CSR computation with arbitrary cross-section of the beam pipe

Work in progress

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KEK Accelerator Seminar

Outline

- Parabolic equation and general formulation of the problem
- Illustration of the method: numerical examples
- Excise of micro-wave instability simulation with Vlasov solver
- Conclusions

Motivation

- Parabolic equation has been solved in FEL, CSR, and Impedance calculations, etc
- Most present codes are limited for simple boundary, for instance, rectangular cross-section for CSR and zero boundary for FEL (need large domain)
- CSR is important for Super-KEKB damping ring. To estimate the threshold of micro-wave instability

Impedance calculation

- **Gennady Stupakov,** *New Journal of Physics* ***8 (2006) 280***

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial \mathcal{E}}{\partial r} - \frac{\mathcal{E}}{r^2} = -2ik\frac{\partial \mathcal{E}}{\partial z}.$$

Axis ymmetric geometry

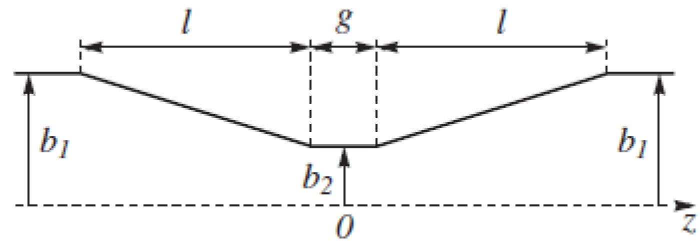
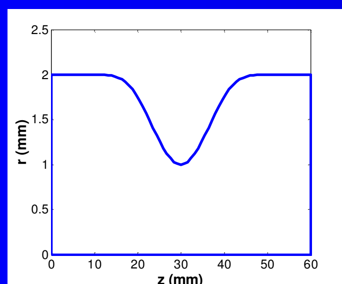


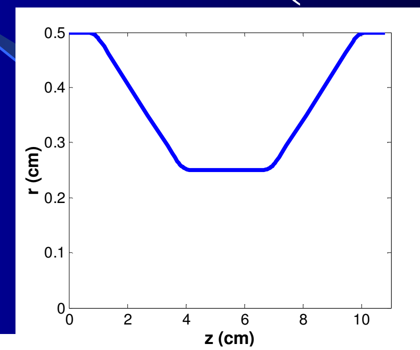
FIG. 1. Geometry of an axisymmetric collimator.

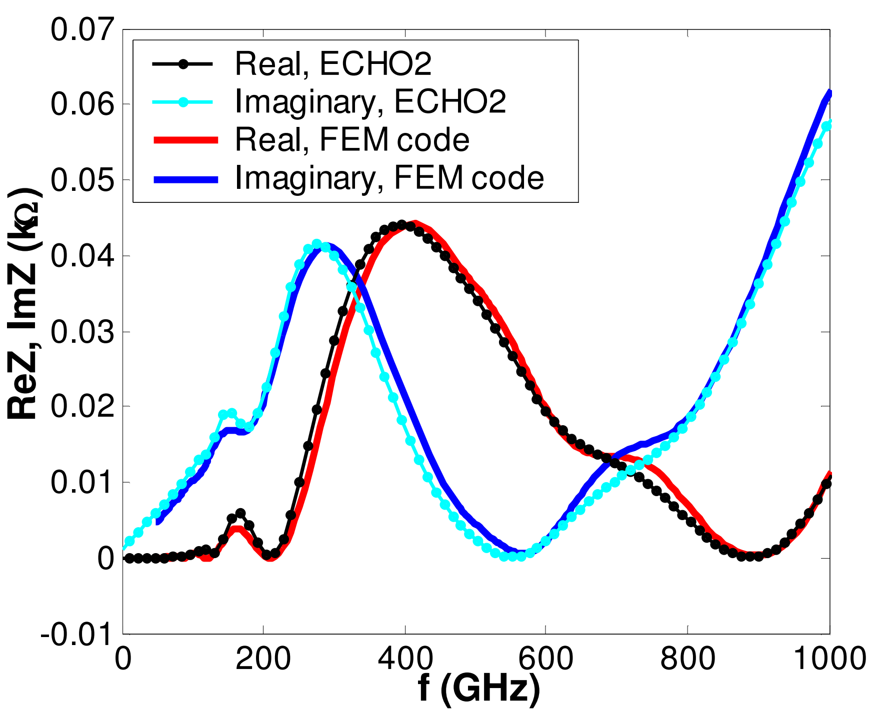
2D parabolic solver for Impedance calculation

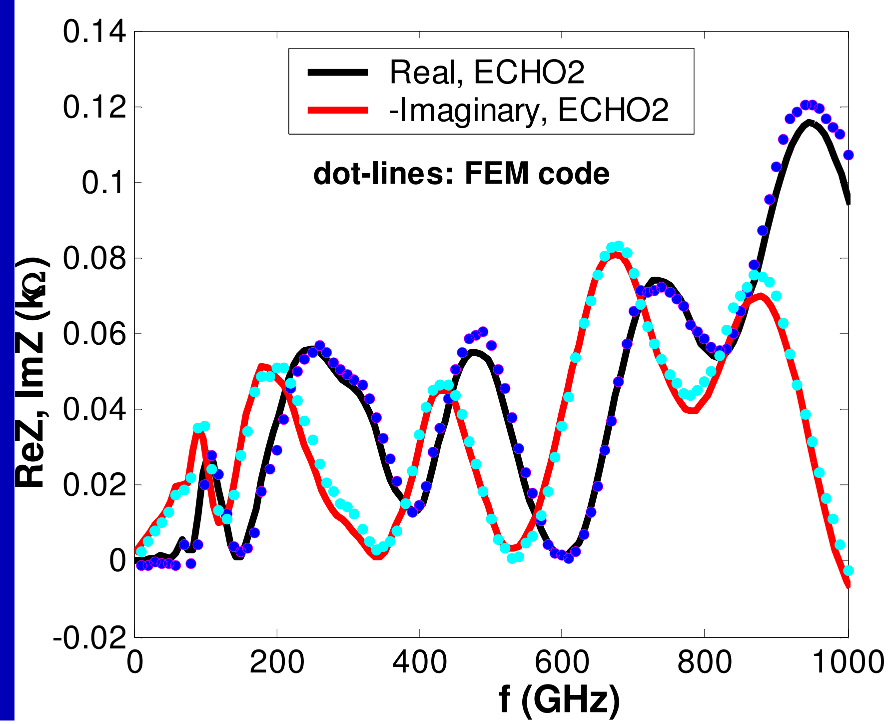
- **L. Wang, L. Lee, G. Stupakov,** *fast 2D solver* **(IPAC10)**



Echo2, sigl=0.1mm







FEL

FEL (for example, Genesis by sven reiche)

$$\left[\Delta_\perp + 2ik\frac{\partial}{\partial z}\right] u = i\frac{e^2\mu_0}{m}\sum_j \delta(\vec{r}-\vec{r}_j)\frac{f_c a_u}{\gamma_j}e^{-i\theta_j}$$

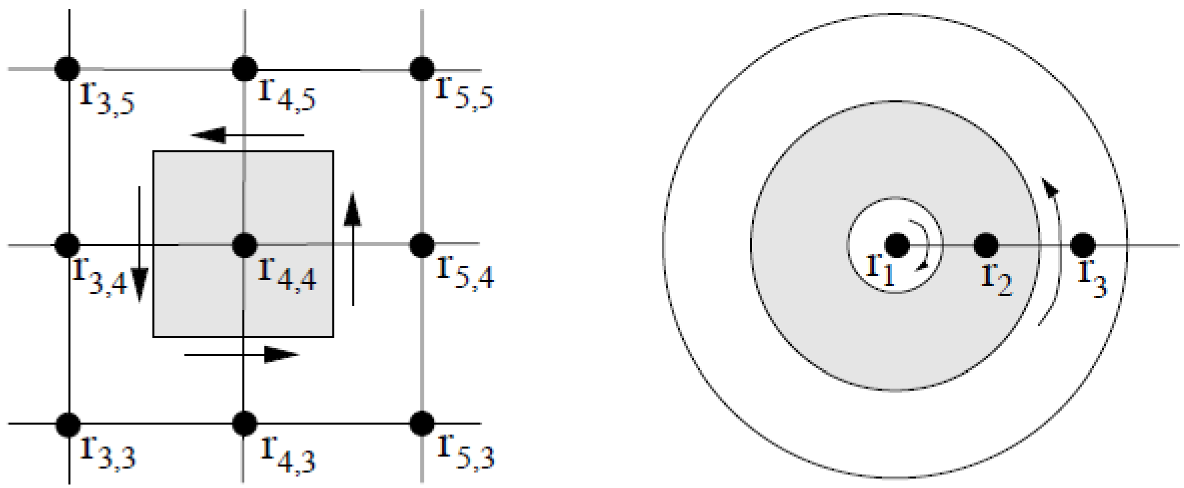


Figure 3.4: *Discretization on a Cartesian and radial mesh, left and right, respectively. The arrows indicate the orientation of the integration path enclosing the grey shaded area of interest.*

Set the field ZERO out the domain of interest

CSR

For example, CSR in bend magnet
(Tomonori Agoh, Phys. Rev. ST Accel. Beams 7, 054403 (2004)

Equation to describe CSR

$$\frac{\partial \boldsymbol{E}_\perp}{\partial s} = \frac{i}{2k}\left[\left(\boldsymbol{\nabla}_\perp^2 + \frac{2k^2 x}{\rho}\right)\boldsymbol{E}_\perp - \mu_0 \boldsymbol{\nabla}_\perp J_0\right]$$

Equation of Evolution

- Agoh, Yokoya, PRSTAB 054403
- Gennady, PRSTAB 104401
- Demin, PH.D thesis
- K. Oide, PAC09

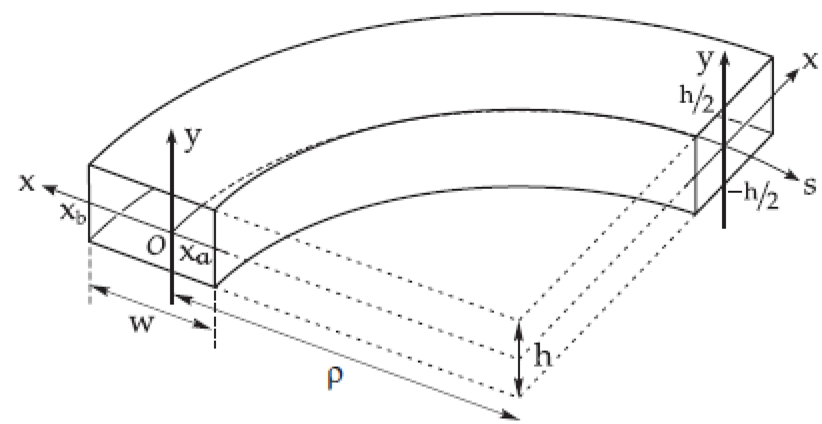
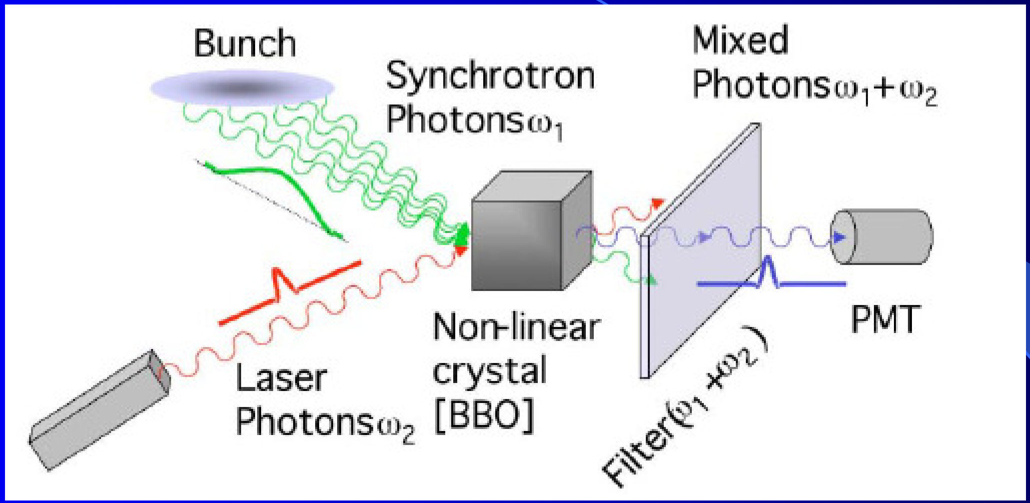
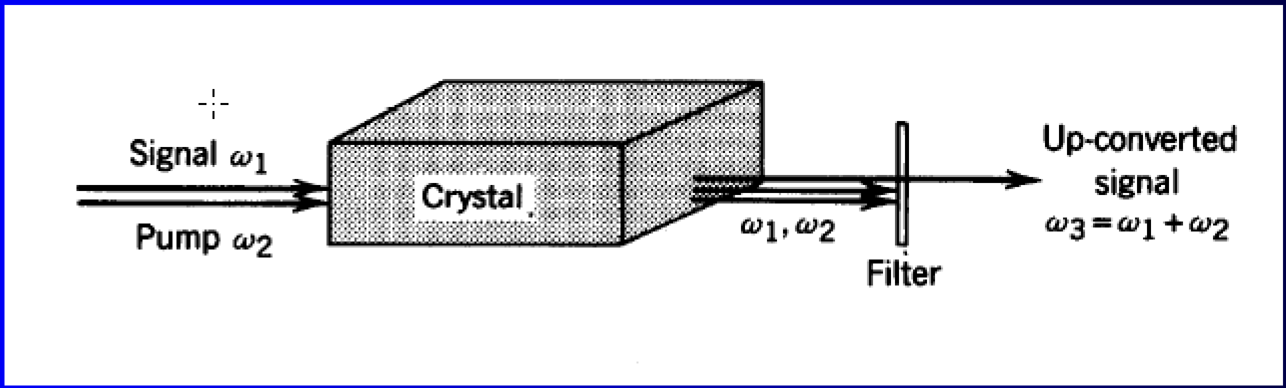


FIG. 1. Coordinate system and vacuum chamber. The refer-

Nonlinear Optics



BUNCH LENGTH MEASUREMENTS WITH LASER/SR CROSS-CORRELATION*



GENERALITY

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\mathbf{E} = \mu_0 \frac{\partial j}{\partial t} + \frac{1}{\varepsilon_0}\nabla\rho$$

IF We neglect the 1st term $\left|E''/E'\right| << k$

$$\frac{\partial^2 E}{\partial z^2} = \left(\frac{\partial^2 \widetilde{E}}{\partial z^2} + 2ik\frac{\partial \widetilde{E}}{\partial z} - k^2 \widetilde{E}\right)e^{ikz}$$

$$\left[\Delta_\perp + 2ik\frac{\partial}{\partial z}\right]u = i\frac{e^2\mu_0}{m}\sum_j \delta(\vec{r} - \vec{r}_j)\frac{f_c a_u}{\gamma_j}e^{-i\theta_j}$$

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial \mathcal{E}}{\partial r} - \frac{\mathcal{E}}{r^2} = -2ik\frac{\partial \mathcal{E}}{\partial z}.$$

$$\frac{\partial \boldsymbol{E}_\perp}{\partial s} = \frac{i}{2k}\left[\left(\boldsymbol{\nabla}_\perp^2 + \frac{2k^2 x}{\rho}\right)\boldsymbol{E}_\perp - \mu_0 \boldsymbol{\nabla}_\perp J_0\right]$$

Why Finite Element Method (FEM)?

Advantages of FEM

Irregular grids

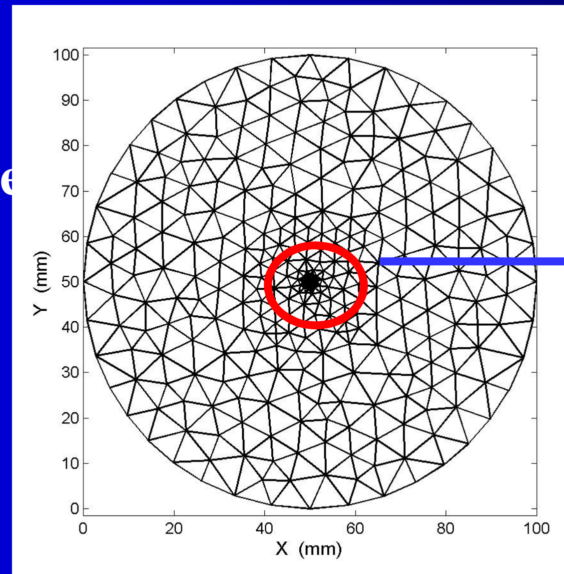
- Arbitrary geometry
- Easy to handle boundary
- Small beam in a large domain (FEL in undulator)
- CPU (strongly depends on the solver)
- Accuracy(higher order element, adaptive mesh, symmetry, etc)

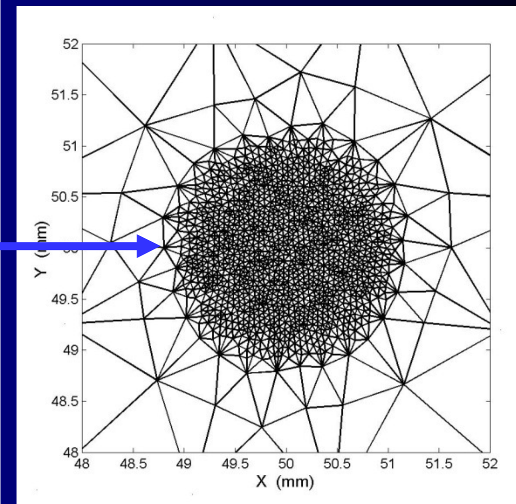
Disadvantage & Challenge:

Complexity in coding (irregular grid, arbitrary geometry, 3D…)

•**Arbitrary geometry of beam pipe**

•**Any shape of beam**





Mesh of chamber & beam

Impedance of Grooved surface

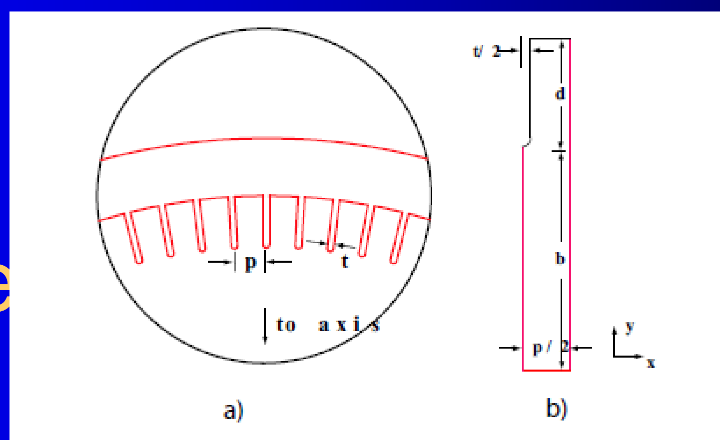
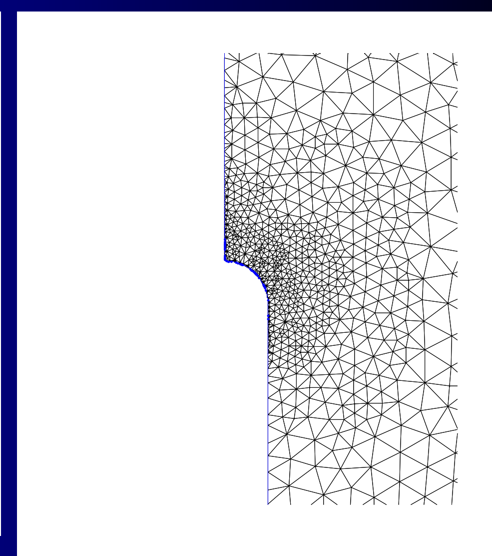


Figure 1: a)—detail of the grooved vacuum chamber wall; dimensions shown are period p and fin thickness t; b)—



Adaptive method

Speedup and improve accuracy

- CSR computation

Straight beam pipe+ Bend magnet + straight beam pipe…

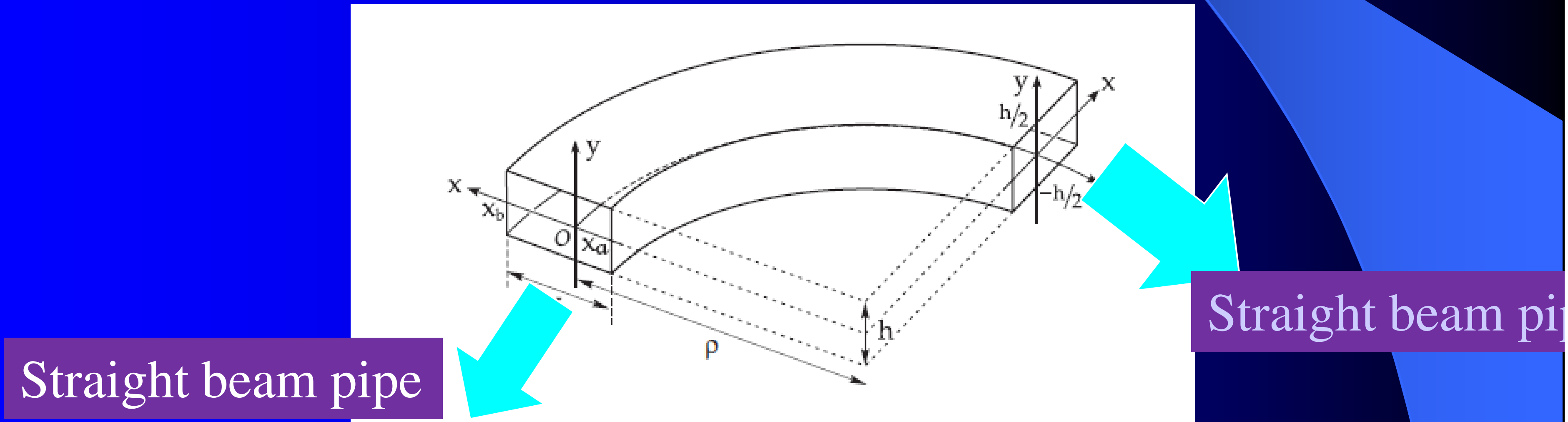


FIG. 1. Coordinate system and vacuum chamber. The refer-

Assumptions of our CSR problem

- We assume perfect conductivity of the walls and relativistic particles with the Lorentz factor $\gamma=\infty$
- The characteristic transverse size of the vacuum chamber a is much smaller than the bending radius R $(a<<R)$
- Constant cross-section of beam chamber

Fourier transform

- The Fourier transformed components of the field and the current is defined as

$$\hat{\mathrm{E}}(x, y, s, \omega) = \int_{-\infty}^{\infty} dt\, \mathrm{e}^{i\omega t - iks}\, \mathrm{E}(x, y, s, t)\,,$$

$$\hat{j}_s(x, y, s, \omega) = \int_{-\infty}^{\infty} dt\, \mathrm{e}^{i\omega t - iks}\, j_s(x, y, s, t)\,,$$

Where $k=\omega/c$ and j_s is the projection of the beam current onto s. The transverse component of the electric field $\tilde{\mathbf{E}}^r_\perp$ is a two-dimensional $\tilde{\mathbf{E}}^r_\perp = (\tilde{\mathbf{E}}^r_x, \tilde{\mathbf{E}}^r_y)$, The longitudinal component is denoted by $\tilde{\mathbf{E}}^r_s$

CSR Parabolic Equation

(Agoh, Yokoya, PRSTAB 054403)

$$\left(\left[\nabla_\perp^2 + \frac{2k^2 x}{R}\right] + 2ki\frac{\partial}{\partial s}\right)\tilde{\mathbf{E}}_\perp = -\frac{e}{\varepsilon_0}\nabla_\perp n_0$$

$$\nabla_\perp^2 \mathbf{E}_\perp^b = \frac{1}{\varepsilon_0}\nabla_\perp \rho_0$$

$$\mathbf{E}_\perp = \mathbf{E}_\perp^r + \mathbf{E}_\perp^b$$

Required for ***Self-consistent*** computation

$$\left(\left[\nabla_\perp^2 + \frac{2k^2 x}{R}\right] + 2ki\frac{\partial}{\partial s}\right)\tilde{\mathbf{E}}_\perp^r = -\frac{2k^2 x}{R}\mathbf{E}_\perp^b$$

Initial field at the beginning of the bend magnet

$$\nabla_\perp^2 \tilde{\mathbf{E}}_\perp^r (s=0) = 0$$

After the bend magnet

$$\left(\nabla_\perp^2 + 2ki\frac{\partial}{\partial s}\right)\tilde{\mathbf{E}}_\perp^r = 0$$

$$\nabla_\perp^2 = \partial_x^2 + \partial_y^2$$

$$\nabla_\perp = (\partial x, \partial y)$$

$$E_r^b = \frac{e}{4\pi\varepsilon_0}\lambda_e \begin{cases} \frac{2}{r} (r > \sigma_r) \\ \frac{2r}{\sigma_r^2} (r < \sigma_r) \end{cases}$$

FEM equation

3D problem (Treat s as one space dimension)

$$\mathbf{M}\tilde{\mathbf{E}}_{\perp}^{r} = \mathbf{J}$$

$$\mathbf{E}_{\perp} \times \vec{n} = 0$$

It's general case, for instance, the cross-section can vary for the geometry impedance computation.

Problem: converge slowly

2D problem (Treat *s* as time)

$$\mathbf{M}\tilde{\mathbf{E}}_{\perp}^{r} + \mathbf{D}\dot{\tilde{\mathbf{E}}}_{\perp}^{r} = \mathbf{J}$$

$$\mathbf{E}_{\perp} \times \vec{n} = 0$$

It converges faster, the stability need to be treated carefully

Current status: Bend only

$$Z(k) = -\frac{1}{I}\int_{0}^{\infty} E_s(x_c, y_c, s)\,ds$$

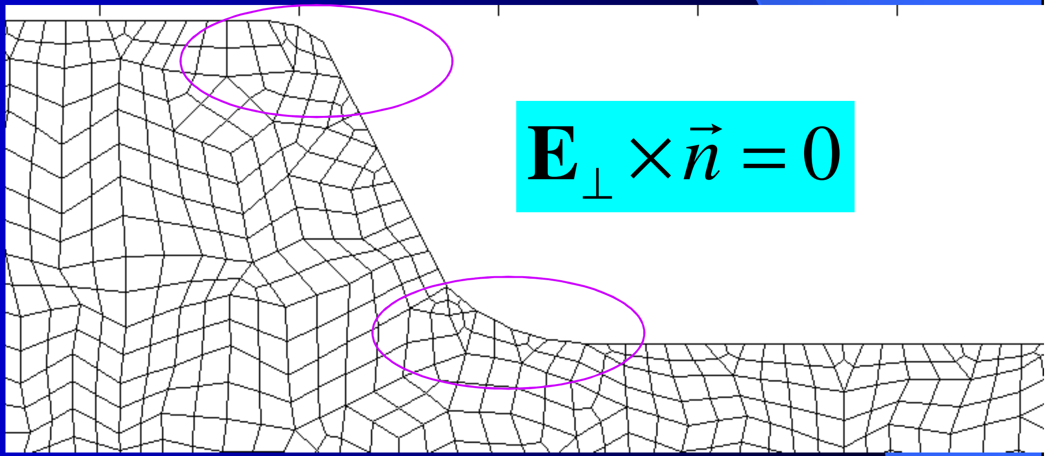
The ways to improve the accuracy

High order element

The (longitudinal) impedance is calculated from the transverse field, which is nonlinear near the beam as shown late,

$$\mathbf{E}_s = \frac{i}{k}\left(\frac{\partial E_x^r}{\partial x} + \frac{\partial E_y^r}{\partial y}\right) = 0$$

Fine girds on the curved boundary



$$\mathbf{E}_\perp \times \vec{n} = 0$$

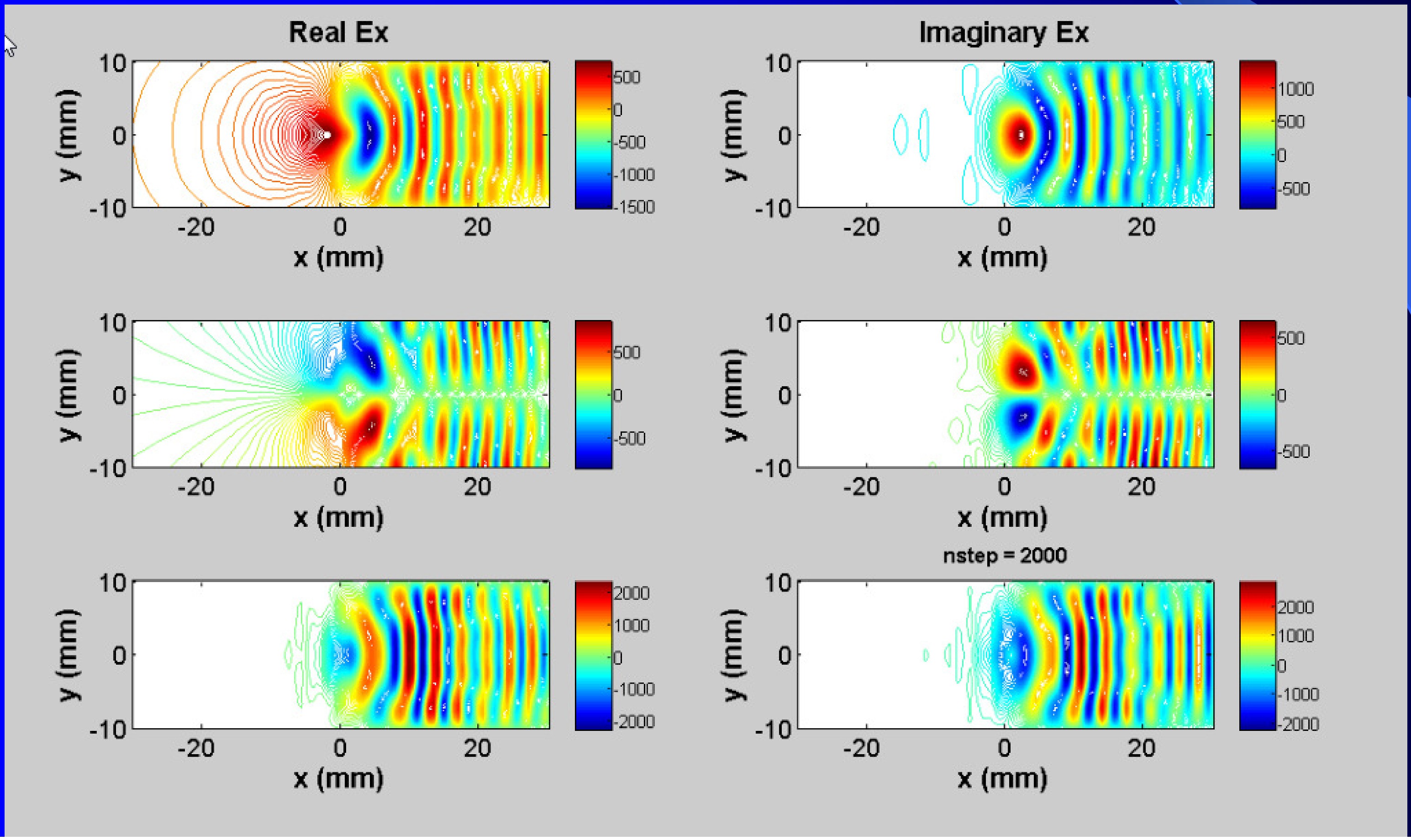
…..

Test of the code

Bend Length =0.2meter, radius =1meter

Rectangular Cross-section with dimension 60mmX20mm

	Wang	Zhou	
$k=10e3m^{-1}$,	(142.2, 119.1)	(140.6, 119.5)	(@end of the bend)



Sample 1

SuperKEKB damping ring bend magnet

Bend	Length[m]	Bending angle	# of elements
B1	.74248	.27679	32
B2	.28654	.09687	38
B3	.39208	.12460	6
B4	.47935	.15218	2

Geometry:

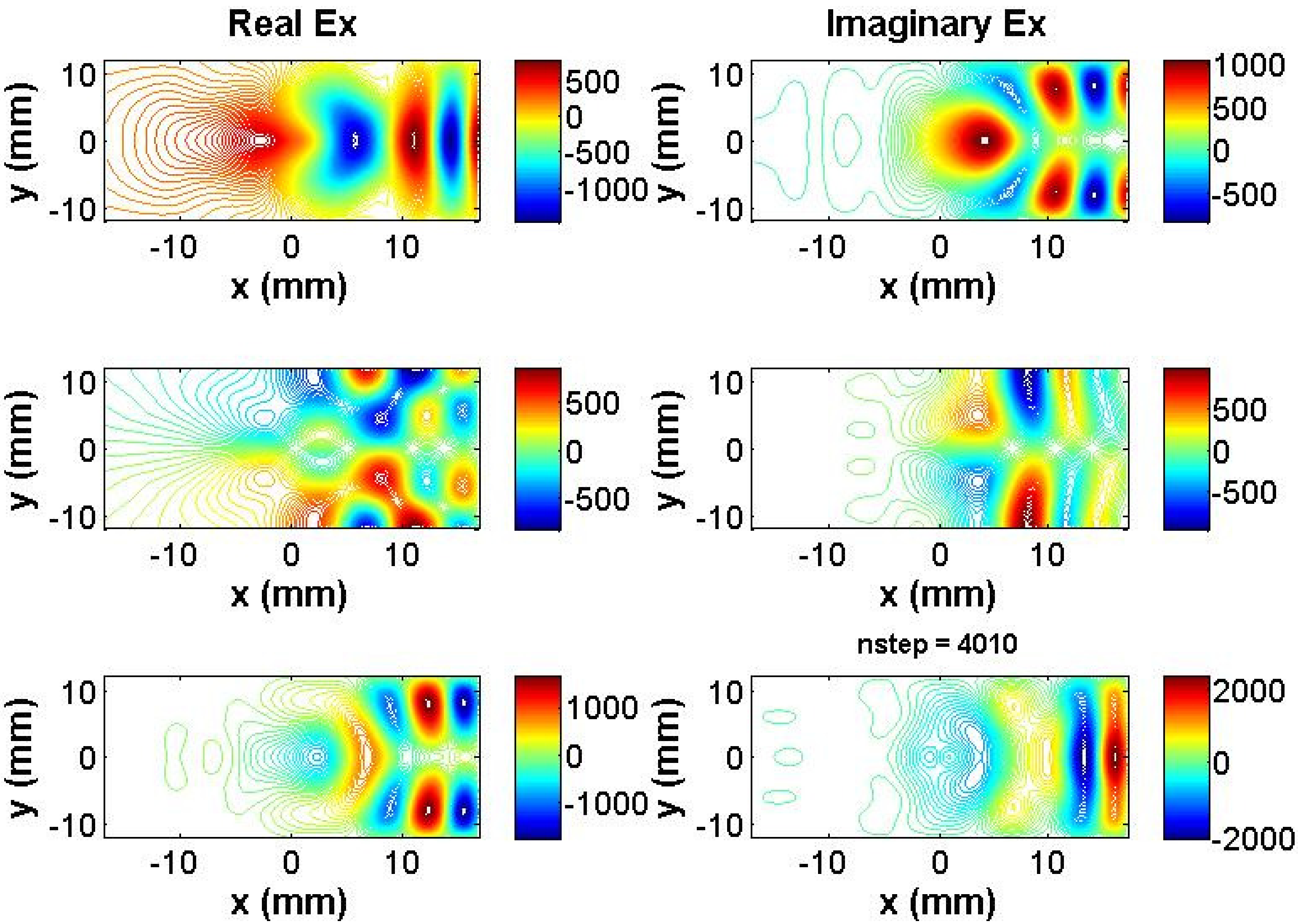
(1) rectangular with width 34mm and height 24mm

(2) Ante-chamber

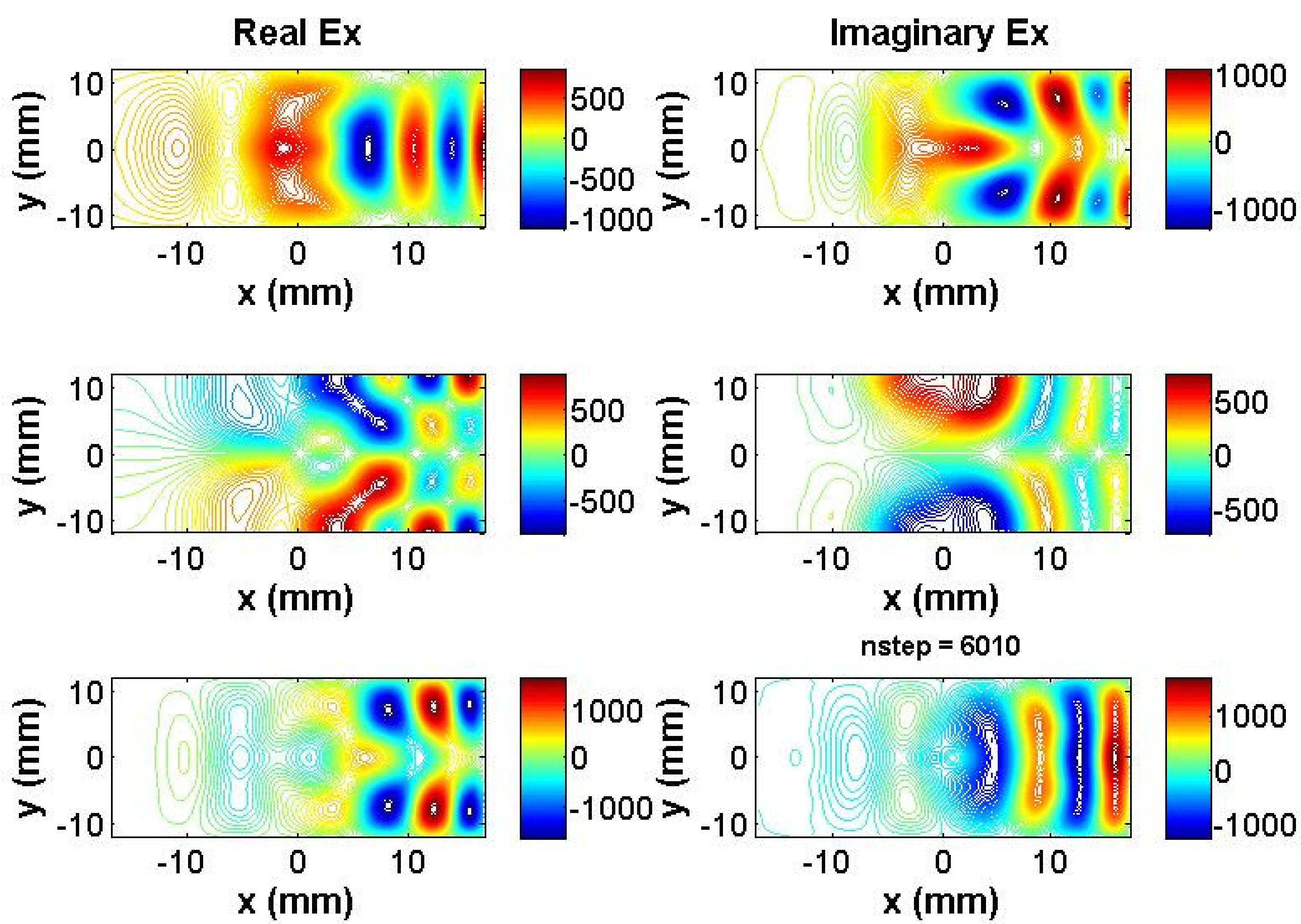
(3) Round beam pipe with radius 25mm

Rectangular(34mmX24mm)

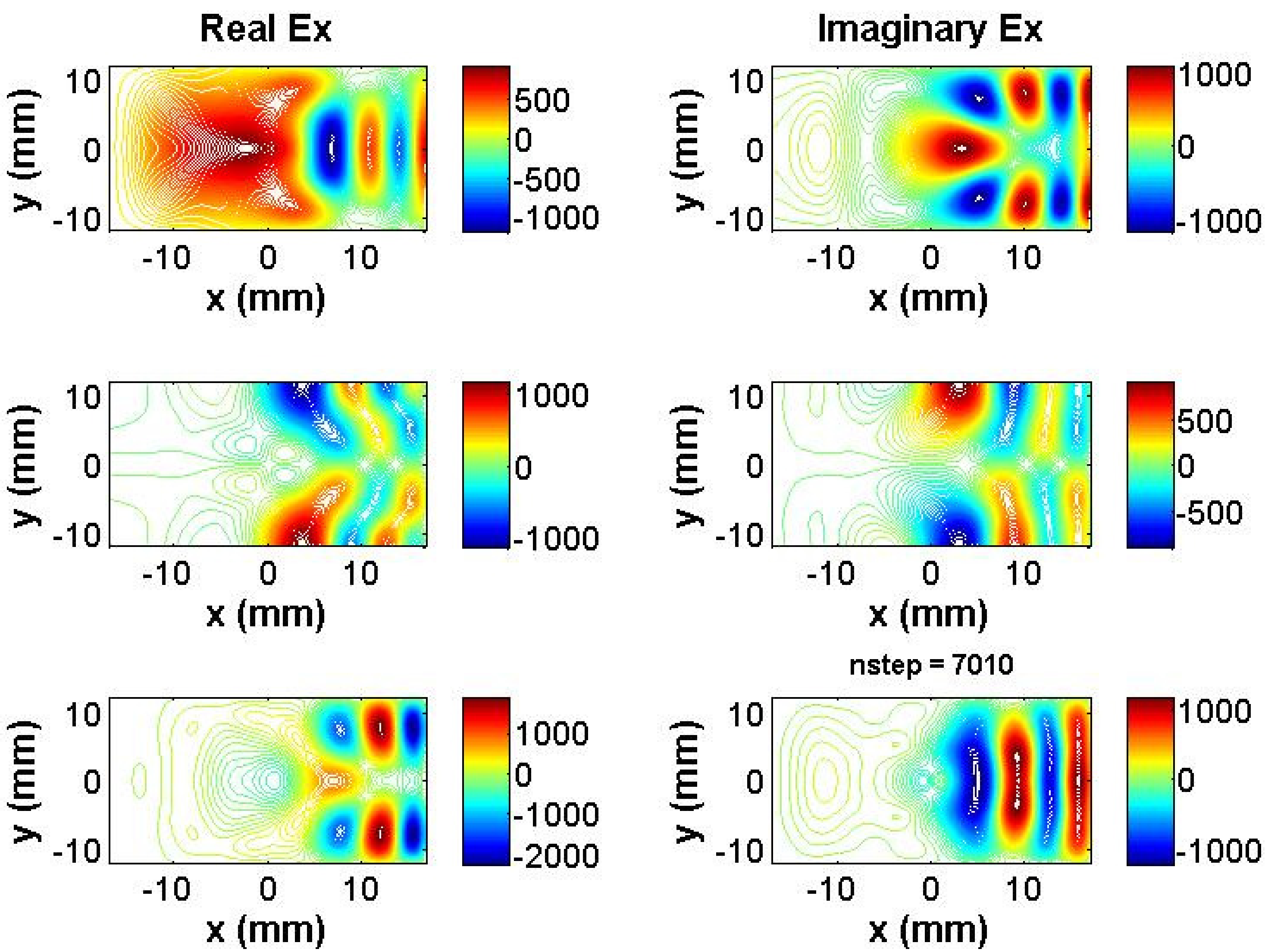
$k=1\times10^{4}m^{-1}$ $s=0.4m$



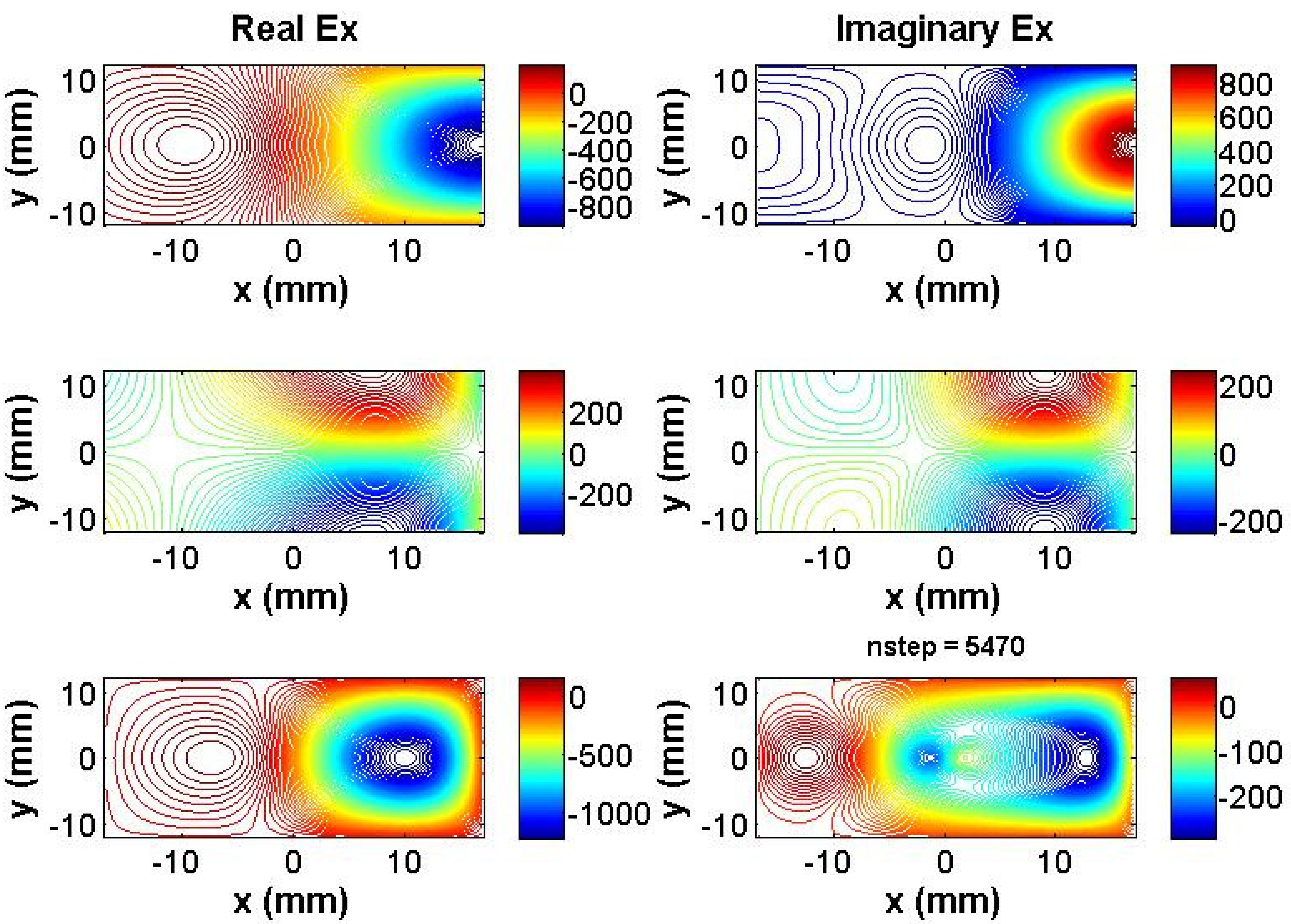
$k=1\times10^4 m^{-1}$ s=0.6m



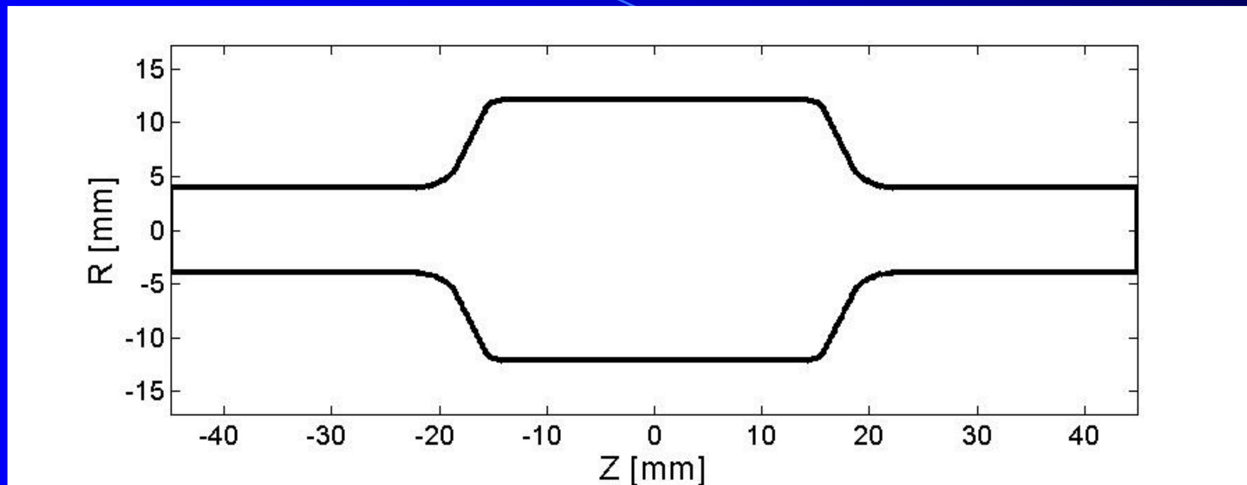
$k=1x10^4m^{-1}$ s=0.7m



$k=2\times10^{3}m^{-1}$ s=0.55m

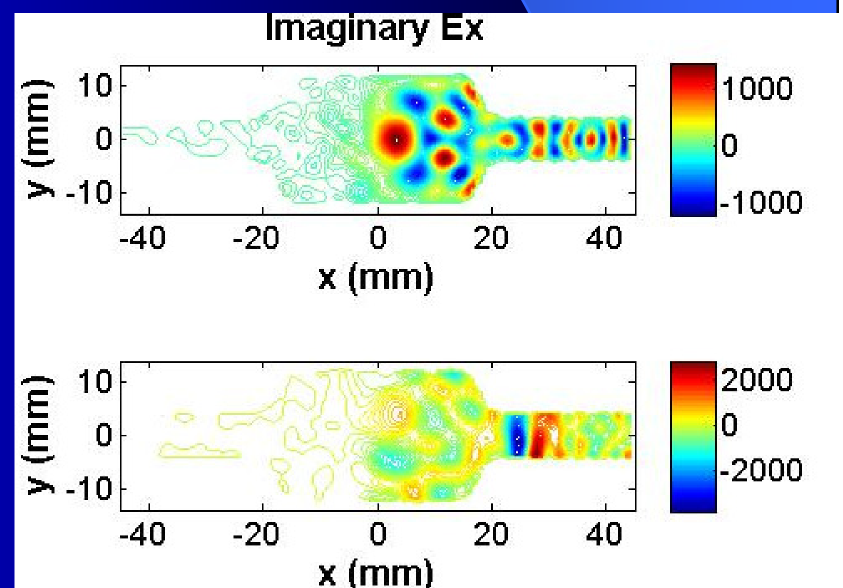


Ante-chamber

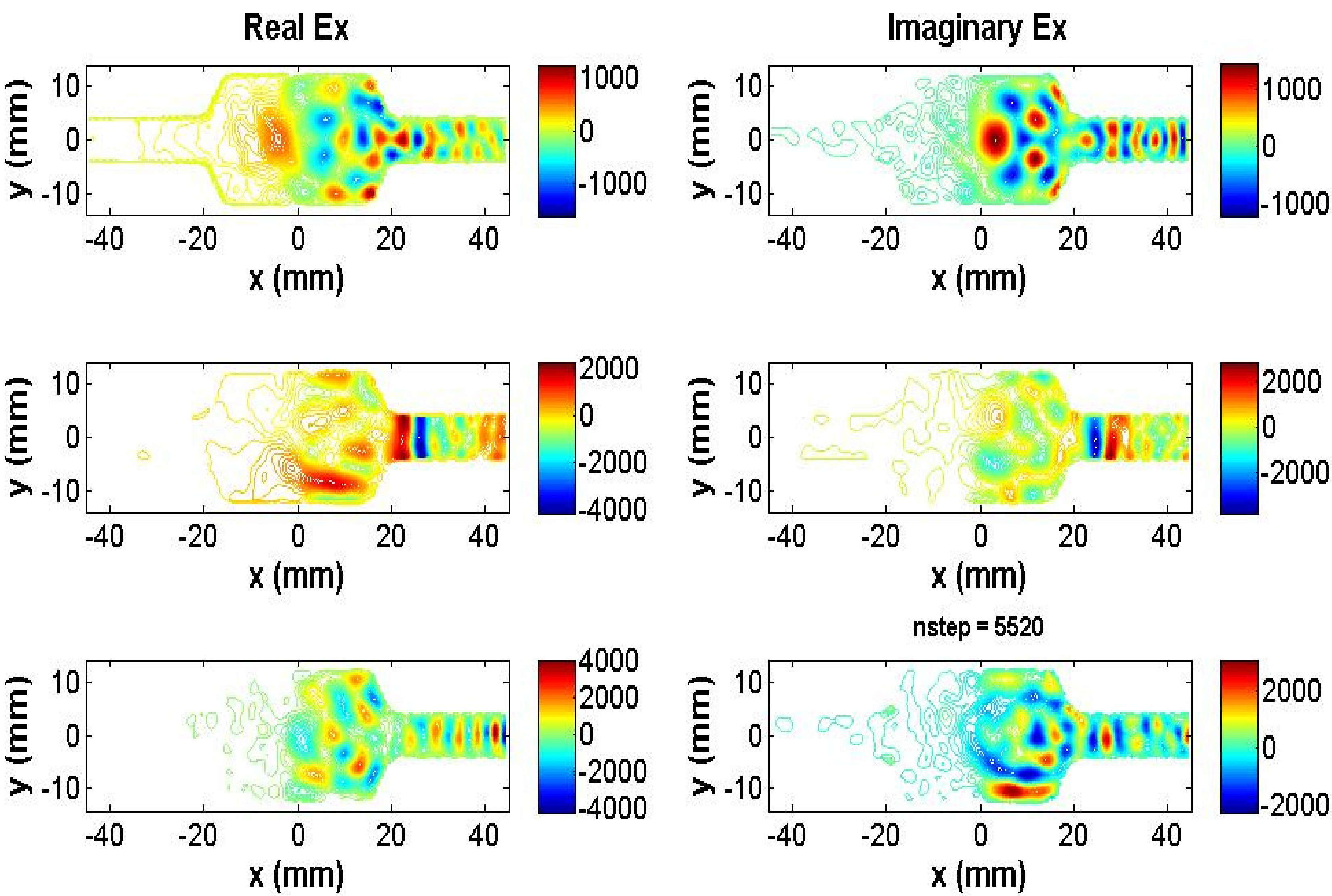


(courtesy K. Shibata)

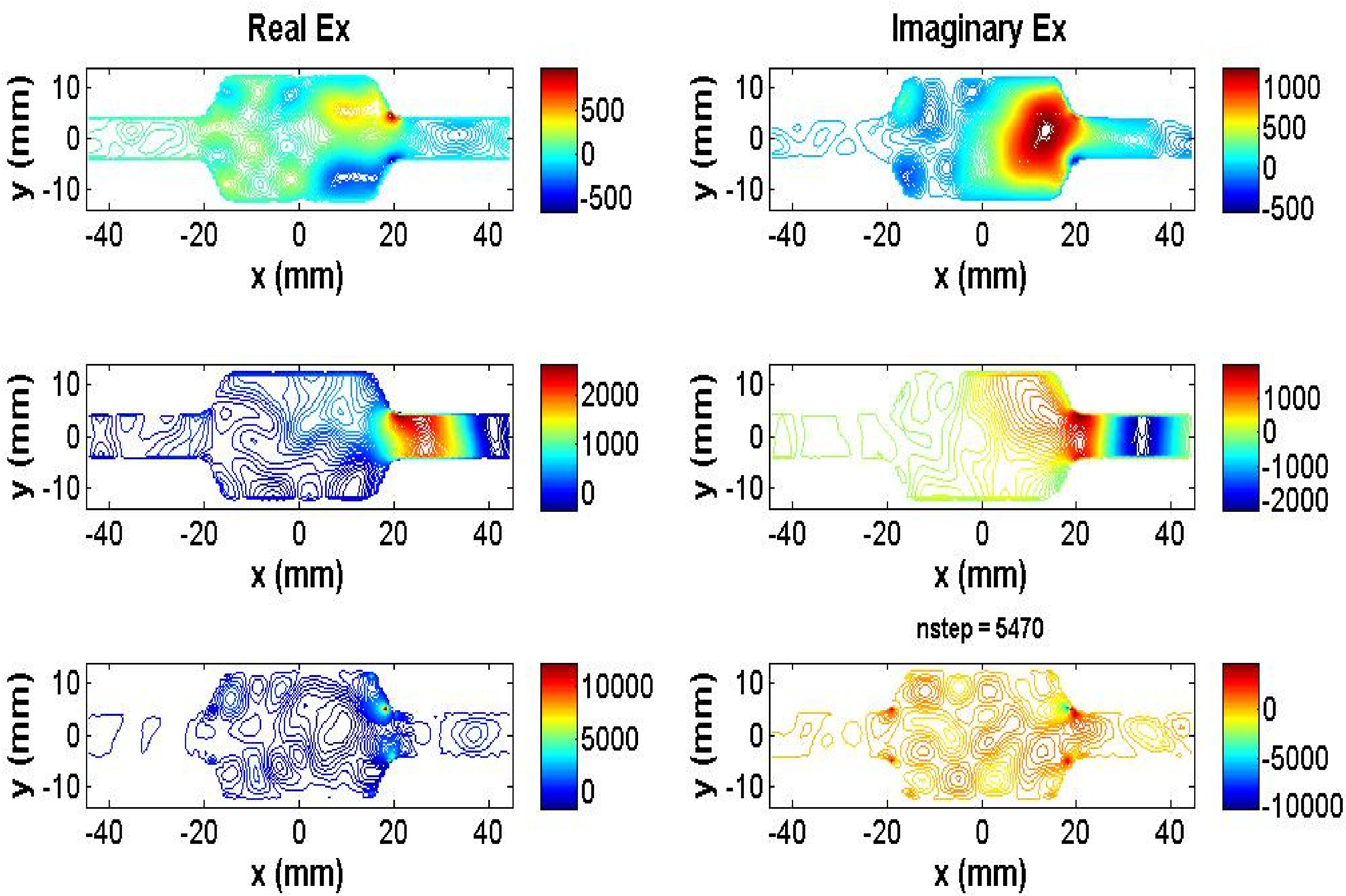
Leakage of the field into ante-chamber, especially high frequency field



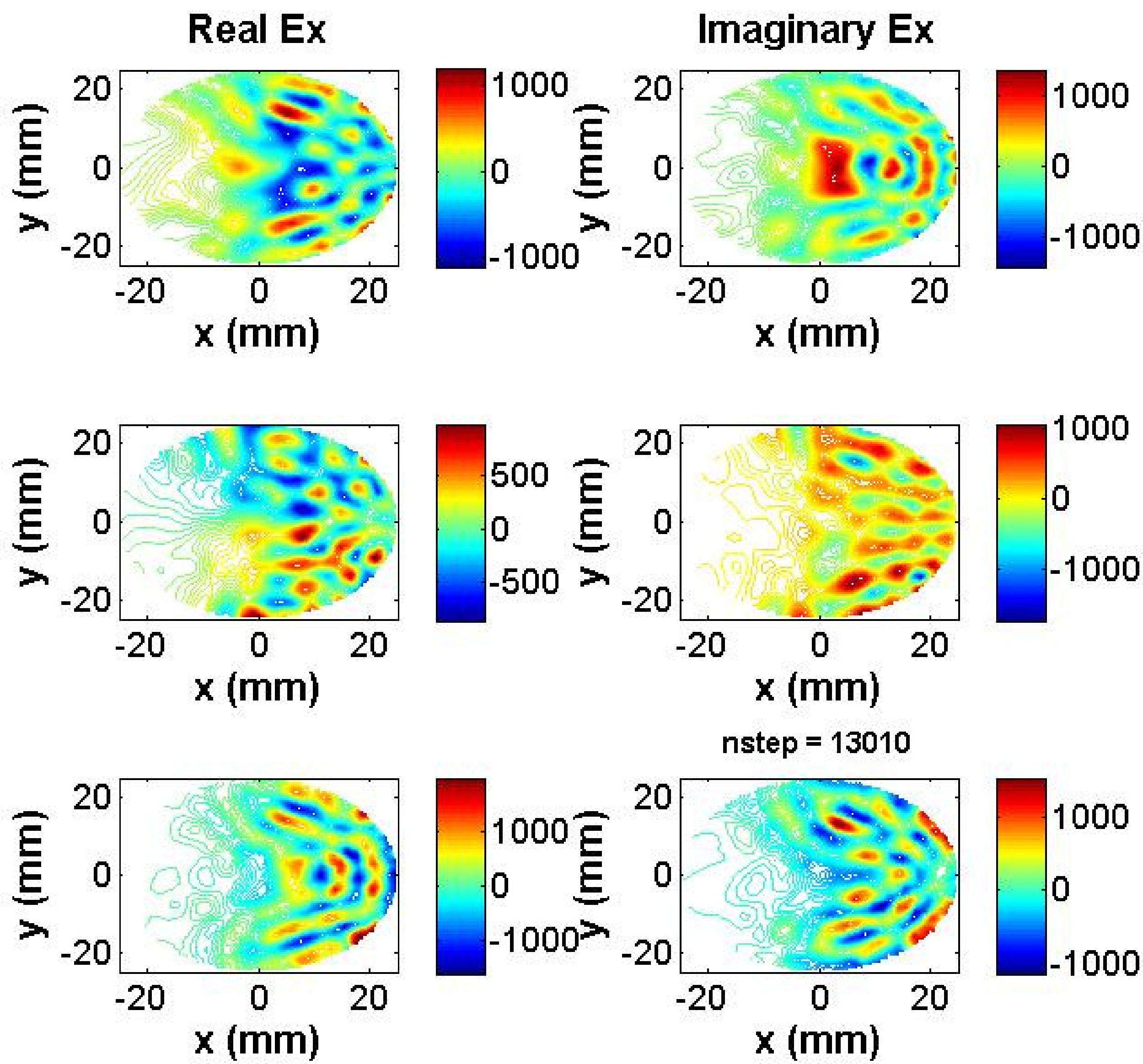
$k=1x10^4m^{-1}$ s=0.55m



$k=2x10^3m^{-1}$ s=0.55m



Round chamber, $k=1\times10^4 m^{-1}$ s=0.65m



Round chamber, radius=25mm

Coarse mesh used for the plot

Simulation of Microwave Instability using

Vlasov-Fokker-Planck code

Y. Cai & B. Warnock's Code, Vlasov solver

(Phys. Rev. ST Accel. Beams 13, 104402 (2010))

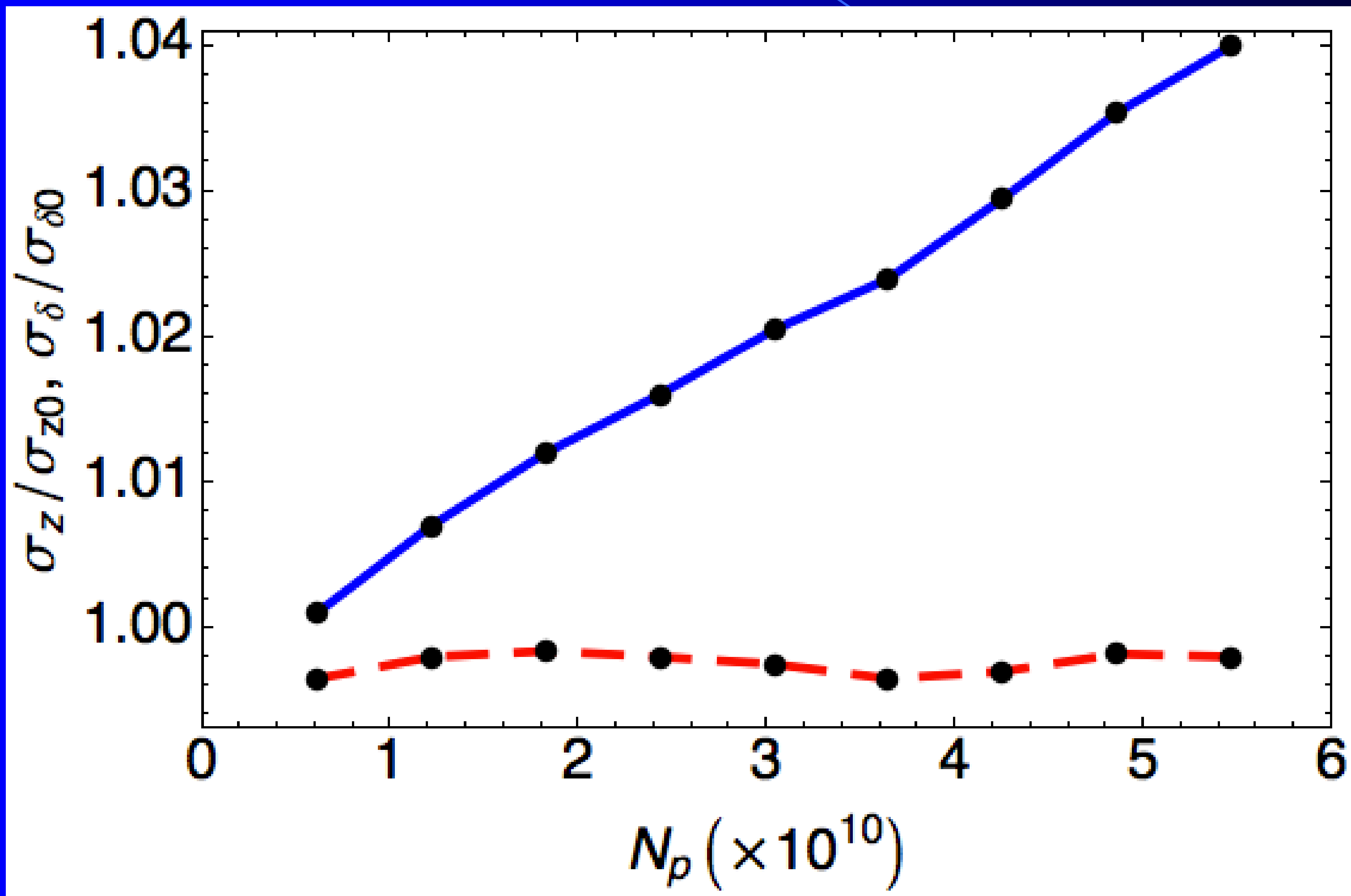
Simulation parameters:

qmax=8, nn=300 (mesh), np=4000(num syn. Period), ndt=1024 (steps/syn.); σz=6.53mm, del=1.654*10^{-4}

CSR wake is calculated using Demin's code with rectangular geometry (34mm width and 24mm height)

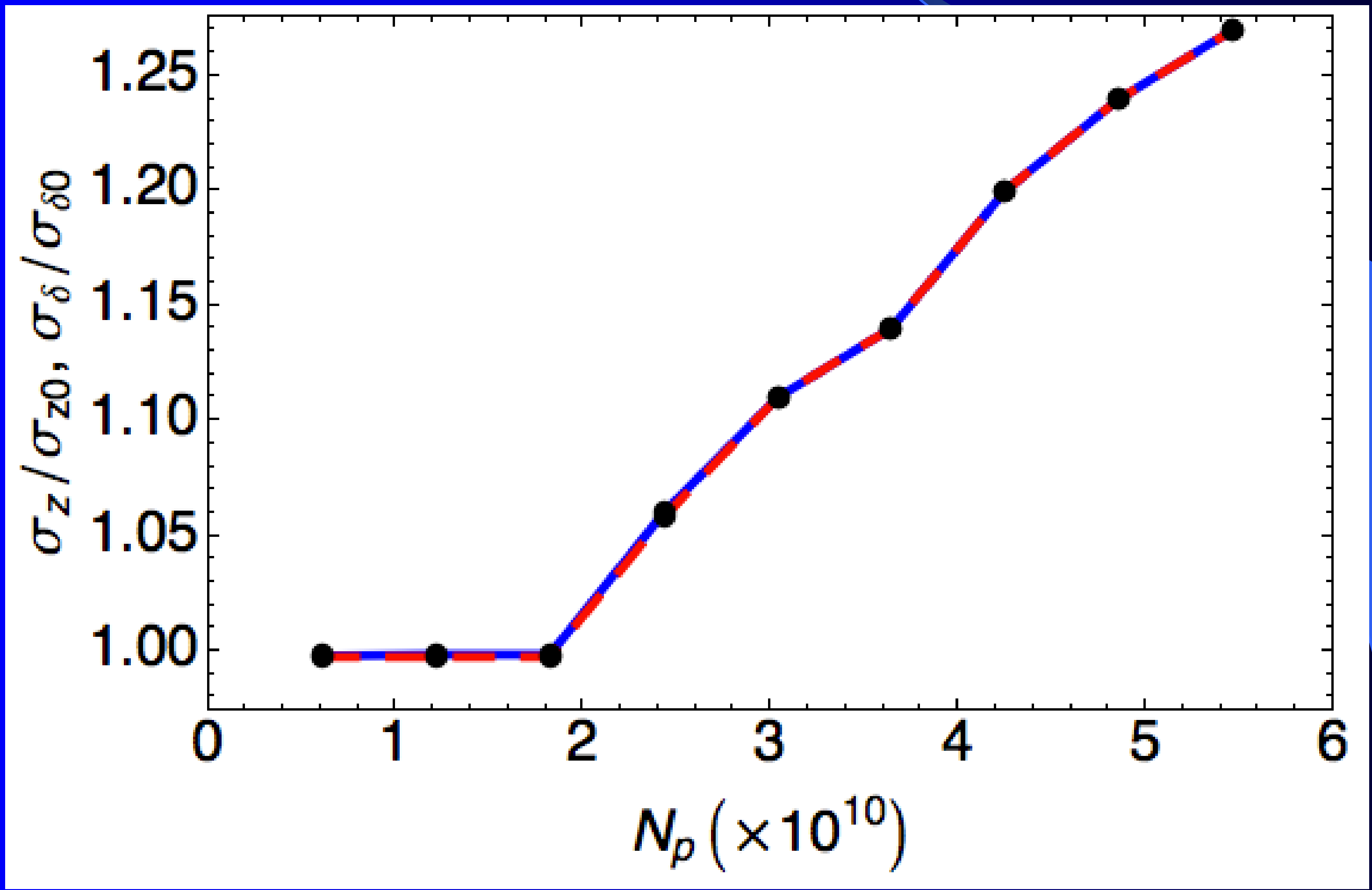
Geometry Wake is from Ikeda-san

Case I: Geometry wake only

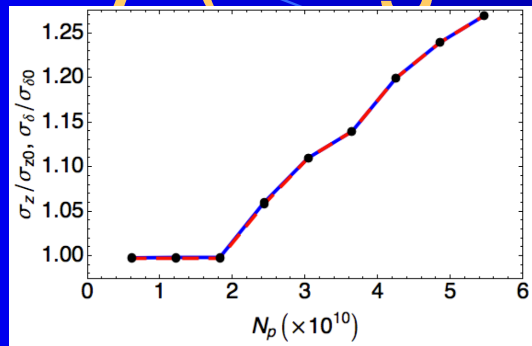


Case II: CSR wake only

Single bend wake computation

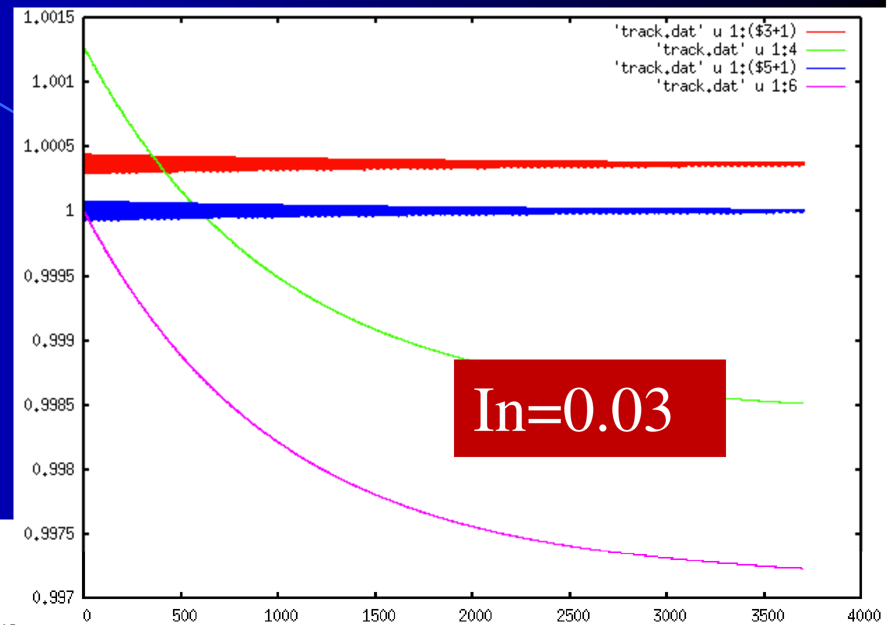


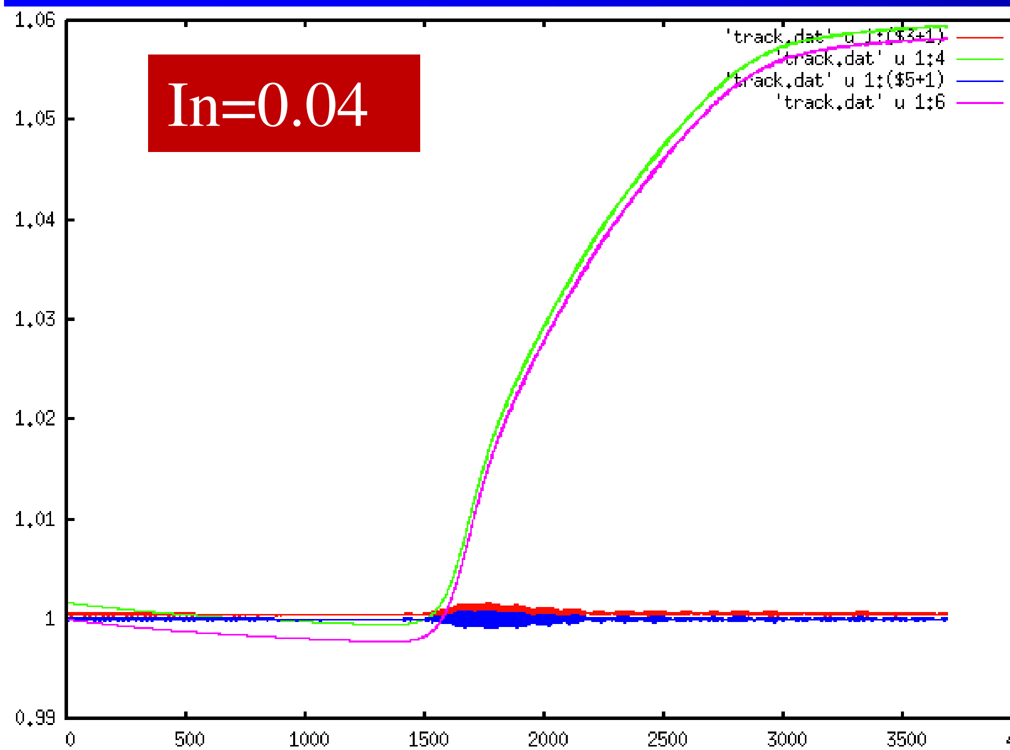
CSR only (single-bend model)

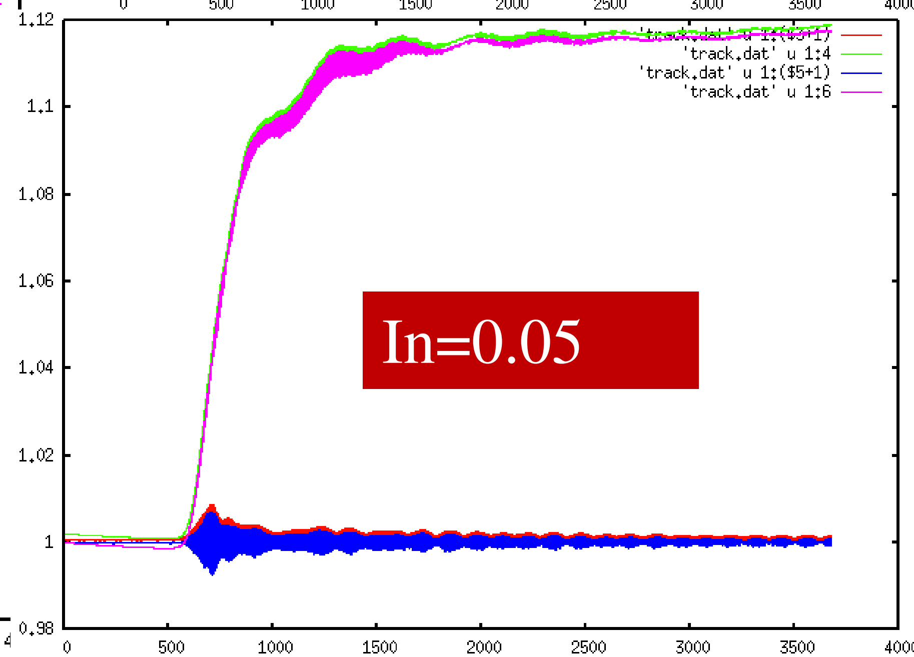


Red: <z>, Blue: <delta_e>,

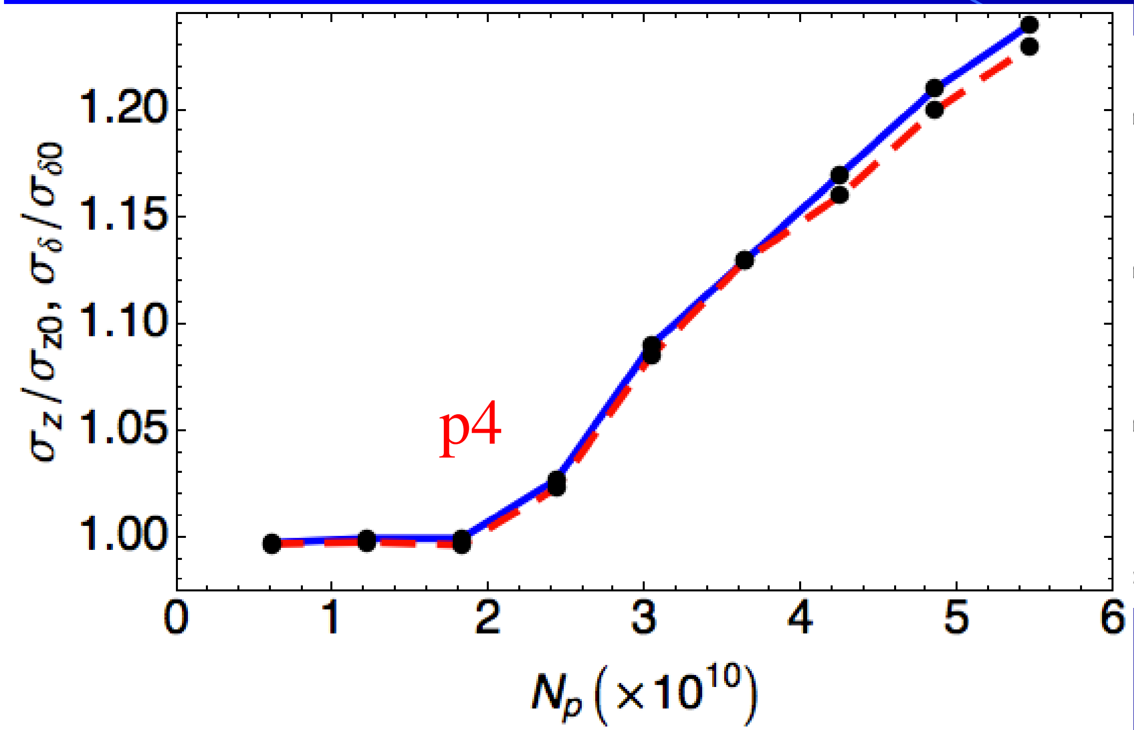
Green: sigmaz; pink: sigmae

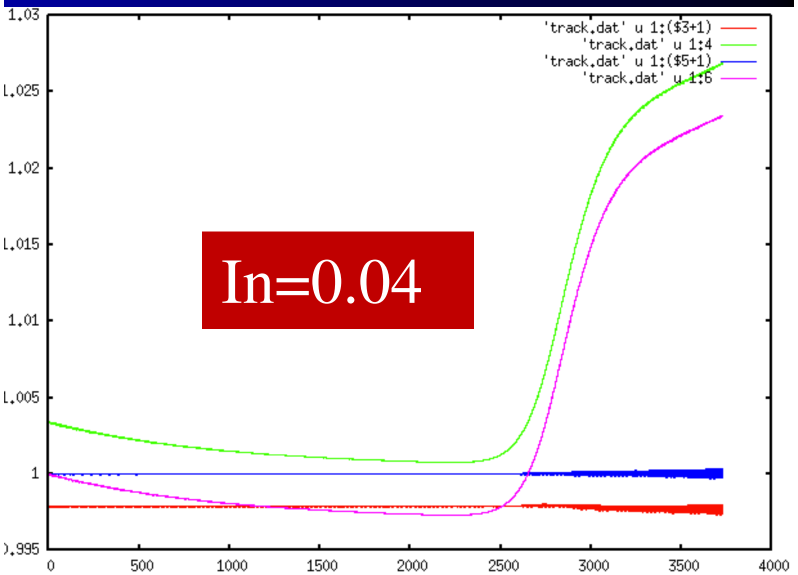


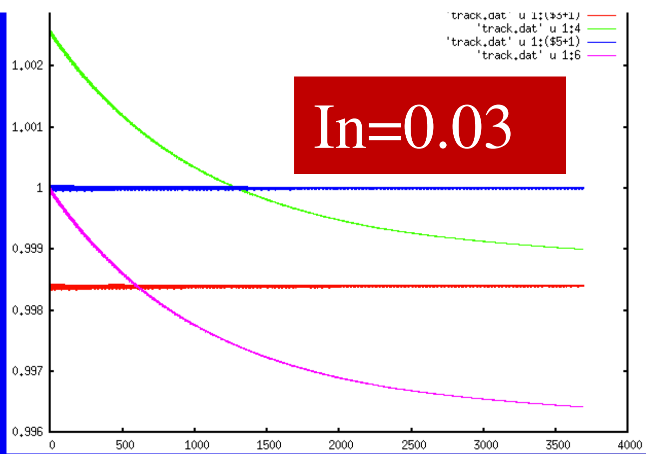


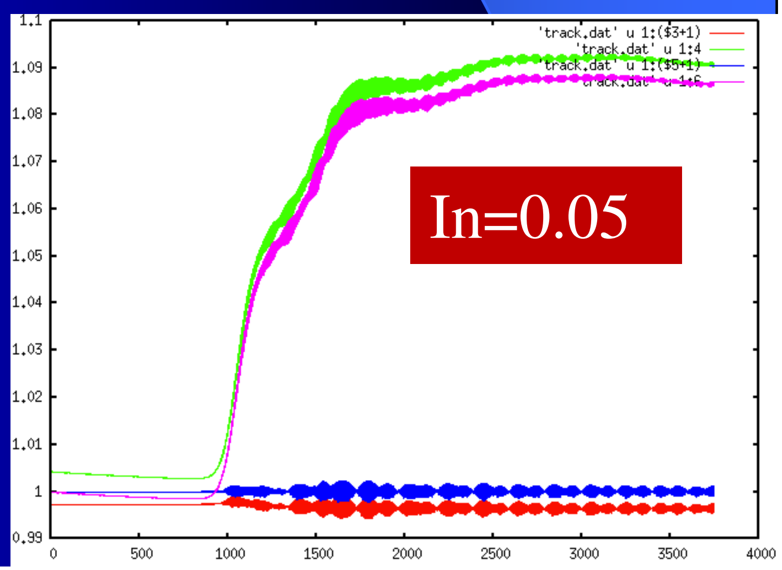


Only CSR (1-cell model)









Inter-bunch Communication through CSR in Whispering Gallery Modes
Robert Warnock

Next steps

•Complete the field solver from end of the bend to infinite

•Accurate computation with fine mesh & high order element, especially for ante-chamber geometry.

•Complete Micro-wave instability simulation with various CSR impedance options (multi-cell csr wake)

Integration of the field from end of bend to infinity

- Agoh, Yokoya, Oide

- In the exit pipe (E_x,E_y) satisfies

$$\frac{dF}{ds} = \frac{i}{2k} M F, \qquad M = \Delta_\perp$$

- This can be solved as

$$F(s) = \exp\left(\frac{i}{2k} M (s - s_{exit})\right) F(s_{exit})$$

$$\int_{s_{exit}}^{\infty} F(s) ds = 2ikM^{-1}F(0)$$

Gennady, PRSTAB 104401 (mode expansion)

$$\hat{E}_s = \sum_{p,m} \sum_{p',m'} \sum_{p'',m''} \left[\iint dxdy \left(\hat{\psi} - \frac{2\pi}{\omega q_{mp}} \hat{j}_s \right) \mathrm{div} \tilde{\mathcal{E}}^*_{m''p'',\perp} \right] \alpha_{mp|m'p'} \alpha^*_{mp|m''p''} (1 - \mathrm{e}^{iq_{mp}l}) \mathrm{e}^{i\bar{q}_{m'p'}(s-l)} \tilde{\mathcal{E}}_{m'p}$$

Other ways??

Summary

A FEM code is developed to calculated the CSR with arbitrary cross-section of the beam pipe

Preliminary results show good agreement. But need more detail benchmark.

Fast solver to integrate from exit to infinite is the next step.

Simulation of the microwave instability with Vlasov code is under the way.

Acknowledgements

Thanks Prof. Oide for the support of this work and discussion; and Prof. Ohmi for his arrangement of this study and discussions

Thanks Ikeda-san and Shibata-san for the valuable information

Thanks Demin for detail introduction his works and great help during my stay.

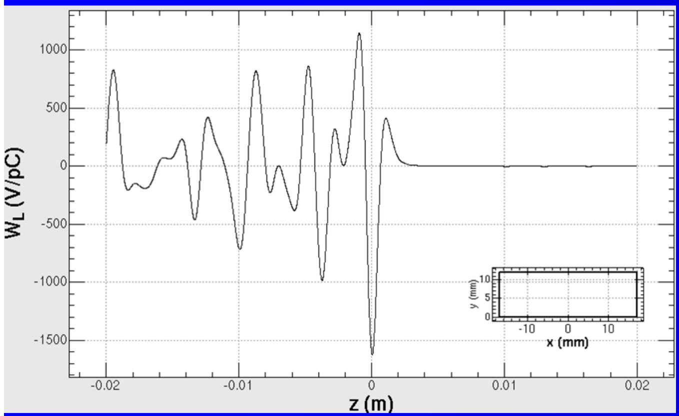
Thanks Agoh -san for the discussions.

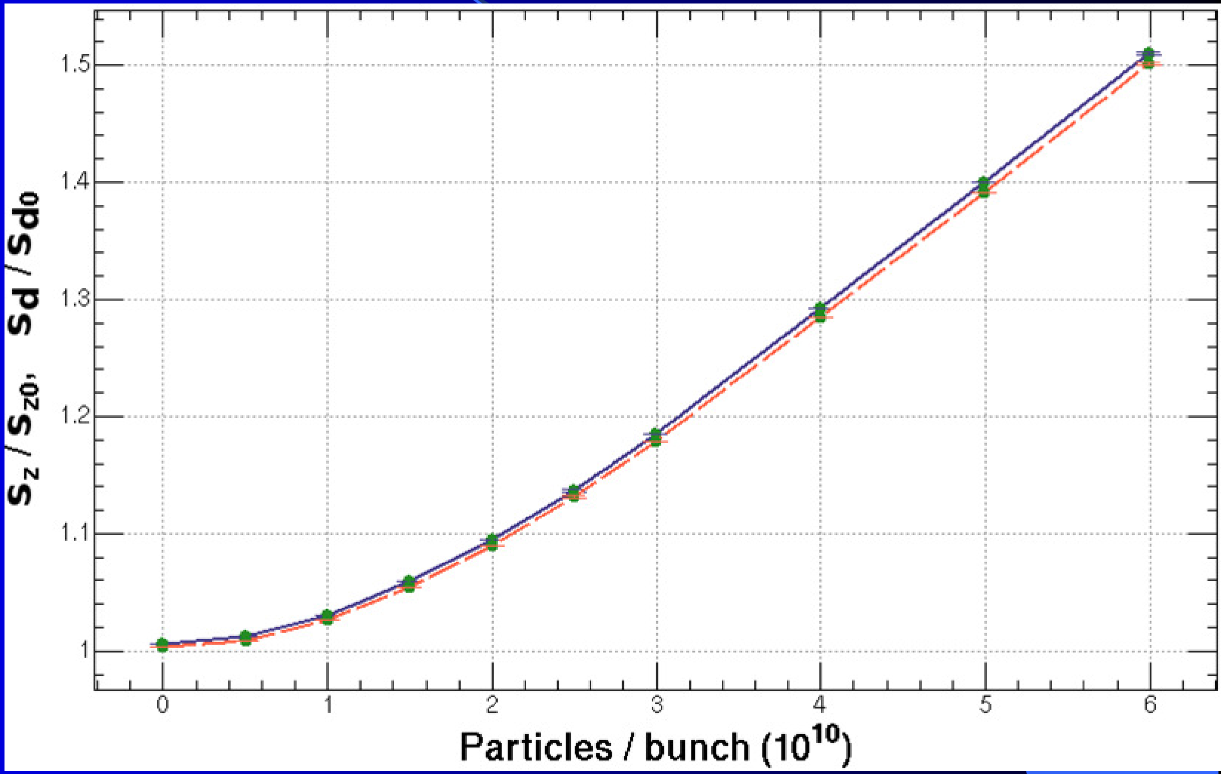
Backup slides

CSR + Geometry wake

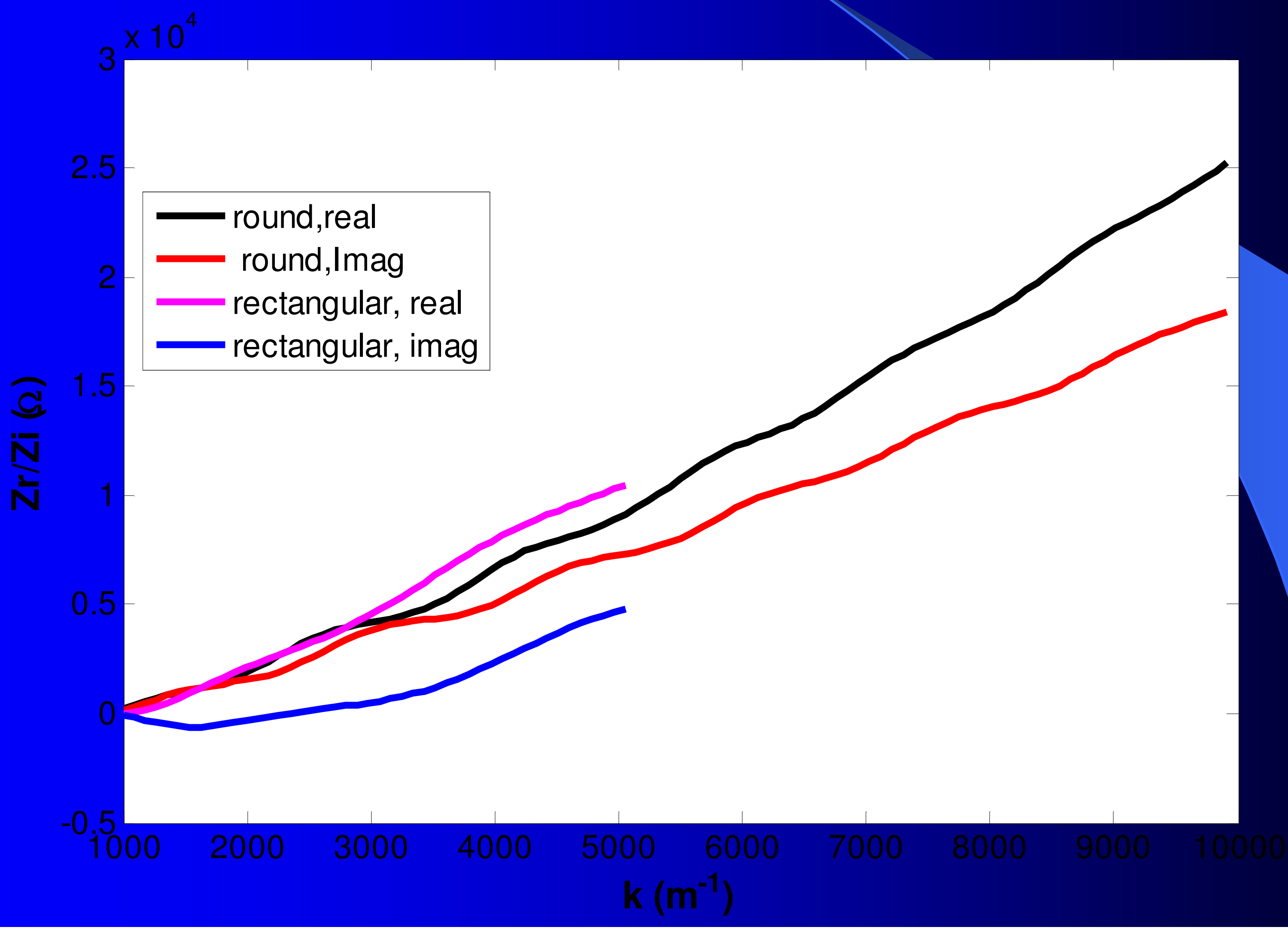
34mmX24mm rectangular

More complete model





(Courtesy, H. Ikeda@ KEKB 17th review)

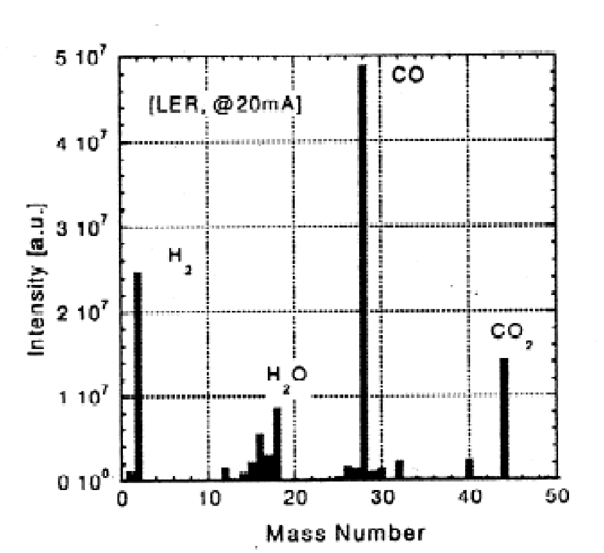

x 10^4
3
2.5
2
1.5
1
0.5
0
-0.5
Zr/Zi (Ω)
round,real
round,Imag
rectangular, real
rectangular, imag
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
k (m^-1)

Fast ion Instability in SuperKEKB Electron ring

Wang, Fukuma and Suetsugu

Ion species in KEKB vacuum chamber (3.76nTorr=5E-7 Pa)

Ion Species	Mass Number	Cross-section (*Mbarn*)	Percentage in Vacuum
H_2	2	0.35	25%
H_2O	18	1.64	10%
CO	28	2.0	50%
CO_2	44	2.92	15%

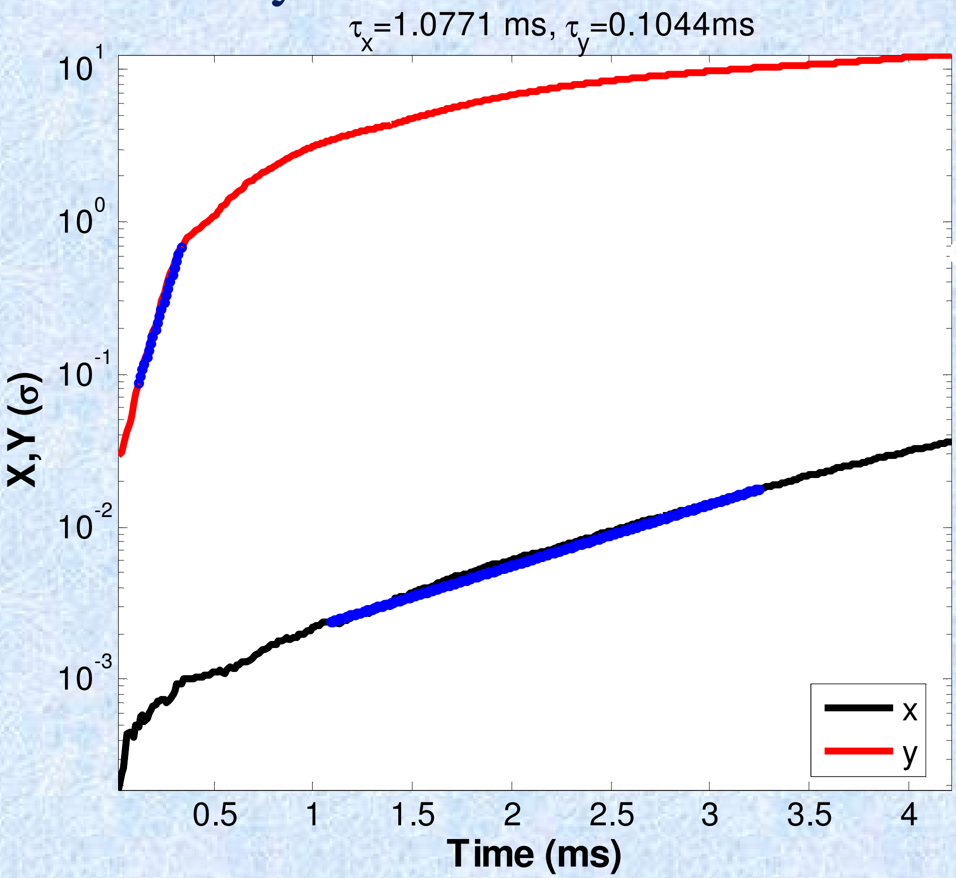


Beam current 2.6A
2500 bunches, 2RF bucket spacing
emiitacex, y 4.6nm/11.5pm

20 bunch trains

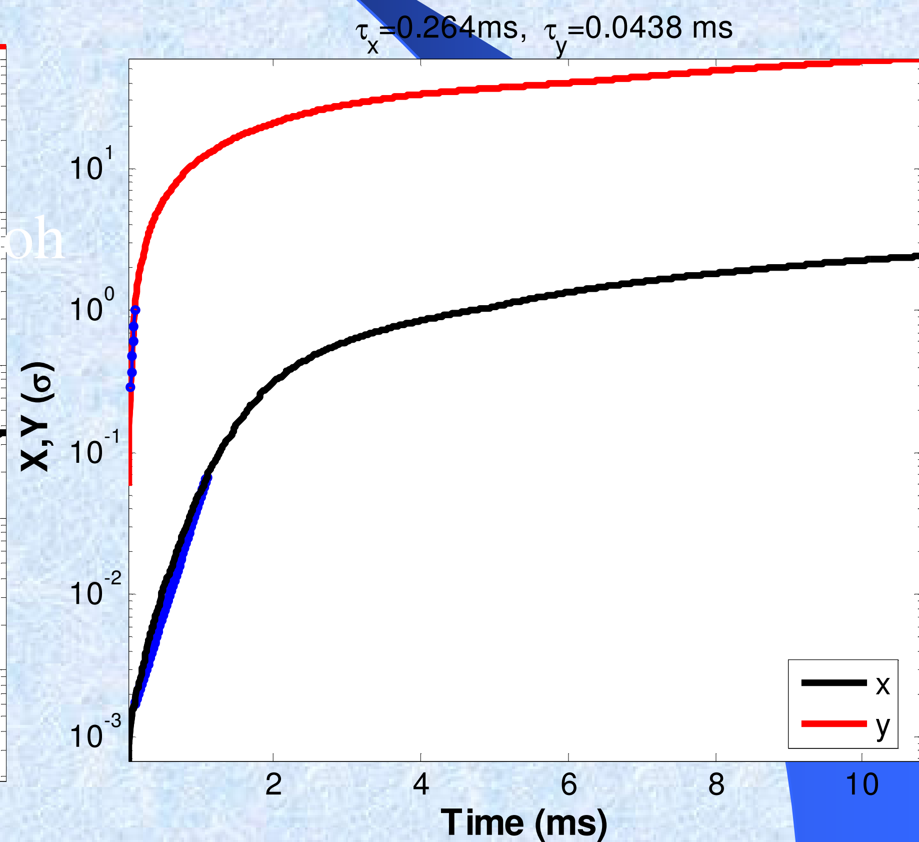
total P=1nTorr

Growth time 1.0ms and 0.1ms in x and y



Total P=3.76nTorr

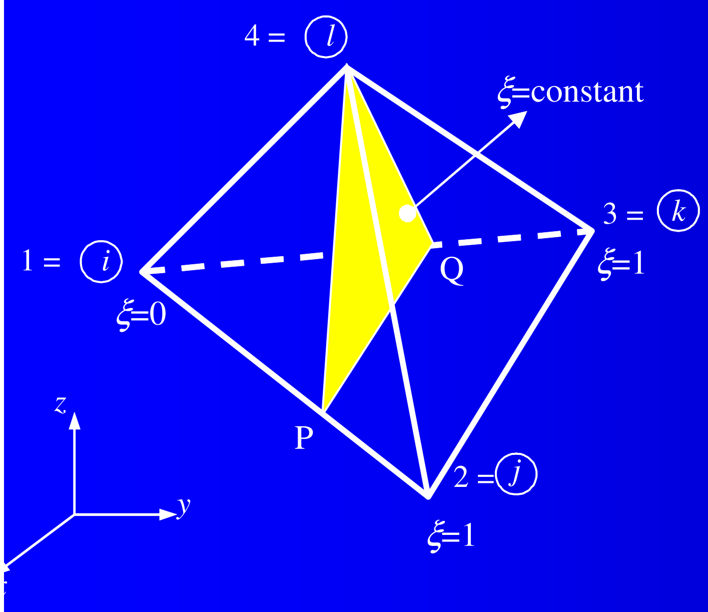
Growth time 0.26ms and 0.04ms in x and y



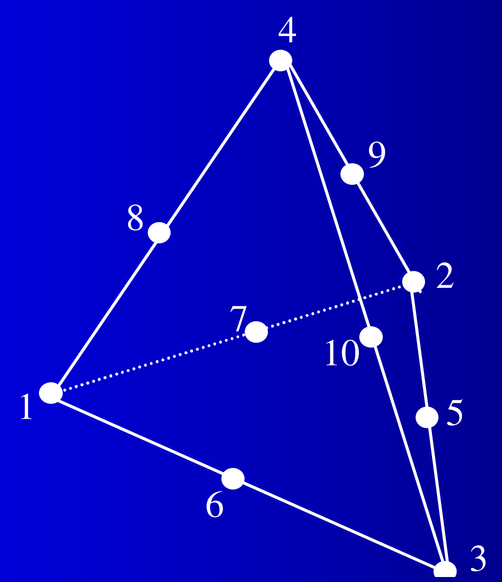
HIGHER ORDER ELEMENTS

- Tetrahedron elements

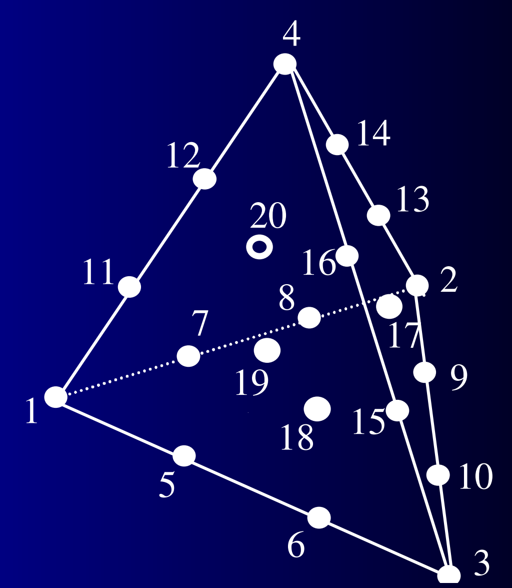
4 nodes, linear:



10 nodes, quadratic:



20 nodes, cubic:



$k=1\times10^{4}m^{-1}$ s=0.46m