CSR calculations using mesh method

Demin Zhou

Thanks to: H. Hama, K. Ohmi, T. Agoh, G. Stupakov, K. Yokoya, K. Oide, M. Kikuchi, H. Ikeda, N. Iida, S. Kramer, ...

ビーム物理研究会2011, 東北大学電子光理学研究センター

Dec. 09, 2011

Outline

1. Introduction

2. CSR field dynamics

3. CSR in SuperKEKB DR

4. CSR in NSLS VUV ring

5. Summary

1. Introduction - Incoherent and coherent SR

Retarded solution for the fields of moving point charge in free space:

$$\vec{E} = \frac{e}{4\pi\epsilon_0}\left\{\frac{\vec{n}-\vec{\beta}}{\gamma^2 K^3 \left|\vec{r}-\vec{r}'\right|^2} + \frac{\vec{n}\times\left[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{cK^3\left|\vec{r}-\vec{r}'\right|}\right\}_{ret} \qquad K = 1-\vec{n}\cdot\vec{\beta}$$

Angular and spectral distribution of the radiation energy by a point charge:

$$\frac{d^2W}{d\Omega dk} = \frac{e^2}{4\pi\epsilon_0}\frac{k^2c^2}{4\pi^2}\left|\int_{-\infty}^{\infty}\vec{n}\times\left(\vec{n}\times\vec{\beta}\right)e^{ik(ct-\vec{n}\cdot\vec{r}(t))}dt\right|^2$$

$$k \equiv \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

Energy spectrum:

$$\frac{dW}{dk} = \frac{e^2}{4\pi\epsilon_0}\sqrt{3}\gamma\frac{k}{k_c}\int_{k/k_c}^{\infty}K_{5/3}(x)dx$$

1. Introduction - Incoherent and coherent SR (cont'd)

In the limit of $k \ll k_c = 3\gamma^3/(2R)$

$$\frac{dW}{dk} = \frac{e^2}{2\pi\epsilon_0} 3^{1/6}\Gamma\left(2/3\right)\left(kR\right)^{1/3}$$

SR impedance per unit length:

$$\frac{\mathrm{Re}Z_{\parallel SR}(k)}{L} = \frac{1}{2\pi R}\frac{\pi}{e^2 c}\frac{dW(k)}{dk} = \frac{Z_0}{4\pi}3^{1/6}\Gamma(2/3)\left(\frac{k}{R^2}\right)^{1/3}$$

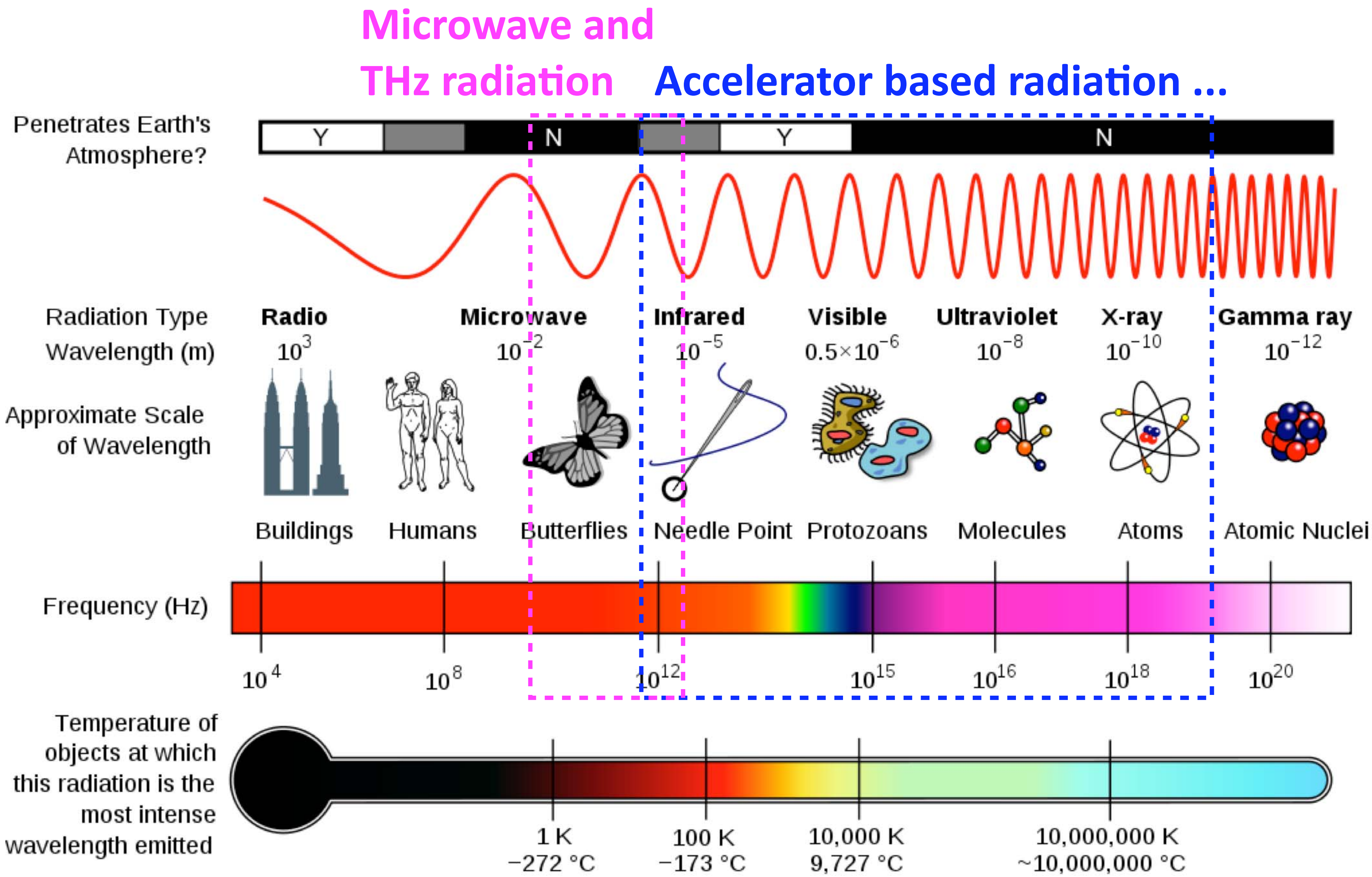
Coherent SR (assuming full transverse coherence):

$$\left.\frac{dW}{dk}\right|_{bunch} = \left[N + N(N-1)\left|\tilde{\lambda}(k)\right|^2\right]\frac{dW}{dk}$$

Wanted and Unwanted ...

Ref. 田中さんの講演, this meeting

1. Introduction - Incoherent and coherent SR (cont'd)



Copied from Wikimedia, Source: http://commons.wikimedia.org/wiki/File:EM_Spectrum_Properties_edit.svg

1. Introduction - Field equations

Parabolic equation in Frenet-Serret coordinate system: $a/R \ll 1$

$$\frac{\partial \vec{E}_\perp}{\partial s} = \frac{i}{2k}\left[\nabla_\perp^2 \vec{E}_\perp - \frac{1}{\epsilon_0}\nabla_\perp \rho_0 + 2k^2\left(\frac{x}{R(s)} - \frac{1}{2\gamma^2}\right)\vec{E}_\perp\right]$$

Longitudinal field:

$$E_s = \frac{i}{k}\left(\nabla_\perp \cdot \vec{E}_\perp - \mu_0 c J_s\right) \qquad J_s = \rho_0 c$$

Longitudinal impedance:

$$k \equiv \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$Z(k) = -\frac{1}{q}\int_0^\infty E_s(x_c, y_c)ds$$

Field separation:

$$\vec{E}_\perp = \vec{E}_\perp^r + \vec{E}_\perp^b \longrightarrow \frac{\partial \vec{E}_\perp^r}{\partial s} = \frac{i}{2k}\left[\nabla_\perp^2 \vec{E}_\perp^r + 2k^2\left(\frac{x}{R(s)} - \frac{1}{2\gamma^2}\right)\left(\vec{E}_\perp^r + \vec{E}_\perp^b\right)\right]$$

 T. Agoh and K. Yokoya, PRST-AB 7, 054403 (2004).

1. Introduction - Chamber model

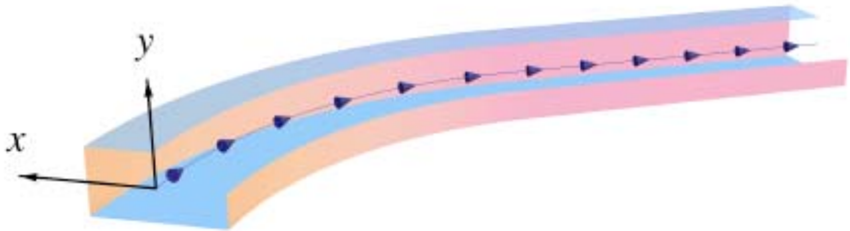
1). The curvature is variable:

Dipoles interleaved with drifts, wiggler, etc.

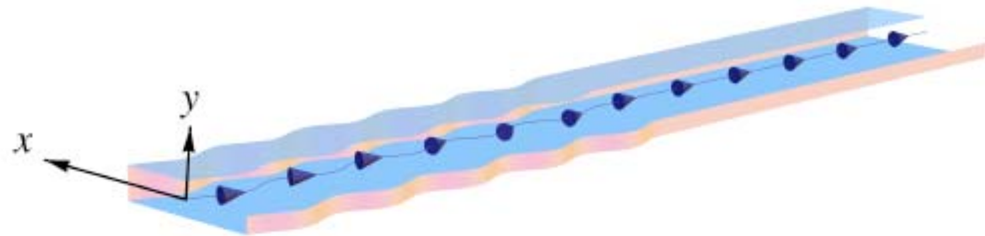
2). Chamber cross-section along the beam orbit:

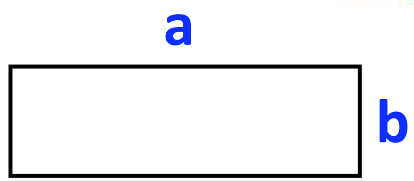
Uniform rectangular cross-section (2D)

Single dipole



Wiggler - "Wiggling pipe"





G. Stupakov and D. Zhou, SLAC-PUB-14332

D. Zhou, PhD thesis, KEK, 2011

Beam line: Line=(Bend)

=(Bend1, Bend2, ...)

=(Bend, Drift)

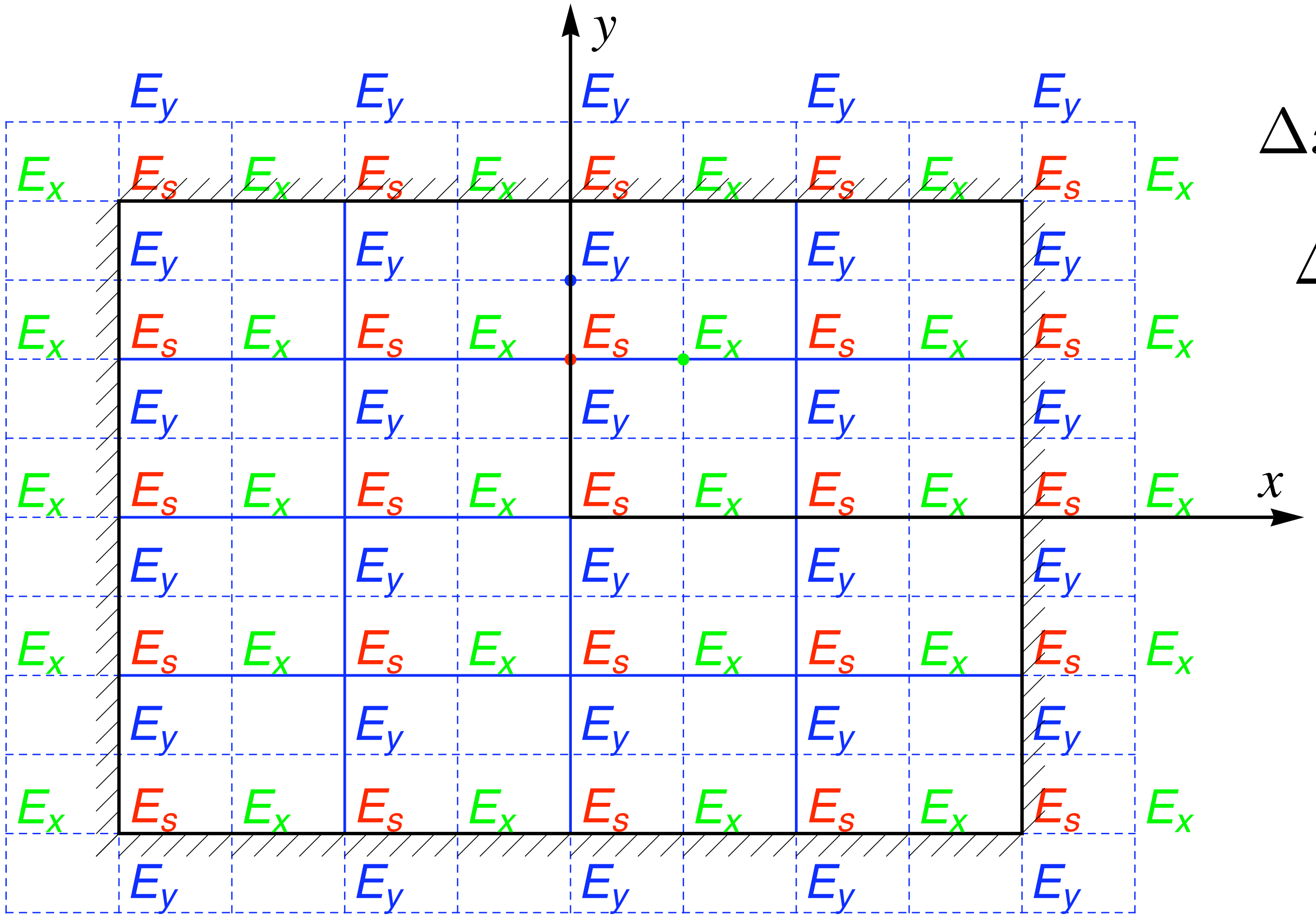
=(Bend1, Drift1, Bend2, Drift2, ...)

1. Introduction - Numerical scheme

Finite-difference discretization:

1. Staggered grid: Central difference → Avoid numerical oscillations

2. Ghost points: Boundary conditions → Avoid numerical damping



$$\Delta x, \Delta y \ll (R/k^2)^{1/3}$$

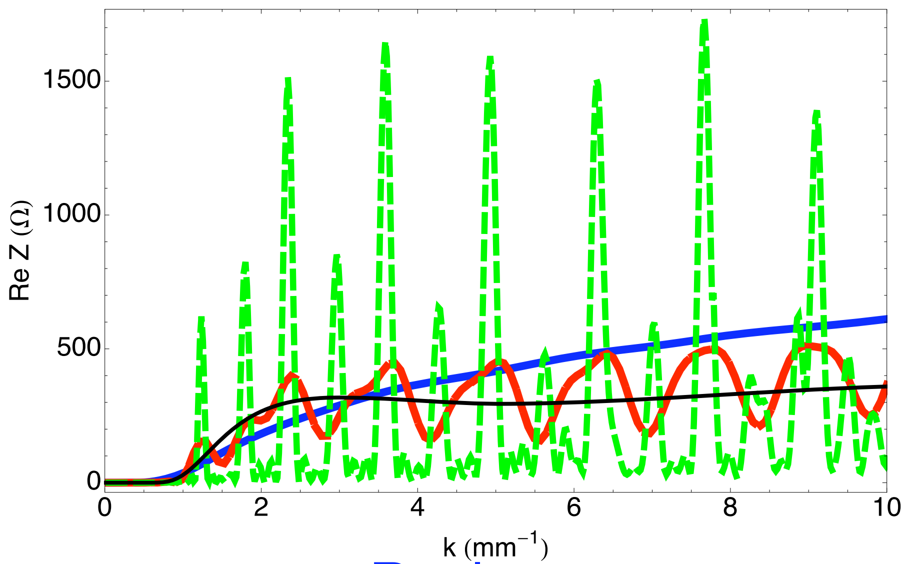
$$\Delta s \ll (R^2/k)^{1/3}$$

T. Agoh and K. Yokoya, PRST-AB 7, 054403 (2004).
D. Zhou, et al., to be published in Jpn. J. Appl. Phys..

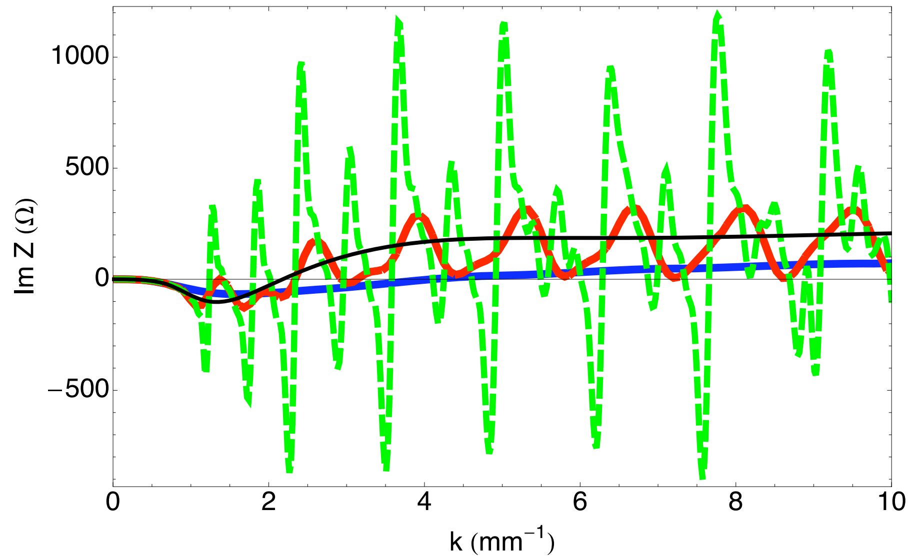
2. CSR field dynamics - Mode excitation

Single dipole:

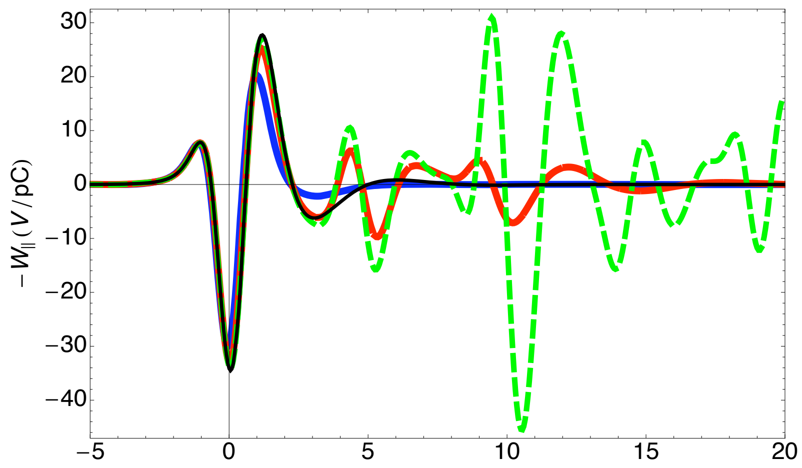
a/b=60/30 mm, R=5 m, L_{bend}=0.5/2/8 m   Bending angle=0.1/0.4/1.6 rad



Real part



Imaginary part



Wake potential

$$\sigma_z = 0.5 \text{ mm}$$

Blue solid lines: L_{bend}=0.5 m

Red dashed lines: L_{bend}=2 m

Green dotted lines: L_{bend}=8 m

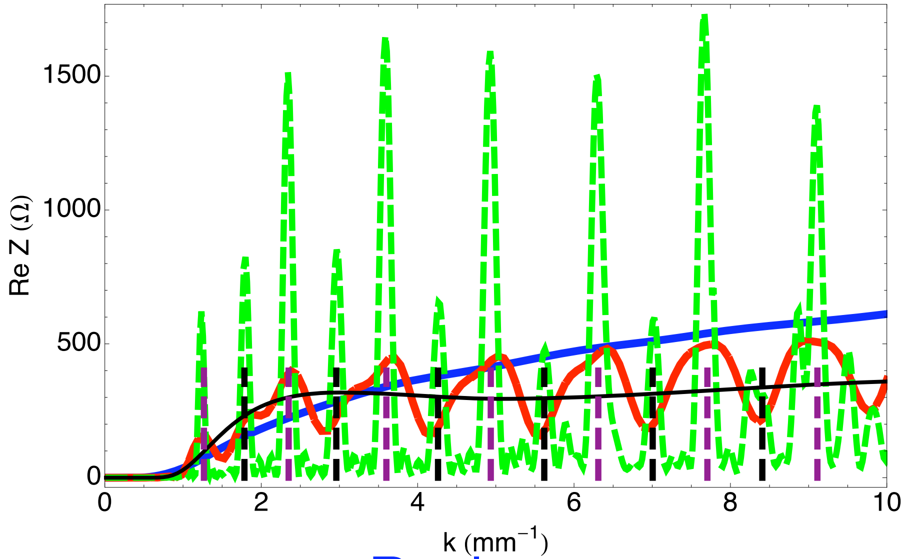
Black solid lines: Parallel plates model

D. Zhou, et al., to be published in Jpn. J. Appl. Phys..

2. CSR field dynamics - Eigenmodes

Single dipole:

a/b=60/30 mm, R=5 m, L_{bend}=0.5/2/8 m



Real part

Vertical black and purple dashed lines: resonant poles of E_y mode and E_x mode (p=1)

Phase matching condition:

$$\Lambda = \frac{\pi^2 p^2}{2k^2 b^2} + \frac{1}{2}\left[\frac{3\pi}{kR}(m \pm 0.25)\right]^{\frac{2}{3}} - \frac{a}{2R} = 0$$

$$k_{mp} = \frac{p\pi}{b}\sqrt{\frac{R}{x_b}}\Upsilon\left(\frac{b(m \pm 0.25)}{p x_b}\right)$$

$$\Upsilon(r) = \left[\left(\sqrt{1 + r^2/3} + 1\right)^{1/3} - \left(\sqrt{1 + r^2/3} - 1\right)^{1/3}\right]^{-3/2}$$

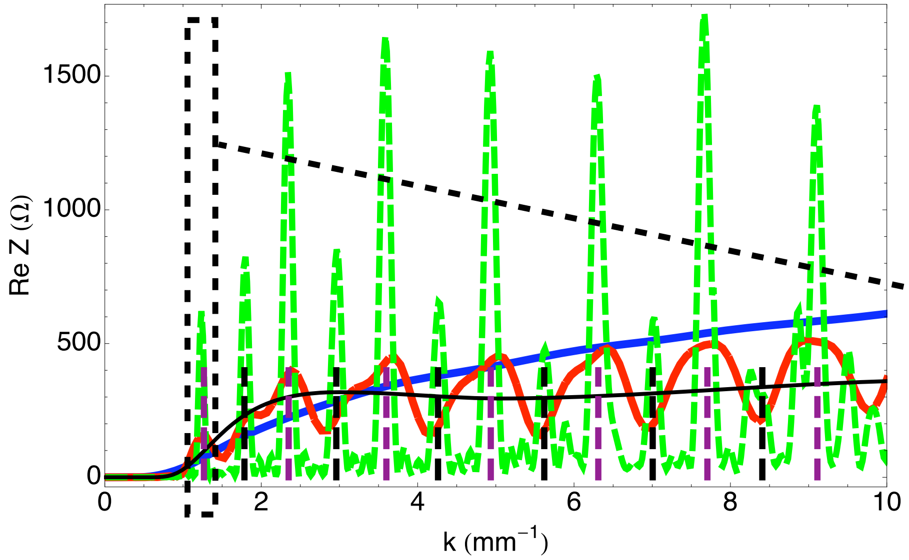
m: horizontal mode index;
p: vertical mode index

R. L. Warnock and P. Morton, Part. Accel. 25, 113 (1990).
G. Stupakov and I. Kotelnikov, PRST-AB 6, 034401 (2003).
T. Agoh, PRST-AB 12, 094402 (2009).
D. Zhou, et al., to be published in Jpn. J. Appl. Phys..

2. CSR field dynamics - Eigenmodes (cont'd)

Radiation fields (E_x^r) on resonance poles (p=1)

a/b=60/30 mm, L_{bend}=8 m, R=5 m

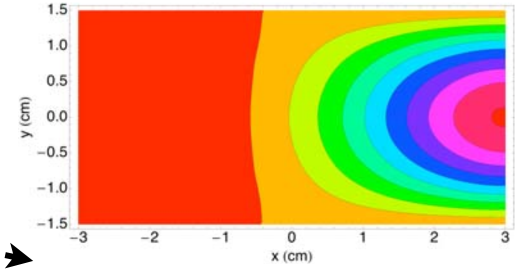


Freq. & index

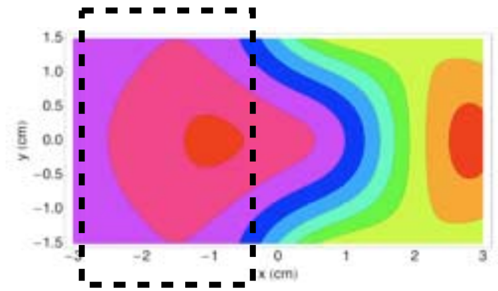
$$k = 1230\ \mathrm{m}^{-1}$$

$$(m, p) = (0, 1)$$

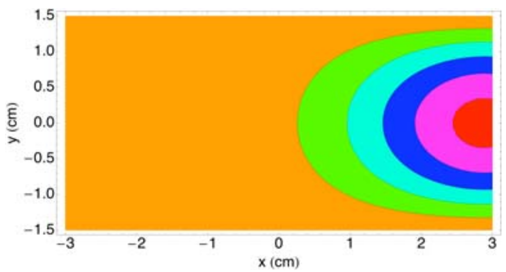
Im. E_x^r:



Re. E_x^r:



Mode pattern:



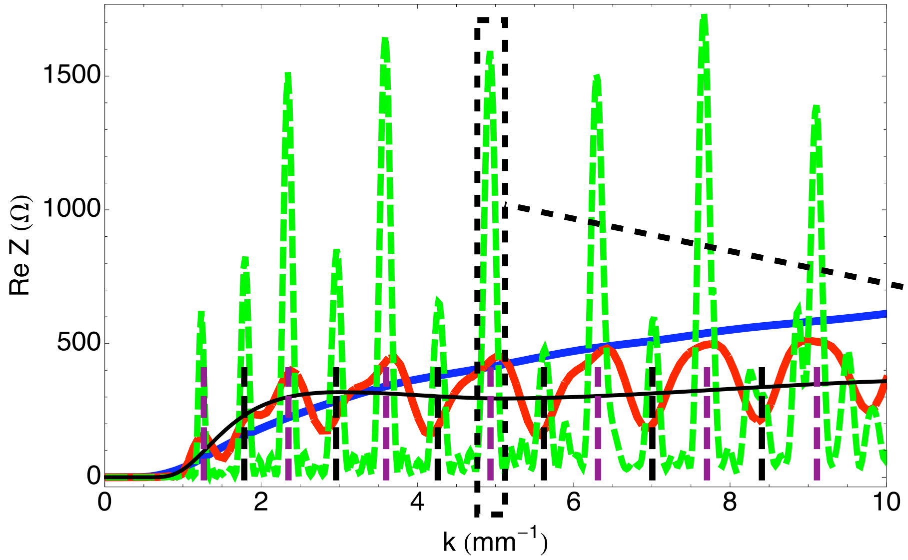
E_x mode: $$E_x(x, y) = E_{x0}\mathrm{Ai}\left(k_y^2\kappa^2 - x/\kappa\right)\sin\left[k_y(y + b/2)\right]$$

G. Stupakov and I. Kotelnikov, PRST-AB 6, 034401 (2003).

2. CSR field dynamics - Eigenmodes (cont'd)

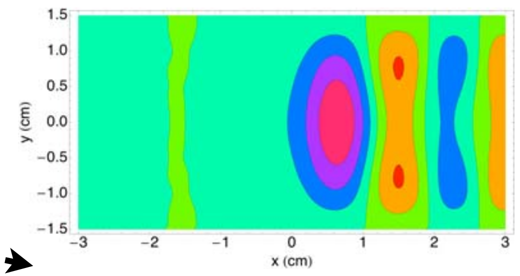
Radiation fields (E_x^r) on resonance poles (p=1)

a/b=60/30 mm, L_{bend}=8 m, R=5 m

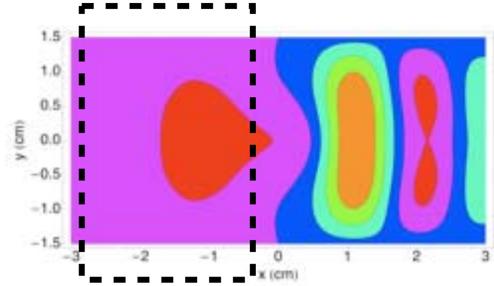


Freq. & index $k = 4930\ \mathrm{m}^{-1}$ $(m, p) = (3, 1)$

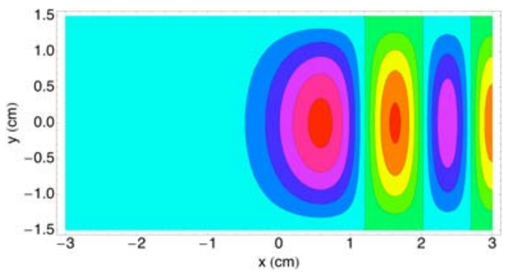
Im. E_x^r:



Re. E_x^r:



Mode pattern:



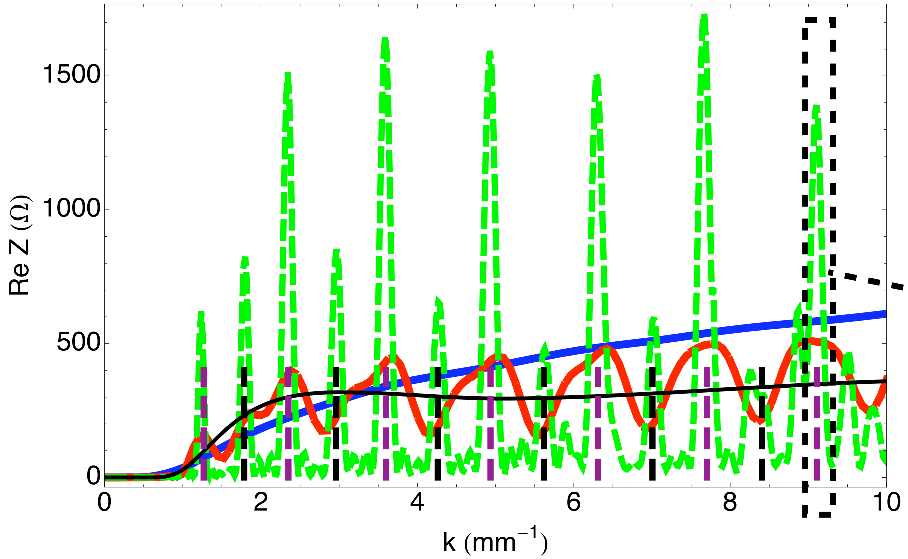
E_x mode: $E_x(x,y) = E_{x0}\mathrm{Ai}\left(k_y^2\kappa^2 - x/\kappa\right)\sin\left[k_y(y+b/2)\right]$

G. Stupakov and I. Kotelnikov, PRST-AB 6, 034401 (2003).

2. CSR field dynamics - Eigenmodes (cont'd)

Radiation fields (E_x^r) on resonance poles (p=1)

a/b=60/30 mm, L_{bend}=8 m, R=5 m

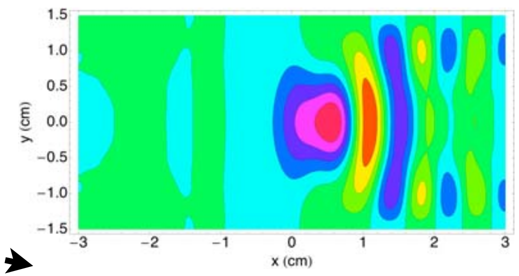


Freq. & index

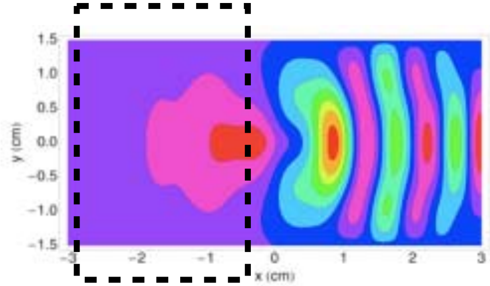
$$k = 9100 \text{ m}^{-1}$$

$$(m, p) = (6, 1)$$

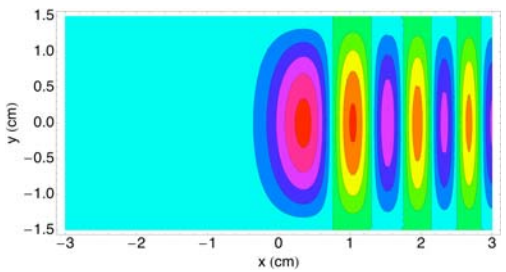
Im. E_x^r:



Re. E_x^r:



Mode pattern:

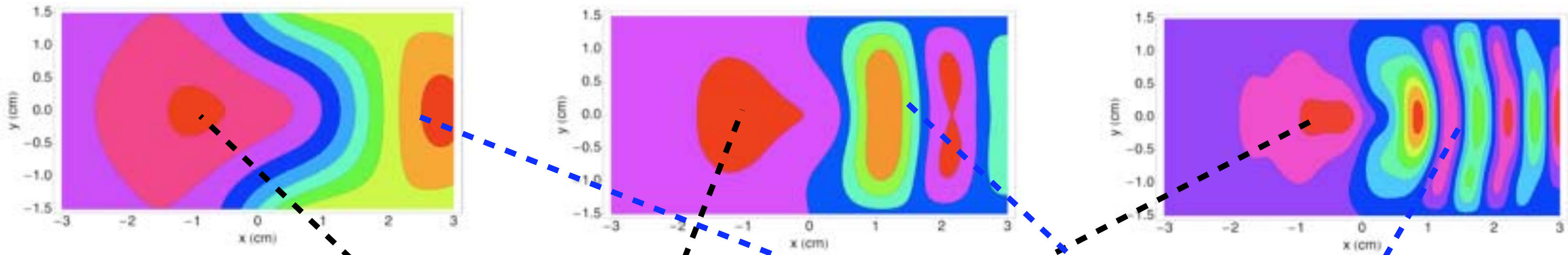


E_x mode: $$E_x(x, y) = E_{x0}\text{Ai}\left(k_y^2\kappa^2 - x/\kappa\right)\sin\left[k_y(y + b/2)\right]$$

G. Stupakov and I. Kotelnikov, PRST-AB 6, 034401 (2003).

2. CSR field dynamics - Velocity and radiation fields

Re. E_x^r:



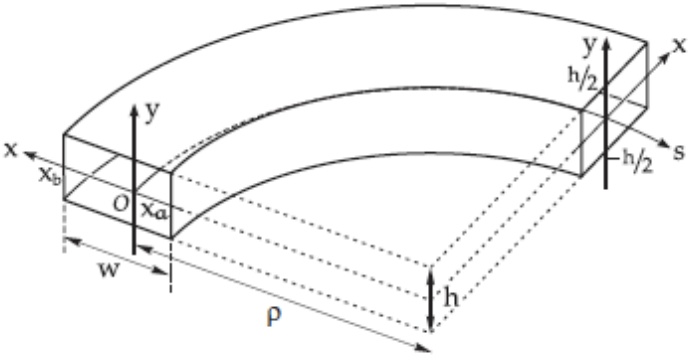
$$\vec{E} = \frac{e}{4\pi\epsilon_0}\left\{\frac{\vec{n}-\vec{\beta}}{\gamma^2 K^3\left|\vec{r}-\vec{r}'\right|^2} + \frac{\vec{n}\times\left[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}\right]}{cK^3\left|\vec{r}-\vec{r}'\right|}\right\}_{ret}$$

Velocity field **Radiation field**

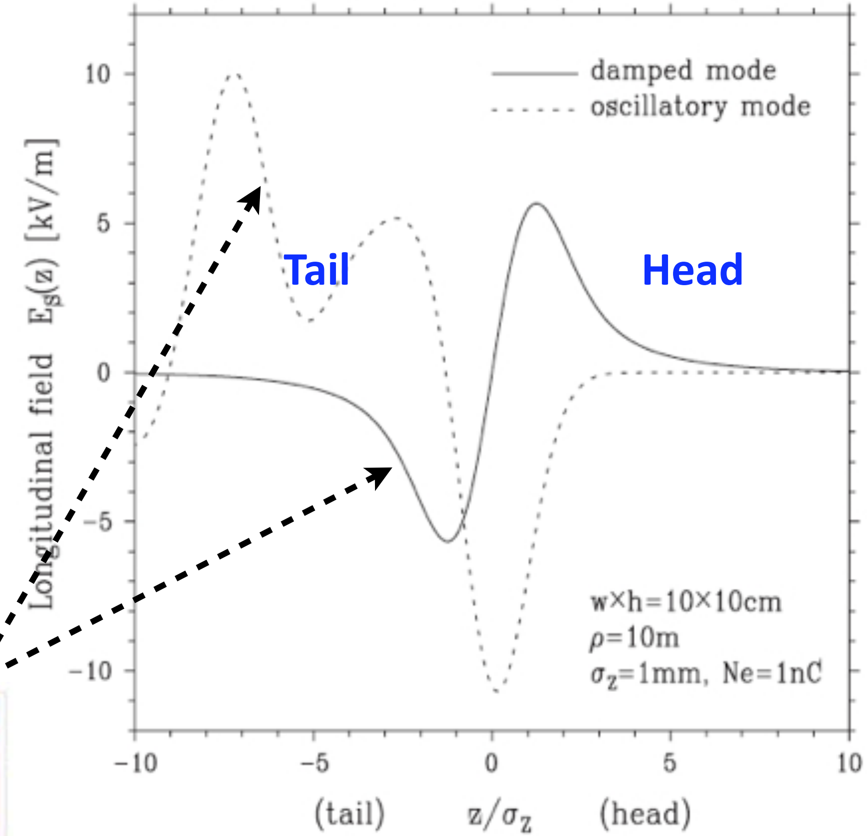
2. CSR field dynamics - Steady-state CSR

CSR fields can be decomposed to a sum of propagating (oscillatory and trailing) and decaying (damped and overtaking) waves in a toroid waveguide [Agoh (2009)].

Geometry



Long. wakefields for a Gaussian bunch



Re. and Im. Poles

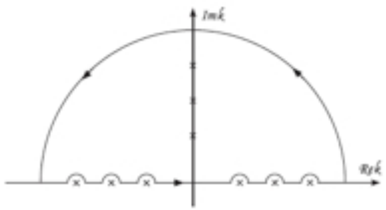


FIG. 7. Contour for $\zeta > 0$.

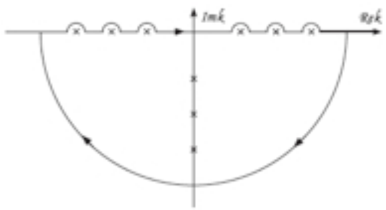
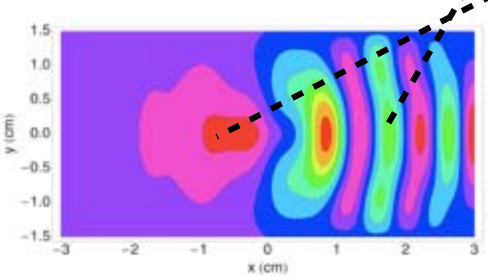


FIG. 8. Contour for $\zeta < 0$.

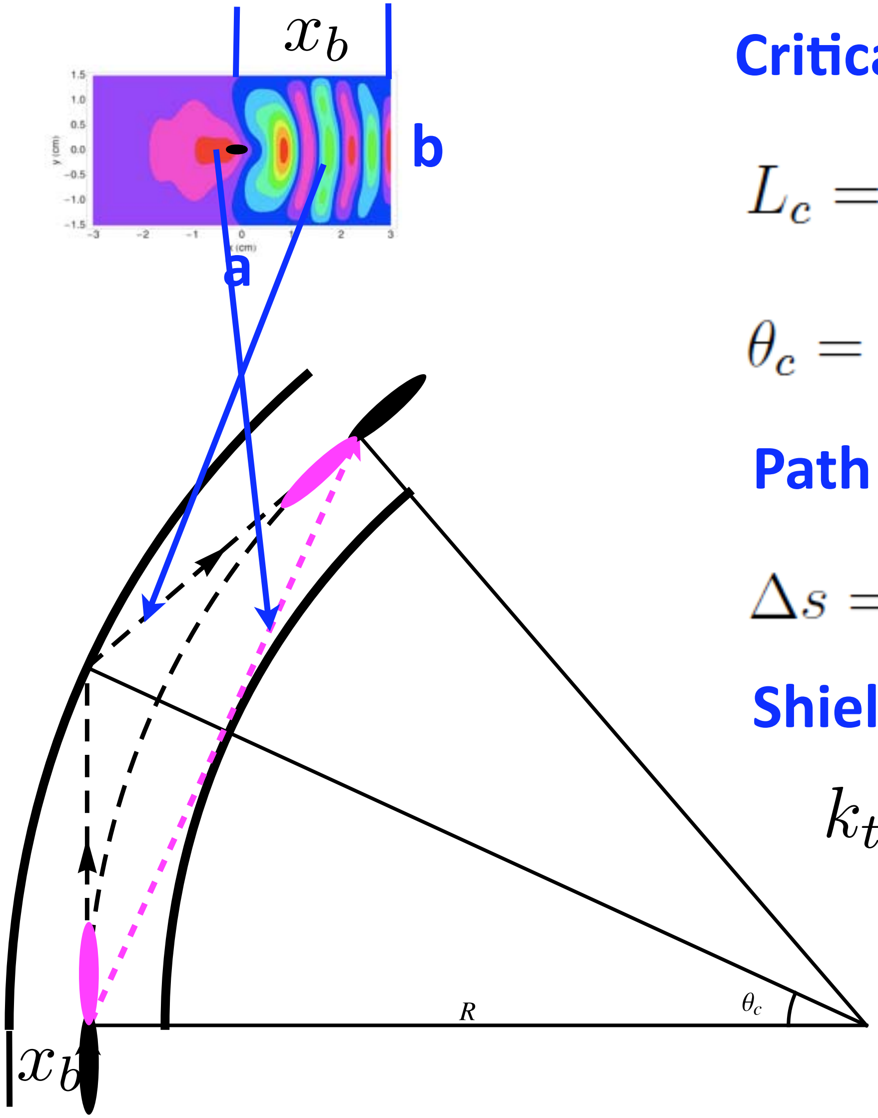


T. Agoh, PRST-AB 12, 094402 (2009).

2. CSR field dynamics - Geometric model

Side-wall reflection can be approximated by a geometric model

[Derbenev (1995), Carr (2001), Sagan (2009), Oide (2010)]



Critical length (Catch-up distance):

$$L_c = 2R\theta_c \approx 2\sqrt{2Rx_b} \qquad x_b \ll R$$

$$\theta_c = \mathrm{ArcCos}\left(R/(R+x_b)\right) \approx \sqrt{2x_b/R}$$

Path difference:

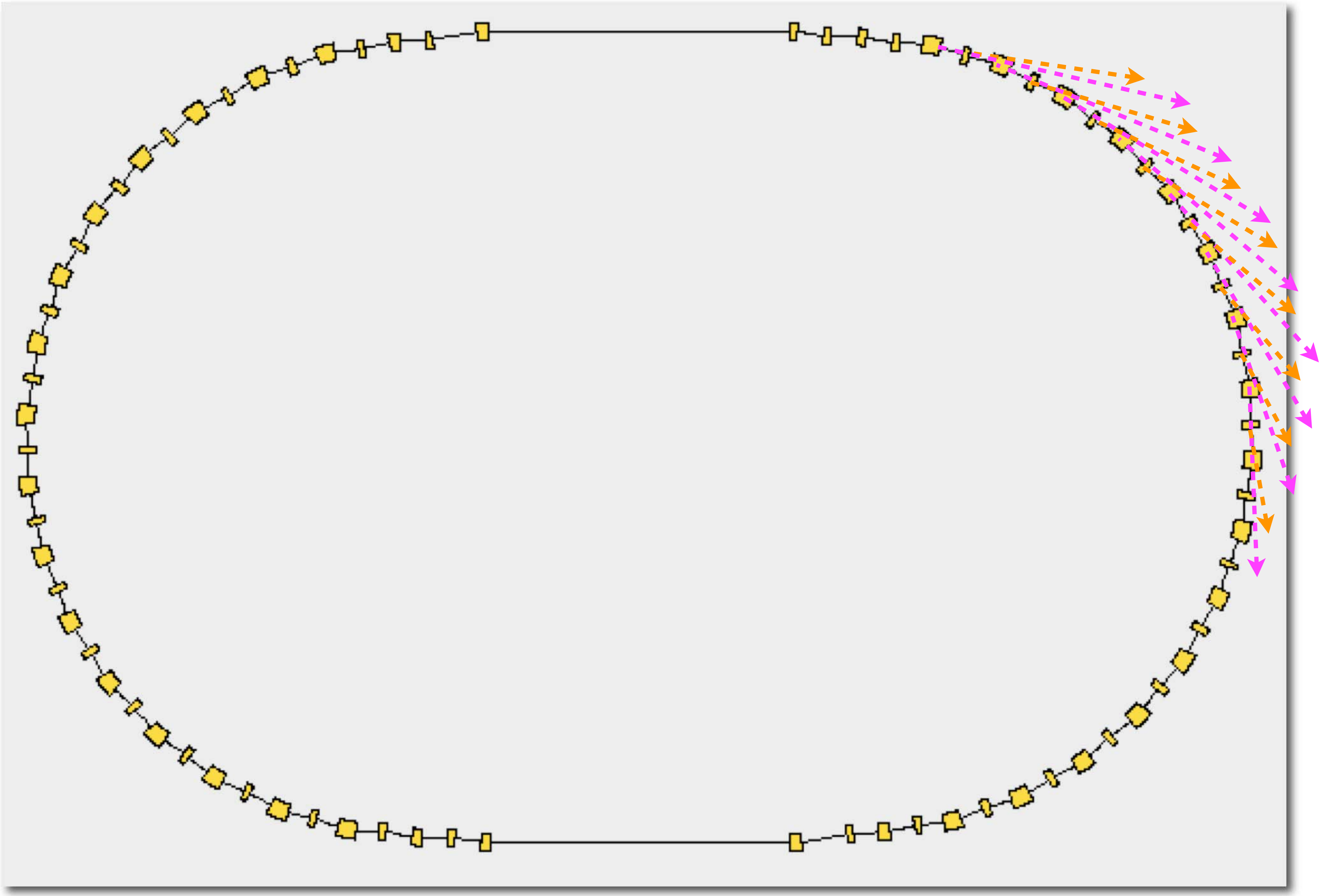
$$\Delta s = 2R(\mathrm{Tan}(\theta_c) - \theta_c) \approx \frac{4}{3}\sqrt{\frac{2x_b^3}{R}}$$

Shielding threshold:

$$k_{th} = \pi\sqrt{R/b^3}$$

Y. S. Derbenev, et al., TESLA FEL-Report 1995-05 (1995).
G. L. Carr, et al., PAC'01, p. 377 (2001).
D. Sagan, et al., PRST-AB 12, 040703 (2009).
K. Oide, Talk at CSR mini-workshop, Nov. 08, 2010.
D. Zhou, et al., to be published in Jpn. J. Appl. Phys..

3. CSR in SuperKEKB DR - Layout



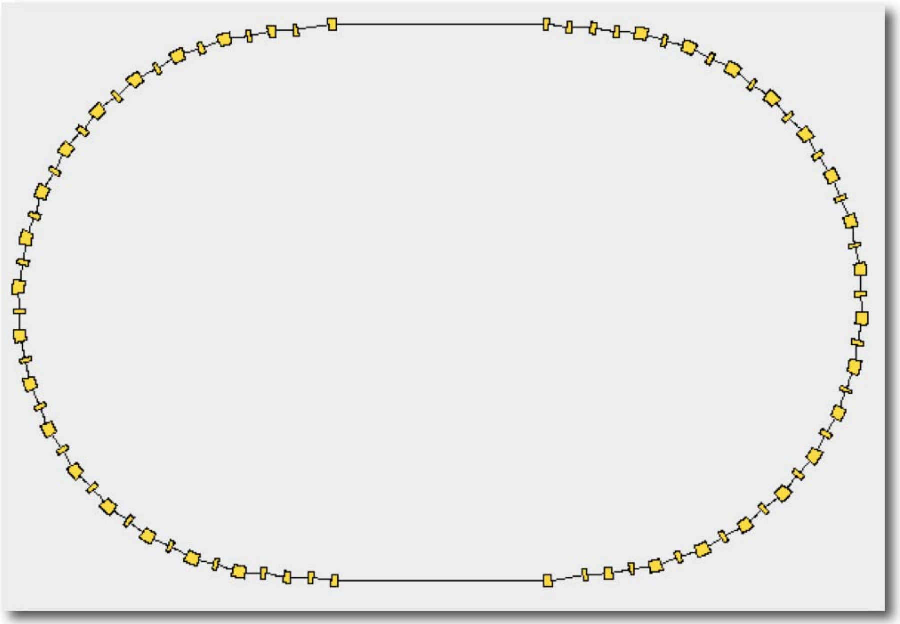
M. Kikuchi, et al., IPAC'10, p. 1641 (2010)

3. CSR in SuperKEKB DR - Parameters

Magnet and chamber parameters:

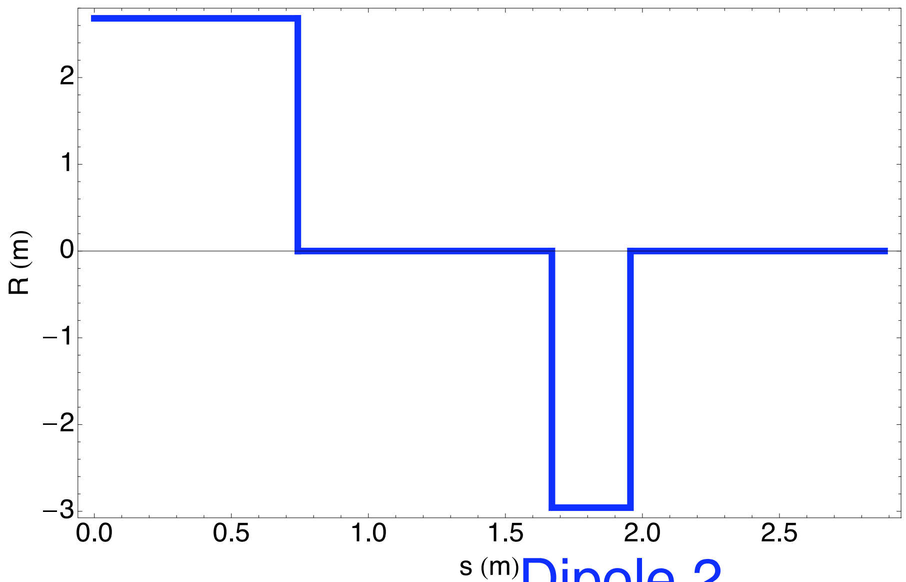
a/b=34/34 mm, L_{bend}=0.74/0.29 m, R=2.7/-3 m (reverse bends)
L_{drift}=0.9 m, N_{cell}=32

The vacuum chamber is curved along the beam orbit



Layout

Dipole 1



Dipole 2

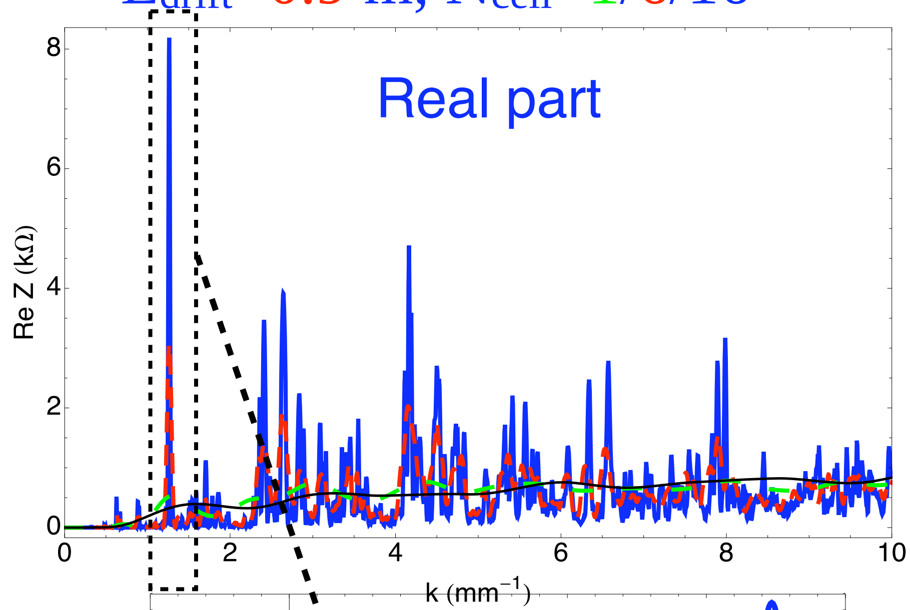
Reverse bends (1 cell)

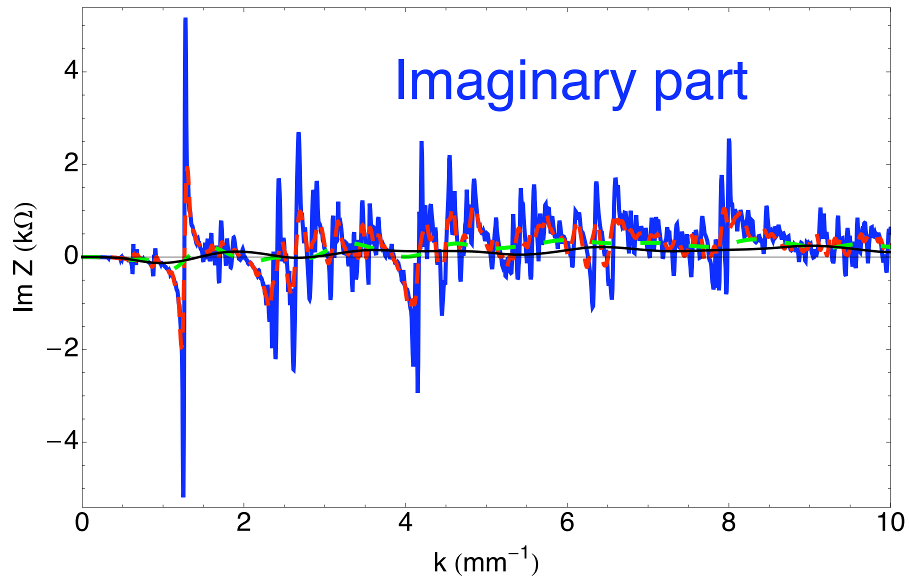
M. Kikuchi, et al., IPAC'10, p. 1641 (2010)

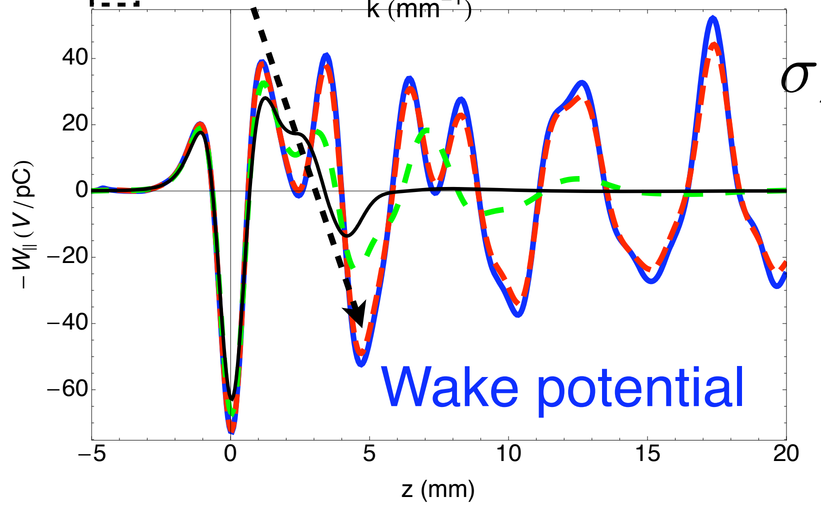
3. CSR in SuperKEKB DR - Multi-bend inerference

SuperKEKB damping ring (one arc section) (Perfect conducting wall)

a/b=34/34 mm, L_{bend}=0.74/0.29 m, R=2.7/-3 m (reverse bends)
L_{drift}=0.9 m, N_{cell}=1/6/16







$$\sigma_z = 0.5 \text{ mm}$$

Blue solid lines: 16 cells
Red dashed lines: 6 cells
Green dotted lines: 1 cell
Black solid lines: single-bend

D. Zhou, et al., to be published in Jpn. J. Appl. Phys..

3. CSR in SuperKEKB DR - Microwave instability

SuperKEKB DR (latest version): CSR instability threshold [Cai (2011)]:

$$\chi = \sigma_z \sqrt{\frac{\rho}{h^3}} \approx 2.9 \qquad \rho = 2.7\ \mathrm{m} \qquad h = 24\ \mathrm{mm}$$

$$I_b = 0.5 * \frac{3\sqrt{2}\alpha\gamma\sigma_\delta^2 I_A \sigma_z}{\pi^{3/2} h} = 0.016\ \mathrm{A} \qquad N_{th} = \frac{I_b C}{ec} \approx 4.6 \times 10^{10}$$

SuperKEKB DR: simulations using Vlasov solver [Ikeda (2011)]:

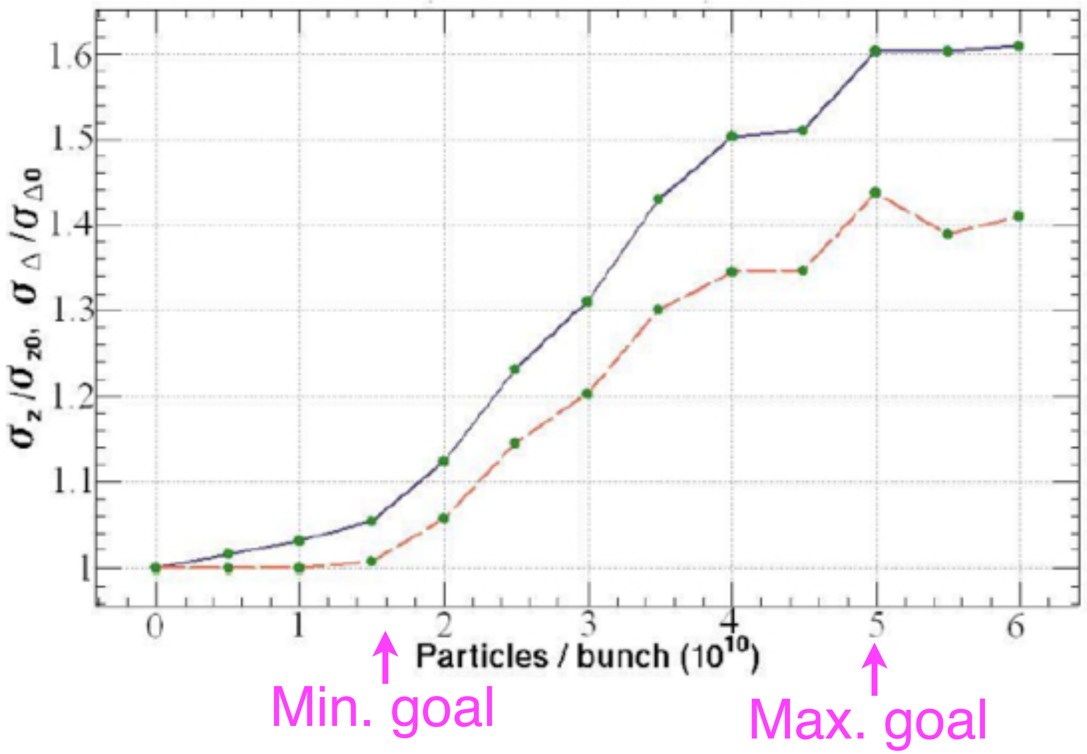


Table 1: Damping ring parameters

Parameter		unit
Energy	1.1	GeV
Maximum bunch charge	8	nC
No. of bunch trains/ bunches per train	2/2	
Circumference	135.5	m
Maximum stored current	70.8	mA
Horizontal damping time	10.9	ms
Injected-beam emittance	1700	nm
Equilibrium emittance(h/v)	41.4/2.07	nm
Maximum x-y coupling	5	%
Emittance at extraction(h/v)	42.5/3.15	nm
Energy band-width of injected beam	±1.5	%
Energy spread	0.055	%
Bunch length	6.53	mm
Momentum compaction factor	0.0141	
Cavity voltage for 1.5 % bucket-height	1.4	MV
RF frequency	509	MHz

Y. Cai, FRXAA01, IPAC'11 (2011)
H. Ikeda, et al., THPZ021, IPAC'11 (2011)
H. Ikeda, this meeting

4. CSR in NSLS VUV ring

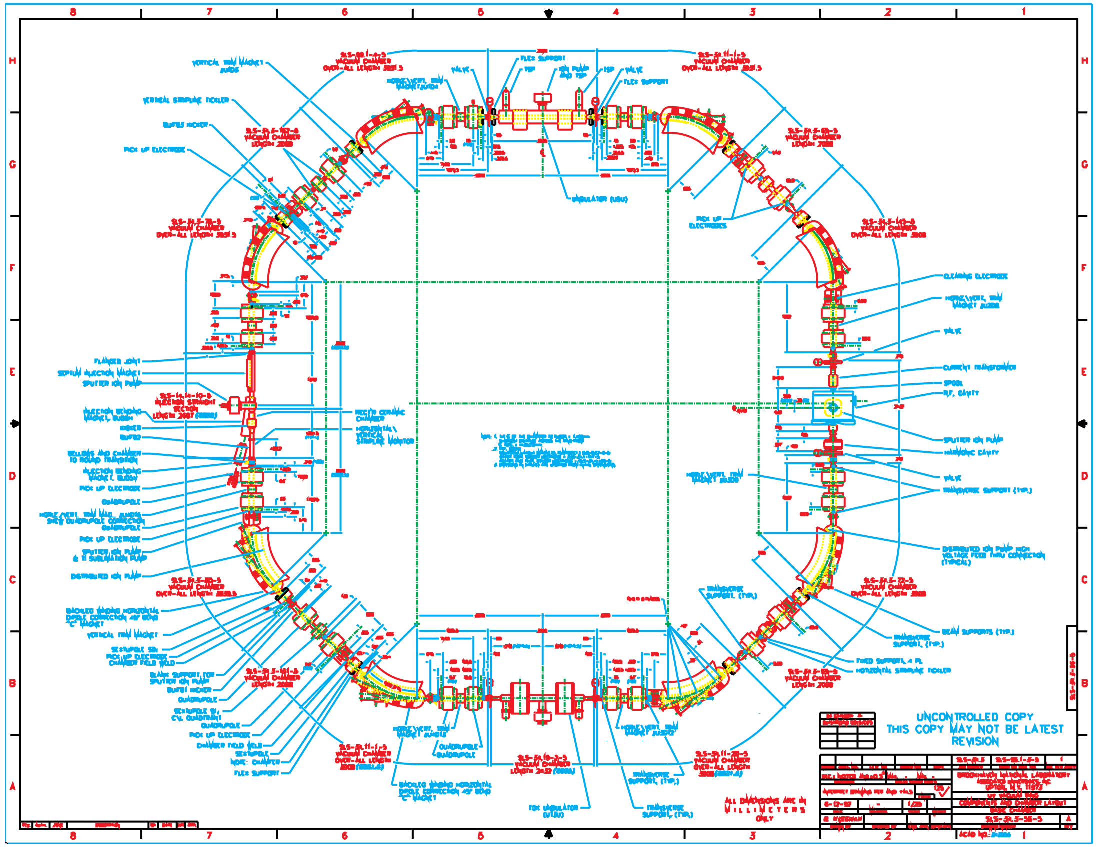
Multi-bend interference considered

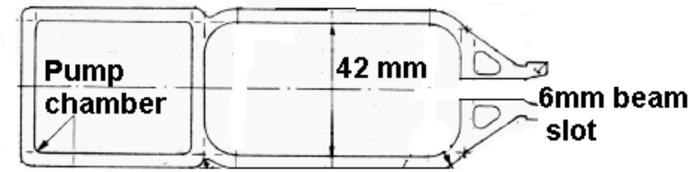
a/b=80/42 mm, L_{bend}=1.5 m, R=1.91 m (reverse bends)

L_{drift}=3.3/6.456 m, N_{cell}=4 (fold)

Beam line:=4×(BD, Drift1, BD, Drift2)

Beam sizes: (σx,σy)=(0.54 mm, 0.06 mm), RMS σ_z = 4.5 to 60 cm



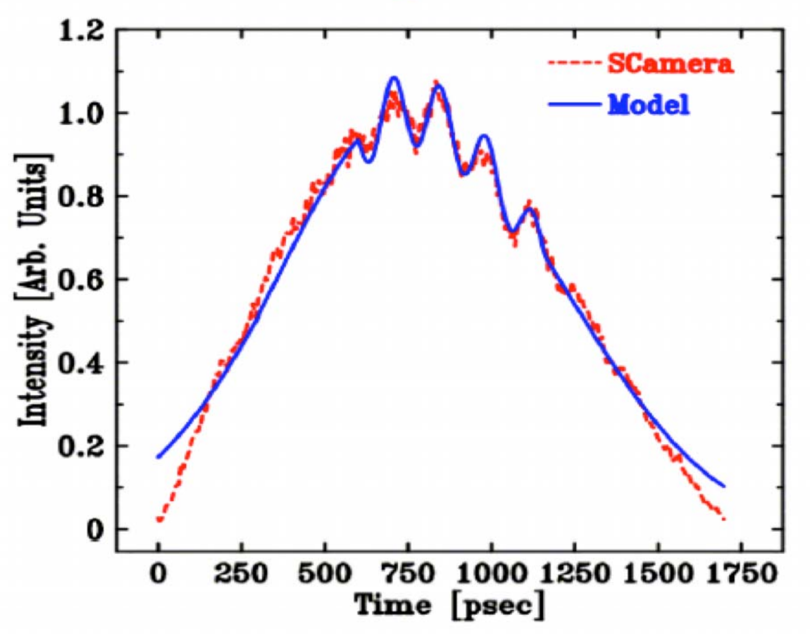


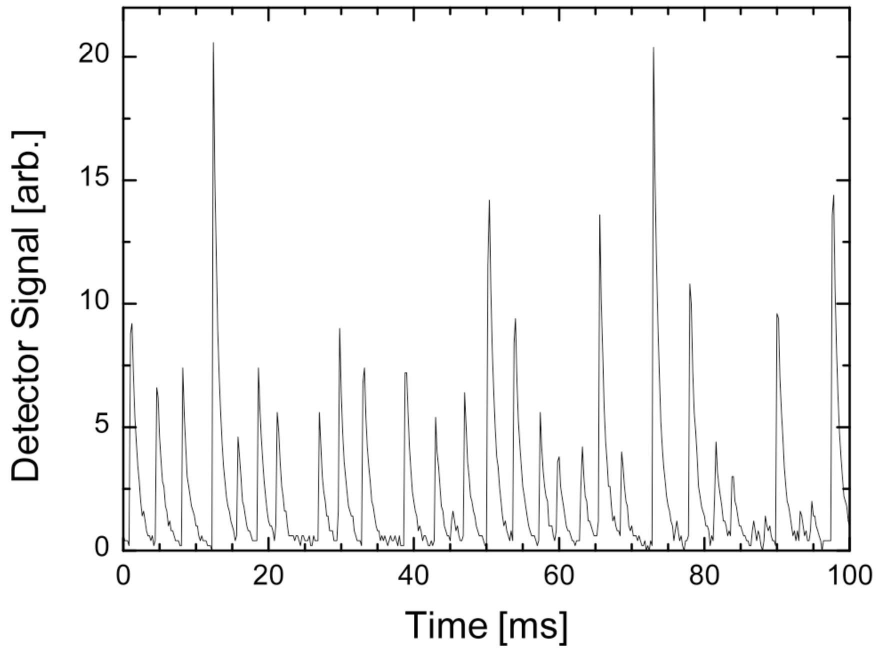
http://www.nsls.bnl.gov/facility/accelerator/vuv/

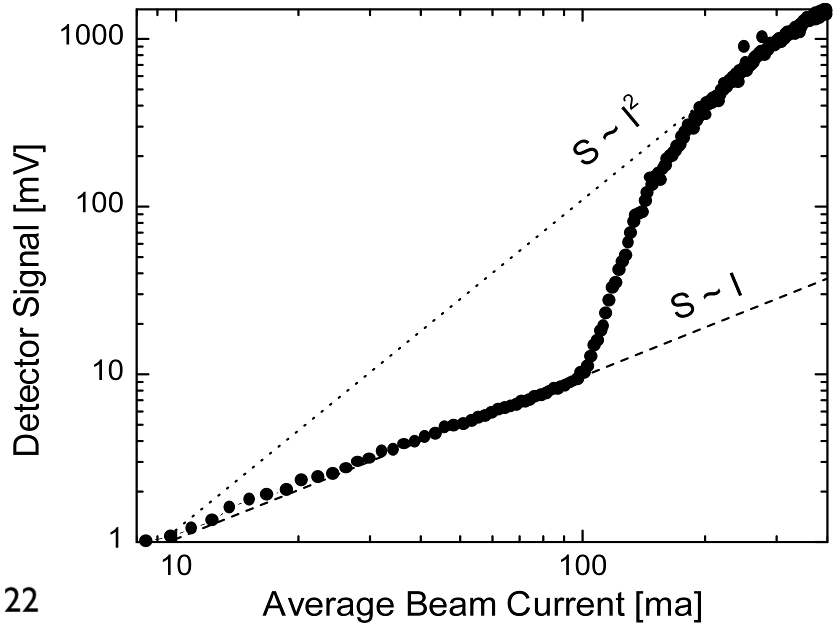
S. Kramer, CSR workshop at Canadian Light source, Nov. 12, 2010

4. CSR in NSLS VUV ring - Measurements

Micro-bunching@Streak Camera



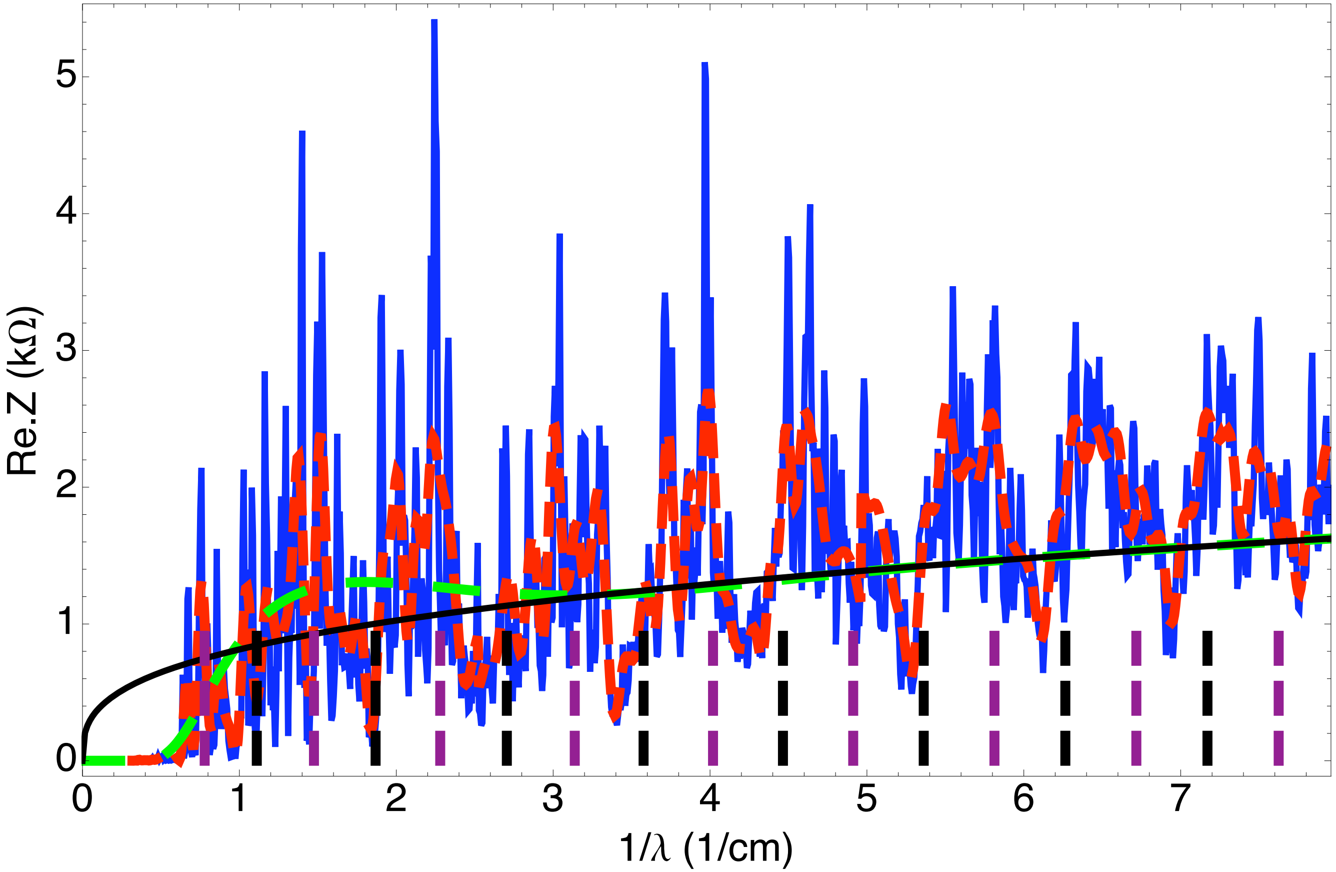




Courtesy of S. Kramer
Talk in CLS CSR workshop,
Nov., 2010

4. CSR in NSLS VUV ring - Impedance calculation

Real part of CSR impedance (∝power spectrum):



Blue solid lines: 4 cells (whole ring)

Red dashed lines: 1 cell (1/4 ring)

Green dotted lines: steady-state Parallel plates model

Black solid lines: steady-state free space model

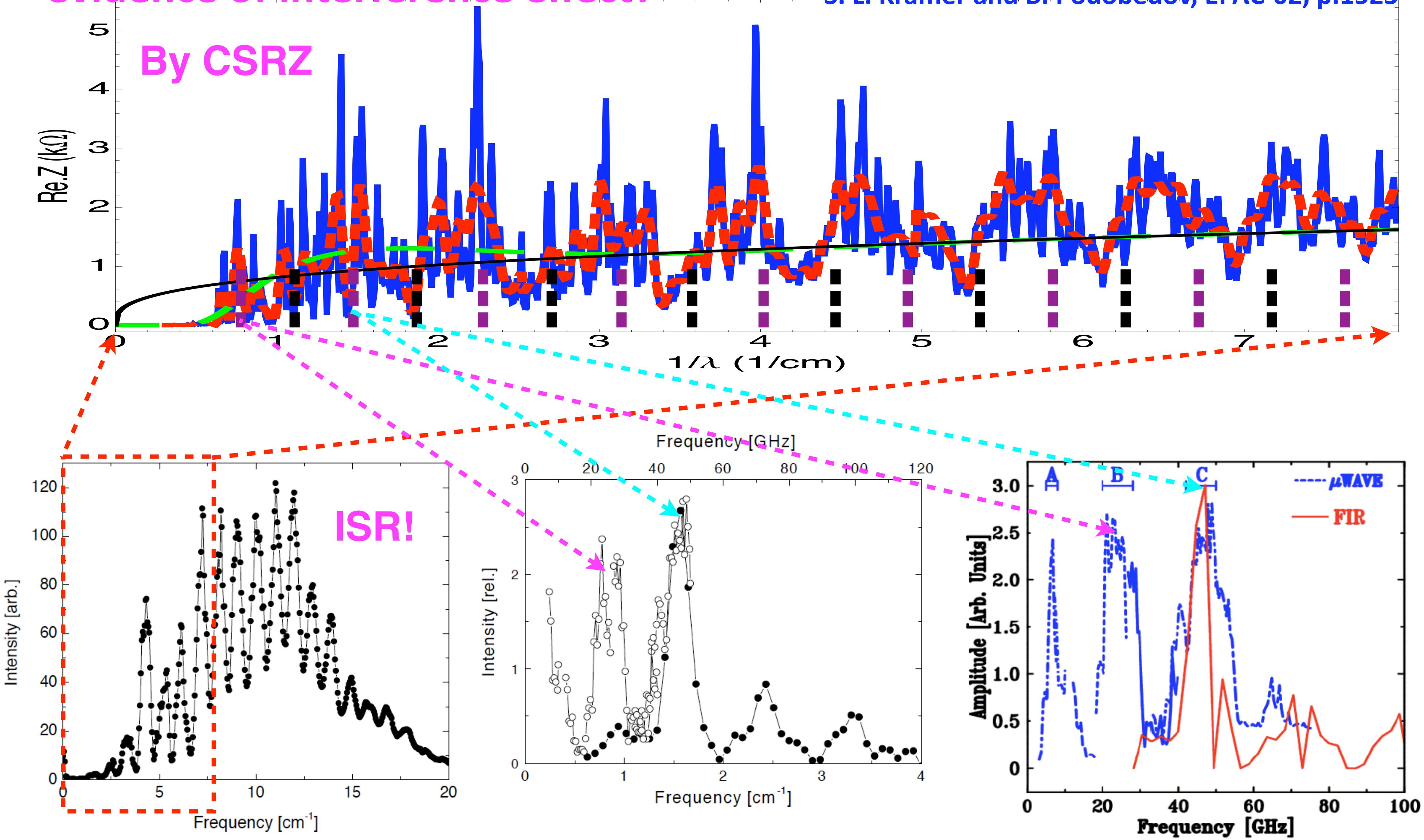
Black/Purple dashed lines: Ex/Ey mode peaks

T. Agoh and K. Yokoya, PRST-AB 7, 054403 (2004).
G. Stupakov and I. Kotelnikov, PRST-AB 6, 034401 (2003).

4. CSR in NSLS VUV ring - Comparison

Measured SR power spectrum: evidence of interference effect?

G. L. Carr, S. L. Kramer, N. Jisrawi, L. Mihaly and D. Talbayev, PAC'01, p.377
S. L. Kramer and B. Podobedov, EPAC'02, p.1523



5. Summary

1). CSR impedance (radiation power spectrum) may exhibit narrow peaks in the presence of chamber, rather than smooth curve. Relevant beam dynamics and radiation performance may differ from steady-state models.

2). CSR fields contains velocity and radiation fields. The velocity fields are overtaking and related to decaying waves. The radiation fields are trailing and related to propagating waves [A proof to T. Agoh's theory (PRST-AB 12, 094402 (2009)**)].**

3). Multi-bend CSR interference appears in small storage rings (small bending radius and short drifts) and may play a role in microwave instability (micro-bunching).

Thank you!