Beam-beam simulations
and
analysis of lattice nonlinearity using PTC

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Acknowledgements:

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Outline

- ➤ **Introduction**
  - FCC-ee design lattices provided by K. Oide
- ➤ **Beam-beam simulations**
  - BBWS: beam-beam + simple linear map
  - SAD: beam-beam + design lattice
  - At present, only consider average turn-by-turn radiation damping/excitation lumped at one point, no quadrupole radiation and no "saw-tooth" effects in orbit
- ➤ **Analysis of lattice nonlinearity (LN)**
  - Lattice translation: SAD => Bmad => PTC [Straightforward]
  - Resonance driving terms (RDTs) calculations using PTC
- ➤ **Summary**

1. Interplay of BB and latt. nonlin.

➤ The idea

- Method demonstrated in D. Zhou et al, TUPE016, IPAC13
- One-turn map:

$$M = M_{\mathrm{RAD}} \circ M_{\mathrm{BB}} \circ M_0$$

- M_0 can be simple matrix or IP-to-IP realistic map from a design lattice
- Interplay of BB, lattice and other issues reflected in luminosity, DA, beam tail, particle loss, etc.
- Separated simulation/analysis of BB and LN help understand the mechanisms of their interplay

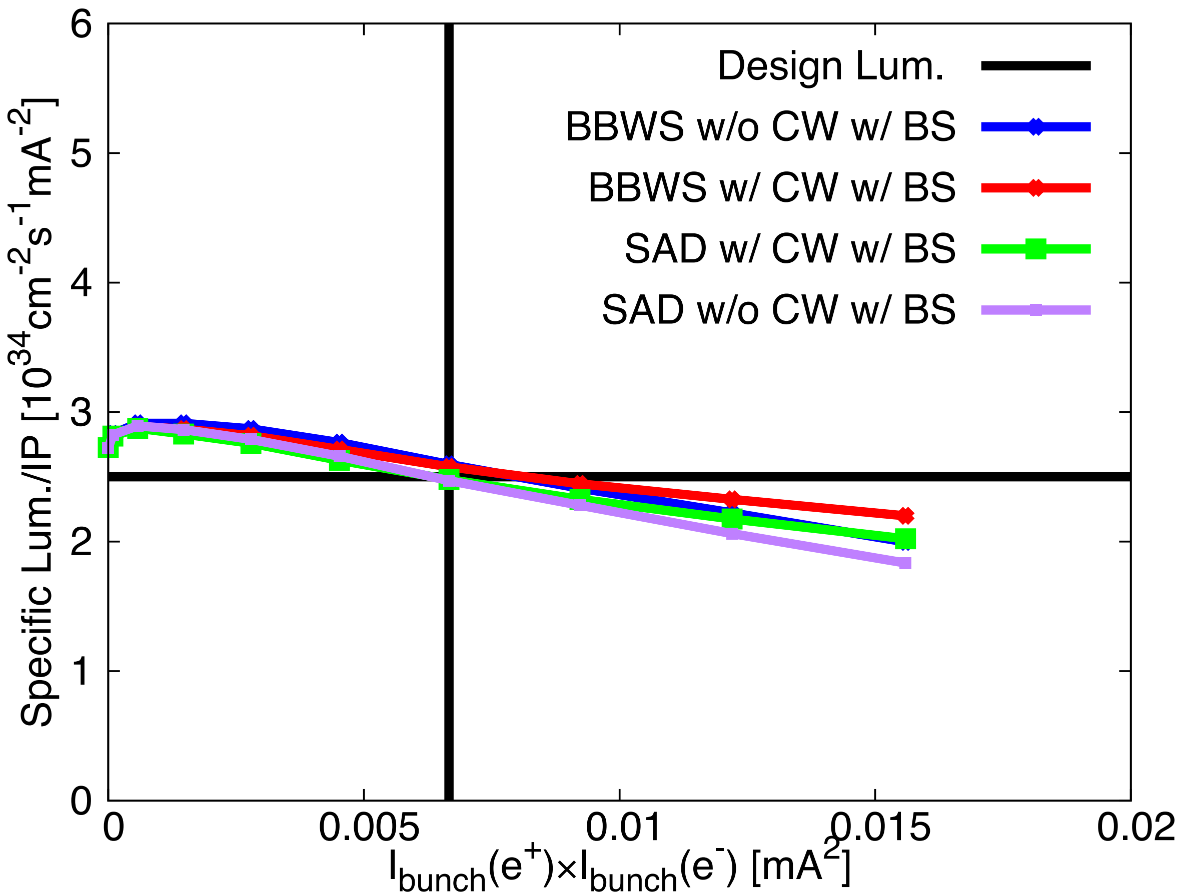
1. Parameters for simulations (half ring)

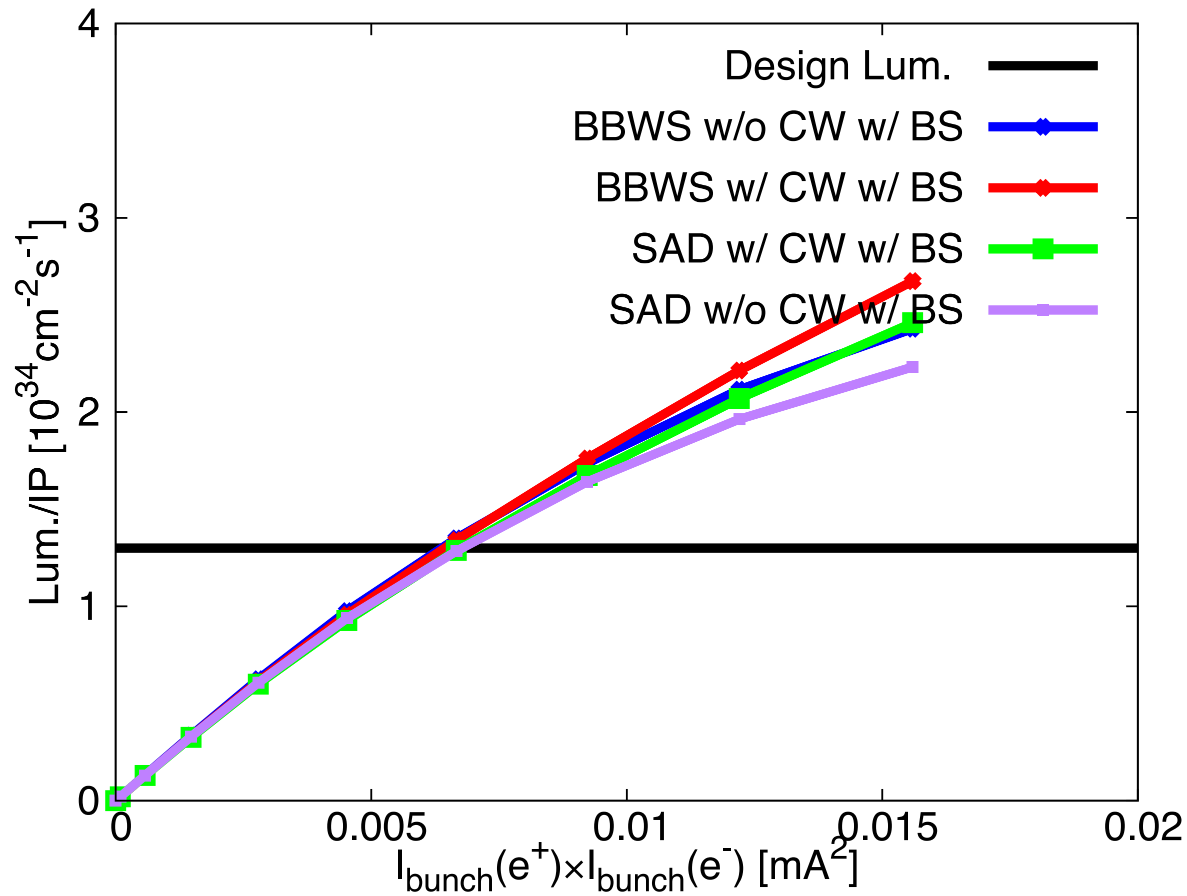
	Z	t
C (km)	49990.9	49990.9
E (GeV)	45.6	175
Number of IPs	1	1
N_b	90300	78
$N_p(10^{11})$	0.33	1.7
Full crossing angle(rad)	0.03	0.03
ε_x (nm)	0.09	1.3
ε_y (pm)	1	2.5
β_x^* (m) [optional]	1 [0.5]	1 [0.5]
β_y^* (mm) [optional]	2 [1]	2 [1]
σ_z (mm)^{SR}	2.7	2.1
$\sigma_\delta(10^{-3})^{SR}$	0.37	1.4
Betatron tune ν_x/ν_y	.55/.57	.54/.57
Synch. tune ν_s	0.0075	0.0375
Damping rate/turn (10^{-2}) [x/y/z]	0.019/0.019/0.038	1.1/1.1/2.2
Lum./IP($10^{34}cm^{-2}s^{-1}$)	68	1.3

Ref. F. Zimmermann, FCC-ee design meeting, Dec. 9, 2015

2. BB simulations: Lum.: t

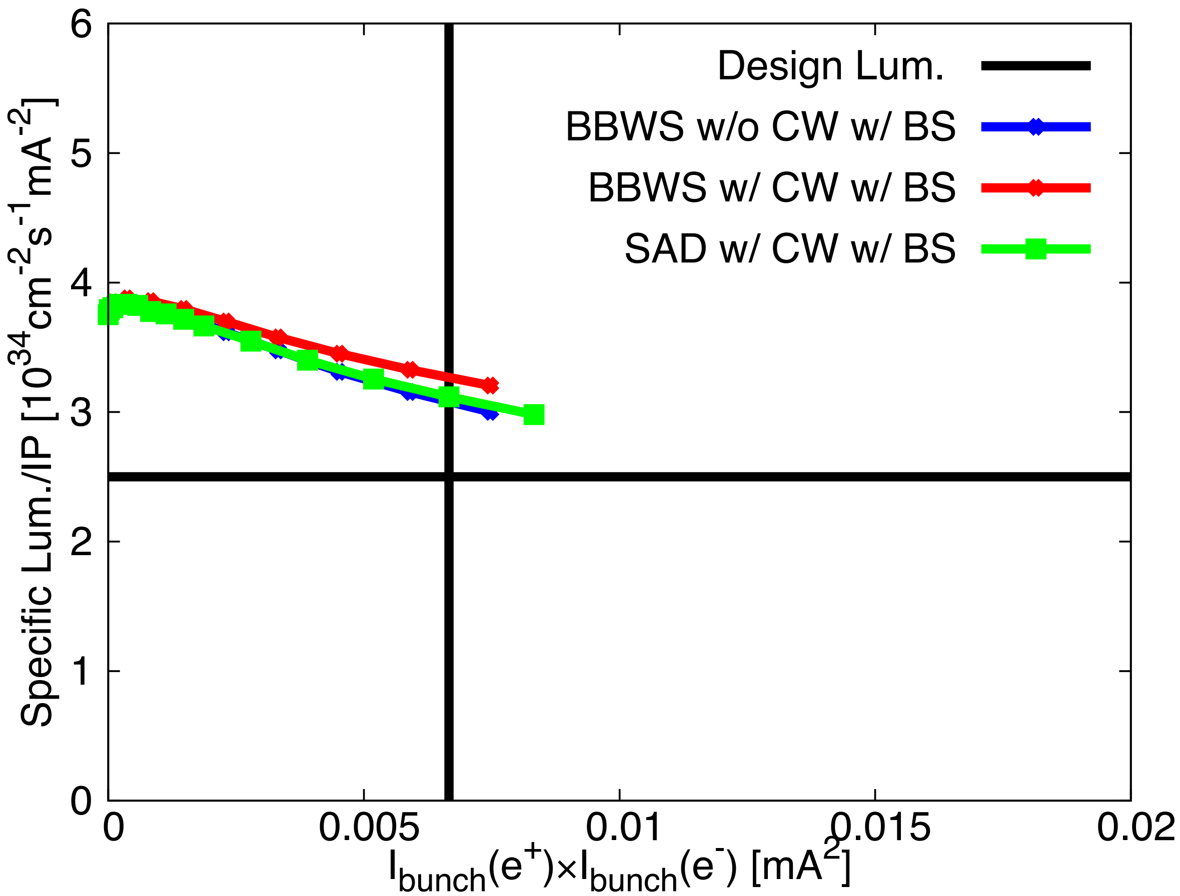
- $\beta_x^*=1$m, $\beta_y^*=2$mm
  - Lattice ver. FCCee_t_65_26_1_2

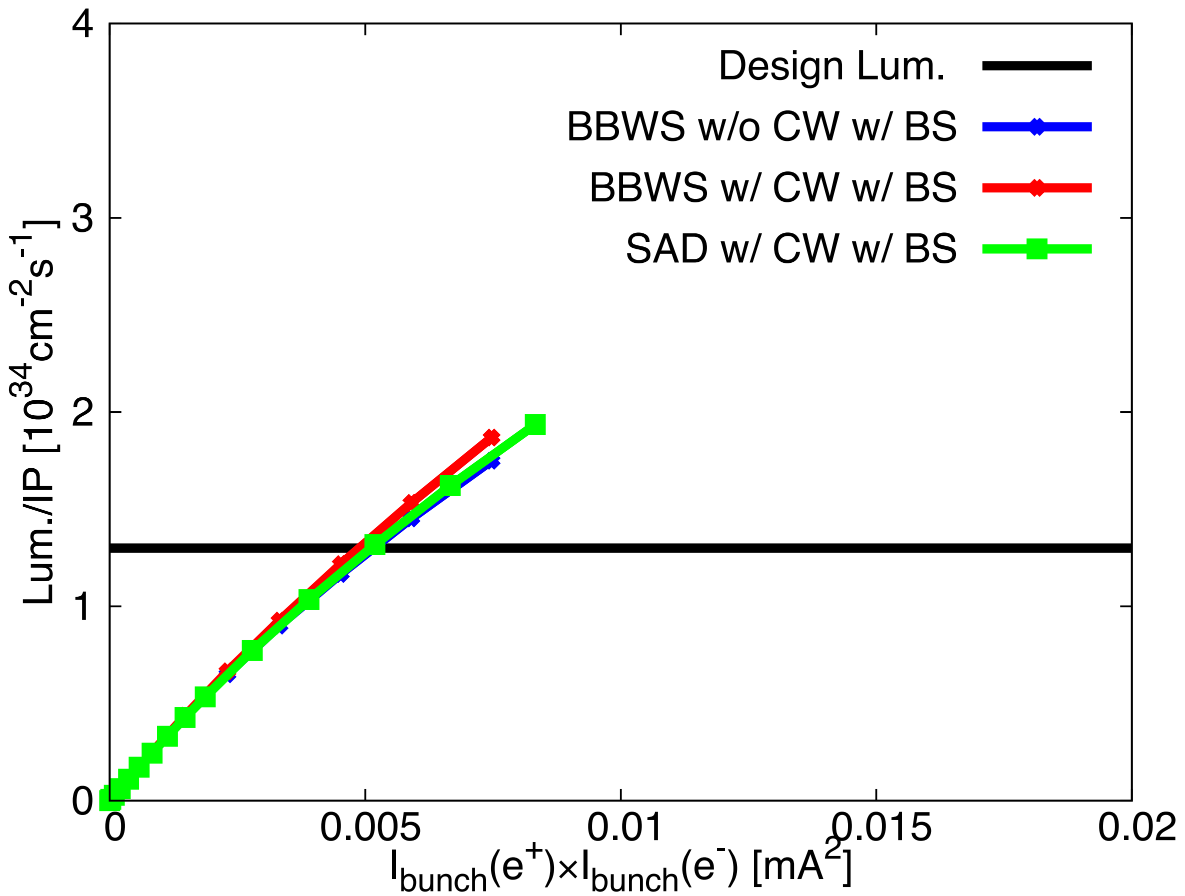




2. BB simulations: Lum.: t

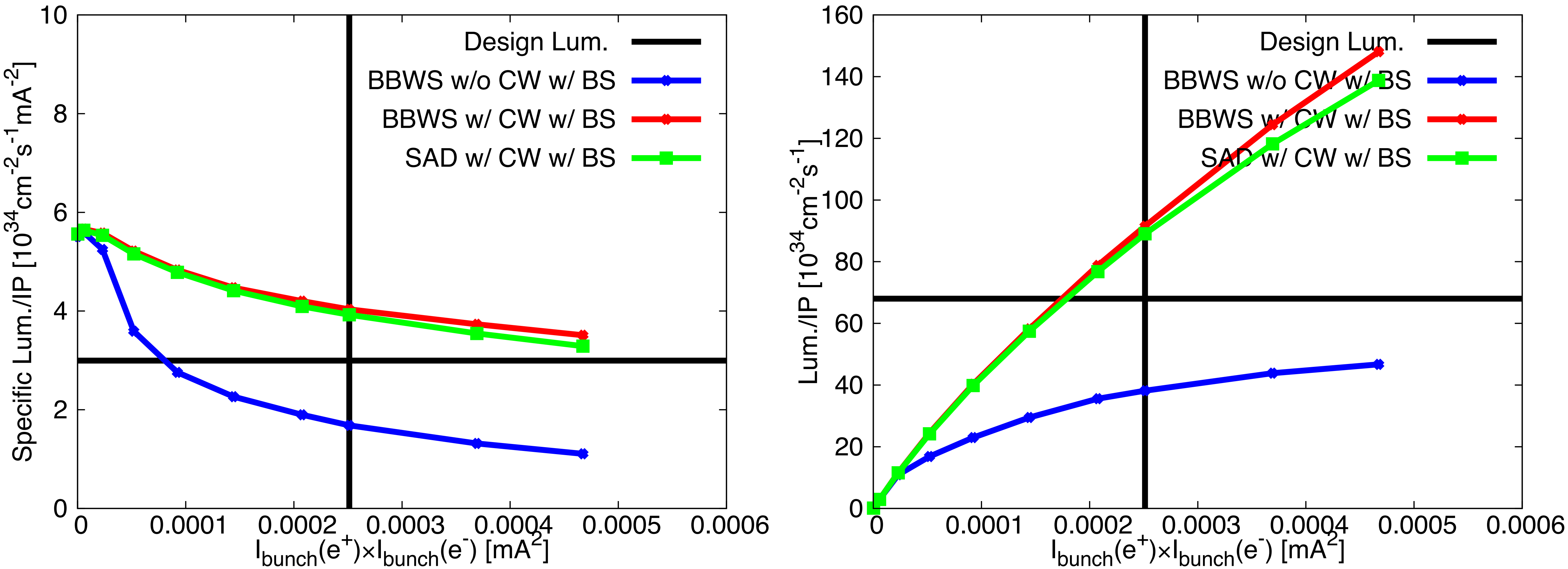
- β_x^*=0.5m, β_y^*=1mm
  - Lattice ver. FCCee_t_65_26





2. BB simulations: Lum.: Z

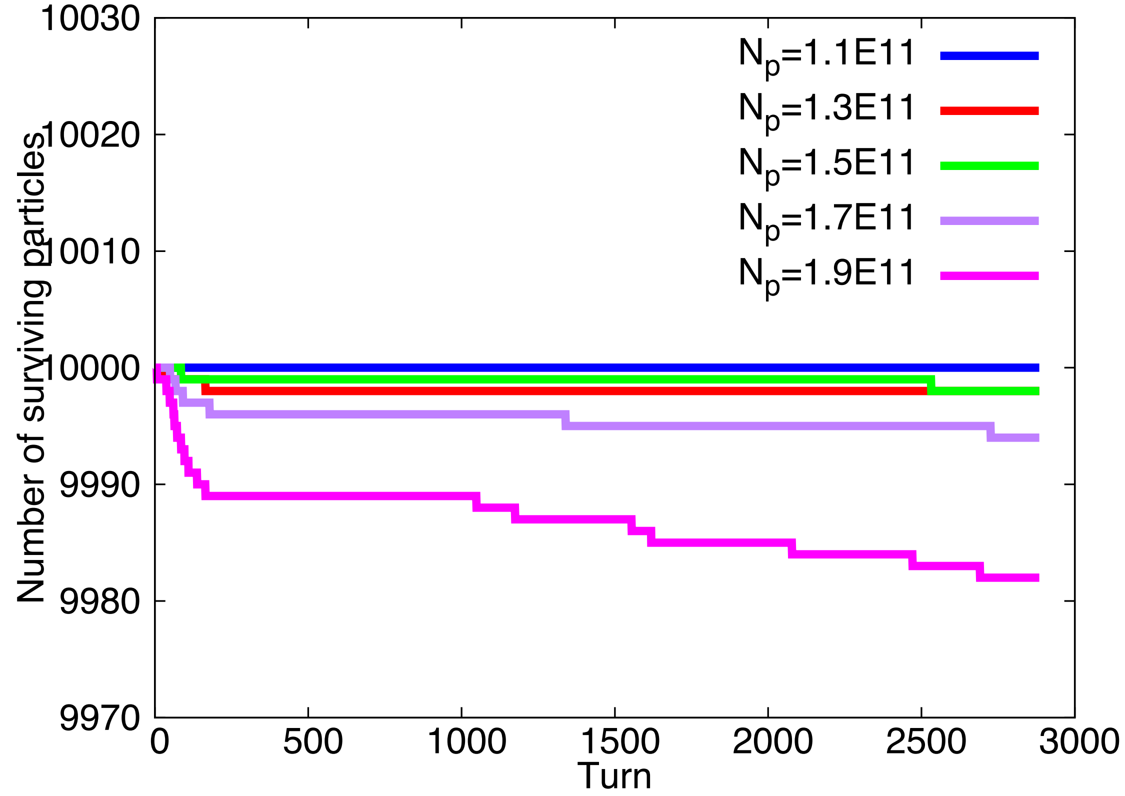
- β_x^*=1m, β_y^*=2mm [Ver. FCCee_z_65_36]
  - Lattice ver. FCCee_z_65_36



2. BB simulations: Particle loss: t

FCCee_t_65_26

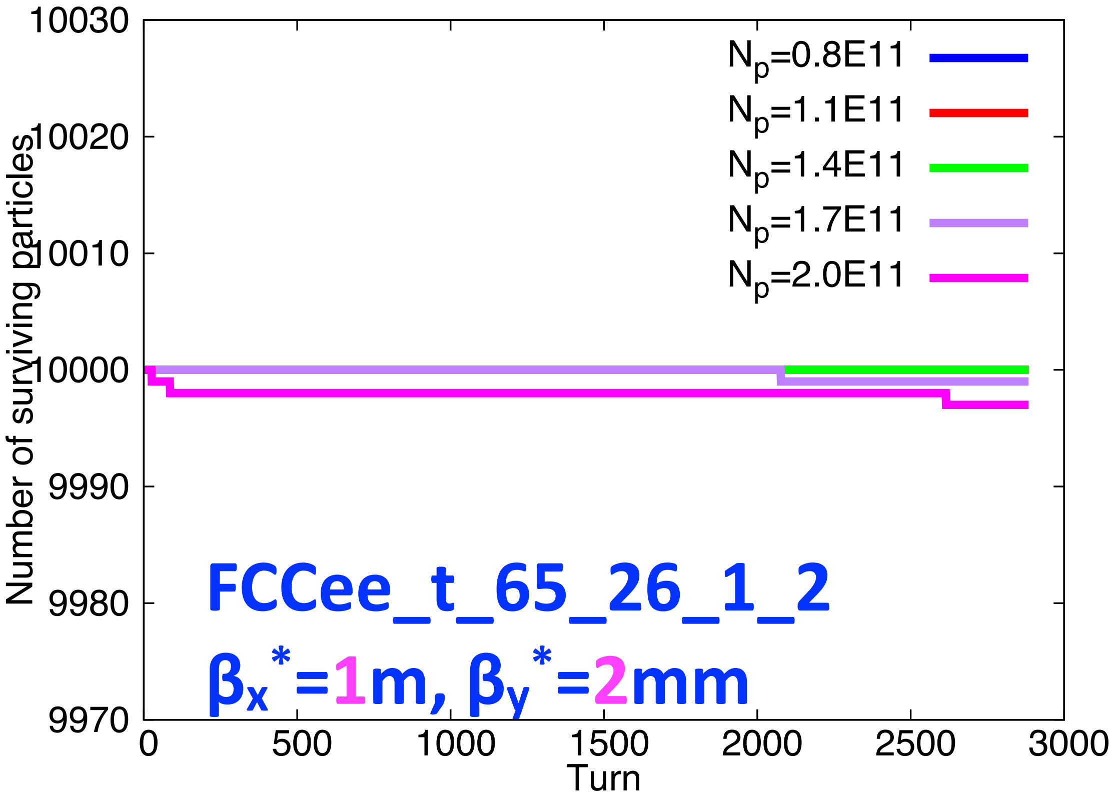
β_x^*=0.5m, β_y^*=1mm

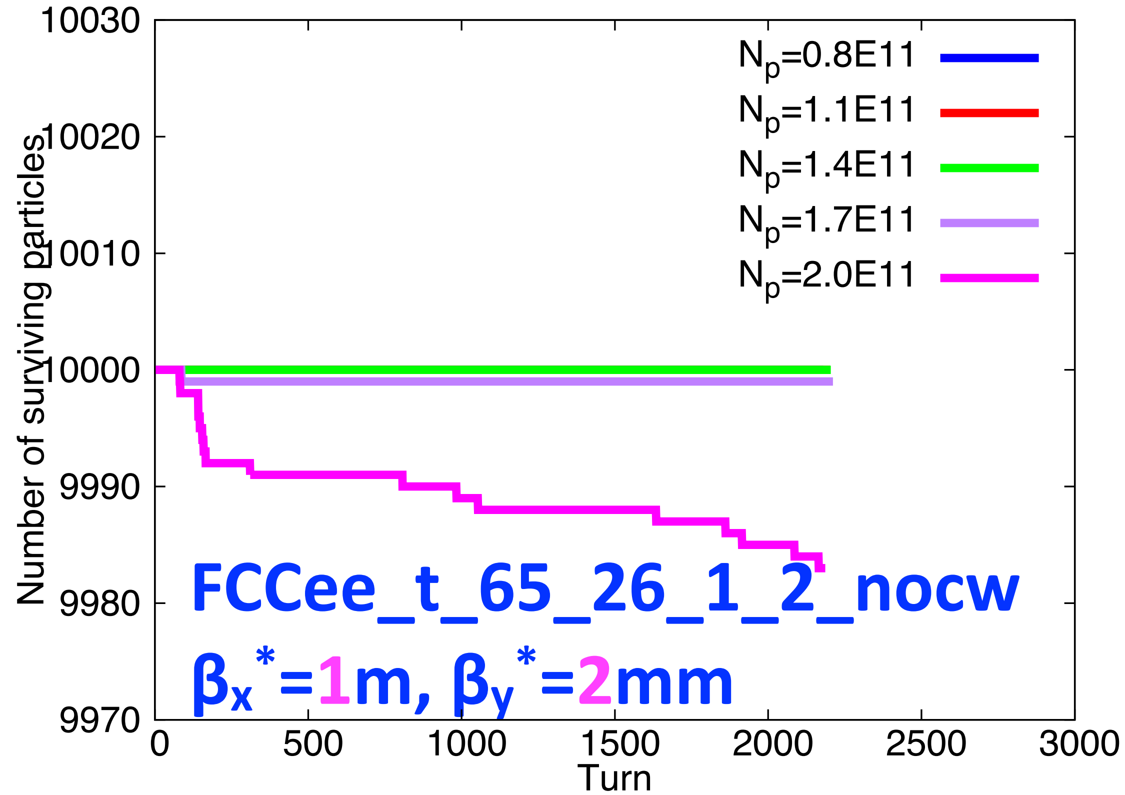


- **Particle losses in tracking**
  - Threshold observed
  - Nominal N_p=1.7E11
  - Loss rate depend on $\beta^*_{x,y}$,
  - Slipping out of RF bucket?
  - Improper setting of simulations?

  Mismatch in transverse beam sizes,

  No crab waist for the strong beam





3. RDTs

➤ Resonance driving terms (RDTs) indicate lattice nonlinearity

The effective Hamiltonian of a ring can be normalized in resonance bases [Ref. E. Forest, *Beam Dynamics – A New Attitude and Framework*, 1998].

For a ring with n elements, one can normalize the one turn map $\mathcal{M}_{1\to n}$ as [Ref. L. Yang *et al.*, Phys. Rev. ST Accel. Beams 14, 054001 (2011)]

$$\mathcal{M}_{1\to n} = \mathcal{A}_1^{-1} e^{:h:} \mathcal{R}_{1\to n} \mathcal{A}_1,$$

with $\mathcal{R}$: rotation, $e^{:h:}$: nonlinear Lie map, $\mathcal{A}_1$: normalizing map. Assume no coupling (the theory can be generalized for nonzero coupling), $\mathcal{A}_i$ in x plane at the ith element can be approximated in perturbation theory as

$$\mathcal{A}_i x = \sqrt{\beta_{x,i}} x + \eta_{x,i} \delta,$$

$$\mathcal{A}_i p_x = \frac{-\alpha_{x,i} x + p_x}{\sqrt{\beta_{x,i}}} + \eta'_{x,i} \delta.$$

3. RDTs

➤ RDTs indicate lattice nonlinearity

In the resonance basis, using action-angle variables (J, ϕ) one can write

$$h_x^{\pm} \equiv \sqrt{2J_x}e^{\pm i\phi_x} = X \mp iP_x,$$

$$\mathcal{R}_{i\to j}h_x^{\pm} = \mathcal{R}_{i\to j}\sqrt{2J_x}e^{\pm i\phi_x} = e^{\pm i\mu_{i\to j,x}}h_x^{\pm},$$

where $\mu_{i\to j,x}$ is the phase advance of $i \to j$. Consequently, the potential of a multipole magnetic field can be expanded in the resonance bases of h_{abcde} as

$$h = \sum h_{abcde}h_x^{+a}h_x^{-b}h_y^{+c}h_y^{-d}\delta^e.$$

Each h_{abcde} (a complex number in general) drives a certain resonance, and is an explicit function of magnet strengths, beta functions and dispersions.

3. RDTs

➤ RDTs indicate lattice nonlinearity

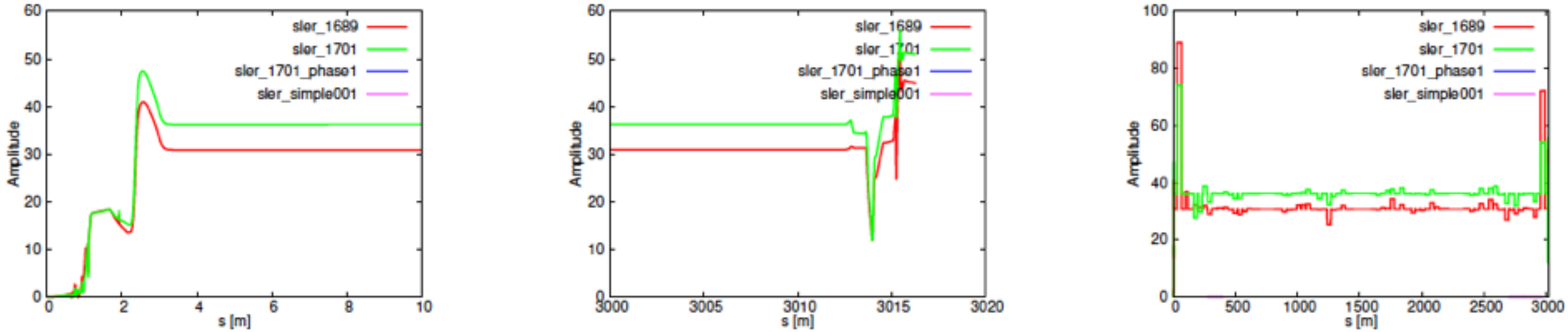
h_{abcde}	Driving effects
h_{11001}, h_{00111}	Linear chromaticity ζ_x, ζ_y
$h_{21000}, h_{12000} \| h_{10110}, h_{01110}$	ν_x $[(J_x)^{3/2}] \| [(J_x)^{1/2}(J_y)]$
$h_{30000}, h_{03000} \| h_{00300}, h_{00030}$	$3\nu_x$ $[(J_x)^{3/2}] \| 3\nu_y$ $[(J_y)^{3/2}]$
$h_{10020}, h_{01200} \| h_{10200}, h_{01020}$	$\nu_x - 2\nu_y \| \nu_x + 2\nu_y$ $[(J_x)^{1/2}(J_y)]$
$h_{20010}, h_{02100} \| h_{20100}, h_{02010}$	$2\nu_x - \nu_y \| 2\nu_x + \nu_y$ $[(J_x)(J_y)^{1/2}]$
$h_{00210}, h_{00120} \| h_{11100}, h_{11010}$	ν_y $[(J_y)^{3/2}] \| [(J_x)(J_y)^{1/2}]$
$h_{22000}, h_{00220}, h_{11110}$	$d\nu_x/dJ_x, d\nu_y/dJ_y, d\nu_{x,y}/dJ_{y,x}$
$h_{40000}, h_{04000} \| h_{00400}, h_{00040}$	$4\nu_x$ $[(J_x)^2] \| 4\nu_y$ $[(J_y)^2]$
$h_{31000}, h_{13000} \| h_{20110}, h_{02110}$	$2\nu_x$ $[(J_x)^2] \| [(J_x)(J_y)]$
$h_{00310}, h_{00130} \| h_{11200}, h_{11020}$	$2\nu_y$ $[(J_y)^2] \| [(J_x)(J_y)]$
$h_{20020}, h_{02200} \| h_{20200}, h_{02020}$	$2\nu_x - 2\nu_y \| 2\nu_x + 2\nu_y$ $[(J_x)(J_y)]$
$h_{30010}, h_{03100} \| h_{30100}, h_{03010}$	$3\nu_x - \nu_y \| 3\nu_x + \nu_y$ $[(J_x)^{3/2}(J_y)^{1/2}]$
$h_{10030}, h_{01300} \| h_{10300}, h_{01030}$	$\nu_x - 3\nu_y \| \nu_x + 3\nu_y$ $[(J_x)^{1/2}(J_y)^{3/2}]$

Table : Low-order driving terms.

3. RDTs: PTC calculation

➤ PTC applied to SuperKEKB: an example

- $2\nu_x - \nu_y$ $[(J_x)(J_y)^{1/2}]$ resonance
- 3D fields near IP [Solenoid and FF quad fringe fields] generate lots of low-order nonlinearities, hard to be compensated using arc multipoles [global correction]
- Simplified lattices show less nonlinearity



(a) IP→s=10m (b) s=3000m→IP (c) Whole ring

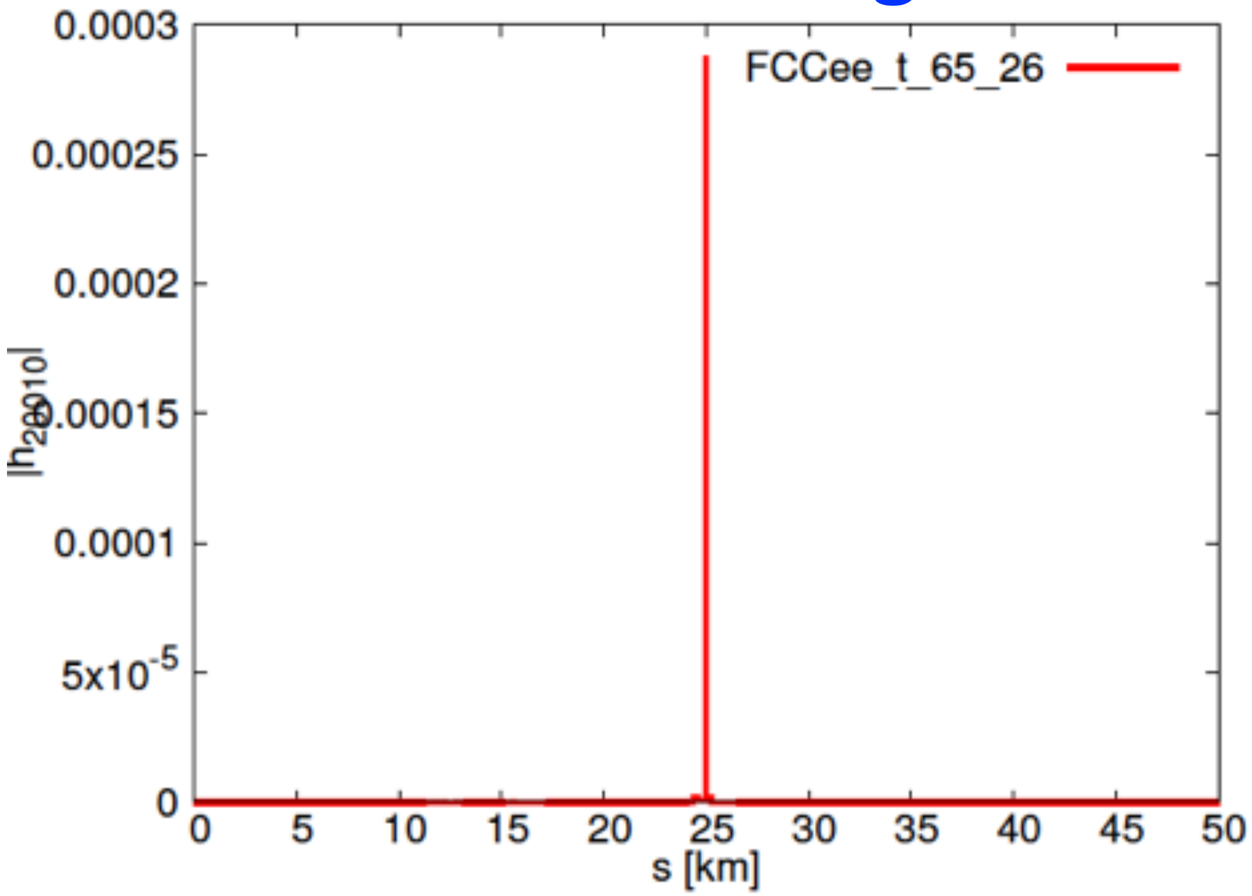
Figure : $|h_{20010}|$ accumulated along the ring.

3. RDTs: PTC calculation

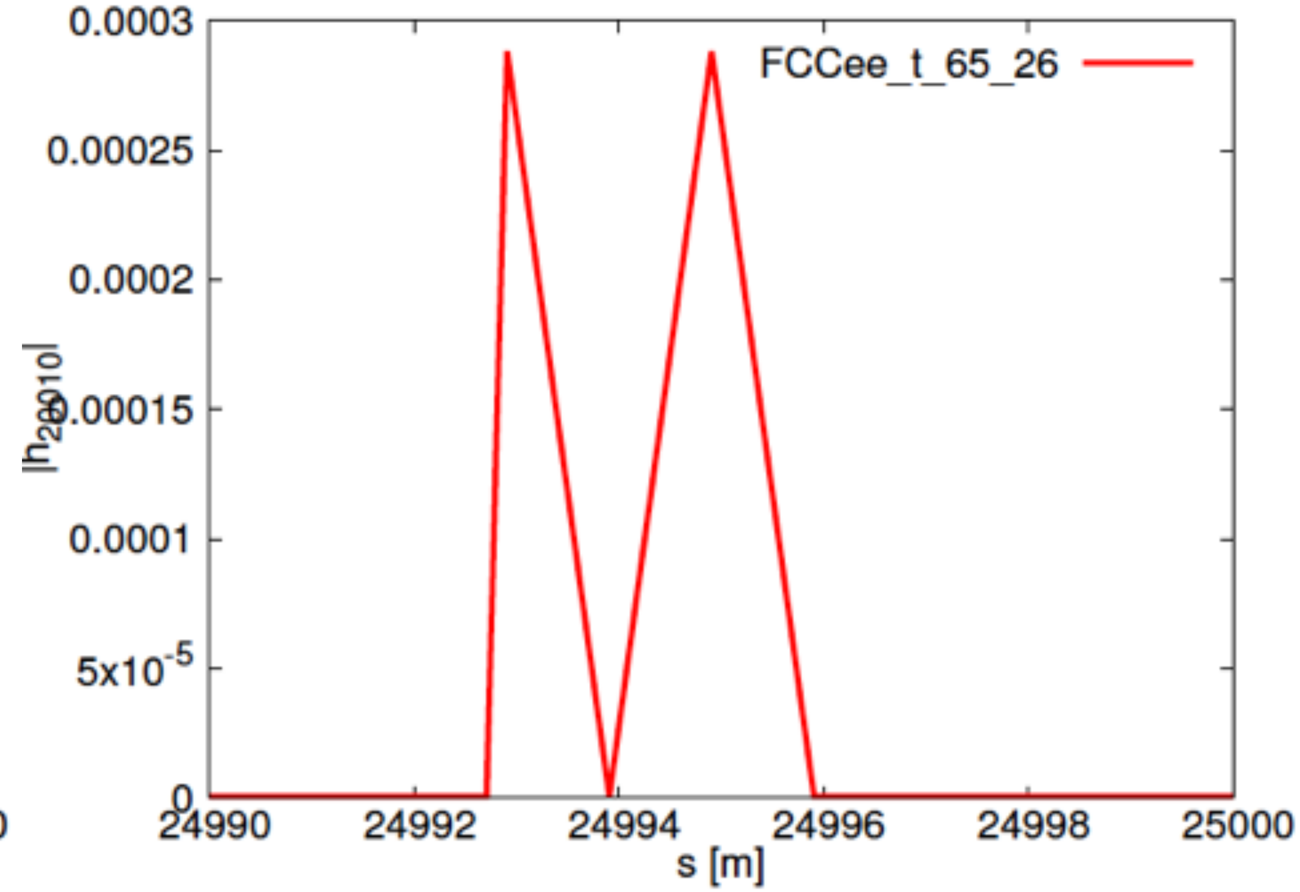
➤ **PTC applied to FCC-ee t lattice: an example**

- $2\nu_x - \nu_y$ $[(J_x)(J_y)^{1/2}]$ resonance for latt. ver. FCCee_t_65_26
- In general, no significant 3rd resonances in FCC-ee lattices

Whole ring



Near IP

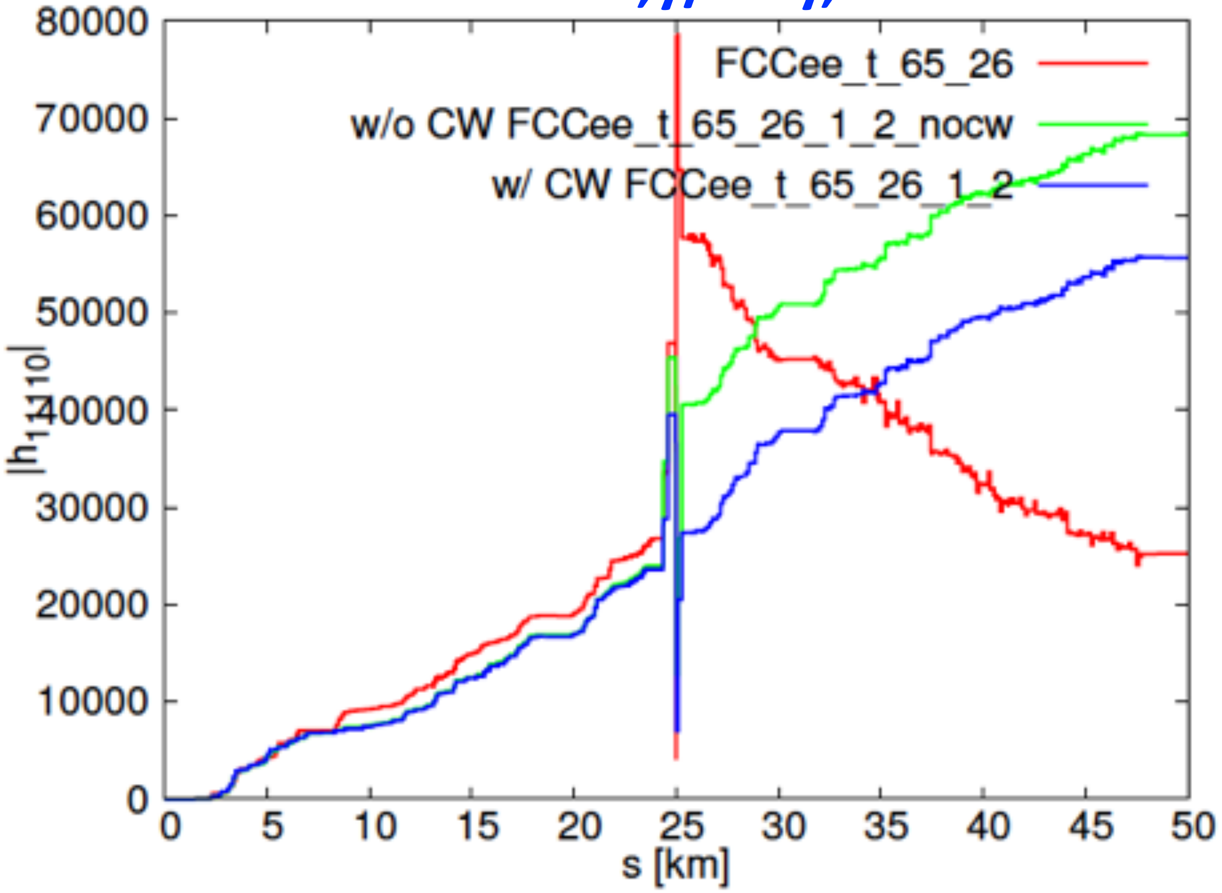


3. RDTs: PTC calculation

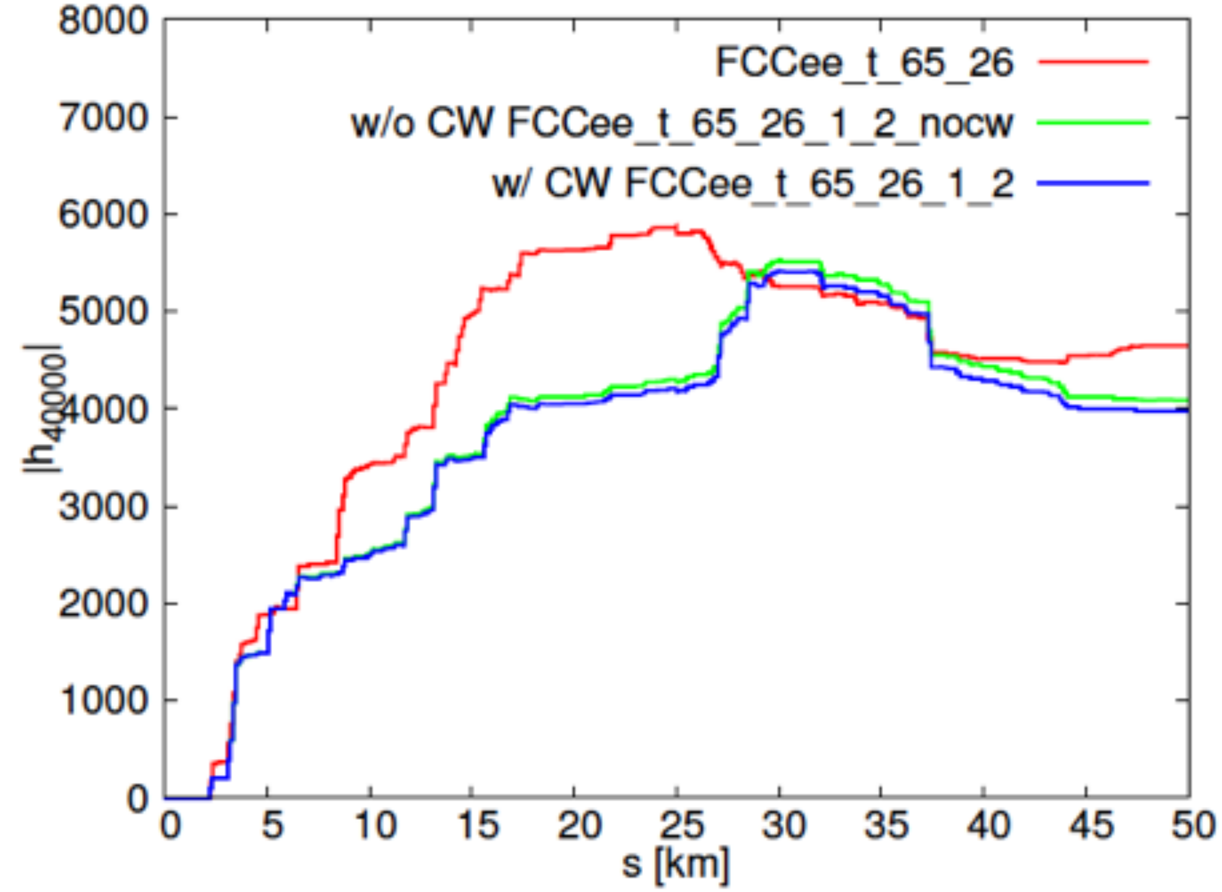
➤ PTC applied to FCC-ee t lattice: an example

- **4th order RDTs for latt. ver. FCCee_t_65_26**
- **Residual 4th order RDTs exist, and depend on lattice design/optimization**

$d\nu_{x,y}/dJ_{y,x}$



$4\nu_x$



4. Summary

➤ Beam-beam simulations

- Small loss [order of a few percent] of luminosity due to BB+Lattice [No limit in luminosity performance, and should be controllable via optics optimization]
- Particle loss in SAD simulations [to be understood]

➤ Lattice analysis using PTC

- PTC is ready for RDTs calculations
- Identify sources of nonlinearity
- Identify dominant nonlinear terms in one-turn-map
- Evaluate lattice designs and optimizations

➤ To do list

- Understand macro-particle losses in beam-beam simulations with lattices
- Simulations for FCC-ee Z lattices with β_x^*=0.5m, β_y^*=1mm
- Simulations with quadrupole/distributed radiation and “saw-tooth” effects