coherent synchrotron radiation for damping rings: assessment of theories & experimental proposal

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Thanks to Karl Bane, Mitsuo Kikuchi, Kazuhito Ohmi, Katsunobu Oide, Yannis Papaphilippou, Demin Zhou

some references

[1] G. Stupakov and S. Heifets, “Beam Instability and Microbunching due to Coherent Synchrotron Radiation,” PRST-AB 5, 054402 (2002); http://prst-ab.aps.org/pdf/PRSTAB/v5/i5/e054402

[2] S. Heifets and G. Stupakov, “Single-mode Coherent Synchrotron Radiation Instability,” PRST-AB 6, 064401 (2003); http://prst-ab.aps.org/pdf/PRSTAB/v6/i6/e064401

[3] K.L.F. Bane, Y. Cai, G. Stupakov, “Comparison of Simulation Codes for Microwave Instability in Bunched Beams,” Proc. IPAC’10, Kyoto Japan (2010) ; http://epaper.kek.jp/IPAC10/papers/tupd078.pdf

[4] D. Zhou et al, “CSR in the SuperKEKB Damping Ring,” IPAC’10 Kyoto (2010), http://epaper.kek.jp/IPAC10/papers/tupeb018.pdf

[5] F. Zimmermann, « Estimates of CSR Instability Thresholds for Various Storage Rings,” CLIC Note to be published

Stupakov-Heifets formulae (2002)

$$\Lambda = \frac{N_b r_e \rho \sqrt{2\pi}}{C|\eta|\sigma_z \gamma \sigma_\delta^2}$$ Stupakov-Heifets parameter

conditions for instability:

$$\frac{\rho}{b} \le \Lambda$$ no shielding by the beam pipe

$$\sigma_z \ge \frac{\rho}{2\Lambda^{3/2}}.$$ sufficiently long bunch (coasting beam approximation)

small horizontal beam size

$$\Lambda << \left(\frac{\rho^2}{\sigma_x \beta_x}\right)^{2/3}$$

negligible effect of velocity spread

$$\frac{N_b r_e}{\sqrt{2\pi}\gamma\sigma_z\sigma_\delta} << 1$$

continuous mode spectrum (2003):

$$C03 \equiv \left(\frac{N_b r_e}{\sqrt{2\pi}\gamma\sigma_z}\right)^5 \left(\frac{2a}{\eta\rho}\right)^3 \frac{1}{\sigma_\delta^8} \frac{1}{\pi^{2/3}} \ge 1$$

not fulfilled!

Stupakov-Heifets formulae [1,2] evaluated for several storage rings

	SuperKEKB LER	SuperKEKB HER	SuperKEKB e+ DR	ATF DR
beam energy [GeV]	4	7	1.1	1.28
slip factor η	0.000274	0.000188	0.017	0.0019
rms mom. spread $\sigma_{\delta,rms}$ [%]	0.08	0.065	0.055	0.06
bunch population [10^9]	90	65	37.5	10
circumference C [m]	3016	3016	135	138.6
bending radius ρ [m]	73.3	104.5	2.65	5.73
vert. beam pipe radius b [cm]	4.7	2.5	1.6	1.2?
Stupakov-Heifets parameter Λ	**1864**	**2905**	**67**	**339**
ρ/b	**1560**	**4180**	**166**	**478**
σ_z [cm]	**0.6**	**0.5**	**0.7**	**0.5**
$\rho/(2\,\Lambda^{3/2})$ [cm]	**0.05**	**0.03**	**0.24**	**0.05**
$N_b r_0/((2\pi)^{1/2}\sigma_z\sigma_\delta\,\gamma)$	0.0027	0.0016	0.0051	0.0015
β_x at bend [m]	10?	10?	1.5	3?
ε_x [nm]	3.2	5.0	2100→ 41	~1.5
σ_x at bend [μm]	179	224	248	67
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	20800	28800	710	2990
τ_x [ms]	37?	56?	11	17.2
C03	**0.0128**	**0.0003**	**0.0033**	**0.0002**
Q_s	-0.025	-0.025	-0.015	-0.0045
$N_{b,thr}$	1.0×10^{11}	1.9×10^{11}	4.5×10^{10}	1.15×10^{10}
$N_b/N_{b,thr}$	**0.89**	**0.35**	**0.83**	**0.86**
$\Lambda b/\rho$	**1.19**	**0.69**	**0.40**	**0.71**

Bane-Cai-Stupakov result (2010)

$$N_{b,thr} \approx \left(\frac{2\pi Q_s \gamma \sigma_{\delta 0}}{r_e} \right) \left(\frac{\sigma_z^{4/3}}{\rho^{1/3}} \right) \left(0.5 + 0.12 \frac{\sigma_{z0} \rho^{1/2}}{b^{3/2}} \right).$$

shielding effects [two parallel plates]

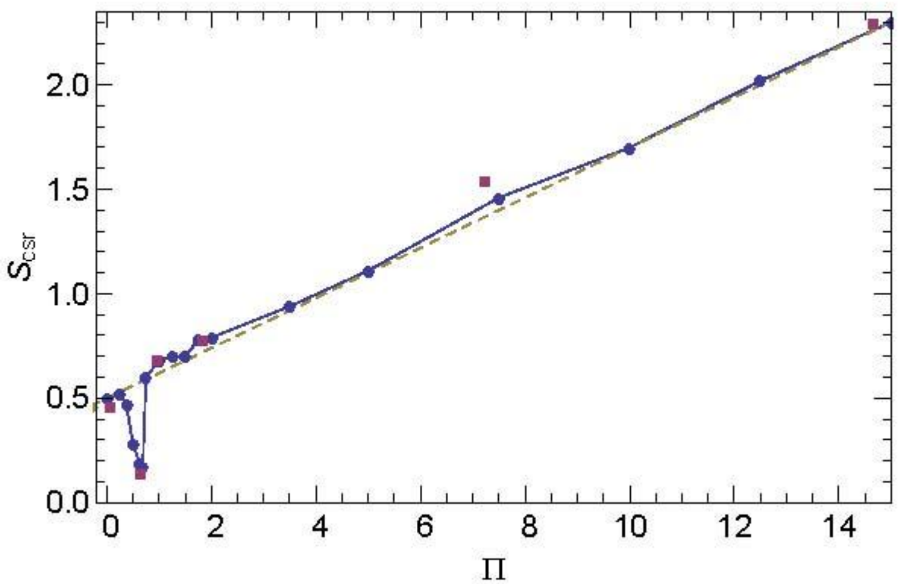


Figure 2: For the CSR wake, threshold value of S_{csr} *vs.* shielding parameter, $\Pi = \rho^{1/2}\sigma_{z0}/h^{3/2}$. Symbols give results of the VFP solver (blue) and the LV code (red).

	SuperKEKB LER	SuperKEKB HER	SuperB LER	SuperB HER	CLIC DR 2009	CLIC DR 2010
beam energy [GeV]	4	7	4.18	6.7	2.86	
slip factor η	0.000274	0.000188	0.00042	0.0004	$6.5\text{x}10^{-5}$	$8\text{x}10^{-5}$
rms mom. spread $\sigma_{\delta,rms}$ [%]	0.08	0.065	0.066	0.062	0.11	0.13
bunch population [10^9]	90	65	57.4	57.4	4.1	
circumference C [m]	3016	3016	1258	1258	493	421
bending radius ρ [m]	73.3	104.5	29.3	80.5	6.9	6.84
vert. beam pipe radius b [cm]	4.7	2.5	2.0	2.5	0.9	1.0
Stupakov-Heifets parameter Λ	**1864**	**2905**	**1254**	**2557**	**915**	**387**
ρ/b	**1560**	**4180**	**1465**	**3220**	**767**	**684**
σ_z [cm]	**0.6**	**0.5**	**0.5**	**0.5**	**0.1**	**0.16**
$\rho/(2\,\Lambda^{3/2})$ [cm]	**0.05**	**0.03**	**0.03**	**0.03**	**0.01**	**0.04**
$N_b r_0/((2\pi)^{1/2}\sigma_z\sigma_\delta\,\gamma)$	0.0027	0.0016	0.0024	0.0016	0.0007	0.0004
β_x at bend [m]	10?	10?	6?	2?	0.2	
ε_x [nm]	3.2	5.0	2.41	2.0	0.09	
σ_x at bend [μm]	179	224	120	63	4	
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	20800	28800	11230	137900	146600	144900
τ_x [ms]	37?	56?	44	29	1.62	1.88
C03	0.0128	0.0003	0.0042	0.0001	0.0052	0.0001
Q_s	-0.025	-0.025	-0.01	-0.01	-0.009	-0.0076
$N_{b,thr}$	$1.0\text{x}10^{11}$	$1.9\text{x}10^{11}$	$5.5\text{x}10^{10}$	$6.7\text{x}10^{10}$	$5.7\text{x}10^{9}$	$1.2\text{x}10^{10}$
$N_b/N_{b,thr}$	**0.89**	**0.35**	**1.04**	**0.85**	**0.72**	**0.33**
$\Lambda b/\rho$	**1.19**	**0.69**	**0.86**	**0.79**	**1.19**	**0.57**

predictions from Stupakov-Heifets & Bane-Cai-Stupakov often similar, but not always!

- e.g., for SuperB LER $\rho/b<\Lambda$, and, yet, the Bane et al formula predicts instability.

2010 version of the CLIC DR more stable than the previous one

both the shielding by the beam pipe and the finite bunch length will prevent any CSR microbunching instability in the KEKB Damping Ring; the instability is also unlikely to appear in the ATF for present operating conditions

Concern:

At the threshold the inequality “C03≥1” of [2] is not fulfilled for any of the example storage rings considered so far!

This could mean that **only a single isolated mode should drive the CSR instability** in all these cases, and arguably that neither the formalism of [1] nor the one of [3] is applicable.

could one make CSR instability appear at ATF?

threshold strongly depends on the momentum spread through the parameter Λ

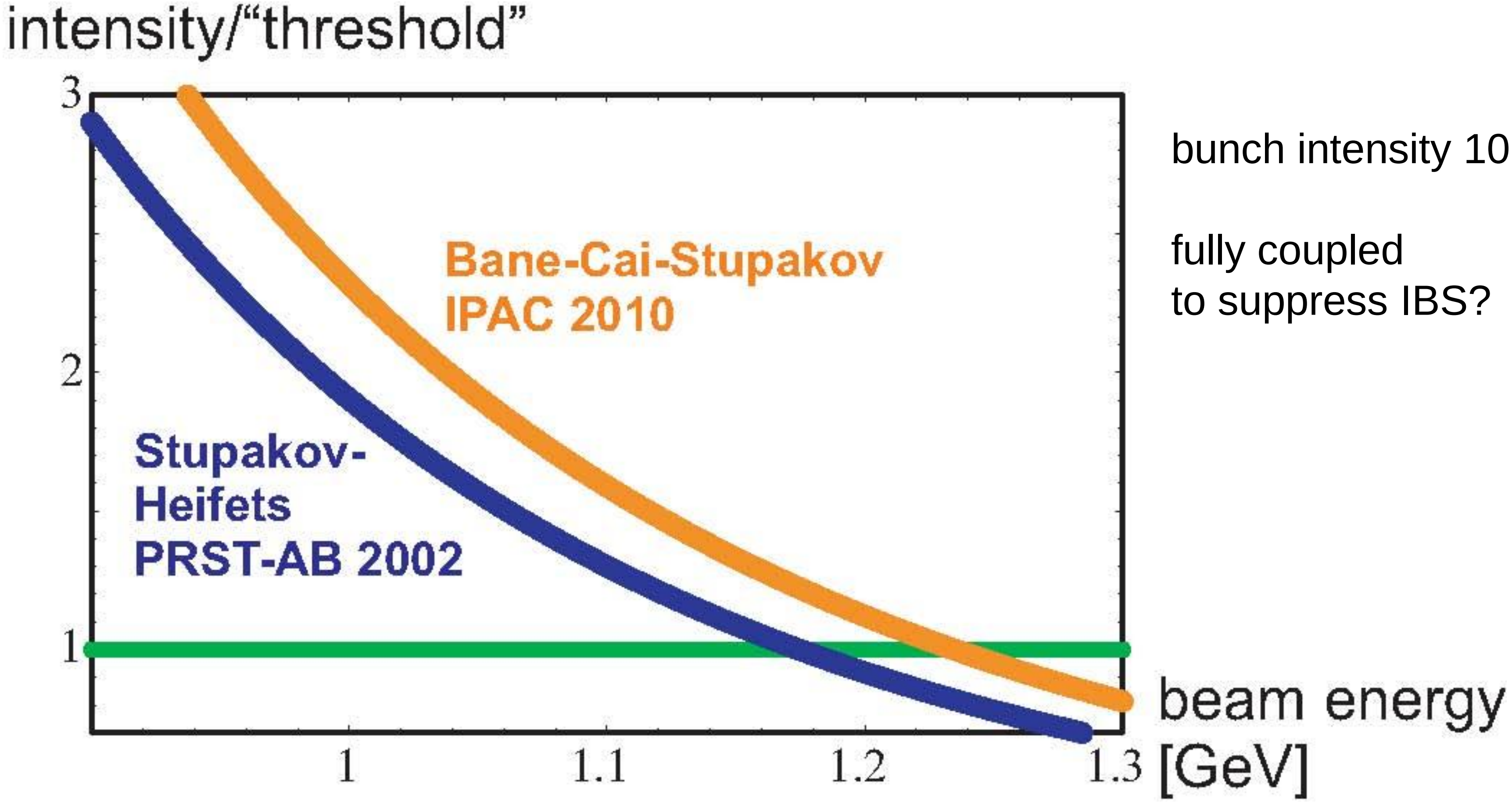
assumed scaling

$\delta_{\mathrm{rms}} \sim E$, $V_{rf} \sim E$, $\sigma_z \sim E$, $\varepsilon_{x,y} \sim E^2$, $\tau \sim 1/E^3$

Beam & CSR-instability related parameters for the ATF DR at 2 different energies.

ATF damping ring	nominal	lower energy
beam energy [GeV]	**1.28**	**1.00**
slip factor η	0.0019	
rms momentum spread $\sigma_{\delta,rms}$ [%]	0.06	0.047
bunch population [10^9]	10	10
circumference C [m]	138.6	
bending radius ρ [m]	5.73	
vert. beam pipe radius b [cm]	1.2?	
Stupakov-Heifets parameter Λ	**339**	**906**
ρ/b	**478**	
σ_z [cm]	**0.5**	**0.39**
$\rho/(2\,\Lambda^{3/2})$ [cm]	**0.05**	**0.011**
$N_b r_0/(\sqrt{2\pi}\,\sigma_z\sigma_\delta\,\gamma)$	0.0015	0.0031
β_x at bend [m]	3?	
ε_x [nm]	~1.5	0.9
σ_x at bend [μm]	67	52
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	2990	3540
τ_x [ms]	17.2	36.1
C03	0.0017	0.0141
Q_s	-0.0045	
$N_{nb,thr}$	1.15×10^{10}	4.3×10^{9}
$N_b/N_{nb,thr}$	**0.86**	**2.32**
$\Lambda b/\rho$	**0.71**	**1.90**

ATF Damping Ring as test bed?
lowering ATF beam energy → CSR instability



bunch intensity 10^{10}

fully coupled
to suppress IBS?

Note: Bane-Cai-Stupakov predict instability in a regime where it is excluded by Stupakov-Heifets
Also note: ATF Damping Ring has initially operated at 0.96 GeV beam energy, in 1997

CSR stability for SuperKEKB Damping Ring designs

	SuperKEKB e+ DR	SuperKEKB e+ DR OLD DESIGN
beam energy [GeV]	1.1	1.0
slip factor η	0.017	0.00343
rms momentum spread $\sigma_{\delta,rms}$ [%]	0.055	0.054
bunch population [10^9]	37.5	37.5
circumference C [m]	135	135.5
bending radius ρ [m]	2.65	2.2
vert. beam pipe radius b [cm]	1.6	1.4?
Stupakov-Heifets parameter Λ	**67**	**430**
ρ/b	**166**	**129**
σ_z [cm]	**0.7**	**0.51**
$\rho/(2\,\Lambda^{3/2})$ [cm]	**0.24**	**0.012**
$N_b r_0/((2\pi)^{1/2}\sigma_z\sigma_\delta\,\gamma)$	0.0051	0.078
β_x at bend [m]	1.5	1.5?
ε_x [nm]	2100→ 41	2100→ 41
σ_x at bend [μm]	248	248?
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	710	553
τ_x [ms]	11	11
C03	0.0033	**7.68?!**
Q_s	-0.015	-0.00788
$N_{b,thr}$	**4.5×10^{10}**	**1.1×10^{10}**
$N_b/N_{b,thr}$	0.83	**3.27**
$\Lambda b/\rho$	0.40	**3.32**

note: C03≥1 here!
so [1] surely applies

calculation shows that
old SuperKEKB DR
would have been CSR-
unstable

Instability threshold in the single-mode regime

For all other cases, C*03*<1, and from [2] there is always an instability if (rewriting the condition "$|\mu|>\eta\omega_0\delta_0$" below Eq. (23) in Ref. [4]):

$$C03b \equiv \frac{\left(\dfrac{r_e N_b 2a}{\gamma\sqrt{a\rho}\sigma_z}\right)^{1/3}}{\left(\eta\pi\sqrt{\dfrac{\rho}{a}}\sigma_\delta\right)} >> 1.$$

Condition "C03b" evaluated for various storage rings

	SuperKEKB LER	SuperKEKB HER	SuperKEKB DR	ATF	ATF at 1 GeV	SuperB LER	SuperB HER	CLIC DR	
								2009	2010
C03b	**239**	**175**	**27**	**75**	**113**	**177**	**100**	**3851**	**540**

If C03b <1 the beam could be stable in the single-mode model, which is not the case for any of the rings considered here.

This would imply that the **HER and LER rings for SuperKEB and SuperB, the two CLIC DR examples, and the new SuperKEKB damping ring are CSR unstable, despite opposite predictions from [1] and [3].**

“think the parallel plate shielded csr doesn’t have discrete modes, unlike in a full torus. Nevertheless, it seems that the two geometries give similar results concerning the instability. I think I heard Agoh-san (who is at KEK) say that somehow the results of a closed torus compared to one with infinite circumference (but with finite rho—it doesn’t really make sense physically) basically agree. (I’ve heard a similar thing from R. Warnock.) So from their point of view it doesn’t seem to matter much whether there are discrete or continuous modes. How does this square with Sam and Gennady’s paper? I don’t know.

…”

An answer from Karl Bane, 15 July 2010

the answer could be in the wake field

D. Zhou, K. Oide, G. Stupakov, et al, 2010

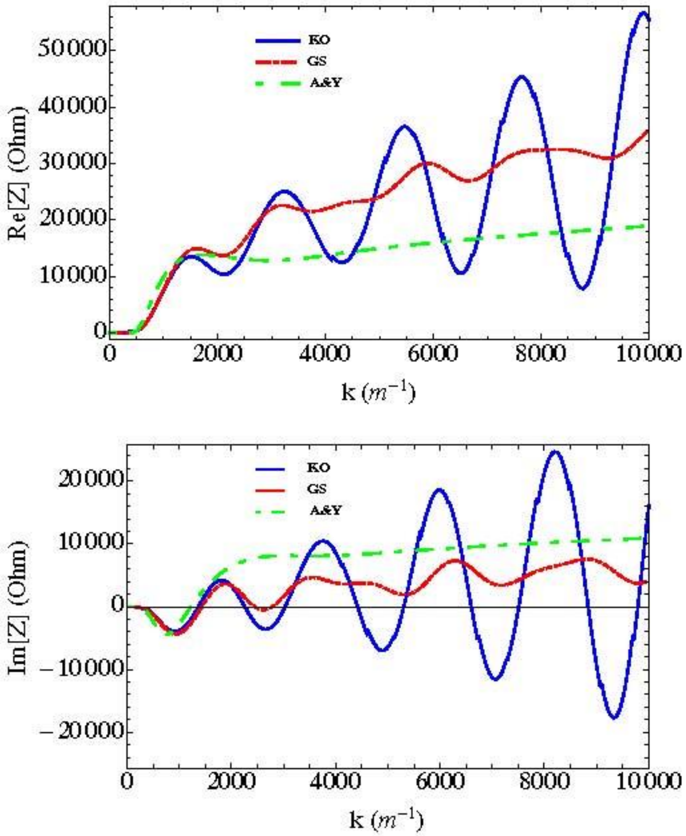


Figure 1: Total CSR impedances calculated by using Agoh and Yokoya's formulae (A&Y), Stupakov's code (GS), and Oide's code (KO).

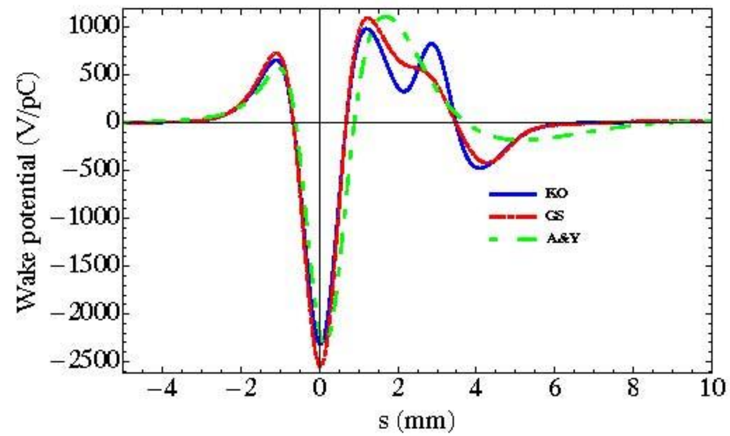


Figure 2: Total wake potentials of CSR with 0.5 mm Gaussian bunch. The head of the bunch is to the left.

should we "close" the ring to find discrete modes?

benchmarking against other existing storage rings

	SLC DR	KEKB LER	KEKB HER
beam energy [GeV]	1.19	3.5	8.0
slip factor η	0.0147	0.00033	0.00034
rms mom. spread $\sigma_{\delta,rms}$ [%]	0.09	0.073	0.067
bunch population [10^9]	40	65	47
circumference C [m]	35.28	3016	3016
bending radius ρ [m]	2.04	15.5	104
vert. beam pipe radius b [cm]	1	4.7	2.5
Stupakov-Heifets parameter Λ	**90**	**389**	**952**
ρ/b	**204**	**330**	**4160**
σ_z [cm]	**0.65**	**0.5**	**0.5**
$\rho/(2\,\Lambda^{3/2})$ [cm]	**0.12**	**0.10**	**0.18**
$N_b r_0/((2\pi)^{1/2}\sigma_z\sigma_\delta\,\gamma)$	0.0033	0.0039	0.001
β_x at bend [m]	1.5	15	15
ε_x [nm]	15	18	24
σ_x at bend [μm]	150	520	600
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	699	983	11300
τ_x [ms]	3.7	90	45
C03	0.0001	**1.5**	4×10^{-6}
Q_s	-0.012	-0.025	-0.021
$N_{b,thr}$	**8.7×10^{10}**	**7.0×10^{10}**	**1.8×10^{11}**
$N_b/N_{b,thr}$	**0.46**	**0.92**	**0.26**
$\Lambda b/\rho$	**0.44**	**1.18**	**0.23**
C03b	**17**	**609**	**81**

SLC DR and KEKB LER should be CSR unstable!

conclusions - 1

- **2002 formulae from Stupakov & Heifets perhaps not applicable in most cases**, except old SuperKEKB DR and KEKB LER which should be unstable
- **unlikely that IPAC10 result from Bane, Cai and Stupakov is applicable for cases with C03<1**
- **C03>1 not fulfilled for most machines → single-mode CSR regime:** for SuperKEKB & SuperB HER & LER, both CLIC DR designs, ATF, KEKB HER, SLC DR, and new SuperKEKB DR) beam predicted to be unstable (C03b>>1)
- indeed SLC DR has been plagued by longitudinal instabilities

conclusions - 2

- **only present KEKB LER & old SuperKEKB DR design operate in regime of [1,3]**; present KEKB LER unstable from [1], but it should be close to CSR instability threshold from [3] - evidence for CSR adding to longitudinal impedance [D. Zhou].
- **safe findings: old SuperKEB DR design unstable, ATF DR unstable for beam energies < 1 GeV**; for all other accelerators conflicting predictions.
- trusting [1] and [3]: new SuperKEKB DR, CLIC DR and ATF at 1.28 GeV stable with regard to CSR.
- **CSR instability and various theories could be studied in ATF damping ring by lowering the ring energy from 1.28 GeV to 1.0 or 1.1 GeV**

CSR study in ATF DR ?!

Email to Junji Urakawa, 17 July .

“I have proposed to use the **ATF DR as test bed for CSR instability by lowering the beam energy to ~1.1 or 1.0 GeV**. ... Do you think it is possible?

Answer from Junji, 30 July 2010

Sorry for my late reply. I think **this experiment proposal has a priority for super KEKB** construction. Several days ago, **Aryshev** asked me the possibility of proposed experiment. I answered if we will make a suitable schedule including ATF2 program, existing ATF-DR studies and this program, we have to do CSR instability study with some priority. Today, I asked **Kikuchi-san** to organize a research group from KEKB because number of ATF staff is very poor.

You know we started the operation of ATF DR at 1.0GeV from 1997. I think we need a time for, I hope, **1.0GeV operation tuning. It takes about three shifts hopefully. Aryshev will prepare the detector system for micro wave measurement. Could you prepare the detail of experimental plan? Terunuma will ask you to present a talk about your proposal and will include about three shifts for this experiment until end of this year**

Sorry for limited machine time because other important studies should be proceeded.

we got three ATF-DR shifts for CSR studies at 1 GeV, and must prepare experimental plan – everybody is welcome to join!

we need a detailed proposal for ATF CSR experiment

some contributors:

Alexander Aryshev, Karl Bane, Hitomi Ikeda, Mitsuo Kikuchi, Anke Susanne Muller, Kazuhito Ohmi, Katsunobu Oide, Nobuhiro Terunuma, Junji Urakawa, Demin Zhou, Frank Zimmermann

First sketch of the ATF CSR experiment

single bunch with N≥10^{10} ;
no small transverse emittance because of IBS; **no careful tuning** ;
operation on linear coupling resonance to blow up the vertical emittance, so that the effect of IBS is not strong, and the emittances almost independent of bunch intensity

Region of CSR instability should be reached at **beam energies below about 1 GeV**. ATF already operated at lower energy, 0.96 GeV, in 1997.

To **observe the CSR instability**:
=> Direct detection of **microwave radiation**
=> Monitoring **bunch length and shape** by streak camera
⇒Extract bunches and measure **energy spread with a wire scanner in the extraction line** by scanning over many bunches, and doing so for different bunch intensities.

We should perhaps hope to see a **kink in the bunch length versus bunch intensity curve, and at the same time an increase in energy spread, plus the emission of microwaves**, all happening at the same bunch intensity.

energy spread & bunch length on coupling resonance

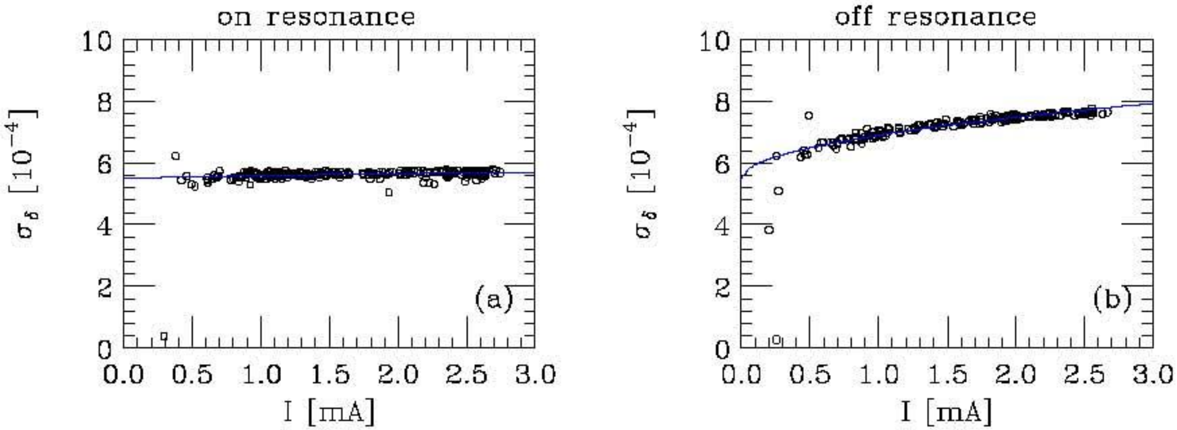


Figure 1: Energy spread as function of current when the ring voltage $V_c =$ 300 kV, with the beam on (a) and off (b) the coupling resonance.

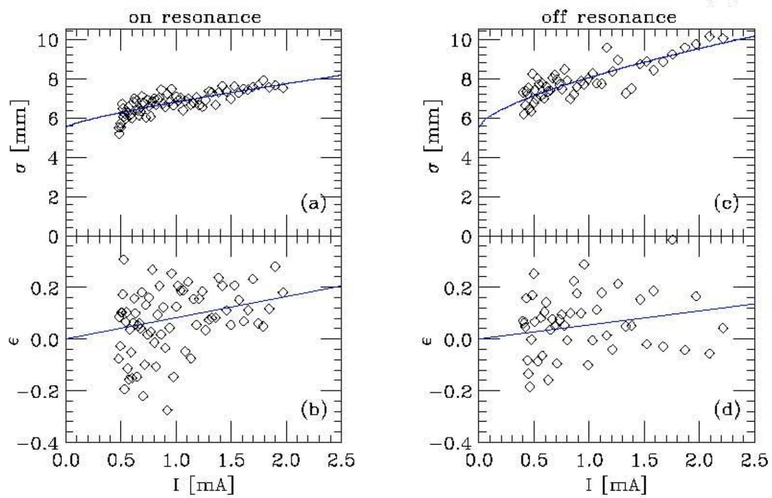


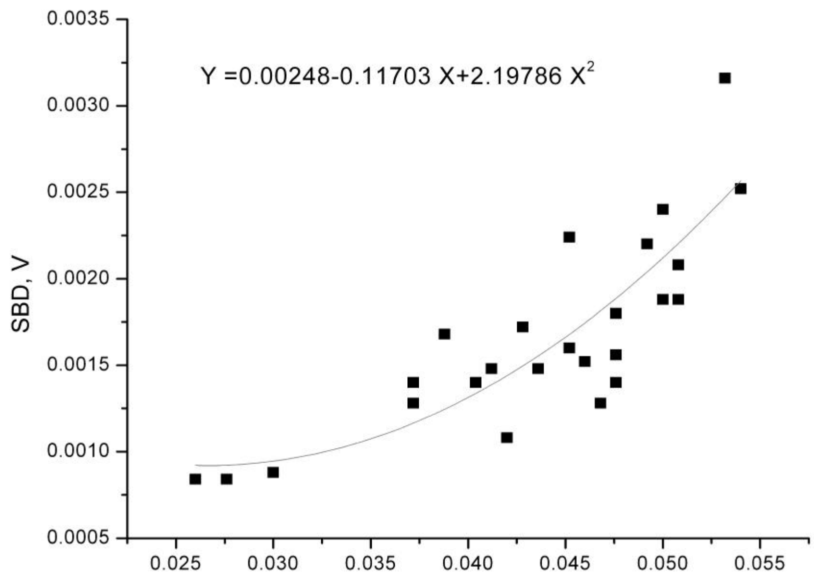
Figure 2: Bunch length (σ) and asymmetry parameter (ϵ) of the asymmetric Gaussian fit to the measured bunch distributions, as functions of current, for the beam on resonance (a,b) and off (c,d). $V_c = 250$ kV. The curves are fits to these results.

from K. Bane et al, "Impedance Analysis of Bunch Length Measurements at the ATF Damping Ring," SLAC=PUB-8846, May 2001

ATF

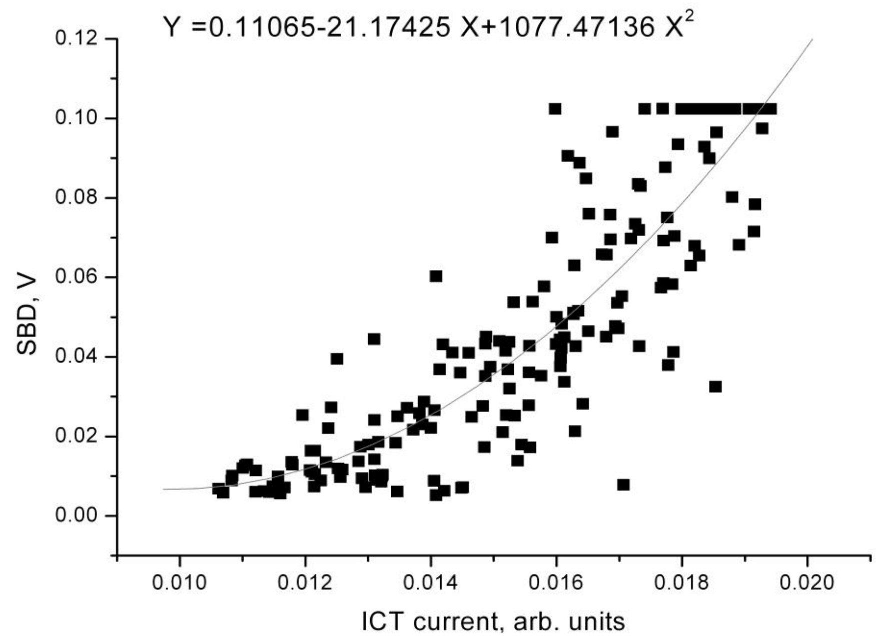
in 2006 CSR signal had been seen at ATF using Schottky barrier diode detector

CSR current dependence



Taken with the floppy

Current dependence



from A. Aryshev et al, “KEK ATF Coherent Synchrotron Radiation Study,” July 2006

ANKA CSR studies (Marit Klein)

Various optics with different α. From 10000-steps optics onward and synchrotron frequencies below 10 kHz coherent radiation is seen. Prediction according to Bane-Cai-Stupakov agrees in the order of magnitude; for short bunches 20% discrepancy in value.

					Models						
Parameters	Formelzeiche	value	units	SI	unsq	6000	8000	10000	12000	14000	15000
	m_0	9,11	10^-31 kg	9,11E-031							
	e			1,60E-019							
	c			3,00E+008							
Beam Energy	E0	1,3	GeV	1,30E+009							
frev	frev	2,7	Mhz	2,70E+006							
rms momentum spread	sigmaE	5,20E-004	per E0	5,20E-004							
			absolut	6,76E+005							
circumference	C	110,4	m	1,10E+002							
bending radius	R	5,56	m	5,56E+000							
vert. beam pipe hight	b	32	mm	3,20E-002							
electron radius	r0	2,82	*10^-15 m	2,82E-015							
sqrt(2*pi)		2,51		2,51E+000							
gamma	gamma	2544		2,54E+003							
sync freq	fs		Hz		3,05E+004	2,40E+004	1,85E+004	9,50E+003	8,20E+003	6,70E+003	5,50E+003
sync tune	Qs				1,13E-002	8,89E-003	6,85E-003	3,52E-003	3,04E-003	2,48E-003	2,04E-003
slip factor / alpha	eta		Ref:	8,00E-003	8,00E-003	4,95E-003	2,94E-003	7,76E-004	5,78E-004	3,86E-004	2,60E-004
bunch length	sigmaz0		mm		5,22E-003	4,11E-003	3,17E-003	1,63E-003	1,40E-003	1,15E-003	9,40E-004
max current	Imax	4	mA	4,00E-003							
min current	Imin	0,1	mA	1,00E-004							
max bunch population	Nmax			9,19E+009							
min	Nmin			2,30E+008							
measured threshold	Ithmeas				5,20E-003	2,98E-003	1,62E-003	3,44E-004	2,41E-004	1,52E-004	9,52E-005
	Nthmeas				1,20E+010	6,84E+009	3,73E+009	7,91E+008	5,54E+008	3,50E+008	2,19E+008
re R sqrt(1pi)/(C g sigd^2)				5,17E-013							
Stupakov-Heifets-parameter	lambda		(N=Nmax)		1,14E+002	2,34E+002	5,10E+002	3,76E+003	5,87E+003	1,07E+004	1,94E+004
			(N=Nmin)		2,85E+000	5,84E+000	1,27E+001	9,39E+001	1,47E+002	2,68E+002	4,86E+002
			(N=Nthmeas)		1,48E+002	1,74E+002	2,07E+002	3,23E+002	3,54E+002	4,08E+002	4,63E+002
R/b				1,74E+002							
R/(2*lambda^3/2)					2,29E-003	7,79E-004	2,42E-004	1,21E-005	6,18E-006	2,51E-006	1,03E-006
					5,79E-001	1,97E-001	6,11E-002	3,05E-003	1,56E-003	6,35E-004	2,59E-004
					1,54E-003	1,21E-003	9,34E-004	4,79E-004	4,17E-004	3,37E-004	2,79E-004
				sigmaz0	5,22E-003	4,11E-003	3,17E-003	1,63E-003	1,40E-003	1,15E-003	9,40E-004
Nb r0/sqrt(2pi) g sigz sigd			(N=Nmax)		1,50E-003	1,90E-003	2,46E-003	4,79E-003	5,58E-003	6,79E-003	8,31E-003
			(N=Nmin)		3,74E-005	4,75E-005	6,16E-005	1,20E-004	1,40E-004	1,70E-004	2,08E-004
			(N=Nthmeas)	1,00E+000	1,95E-003	1,41E-003	1,00E-003	4,12E-004	3,37E-004	2,59E-004	1,98E-004
(Nb r0/sqrt(2pi) g sigz)^5			(N=Nmax)		2,86E-031	9,44E-031	3,46E-030	9,62E-029	2,06E-028	5,50E-028	1,51E-027
			(N=Nmin)		2,79E-039	9,22E-039	3,38E-038	9,40E-037	2,01E-036	5,38E-036	1,47E-035
			(N=Nthmeas)		1,06E-030	2,15E-031	3,81E-032	4,52E-034	1,64E-034	4,42E-035	1,15E-035
(2a/eta R)^3					2,98E+000	1,26E+001	5,98E+001	3,26E+003	7,89E+003	2,65E+004	8,67E+004
'1/(sigd^8 pi^2/3)				8,72E+025							
C03			(N=Nmax)		7,42E-005	1,03E-003	1,81E-002	2,74E+001	1,42E+002	1,27E+003	1,14E+004
			(N=Nmin)		7,25E-013	1,01E-011	1,76E-010	2,67E-007	1,38E-006	1,24E-005	1,11E-004
			(N=Nthmeas)	1	2,76E-004	2,36E-004	1,99E-004	1,29E-004	1,13E-004	1,02E-004	8,71E-005
2pi Qs g sigd/re					3,33E+013	2,62E+013	2,02E+013	1,04E+013	8,96E+012	7,32E+012	6,01E+012
sigz^4/3 /R^1/3					5,11E-004	3,72E-004	2,63E-004	1,08E-004	8,84E-005	6,80E-005	5,20E-005
0,5+0,12 sigz R^1/2 /b^3/2					7,58E-001	7,03E-001	6,57E-001	5,81E-001	5,69E-001	5,57E-001	5,46E-001
Nb,thr					1,29E+010	6,85E+009	3,49E+009	6,52E+008	4,51E+008	2,77E+008	1,71E+008
				Nthmeas	1,20E+010	6,84E+009	3,73E+009	7,91E+008	5,54E+008	3,50E+008	2,19E+008

thank you for your attention!