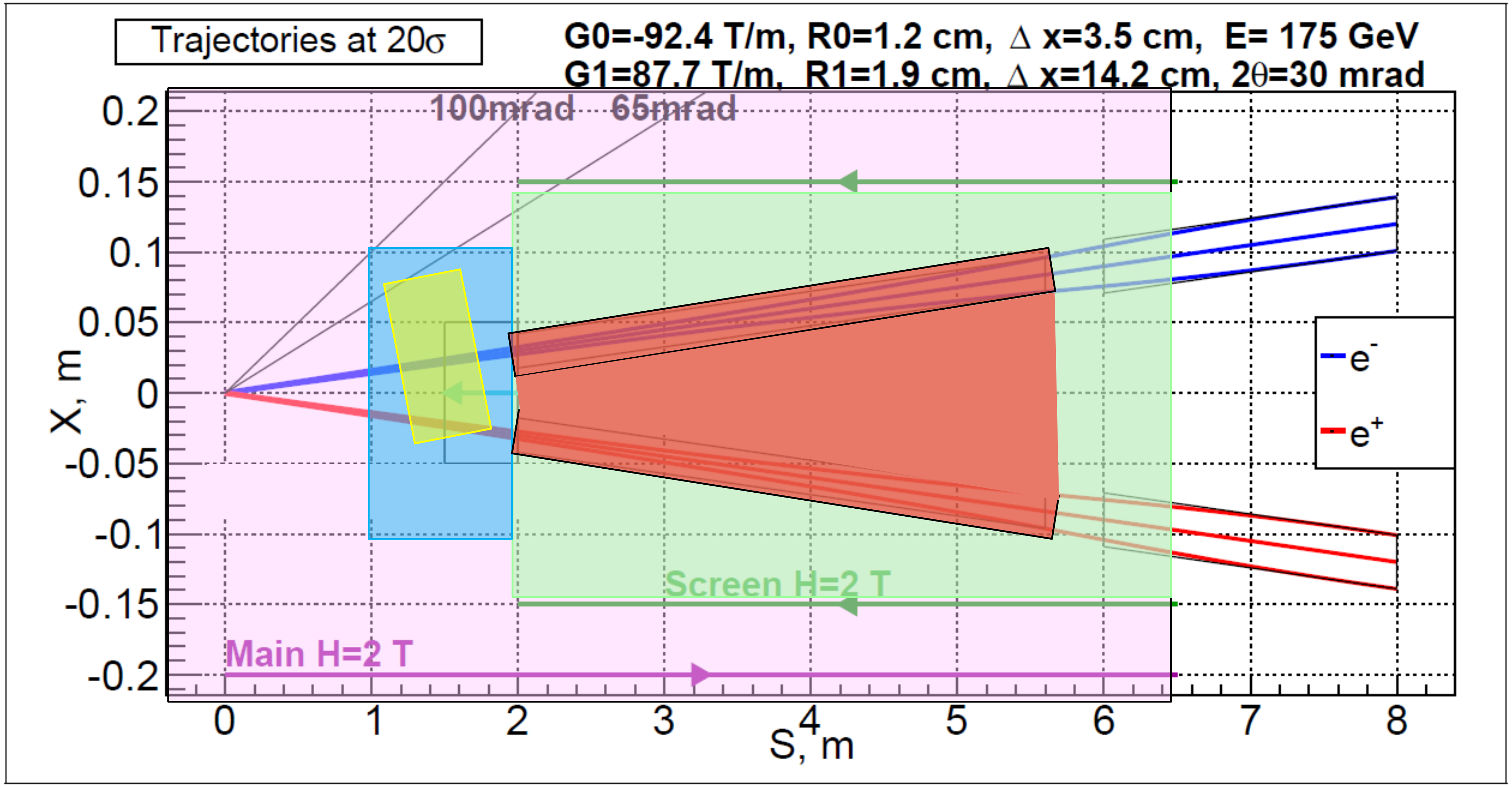
Solenoid compensation / shielding at the IR



Final quads

Main detector solenoid

Quad screening solenoid

Compensating solenoid, -4T

Luminosity counter?

Favoured design at the moment. (it is not clear that a luminometer can fit inside the compensating solenoid)

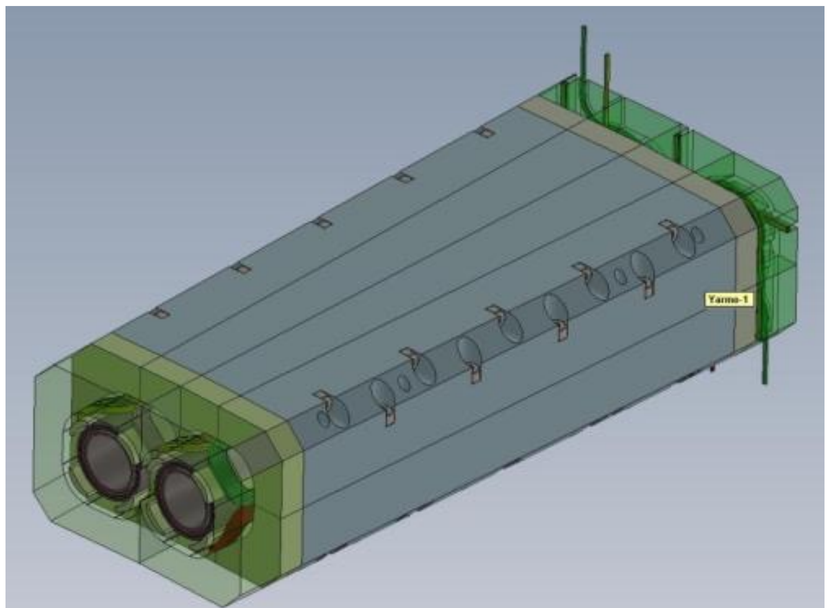
- The effect of the solenoids are locally compensated within ±2 m around the IP.
- The final quads are shielded.

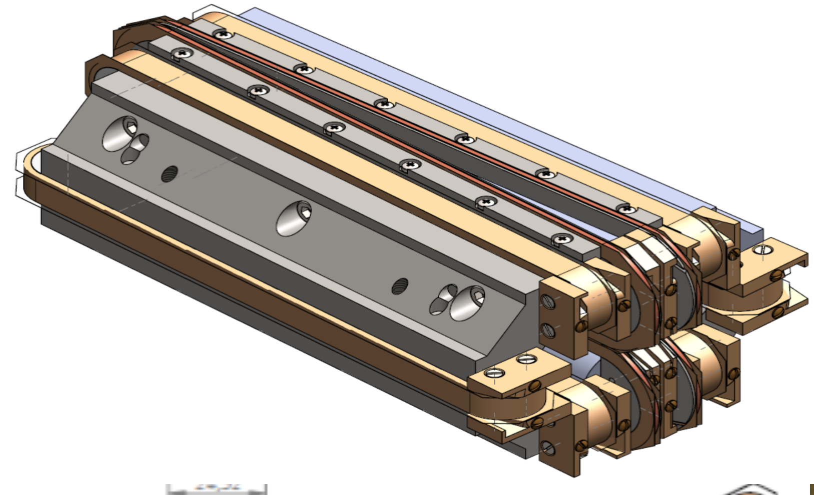
M. Koratzinos

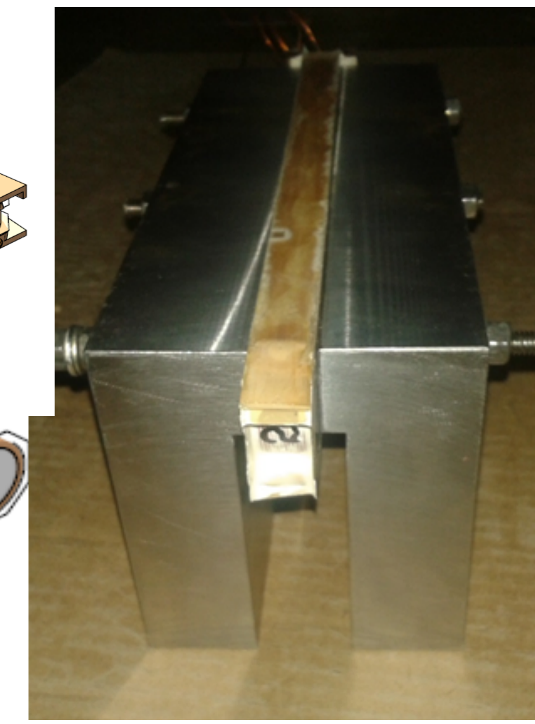
SC final focus quadrupole at BINP

Main contributors are Ivan Okunev and Pavel Vobly

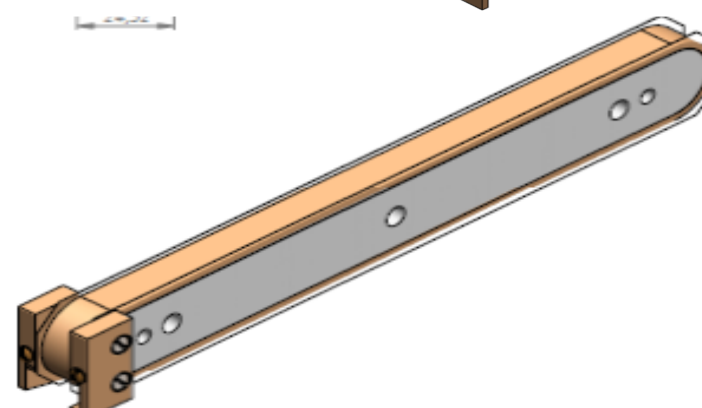
Two versions of the FF twin-aperture iron yoke quad prototype with 2 cm aperture and 100 T/m gradient are in production.







Saddle-shaped coils, complicated in production, the first coil failed. New winding device is in development.



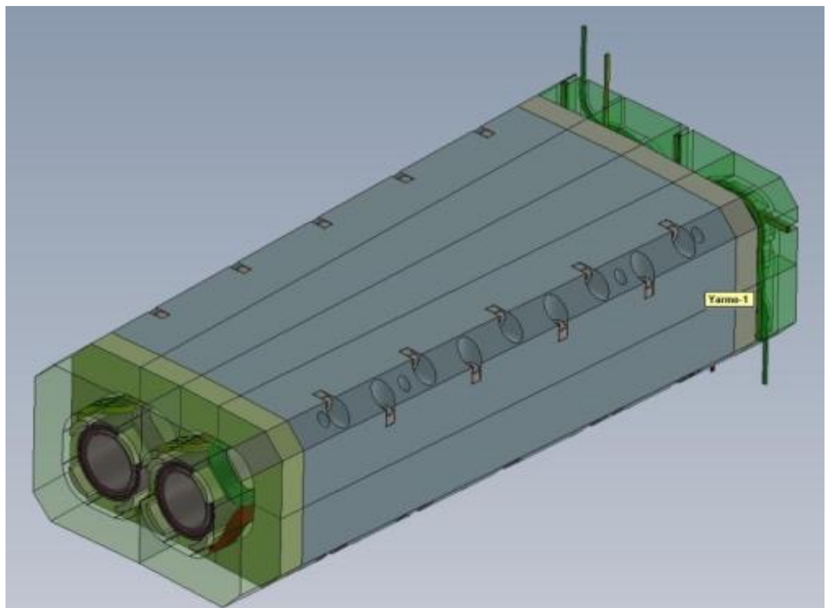
Straight coil, successfully wound and tested (650 A instead of the nominal 400 A)

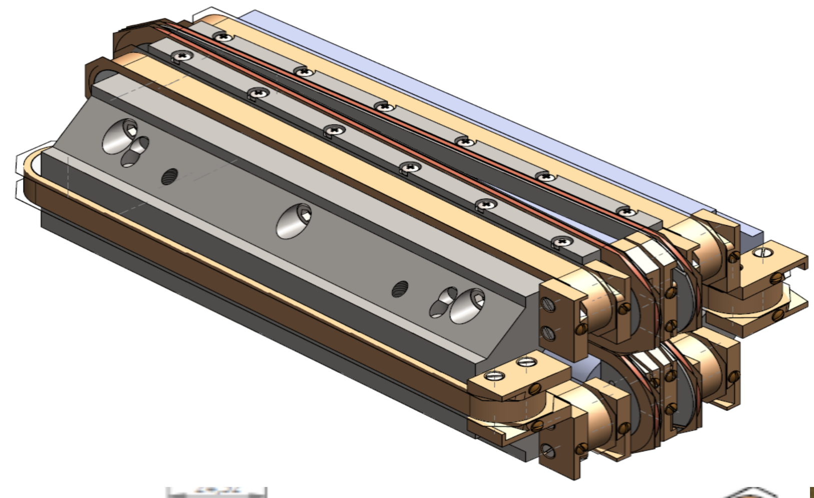
E. Levitchev

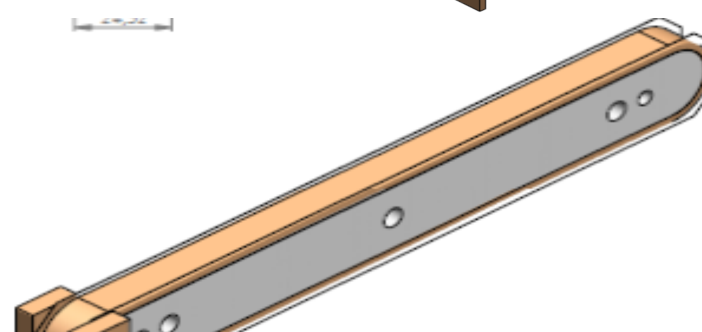
SC final focus quadrupole at BINP

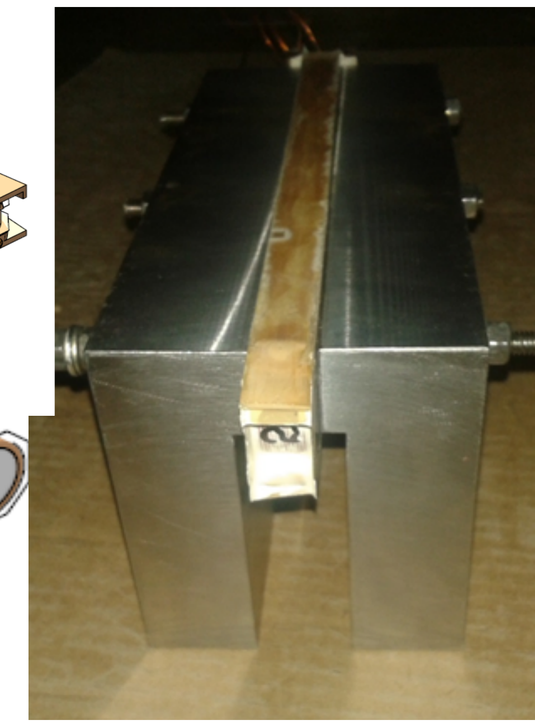
Main contributors are Ivan Okunev and Pavel Vobly

Two versions of the FF twin-aperture iron yoke quad prototype with 2 cm aperture and 100 T/m gradient are in production.









Saddle-shaped coils
con
pro
faile
dev

Also prototyping of CCT quadrupoles has started at CERN (M. Koratzinos, G. Kirby).

successfully wound
50 A instead of the
A)

E. Levitchev

Another QD0 prototype

A new version of QD0 was developed at BINP recently and a single-aperture prototype was manufactured.

Main parameters:

Max.gradient 100 T/m

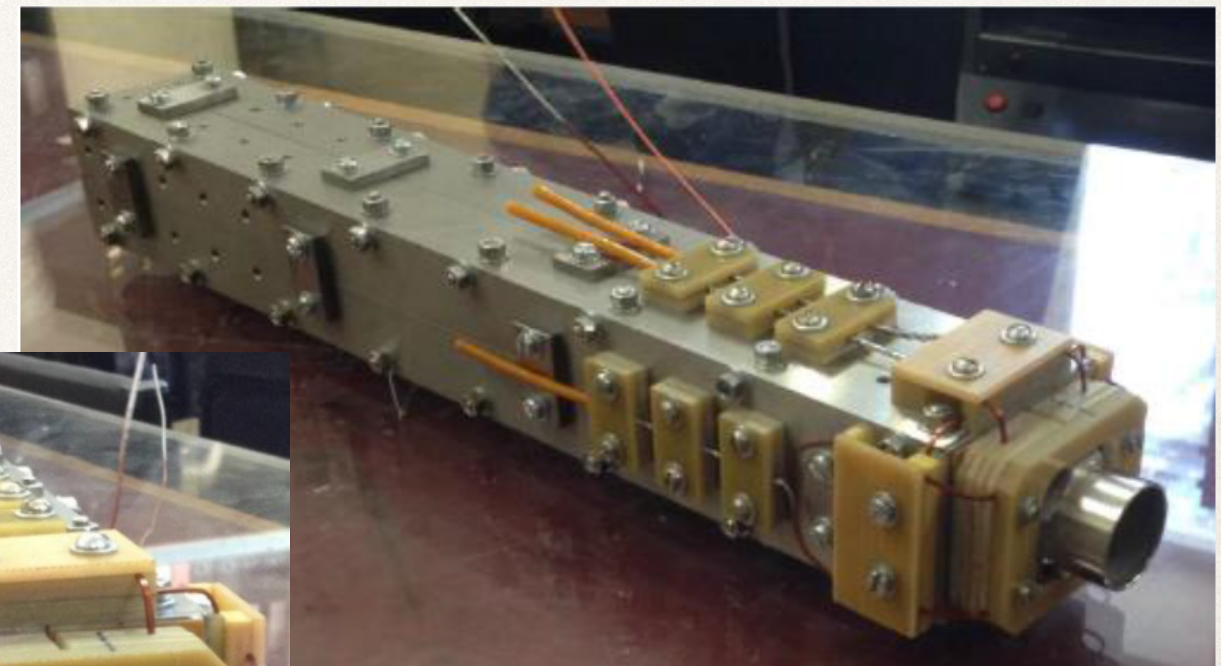
Max.current 1100 A

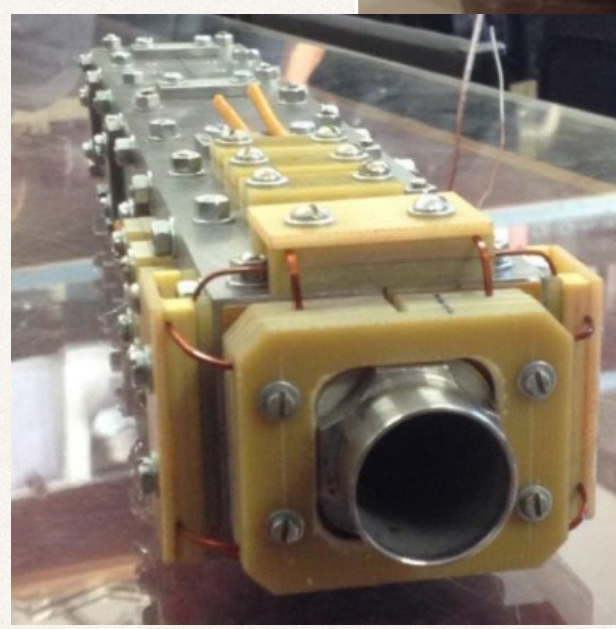
Length 40 cm

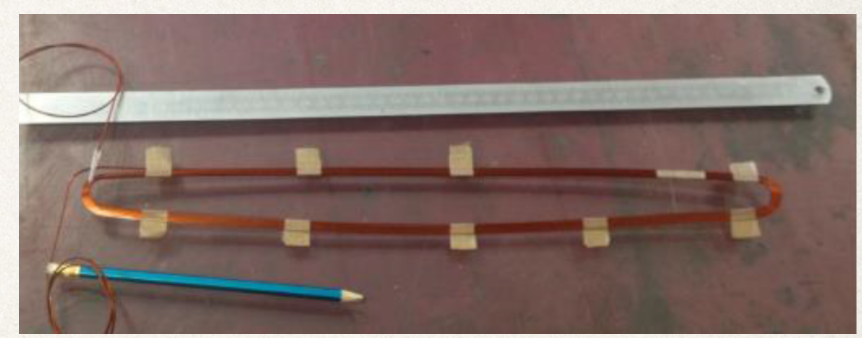
Aperture 2 cm

NbTi 1.8 x 1.4 mm^2

Saddle-type coils

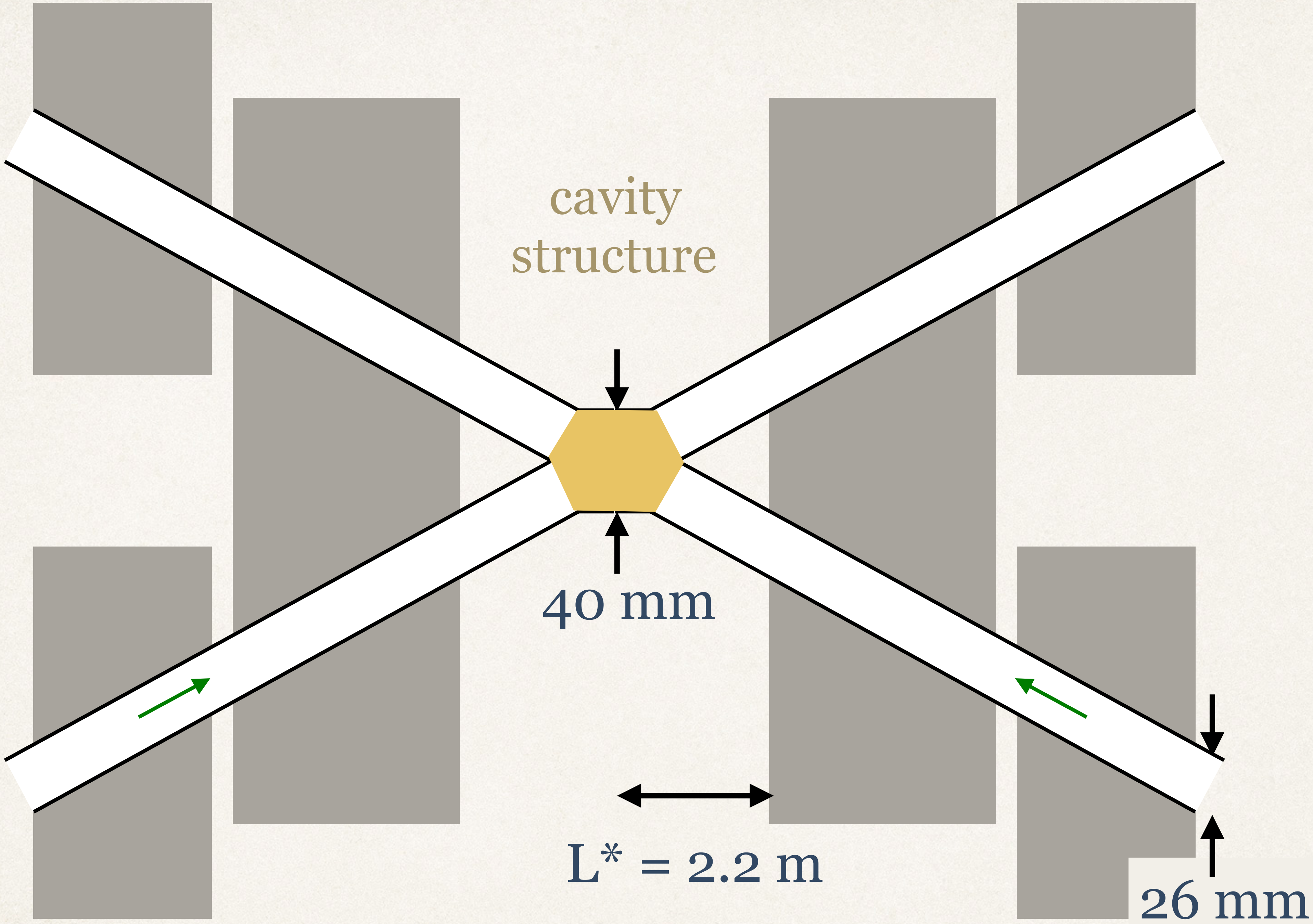






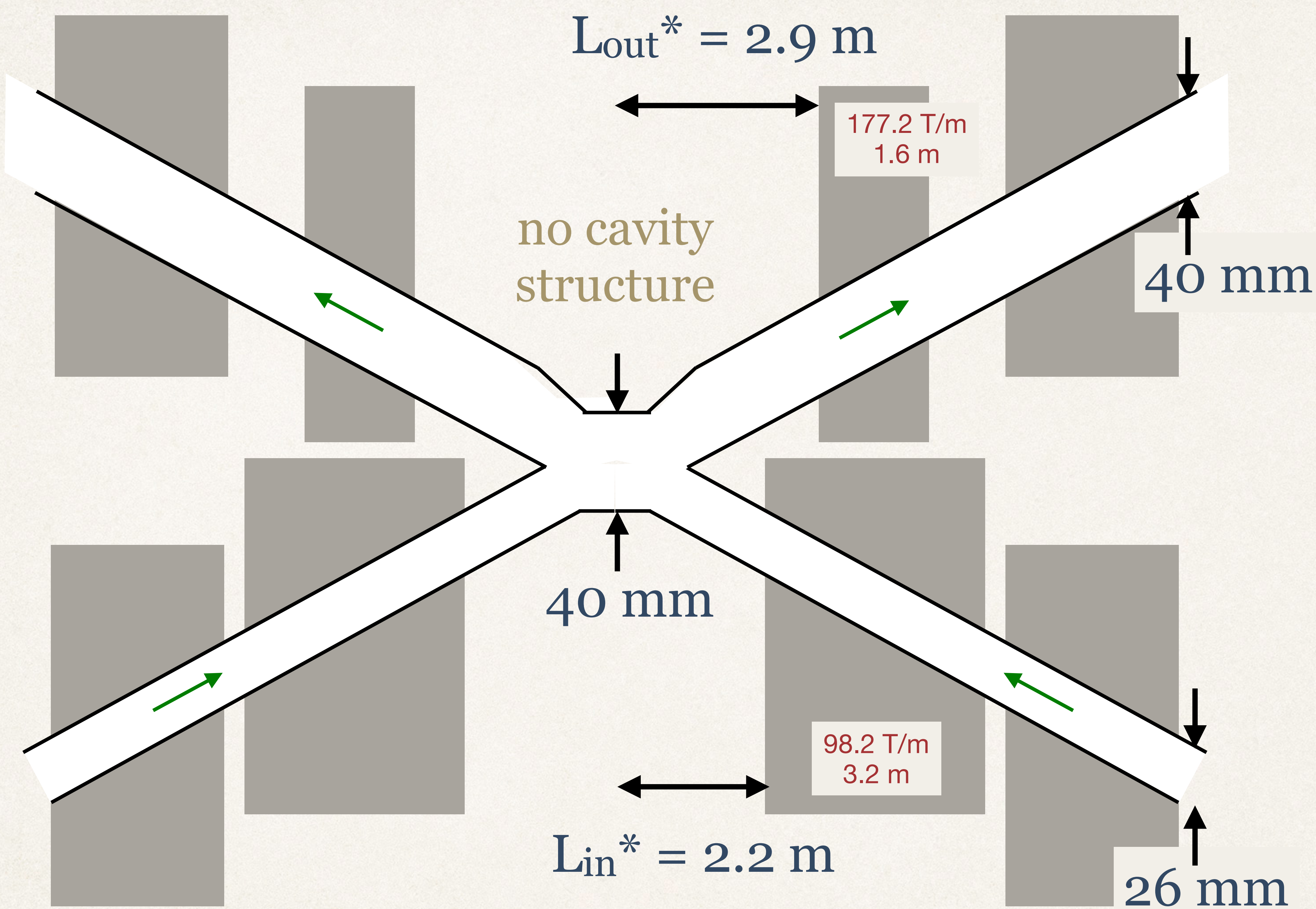
During the first cryo-test (01.02.16) the current of 1060 A was achieved after 3 quenches.

HOM trapping by the cavity structure at IP



- HOM is trapped in the IP beam pipe, if all beam pipes are narrower than the IP, which needs to be larger that 40 mm (M. Sullivan).
- Heating, esp. at Z.
- Leak of HOM to the detector, through the thin Be beam pipe at the IP.

Asymmetric L*: larger outgoing beam pipe & thinner final quads

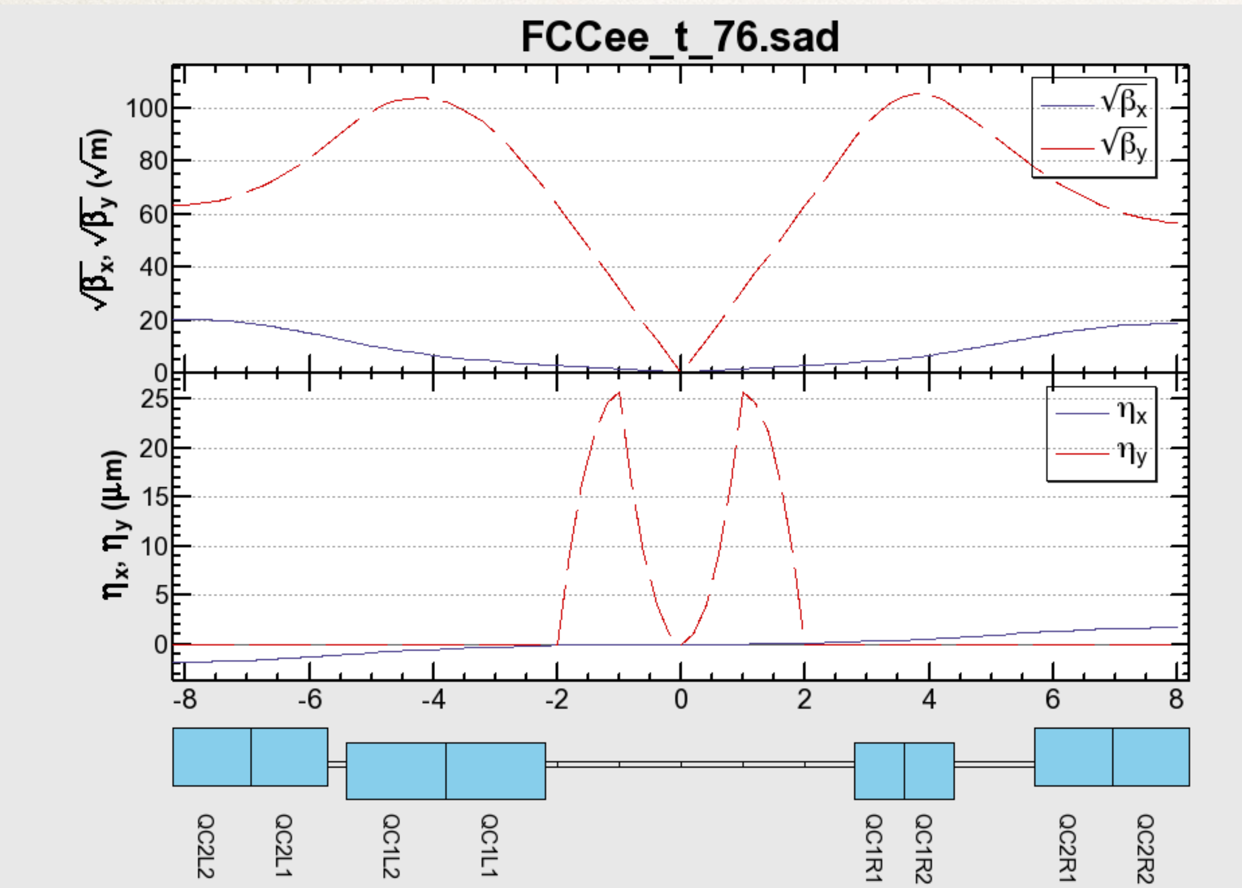


- The HOM can escape to the outside through the outgoing beam pipe, which has a diameter not smaller than IP.
- The outgoing final quad becomes thinner and stronger (E. Levichev, S. Sinyatkin).

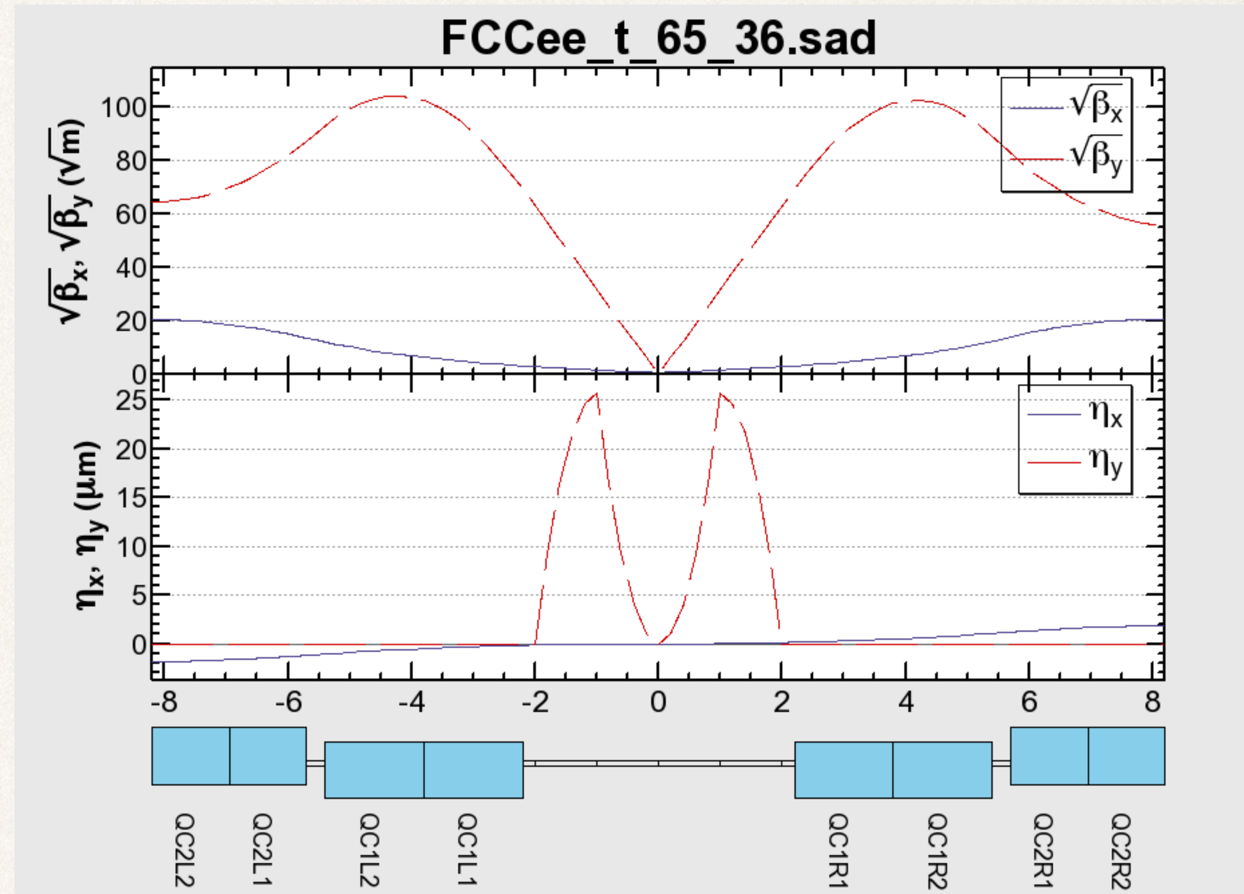
Optics at the IP

175 GeV, $\beta^*_{x,y}$ = (0.5 m, 1 mm)

Asymmetric L*

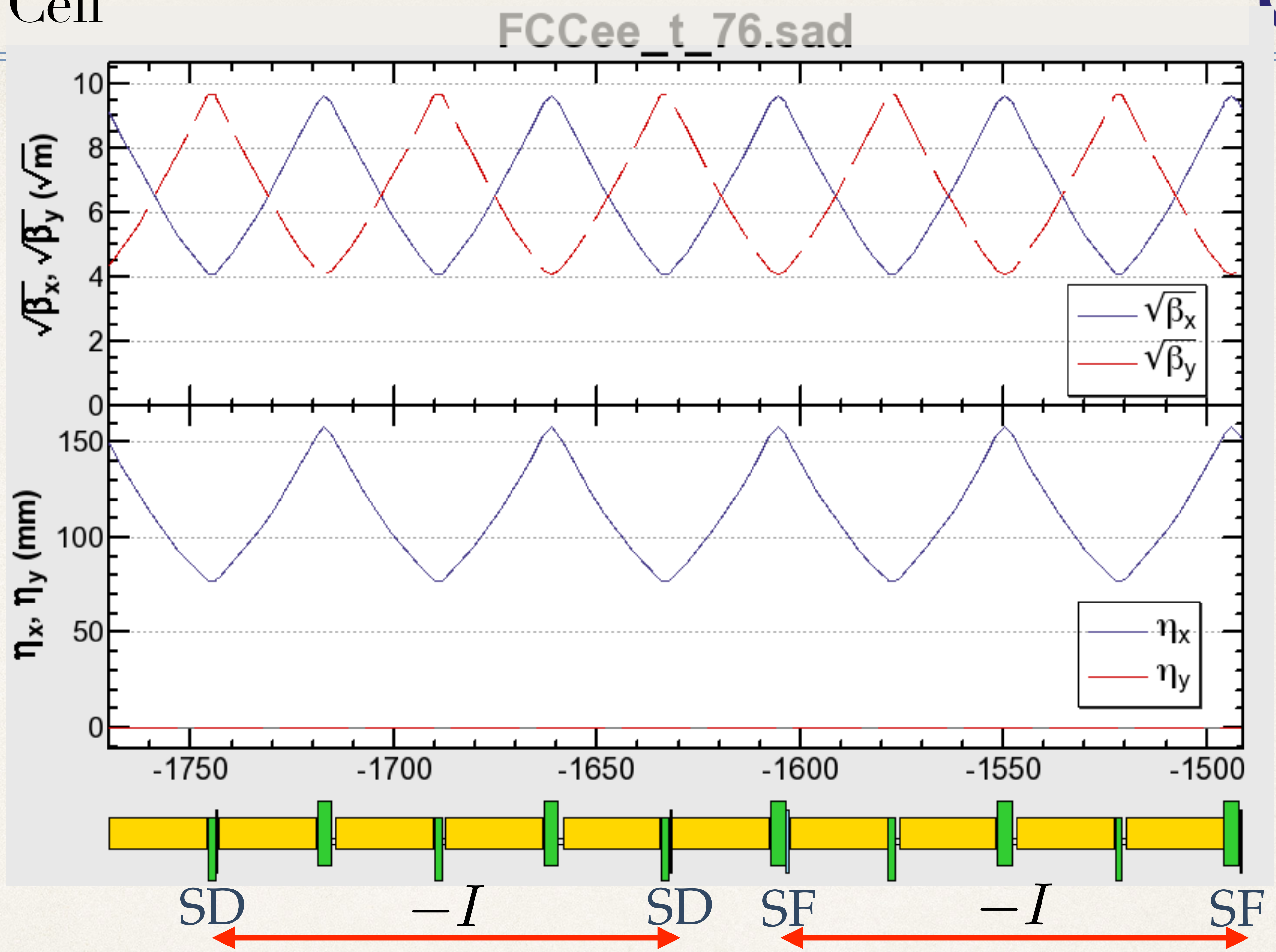


Symmetric L*



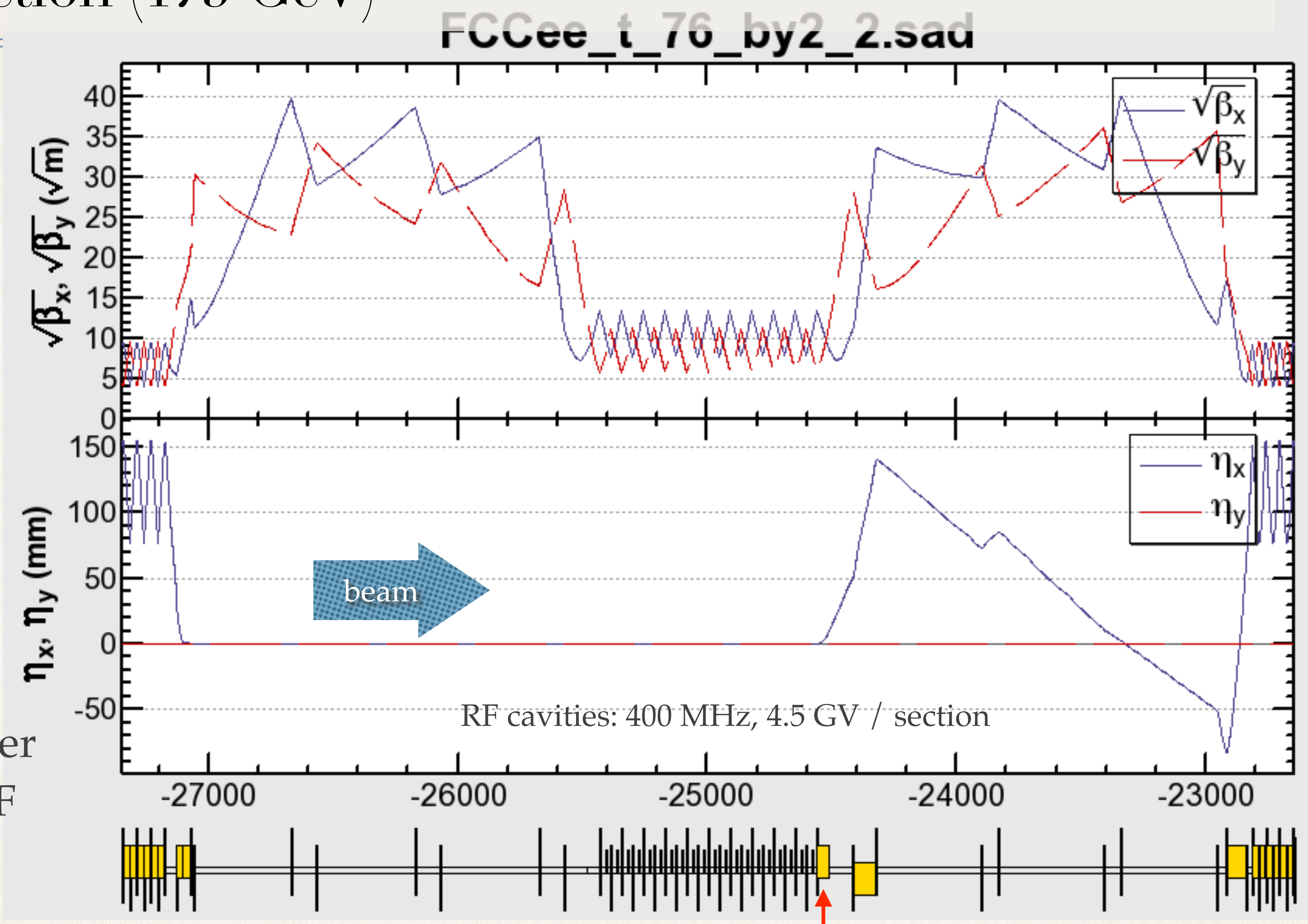
- Even with the asymmetric L*, the optics, so as the chromaticity, look similar.
- The solenoid compensation is unchanged: locally compensated up to 2.2 m from the IP.
- Longer L* downstream may give a space for a luminometer.

The Arc Cell



- Basically a 90/90 degree FODO cell.
- The quadrupoles QF/QD are 3.5 m/1.8 m long, respectively, to reduce the synchrotron radiation. They also depends on the design of quads and the beam pipe (A. Milanese, F. Zimmermann).
- All sextupoles are paired with -*I* transformation.
- 292 sextupole pairs per half ring.

The RF section (175 GeV)



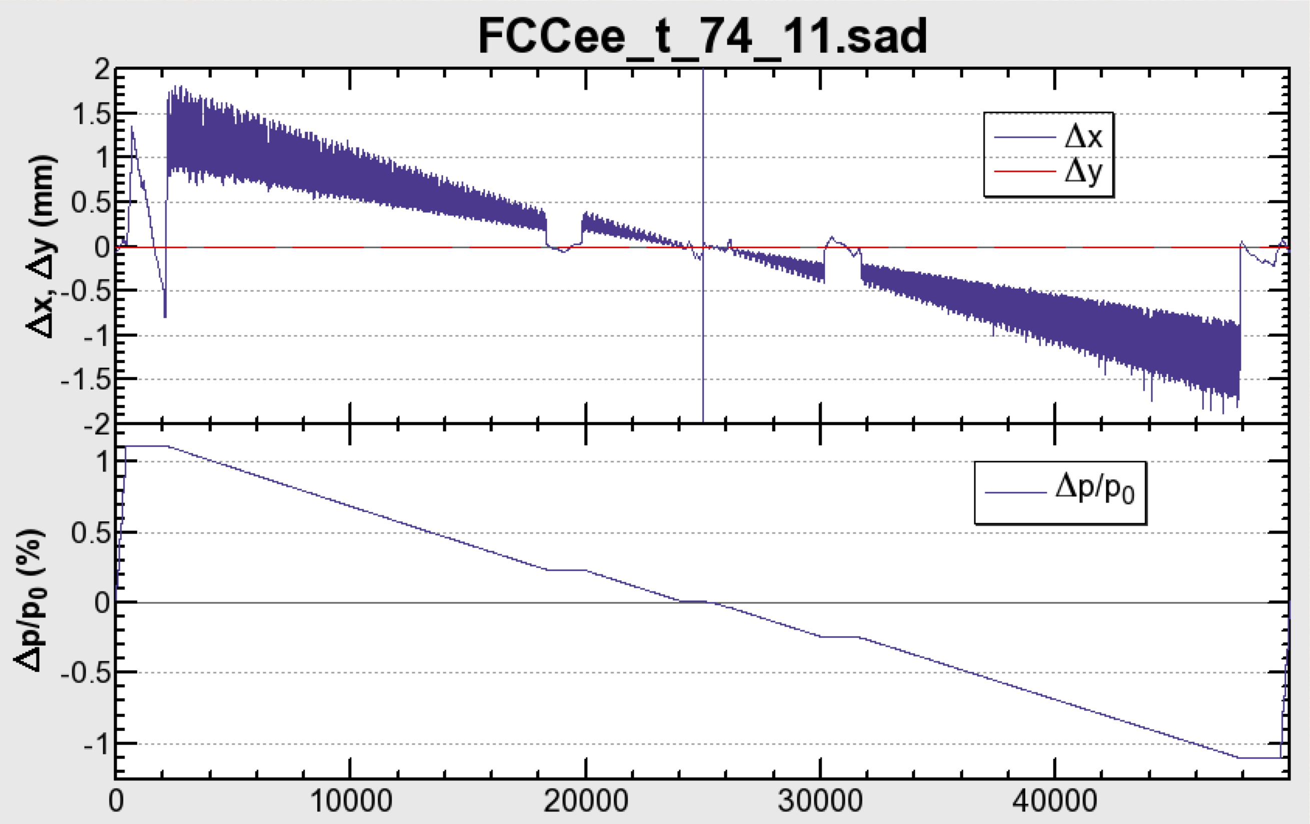
Beams cross over through the RF section.

An electrostatic separator, combined with a dipole magnet

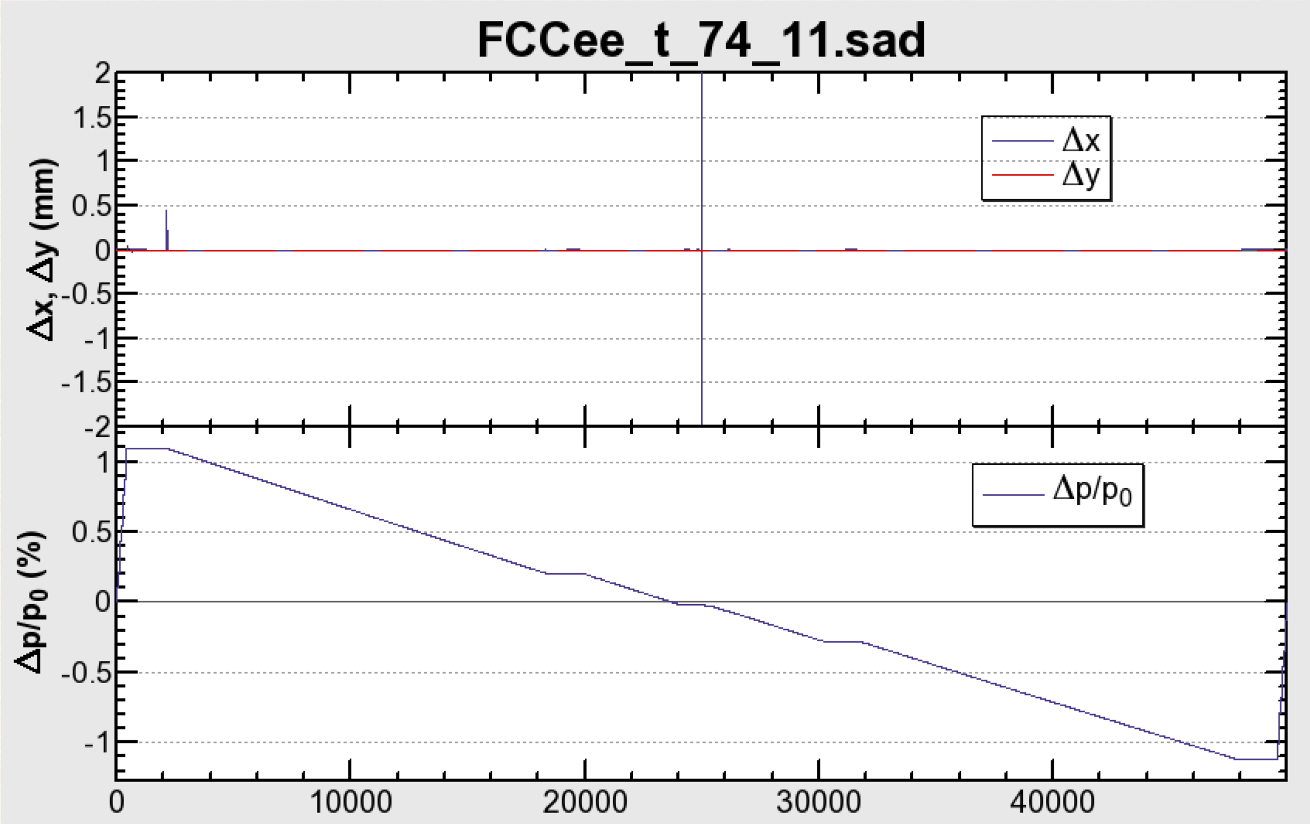
- The usage of the straights on the both sides of the RF is to be determined.
- If the nominal strengths of quads are symmetrical in the common section, it matches to the optics of both beam.
- This section is compatible with the RF staging scenario. For lower energy, the common RF and cross over will not be necessary.

The Sawtooth & Tapering (175 GeV)

No Taper



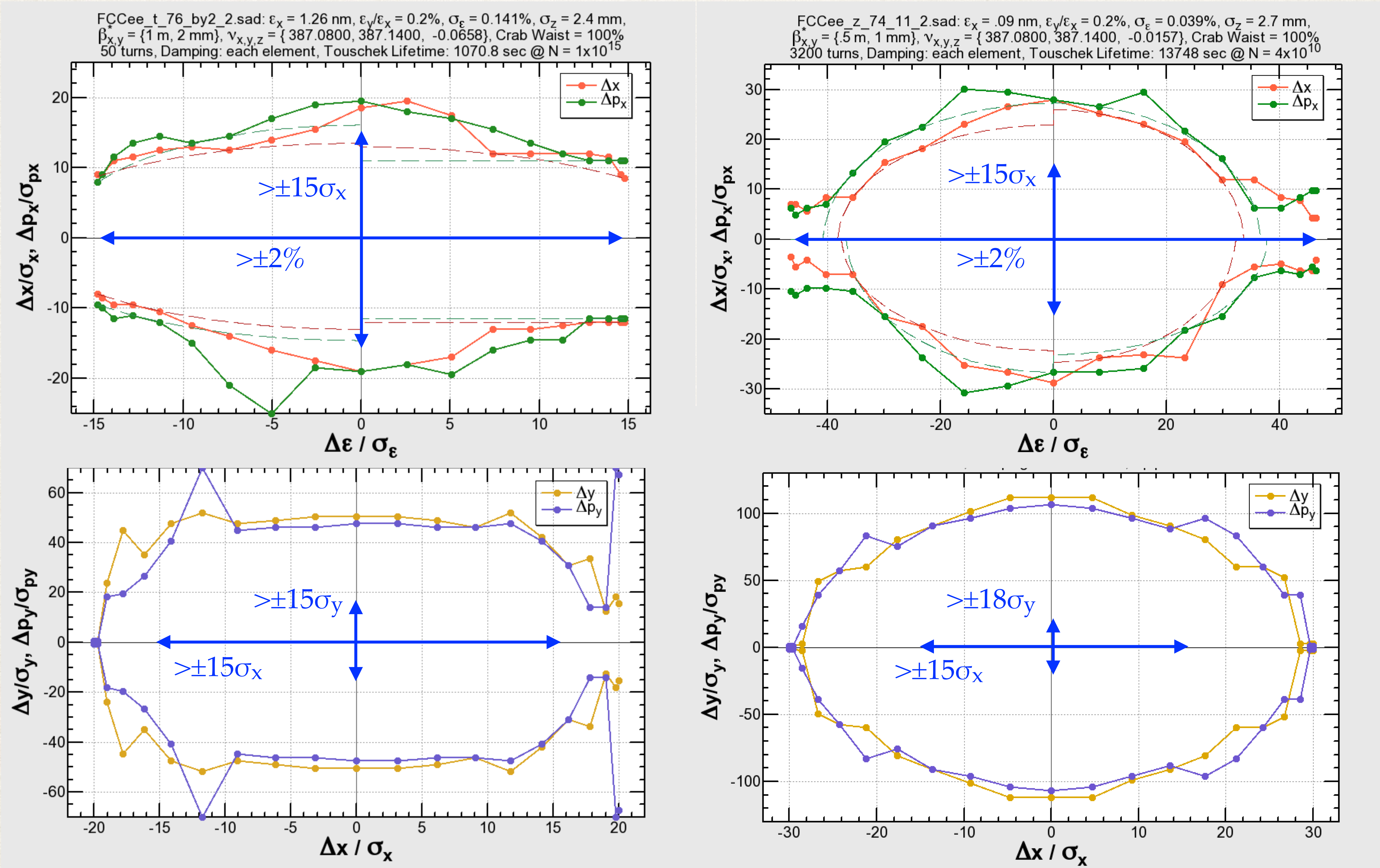
Tapered



- The change of the orbit due to energy loss along the arc causes serious deformation on the optics, causing the loss of the dynamic aperture.
- Everything can be cured almost completely by "tapering", i.e. scaling the strengths of all magnets along the local energy of the beam: this is one of the best merits of a double-ring collider (F. Zimmermann).

Dynamic Aperture satisfies the requirements.

175 GeV, $\beta^*_{x,y}$ = (1 m, 2 mm)   45.6 GeV, $\beta^*_{x,y}$ = (0.5 m, 1 mm)

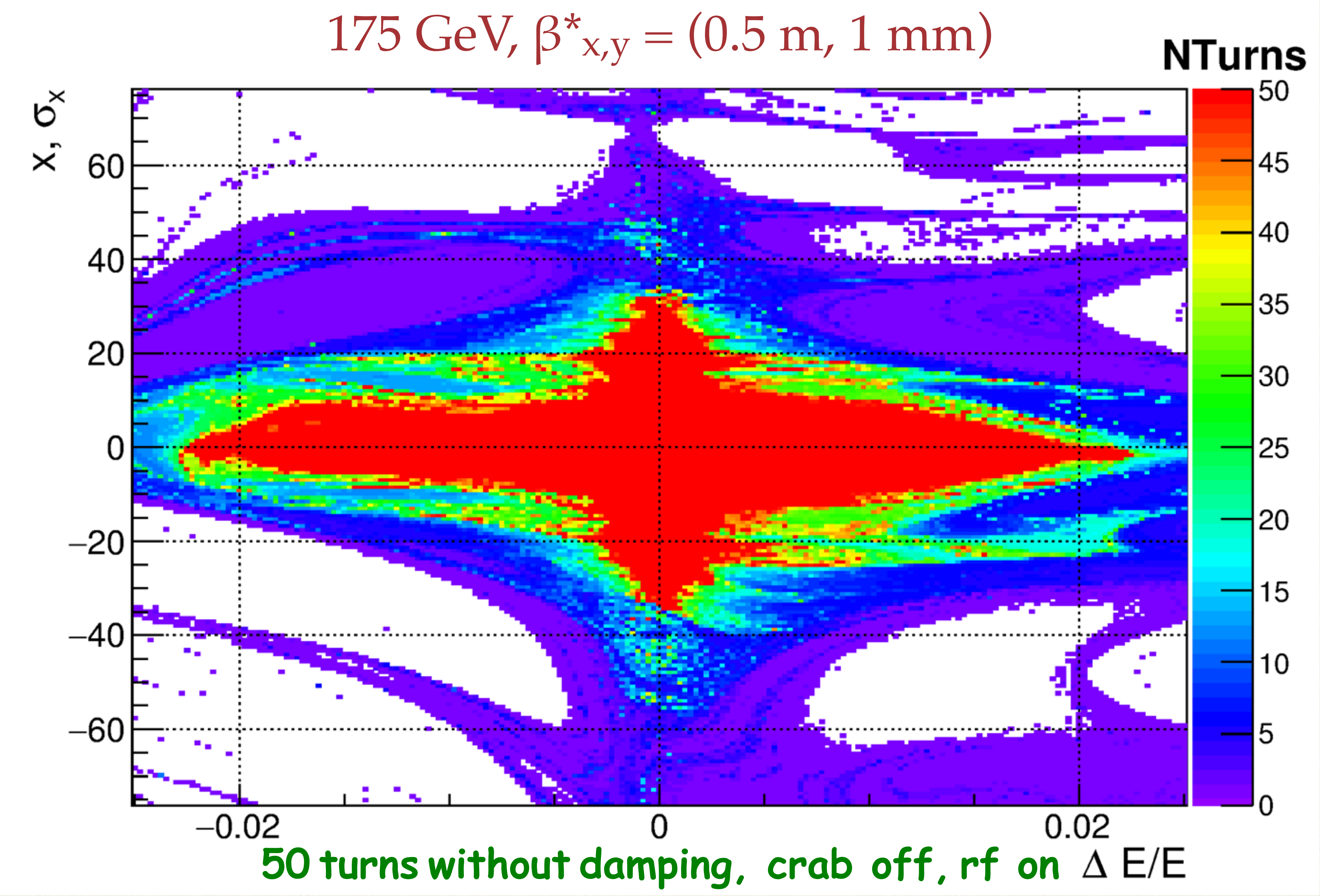


Requirements assuming the same horizontal emittance as the collider and 1% coupling from the booster:
$\Delta p/p$ > ±2%, Δx > 15σ_x, Δy > 15σ_y @ 175 GeV, $\Delta p/p$ > ±2%, Δx > 15σ_x, Δy > 18σ_y @ 45.6 GeV (See M. Aiba's talk).

Effects included in the dynamic aperture survey

Effects	included?	significance for DA in FCC-ee @ 175 GeV
synchrotron motion	yes	**essential**
radiation damping (turn by turn)	yes	**essential** aperture↑
radiation damping (each element, esp. quads)	yes (no fluctuation yet)	**essential** aperture↓
"tapering"	yes	**essential**
crab waist	yes	yes, aperture↓
solenoids	yes	minimal, if locally compensated
Maxwellian fringe field	yes	small
kinematical terms	yes	small
beam-beam	yes (weak-strong)	yes, esp. on lifetime (D. Zhou)
errors/misalignments	not yet	**essential**, correction schemes must be developed

Dynamic Aperture for the AB Lattice

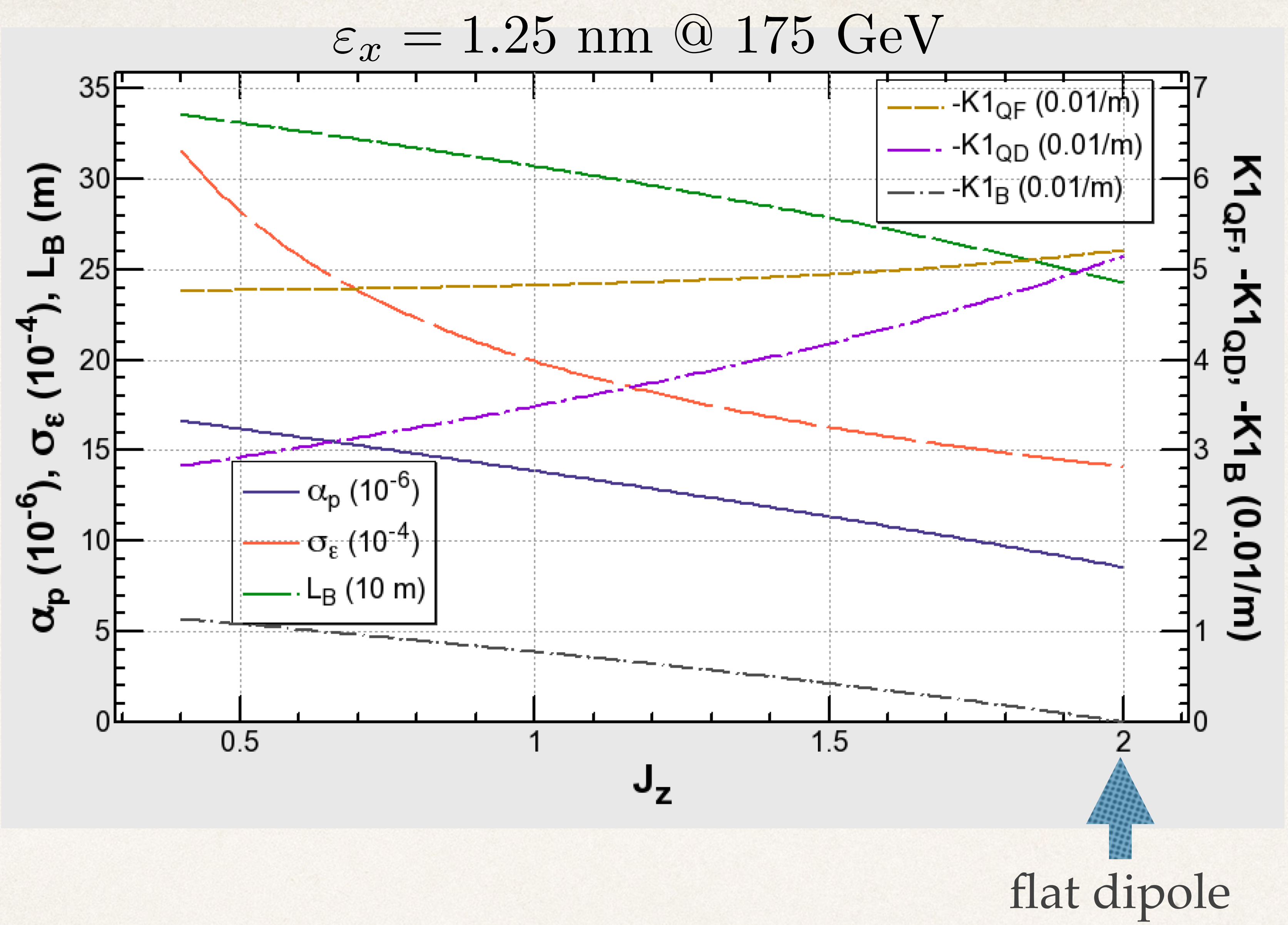


ε_x=1.4 nm·rad, 0.2% coupling, τ_x=44 turns, ν_s=0.0807, Urf=11 GV

The dynamic aperture for the AB lattice is under optimization, and looks promising so far.

P. Piminov, A. Bogomyagkov

A possibility of combined function dipole in the arc



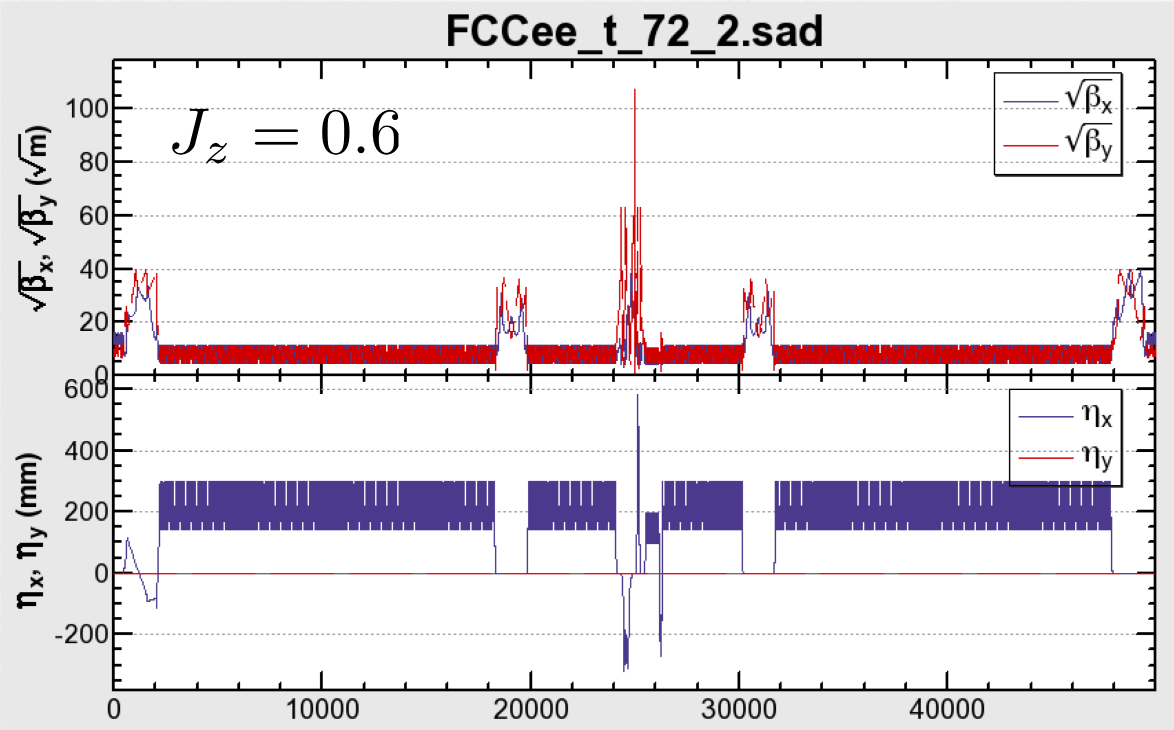
A negative field gradient in the main dipole of the unit cell provides:

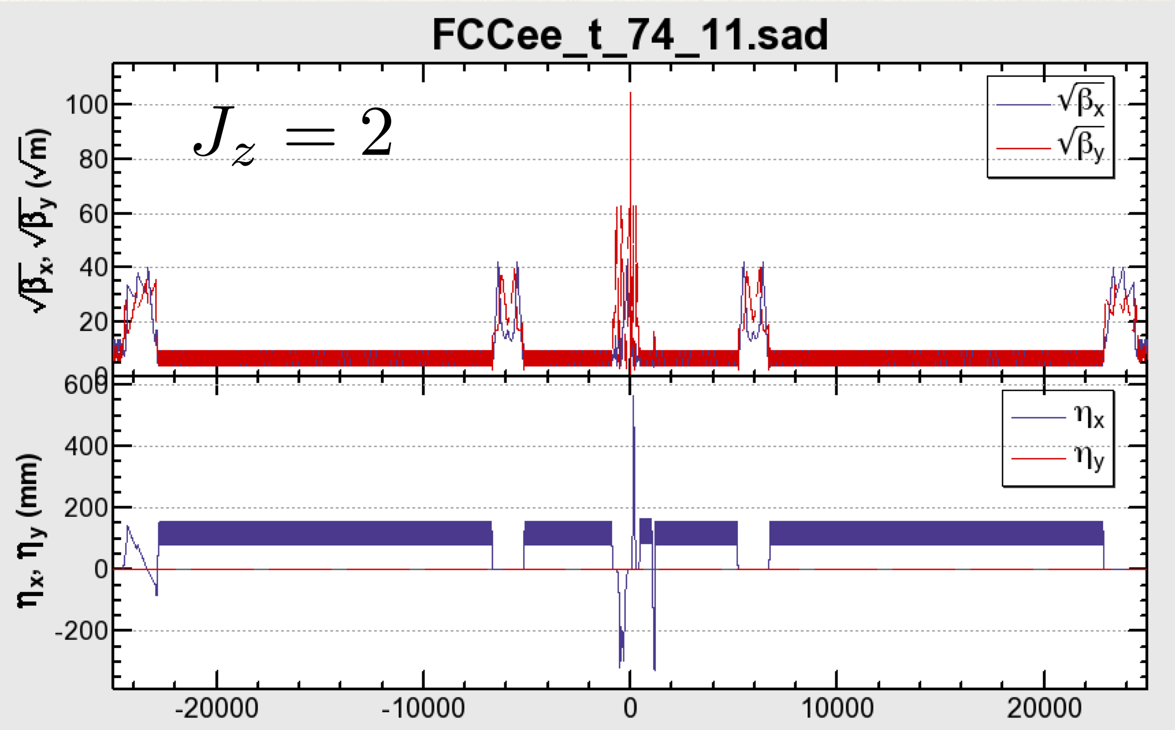
- longer cell length for a given emittance / better packing factor
- larger momentum compaction (longer bunch length for a same RF voltage)
- larger energy spread
- larger dispersion
- weaker sextupoles

Suggested by E. Levechev

An example of combined function: $J_z = 0.6$ @ 175 GeV

Jz	0.6	2
# of FODO cells	**1062**	1442
Length of dipole (m)	33.9	23.1
H dispersion at SF (cm)	**29.6**	16.3
1 turn energy loss (GV)	**7.09**	7.74
momentum spread (%)	0.24	0.14
momentum compaction (10^{-6})	**12.8**	7.2
bunch length (mm)	5.0	2.4
RF voltage (GV)	9.6	9.4
synchrotron tune	-0.10	-0.068

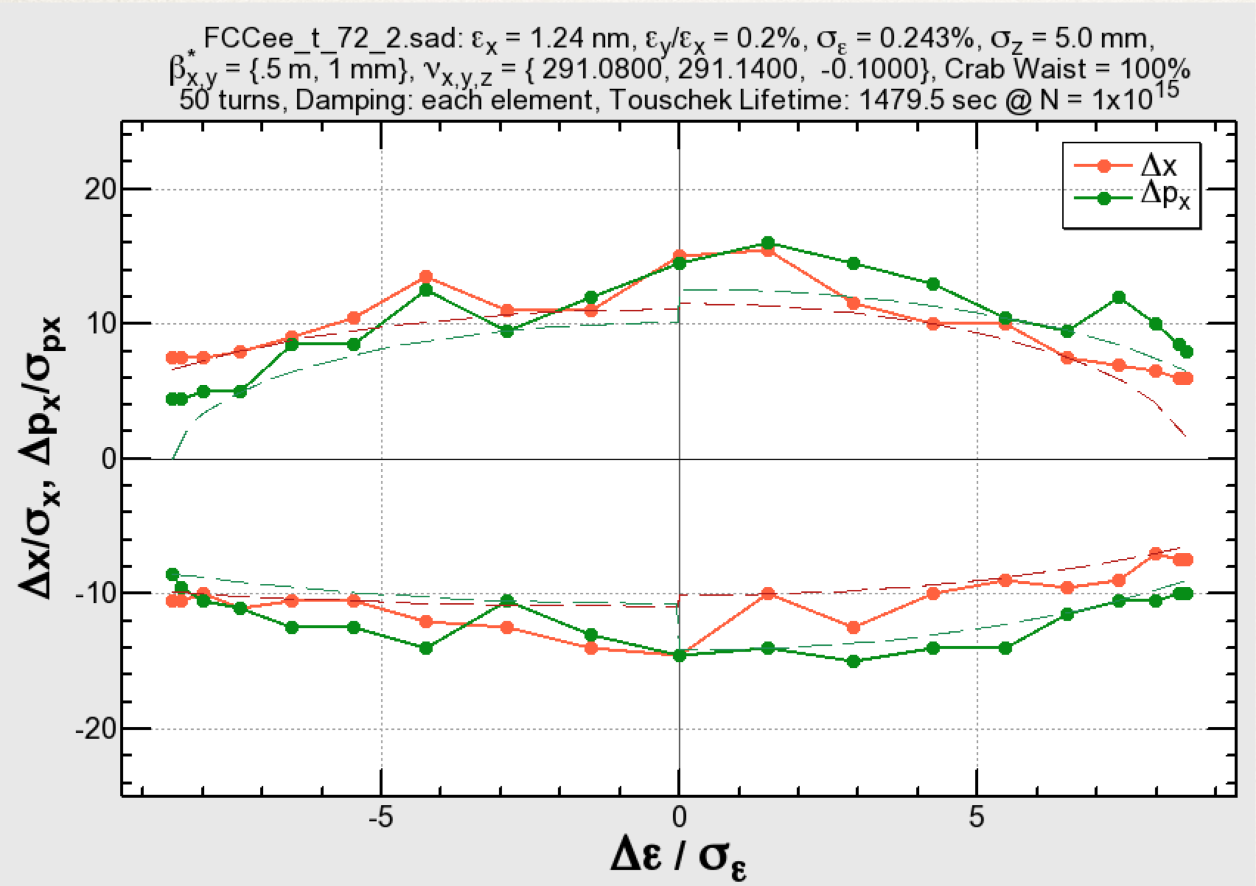




Dynamic aperture of combined function lattice.

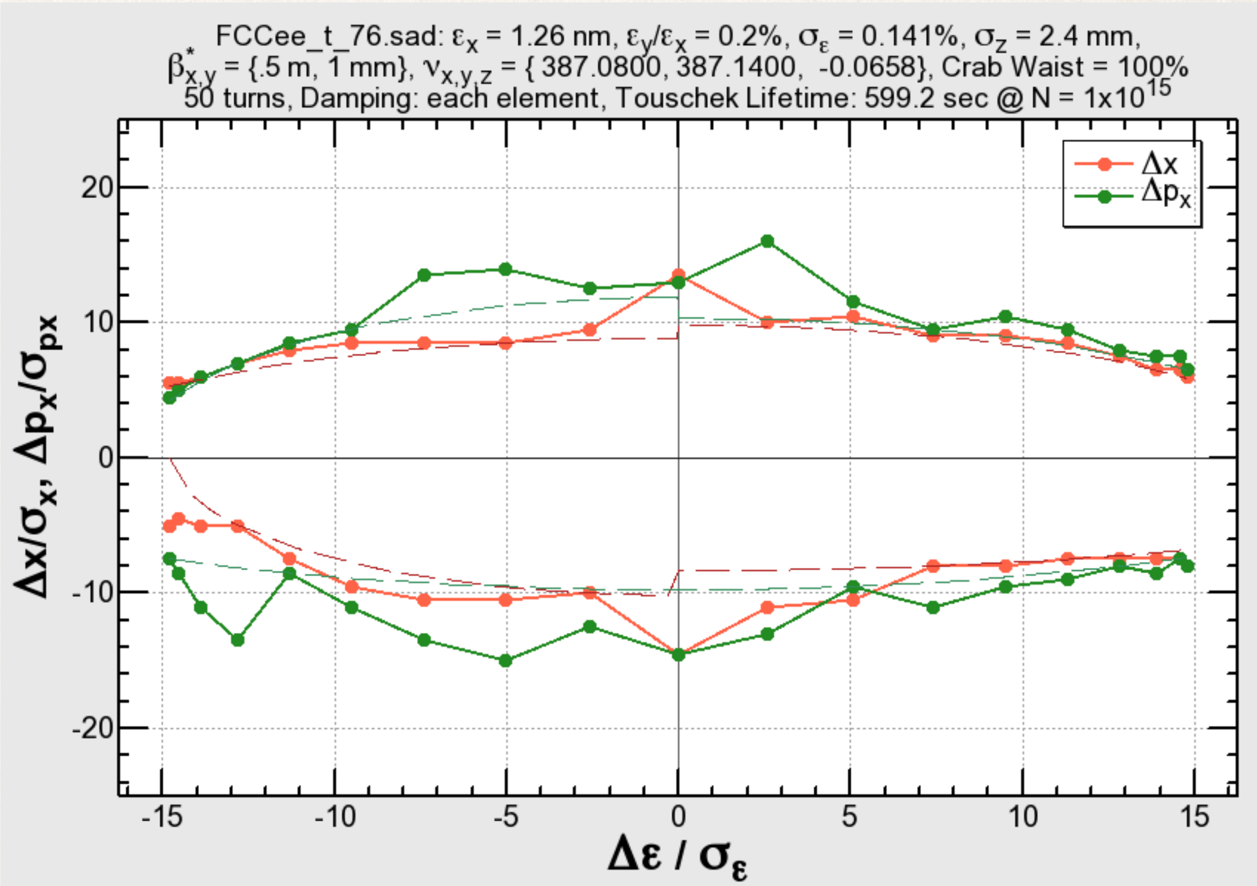
175 GeV, $\beta^*_{x,y} = (0.5\text{ m}, 1\text{ mm})$

Combined function dipole



±2%

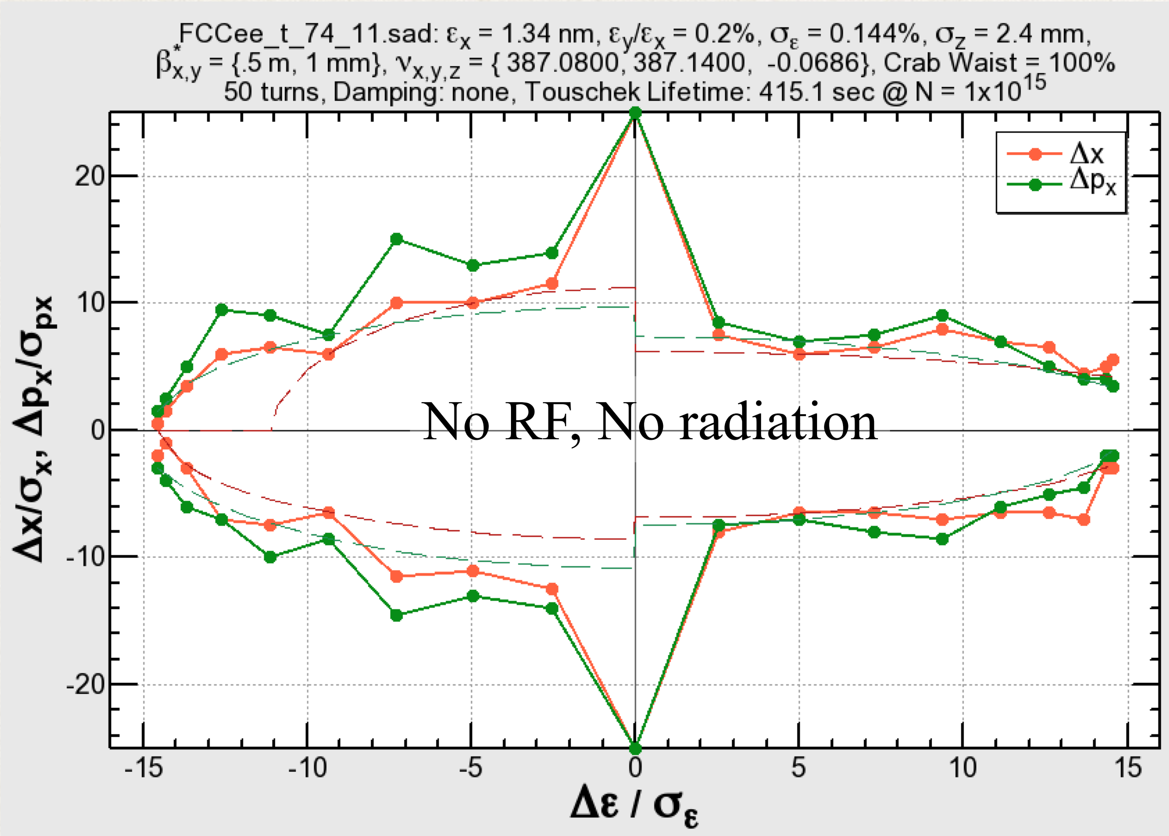
Flat dipole

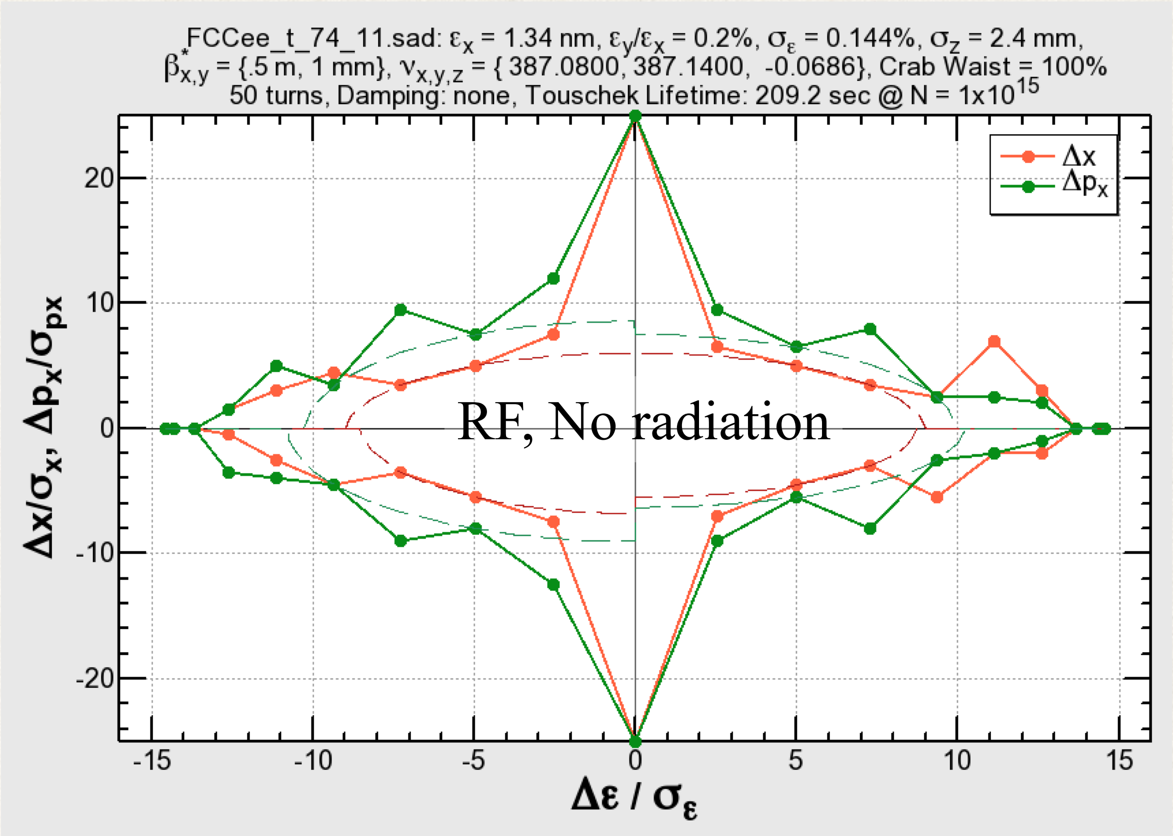


±2%

- The dynamic aperture is comparable to the flat-dipole lattice.
- Looking for beam-beam simulation and hardware solution of the dipole.

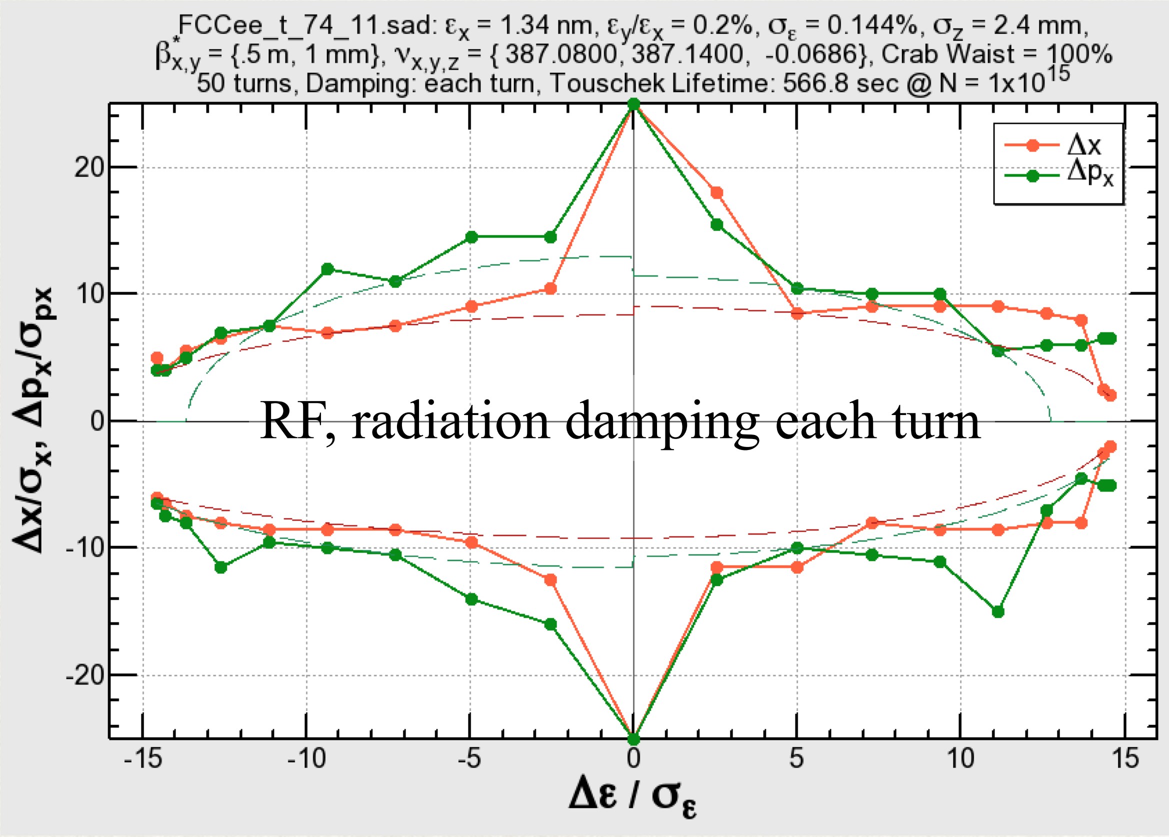
Several effects on the dynamic aperture

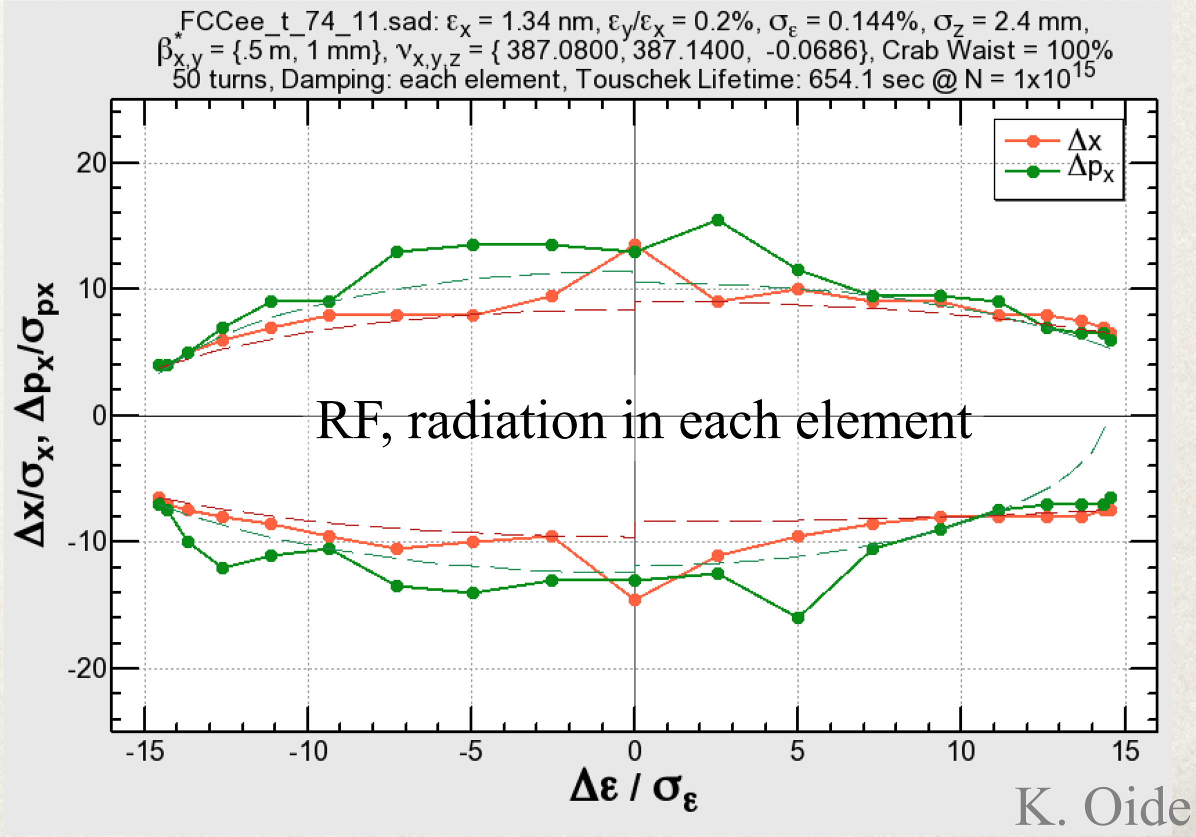




±2%

±2%

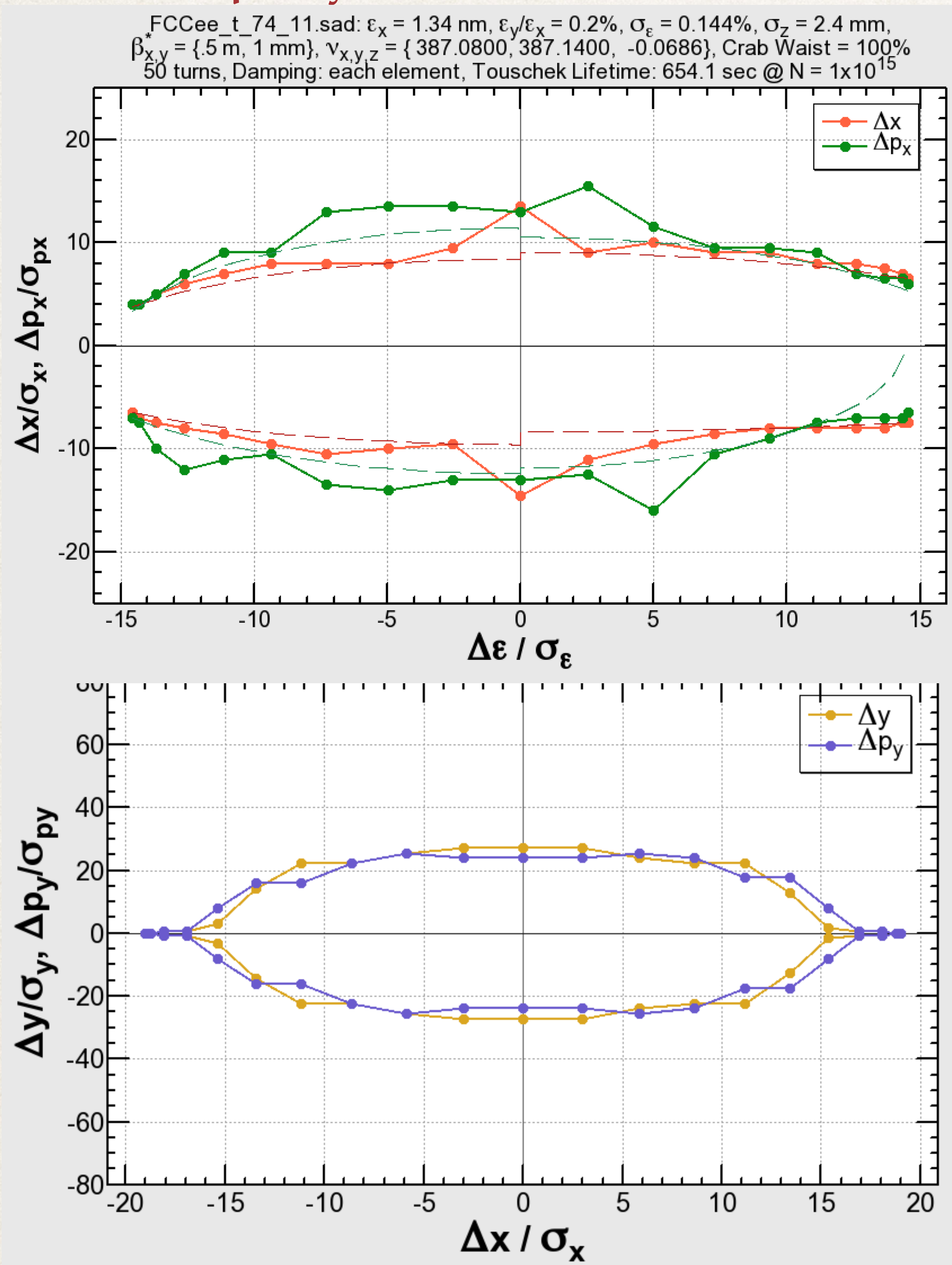




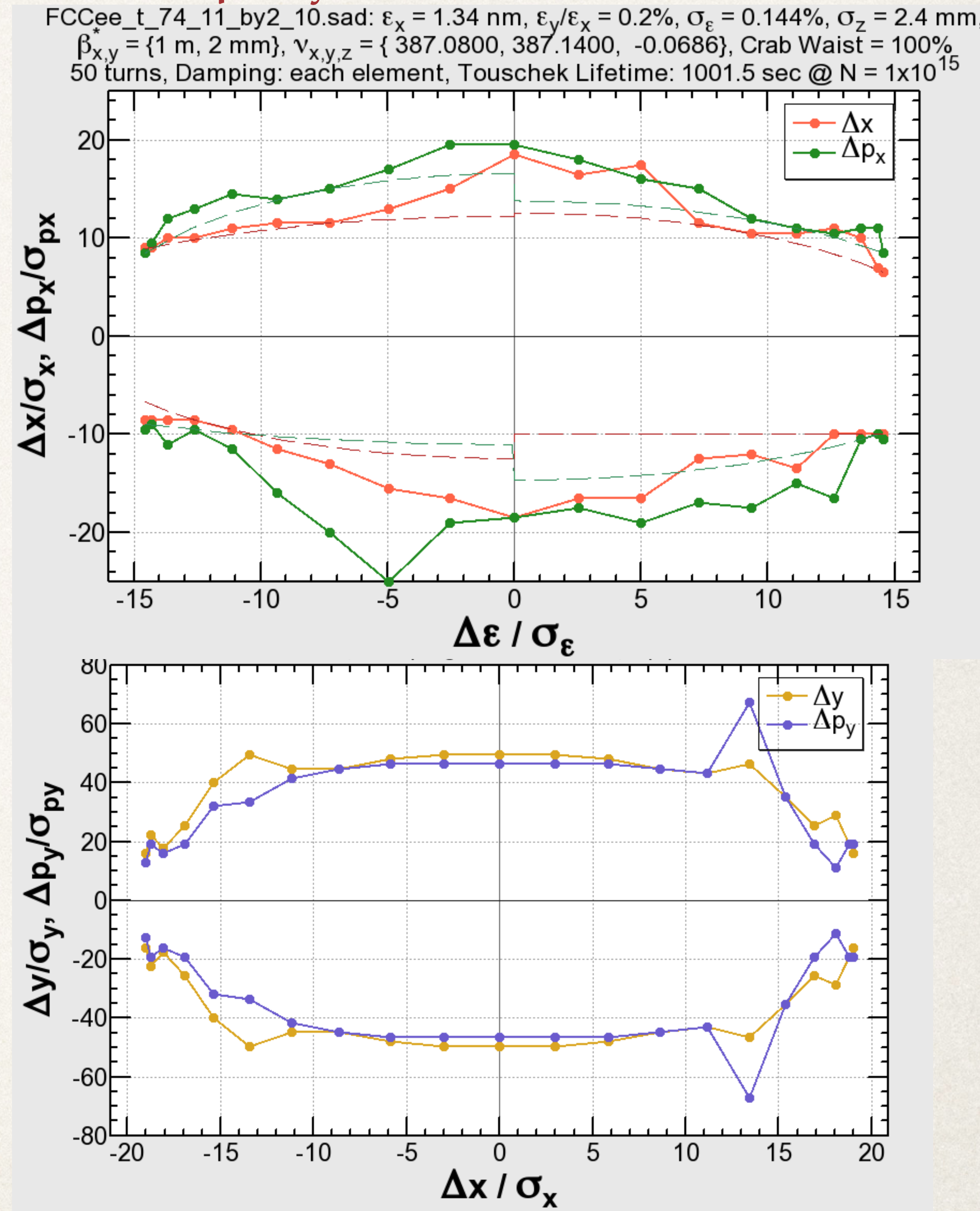
K. Oide

Several effects on the dynamic aperture (2)

$\beta^*_{x,y} = (0.5\text{ m}, 1\text{ mm})$



$\beta^*_{x,y} = (1\text{ m}, 2\text{ mm})$

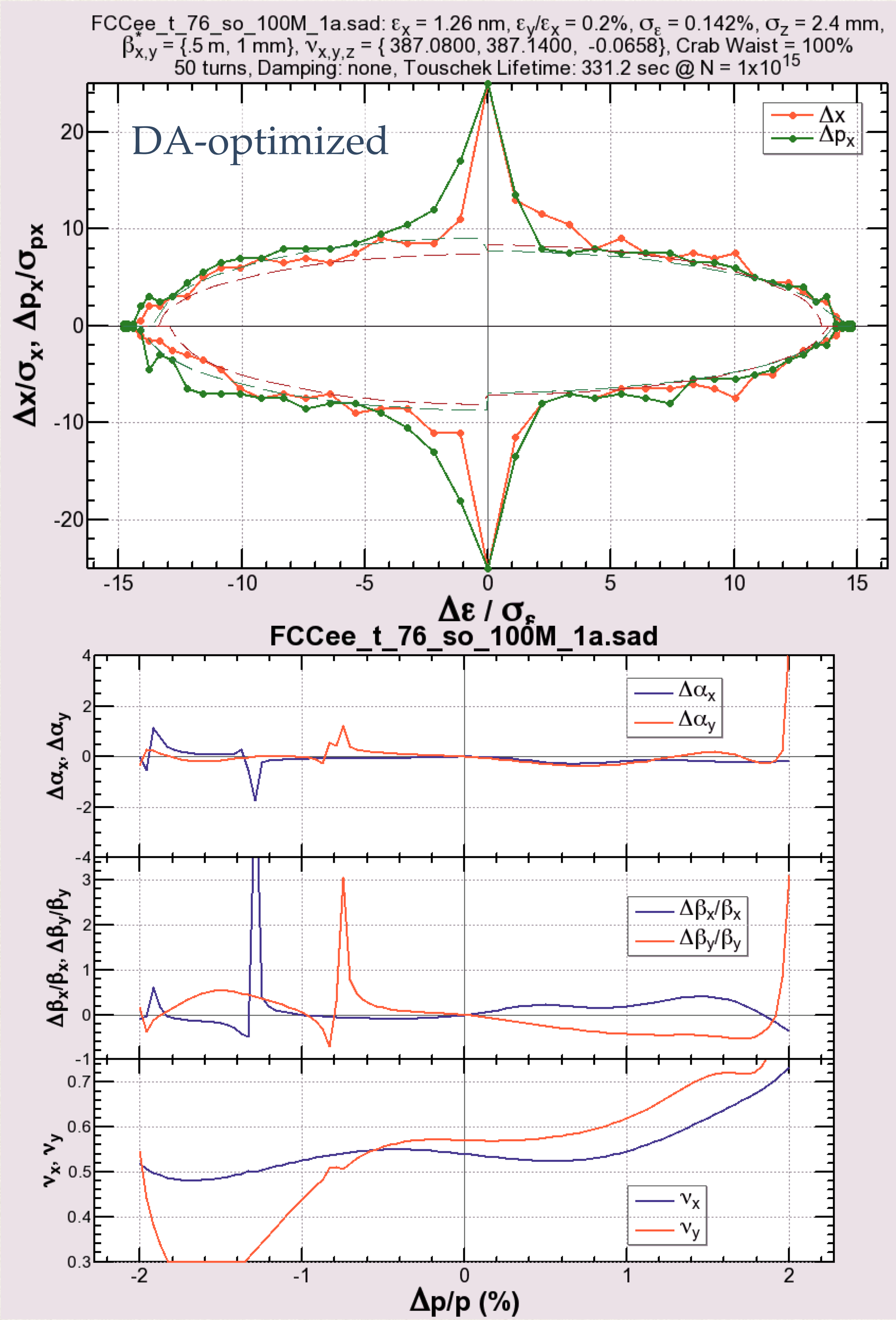


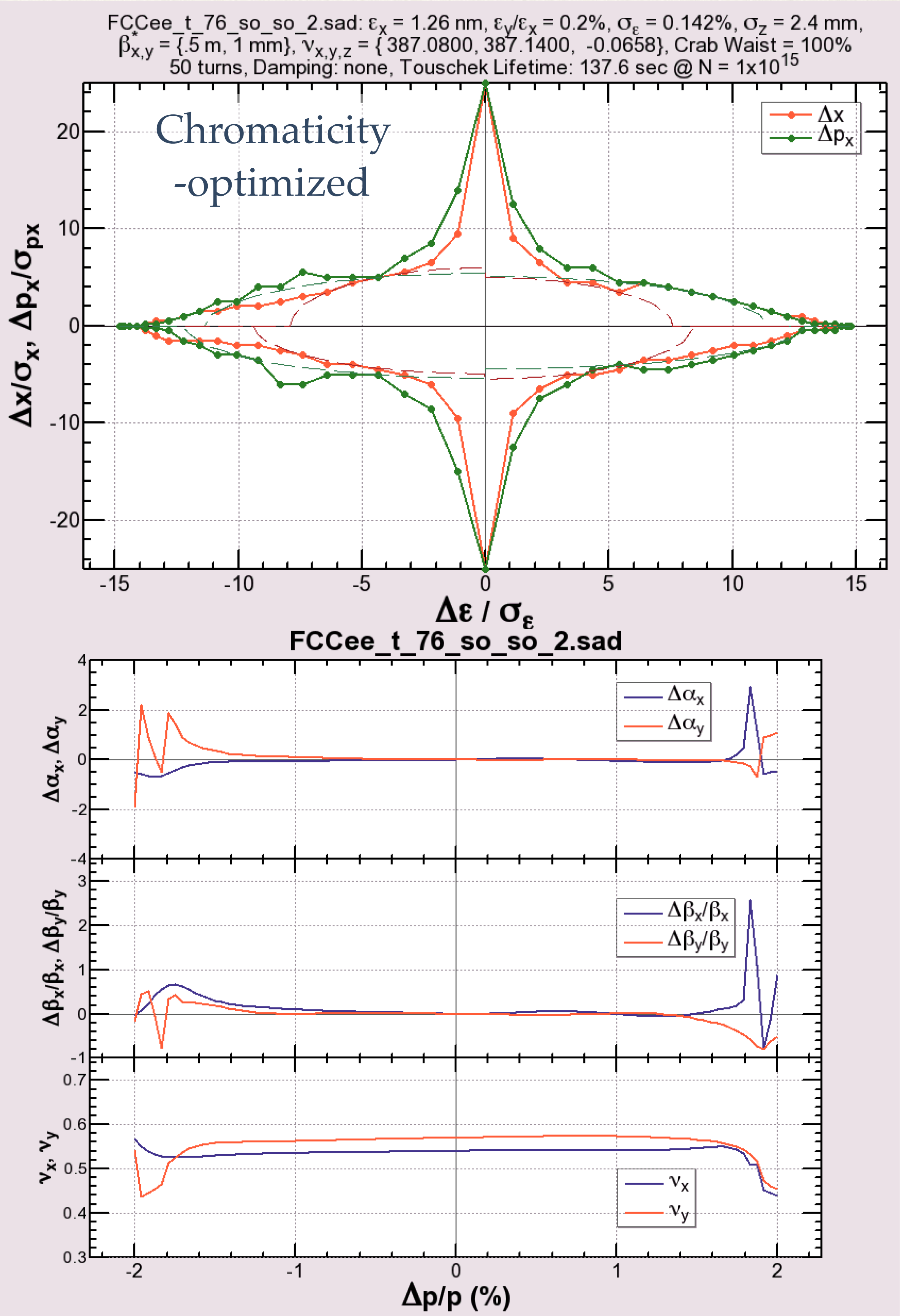
The reduction of the vertical aperture for $\beta^*_y = 1$ mm is due to the synchrotron radiation in the final quads.

K. Oide

Less chromaticity ≠ better dynamic aperture

$\beta^*_{x,y}$ = (0.5 m, 1 mm), no radiation damping





Transvers Optics in a period of synchrotron motion

Consider a transverse transfer matrix along a synchrotron motion:

$$M(\Delta_s) = M(\delta_n) \cdot M(\delta_{n-1}) \cdots M(\delta_0) \ , \tag{1}$$

where δ_k is the momentum deviation at the k-th turn of the synchrotron motion with an amplitude Δ_s:

$$\delta_k = \Delta_s \sin(k\mu_s) \ , \tag{2}$$

assuming a simple sinusoidal synchrotron motion. If we can approximate the synchrotron tune $\nu_s = \mu_s/2\pi$ by a rational number m/n, the transfer matrix $M(\delta_n)$ becomes periodic over m times the synchrotron period, then we can calculate Twiss parameters in a similar way to a usual periodic optic. Such an approximation can be done with a continued fraction of the synchrotron tune:

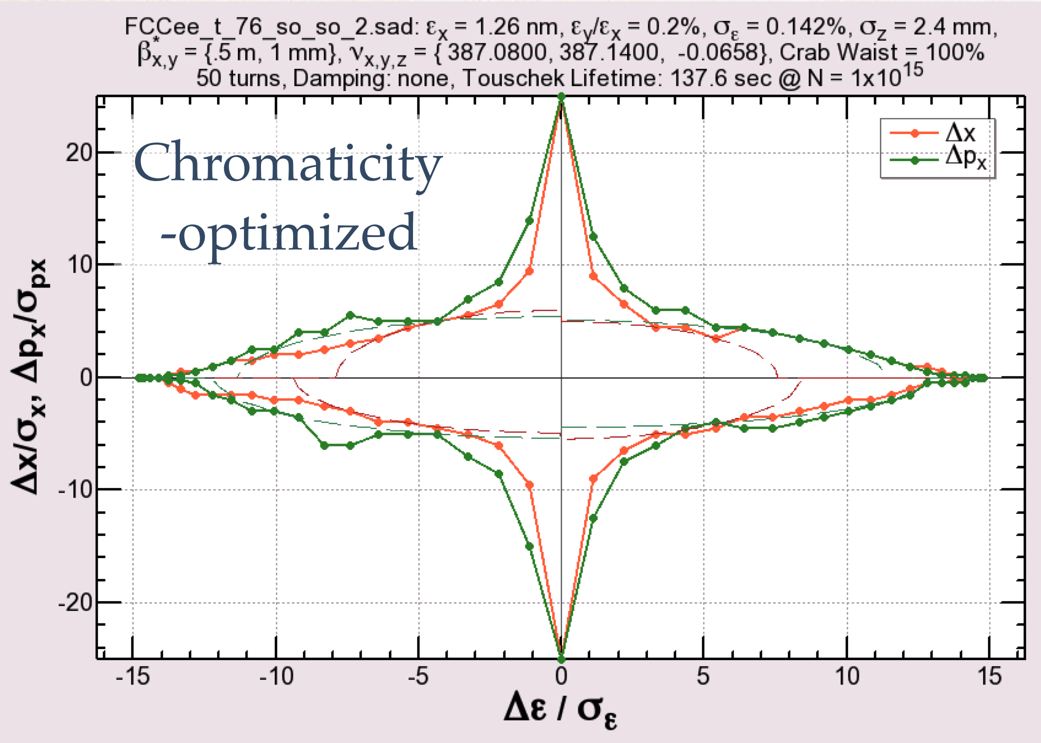
$$\nu_s = n_0 + \frac{1}{n_1 + \frac{1}{n_2 + \ldots}} \ . \tag{3}$$

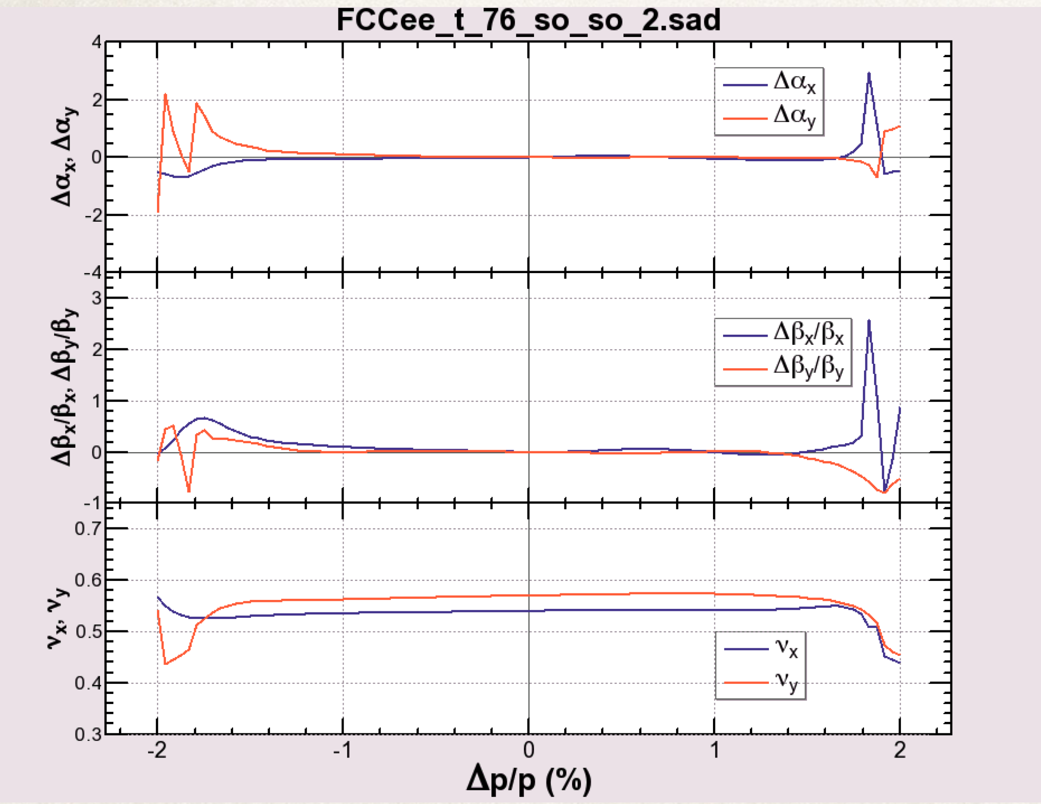
In these example optics we use,

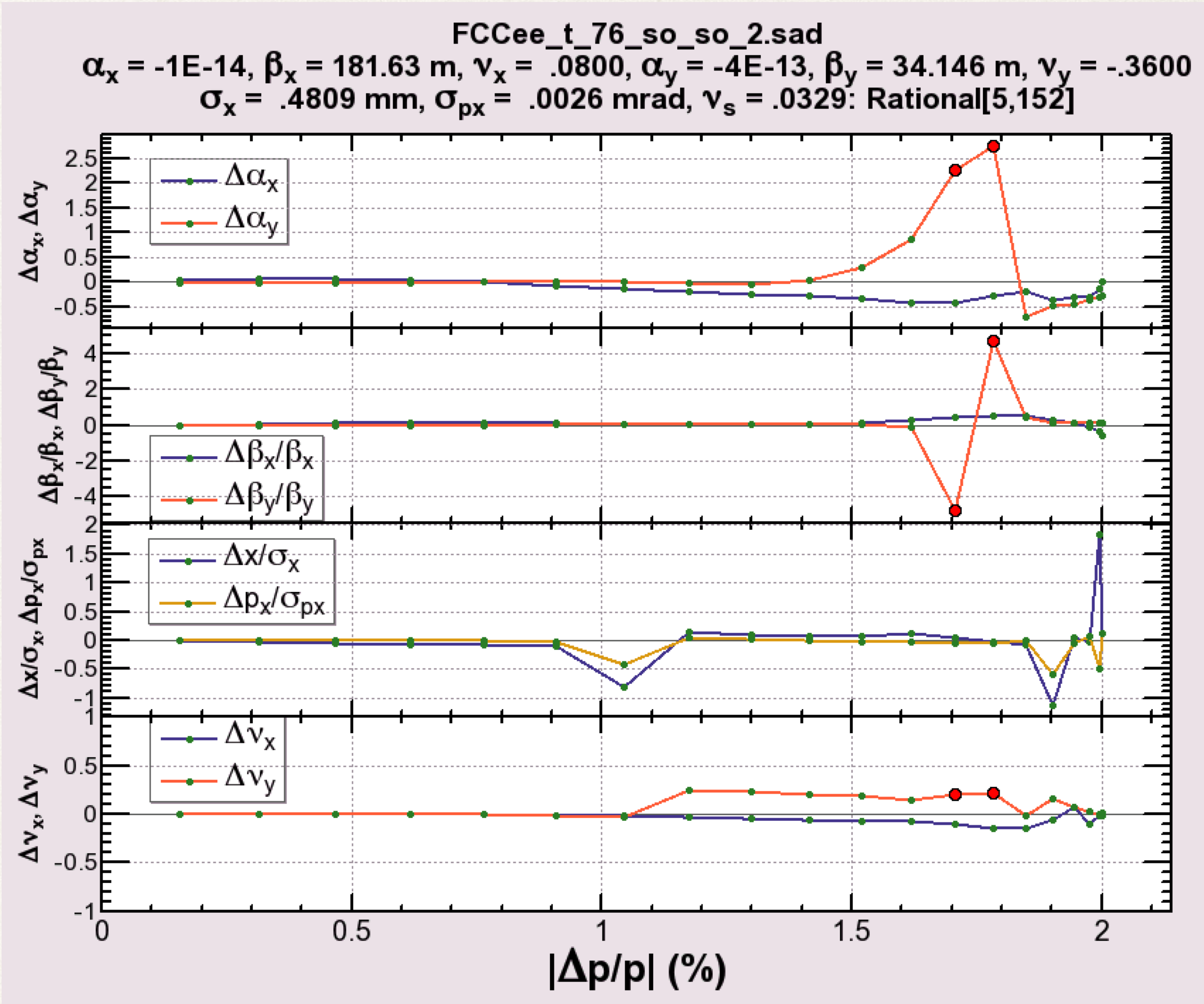
$$\nu_s = 0.0328756 \approx \frac{1}{30 + \frac{1}{2 + \frac{1}{2}}} = \frac{5}{152} = 0.0328947 \ . \tag{4}$$

Synchrotron-optics

$\beta^*_{x,y} = (0.5\ \mathrm{m}, 1\ \mathrm{mm})$, no radiation damping



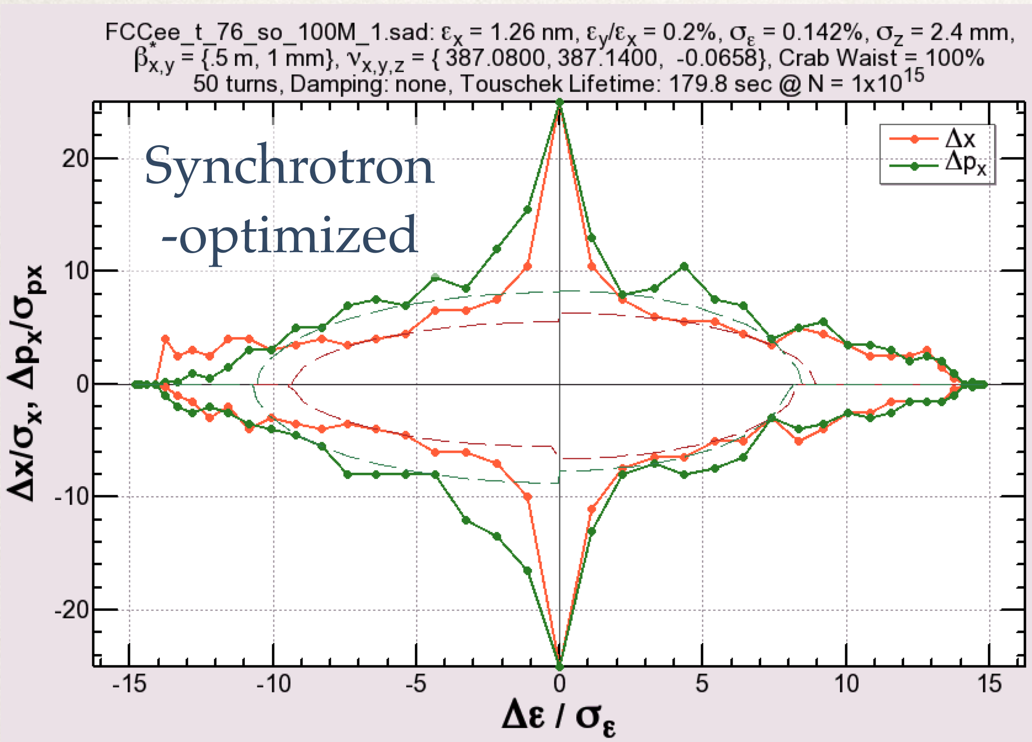


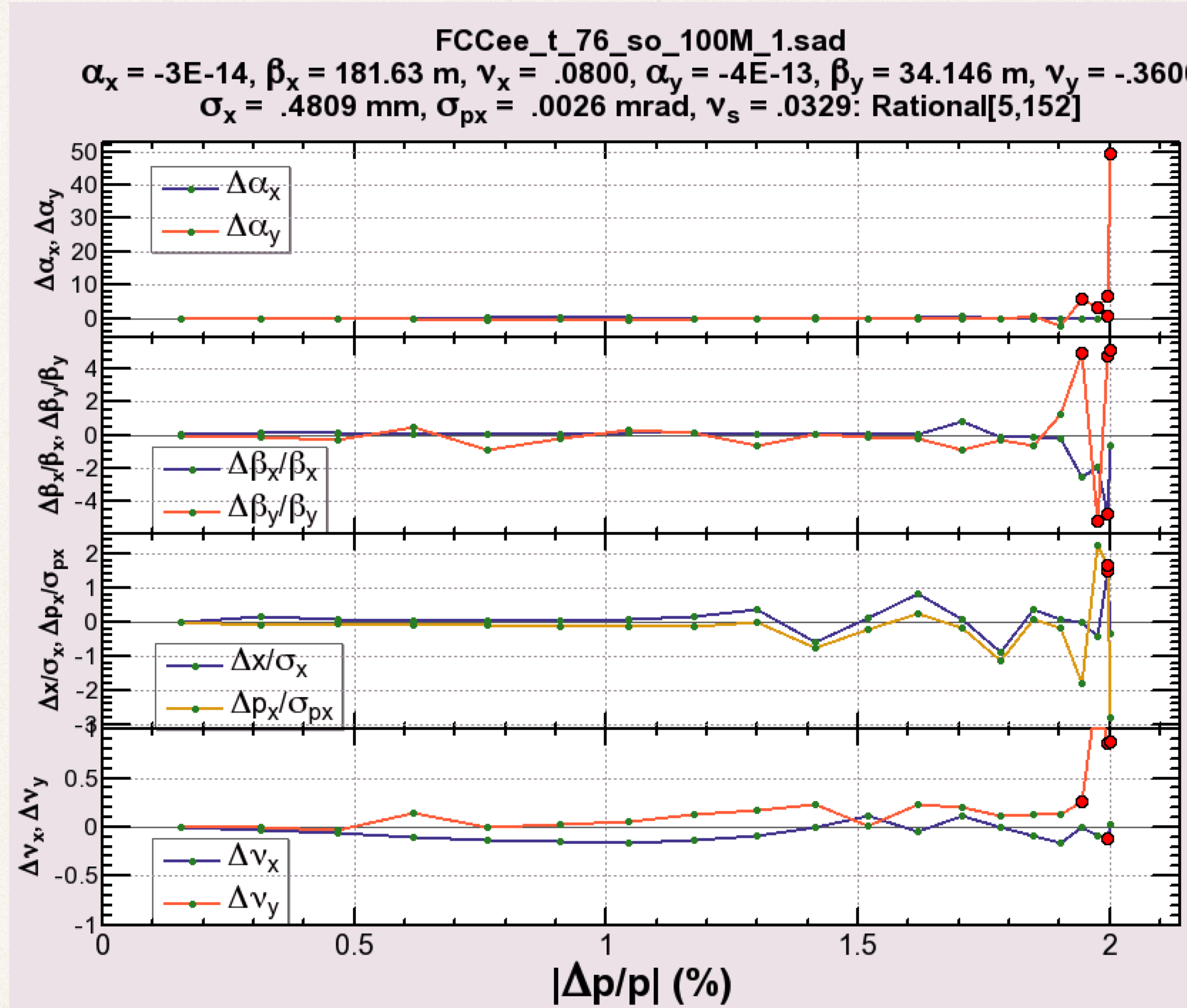


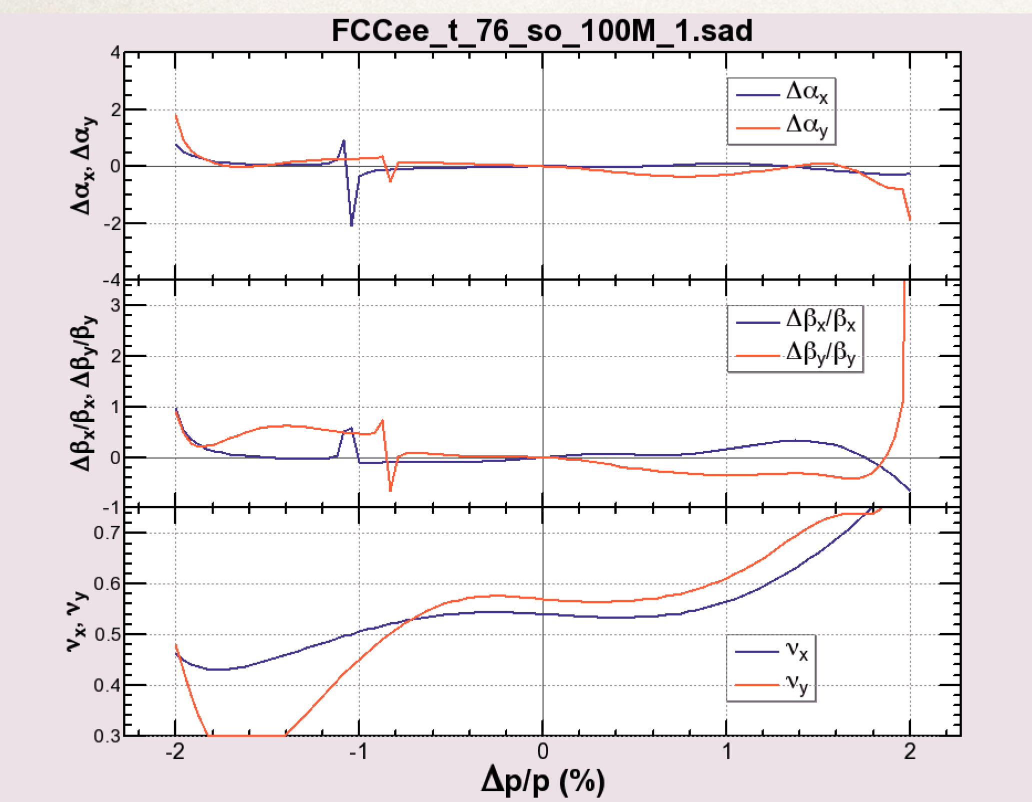
- The synchrotron optics becomes unstable at $\Delta p/p = 1.7$ - 1.8% in this chromaticity-optimized solution.

Optimized synchrotron-optics

$\beta^*_{x,y}$ = (0.5 m, 1 mm), no radiation damping



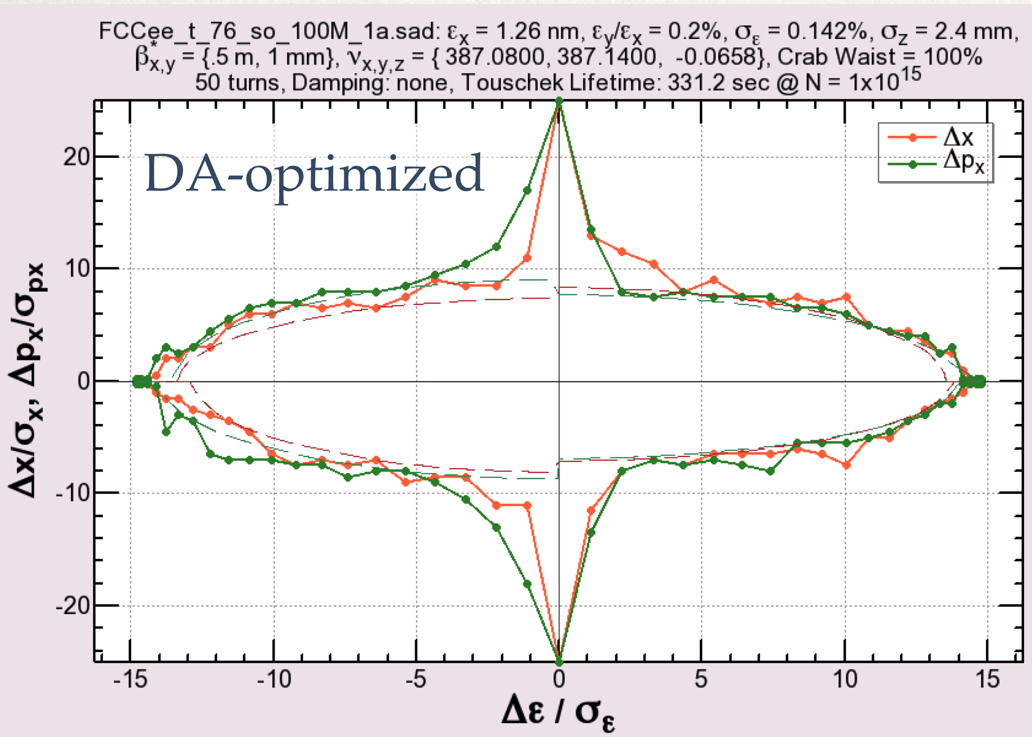


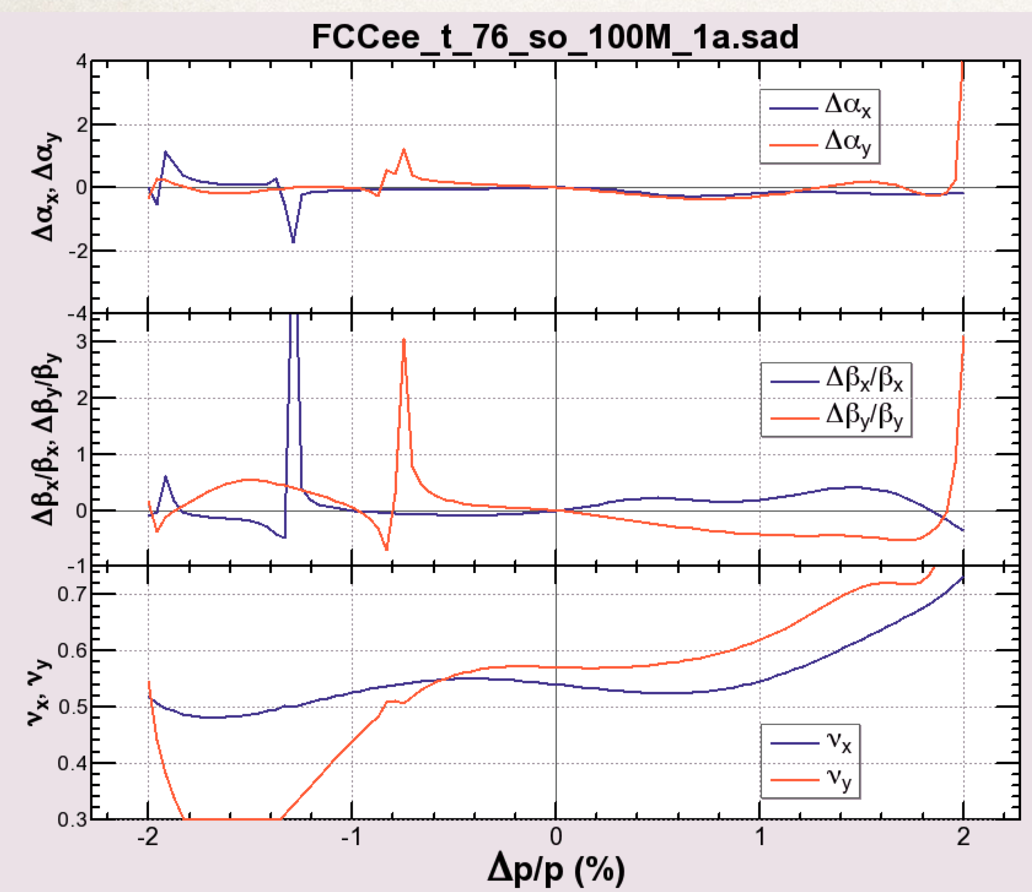


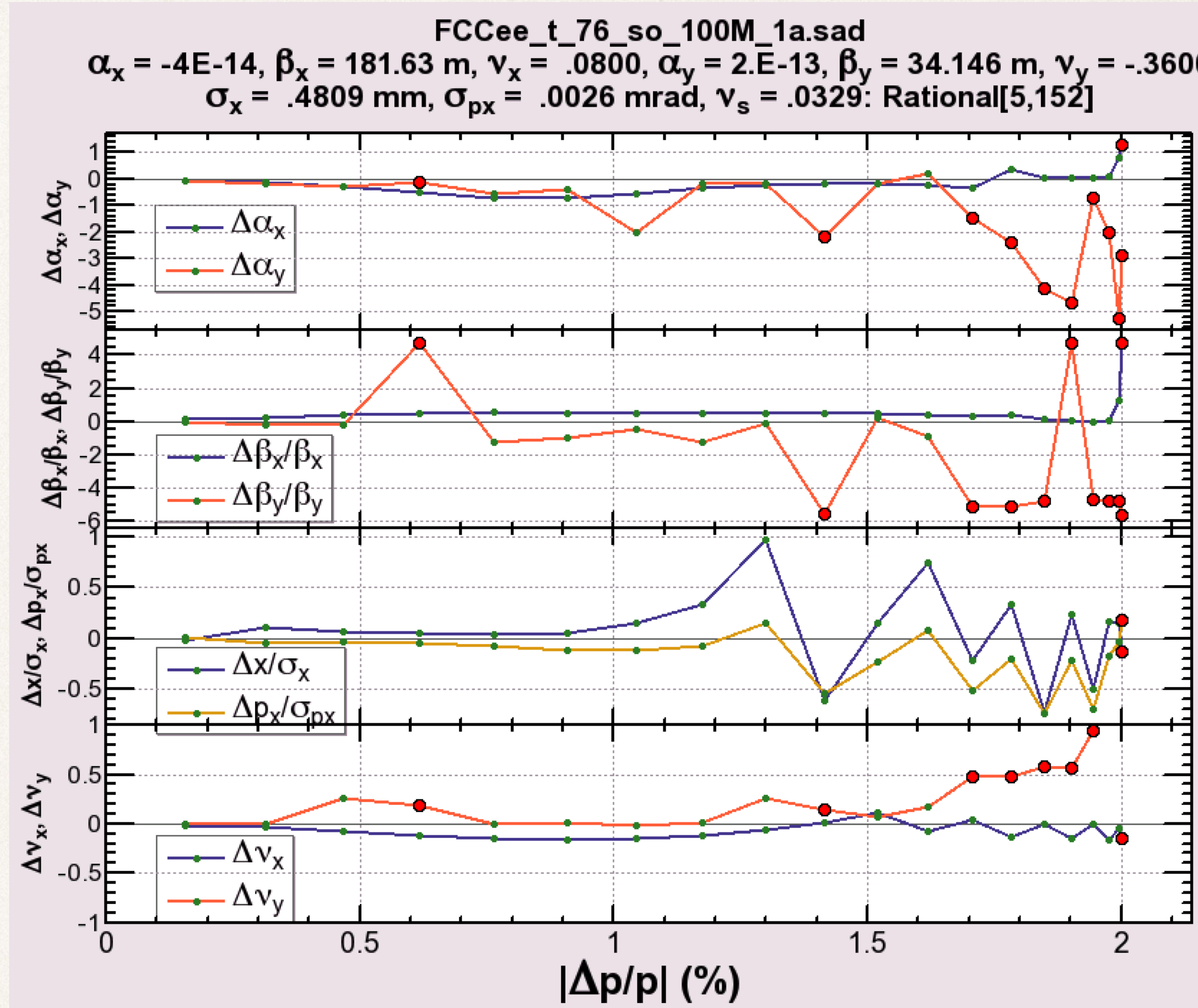
- A synchrotron-optimized solution can be obtained like above.
- Strong 3rd+ orders appear in chromaticity.
- The rf frequency is reduced to 100 MHz (actual: 400 MHz) for a more constant synch. freq.

DA-optimized synchrotron-optics

$\beta^*_{x,y}$ = (0.5 m, 1 mm), no radiation damping







- The stability of synchrotron-optics becomes worse for DA-optimized optics.

Any correlation?

Optimization	>3rd order chromaticity	Synchrotron optics	Dynamic aperture
Chromaticity	small	unstable @ 1.7- 1.8%	thin
Synchrotron optics	large	stable up to 1.9%	thin+
Dynamic aperture	large	largely unstable	thick

Summary

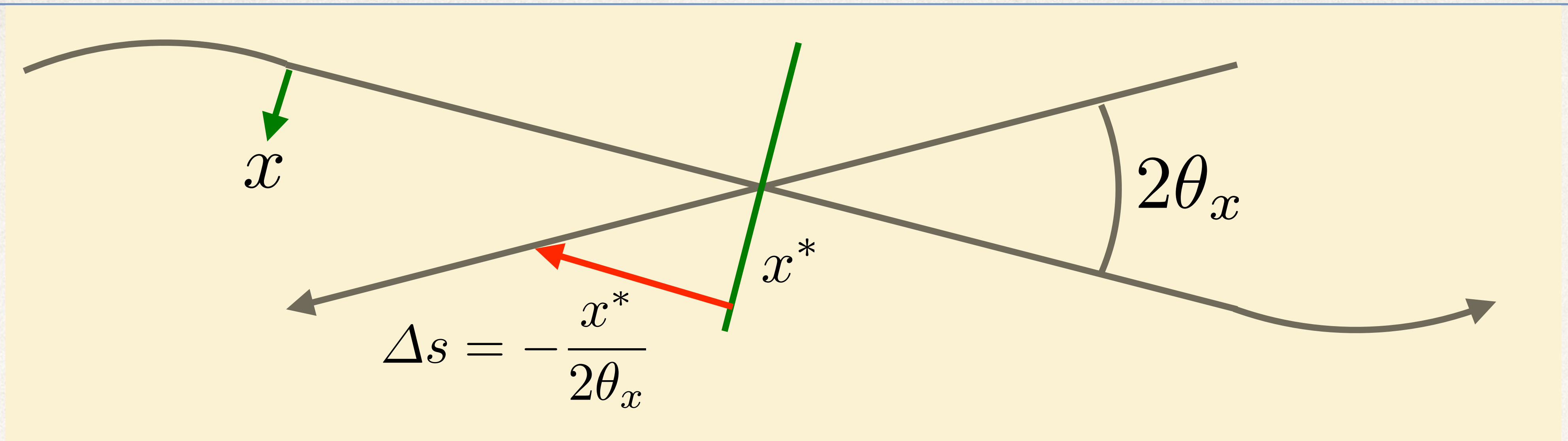
- **Optics for FCC-ee are presented, considering:**
  - 2 IPs/ring, with 30 mrad crossing angle.
  - Local chromaticity correction with crab waist.
  - Suppression of synchrotron radiation in the IR below 100 keV up to 510 m from the IP.
  - Solenoid at IP & its compensation.
  - Possible asymmetric L* for wider outgoing beam pipe at the IP.
  - Element-by-element synchrotron radiation.
  - Tapering of all magnets according to the local beam energy to suppress sawtooth.
  - Common RF sections with cross-over of two beams (at least at tt).
  - Optimization of dynamic aperture with hundreds of sextuple families .
  - Geometrical fitting to the FCC-hh tunnel.
  - Combined function dipole in the arc will bring a number of merits, if realized.
- **Resulting dynamic aperture almost satisfies the requirements.**
- **Things need further investigation:**
  - Field quality, more realistic profile of magnetic field.
  - Tolerances / tuning scheme for machine errors, misalignments.
  - 4 IPs.
  - and more…

Summary (cont'd)

- **Optics calculation over synchrotron periods is tried**
    - A better chromaticity does not provide a better dynamic aperture.
    - DA-optimized solutions show large 3rd+ orders chromaticity.
    - Synchrotron-optimized solutions indeed increase such 3rd+ orders.
    - The correlation between the synchrotron optics and dynamic aperture has not been clear.

Backups

Crab waist sextuple within CCS



The crab waist scheme shifts the vertical waist of a beam by

$$\Delta s = -\frac{x^*}{2\theta_x} \; . \tag{1}$$

Thus the associated transformation is

$$y^* \to y^* - p_y^* \Delta s = y^* + \frac{p_y^* x^*}{2\theta_x} \; , \tag{2}$$

which is performed by a Hamiltonian at the IP:

$$H^* = \frac{x^* p_y^{*2}}{4\theta_x} \; . \tag{3}$$

If there are the phase relations between the IP and the sextupoles:

$$\Delta\psi_x = 2\pi \quad \text{and} \quad \Delta\psi_y = 2.5\pi, \tag{4}$$

then the variables at the IP (x^*, p_y^*) are expressed in those at the sext (x, y):

$$x^* = \sqrt{\frac{\beta_x^*}{\beta_x}} x, \quad p_y^* = \frac{y}{\sqrt{\beta_y^* \beta_y}} \; . \tag{5}$$

Thus the Hamitonian at the IP is equivalent to a Hamiltonian at the sext:

$$H = \frac{x y^2}{4\theta_x \beta_y^* \beta_y} \sqrt{\frac{\beta_x^*}{\beta_x}} \; , \tag{6}$$

which can be approximated by a Hamiltonian of a sextupole:

$$H_s = \frac{k_2}{6} \left(x^3 - 3xy^2 \right) \; , \tag{7}$$

with

$$k_2 = -\frac{1}{2\theta_x \beta_y^* \beta_y} \sqrt{\frac{\beta_x^*}{\beta_x}} \; . \tag{8}$$

Chromogeometric aberration of CCS

Consider the transfer matrix between sexts of YCCS with momentum dependence:

$$m_y = \begin{pmatrix} -1+8\delta^2 & -2\delta\ell(-4-4\sqrt{2}+(6+5\sqrt{2})\delta) \\ \delta/\ell(-4-4\sqrt{2}+(14+9\sqrt{2})\delta) & -1+8\delta^2 \end{pmatrix} + O(\delta^3)\,, \tag{1}$$

and similar for m_x. Then the emittance increment due to the sextupole kick is calculated as:

$$R\delta^2 \equiv \frac{\Delta\varepsilon_y}{\varepsilon_y} = \frac{256\delta^2}{\beta_x\beta_y^2\eta_x^2\ell^2}(2\sqrt{2}\beta_y+\ell\xi_y)^2 \tag{2}$$

$$\times \Big((6-4\sqrt{2})\beta_y^2\varepsilon_x\ell^2 + ((9-6\sqrt{2})\beta_y^2 - 2\ell^2)\beta_x\beta_y\varepsilon_y \tag{3}$$

$$+ ((2\sqrt{2}-1)\beta_y^2 + (14+8\sqrt{2})\ell^2)\beta_x^2\varepsilon_x\Big)\,, \tag{4}$$

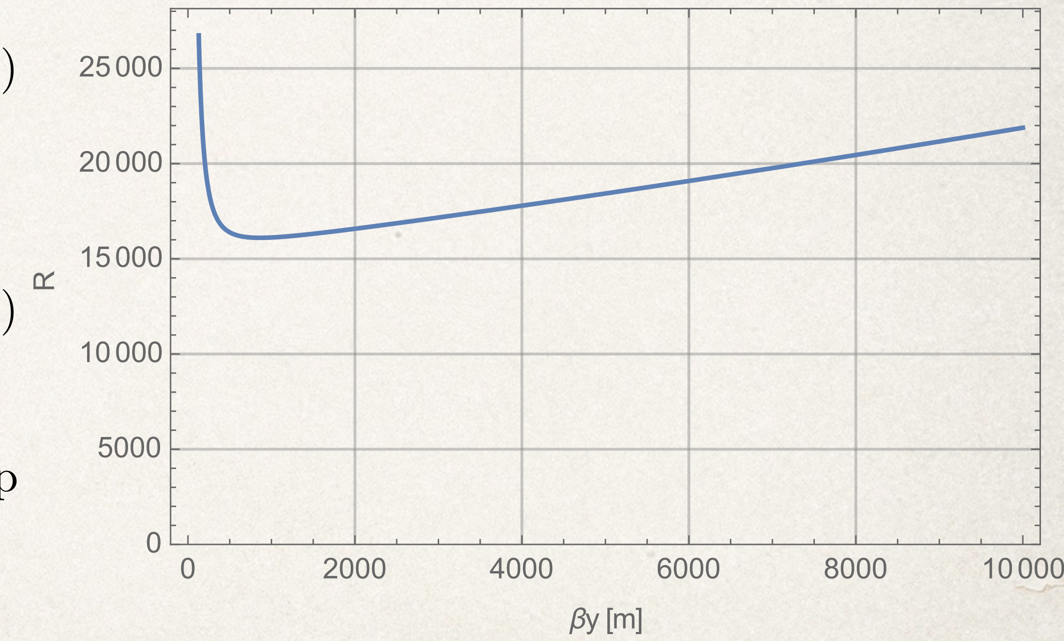
where β_x, β_y,and η_x are the values at the sextupole, ℓ the separation of quads, and we have assumed

$$-k_2\beta_y\eta_x = \xi_y + 2\sqrt{2}\beta_y/\ell\,. \tag{5}$$

If we plug in the numbers:

$$\beta_x = 15\text{ m},\ \varepsilon_x = 1.2\text{ nm},\ \varepsilon_y = 2.4\text{ pm},\ \eta_x = 0.16\text{ m},\ \ell = 58\text{ m},\ \xi_y = 3,500\,, \tag{6}$$

we get the following graph.



Although the optimum is at around $\beta_y = 600$ m, the increment is small up to $\beta_y \lesssim 8,000$ m.

- Above are just tentative optics.
- Usage of these sections is to be determined.

A rough estimation of radiation by arc quads

- The radiation power:

$$P \propto \gamma^2 B^2 \ell$$

- Ratio of powers by dipoles and quadrupoles per unit cell:
  - dipole: $$P_d \propto \gamma^2 \left(\frac{B\ell_{\rm cell}}{B\rho}\right)^2 \left(\frac{B\rho}{\ell_{\rm cell}}\right)^2 \ell_{\rm cell} \propto \gamma^4 \frac{\theta^2}{\ell_{\rm cell}}$$
  - quadrupole: $$P_q \propto \frac{\gamma^2}{2} \left(\frac{B'\Delta x \ell_q}{B\rho}\right)^2 \left(\frac{B\rho}{\ell_q}\right)^2 \ell_q \propto \frac{\gamma^4}{2} \frac{k_1^2 \Delta x^2}{\ell_q}$$
  - ratio: $$\frac{P_q}{P_d} = \frac{(k_1\ell_{\rm cell})^2}{2} \frac{\beta_{xq}}{\ell_{\rm cell}} \frac{n^2\varepsilon_x}{\theta^2\ell_q}\,, \qquad \Delta x^2 = n^2\beta_{xq}\varepsilon_x$$
- In the case of a 90° cell, $k_1\ell_{\rm cell} = 2\sqrt{2}$, $\beta_{xq}/\ell_{\rm cell} = 1 + \frac{1}{\sqrt{2}}$, then:

$$\frac{P_q}{P_d} = (4 + 2\sqrt{2}) \frac{n^2\varepsilon_x}{\theta^2\ell_q}$$

- or a particle with an amplitude of $n\sigma_x$ will receive an energy loss per every turn:

$$\frac{\Delta p_1}{p_0} = \frac{P_q}{P_d} \times \frac{U_0}{E} = (4 + 2\sqrt{2}) \frac{n^2\varepsilon_x}{\theta^2\ell_q} \alpha_\varepsilon \quad (\alpha_\varepsilon\text{: long. damping per turn})$$

- which causes a synchrotron motion with a momentum amplitude $\pm\Delta p/p_0$:

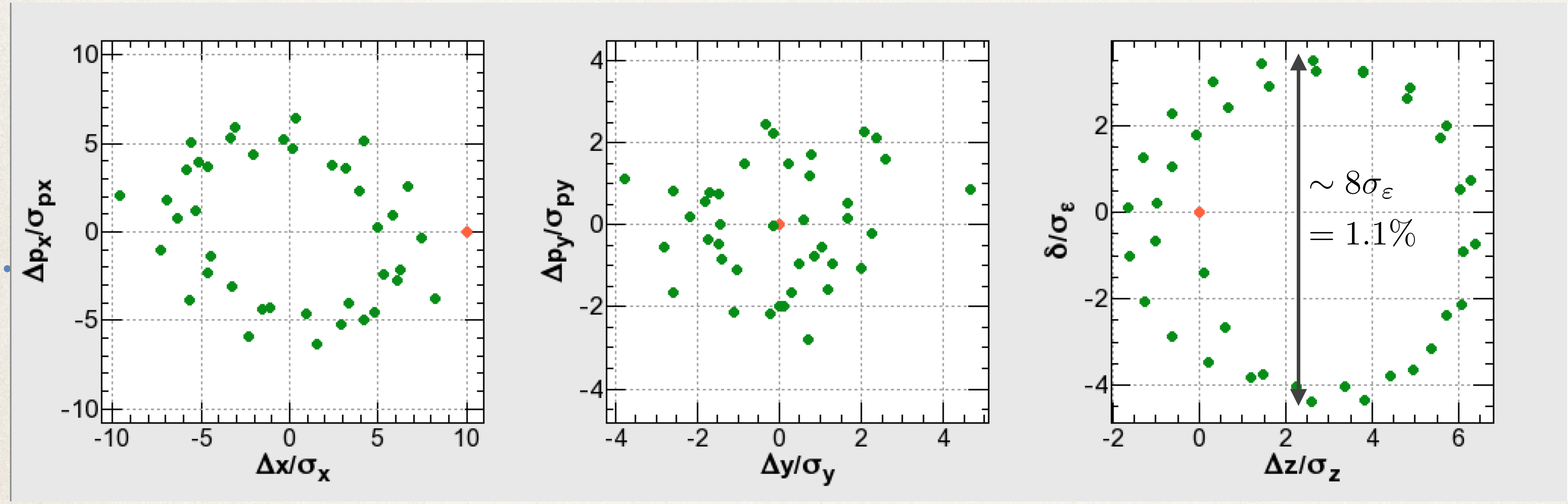
$$\frac{\Delta p}{p_0} = \frac{1}{2\pi\nu_s} \frac{\Delta p_1}{p_0} = \left(2 + \sqrt{2}\right) \frac{n^2\varepsilon_x}{\pi\theta^2\ell_q} \frac{\alpha_\varepsilon}{\nu_s}$$

A rough estimation of radiation by arc quads (cont'd)

$$\varepsilon_x = 2 \text{ nm}, \theta = 2\pi/1240, \alpha_\varepsilon/\nu_s = 0.41 \quad \text{gives}$$

$$\frac{\Delta p}{p_0} = 0.58\% \left(\frac{n}{10}\right)^2 \left(\frac{0.6 \text{ m}}{\ell_q}\right)$$

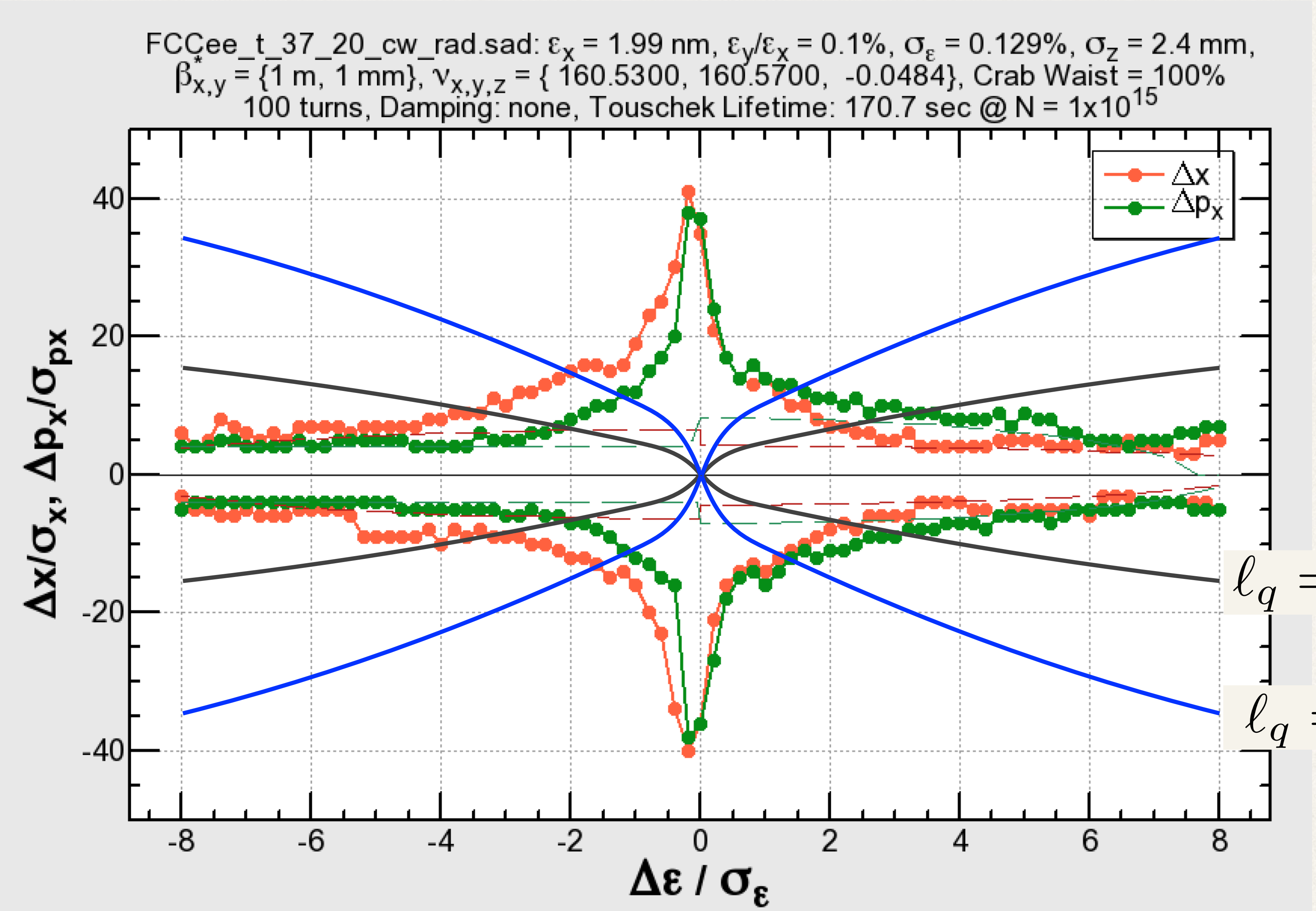
- If we plug-in the number for FCC-ee-tt:



* only damping, no fluctuation, is taken into account in simulations in these slides.

Cf. Barbarin, F ; Iselin, F Christoph ; Jowett, John M, 4th European Particle Accelerator Conference, London, UK, 27 Jun - 1 Jul 1994, pp.193-195

The effect on the dynamic aperture



$\ell_q = 0.6 \text{ m}$

$\ell_q = 3 \text{ m}$

$\Delta x/\sigma_x$