Beam-beam simulations
and
nonlinear lattice analysis for FCC-ee

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Acknowledgements:

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38th FCC-ee Optics design meeting, CERN

Sep. 09, 2016

Outline

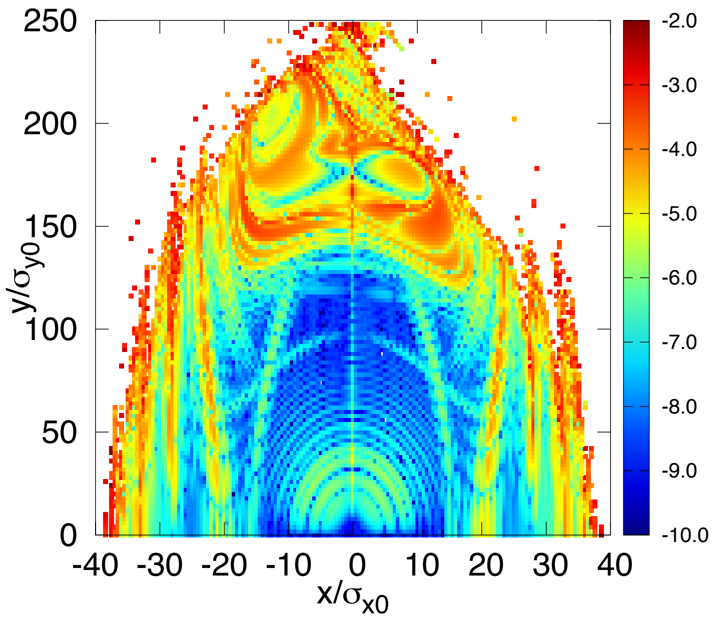
- **Lattice**
  - Lattice version: FCCee_t_82_by2_1a_nosol_DS_2 by K. Oide
  - Error seeds in vertical offsets of S{DF}* to generate vertical emittance
- **Analysis of lattice nonlinearity (LN)**
  - FMA
  - Resonance driving terms (RDTs) calculations using PTC
- **Beam-beam simulations**
  - SAD: beam-beam + lattice
- **Summary**

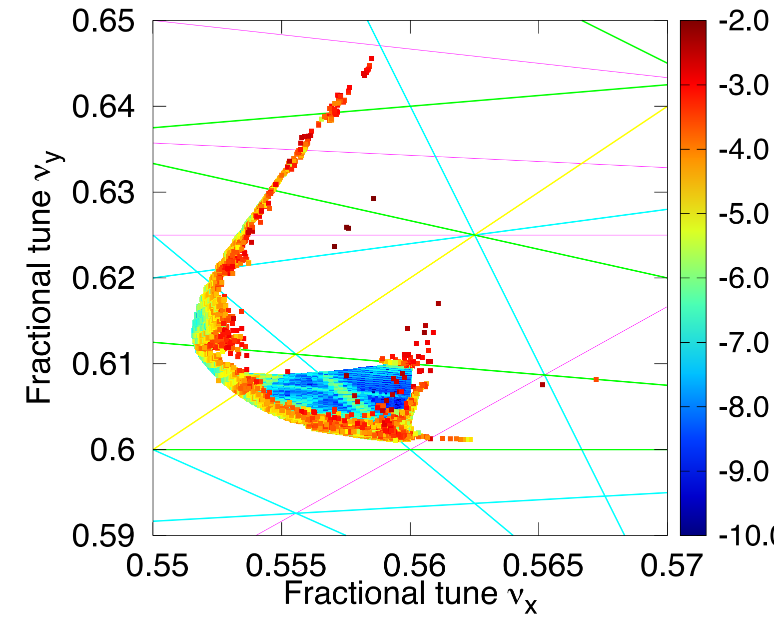
1. FMA

➤ **Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2**

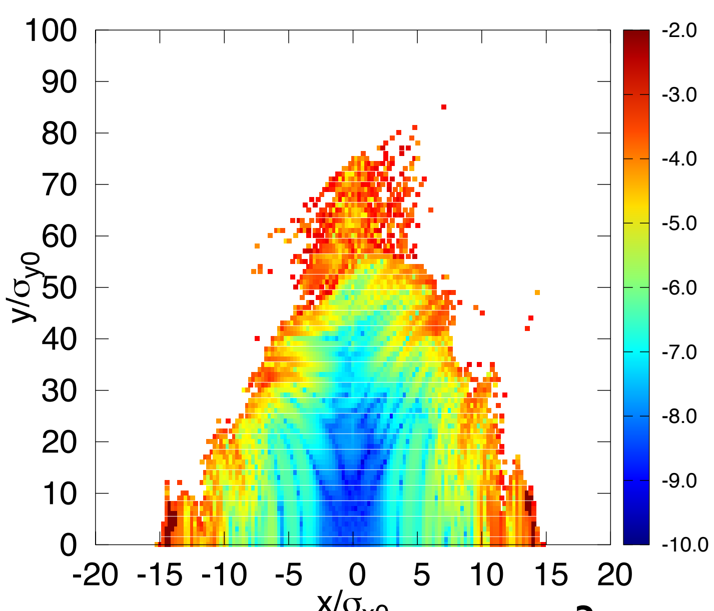
- **Bare lattice: β_x^*=1m, β_y^*=2mm**
- **Start point for tracking: FRF**
- **Tracking: SAD + NAFF, 1024 turns**

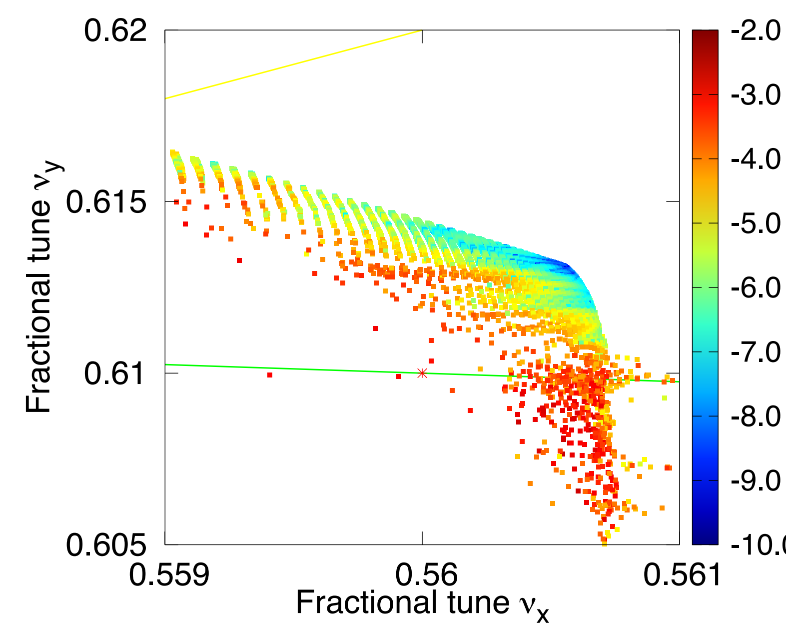
δ=0:





δ=2σ_p:



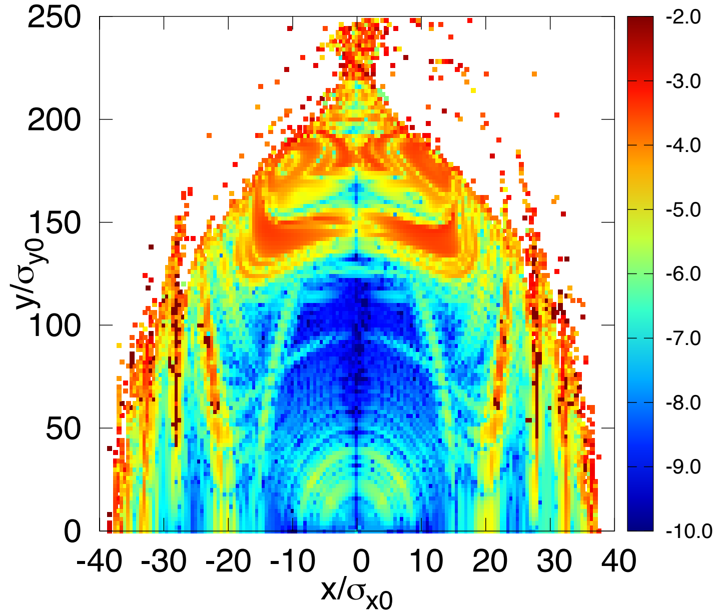


1. FMA

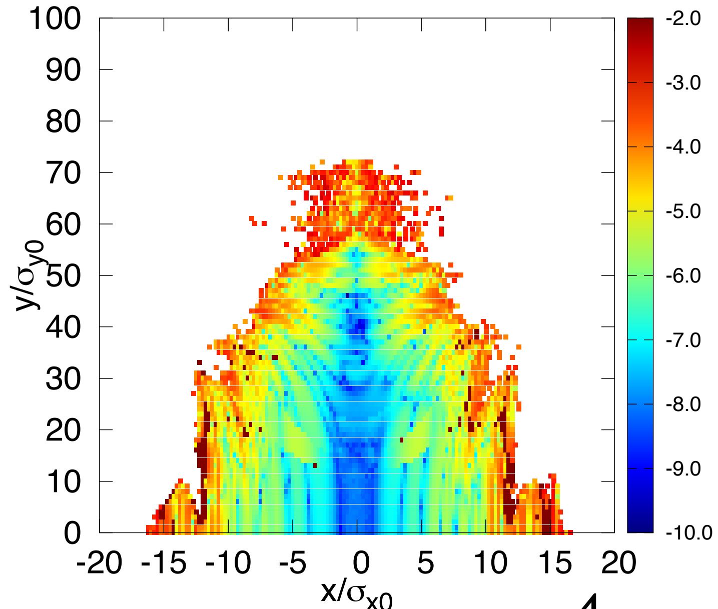
➤ **Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2**

- **Bare lattice: β_x^*=1m, β_y^*=2mm**
- **Start point for tracking: IP**
- **Tracking: SAD + NAFF, 1024 turns**

δ=0:



δ=2σ_p:



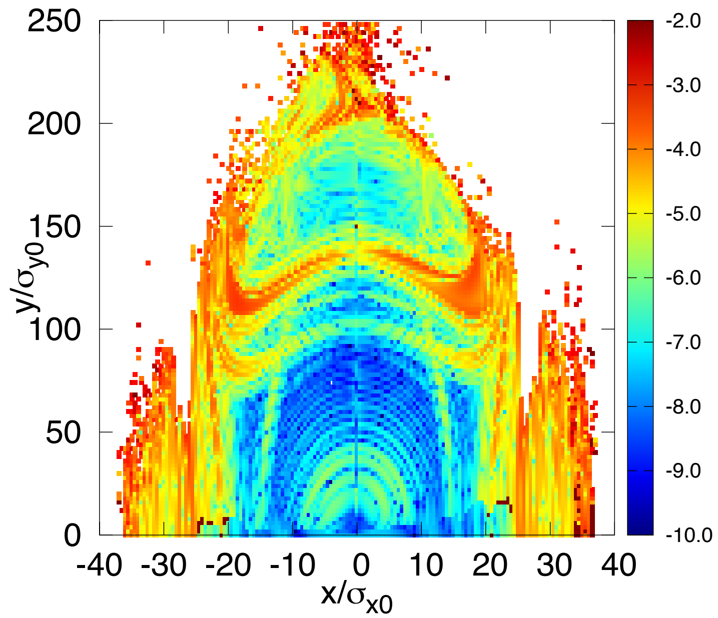
Footprint in tune space is hard to obtain, indicating that starting tracking from IP is not good.

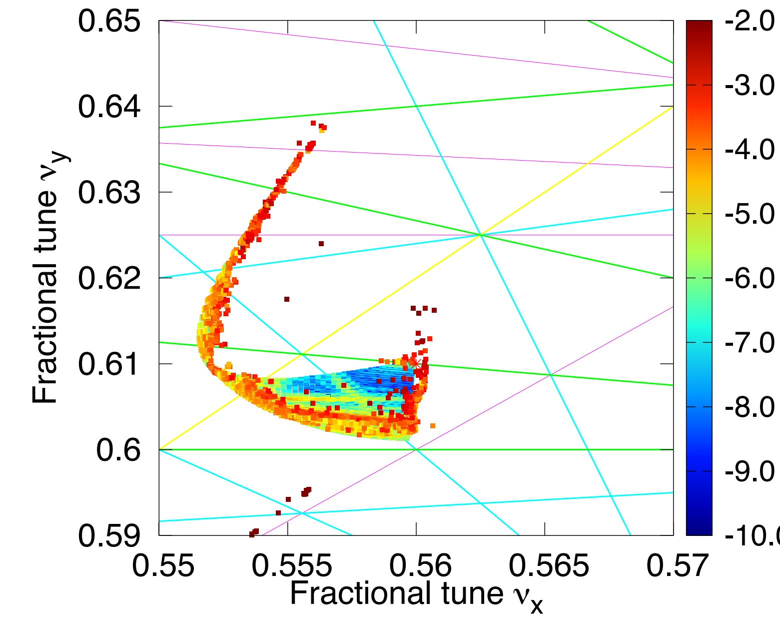
1. FMA

➤ **Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2**

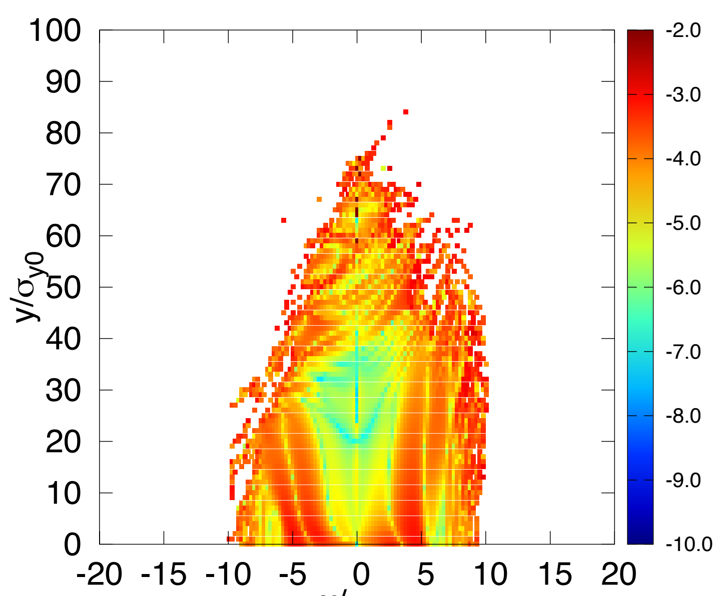
- **Bare lattice + Error seed #25**
- **Start point for tracking: FRF**
- **Tracking: SAD + NAFF, 1024 turns**

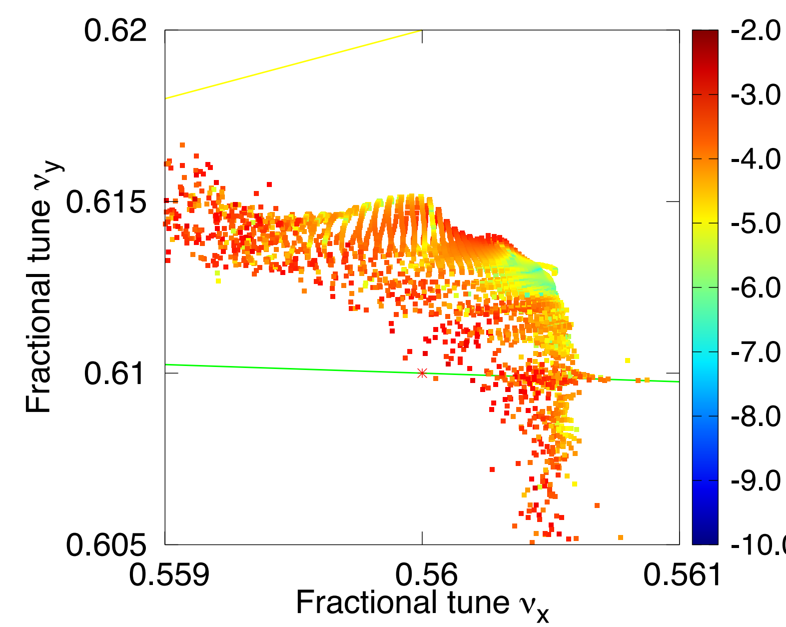
$\delta=0$:





$\delta=2\sigma_p$:

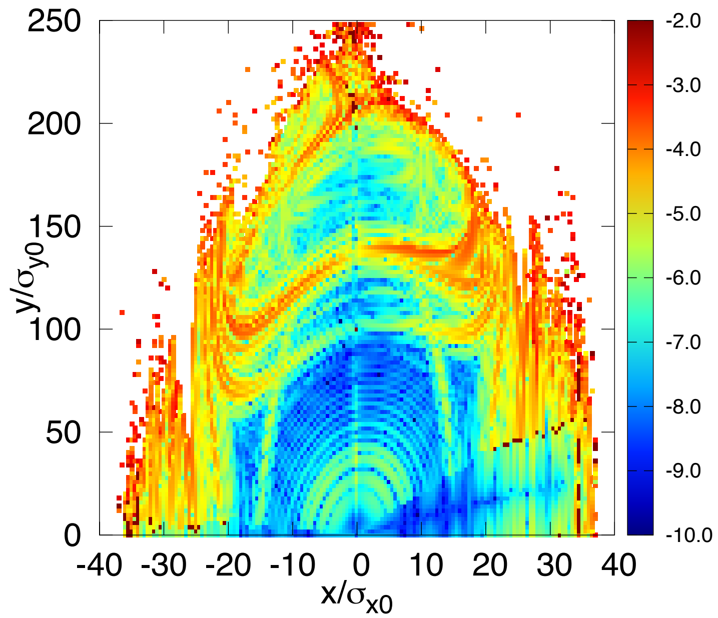


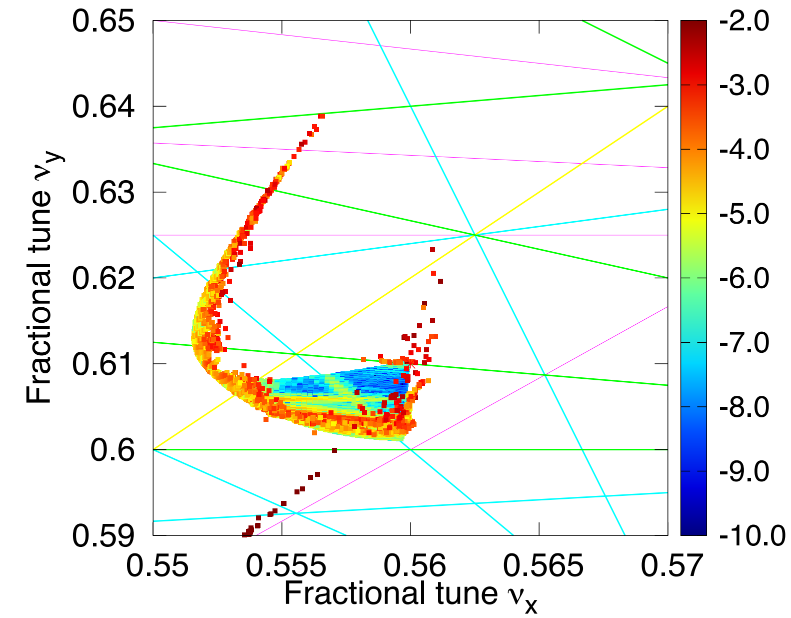


1. FMA

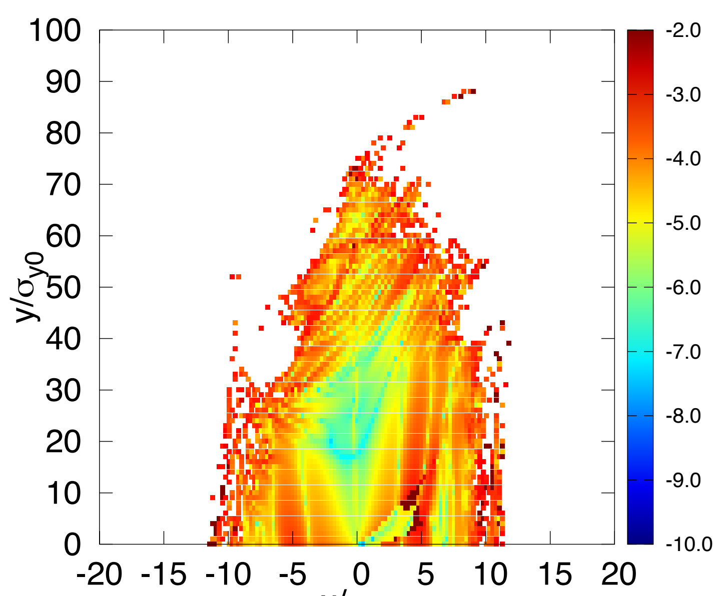
- **Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2**
  - **Bare lattice + Error seed #3**
  - **Start point for tracking: FRF**
  - **Tracking: SAD + NAFF, 1024 turns**

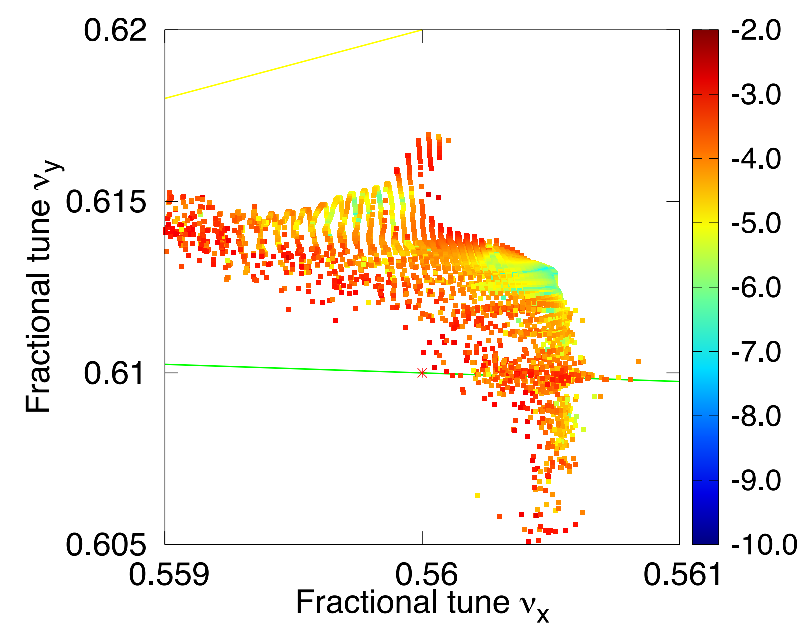
δ=0:





δ=2σ_p:





1. FMA

➤ **Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2**

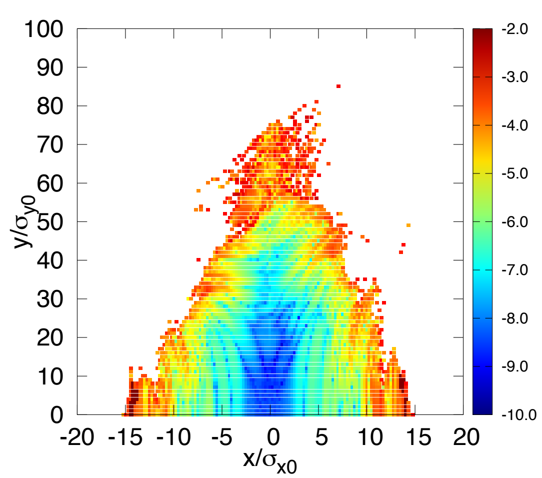
- **Here error seeds in vertical offsets of S{DF}* are used to generate vertical emittance**
- **But optics corrections might also be necessary, such as local coupling at IP, dispersion function…**

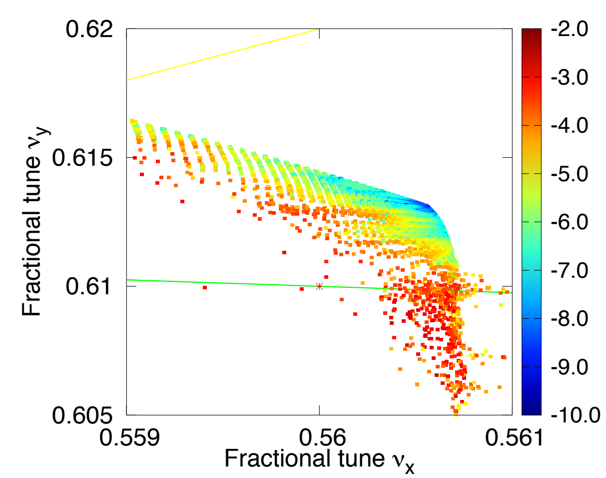
1. FMA

➤ Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2

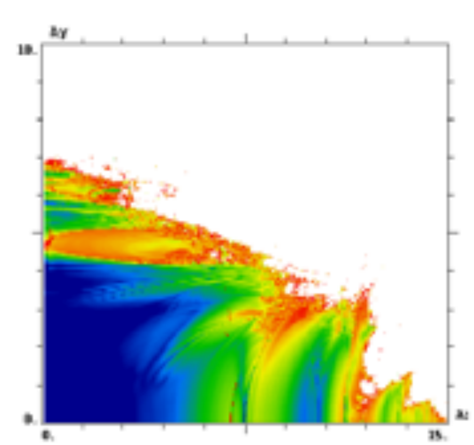
- **Compare with D. Shatilov's results [Refer to talk at 36th FCC-ee optics design meeting]: Aperture almost agrees, but some discrepancy in footprint in tune space.**
- **Hamiltonian (maps) for each type of element**
- **Integration algorithm and Number of slices**

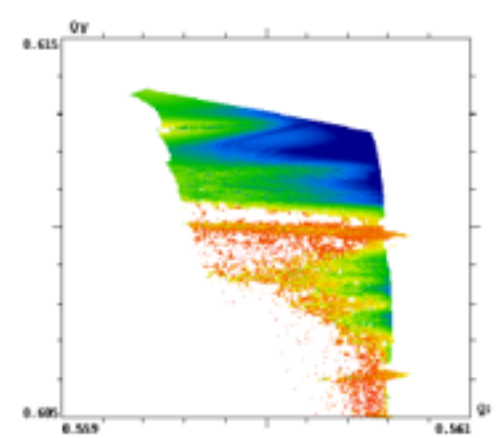
SAD:





Acceleraticum by P. Piminov:



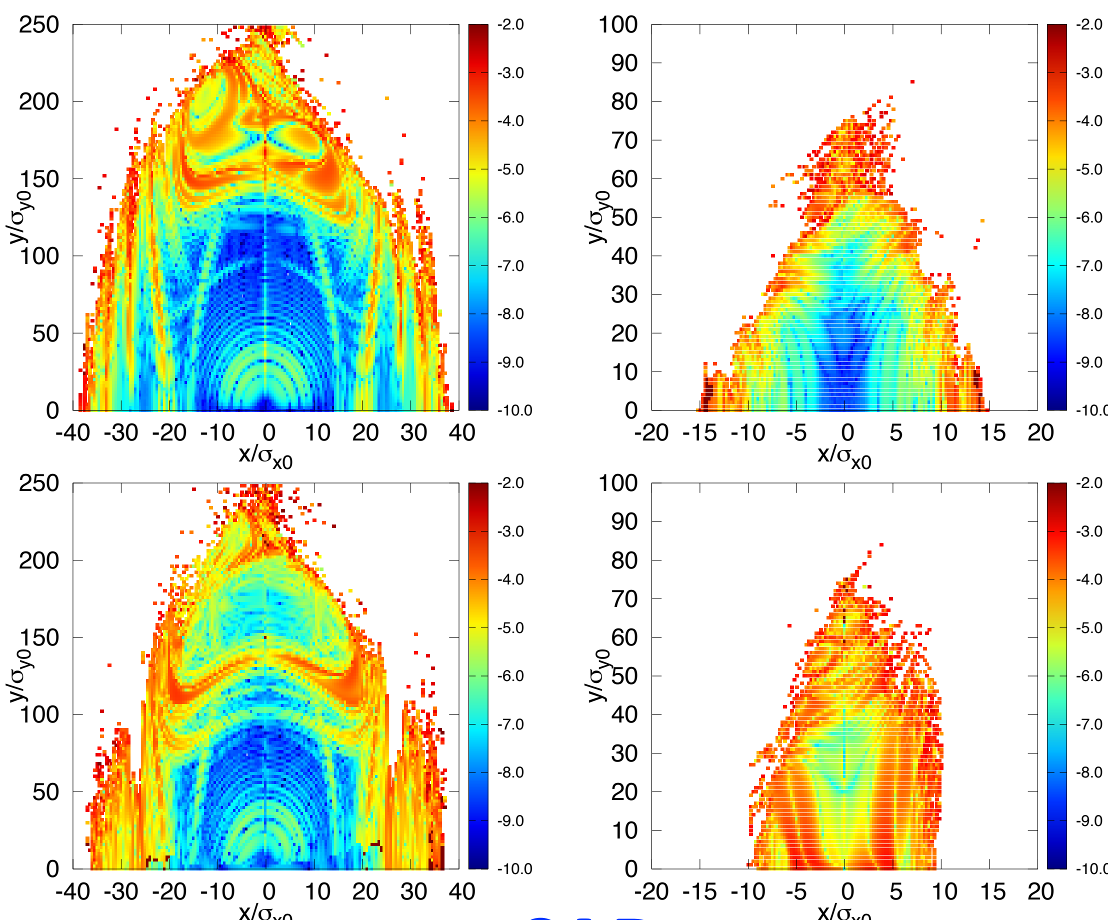


Refer to D. Shatilov, 36th FCC-ee optics design meeting

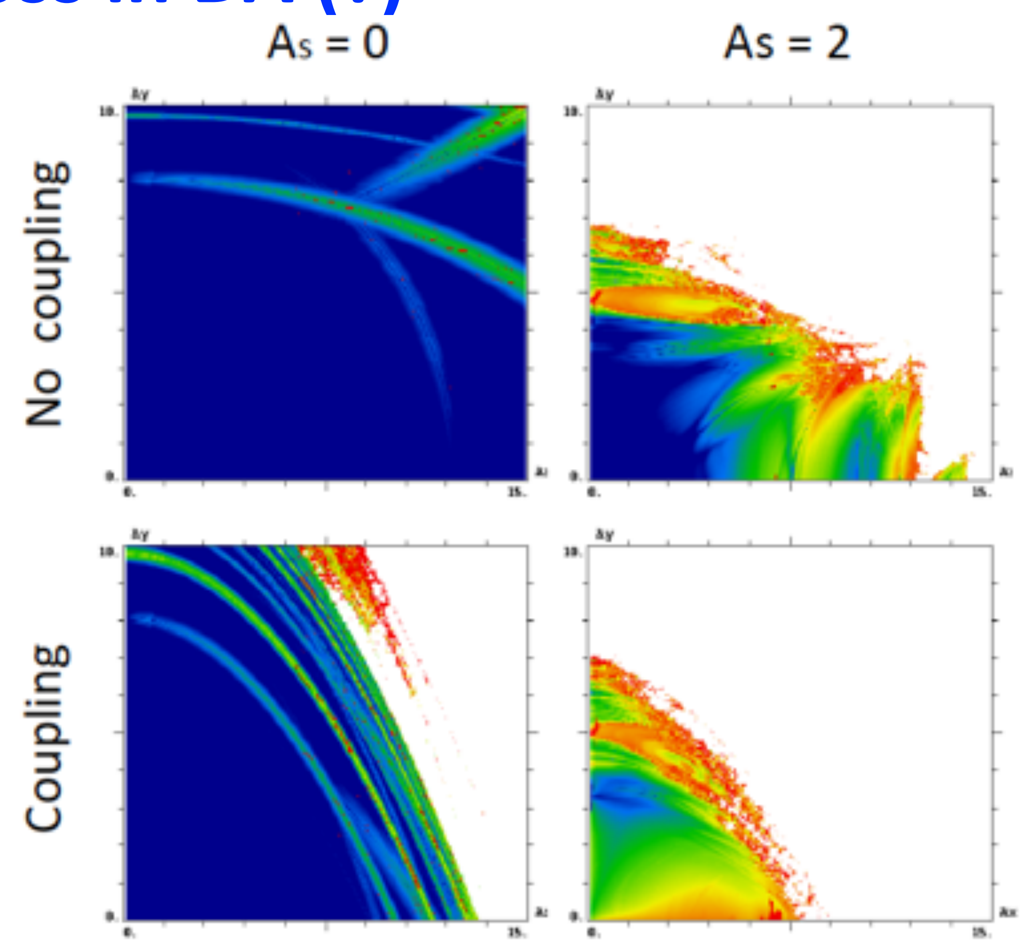
1. FMA

➤ Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2

- Compare with D. Shatilov's results [Refer to talk at 36th FCC-ee optics design meeting]: Loss of DA is smaller in SAD.
- DA also depends on tracking turns: 1024 in SAD and 2048 in thin slices tracking
- Since damping turns are the order of 100, likely we can conclude that coupling does not cause much loss in DA (?)



SAD



Thin slices [D. Shatilov]

2. RDTs

➤ Resonance driving terms (RDTs) indicate lattice nonlinearity

The effective Hamiltonian of a ring can be normalized in resonance bases [Ref. E. Forest, *Beam Dynamics – A New Attitude and Framework*, 1998].

For a ring with n elements, one can normalize the one turn map $\mathcal{M}_{1\to n}$ as [Ref. L. Yang *et al.*, Phys. Rev. ST Accel. Beams 14, 054001 (2011)]

$$\mathcal{M}_{1\to n} = \mathcal{A}_1^{-1} e^{:h:} \mathcal{R}_{1\to n} \mathcal{A}_1,$$

with $\mathcal{R}$: rotation, $e^{:h:}$: nonlinear Lie map, $\mathcal{A}_1$: normalizing map. Assume no coupling (the theory can be generalized for nonzero coupling), $\mathcal{A}_i$ in x plane at the ith element can be approximated in perturbation theory as

$$\mathcal{A}_i x = \sqrt{\beta_{x,i}} x + \eta_{x,i}\delta,$$

$$\mathcal{A}_i p_x = \frac{-\alpha_{x,i} x + p_x}{\sqrt{\beta_{x,i}}} + \eta'_{x,i}\delta.$$

2. RDTs

➤ RDTs indicate lattice nonlinearity

In the resonance basis, using action-angle variables (J, ϕ) one can write

$$h_x^{\pm} \equiv \sqrt{J_x} e^{\pm i\phi_x} = \frac{X \mp iP_x}{\sqrt{2}},$$

$$\mathcal{R}_{i\to j} h_x^{\pm} = \mathcal{R}_{i\to j} \sqrt{J_x} e^{\pm i\phi_x} = e^{\pm i\mu_{i\to j,x}} h_x^{\pm},$$

where $\mu_{i\to j,x}$ is the phase advance of $i \to j$. Consequently, the potential of a multipole magnetic field can be expanded in the resonance bases of h_{abcde} as

$$h = \sum h_{abcde} h_x^{+a} h_x^{-b} h_y^{+c} h_y^{-d} \delta^e.$$

Each h_{abcde} (a complex number in general) drives a certain resonance, and is an explicit function of magnet strengths, beta functions and dispersions.

2. RDTs

➤ **RDTs indicate lattice nonlinearity**

The effective Hamiltonian corresponding to chromaticity is

$$h_c = \sum h_{1100e} h_x^{+1} h_x^{-1} \delta^e + \sum h_{0011e} h_y^{+1} h_y^{-1} \delta^e,$$

$$h_c = J_x \sum h_{1100e} \delta^e + J_y \sum h_{0011e} \delta^e.$$

Then the tunes are calculated as

$$\nu_x = -\frac{1}{2\pi} \frac{\partial h_c}{\partial J_x} = -\frac{1}{2\pi} \sum h_{1100e} \delta^e,$$

$$\nu_y = -\frac{1}{2\pi} \frac{\partial h_c}{\partial J_y} = -\frac{1}{2\pi} \sum h_{0011e} \delta^e.$$

Therefore the RDTs of h_{1100e} and h_{0011e} correspond to linear and high-order chromaticity.

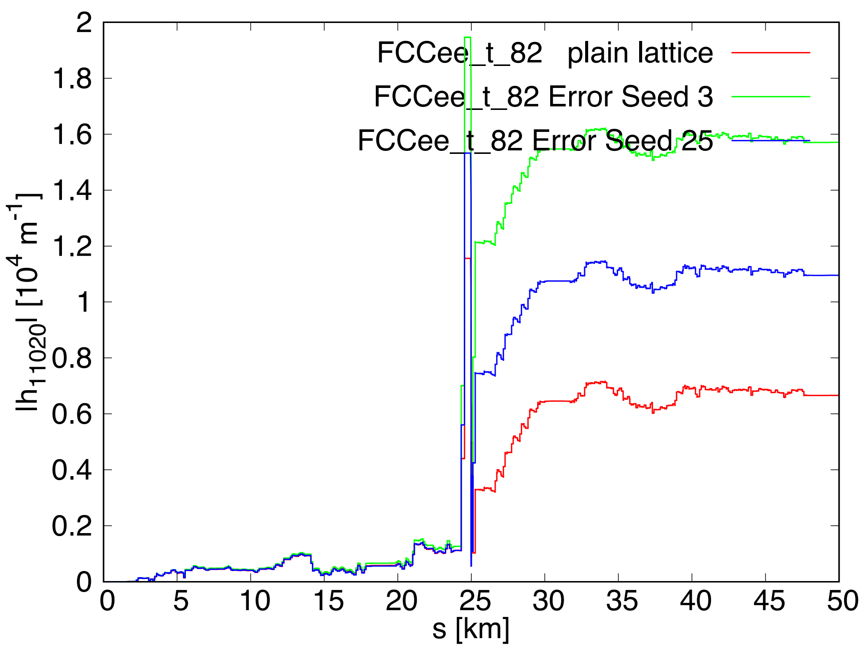
2. RDTs

➤ Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2

● No significant changes in RDTs up to 4th order due to errors in vertical offsets of S{DF}*. 5th and higher-order RDTs may dominate the nonlinear dynamics

● It might be interesting to study lattice nonlinearity after optics corrections with more errors in magnets.

$2\nu_y$ [$(J_x)(J_y)$]



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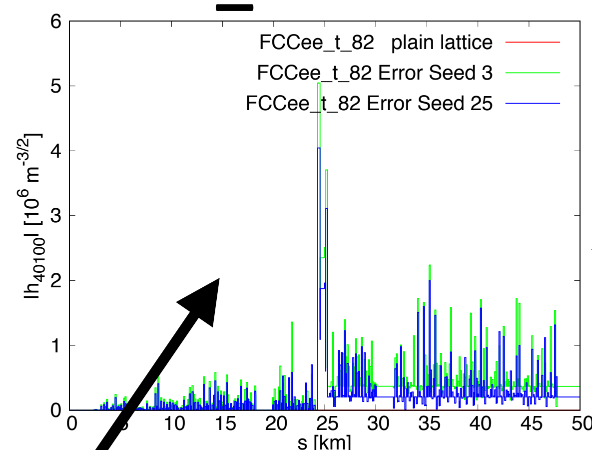

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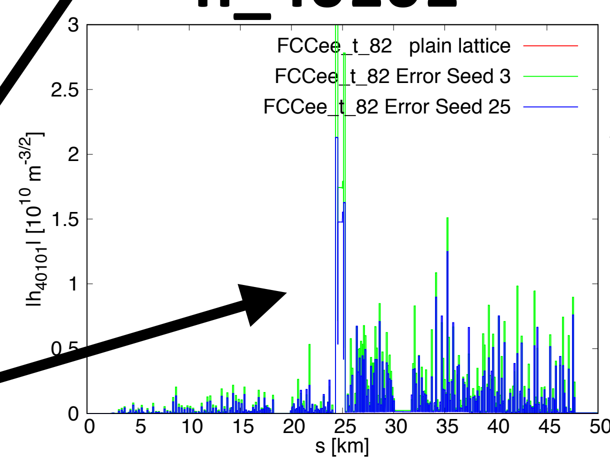
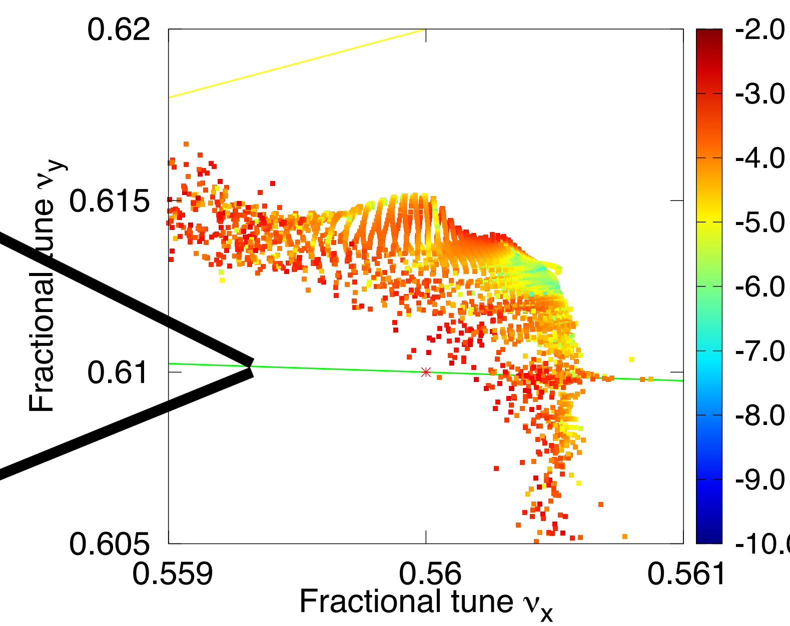




Green: 5th order

Yellow: 6th order

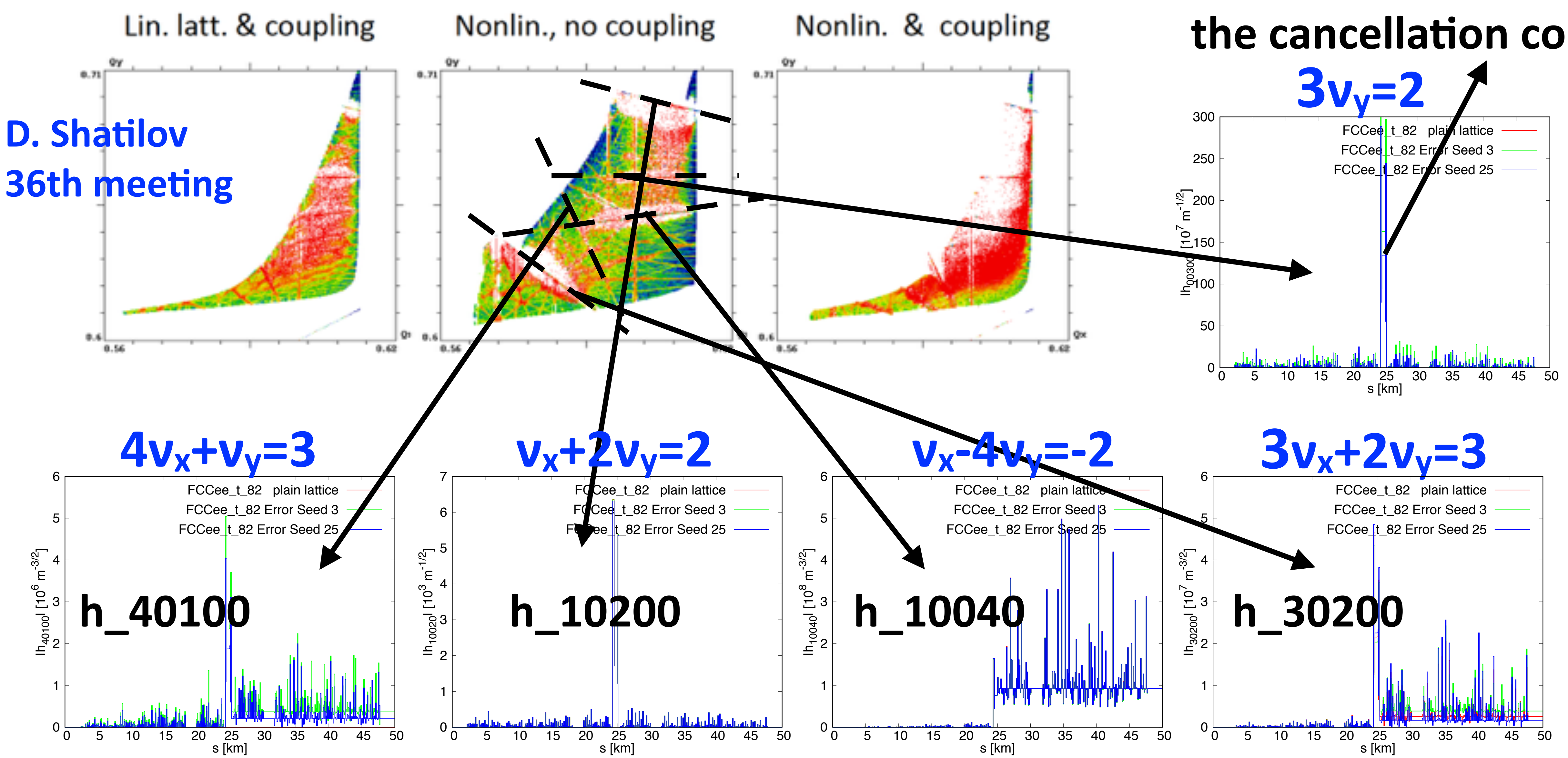
Small amplitude in bare lattice

2. RDTs

➤ **Lattice ver. FCCee_t_82_by2_1a_nosol_DS_2**

- **Possible interplay of beam-beam resonances and RDTs in lattice**
- **Emittance growth due to beam-beam resonances [K. Ohmi has a code to analyse it]**

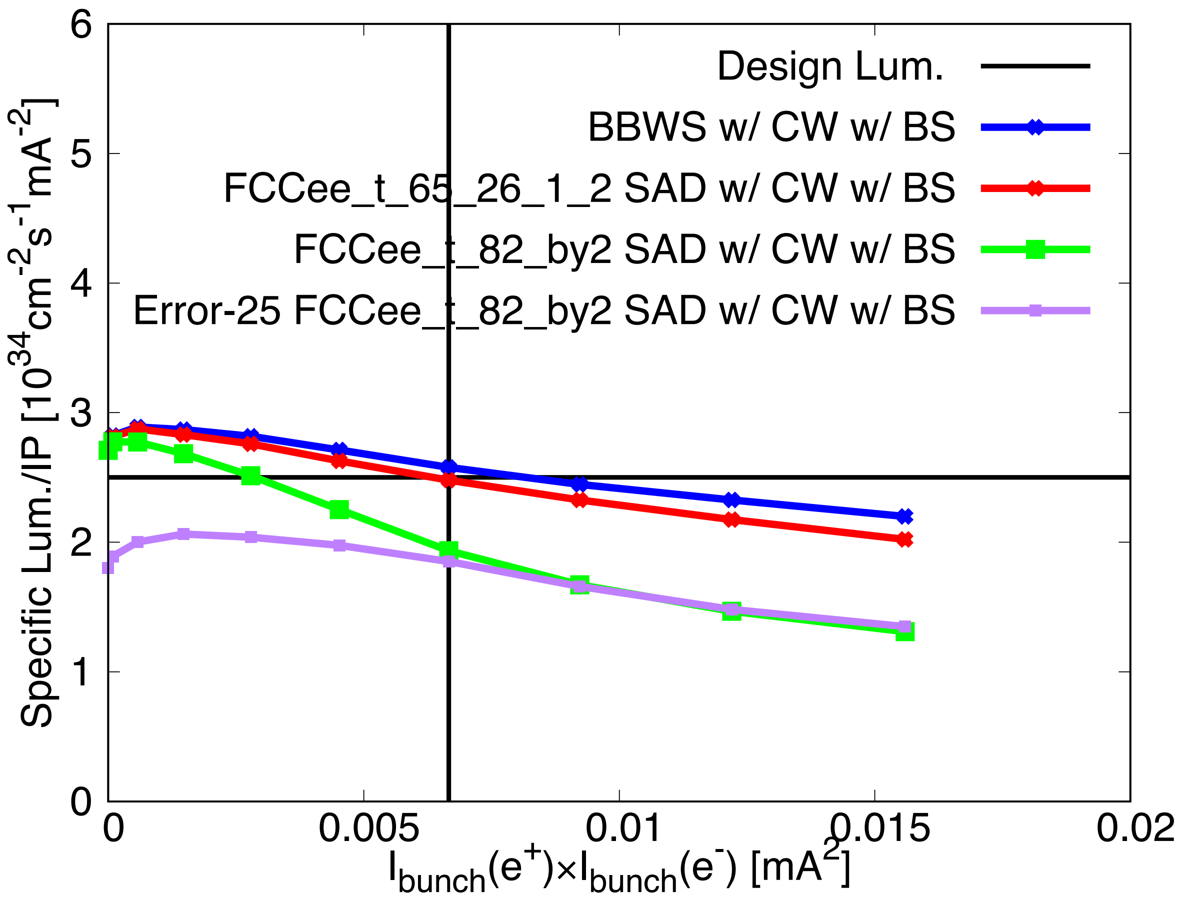
Q: Will beam-beam break the cancellation condition?

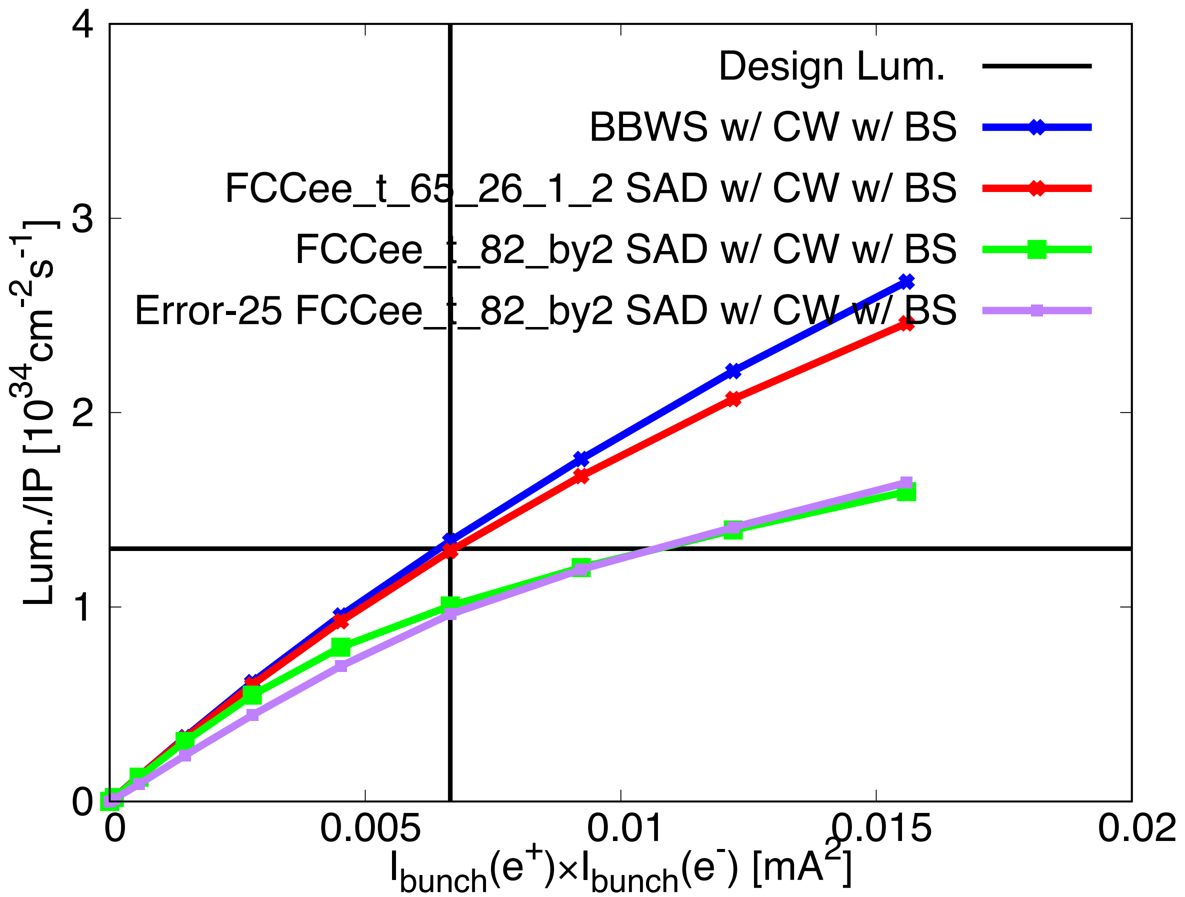


3. BB simulations

➤ **FCCee_t_82_by2_1a_nosol_DS_2**

- **Significant lum. loss due to lattice nonlinearity [to be understood]**
- **With errors: Local coupling and dispersion at IP not corrected; Use radiation damping/excitation matrices**



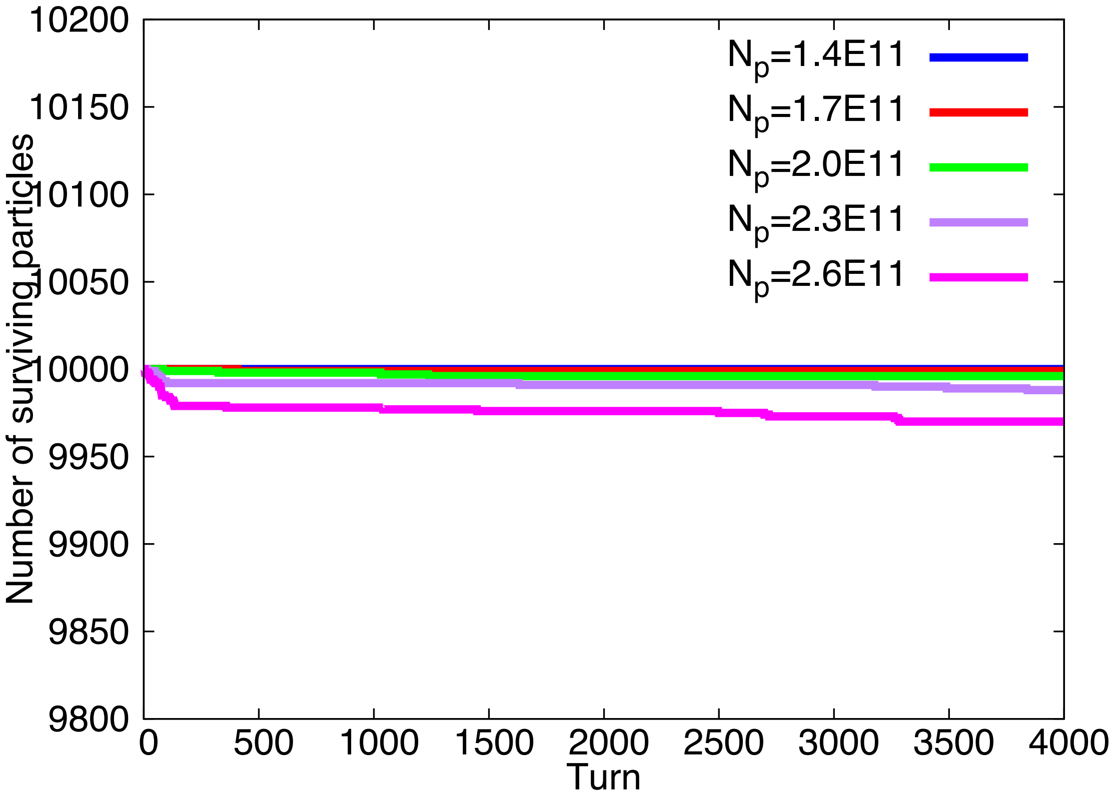


3. BB simulations

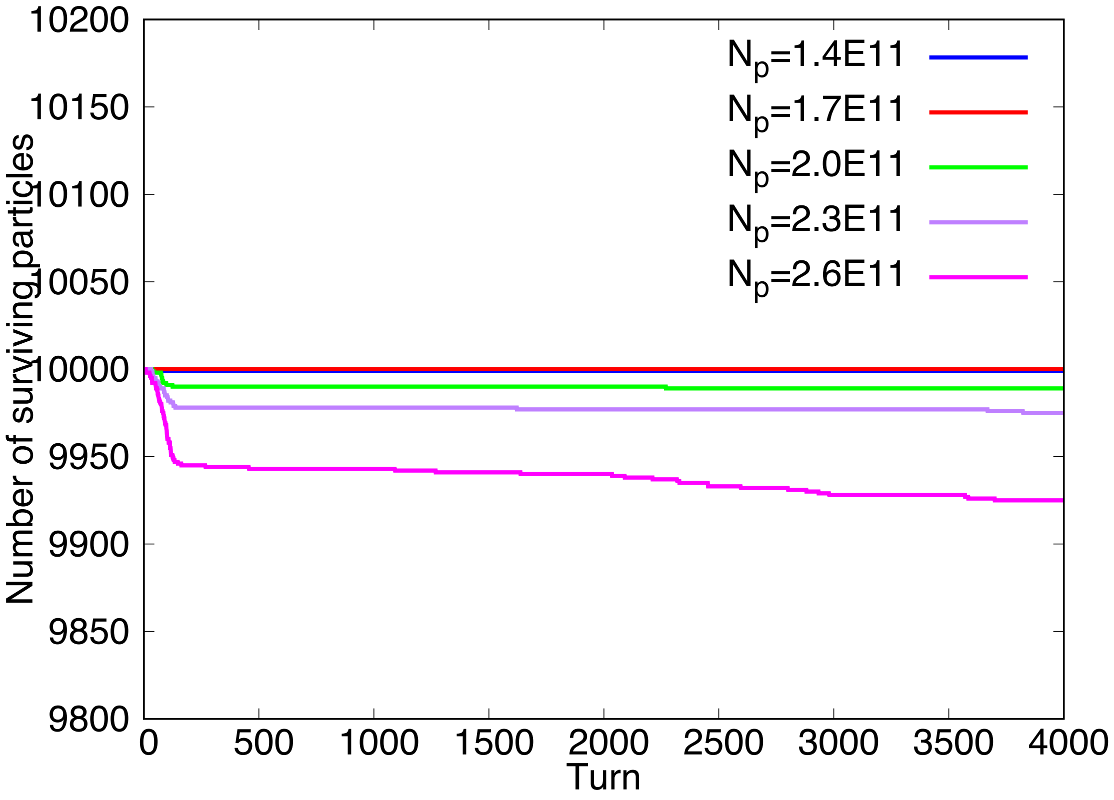
➤ Particle losses in tracking

- Nominal N_p=1.7E11, β_x^*=1m, β_y^*=2mm
- Beamstrahlung effect with finite DA cause particle losses
- Loss rate depend on $\beta^*_{x,y}$, and DA
- Lifetime seems acceptable from previous studies

FCCee_t_65_26_1_2



FCCee_t_82_by2_1a_nosol_DS_2



Refer to D. Zhou, FCC week 2016

3. BB simulations

➤ Particle losses in tracking

- **Alternative estimate: weak-strong simulation [K. Ohmi]**

Lifetime given by weak-strong simulation

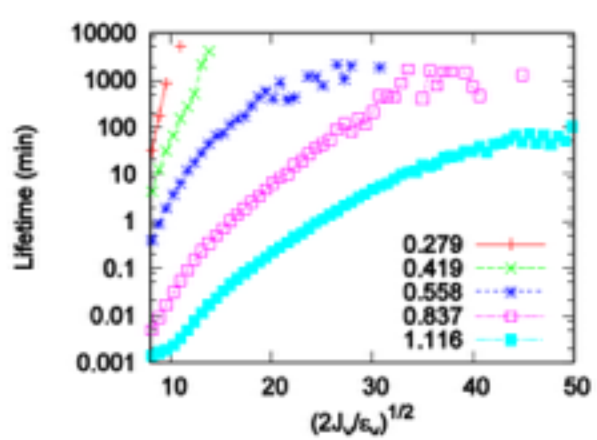
- In equilibrium, particles escape a boundary is the same number as damping from the boundary. [M. Sands, SLAC-R-121 (1970)]

$$\frac{dN}{dt} = f(J_i)\frac{dJ_i}{dt} \qquad \frac{dJ_i}{dt} = -\frac{2J_i}{\tau_i}$$

$$\tau_\ell = \frac{N}{\frac{dN}{dt}} = \frac{t_i}{2J_{i,max}f(J_{i,max})}$$

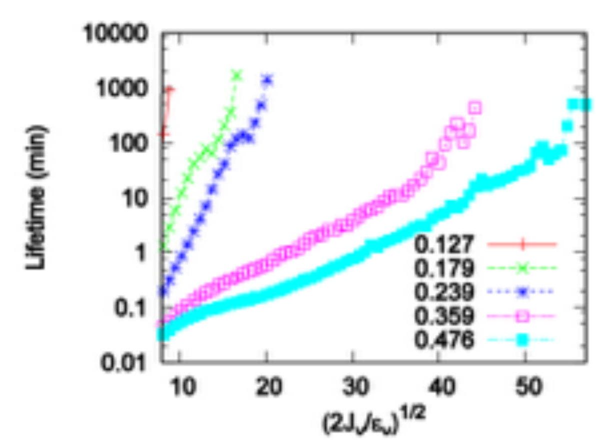
f(J): equilibrium beam distribution. For example f(J)=exp(-J/ε) for Gaussian. f(J)=N(J)/N_0 in the last slide.

Zero crossing angle



vertical aperture

Large crossing angle with crab waist



vertical aperture

Refer to K. Ohmi, 37th FCC-ee optics design meeting

4. Summary

➤ FMA

- Compared with D. Shatilov's results, discrepancies seem understandable

➤ RDTs

- 5th and high-order RDTs from errors in vertical offsets of S{DF}*
- Possible to study interplay of RDTs and beam-beam resonances

➤ Luminosity and lifetime

- Lum. loss for new lattice to be understood
- Need optics correction with errors in vertical offsets of S{DF}*