CSR Instability Studies for SuperKEKB

Frank Zimmermann

SuperKEKB Damping Ring Meeting

Friday 16 July 2010

Thanks to

Karl Bane, Mitsuo Kikuchi, Kazuhito Ohmi, Katsunobu Oide, Demin Zhou

disclaimer

I had not done any work on CSR since 1997;
so this study is not very deep

some references

[1] G. Stupakov and S. Heifets, "Beam Instability and Microbunching due to Coherent Synchrotron Radiation," PRST-AB 5, 054402 (2002); http://prst-ab.aps.org/pdf/PRSTAB/v5/i5/e054402

[2] S. Heifets and G. Stupakov, "Single-mode Coherent Synchrotron Radiation Instability," PRST-AB 6, 064401 (2003); http://prst-ab.aps.org/pdf/PRSTAB/v6/i6/e064401

[3] K.L.F. Bane, Y. Cai, G. Stupakov, « Comparison of Simulation Codes for Microwave Instability in Bunched Beams," Proc. IPAC'10, Kyoto Japan (2010) ; http://epaper.kek.jp/IPAC10/papers/tupd078.pdf

[4] D. Zhou et al, "CSR in the SuperKEKB Damping Ring," IPAC'10 Kyoto (2010), http://epaper.kek.jp/IPAC10/papers/tupeb018.pdf

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **5**, 054402 (2002)

Beam instability and microbunching due to coherent synchrotron radiation

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(Received 7 February 2002; published 21 May 2002)

A relativistic electron beam moving in a circular orbit in free space can radiate coherently if the wavelength of the synchrotron radiation exceeds the length of the bunch. In accelerators coherent synchrotron radiation of the bunch is usually suppressed by the shielding effect of the conducting walls of the vacuum chamber. However an initial density fluctuation with a characteristic length much shorter than the bunch length can radiate coherently. If the radiation reaction force results in the growth of the initial fluctuation, one can expect an instability which leads to microbunching of the beam and an increased coherent radiation at short wavelengths. Such an instability is studied theoretically in this paper.

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **6**, 064401 (2003)

Single-mode coherent synchrotron radiation instability

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(Received 27 January 2003; published 25 June 2003)

The microwave instability driven by the coherent synchrotron radiation (CSR) has been previously studied [S. Heifets and G.V. Stupakov, Phys. Rev. ST Accel. Beams **5**, 054402 (2002)] neglecting effect of the shielding caused by the finite beam pipe aperture. In practice, the unstable mode can be close to the shielding threshold where the spectrum of the radiation in a toroidal beam pipe is discrete. In this paper, the CSR instability is studied in the case when it is driven by a single synchronous mode. A system of equations for the beam-wave interaction is derived and its similarity to the 1D free-electron laser theory is demonstrated. In the linear regime, the growth rate of the instability is obtained and a transition to the case of continuous spectrum is discussed. The nonlinear evolution of the single-mode instability, both with and without synchrotron damping and quantum diffusion, is also studied.

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COMPARISON OF SIMULATION CODES FOR MICROWAVE INSTABILITY IN BUNCHED BEAMS *

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CSR IN THE SUPERKEKB DAMPING RING

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Stupakov-Heifets formulae (2002)

$$\Lambda \equiv \frac{N_b r_e \rho \sqrt{2\pi}}{C|\eta|\sigma_z \gamma \sigma_\delta^2}$$ Stupakov-Heifets parameter

conditions for instability:

$$\frac{\rho}{b} \leq \Lambda$$ no shielding by the beam pipe

$$\sigma_z \geq \frac{\rho}{2\Lambda^{3/2}}$$ sufficiently long bunch

small horizontal beam size

$$\Lambda \ll \left(\frac{\rho^2}{\sigma_x \beta_x}\right)^{2/3}$$

negligible effect of velocity spread

$$\frac{N_b r_e}{\sqrt{2\pi}\gamma \sigma_z \sigma_\delta} \ll 1 ,$$

continuous mode spectrum (2003): $$C03 \equiv \left(\frac{N_b r_e}{\sqrt{2\pi}\gamma \sigma_z}\right)^5 \left(\frac{2\alpha}{\eta\rho}\right)^3 \frac{1}{\sigma_\delta^8} \frac{1}{\pi^{2/3}} \geq 1$$

not fulfilled!

Table 1: Beam and CSR-instability related parameters for four storage rings.

	SuperKEKB LER	SuperKEKB HER	SuperKEKB e+ DR	ATF DR
beam energy [GeV]	4	7	1.1	1.28
slip factor η	0.000274	0.000188	0.017	0.0019
rms momentum spread $\sigma_{\delta,rms}$ [%]	0.08	0.065	0.055	0.06
bunch population [10^9]	90	65	37.5	10
circumference C [m]	3016	3016	135	138.6
bending radius ρ [m]	73.3	104.5	2.65	5.73
vert. beam pipe radius b [cm]	4.7	4.7	1.6	1.2?
Stupakov-Heifets parameter Λ	**1864**	**2905**	**67**	**339**
ρ/b	**1560**	**2223**	**166**	**478**
σ_z [cm]	**0.6**	**0.5**	**0.7**	**0.5**
$\rho/(2\,\Lambda^{3/2})$ [cm]	**0.05**	**0.03**	**0.24**	**0.05**
$N_b r_0/(\sqrt{2\pi}\ \sigma_z\sigma_\delta\ \gamma)$	0.0027	0.0016	0.0051	0.0015
β_x at bend [m]	10?	10?	1.5	3?
ε_x [nm]	3.2	5.0	2100→ 41	~1.5
σ_x at bend [μm]	179	224	248	67
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	20800	28800	710	2990
τ_x [ms]	37?	56?	11	17.2
C03	0.0128	0.0021	0.0033	0.0002
Q_s	-0.025	-0.025	-0.015	-0.0045
$N_{nb,thr}$	$1.0\text{x}10^{11}$	$1.0\text{x}10^{11}$	$4.5\text{x}10^{10}$	$1.15\text{x}10^{10}$
$N_b/N_{nb,thr}$	**0.89**	**0.65**	**0.83**	**0.86**
$\Lambda b/\rho$	**1.19**	**1.31**	**0.40**	**0.71**

$dP/d\omega = 0.52(e^2/R)(kR)^{1/3}$, and the loss factor $\chi(\omega)$ is estimated in Ref. [11] as $\chi \sim a^{-2}(ck_0/\omega)^{1/3}$, which gives for $\Delta\omega$

$$\Delta\omega \sim ck_0\left(\frac{ck_0}{\omega}\right)^{2/3}.$$

The width of the mode $\Delta\omega_{\rm mode}$ can be estimated as $c\Delta q$ where Δq can be found from the second of Eq. (25) as $\Delta q \sim \Delta y \mu/(1-\beta_g)$. Observing from Fig. 1 that $\Delta y \sim 1$, we conclude that $\Delta q \sim \mu/(1-\beta_g)$. It is interesting to note that $\Delta\omega_{\rm mode} \gg \Delta\Omega$. The overlapping takes place when $c\mu/(1-\beta_g) \gtrsim ck_0(ck_0/\omega)^{2/3}$, or

$$\Lambda(\eta\delta_0)^2\left(\frac{k}{k_0}\right)^{2/3}(ka)^2 \gtrsim 1, \qquad (26)$$

where the parameter Λ is defined by Eq. (1) and $k = \omega/c$.

The growth rate of the instability $\Gamma_{\rm inst}$ for a cold beam in the continuous spectrum model [1] can be estimated as

$$\left(\frac{\Gamma_{\rm inst}}{c}\right)_{\rm cont} = \Lambda^{1/2}\left(\frac{\eta\delta_0}{a}\right)\left(\frac{k}{k_0}\right)^{2/3}.$$

It is easy to check that at Λ given by Eq. (26), $\Gamma_{\rm inst} \sim \mu$, which means that the growth rates in both theories match at the boundary of their validity regions.

Equation (26) shows that the mode overlapping occurs easier for high-frequency modes. In the continuous spectrum model, the maximum growth rate is achieved for $kR \simeq \Lambda^{3/2}$ [4]. Equation (26) gives the critical linear bunch density $n_{\rm cr}$ at which overlapping occurs for this frequency,

$$n_{\rm cr} \sim \frac{\gamma\delta_0}{r_e}(\eta\delta_0)^{3/5}\left(\frac{R}{a}\right)^{3/5}.$$

The model of Ref. [4] is valid if the beam linear density n_b is larger than $n_{\rm cr}$. It describes the instability of higher modes where the shielding effect of the walls can be neglected. At the same time, the lowest toroidal modes are described by the single-mode model developed in this paper.

VI. NONLINEAR REGIME OF THE INSTABILITY

When the amplitude of the unstable mode becomes large, the linear theory is not valid anymore and one has to use the full Vlasov equation for the distribution function $f(z, \delta, t)$:

$$\frac{\partial f}{\partial t} - \eta c\delta\frac{\partial f}{\partial z} + \frac{e}{\gamma mc}\mathcal{E}(z,t)\frac{\partial f}{\partial \delta} = 0. \qquad (27)$$

An important approximation that we make in the nonlinear regime is that the evolution of the instability is governed by a single mode with a wave number q_w. One would expect that this wave number is equal to q_0—the mode that has the maximum growth rate in the linear regime—however, for the sake of generality, we treat q_w as arbitrary (but close to q_0). The derivation of the equation for $\mathcal{E}(z,t)$ describing the interaction of the beam with the mode is presented in Appendix B. The result is given by Eq. (B7) which we reproduce here:

Bane-Cai-Stupakov result (2010)

$$N_{nb,thr} \approx \left(\frac{2\pi Q_s \gamma \sigma_{\delta 0}}{r_e}\right)\left(\frac{\sigma_z^{4/3}}{\rho^{1/3}}\right)\left(0.5 + 0.12\frac{\sigma_{z0}\rho^{1/2}}{b^{3/2}}\right)$$

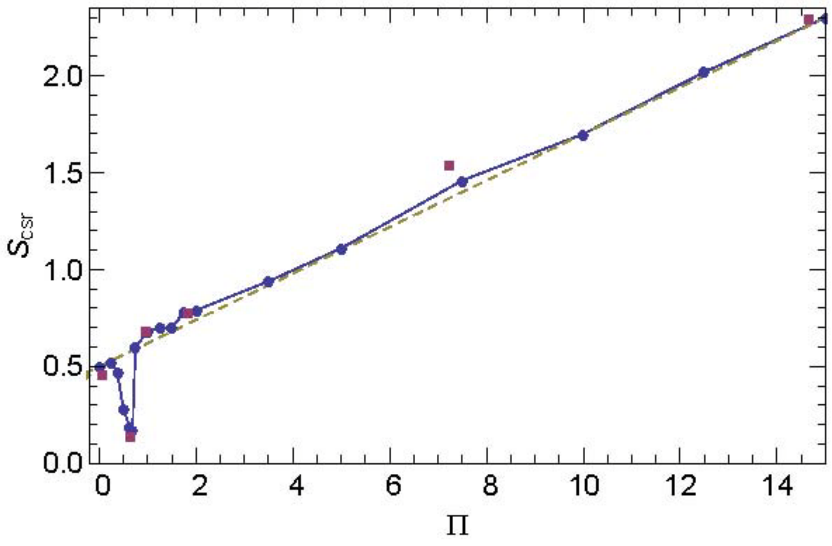


Figure 2: For the CSR wake, threshold value of $S_{\rm csr}$ *vs.* shielding parameter, $\Pi = \rho^{1/2}\sigma_{z0}/h^{3/2}$. Symbols give results of the VFP solver (blue) and the LV code (red).

Bane
Cai
Stupakov
2010

CSR Wake

We consider the CSR wakefield generated by an electron moving on a circular orbit with bending radius ρ in the middle of two parallel plates [7]. In the case of no shielding the wake is non-zero only for positive q (*i.e.* the test particle ahead of the driving charge); it is given by

$$w_0(q) = -\frac{4\pi}{3^{4/3}} H(q) \frac{\rho^{1/3}}{(q\sigma_{z0})^{4/3}}. \qquad (6)$$

This wake is singular and requires special care. In the simulations, we obtain the bunch wake v_{ind} by convoluting with the bunch shape λ. For such a singular wake, however, we integrate by parts and discard the boundary term; *i.e.* we let the bunch wake $v_{\text{ind}}(q) = \int s(q')\lambda'(q-q')\,dq'$, where $s(q) = \int_{-\infty}^{q} w(q')\,dq'$ and λ' is the derivative of the bunch distribution (for a justification, see *e.g.* Ref. [8]). Because of the λ' in the integral, simulation with such a wake is more sensitive to numerical errors or noise, and obtaining reliable results becomes more challenging.

With shielding, the wake $w(q) = w_0(q) + w_1(q)$, with

$$w_1(q) = -\rho^{1/3}\left(\frac{\Pi}{\sigma_{z0}}\right)^{4/3} G(\Pi q), \qquad (7)$$

where the shielding parameter $\Pi = \sigma_{z0}\rho^{1/2}/h^{3/2}$, and $2h$ is the separation between the two plates. The term $w_1(q)$ is the contribution to the wake of the image charges generated by the metal plates; note that it is in general non-zero for both signs of argument. The function G is given by

$$G(\zeta) = 8\pi \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} \frac{Y_k(\zeta)[3 - Y_k(\zeta)]}{[1 + Y_k(\zeta)]^3}, \qquad (8)$$

where Y_k is a root of the equation

$$Y_k - \frac{3\zeta}{k^{3/2}} Y_k^{1/4} - 3 = 0. \qquad (9)$$

We have performed stability calculations for this model, for shielding parameter Π up to 15. The threshold results are given in Fig. 2. We find good agreement between the VFP and LV results. With no shielding $S_{\text{csr}} = 0.50$; there is a deep dip in the curve in the vicinity of $\Pi = 0.7$ where $S_{\text{csr}} = 0.17$; then most of the results follow closely the straight line $S_{\text{csr}} = 0.5 + 0.12\Pi$ (the dashes). In Fig. 3 we plot the Haïssinski solution at threshold and the wake induced voltage v_{ind} for selected values of shielding parameter, Π. With no shielding the bunch shape is markedly triangular; with increasing shielding it moves gradually toward that of the unperturbed Gaussian. We see that v_{ind}, in amplitude, drops quickly as Π increases from zero; by $\Pi \gtrsim 1.5$ this function, in addition, has become largely inductive.

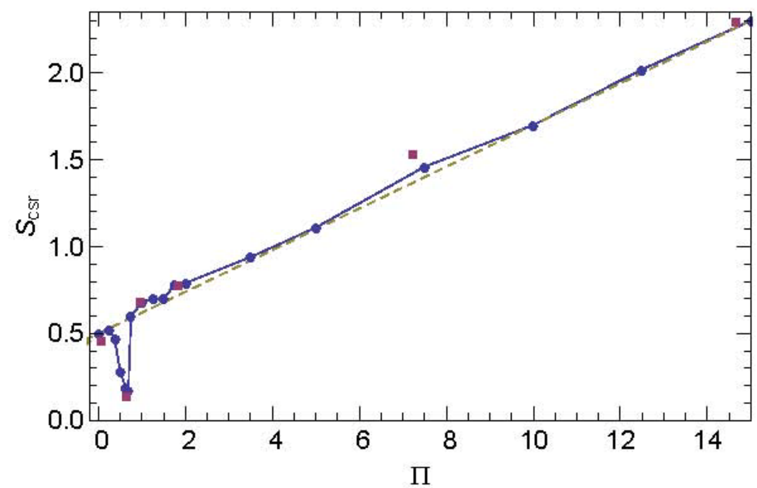
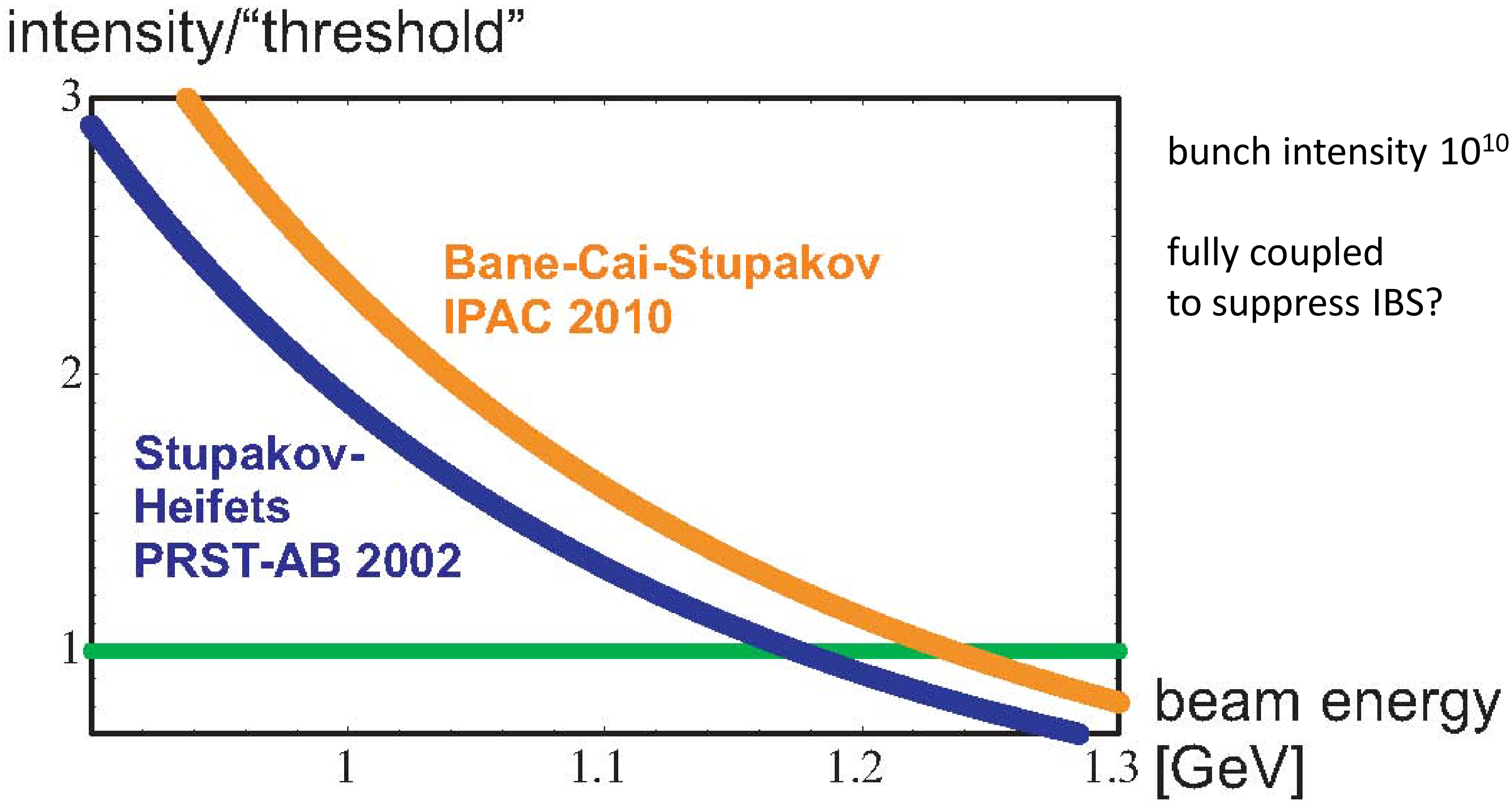


Figure 2: For the CSR wake, threshold value of S_{csr} *vs.* shielding parameter, $\Pi = \rho^{1/2}\sigma_{z0}/h^{3/2}$. Symbols give results of the VFP solver (blue) and the LV code (red).

Table 2: Beam and CSR-instability related parameters for the ATF DR at two different energies.

ATF damping ring	nominal	lower energy
beam energy [GeV]	**1.28**	**1.00**
slip factor η	0.0019	
rms momentum spread $\sigma_{\delta,rms}$ [%]	0.06	0.047
bunch population [10^9]	10	10
circumference C [m]	138.6	
bending radius ρ [m]	5.73	
vert. beam pipe radius b [cm]	1.2?	
Stupakov-Heifets parameter Λ	**339**	**906**
ρ/b	**478**	
σ_z [cm]	**0.5**	**0.39**
$\rho/(2\,\Lambda^{3/2})$ [cm]	**0.05**	**0.011**
$N_b r_0/(\sqrt{2\pi}\,\sigma_z\sigma_\delta\,\gamma)$	0.0015	0.0031
β_x at bend [m]	3?	
ε_x [nm]	~1.5	0.9
σ_x at bend [μm]	67	52
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	2990	3540
τ_x [ms]	17.2	36.1
C03	0.0017	0.0141
Q_s	-0.0045	
$N_{nb,thr}$	1.15×10^{10}	4.3×10^{9}
$N_b/N_{nb,thr}$	**0.86**	**2.32**
$\Lambda b/\rho$	**0.71**	**1.90**

ATF Damping Ring as test bed? lowering ATF beam energy → CSR instability



bunch intensity 10^{10}

fully coupled
to suppress IBS?

Note: Bane-Cai-Stupakov predict instability in a regime where it is excluded by Stupakov-Heifets
Also note: ATF Damping Ring has initially operated at 0.96 GeV beam energy, in 1997

Table 3: Beam and CSR-instability related parameters for five storage rings.

	SuperKEKB LER	SuperKEKB HER	SuperB LER	SuperB HER	CLIC DR
beam energy [GeV]	4	7	4.18	6.7	2.86
slip factor η	0.000274	0.000188	0.00042	0.00040	0.000065
rms momentum spread $\sigma_{\delta,rms}$ [%]	0.08	0.065	0.066	0.062	0.11
bunch population [10^9]	90	65	57.4	57.4	4.1
circumference *C* [m]	3016	3016	1258	1258	493
bending radius ρ [m]	73.3	104.5	29.3 (13.75)	80.5 (165)	6.9
vert. beam pipe radius *b* [cm]	4.7	4.7	2.0	2.5	0.9
Stupakov-Heifets parameter Λ	**1864**	**2905**	**1254**	**2557**	**915**
ρ/b	**1560**	**2223**	**1465**	**3220**	**767**
σ_z [cm]	**0.6**	**0.5**	**0.5**	**0.5**	**0.1**
$\rho/(2\Lambda^{3/2})$ [cm]	**0.05**	**0.03**	**0.03**	**0.03**	**0.01**
$N_b r_0/(\sqrt{2\pi}\,\sigma_z\sigma_\delta\,\gamma)$	0.0027	0.0016	0.0024	0.0016	0.0007
β_x at bend [m]	10?	10?	6?	2?	0.2?
ε_x [nm]	3.2	5.0	2.41	2.0	0.09
σ_x at bend [μm]	179	224	120	63	4
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	20800	28800	11230	137900	146600
τ_x [ms]	37?	56?	44	29	1.62
C03	0.0128	0.0021	0.0042	0.0001	0.0052
Q_s	-0.025	-0.025	-0.01	-0.01	-0.009
$N_{nb,thr}$	1.0×10^{11}	1.0×10^{11}	5.5×10^{10}	6.7×10^{10}	5.7×10^{9}
$N_b/N_{nb,thr}$	**0.89**	**0.65**	**1.04**	**0.85**	**0.72**
$\Lambda b/\rho$	**1.19**	**1.31**	0.86	0.79	**1.19**

comment & questions

At the threshold the inequality (26) in the 2003 paper by Sam Heifets and Gennady Stupakov "Single-mode Coherent Synchrotron Radiation Instability," PRST-AB 6, 064401 (2003)is not fulfilled for any of the example storage rings considered here.

I think this means that only a single isolated mode should drive the CSR instability.

Three questions:

- Is the treatment from the recent two codes of Bane-Cai-Stupakov still applicable for such cases, and is the fact that there may be only a single mode of interaction somehow included in the shielding function?
- Or are the present codes not applicable?
- Or should the 2003 paper not be interpreted in this sense?

I contacted Karl Bane about this question “think the parallel plate shielded csr doesn’t have discrete modes, unlike in a full torus. Nevertheless, it seems that the two geometries give similar results concerning the instability. I think I heard Agoh-san (who is at KEK) say that somehow the results of a closed torus compared to one with infinite circumference (but with finite rho—it doesn’t really make sense physically) basically agree. (I’ve heard a similar thing from R. Warnock.) So from their point of view it doesn’t seem to matter much whether there are discrete or continuous modes. How does this square with Sam and Gennady’s paper? I don’t know.

… ”

Answer from Karl Bane, 15 July 2010

the answer must be in the wake field

D. Zhou, K. Oide, G. Stupakov, et al, 2010

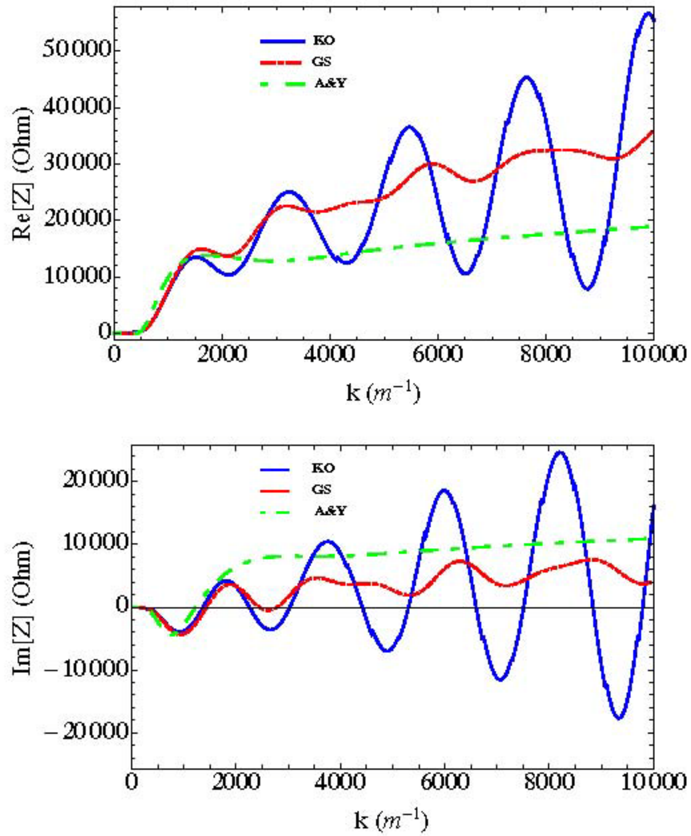


Figure 1: Total CSR impedances calculated by using Agoh and Yokoya's formulae (A&Y), Stupakov's code (GS), and Oide's code (KO).

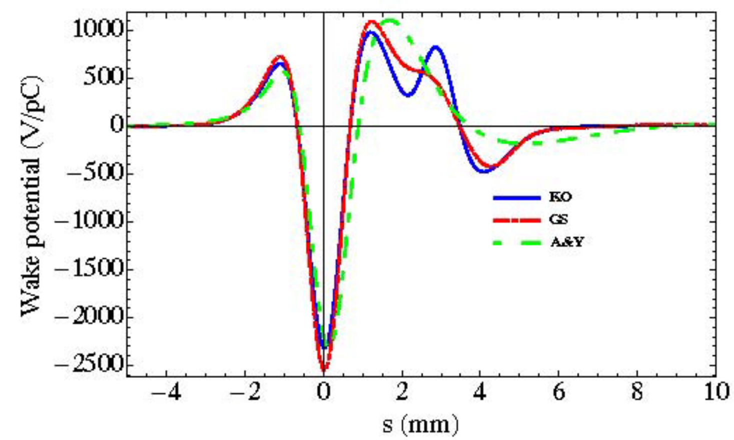


Figure 2: Total wake potentials of CSR with 0.5 mm Gaussian bunch. The head of the bunch is to the left.

should we "close" the ring to find discrete modes?

what happens in the single-mode regime?

$$\Delta\Omega - v_g \Delta q = -\frac{n_b \lambda v_g}{\eta\omega_0\delta_0^2}\int d\xi \frac{d\rho_0}{d\xi}\left(\frac{\Delta\Omega - c\Delta q}{\eta\omega_0\delta_0} + \xi + i\epsilon\right)^{-1}, \quad (20)$$

Heifets-Stupakov 2003

where $\Delta q = q - q_0$.

Depending on the ratio $\Delta\Omega/\eta\omega_0\delta_0$, there are two possible regimes for the instability: a large energy spread regime, when $|\Delta\Omega| \ll |\eta\omega_0\delta_0|$, and a "cold beam" approximation when the opposite inequality holds. We consider here the latter case only, as more relevant to the parameters of the existing accelerators. In this case, we can evaluate the integrand in Eq. (20) asymptotically in the limit $|(\Delta\Omega - c\Delta q)/\eta\omega_0\delta_0| \gg 1$, which results in the cubic dispersion equation:

$$(\Delta\Omega - \Delta q v_g)(\Delta\Omega - \Delta q c)^2 = -n_b \lambda v_g \eta\omega_0. \quad (21)$$

For $\Delta q = 0$, one of the roots has a positive imaginary part:

$$\Delta\Omega = \mu e^{i\pi/3}, \quad (22)$$

where we introduced the parameter μ

$$\mu = (n_b \lambda v_g \eta\omega_0)^{1/3} = c\left[\frac{r_e n_b \omega_0 \eta \chi}{c\gamma}(1 - \beta_g)\right]^{1/3}. \quad (23)$$

Note that for a cold beam there is no threshold for the instability. The estimate of the integral term in the dispersion equation used above neglects the Landau damping and is valid provided $|\mu| \gg \eta\omega_0\delta_0$.

According to [2] in the single-mode driven case Landau damping can be neglected and there is always an instability if

$$C03b \equiv \frac{\left(\frac{r_e n_b 2}{\gamma\sqrt{a\rho}}\right)^{1/3}}{\left(\eta\frac{\pi}{a}\sqrt{\frac{\rho}{a}}\sigma_\delta\right)} \gg 1$$

This condition may be fulfilled for the SuperKEKB and SuperB HER and LER rings, for CLIC and the old version of the SuperKEKB Damping Ring (where however this theory does not apply). It is not fulfilled for the new SuperKKB damping ring, and for the ATF Damping Ring at either 1.28 or 1.0 GeV, as illustrated in the following table.

	SuperKEKB LER	SuperKEKB HER	SuperKEKB DR	ATF	ATF lower energy	SuperB LER	SuperB HER	CLIC DR	KEKB DR OLD
C03b	5.6	5.9	0.3	0.7	0.9	2.2	1.5	3.7	(2.1)

	SuperKEKB e+ DR NOW	SuperKEKB e+ DR OLD DESIGN
beam energy [GeV]	1.1	1.0
slip factor η	0.017	0.00343
rms momentum spread $\sigma_{\delta,rms}$ [%]	0.055	0.054
bunch population [10^9]	37.5	37.5
circumference C [m]	135	135.5
bending radius ρ [m]	2.65	2.2
vert. beam pipe radius b [cm]	1.6	1.4?
Stupakov-Heifets parameter Λ	**67**	**430**
ρ/b	**166**	**129**
σ_z [cm]	**0.7**	**0.51**
$\rho/(2\,\Lambda^{3/2})$ [cm]	**0.24**	**0.012**
$N_b r_0/(\sqrt{2\pi}\,\sigma_z\sigma_\delta\,\gamma)$	0.0051	0.078
β_x at bend [m]	1.5	1.5?
ε_x [nm]	2100→ 41	2100→ 41
σ_x at bend [μm]	248	248?
$\rho^{4/3}/(\sigma_x\beta_x)^{2/3}$	710	553
τ_x [ms]	11	11
C03	0.0033	7.68??
Q_s	-0.015	-0.00788
$N_{nb,thr}$	**$4.5\text{x}10^{10}$**	**$1.1\text{x}10^{10}$**
$N_b/N_{nb,thr}$	0.83	**3.27**
$\Lambda b/\rho$	0.40	**3.32**

For the **earlier version of the SuperKEKB DR** all indicators strongly suggest that the CSR instability should occur. Remarkably this is the **only case where the inequality (26) or [2] is fulfilled**. So we can be certain that the formalism of [1] and result of [3] are valid!

conclusions

- 2002 formulae from Stupakov & Heifets perhaps not applicable in most cases, except old SuperKEKB DR which is predicted to be clearly unstable
- is IPAC10 result from Bane, Cai and Stupakov applicable?
- in this case SuperKEKB LER, HER and DR are predicted to be stable with respect to CSR , while SuperB LER might be marginally unstable (but only 4% above threshold)
- CSR instability could be studied in ATF damping ring by lowering the ring energy from 1.28 GeV to 1.0 or 1.1 GeV