CSR effect at USSR

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Acknowledgements

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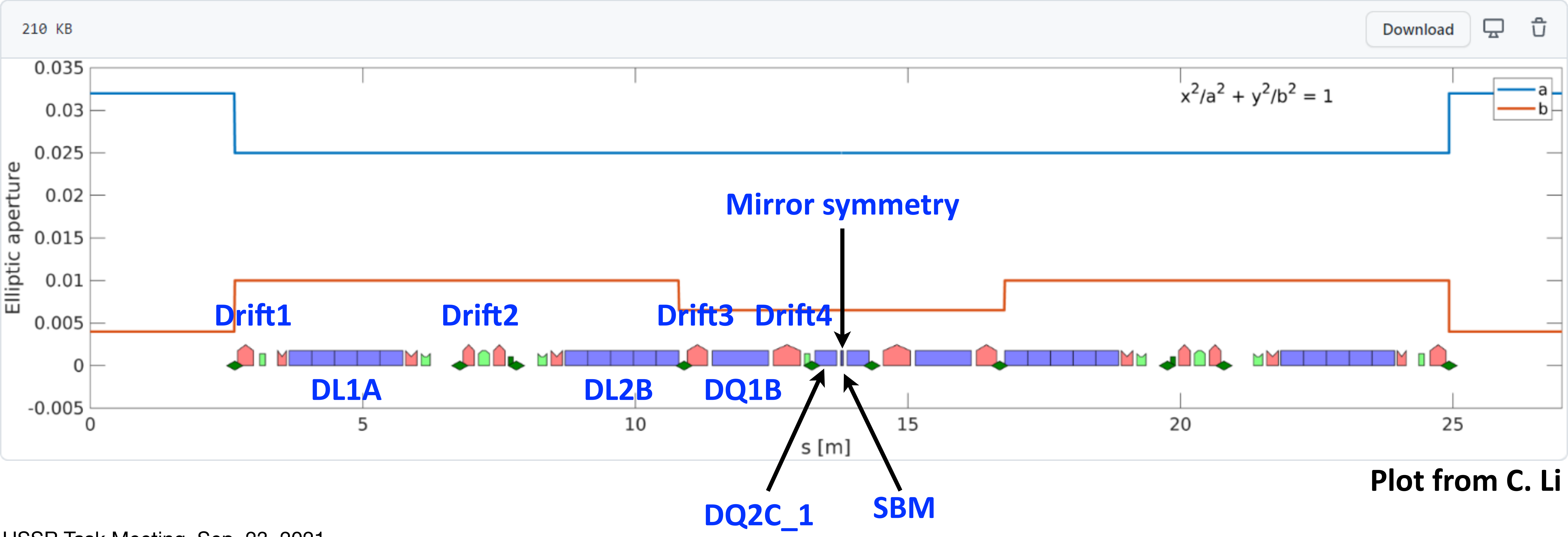
Outline

- CSR impedance calculation using CSRZ
- Threshold of CSR instability
- Summary

CSR impedance calculation

- Lattice configuration
  - Elliptical chamber a/b=25/10 mm (high profile) and a/b=25/6.5 mm (low profile) [1].
  - LGB: DL1A and DL2B (high profile); Single bend: DQ1B, DQ2C, and SBM (low profile).
  - Calculation-1: Calculate CSR impedance of each LGB or single bend.
  - Calculation-2: Calculate CSR impedance of one lattice cell.
  - 40 cells in total.



Plot from C. Li

[1] Chao Li, USSR Task Meeting, Sep. 23, 2021.

CSR impedance calculation

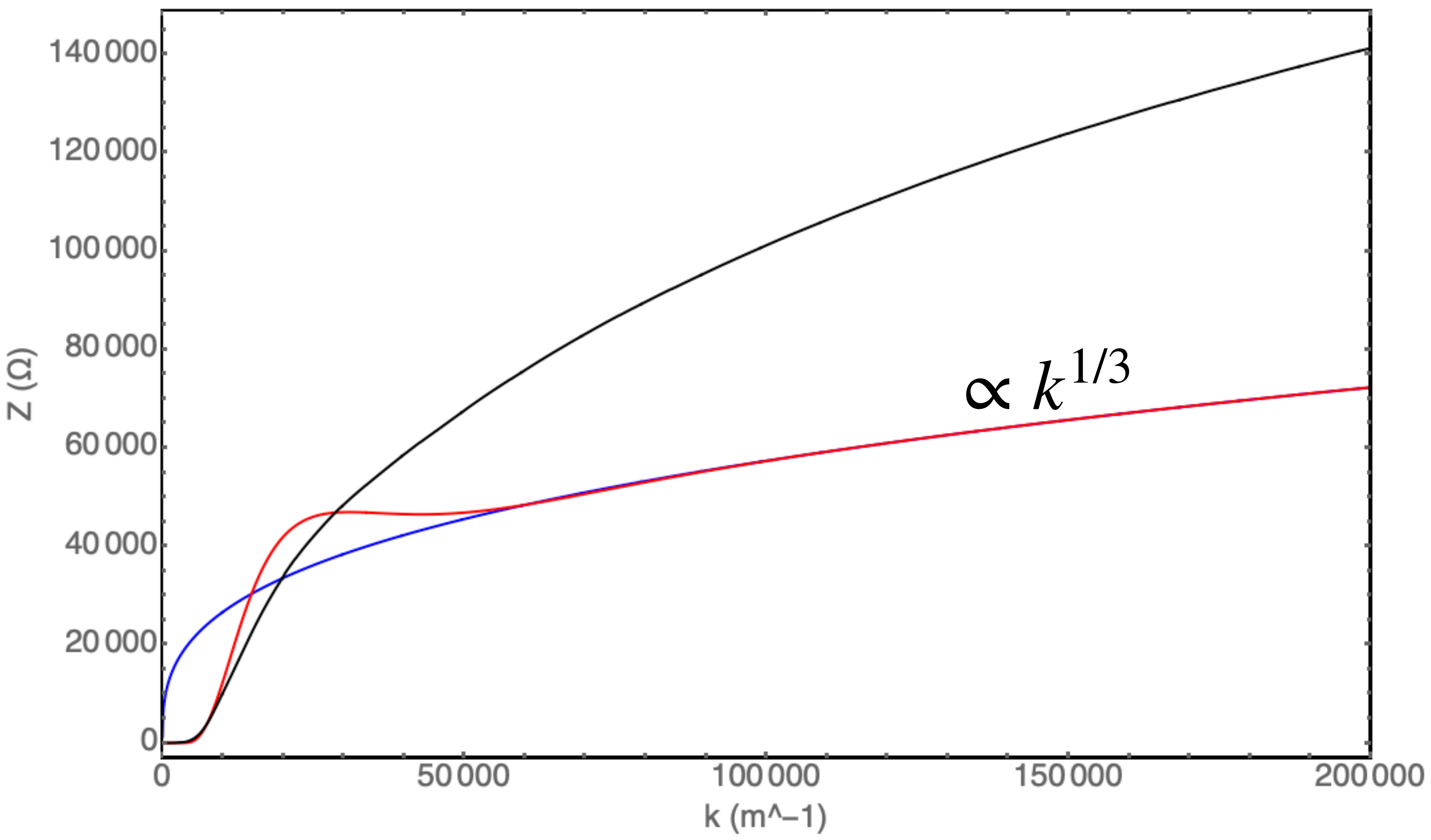
Critical frequency of SR:

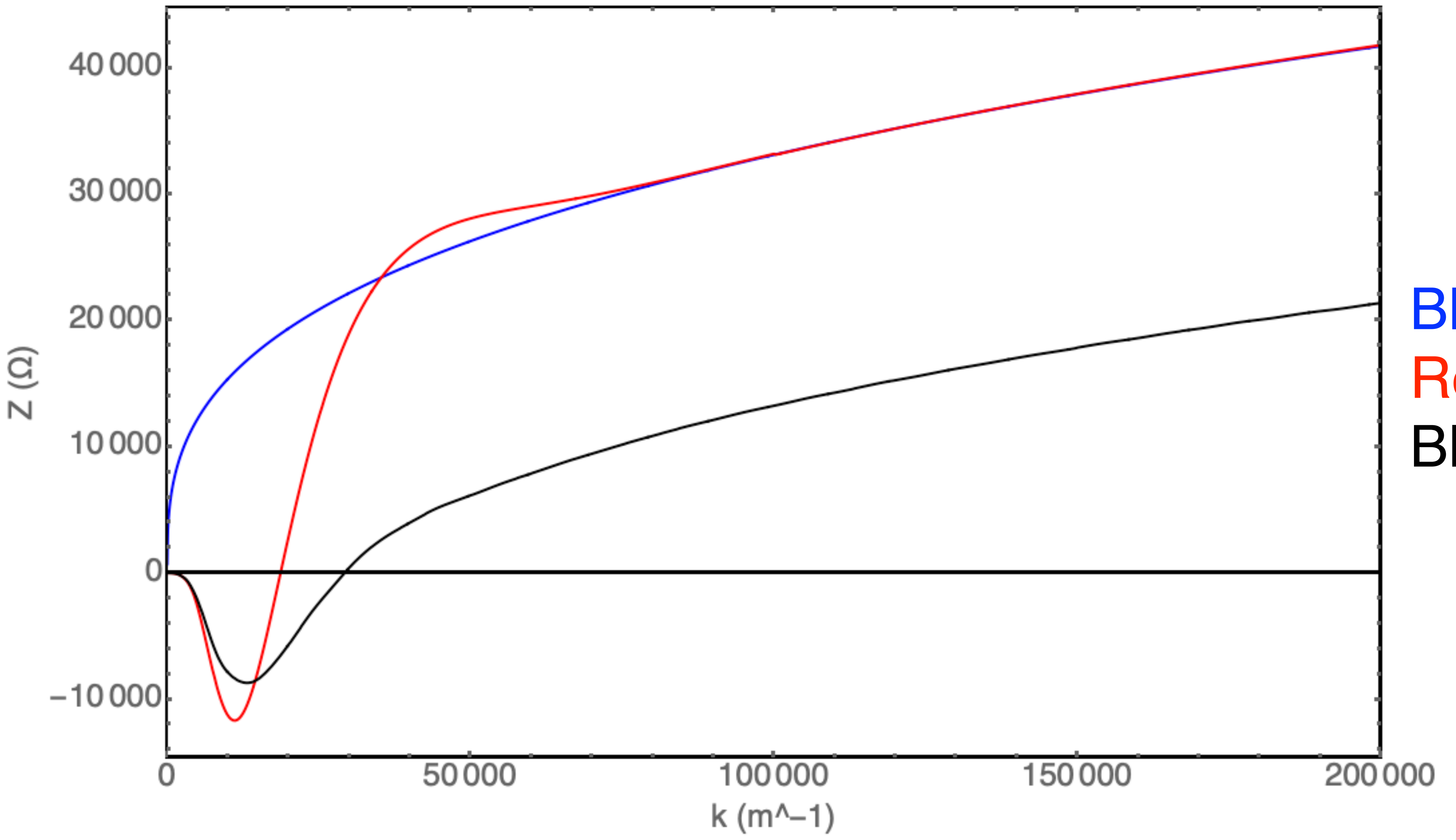
$$k_c = \frac{3\gamma^3}{2R} \approx 3.7 \times 10^{10}\ \text{m}^{-1}$$

with $R = 65.7$ m

- Results of calculation-1
  - Compared with free-space (FS) and parallel-plates (PP) models of CSR impedance.
  - Calculation-1: Calculate CSR impedance of each LGB or single bend.
  - Sum up impedances of all bends.
  - Comment-1: Similar low-frequency shielding between CSRZ and PP model.
  - Comment-2: Big difference at high frequencies between CSRZ and FS models. This is due to transient effects at entrance and exit of the bends (short bends are used).

$$\frac{2\pi R}{R^{2/3}} = \sum_i \frac{L_i}{R_i^{2/3}}$$





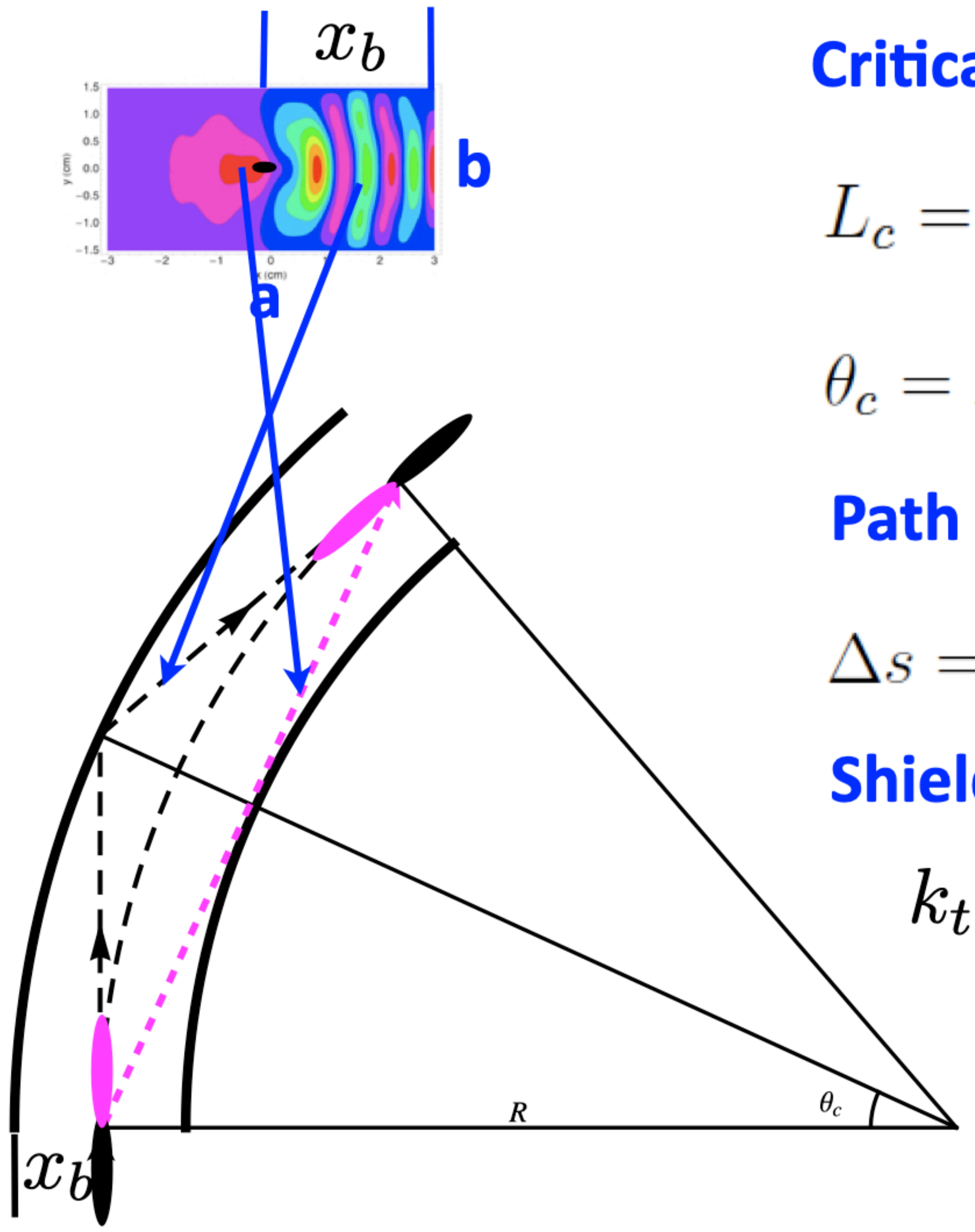
Blue: FS
Red: PP
Black: CSRZ

$k = 2 \times 10^5\ \text{m}^{-1}$ corresponds to $f = \frac{kc}{2\pi} = 9.54 \times 10^{12}$ Hz.

CSR impedance calculation

- Results of calculation-1
  - The aspect ratio of USSR chamber is larger than 2 (25/10 for high profile and 20/6.5 for low profile), therefore the outer-wall reflection has weak effect on CSR impedance with single-bend model.
  - The critical length of a single bend can be calculated by $L_c \approx 2\sqrt{2Rx_b} \approx 3.6$ m with $R \approx 65.7$ m and $x_b = 25$ mm. Most of the dipole magnets in USSR have lengths smaller than 3.6 m, so the transient effects at the entrance and exit are significant.

Outer-wall reflection can be well approximated by a geometric model [Derbenev (1995), Carr (2001), Sagan (2009), Oide (2010)]



Critical length (Catch-up distance):

$$L_c = 2R\theta_c \approx 2\sqrt{2Rx_b} \qquad x_b \ll R$$

$$\theta_c = \mathrm{ArcCos}\left(R/(R+x_b)\right) \approx \sqrt{2x_b/R}$$

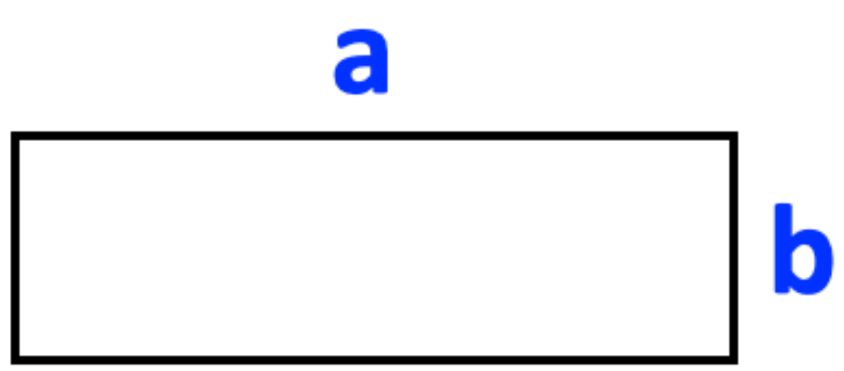
Path difference:

$$\Delta s = 2R(\mathrm{Tan}(\theta_c) - \theta_c) \approx \frac{4}{3}\sqrt{\frac{2x_b^3}{R}}$$

Shielding threshold:

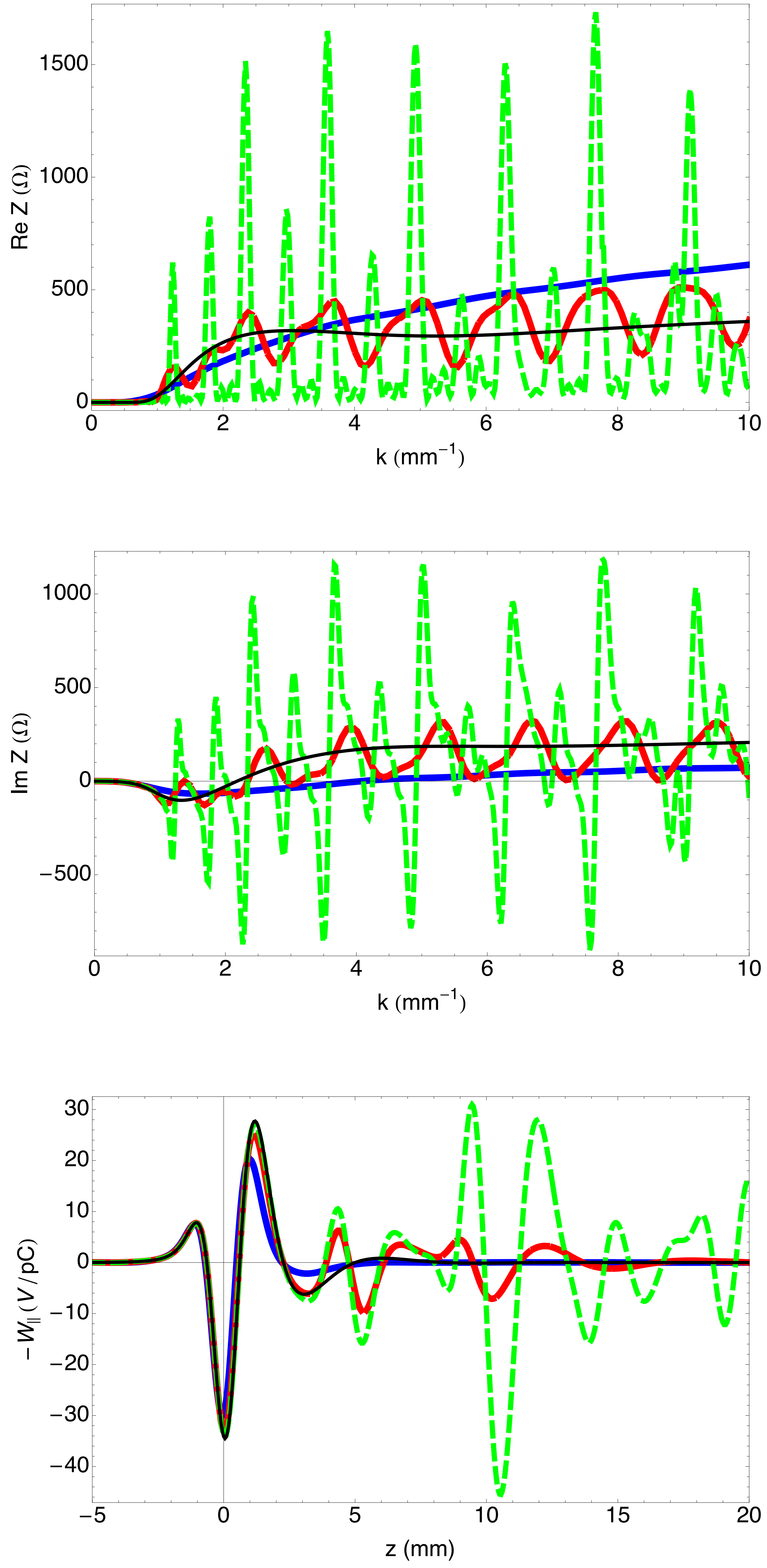
$$k_{th} = \pi\sqrt{R/b^3}$$

Y. S. Derbenev, et al., TESLA FEL-Report 1995-05 (1995).
G. L. Carr, et al., PAC'01, p. 377 (2001).
D. Sagan, et al., PRST-AB 12, 040703 (2009).
K. Oide, Talk at CSR mini-workshop, Nov. 08, 2010.
D. Zhou, et al., Jpn. J. Appl. Phys. 51 (2012) 016401.



CSR impedance calculation

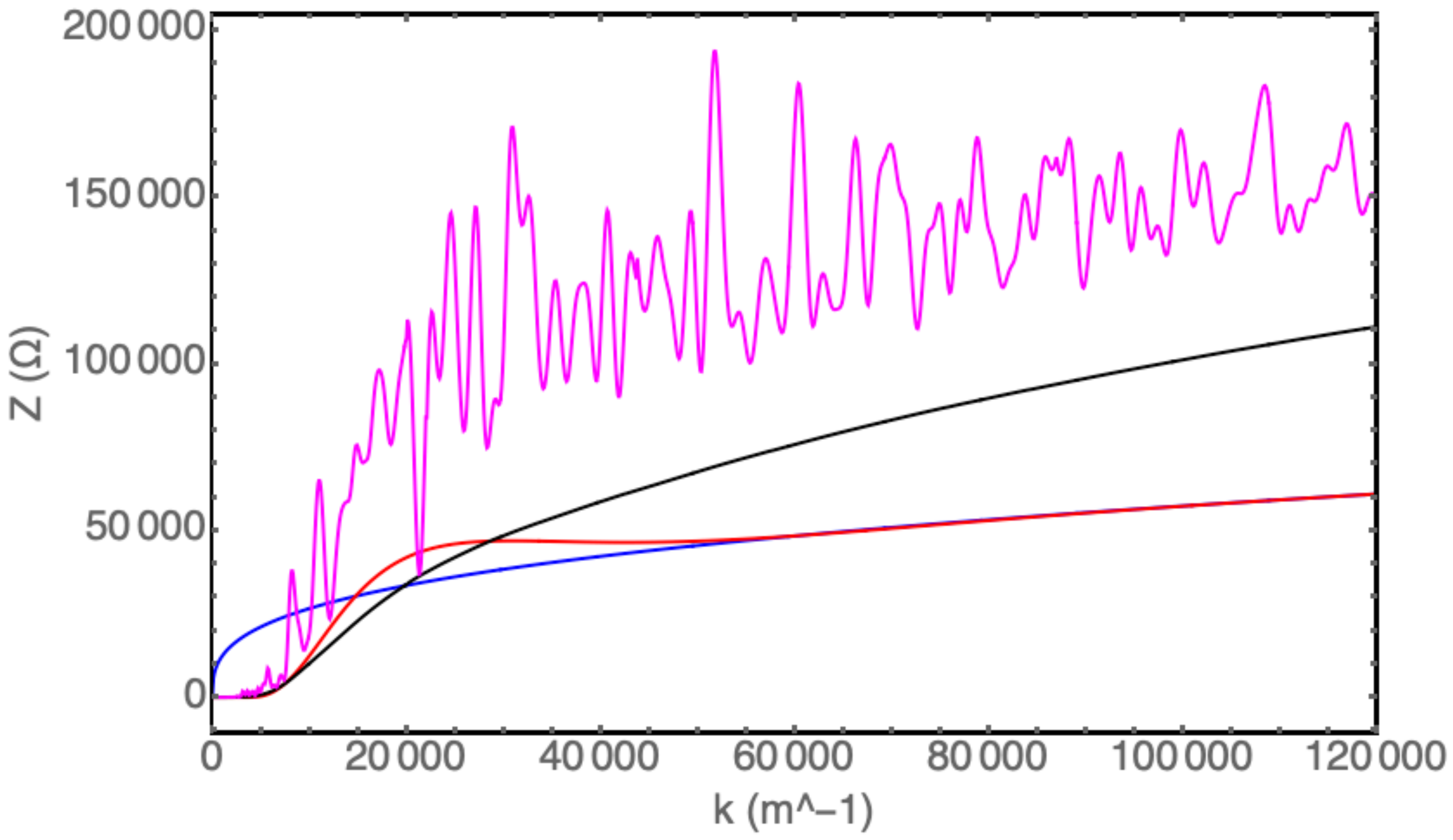
- Examples of CSR impedance by CSRZ
  - A single bend with varied length: a/b=60/30 mm, R=5 m, L_{bend}=0.5/2/8 m.
  - Black/Blue/Red/Green lines: Steady-state parallel-plates/L=0.5/ L=2/L=8 m. For convenience of comparison, the impedance amplitude is scaled to L=1 m.
  - “Short bend”: Transient effect at the entrance and exit is important.
  - “Long bend”: Excited eigenmodes of a toroidal chamber (or “whispering gallery modes” by R. Warnock [8]).
  - “Overtaking field”: Short-range wake fields, space charge like.
  - “Trailing field”: Long-range wake fields, relevant to excited eigenmodes.

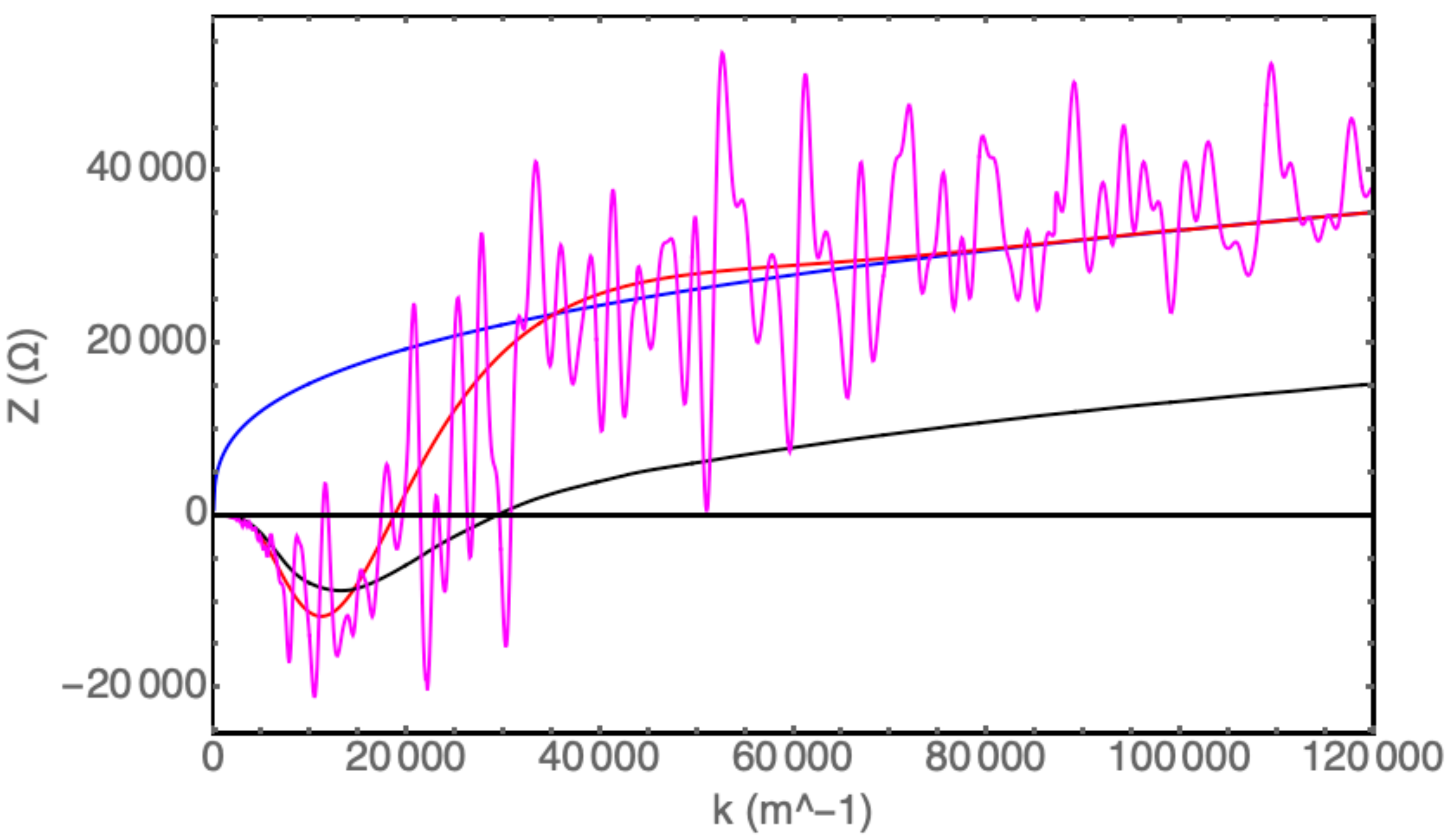


[8] R. Warnock, arXiv:1708.05500

CSR impedance calculation

- Results of calculation-2
  - Compared with free-space (FS) and parallel-plates (PP) models of CSR impedance.
  - Calculation-2: Calculate CSR impedance of 1 lattice cell.
  - Sum up impedances of all cells.
  - Comment-1: High profile of chamber is used since CSRZ cannot handle transitions of chambers.
  - Comment-2: Computing time is long so I used crude meshes in calculations at high frequencies. Numerical errors can be improved by refining meshes.
  - Comment-3: Is there “geometric” impedance from a long curved chamber?





Threshold of CSR instability

- Stupakov-Heifets (S-H) theory [2]
  - Beam becomes unstable when $(\pi R/(2b))^{3/2} \leq kR < 2\Lambda^{3/2}$. The right inequality is obtained by solving the dispersion relation with steady-state free-space CSR impedance. The left inequality is from chamber shielding.
  - For Gaussian bunch, the theory is valid when $k\sigma_z \gg 1$ (coasting-beam approximation).
  - The S-H theory was translated to bunch current threshold [2]:

$$I_b > \frac{\pi^{1/6}}{\sqrt{2}} \frac{ec}{r_0} \frac{\gamma}{\rho^{1/3}} \alpha_p \delta_0^2 \sigma_z \frac{1}{\lambda^{2/3}}$$

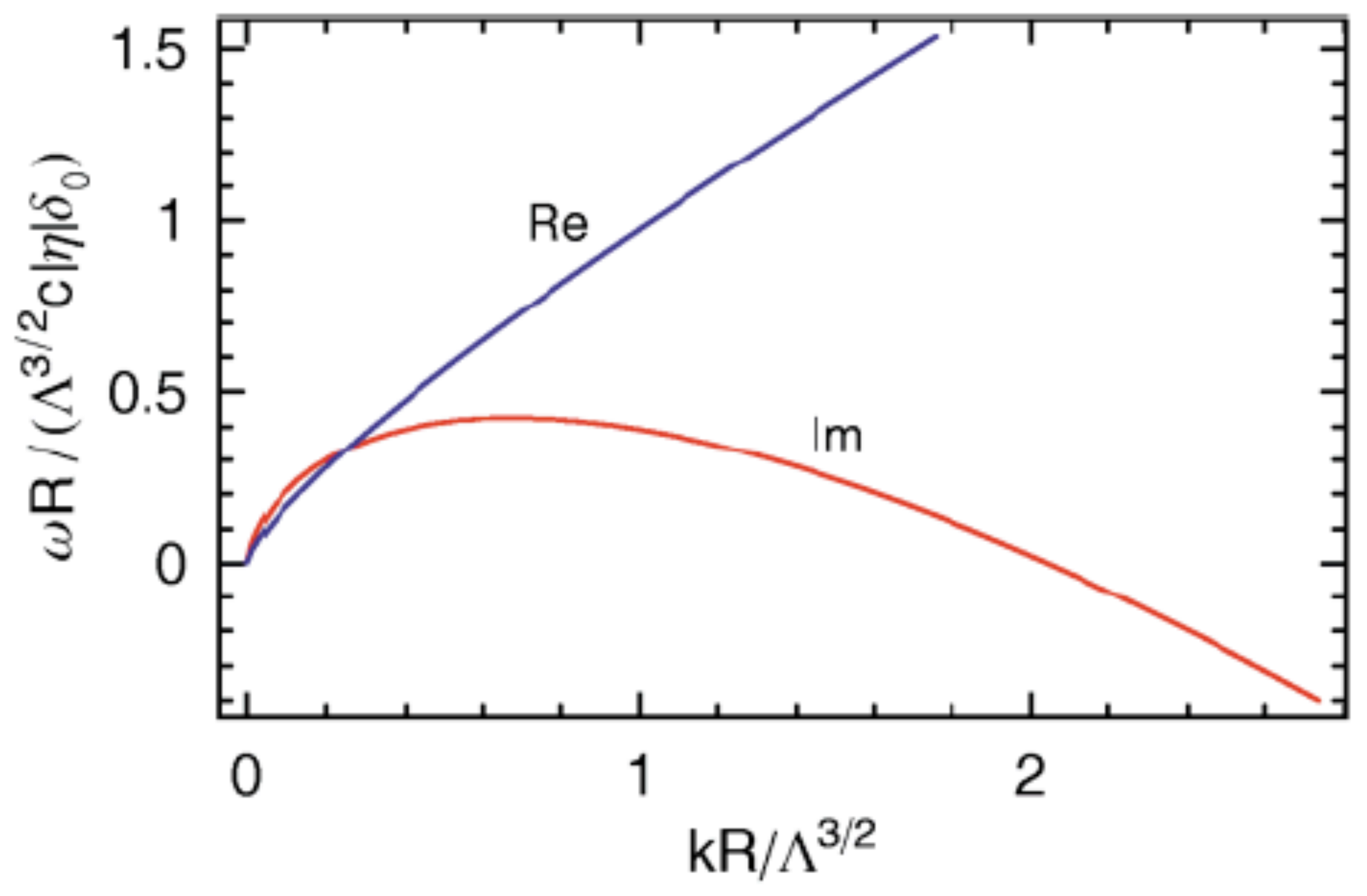


FIG. 1. (Color) The imaginary (Im) and real (Re) parts of the frequency ω as functions of $kR/\Lambda^{3/2}$, for a positive value of η. For negative values of k, the frequency can be found from the relation $\omega(-k) = -\omega^*(k)$ which follows from Eq. (9).

$$1 = \frac{ir_0 cZ(k)}{\gamma} \int \frac{d\delta\,(d\rho_0/d\delta)}{\omega + ck\eta\delta}, \qquad (9)$$

[2] G. Stupakov and S. Heifets, PRST-AB 5, 054402 (2002).

Threshold of CSR instability

- Estimate of threshold (preliminary results)
  - Beam parameters referred to [1].
  - FS CSR impedance model:

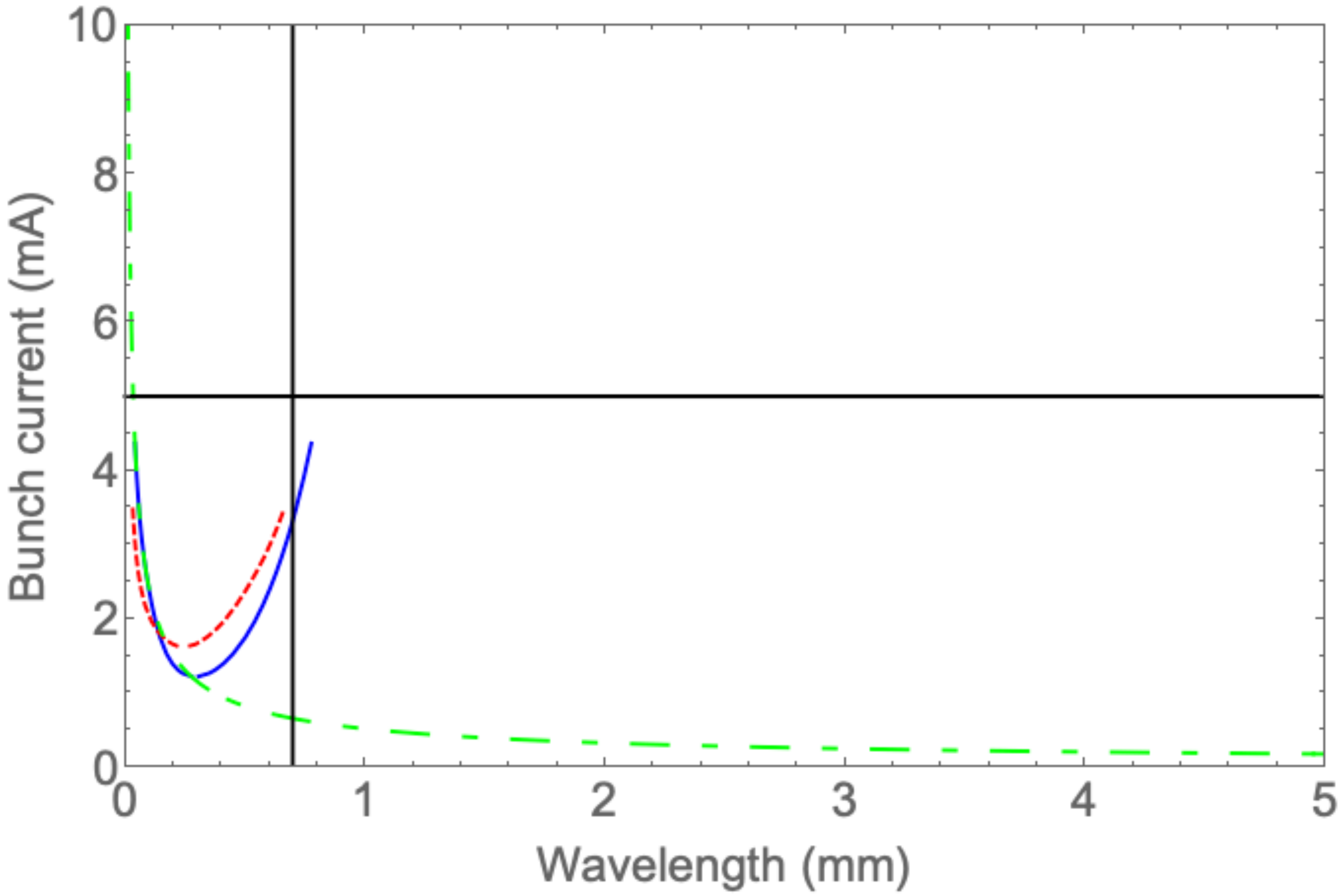
$$\frac{Z_L(k)}{L} = \frac{Z_0}{2\pi \cdot 3^{1/3}} \Gamma\left(\frac{2}{3}\right) \left[\frac{ik}{R^2}\right]^{1/3} \qquad \frac{2\pi R}{R^{2/3}} = \sum_i \frac{L_i}{R_i^{2/3}}$$

  - Shielding threshold for wavelength (assume b=10 mm for instability analysis):

$$\lambda_{th} = 2\sqrt{\frac{(2b)^3}{R}}$$

  - Analysis is done for the most serious case: $\sigma_{z0} = 2.9$ mm, $I_{bunch} = 5$ mA with 40 bunches [1].
  - Estimated threshold is around 1.2 mA for PP model, and 1.6 mA for CSRZ model.
  - Interplay of CSR and other wakes is expected.

Parameters	Values
Circumference (m)	1110.537
Energy (GeV)	6
Mom. compaction	5.758E-05
Energy spread(10^{-4})	8.6
Bunch length (mm)	2.9
Bunch current (mA)	5



Blue: PP
Red: CSRZ
Green: FS

[2] G. Stupakov and S. Heifets, PRST-AB 5, 054402 (2002).

Summary

- Preliminary CSR impedance calculation using CSRZ was done.
  - Transient effect makes single-bend CSR impedance different from FS steady-state CSR model.
  - Multi-bend interference needs further investigations.
- Preliminary results of CSR instability analysis was done.
  - Interplay of CSR and other wakes (geometric and RW) is an issues to be studied.