Numerical Calculations of CSR Impedance

Demin Zhou

Thanks to G. Stupakov and Y. Cai for hosting my visits

and

Acknowledgements: K. Ohmi, K. Oide, Y.H. Chin, T. Agoh, K. Yokoya, M. Kikuchi, K. Shibata, H. Ikeda

SLAC, Mar. 11, 2011

Outline

1. Introduction
2. Algorithms
3. Numerical calculations
    3.1 Single dipole
    3.2 Fringe field
    3.3 Wiggler/Undulator
    3.4 Interference
    3.5 Resistive wall
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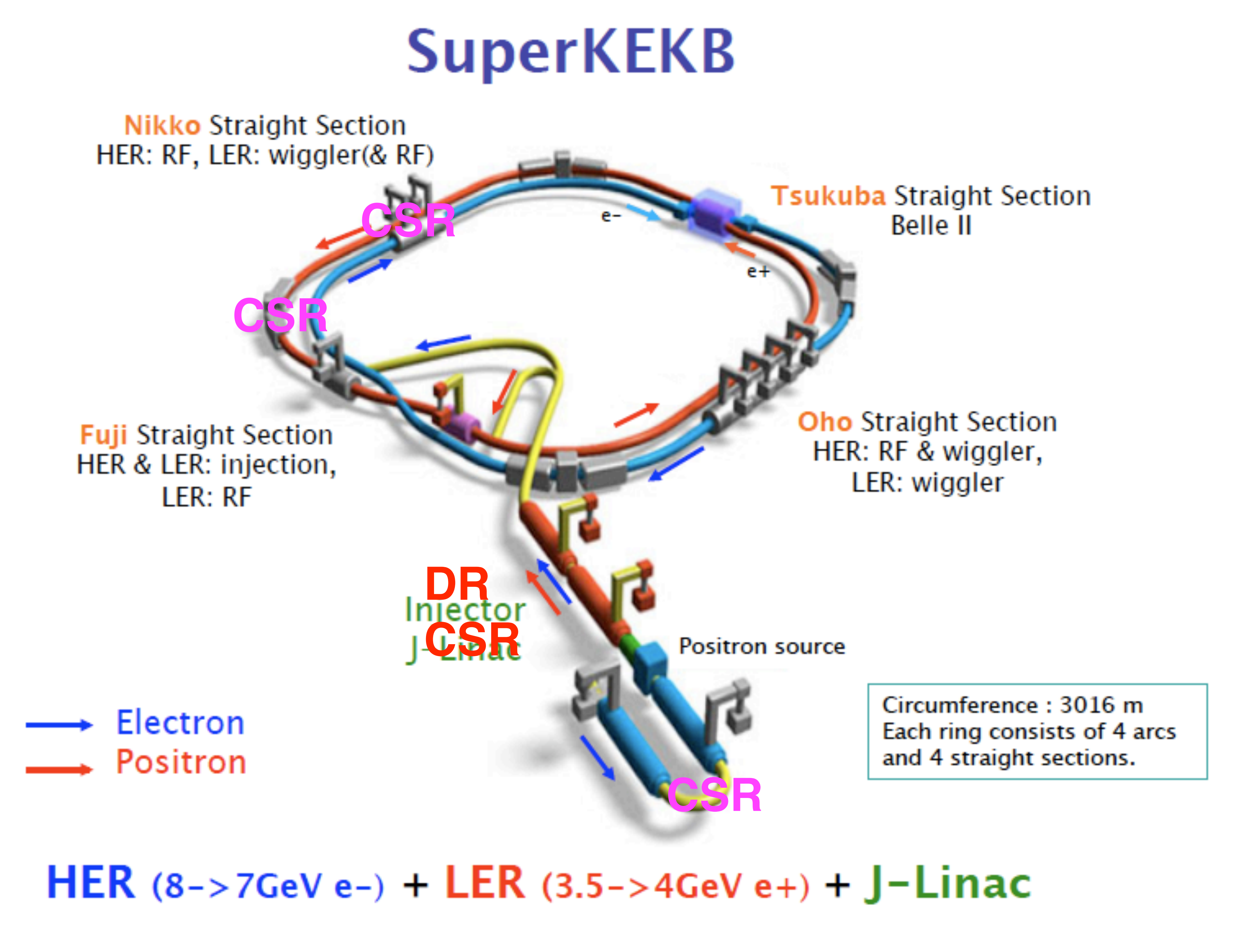
Introduction



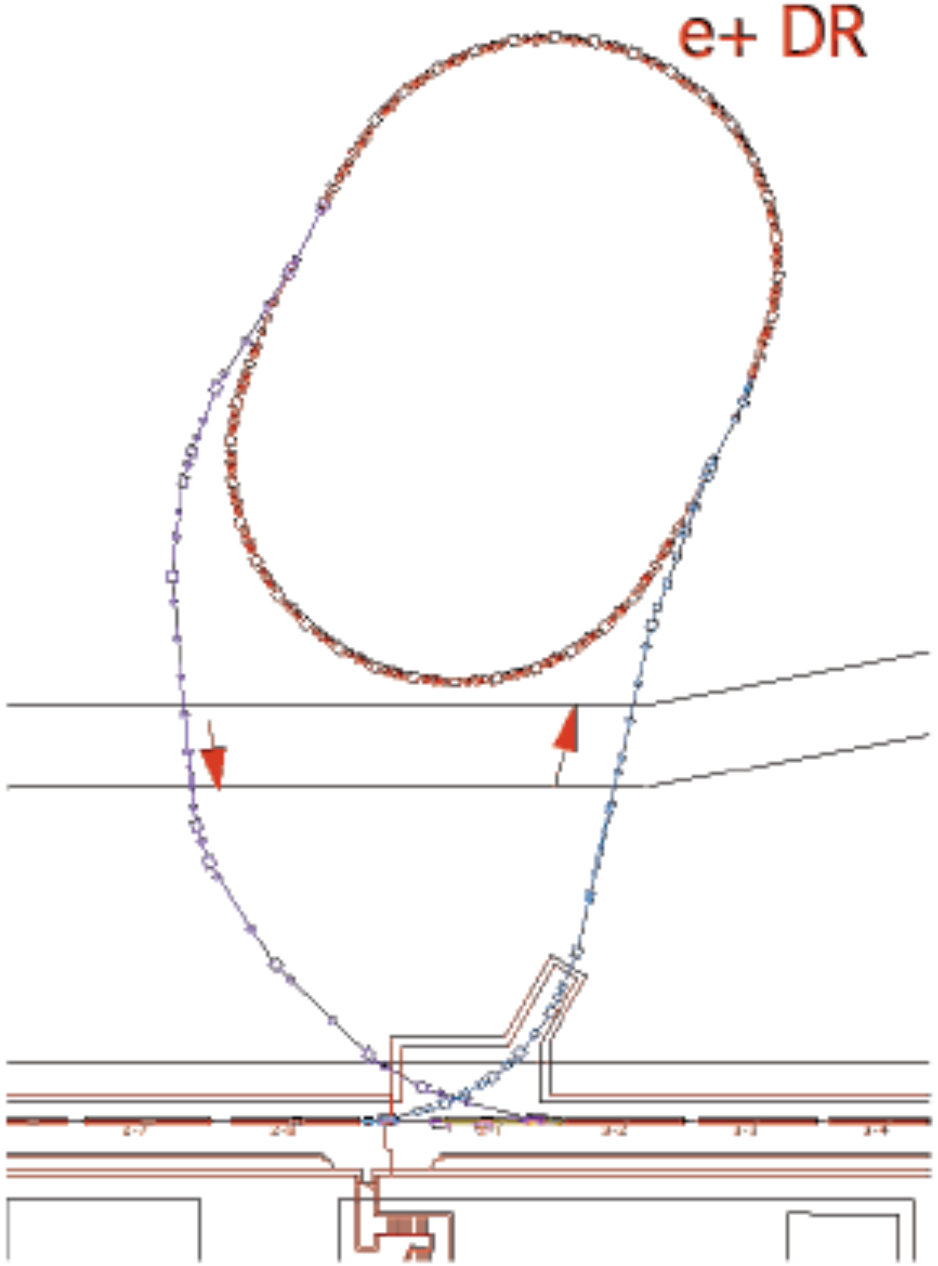
Ref. H. Koiso, 16th KEKB Review, Feb. 07, 2011

Introduction

	KEKB Design	KEKB Achieved : with crab	SuperKEKB Nano-Beam
Energy (GeV) (LER/HER)	3.5/8.0	3.5/8.0	4.0/7.0
β_y^* (mm)	10/10	5.9/5.9	0.27/0.30
β_x^* (mm)	330/330	1200/1200	32/25
ε_x (nm)	18/18	18/24	3.2/5.3
$\varepsilon_y/\varepsilon_x$ (%)	1	0.85/0.64	0.27/0.24
σ_y(μm)	1.9	0.94	0.048/0.062
ξ_y	0.052	0.129/0.090	0.09/0.081
σ_z (mm)	4	6 - 7	6/5
I_{beam} (A)	2.6/1.1	1.64/1.19	3.6/2.6
$N_{bunches}$	5000	1584	2500
Luminosity (10^{34} cm^{-2} s^{-1})	1	2.11	80

Ref. H. Koiso, 16th KEKB Review, Feb. 07, 2011

Introduction



Beam energy (GeV)	1.1		
Circumference (m)	135		
# of train	2		
# of bunches/train	2		
Maximum stored current (mA)	70.8		
Horizontal damping time (ms)	11		
Injected-beam emittance (μm)	1.7		
Emittance @ extraction (H/V) (nm)	42.5 / 2.07		
Cavity voltage (Vc) (MV)	0.5	1.0	1.4
Bunch length (mm)	11.1	7.7	6.5
Momentum compaction (α)	0.0141		
Energy spread (%)	0.055		

The e+ DR is to mitigate the problems of lifetime and injection aperture in LER

Ref. H. Koiso and M. Kikuchi
16th KEKB Review, Feb. 07, 2011

Introduction

Motivations for this work:

1. To figure out the unknown source of longitudinal impedance which drive the microwave instability (MWI) in the KEKB LER
2. To work out a reliable impedance model for SuperKEKB
3. CSR in wigglers, in dipoles with interference, or with resistive wall

Existing publications on numerical calculations of CSR impedance:

1. T. Agoh and K. Yokoya, PRST-AB 7, 054403 (2004) and T. Agoh, PhD. Thesis (2004)
2. K. Oide, Presentations at KEKB ARC 2009 and CSR mini-workshop (Nov. 2010 at KEK); PAC09
3. G. Stupakov and I. Kotelnikov, PRST-AB 12, 104401 (2009)

Algorithms - Fundamental equations

Parabolic equation in curvilinear coordinate system:

$$\frac{\partial \vec{E}_\perp}{\partial s} = \frac{i}{2k}\left(\nabla_\perp^2 \vec{E}_\perp - \mu_0 c^2 \nabla_\perp \rho_0 + \underline{\frac{2k^2 x}{\rho(s)}} \vec{E}_\perp\right)$$

β=1

Field separation:

$$\vec{E}_\perp = \vec{E}_\perp^r + \vec{E}_\perp^b$$

$$\frac{\partial \vec{E}_\perp^r}{\partial s} = \frac{i}{2k}\left[\nabla_\perp^2 \vec{E}_\perp^r + \frac{2k^2 x}{\rho}(\vec{E}_\perp^r + \vec{E}_\perp^b)\right]$$

Beam field in free space (independent of s):

$$\frac{\partial^2 E_x^b}{\partial x^2} + \frac{\partial^2 E_x^b}{\partial y^2} = \mu_0 c^2 \frac{\partial \rho_0}{\partial x}$$

$$\frac{\partial^2 E_y^b}{\partial x^2} + \frac{\partial^2 E_y^b}{\partial y^2} = \mu_0 c^2 \frac{\partial \rho_0}{\partial y}$$

Ref. T. Agoh and K. Yokoya, PRST-AB 7, 054403 (2004)

Algorithms - Beam field

Beam field in free space with finite beam sizes (Bassetti-Erskine formula)

Typical beam size: σ_x=0.5mm, σ_y=0.01mm (bi-gaussian)

$$E_x^b(x,y) = \frac{\lambda(k)}{2\epsilon_0\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}}\mathrm{Im}[F(x,y)]$$

$$E_y^b(x,y) = \frac{\lambda(k)}{2\epsilon_0\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}}\mathrm{Re}[F(x,y)]$$

$$F(x,y) = w(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}) - e^{-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}}w(\frac{\frac{\sigma_y}{\sigma_x}x+i\frac{\sigma_x}{\sigma_y}y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}})$$

$$w(z) = e^{-z^2}(1+\frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt)$$

Algorithms - Initial conditions

Parabolic equation:

$$\frac{\partial \vec{E}_{\perp}^{r}}{\partial s} = \frac{i}{2k}[\nabla_{\perp}^{2}\vec{E}_{\perp}^{r} + \frac{2k^2 x}{\rho}(\vec{E}_{\perp}^{r} + \vec{E}_{\perp}^{b})]$$

Poisson's equation:

$$\nabla^2\phi = -\frac{\rho_0}{\epsilon_0}$$

$$\phi = \phi^r + \phi^b$$

Laplace's equation:

$$\nabla^2\phi^r = 0$$

$$\phi^r|S = -\phi^b|S$$

$$\phi^b(x,y) = \frac{Q}{2\pi\epsilon_0}\int_r^1 \frac{e^{-(1-t^2)(A+\frac{B}{t^2})} - 1}{1-t^2}dt$$

$$A(x) = \frac{x^2}{2(\sigma_x^2 - \sigma_y^2)}$$

$$B(y) = \frac{y^2}{2(\sigma_x^2 - \sigma_y^2)}$$

$$r = \frac{\sigma_y}{\sigma_x}$$

Ref. R. Alves-Pires, CERN PS/87-66

Algorithms - Beam pipe

Model of the beam pipe:

1. The bending radius can be arbitrarily s-dependent, which allows for treating fringe field, wigglers or a series of dipole magnets
2. Uniform rectangular cross section along the beam orbit (simply the calculation)

Field integration along s:

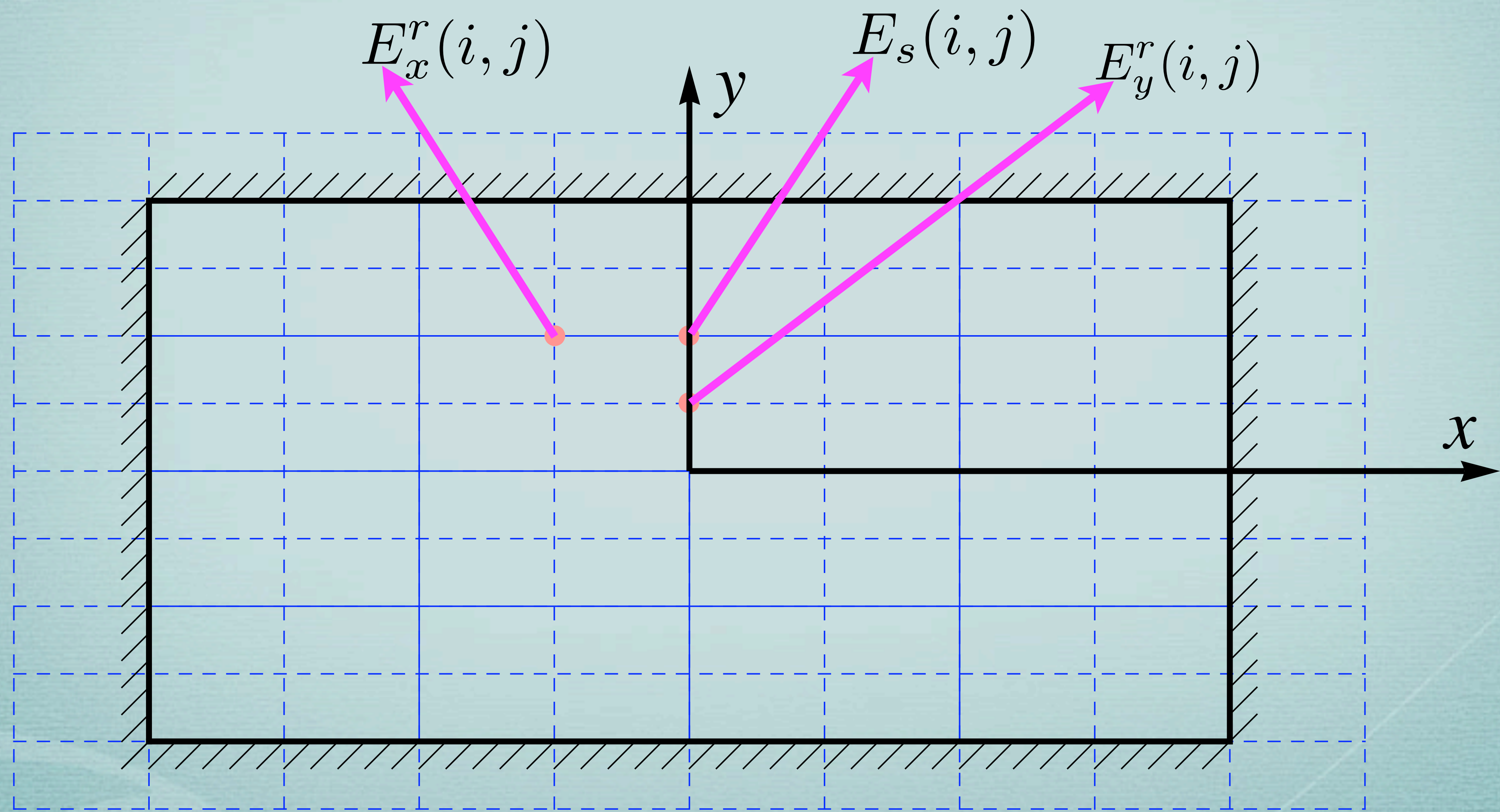
1. Toroidal part: Numerical integration
2. Straight pipe: mode expansion

Ref. T. Agoh, Ph.D. Thesis (2004)
G. Stupakov and I. Kotelnikov, PRST-AB 12, 104401 (2009)

Algorithms - Mesh

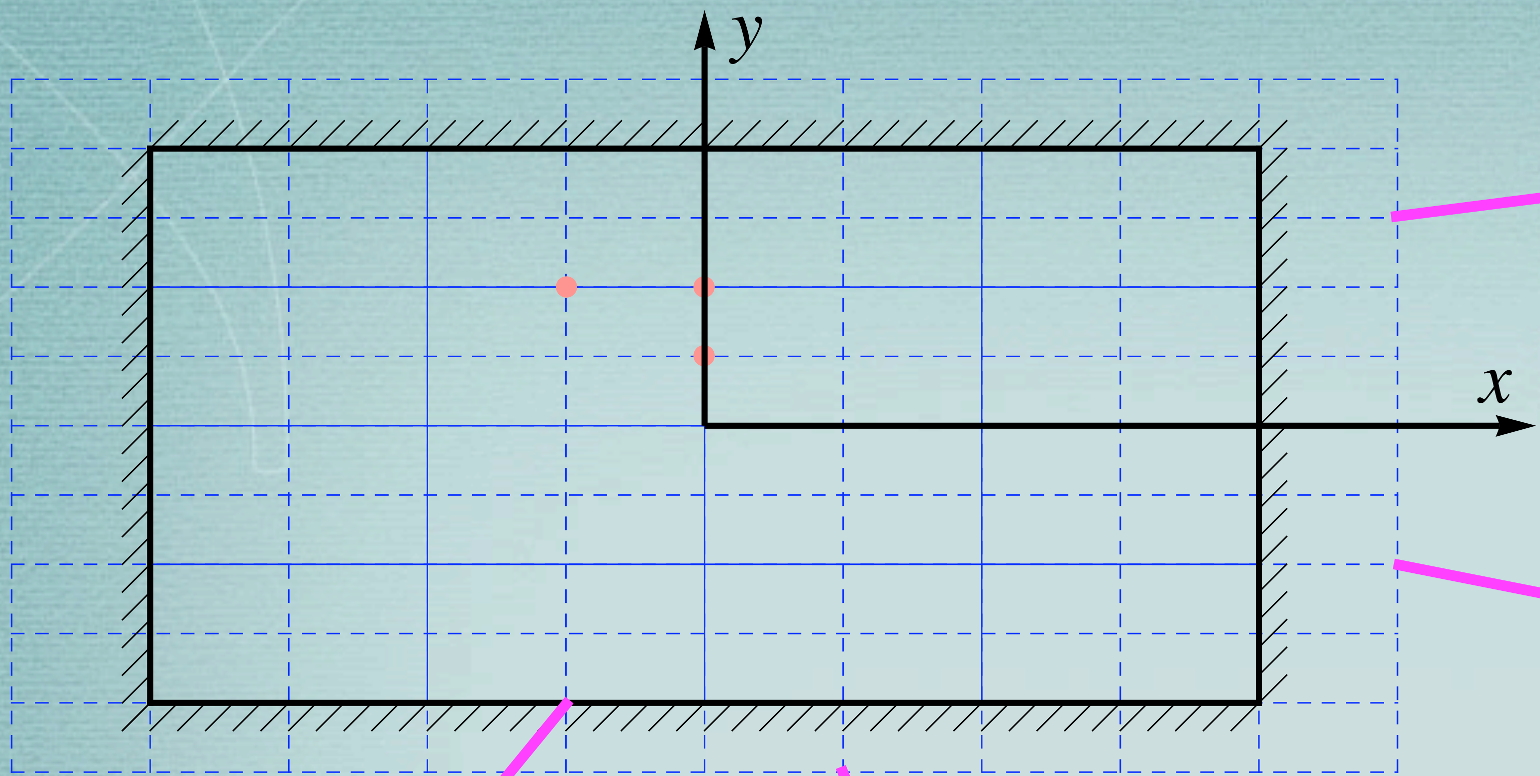
Finite-difference discretization:

1. Staggered grid: Central difference
2. Ghost point: Boundary conditions



Ref. T. Agoh, Ph.D. Thesis (2004)

Algorithms - Boundary conditions



$$E_y^r(M,j) = -E_y^b(M,j)$$

$$E_x^r(M+1,j)$$

$$E_x^r(i,0) = -E_x^b(i,0)$$

$$E_y^r(i,0)$$

$$\frac{\partial E_x^r}{\partial x} + \frac{\partial E_y^r}{\partial y} + \frac{1}{\rho g} E_x^r + \frac{1}{g} ik E_s = 0$$

Leontovich boundary condition (Resistive wall):

$$\vec{E}_{||} = -\sqrt{\frac{\mu_c \omega}{2\sigma_c}}(1-i)(\vec{n} \times \vec{H}_{||})$$

$$g = 1 + \frac{x}{\rho(s)}$$

Gauss's law:

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{1}{\rho g} E_x + \frac{1}{g}(ikE_s + \frac{\partial E_s}{\partial s}) = \frac{\rho_0}{\epsilon_0}$$

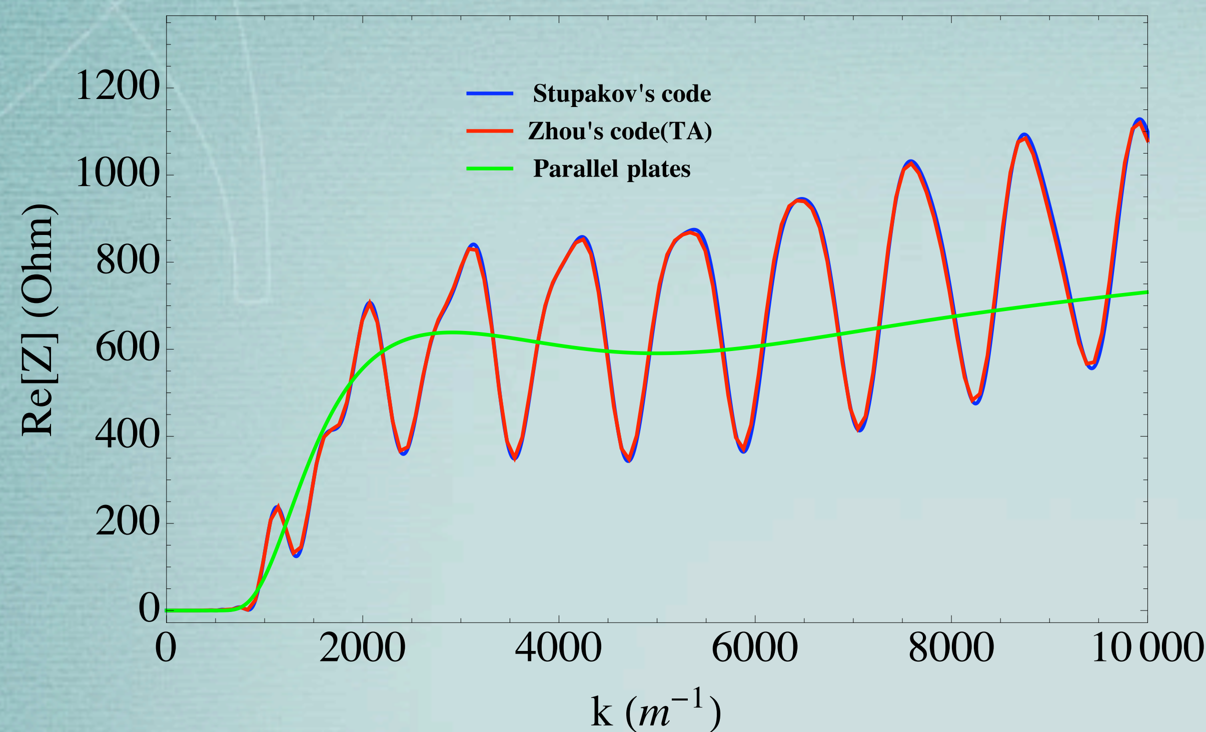
Faraday's law:

$$\vec{n} \times \vec{E} = 0$$

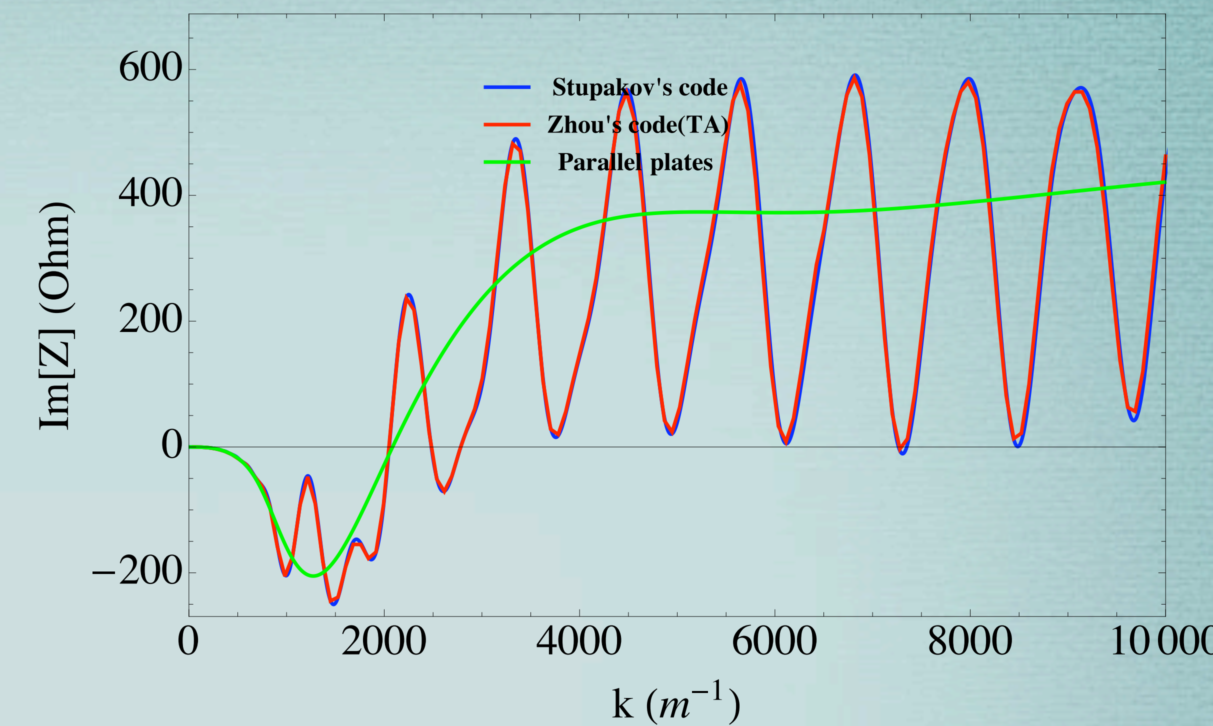
Single dipole - ANKA

Collaborate with K. Marit

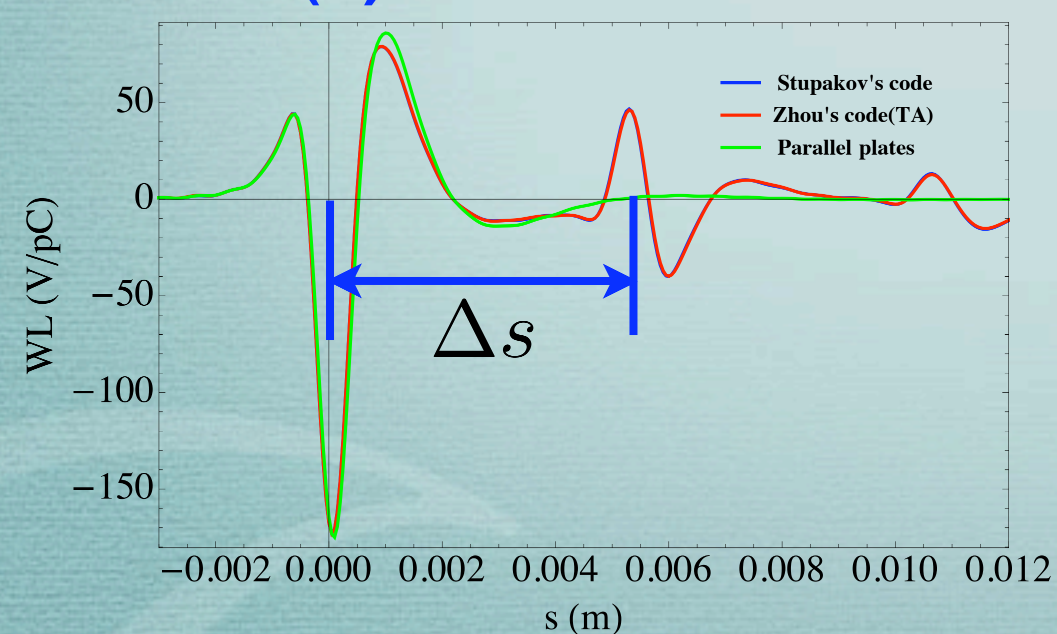
$Re.Z_L(k)$



$Im.Z_L(k)$



$W_L(s)$ with σ_z=0.3mm



Fluctuation in impedance is due to reflections of side walls (Oide)

ANKA

w/h=70/32mm

L_{bend}=2.183m

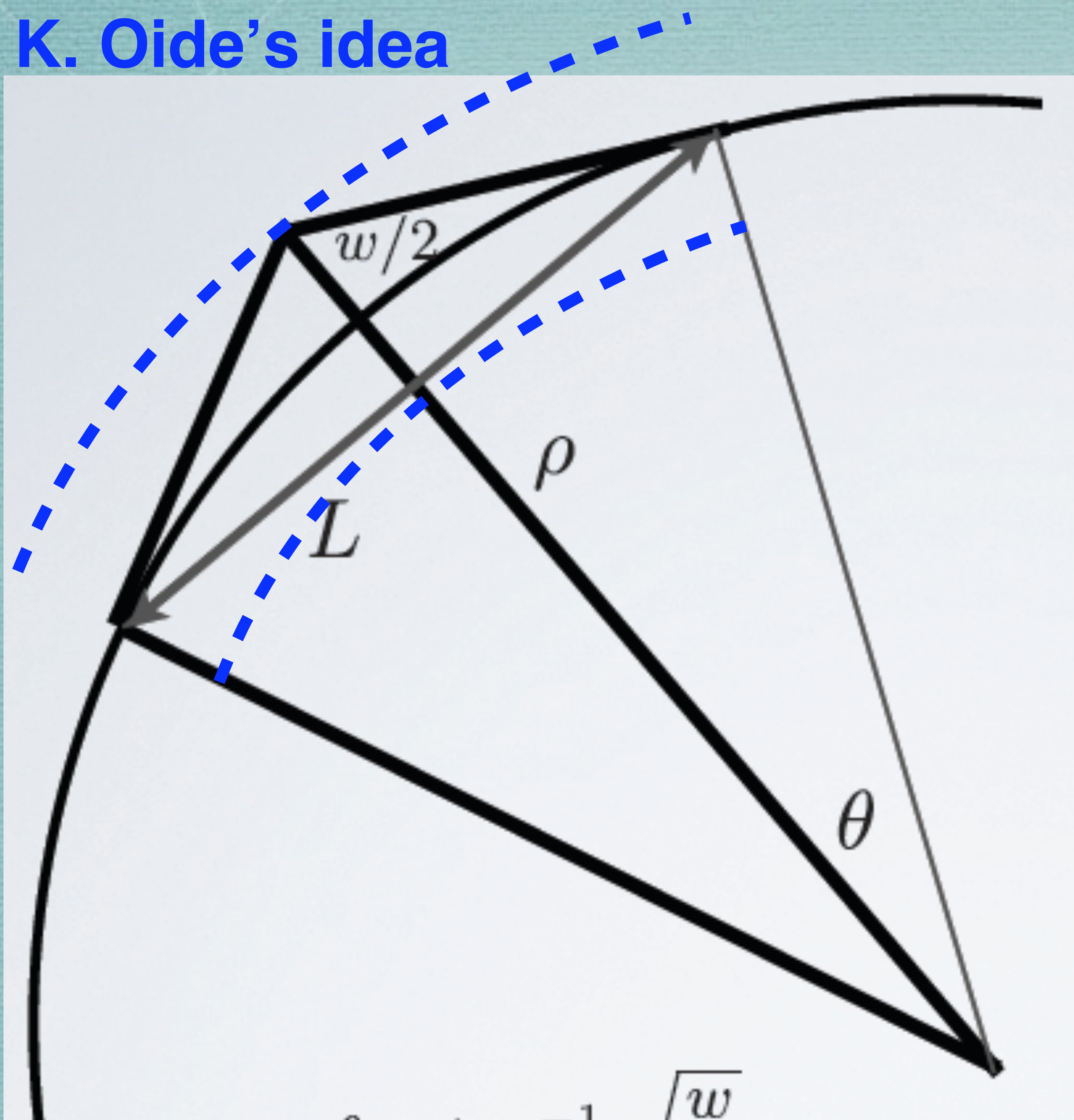
ρ=5.559m

L_{exit}=Infinity (pipe after exit)

X_{offset}=0mm

Single dipole - Phase matching condition

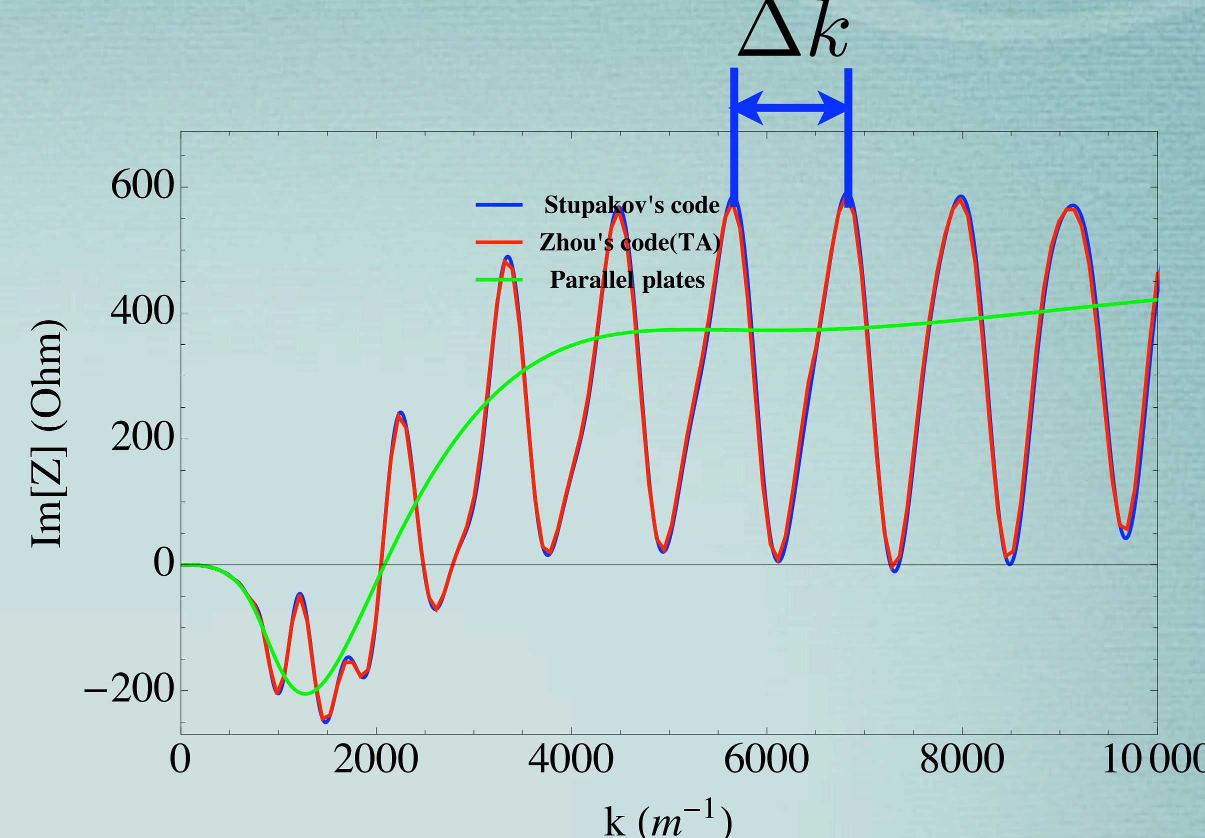
K. Oide's idea



$$\theta \approx \tan^{-1}\sqrt{\frac{w}{\rho}}$$

$$L = 2\rho\theta \approx 2\sqrt{\rho w}$$

$$\Delta s = 2\rho(\tan\theta - \theta) \approx 2\rho\frac{\theta^3}{3} \approx \frac{2w^{3/2}}{3\rho^{1/2}}$$



$$\Delta k = \frac{2\pi}{\Delta s} \approx 1200\ \mathrm{m}^{-1}$$

$$L = 1.24\ \mathrm{m}$$

$$\Delta s = 5.2\ \mathrm{mm}$$

ANKA

w/h=70/32mm

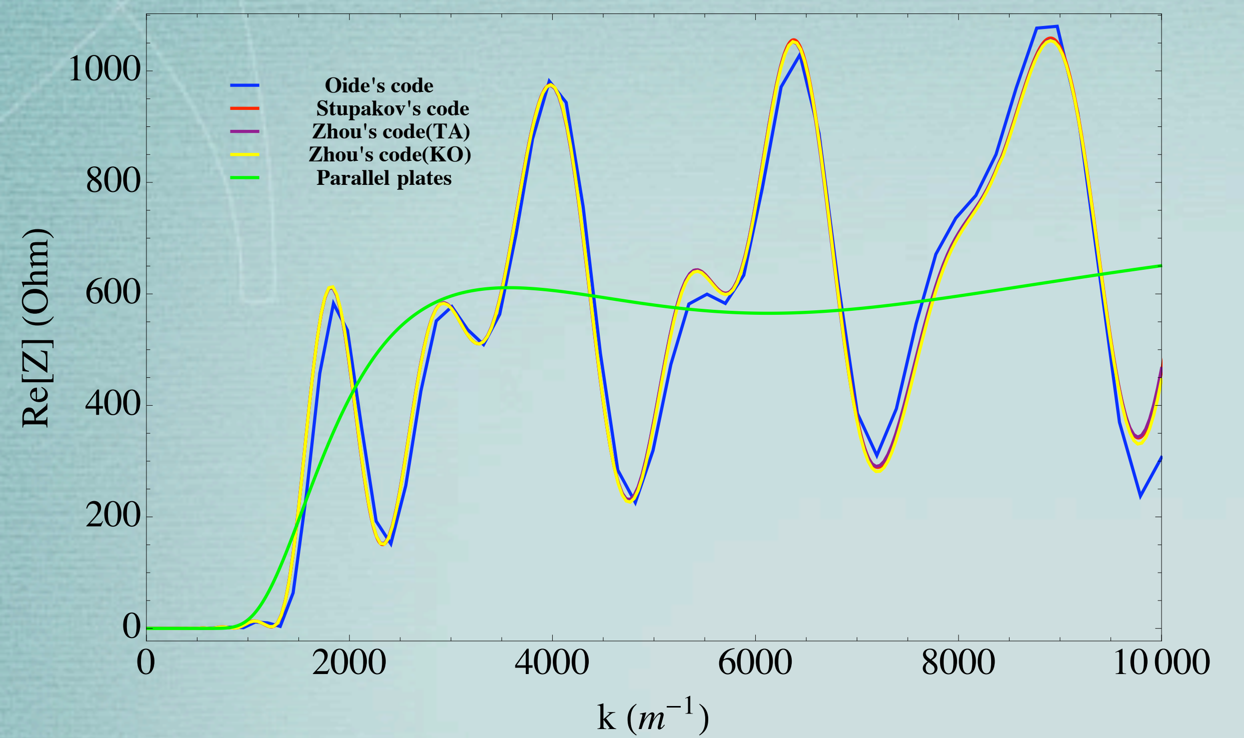
L_{bend}=2.183m

ρ=5.559m

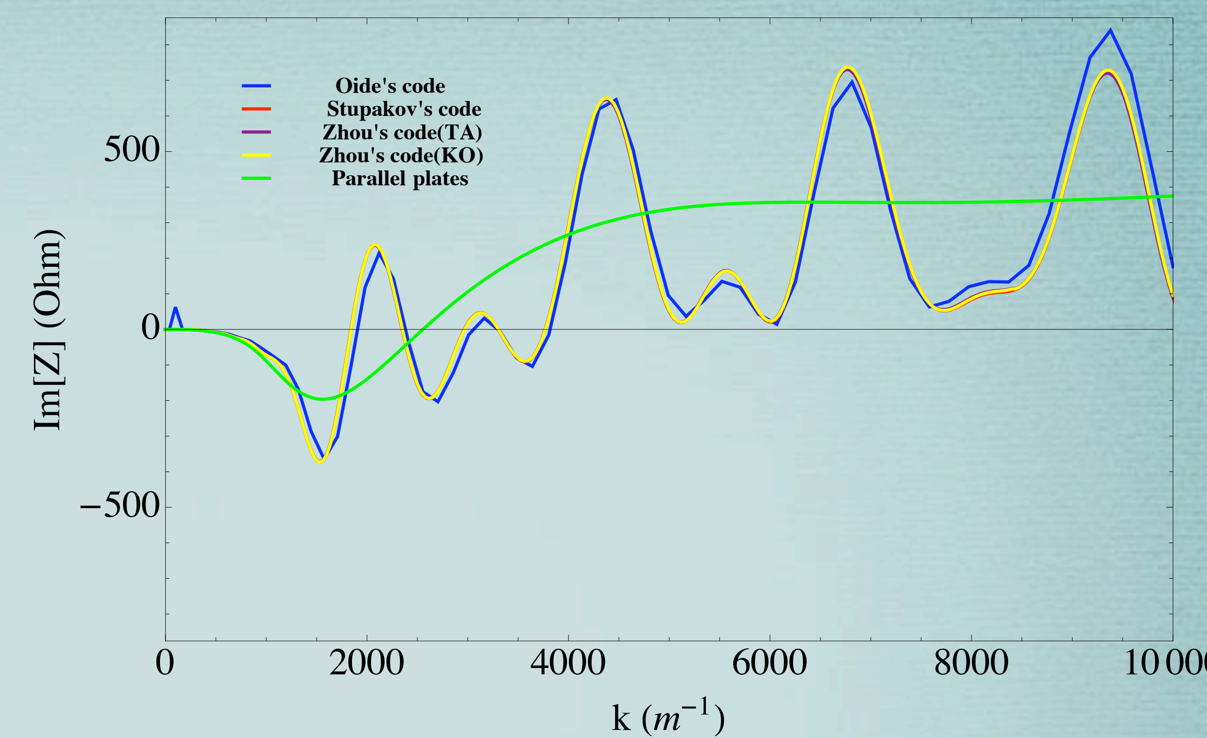
Ref. K. Oide, Presentation to CSR mini-workshop, Nov. 08, 2010, KEK

Single dipole - Benchmark example

Re.Z_L(k)

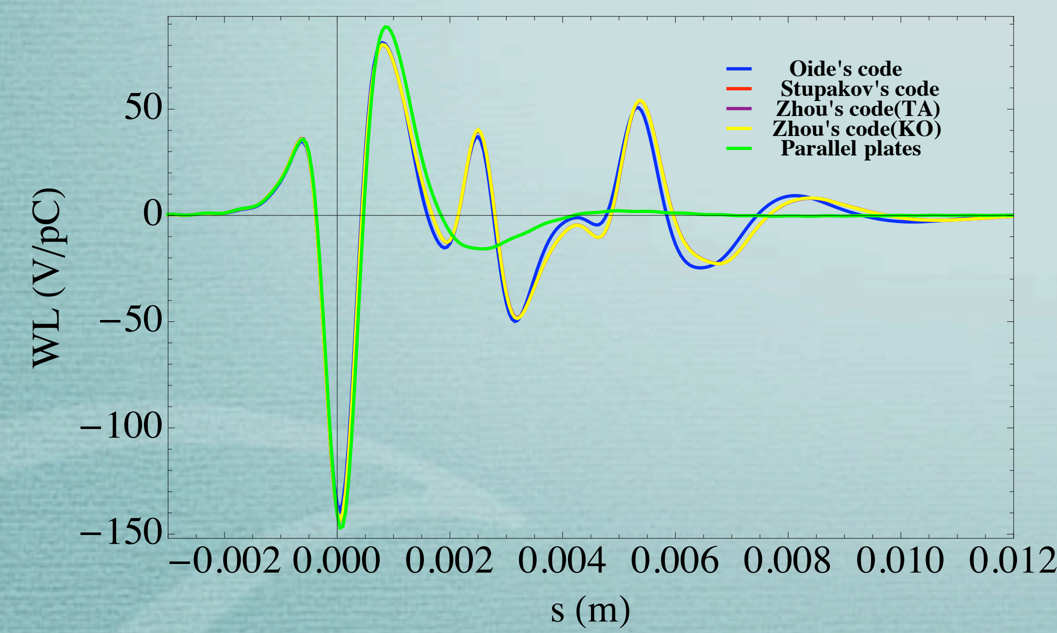


Im.Z_L(k)



TA: Agoh's algorithm
KO: Oide's algorithm

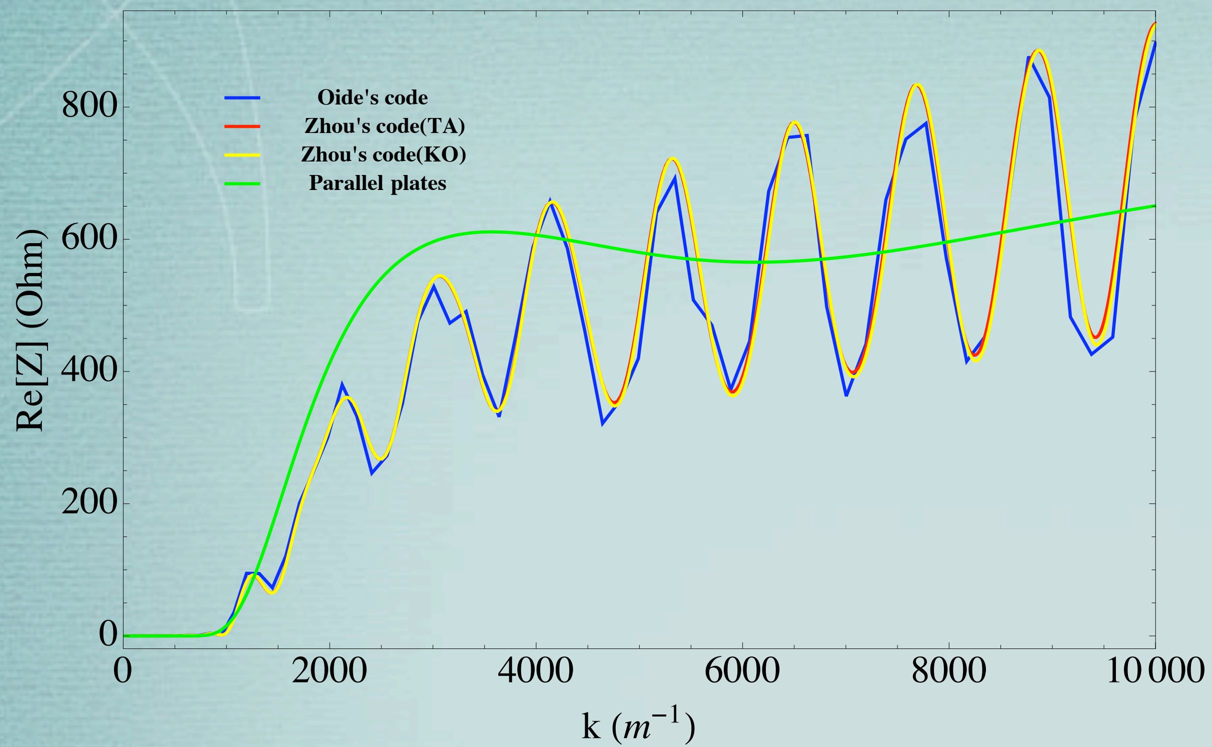
W_L(s) with σ_z=0.3mm



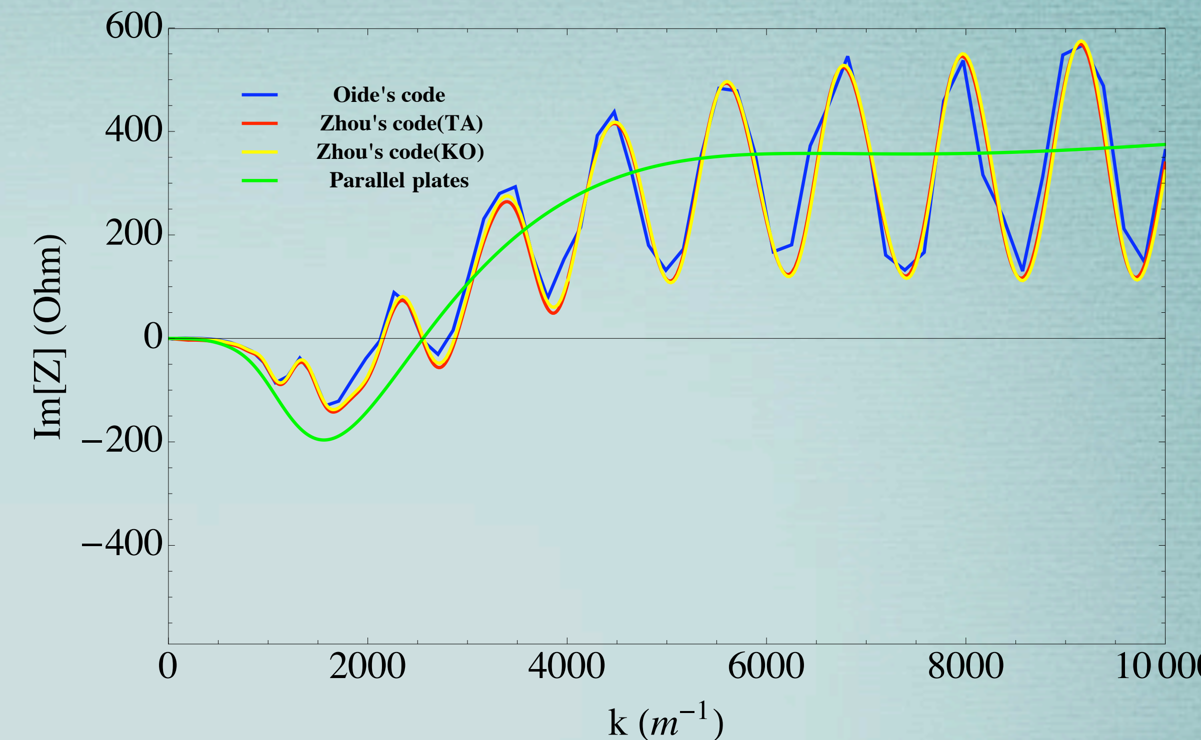
w/h=60/40mm
L_{bend}=4m
ρ=16.3m
L_{exit}=Infinity (pipe after exit)
X_{offset}=0mm

Single dipole - Benchmark example

Re.$Z_L(k)$

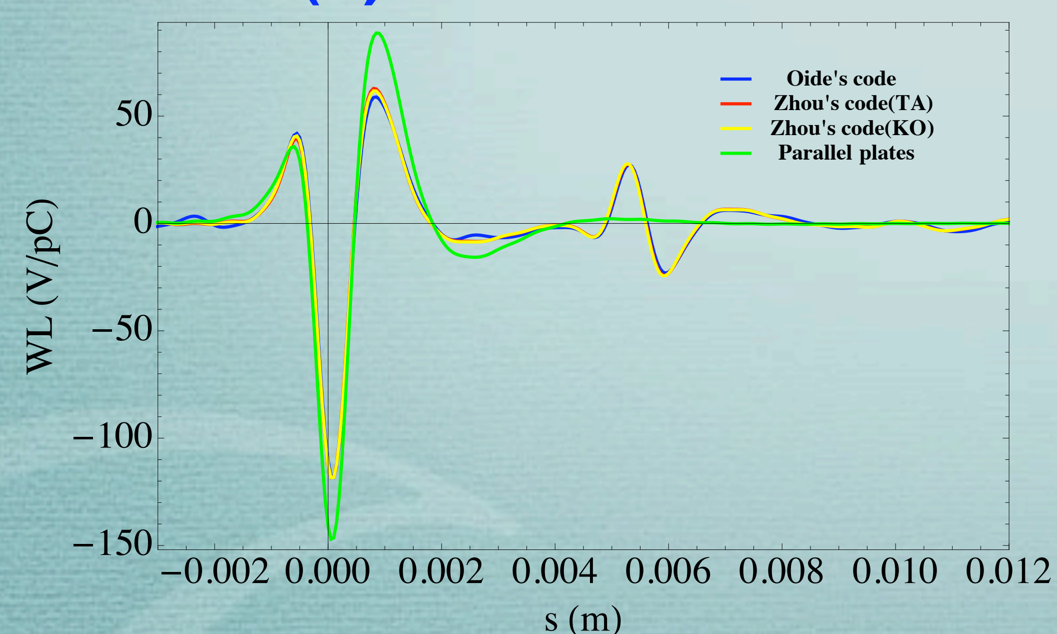


Im.$Z_L(k)$



TA: Agoh's algorithm
KO: Oide's algorithm

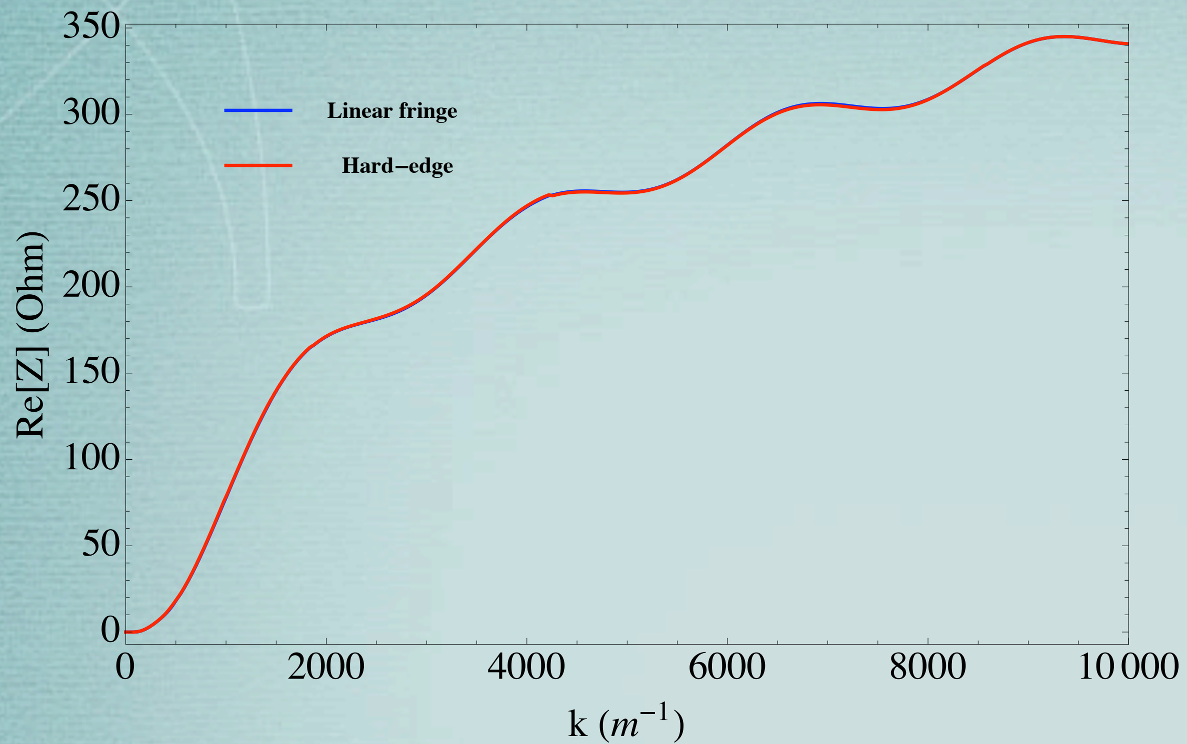
$W_L(s)$ with σ_z=0.3mm



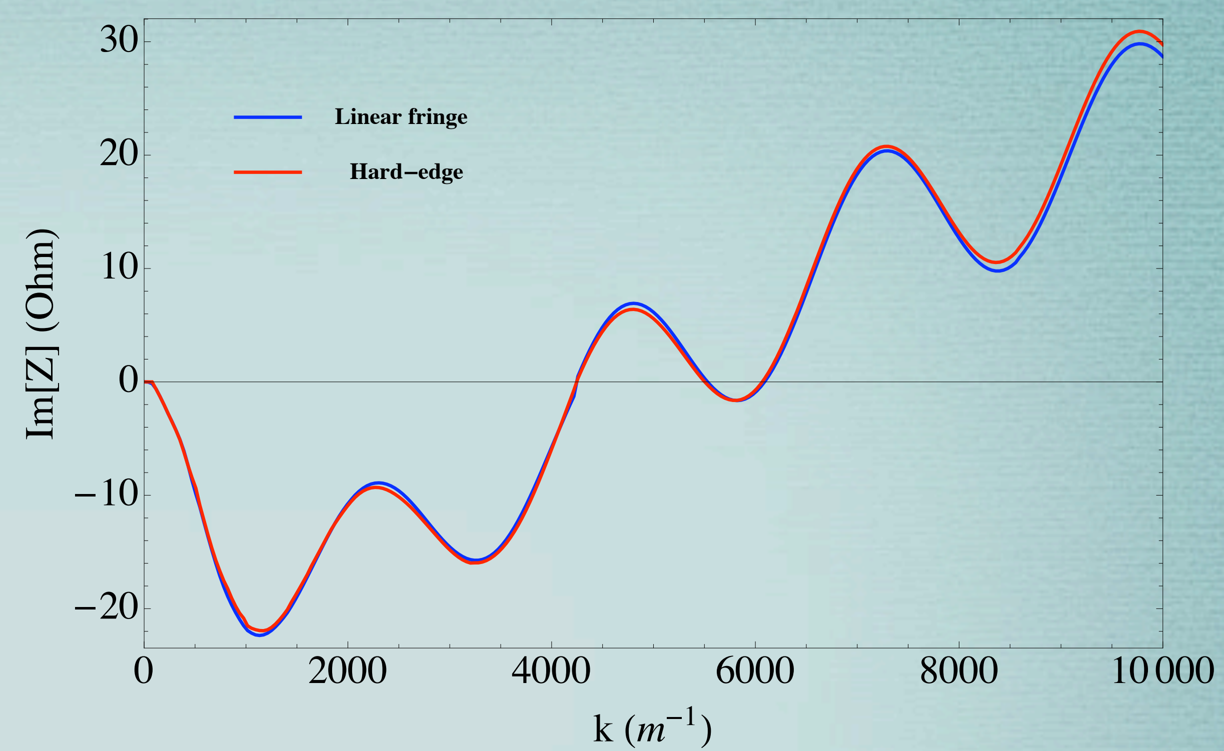
w/h=60/40mm
L_{bend}=4m
ρ=16.3m
L_{exit}=Infinity (pipe after exit)
X_{offset}=-20mm (To inner wall)

Fringe field - KEKB LER

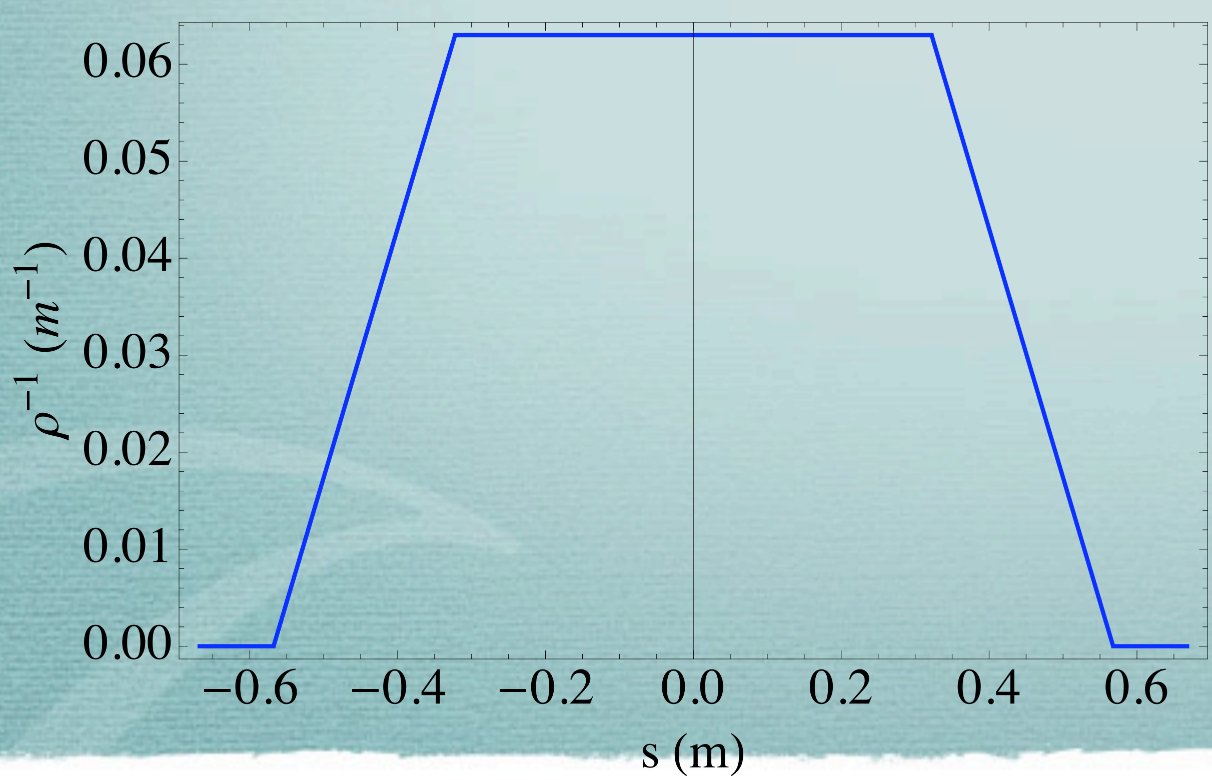
Re.Z_L(k)



Im.Z_L(k)



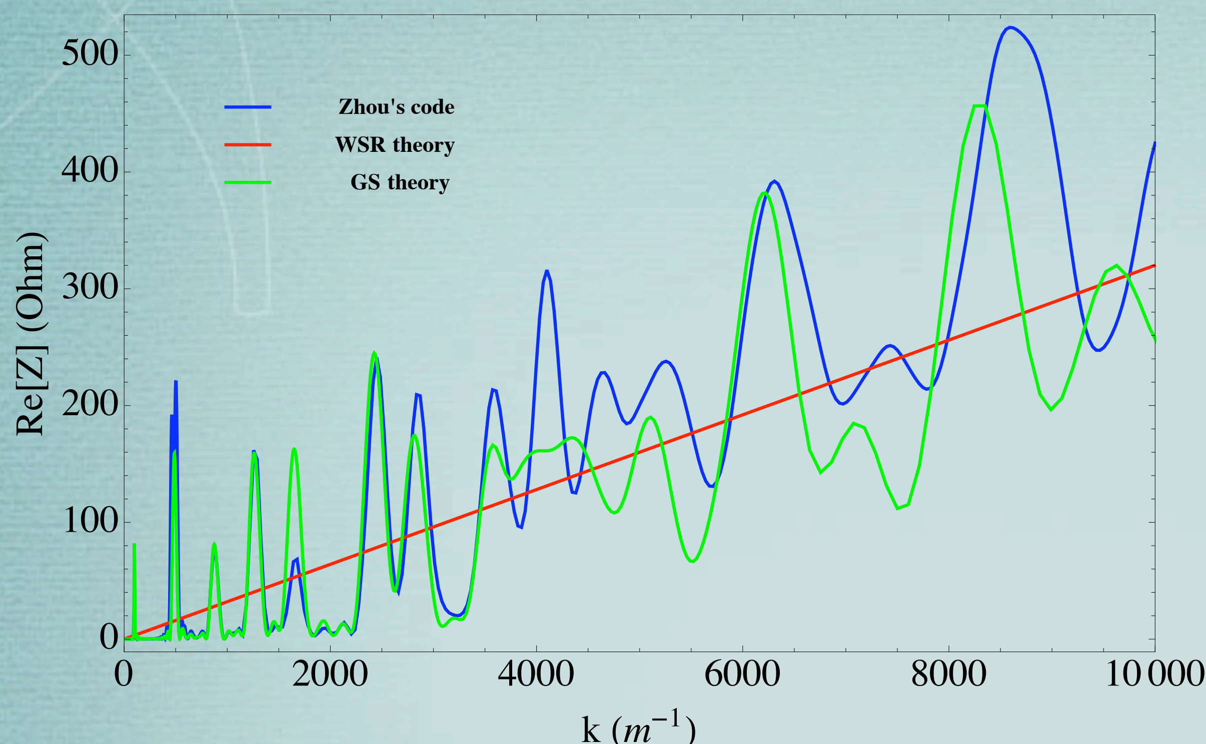
Field distribution



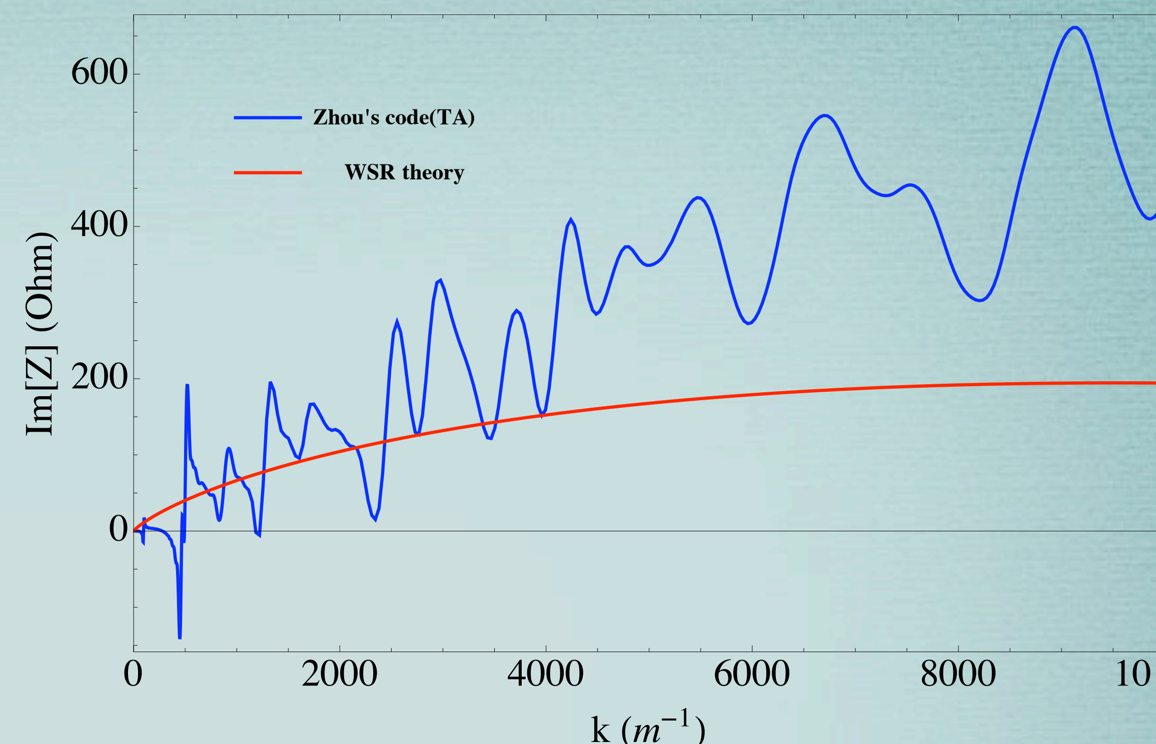
"Fringe effect" is negligible?

Wiggler - Benchmark example

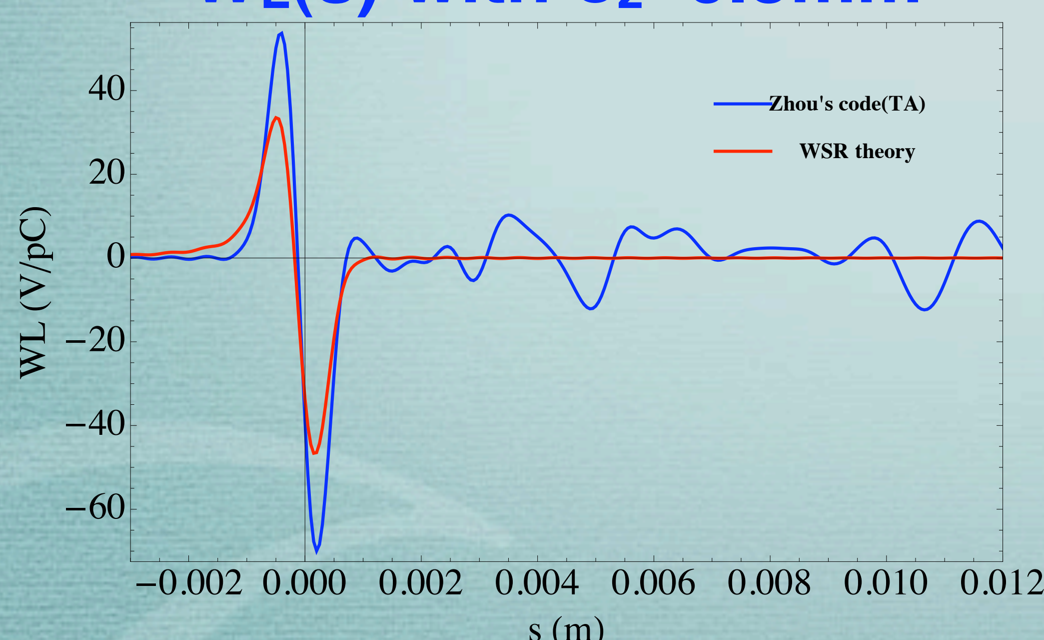
$Re.Z_L(k)$



$Im.Z_L(k)$



$W_L(s)$ with σ_z=0.3mm



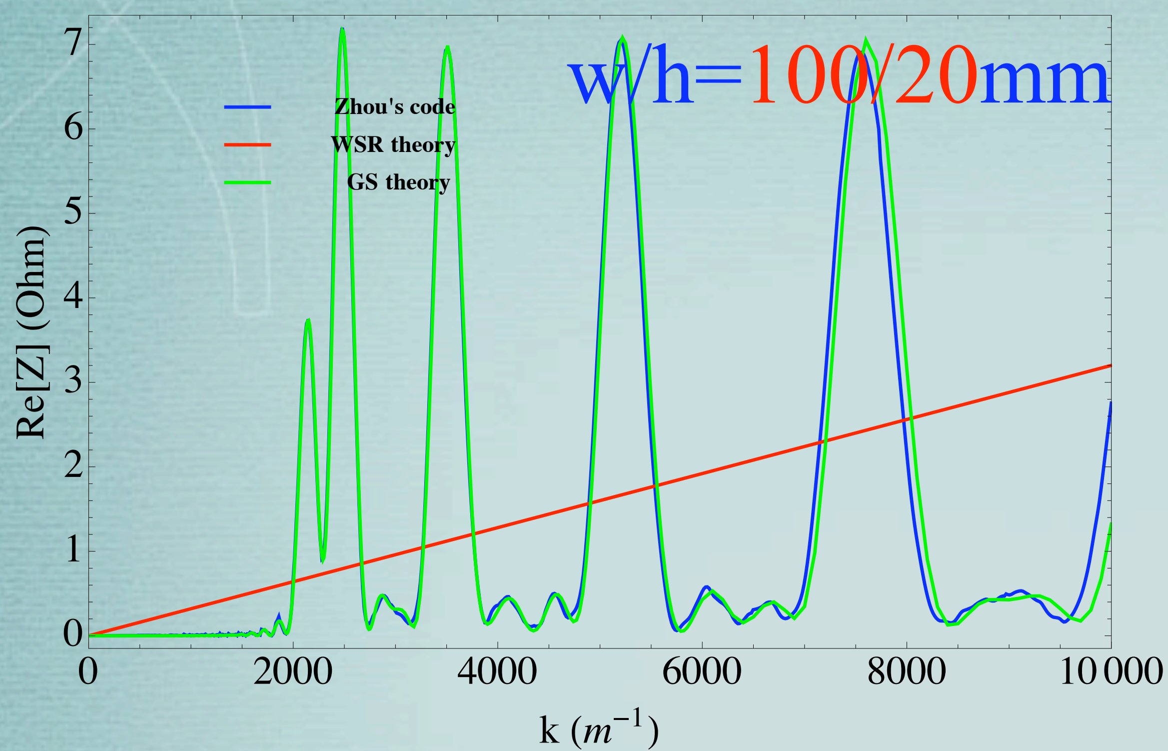
WSR: Wu-Stupakov-Raubenheimer theory
J. Wu et al., PRST-AB 6, 040701 (2003)

N_{period}=10
w/h=94/94mm
λ_w=1.088m
ρ=15.483m
L_{exit}=Infinity (pipe after exit)
Xoffset=0mm

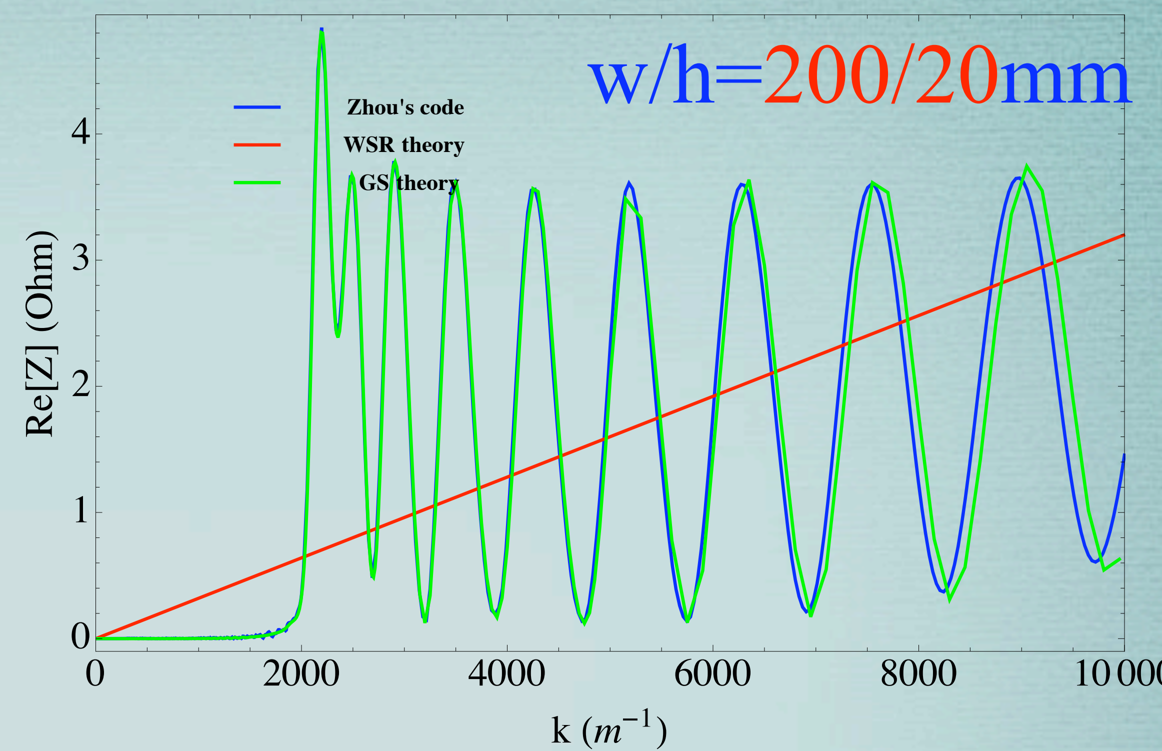
KEKB-LER type

Wiggler - Benchmark example

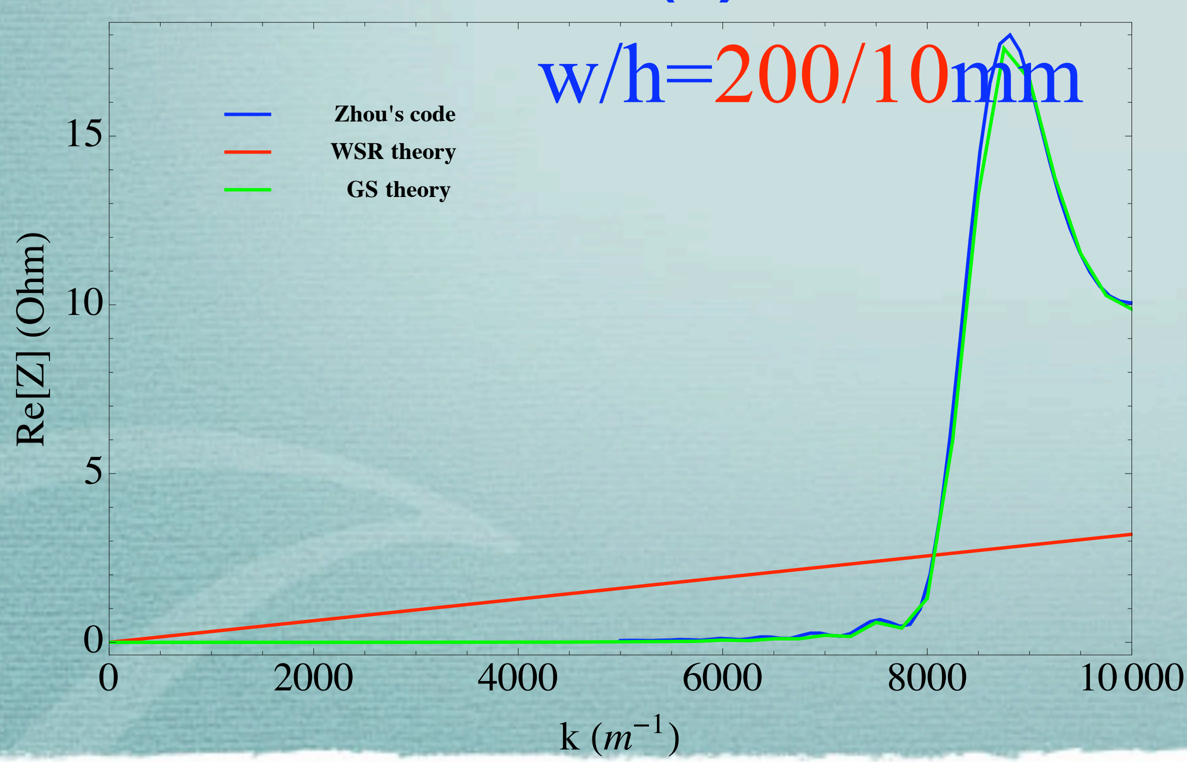
Re.Z_L(k)



Re.Z_L(k)



Re.Z_L(k)



N_{period}=10
λ_w=1.088m
ρ=154.83m
L_{exit}=Infinity (pipe after exit)
Xoffset=0mm

Wiggler - phase matching condition

$$k - pk_w - k_z = 0$$

$$k_z = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

p **: harmornic number**

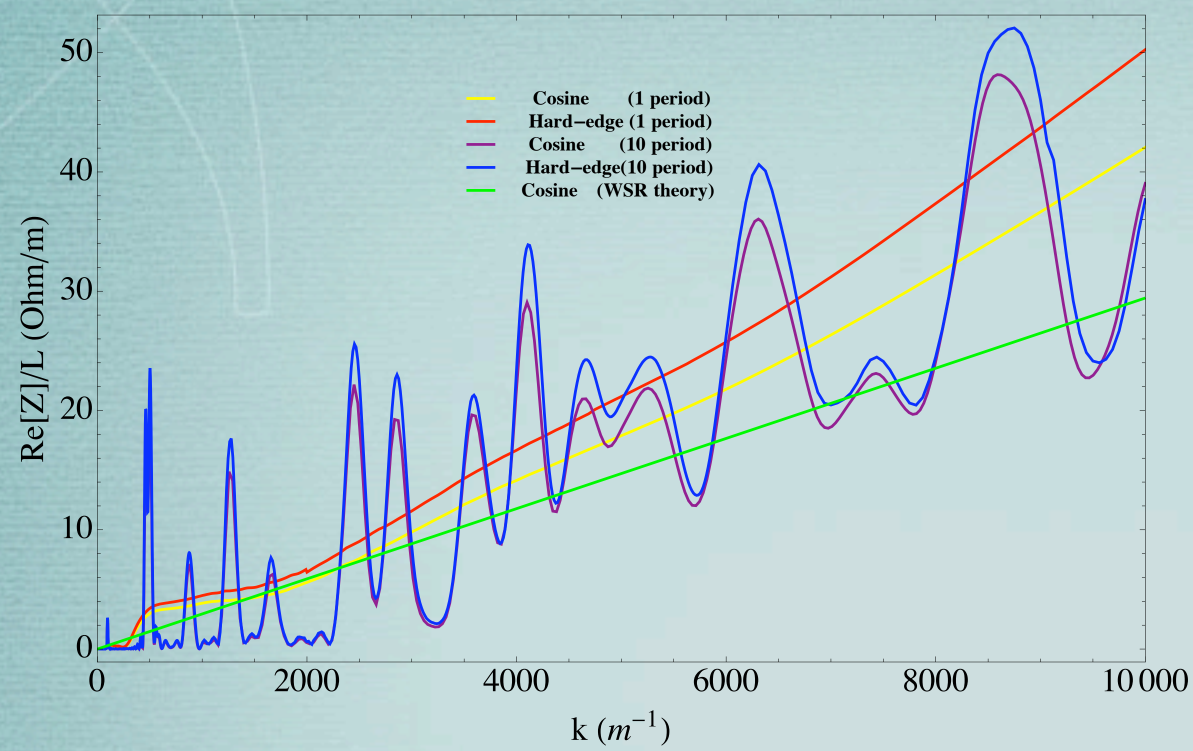
k_w**: wiggler wavenumber**

k_z **: longitudinal wavenumber of the eigenmodes in a rectangular waveguide**

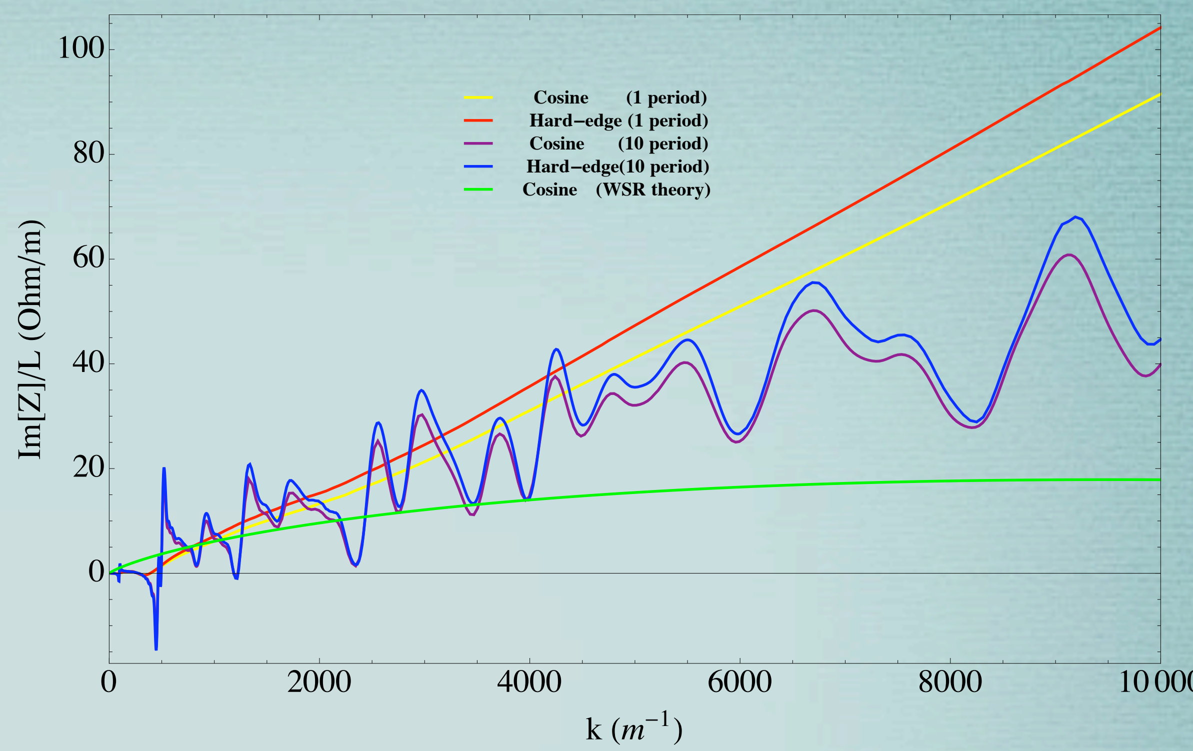
Ref. G. Stupakov and D. Zhou, SLAC-PUB-14332

Wiggler - Hard-edge approximation

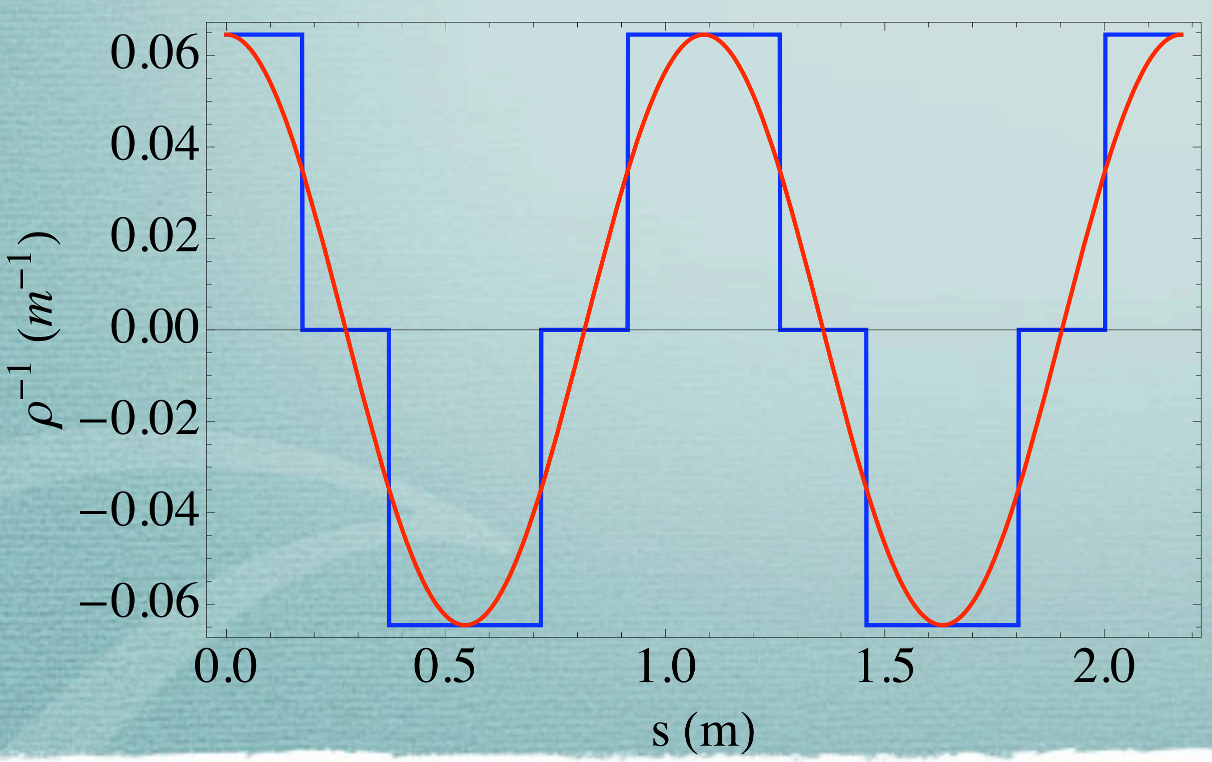
Re.Z$_L$(k)



Im.Z$_L$(k)



Field distribution



H.E. model looks to be good?

KEKB-LER

N_{period}=1/10

w/h=94/94mm

λ_w=1.088m

ρ=15.483m

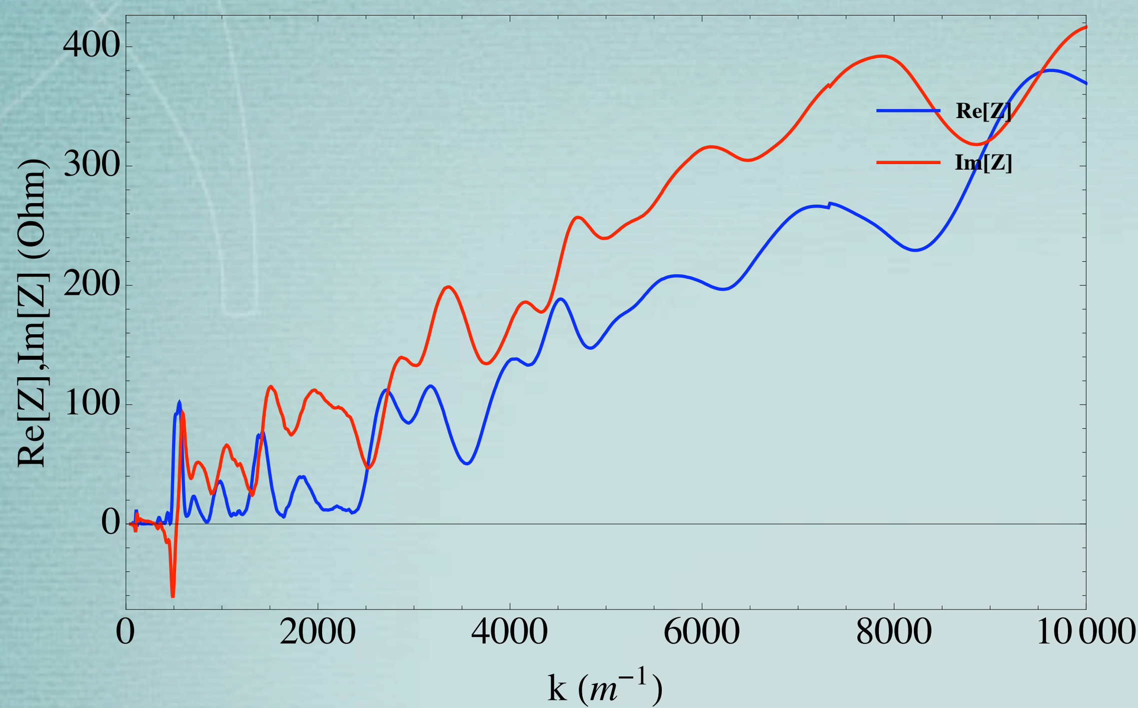
L_{exit}=Infinity (pipe after exit)

Xoffset=0mm (To inner wall)

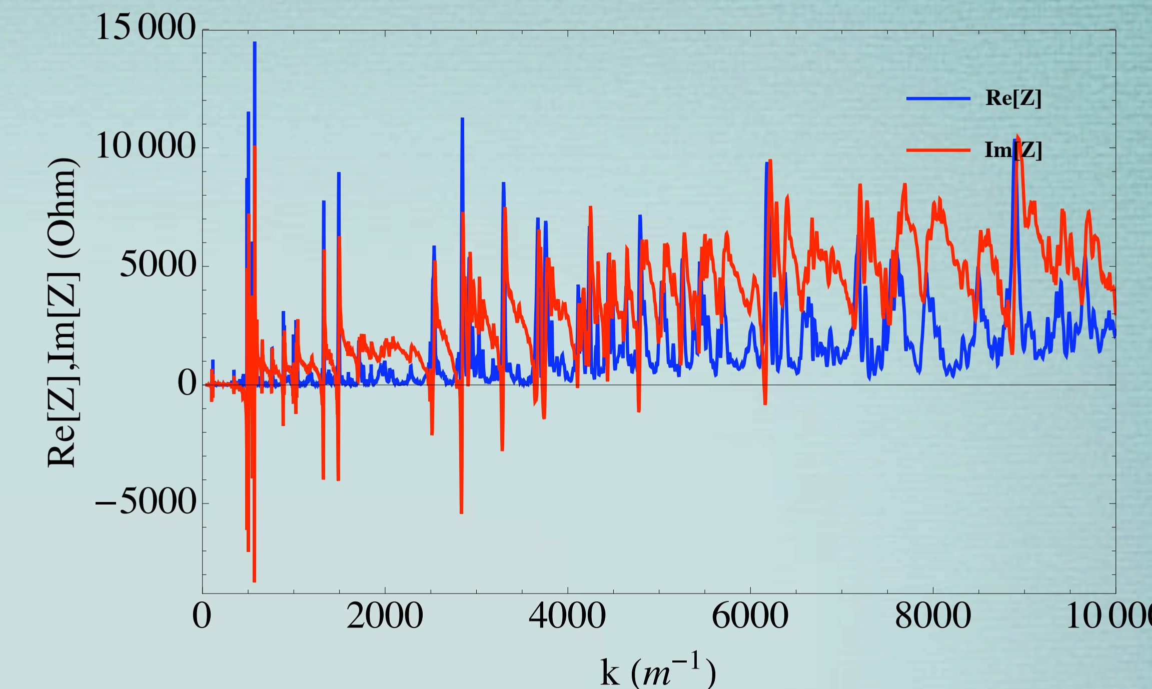
Field distribution: Cosine/H.E.

Wigglers - SuperKEKB LER

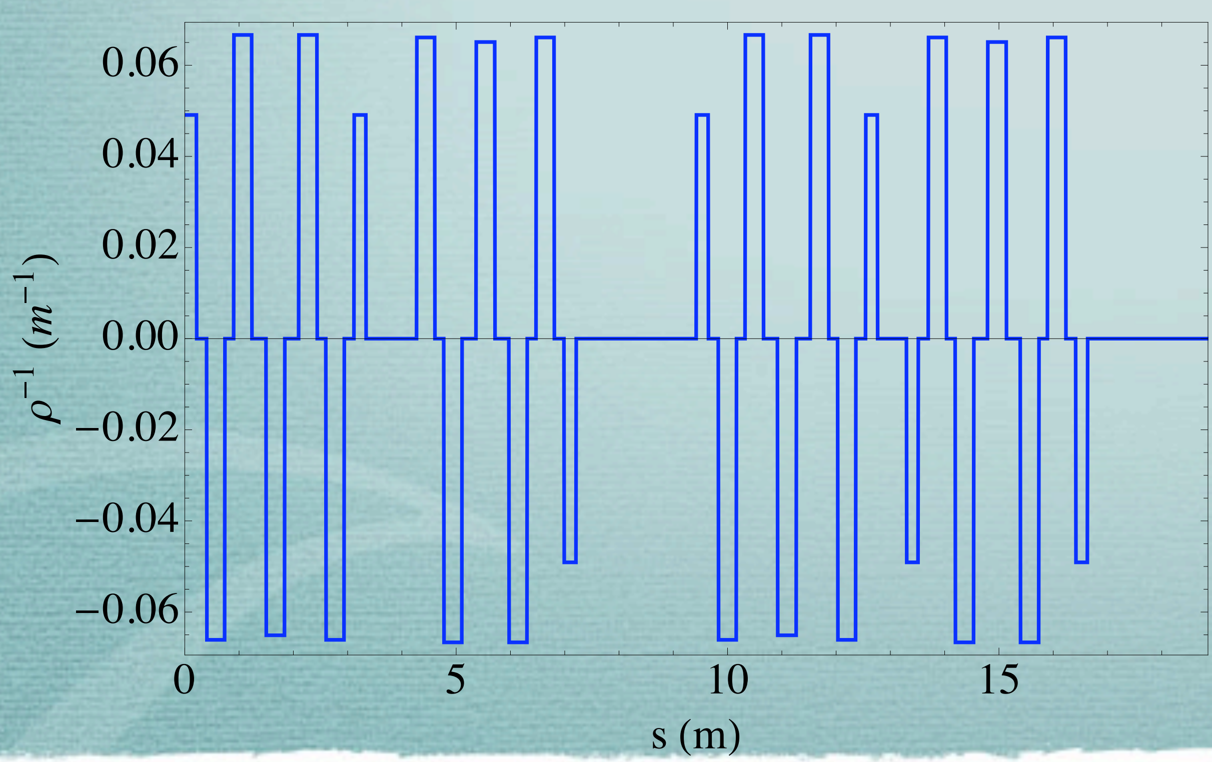
$Z_L(k)$,1 Super-period



$Z_L(k)$,15 Super-periods



Field distribution



$N_{super\text{-}period}$=1/15

w/h=90/90mm

L_w=140m

ρ≅15m

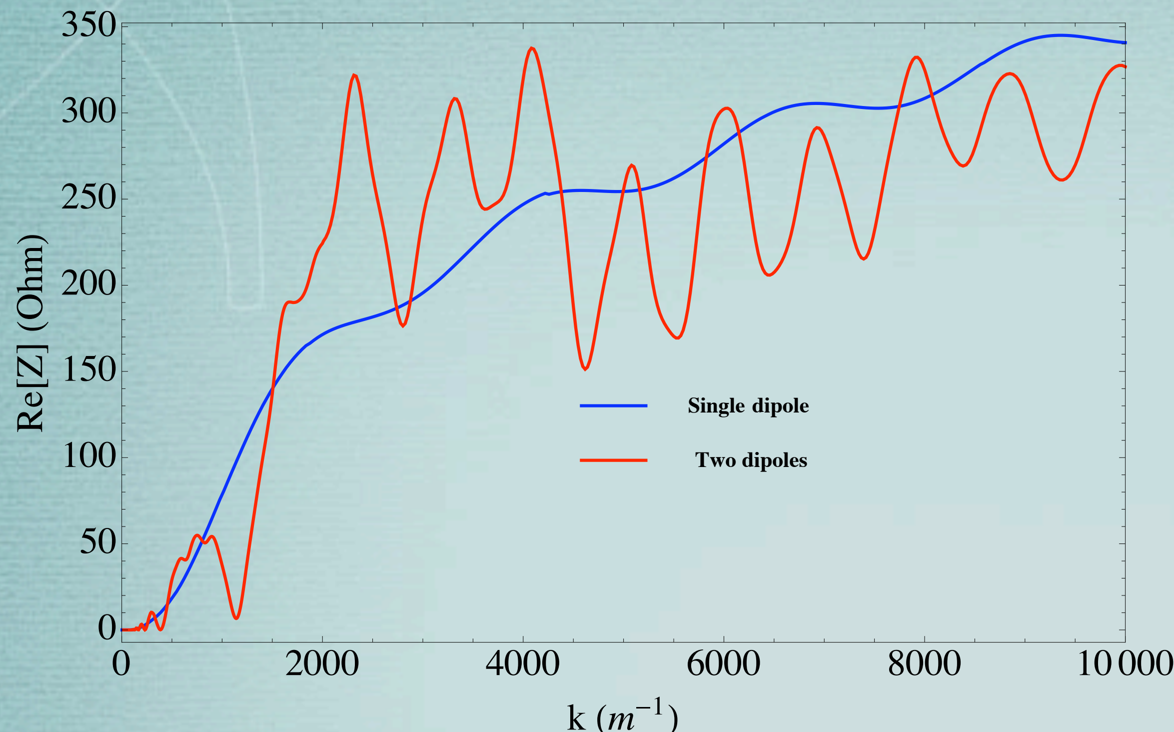
L_{exit}=Infinity (pipe after exit)

Xoffset=0mm

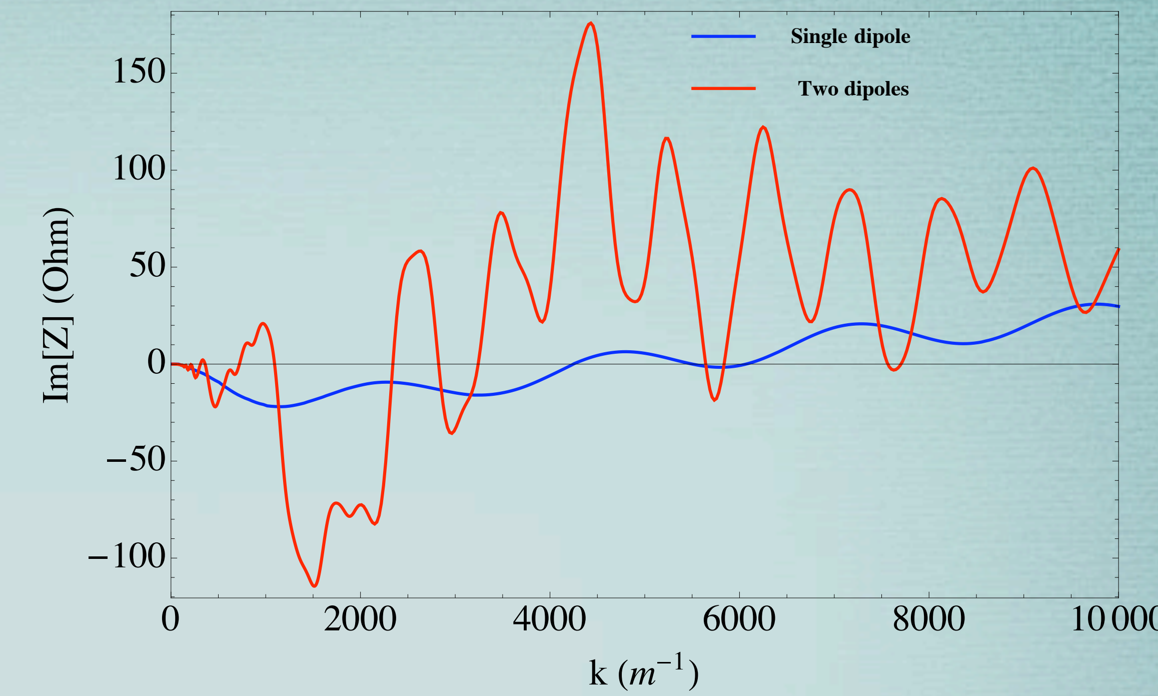
Field distribution: Hard-edge

Interference - KEKB LER

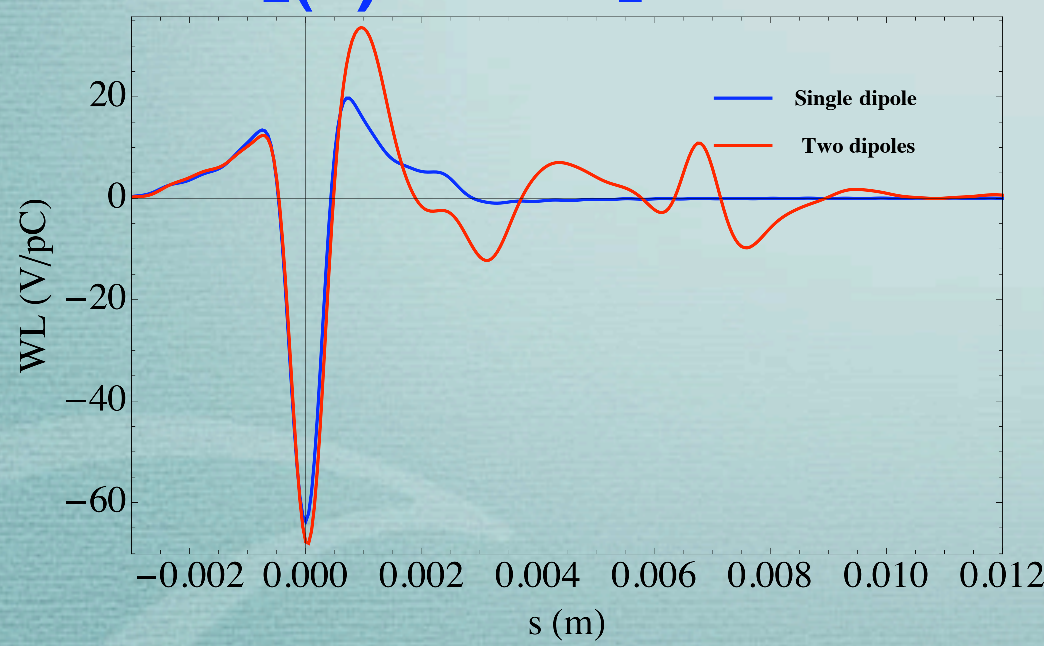
Re.Z_L(k)



Im.Z_L(k)



W_L(s) with σ_z=0.3mm



Two dipoles

w/h=94/94mm

L_{bend}=0.89m

L_{drift}=5.65m

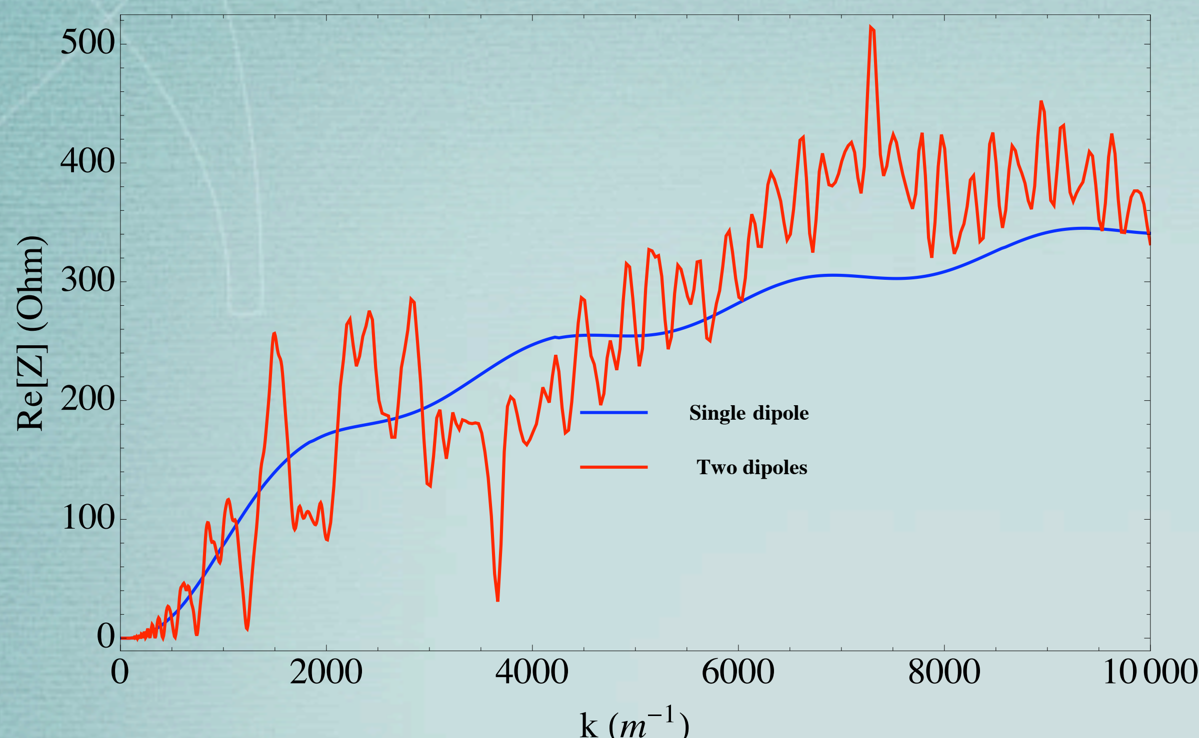
ρ=15.872m

L_{exit}=Infinity (pipe after exit)

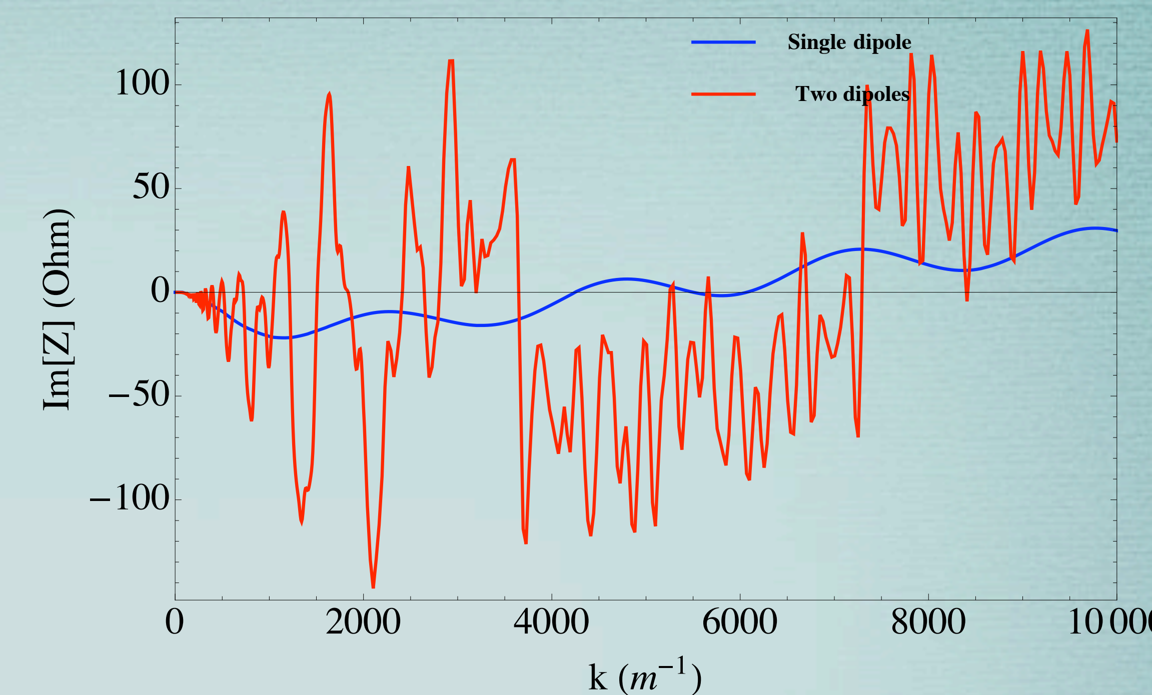
Xoffset=0mm

Interference - KEKB LER

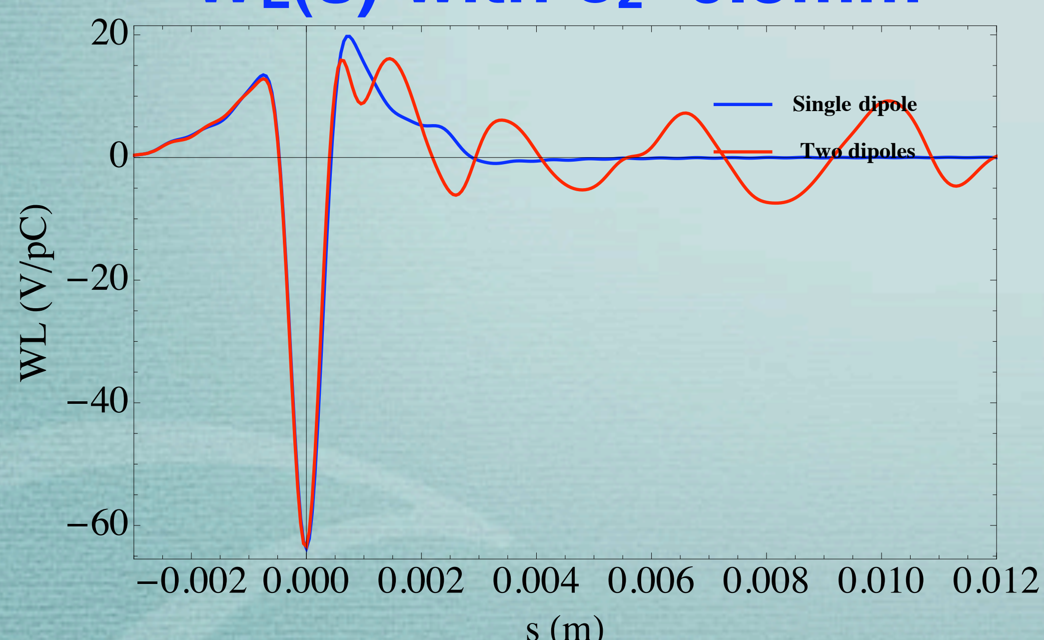
Re.Z_L(k)



Im.Z_L(k)



W_L(s) with σ_z=0.3mm



Two dipoles

w/h=94/94mm

L_{bend}=0.89m

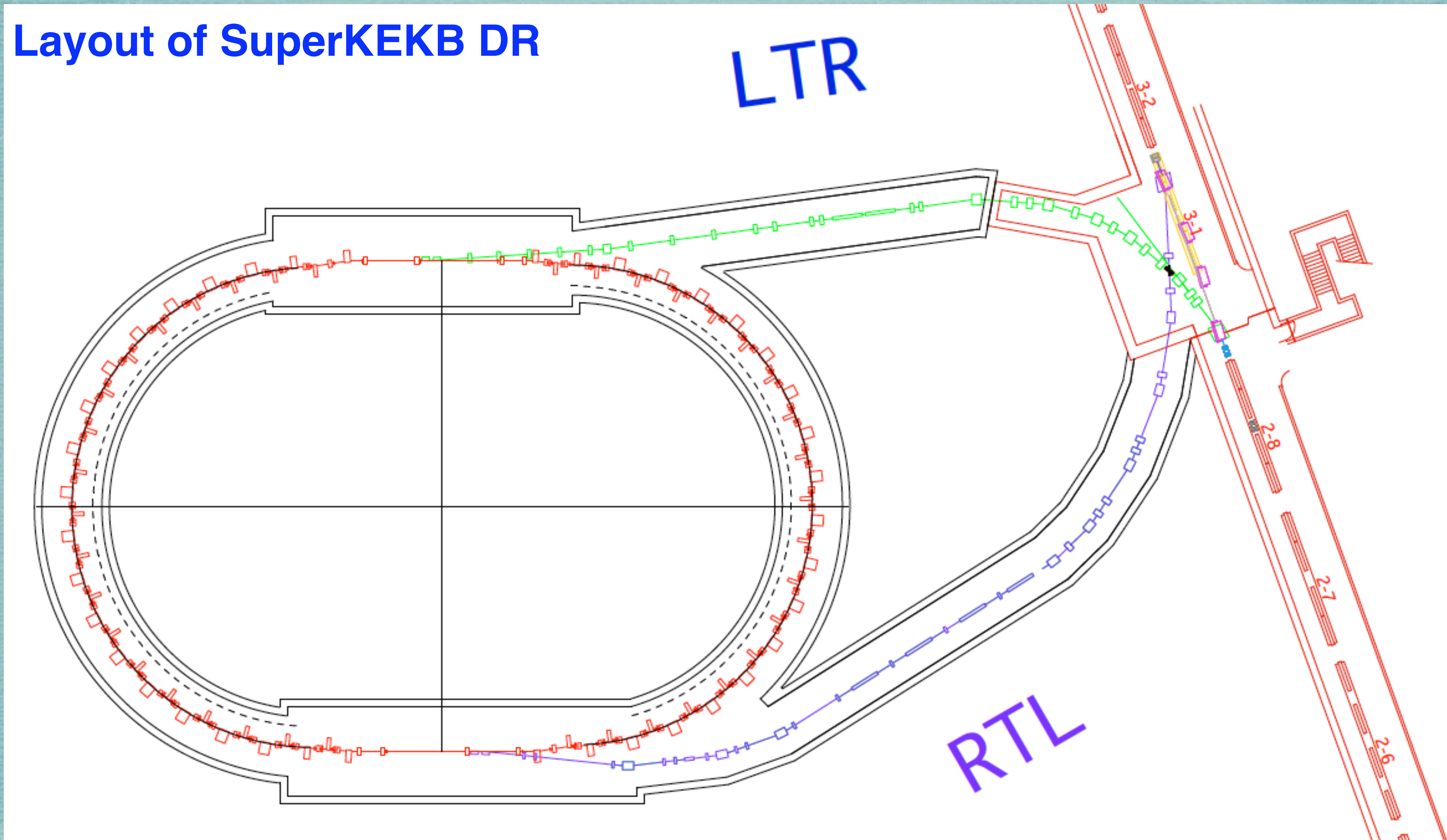
L_{drift}=20m

ρ=15.872m

L_{exit}=Infinity (pipe after exit)

Xoffset=0mm

Interference - SuperKEKB DR



M. Kikuchi

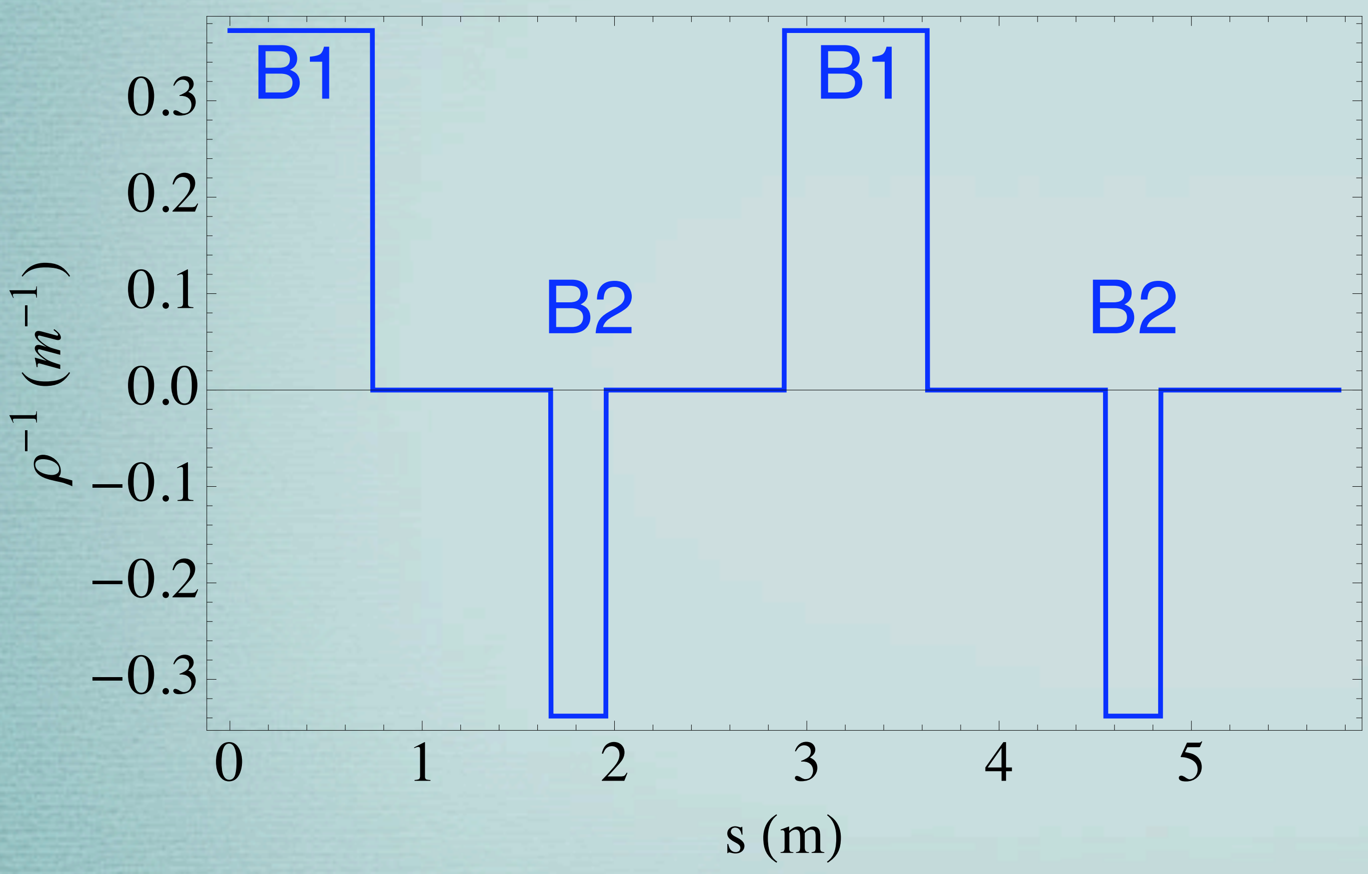
Interference - SuperKEKB DR

SuperKEKB DR parameters

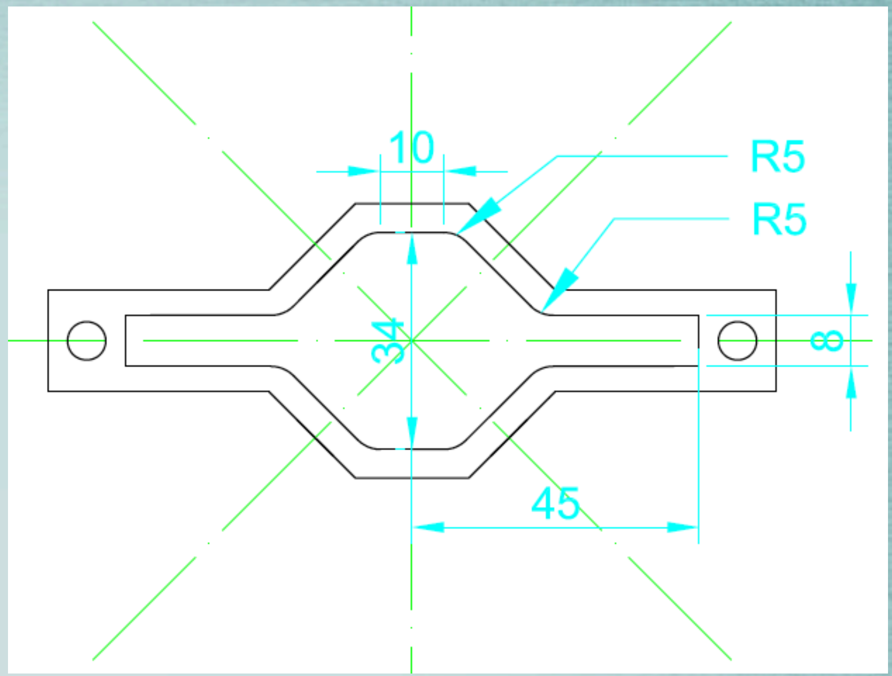
parameter	Value
Beam energy (GeV)	1.1
Circumference (m)	135.502
Bunch Length (mm)	11.1
Rel. Energy spread (10^{-4})	5.53
Beam pipe height in bends (mm)	34
Beam pipe width in bends w/o antechamber (mm)	34
Effective Length of bends (B1/B2/B3/B4)	0.74248/0.28654/0.39208/.47935
Number of bends (B1/B2/B3/B4)	32/38/4/4
Bending radius (m) (B1/B2/B3/B4)	2.68/2.96/3.15/3.15

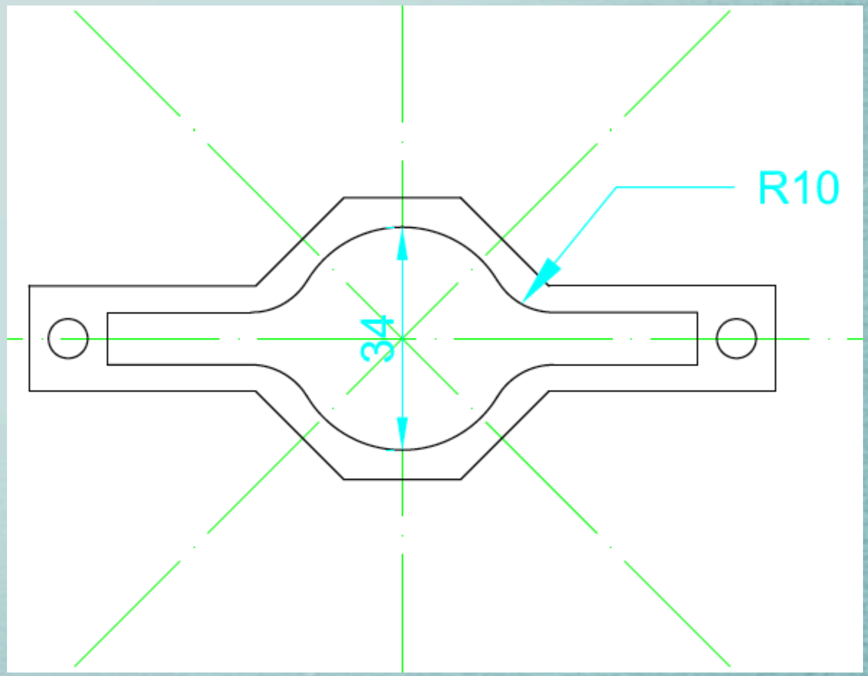
Interference - SuperKEKB DR

Field distribution (2 cells)
"Reverse-bend FODO"



Vacuum chamber (candidates)

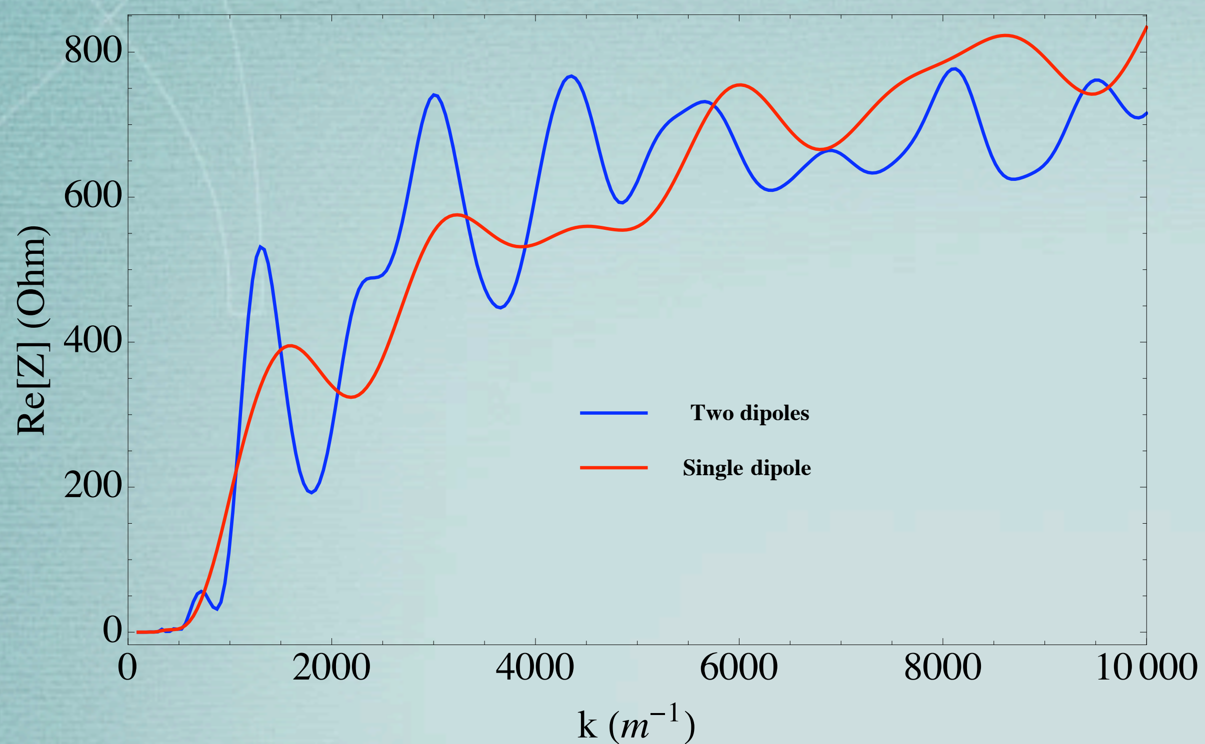




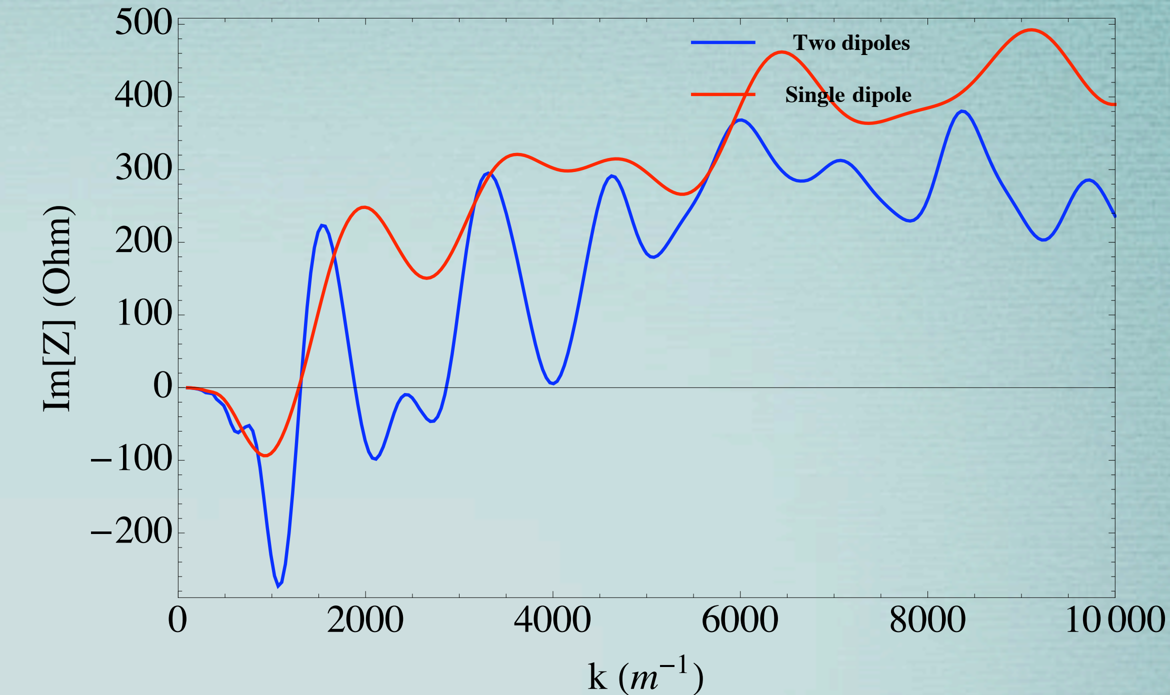
K. Shibata

Interference - SuperKEKB DR

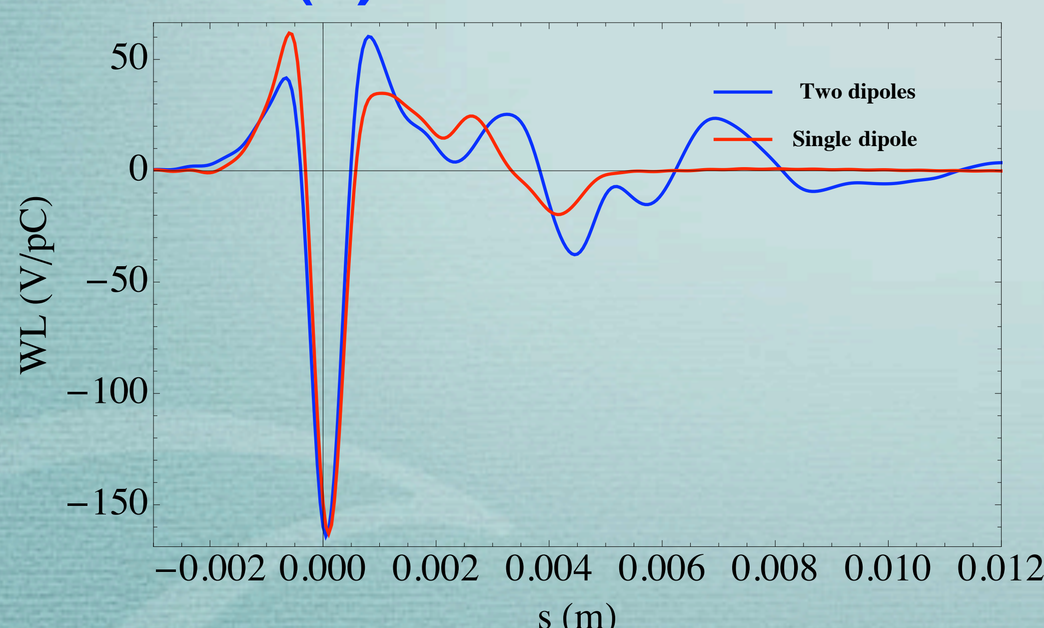
Re.Z_L(k)



Im.Z_L(k)



W_L(s) with σ_z=0.3mm



1 cell

w/h=34/34mm

B1+B2

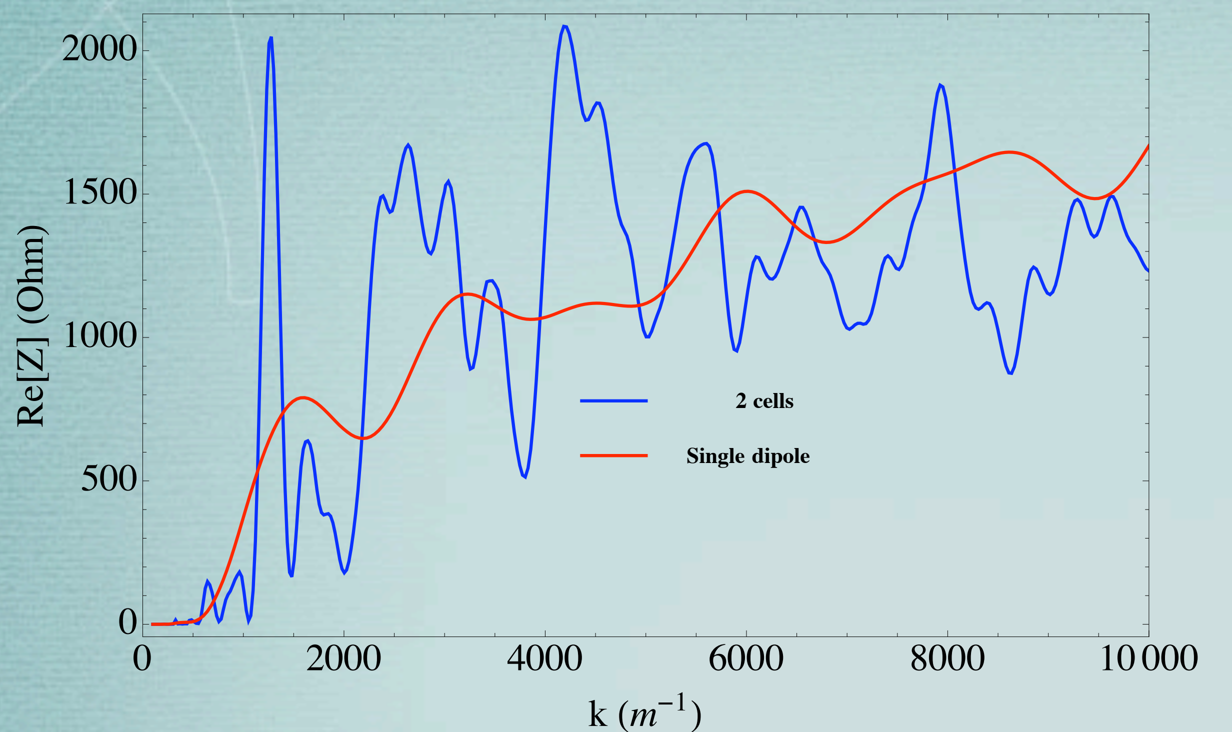
L_{drift}=0.93m

L_{exit}=Infinity (pipe after exit)

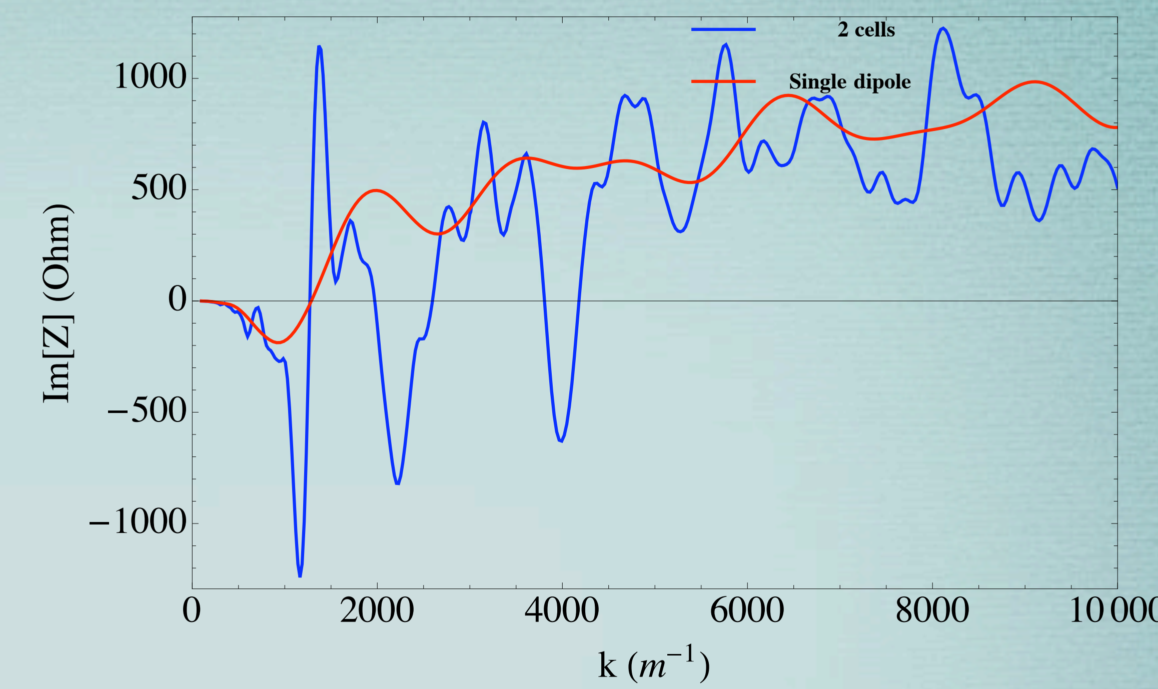
Xoffset=0mm

Interference - SuperKEKB DR

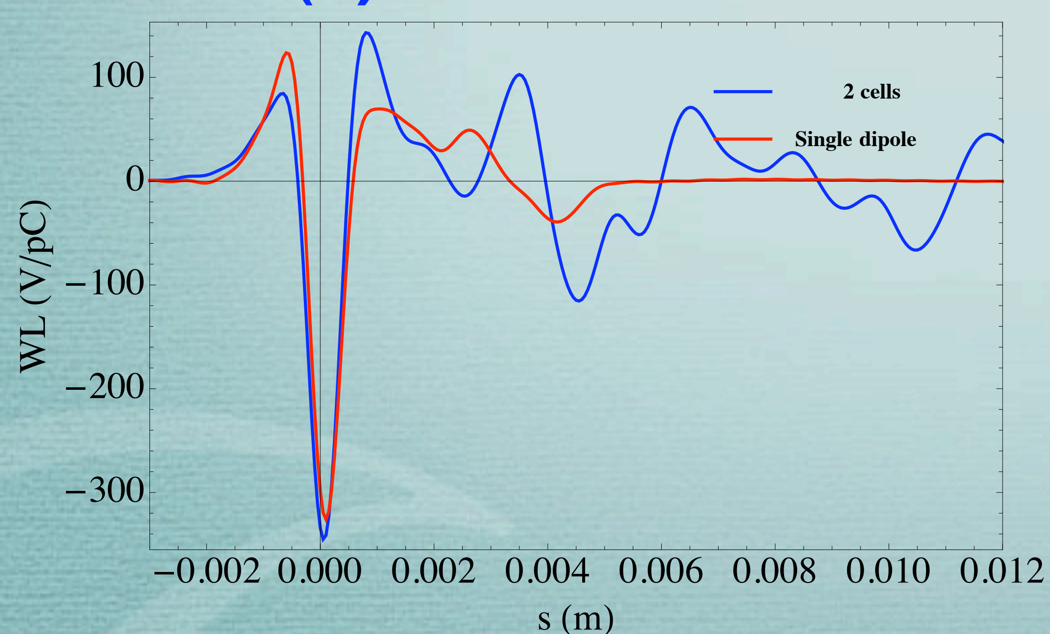
$Re.Z_L(k)$



$Im.Z_L(k)$



$W_L(s)$ with σ_z=0.3mm



2 cells

w/h=34/34mm

2×(B1+B2)

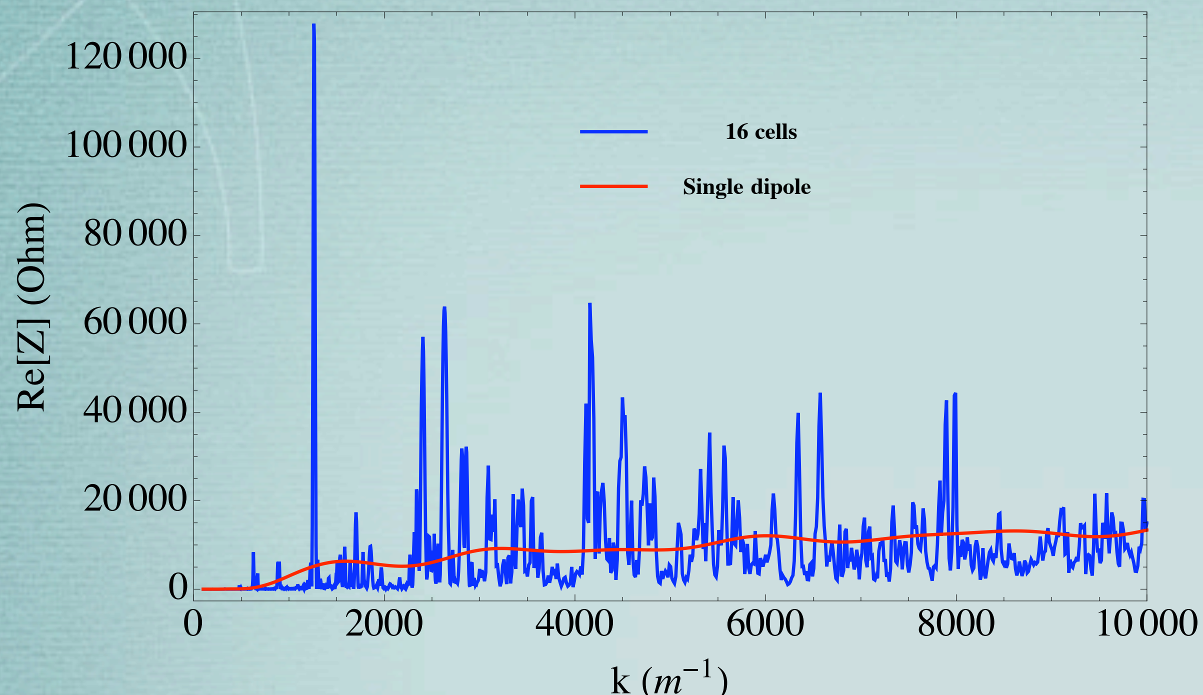
L_{drift}=0.93m

L_{exit}=Infinity (pipe after exit)

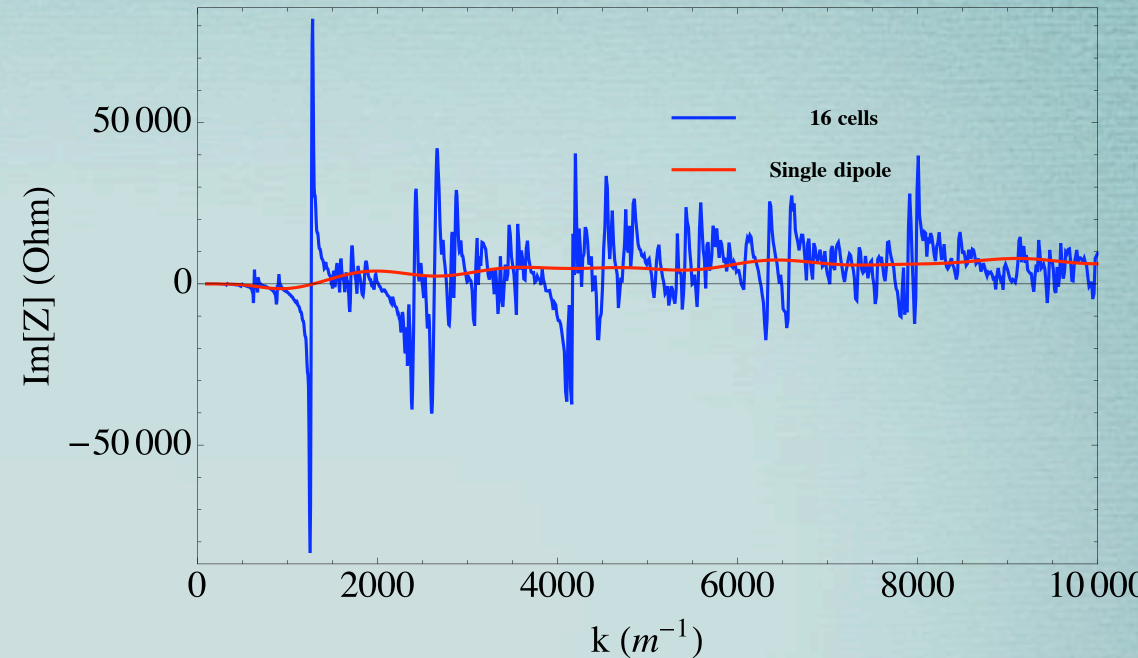
Xoffset=0mm

Interference - SuperKEKB DR

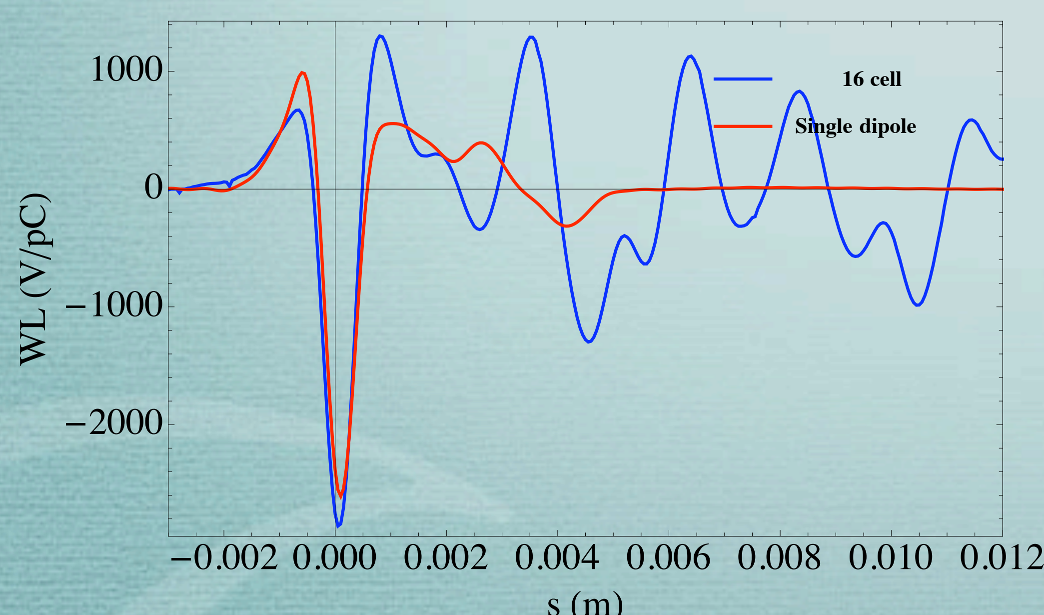
Re.$Z_L(k)$



Im.$Z_L(k)$



$W_L(s)$ with σ_z=0.3mm



16 cells

w/h=34/34mm

16×(B1+B2)

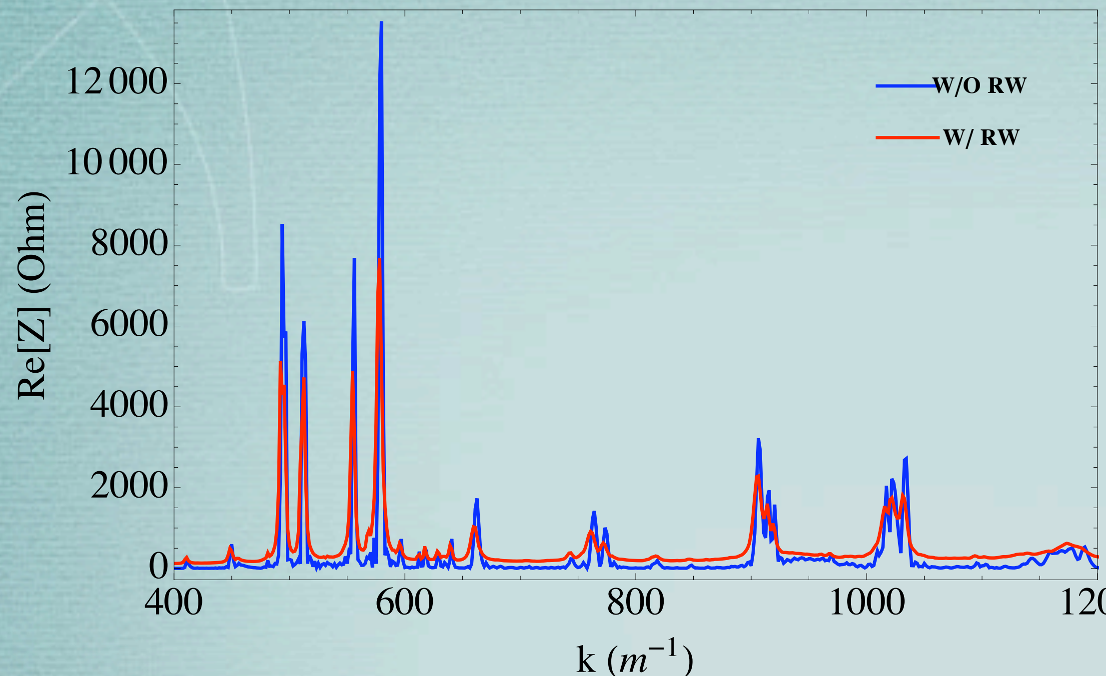
L_{drift}=0.93m

L_{exit}=Infinity (pipe after exit)

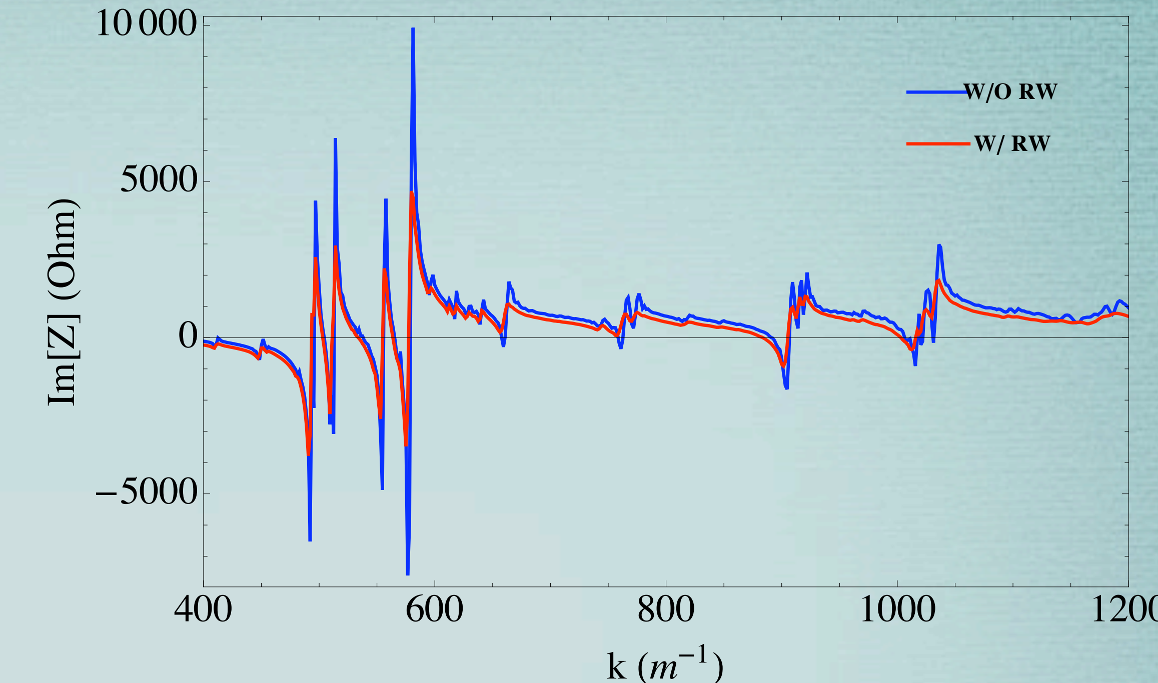
Xoffset=0mm

Resistive wall - SuperKEKB LER wiggler section

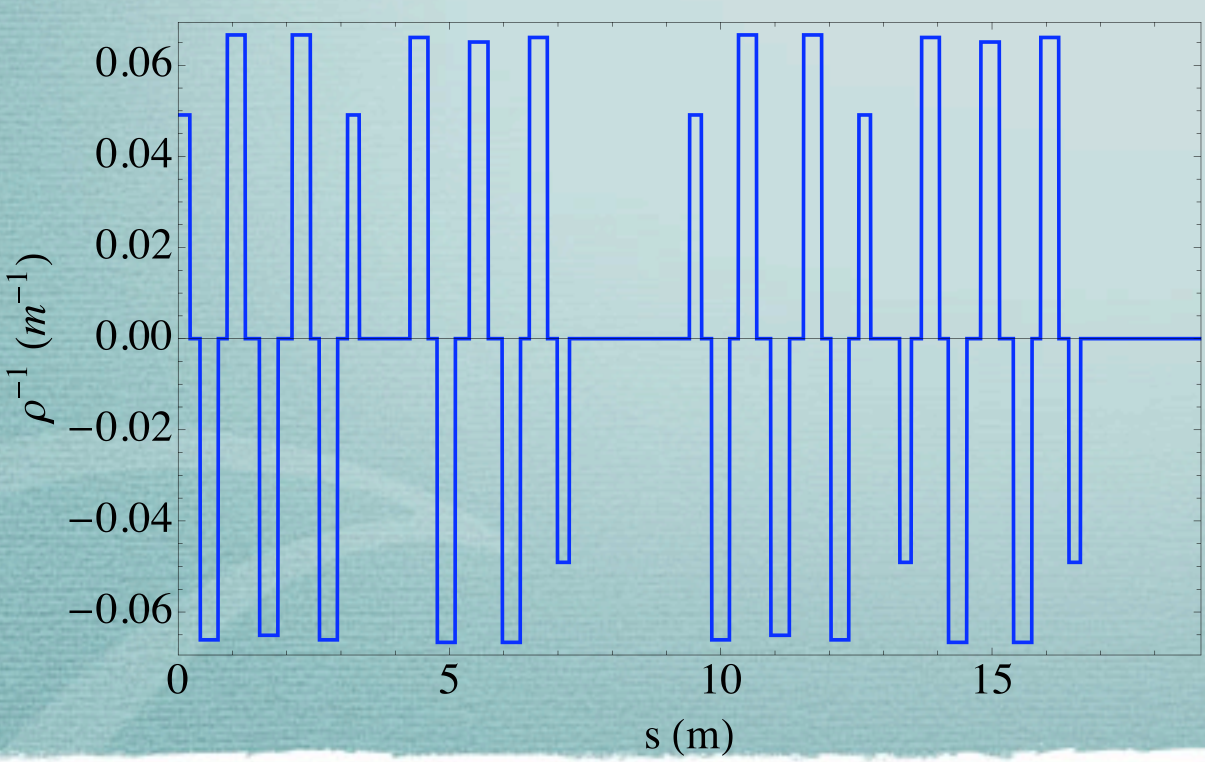
Re.Z_L(k)



Im.Z_L(k)



Field distribution



$N_{super\text{-}period}$=15
w/h=90/90mm
L_W=140m
ρ≅15m
L_{exit}=Infinity (pipe after exit)
Xoffset=0mm
Field distribution: Hard-edge

Summary

1. Features of the new CSR code (CSRZ):

1.1 Low noise level

1.2 Allow for s-dependent bending radius (fringe field, wigglers, interference between consecutive dipoles)

1.3 Allow for resistive wall (to be benchmarked)

2. Findings

2.1 Narrow-band impedances (spikes) due to CSR in wigglers were observed

2.2 Interference between consecutive dipoles can be significant and lead to narrow-band CSR impedances (to be benchmarked)

2.3 In the SuperKEKB project, CSR is still an important issue (beam instabilities not discussed in this talk)

3. Problems to be solved

3.1 Computing time is not quite acceptable at high freq. or very long components which require refinements in meshes or huge integration steps

3.2 “Wiggling pipe” is not a good approximation

3.3 A new code to treat the pipe with arbitrary cross section is needed.