# CSR effect in electron storage rings

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Acknowledgements

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# Outline

- Analytic theories of CSR driven microwave instability threshold
- CSR impedance calculation
- Prediction of CSR effects using tracking codes or Vlasov solver
- Summary

# Analytic theories of CSR driven MWI threshold

- Stupakov-Heifets (S-H) theory [1]
  - Beam becomes unstable when $(\pi R/(2b))^{3/2} \leq kR < 2\Lambda^{3/2}$. The right inequality is obtained by solving the dispersion relation with steady-state free-space CSR impedance. The left inequality is from chamber shielding.
  - For Gaussian bunch, the theory is valid when $k\sigma_z \gg 1$ (coasting-beam approximation).
  - The S-H theory was translated to bunch current threshold [2]:

$$I_b > \frac{\pi^{1/6}}{\sqrt{2}} \frac{ec}{r_0} \frac{\gamma}{\rho^{1/3}} \alpha_p \delta_0^2 \sigma_z \frac{1}{\lambda^{2/3}}$$

- Improvements on S-H theory
  - Simulation of MWI with steady-state parallel-plates model of CSR impedance [3].
  - Simulation of MWI with steady-state rectangular-chamber model of CSR impedance [4].

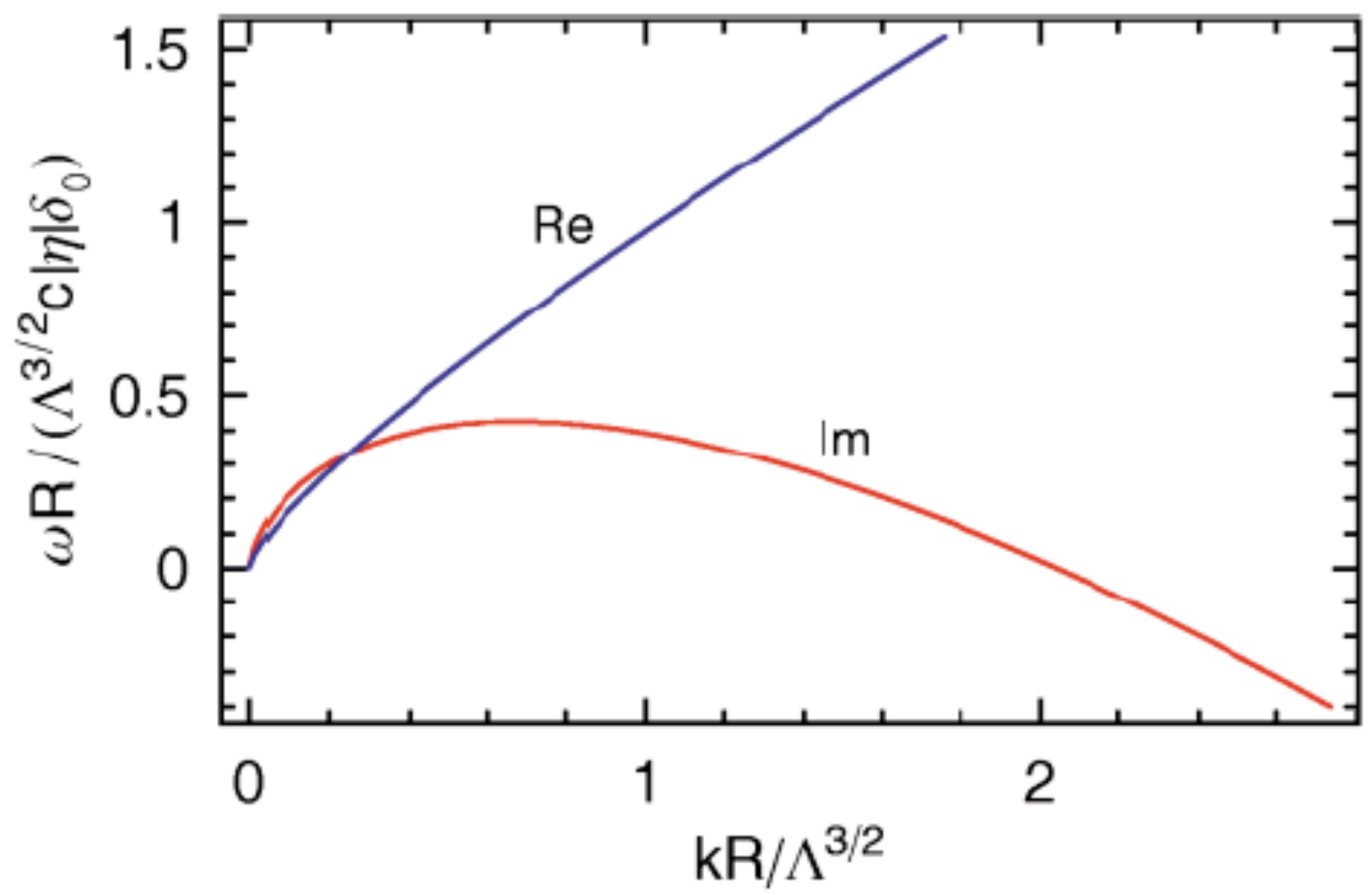



FIG. 1. (Color) The imaginary (Im) and real (Re) parts of the frequency $\omega$ as functions of $kR/\Lambda^{3/2}$, for a positive value of $\eta$. For negative values of $k$, the frequency can be found from the relation $\omega(-k) = -\omega^*(k)$ which follows from Eq. (9).

$$1 = \frac{ir_0 cZ(k)}{\gamma} \int \frac{d\delta\,(d\rho_0/d\delta)}{\omega + ck\eta\delta}, \qquad (9)$$


[1] G. Stupakov and S. Heifets, PRST-AB 5, 054402 (2002).

[2] J. Byrd, et al., PRL 89, 22, Nov. 2002.

[3] K.L. F. Bane, Y. Cai, and G. Stupakov, Phys. Rev. ST Accel. Beams **13**, 104402 (2010).

[4] Y. Cai, Phys. Rev. ST Accel. Beams **17**, 020702 (2014).

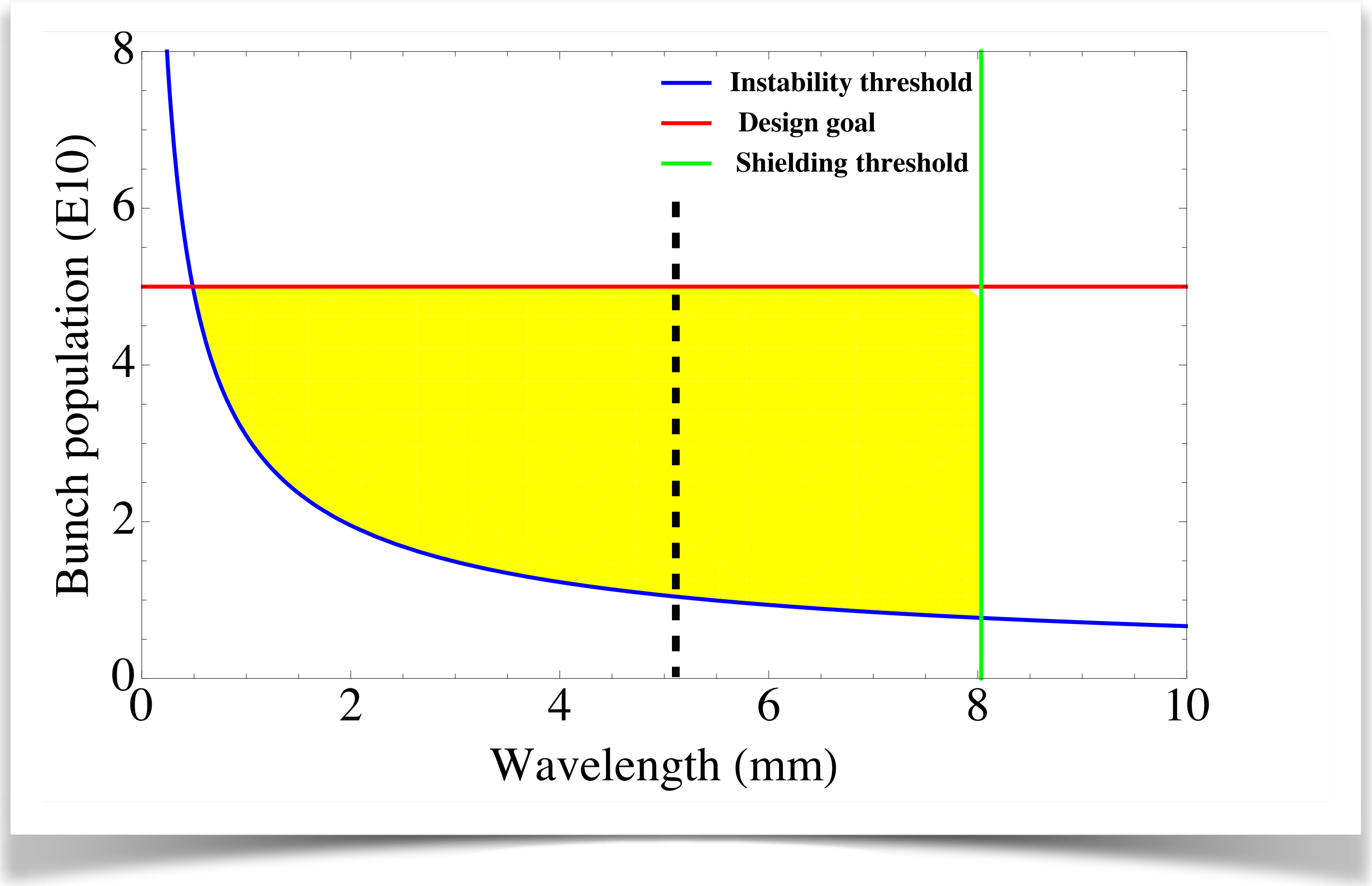
# Analytic theories of CSR driven MWI threshold (cont'd)

- Apply S-H theory to electron storage rings
  - Quick estimate of CSR instability.
  - Very useful in the design stage of a storage ring.
  - "Yellow region" indicates "severity of instability".
  - For rings where CSR is marginally of concern, MWI simulations are required.

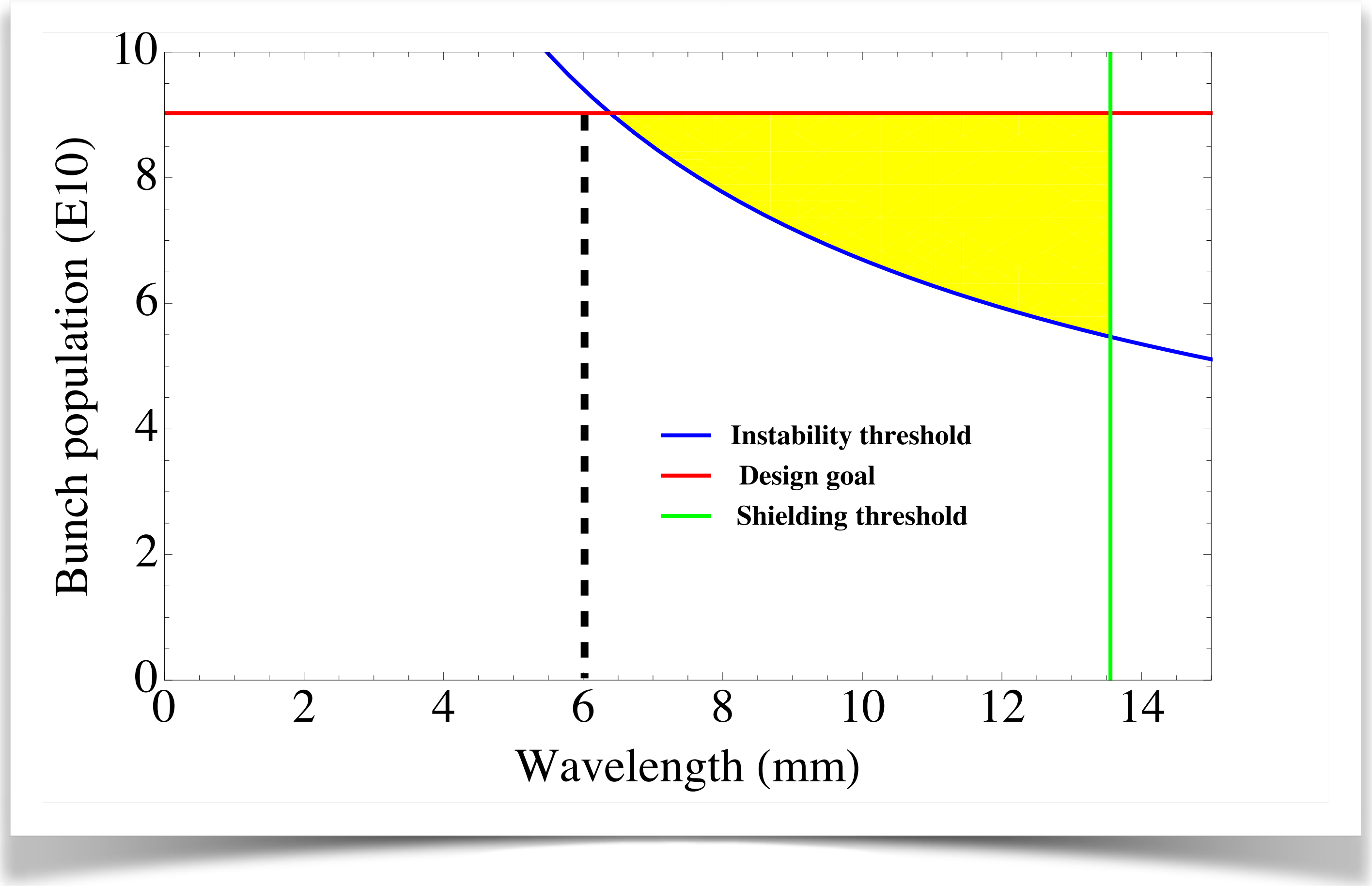
| Parameters | SuperKEKB DR[1] | SLC DR[2] | ATF[3] | SuperKEKB LER[4] | SuperKEKB LER[5] | PEP-II LER[6] | ALS[6] | KEKB LER[7] |
|---|---|---|---|---|---|---|---|---|
| Circumference (m) | 135.5 | 35.27 | 138.6 | 3016 | 3016 | 2200 | 196 | 3016 |
| Energy (GeV) | 1 | 1.21 | 1.54 | 4 | 3.5 | 3.1 | 1.5 | 3.5 |
| Bending radius | 2.43623 | 2.0372 | 5.73 | 15.87 | 15.87 | 13.7 | 4 | 15.87 |
| Mom. compaction | 3.43E-03 | 0.01814 | 2.17E-03 | 2.74E-04 | 2.74E-04 | 1.31E-03 | 1.41E-03 | 3.31E-04 |
| Energy spread($10^{-4}$) | 5.44 | 7.3 | 5.56 | 8.14 | 7.13 | 8.1 | 7.1 | 7.27 |
| Bunch length (mm) | 5.1 | 5.9 | 5 | 6 | 3 | 10 | 7 | 4.58 |
| Bunch population ($10^{10}$) | 5 | 5 | 2 | 9.03 | 11.7 | 9.16 | 12.3 | 6.47 |
| Pipe height@bends (mm) | 34 | 15.6 | 24 | 90 | 90 | 50 | 40 | 94 |
| Total bend. radius($2\pi$)[8] | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

1) Design Version 1.140, Apr. 2010
2) SLC design handbook, Dec. 1984
3) ATF design and study report, KEK Internal 95-4
4) Nano-beam option design, Feb. 2008
5) High-current option design
6) G. Stupakov and S. Heifets, PRST-AB 5, 054402 (2002)
7) Machine operating parameters, Jun.17, 2009
8) Assumed

**SuperKEKB DR (Design Ver. 1.140)**




**SuperKEKB LER (Design nano-beam option)**

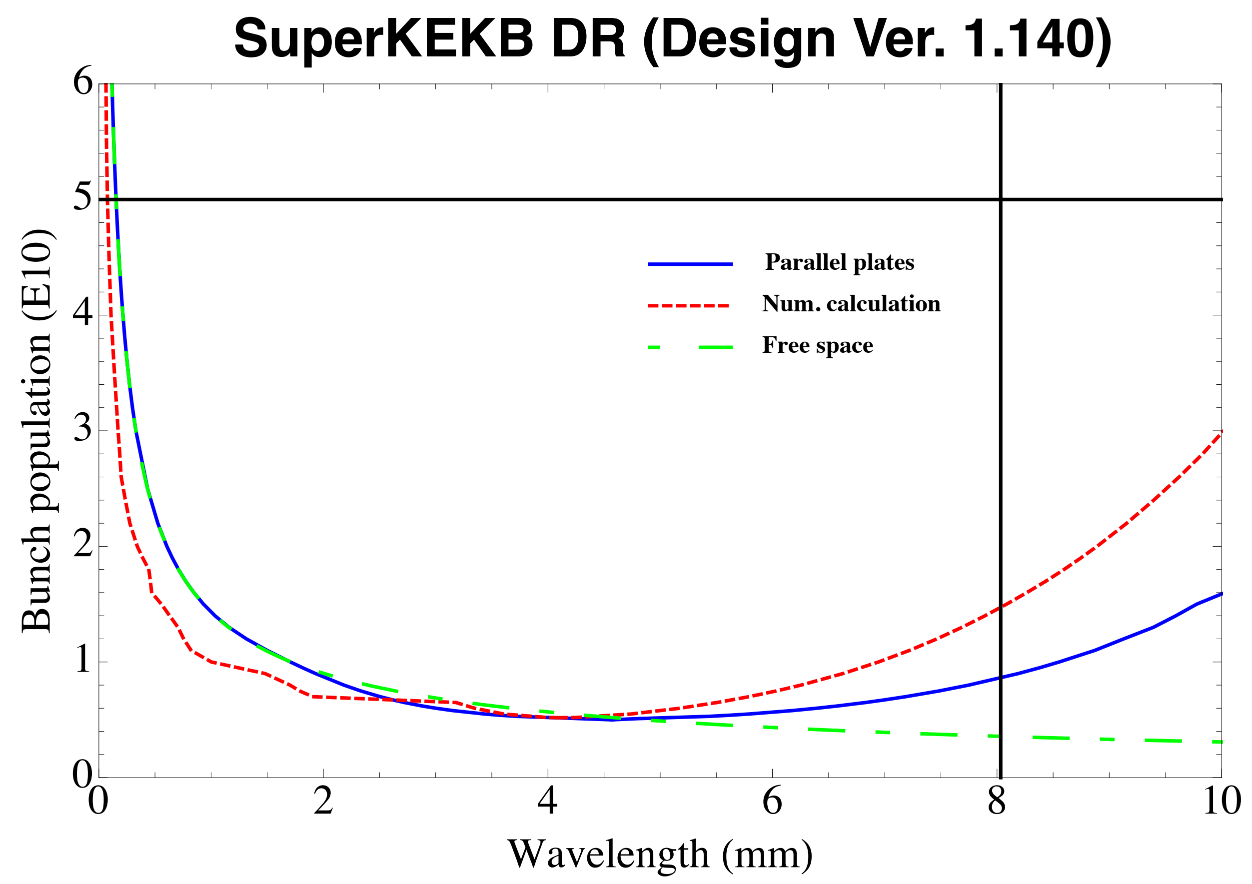


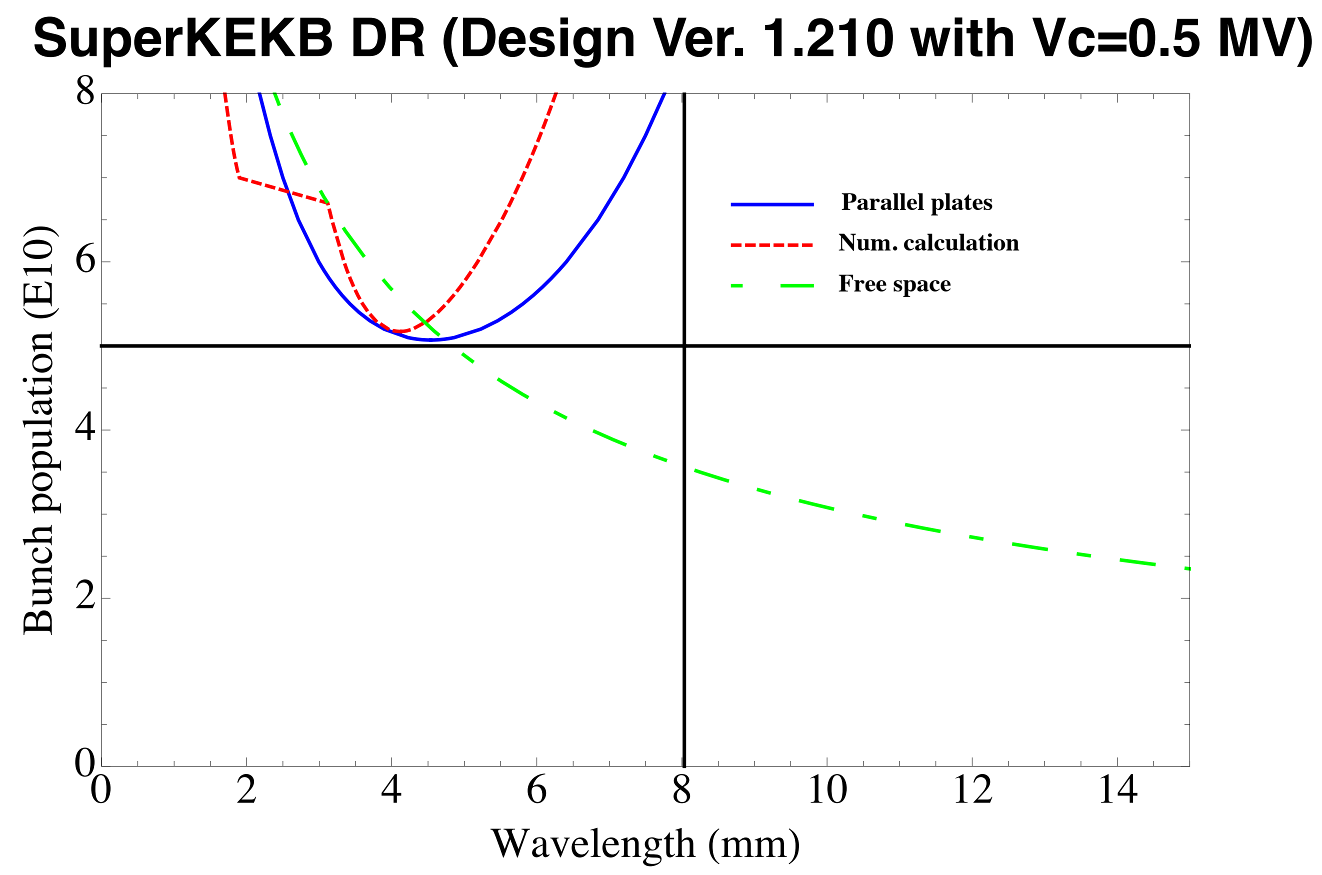
# Analytic theories of CSR driven MWI threshold (cont'd)

- The case of arbitrary impedance $Z(k)$
  - An alternative way of extending S-H theory is to solve the dispersion relation numerically:

$$1 = -i\frac{4\pi\epsilon_0 c\Lambda}{\sqrt{2\pi}} \cdot \frac{Z(k)}{kC}\int_{-\infty}^{\infty} dp \frac{pe^{-p^2/2}}{\Omega + p}$$

  - The impedance $Z(k)$ can be obtained by analytical or numerical methods, including transient effects and/or chamber shielding.
  - For the impedance $Z(k)$, it is good to use data as a smooth function of $k$ (personal opinion).
  - For CSR impedance in storage rings, the low-frequency part of $Z(k)$ is mainly determined by chamber shielding, the high-frequency part is mainly determined by transient effects.




[5] D. Zhou et al., "CSR in the SuperKEKB Damping Ring", in Proceedings of IPAC'10, Kyoto, Japan.

# CSR impedance calculation

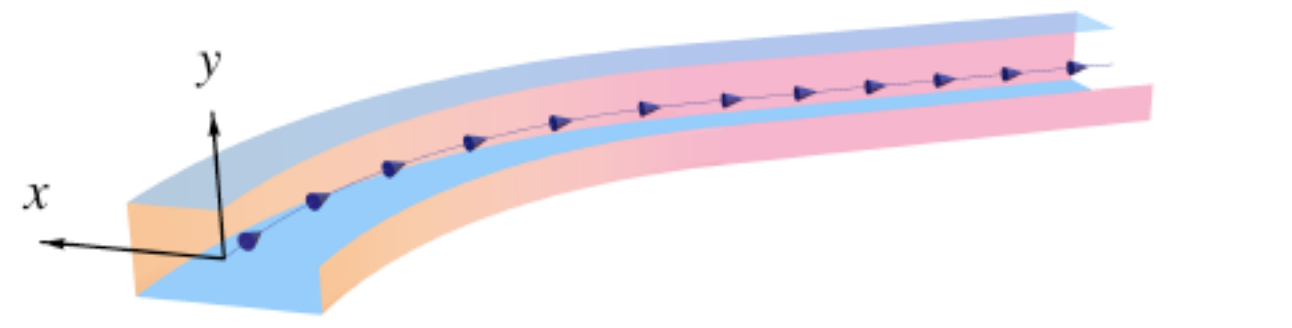
- Impedance calculation based on parabolic equation
  - G. Stupakov, T. Agoh et al. developed the method of calculating CSR impedance using parabolic equation (PE).
  - The CSRZ code following this line to solve PE:

$$\frac{\partial \vec{E}_\perp}{\partial s} = \frac{i}{2k}\left[\nabla_\perp^2 \vec{E}_\perp - \frac{1}{\epsilon_0}\nabla_\perp \rho_0 + 2k^2\left(\frac{x}{R(s)} - \frac{1}{2\gamma^2}\right)\vec{E}_\perp\right]$$

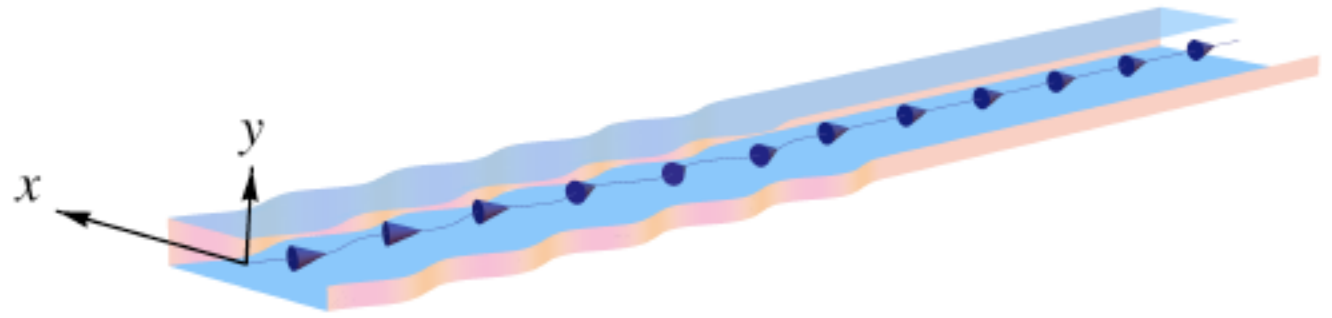
$$E_s = \frac{i}{k}\left(\nabla_\perp \cdot \vec{E}_\perp - \mu_0 c J_s\right) \qquad Z(k) = -\frac{1}{q}\int_0^\infty E_s(x_c, y_c)ds$$

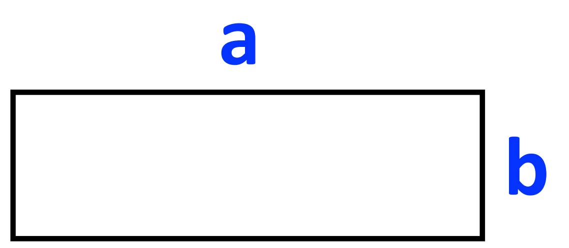
  - CSRZ takes into account: Arbitrary curvature of beam orbit $R(s)$ (CSR), finite beam energy $\gamma$ (space charge effects, SC), and resistive wall (RW). The total impedance is not a simple sum of $Z_{CSR} + Z_{SC} + Z_{RW}$, but includes their interaction.
  - CSRZ uses Gaussian charge distribution in x-y plane, assuming $\sigma_x > \sigma_y$. Self-field is calculated by Bassetti-Erskine formulae.
  - Currently, CSRZ assumes uniform rectangular chamber.
  - See Ref.[6] for an overview and Ref.[7] for details of CSRZ code and examples of its application.

**A single bend**




**A wiggler with "wiggling" chamber**



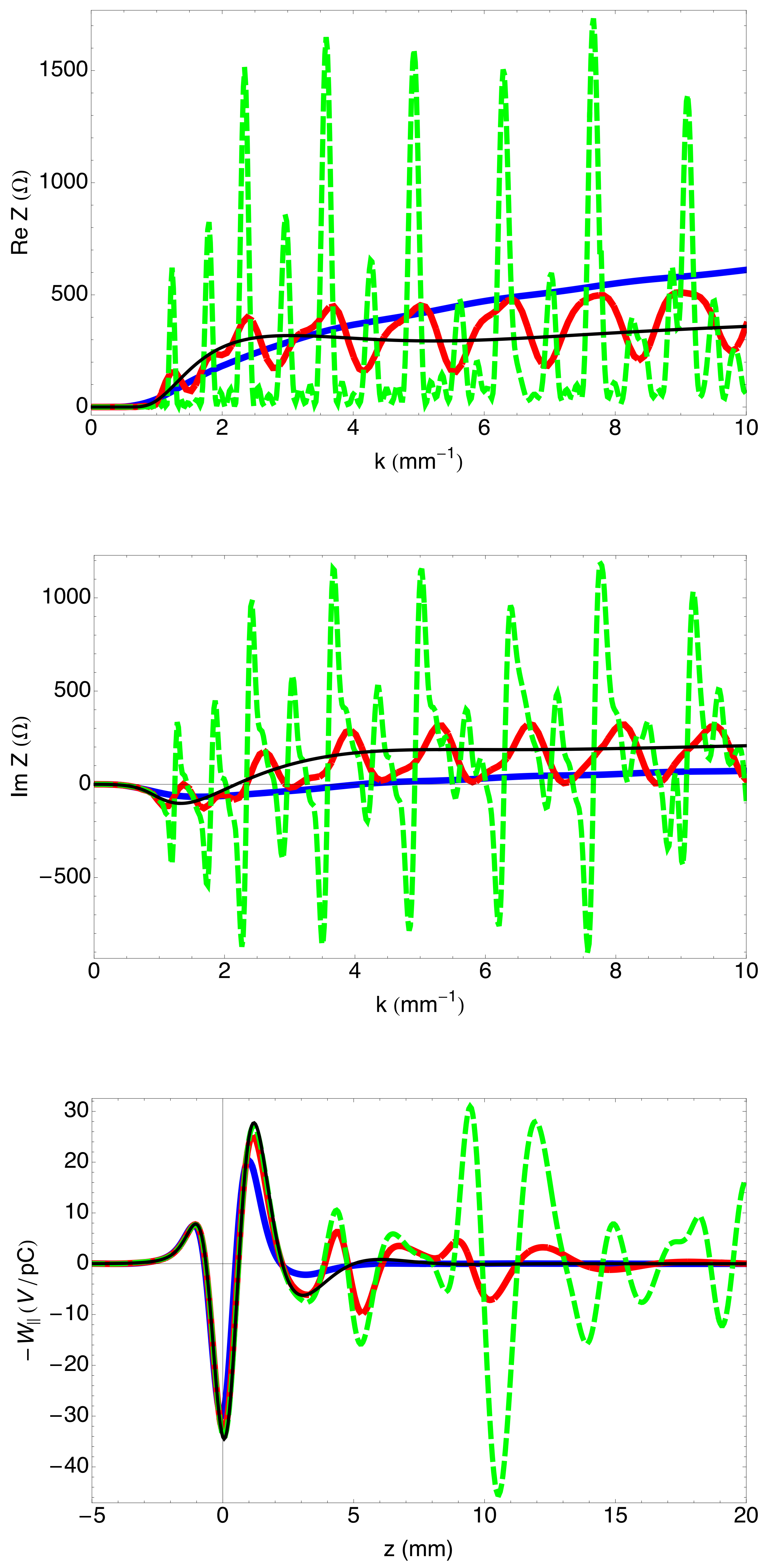


[6] D. Zhou et al., "An Alternative 1D Model for CSR with Chamber Shielding", in Proceedings of IPAC'12, New Orleans, Louisiana, USA.

[7] D. Zhou, Coherent Synchrotron Radiation and Microwave Instability in Electron Storage Rings, Ph.D. thesis, SOKENDAI and KEK, 2011.

# CSR impedance calculation (cont'd)

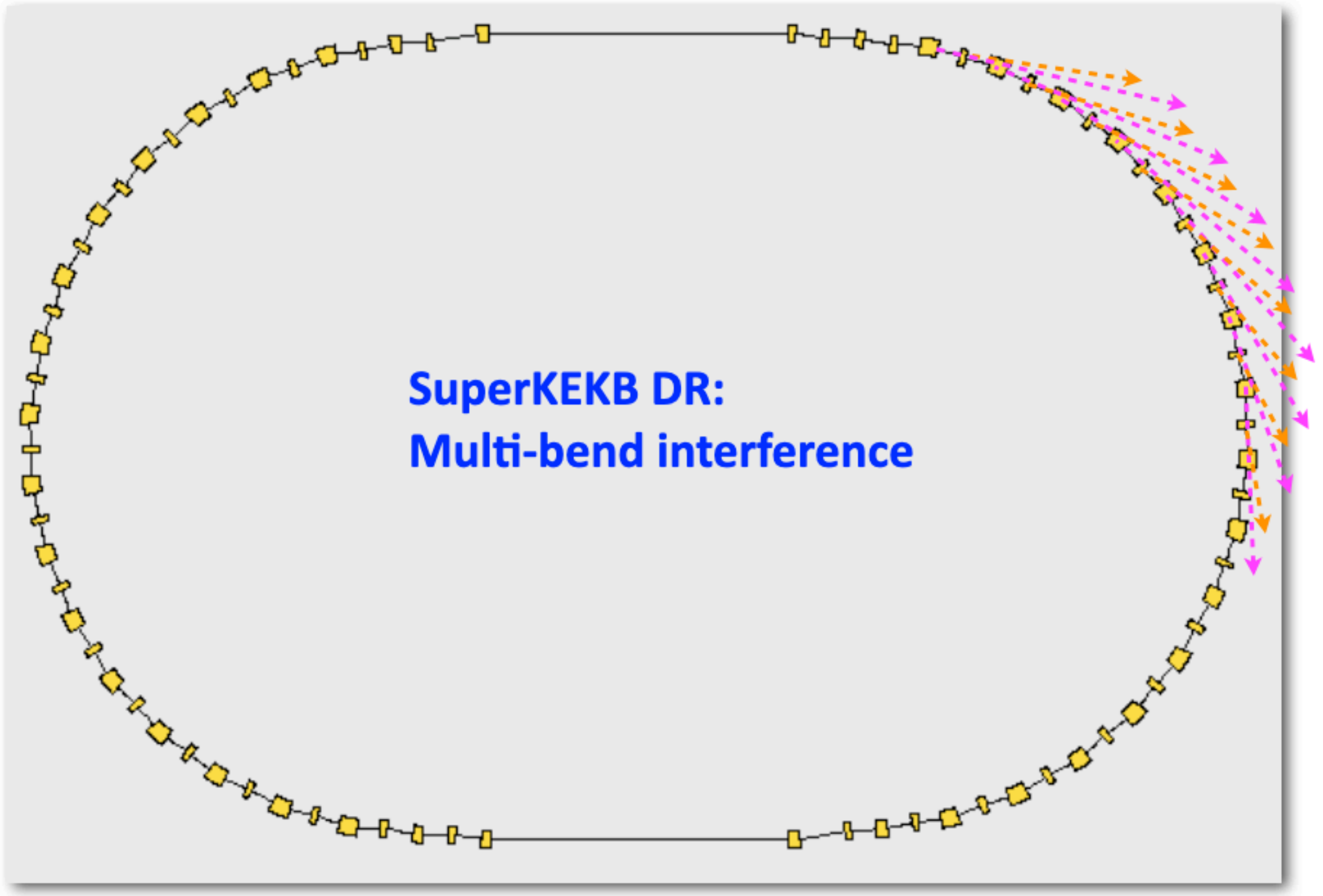
- Examples of CSR impedance by CSRZ
  - A single bend with varied length: a/b=60/30 mm, R=5 m, $L_{bend}$=0.5/2/8 m.
  - Black/Blue/Red/Green lines: Steady-state parallel-plates/L=0.5/ L=2/L=8 m. For convenience of comparison, the impedance amplitude is scaled to L=1 m.
  - "Short bend": Transient effect at the entrance and exit is important.
  - "Long bend": Excited eigenmodes of a toroidal chamber (or "whispering gallery modes" by R. Warnock [8]).
  - "Overtaking field": Short-range wake fields, space charge like.
  - "Trailing field": Long-range wake fields, relevant to excited eigenmodes.

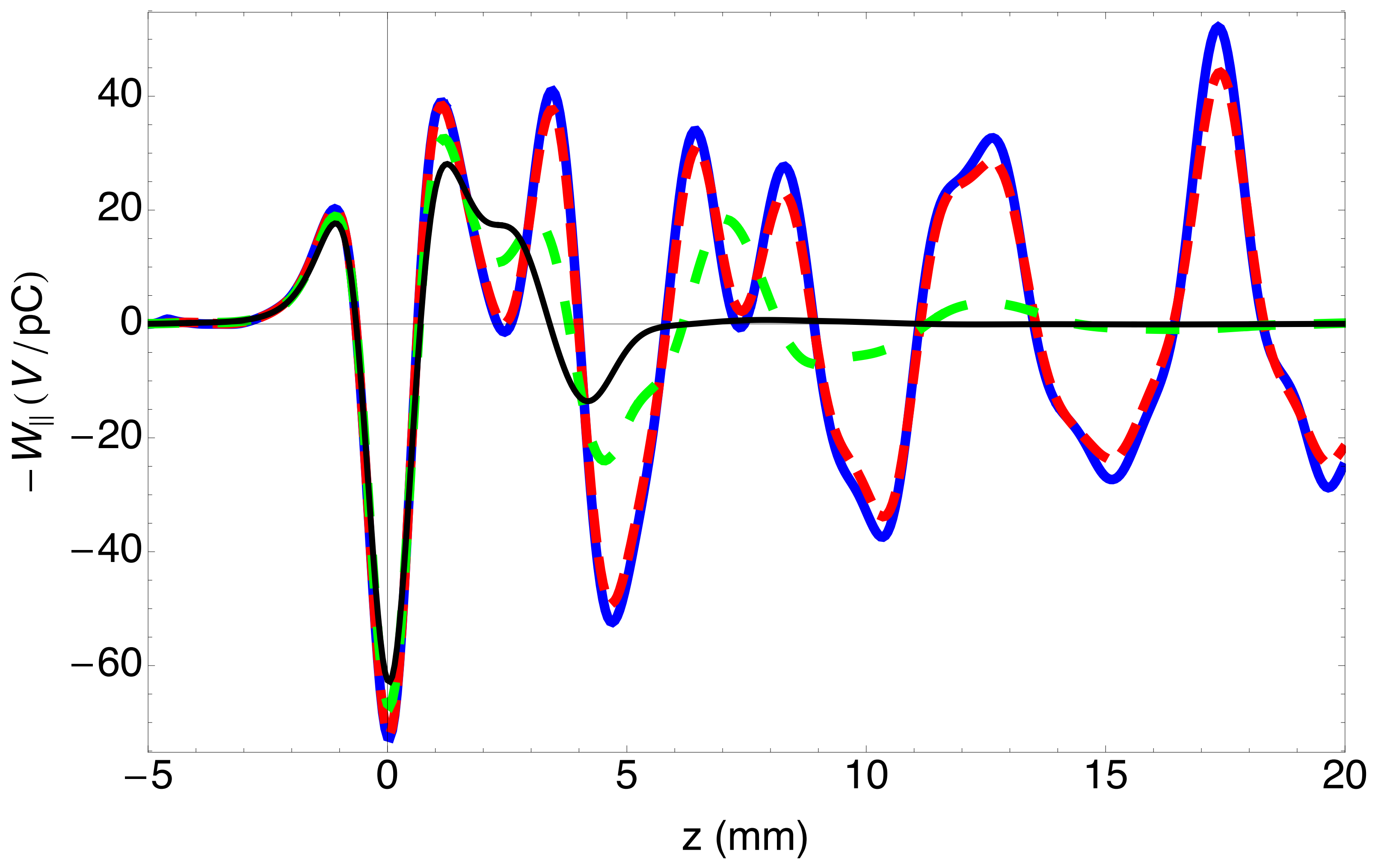


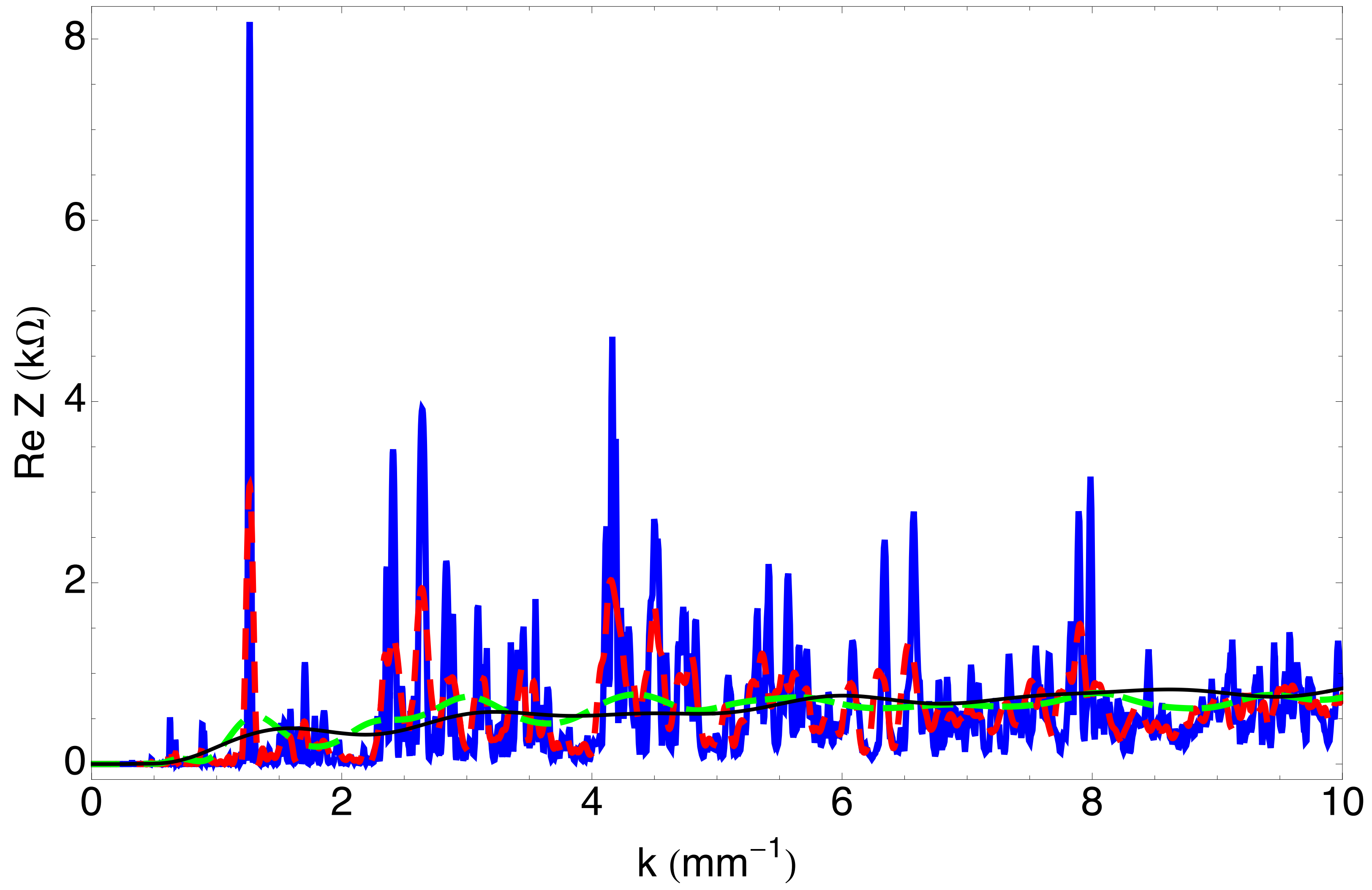

[8] R. Warnock, arXiv:1708.05500

# CSR impedance calculation (cont'd)

- Examples of CSR impedance by CSRZ
  - A realistic ring with multiple-bends: assuming smooth chamber.
  - SuperKEKB DR as an example: a/b=34/34 mm, $L_{bend}$=0.74/0.29 m, R=2.7/-3 m (reverse bends), $L_{drift}$=0.9 m, $N_{cell}$=1/6/16.
  - Multi-bend interference: CSR fields generated by multiple bends propagate along the chamber together with the beam. The fields interfere to produce a pattern of "narrow-band spikes".
  - The real part of CSR impedance should correspond to SR spectrum in measurement.



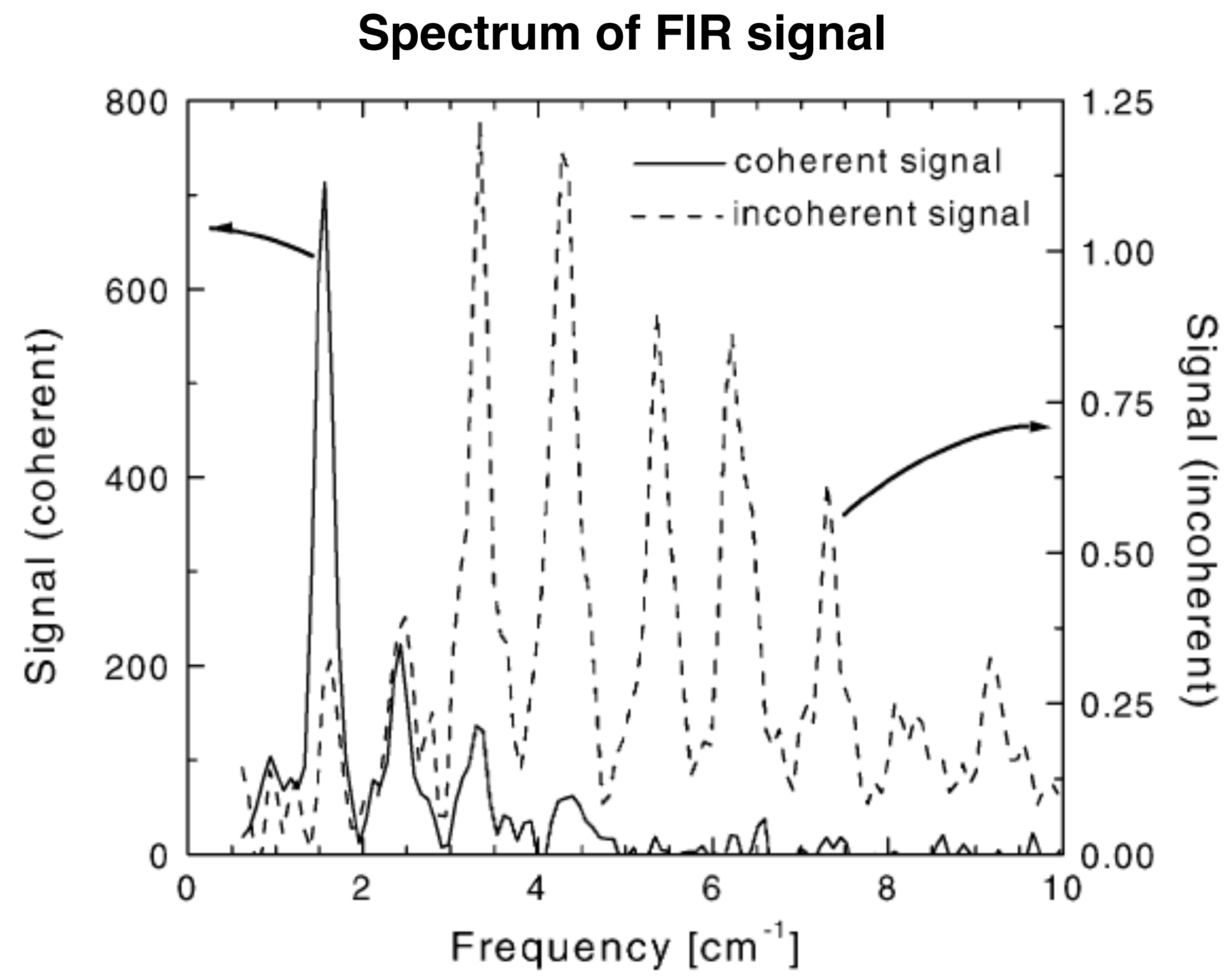



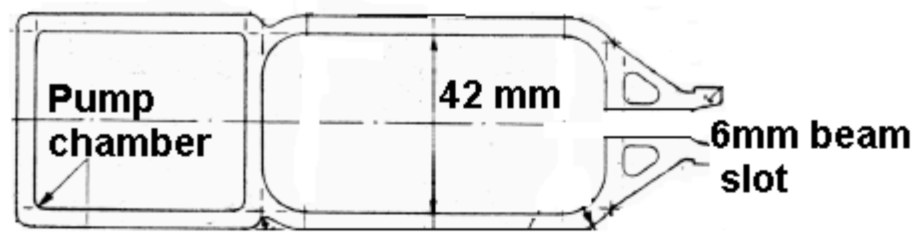



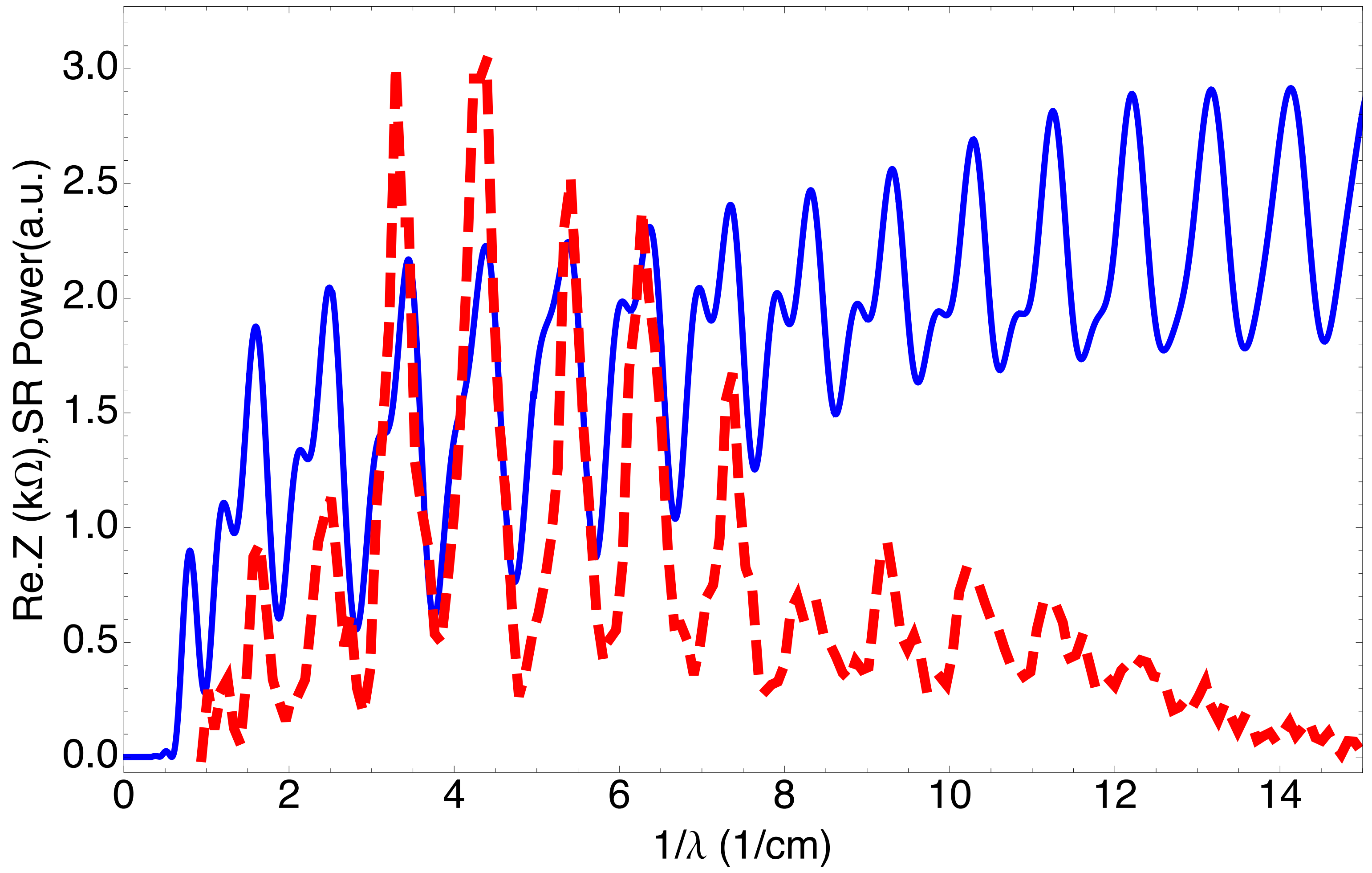

[9] D. Zhou, et al., Jpn. J. Appl. Phys. 51 (2012) 016401.

# CSR impedance calculation (cont'd)

- Examples of CSR impedance by CSRZ
  - NSLS VUV as an example: a/b=80/42 mm, Lbend=1.5 m, R=1.91 m (Collaboration with S. Kramer)
  - Measured SR spectrum showed similar pattern of CSR impedance [6,8,10]. This is an evidence of multi-bend interference of CSR, or CSR in "whispering gallery modes".





[10] G. Carr et al., "Observation of coherent synchrotron radiation from the NSLS VUV ring", Nucl. Instrum. Methods Phys. Res., Sect. A **463**, 387 (2001).

# CSR impedance calculation (cont'd)

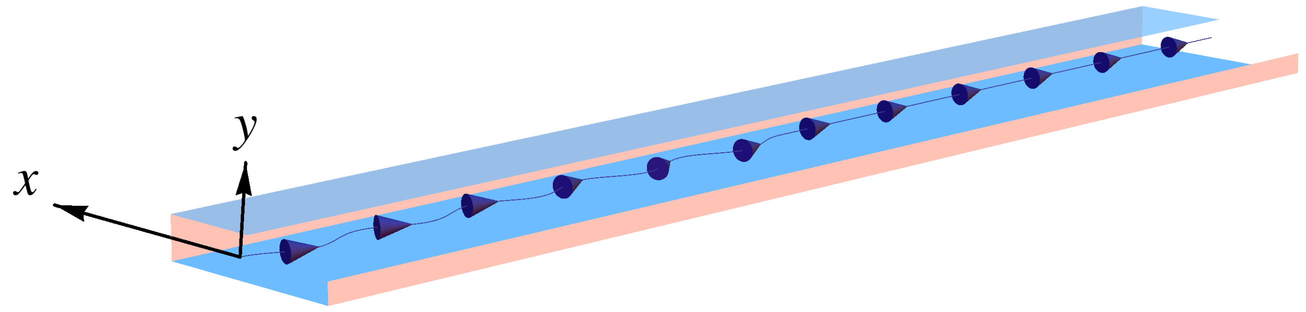
- Examples of CSR impedance by CSRZ
  - CSR in a wiggler/undulator: Coherent wiggler/undulator radiation (CWR or CUR).
  - The CWR spectrum can be calculated analytically (for example, see Refs.[11,12]):

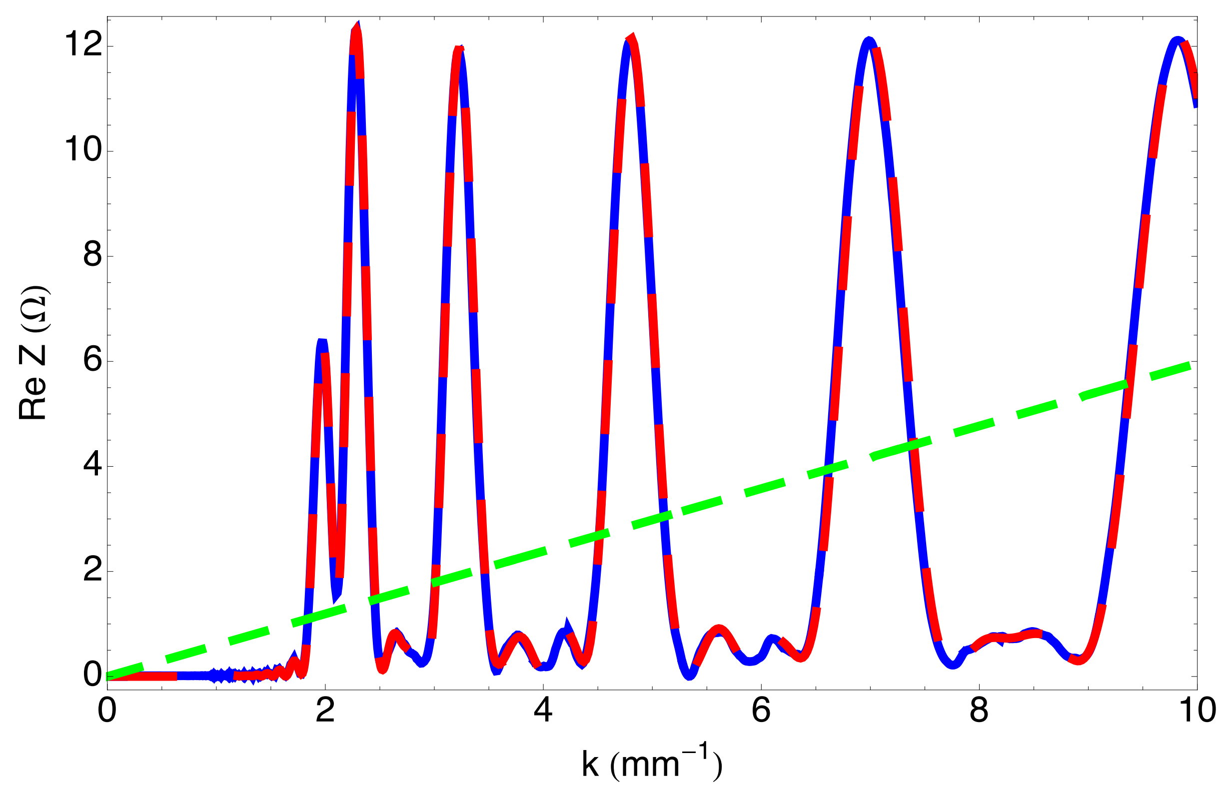
$$\mathrm{Re}\ Z(k) = \frac{4Z_0}{abR_0^2}\sum_{m=0}^{\infty}\sum_{p=1}^{\infty}\frac{k}{(1+\delta_{m0})k_z}\frac{\sin^2\left((k-k_z-k_w)L_w/2\right)}{(k-k_z)^2-k_w^2}$$

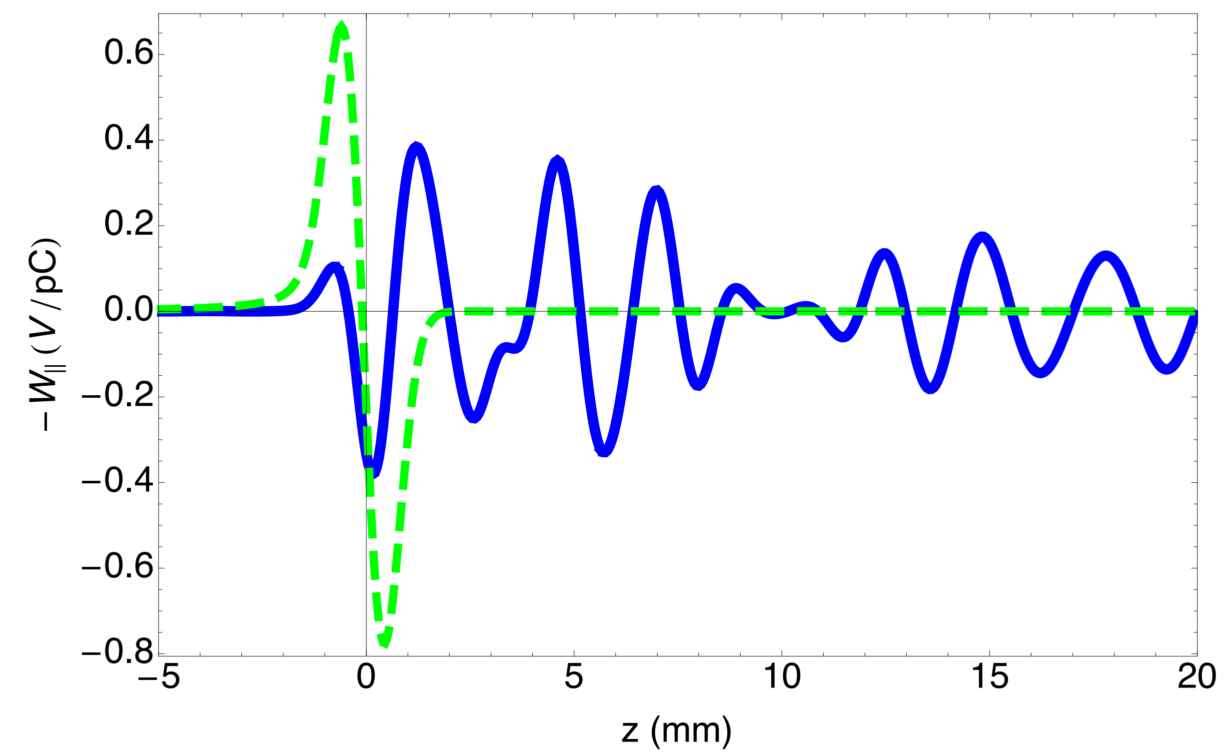
  - A weak wiggler: a/b=100/20 mm, λw=1 m, R0=100 m, $N_{\mathrm{period}}$=10
  - Blue line by CSRZ; Red line by analytic theory with rectangular chamber; Green line by analytic theory in free space [13].

$$Z(k) = \frac{1}{4}Z_0 L_w k\frac{k_w}{k_0}\left(1-\frac{2i}{\pi}\left(\log\frac{4k}{k_0}+\gamma_E\right)\right)$$

  - For storage-ring light sources or THz FELs, it might be interesting to look at the interference of CUR + SC + RW.





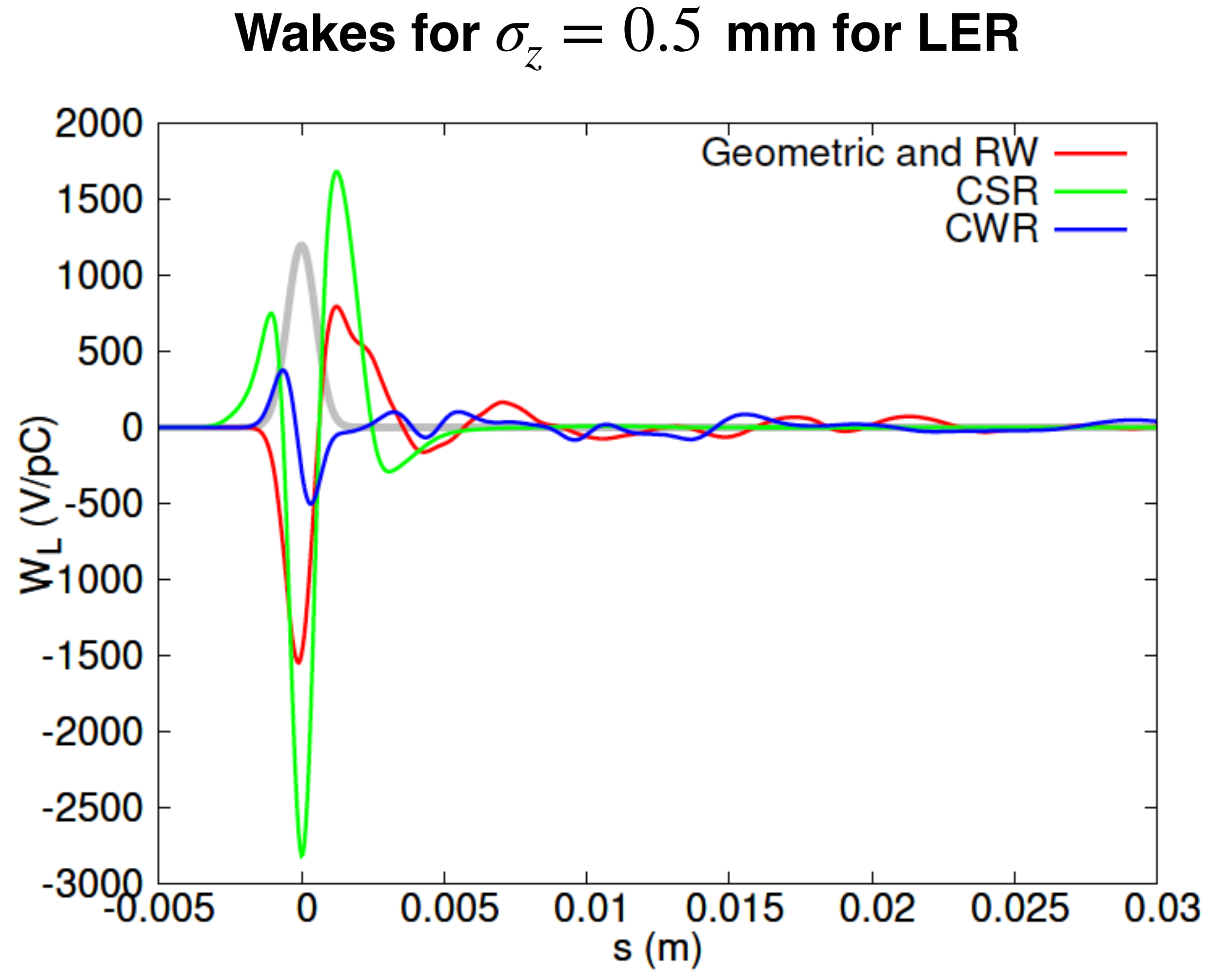
[11] Y. Chin, LBL-29981, 1990.

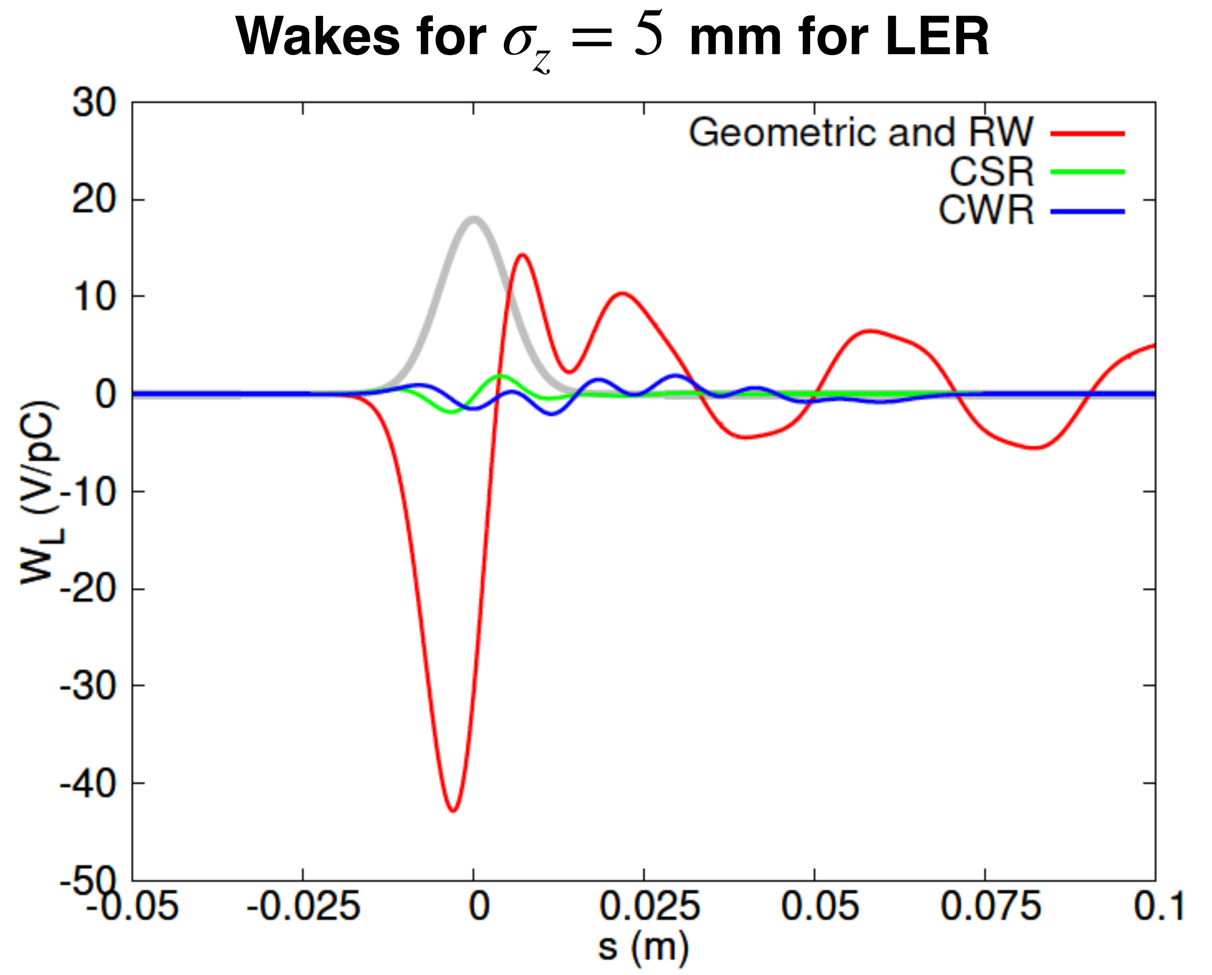
[12] G. Stupakov and D. Zhou, KEK Preprint 2010-43.

[13] J. Wu et al., Phys. Rev. ST Accel. Beams **6**, 040701 (2003).

# Prediction of CSR effects using tracking codes or Vlasov solver

- MWI simulations including CSR
  - By solving Vlasov-Fokker-Planck (VFP) equation, the MWI threshold due to CSR can be determined. Alternative models of CSR can be used (see Refs.[3,4] for example).
  - In realistic storage rings, impedances from geometric discontinuities and resistive walls might dominate MWI. Practically, one needs to do simulations with all impedances taken into account, including CSR and CWR.
  - For DR and MRs of SuperKEKB, we constructed pseudo-Green function wake model (use 0.5 mm Gaussian bunch, which is about 1/10 of the nominal bunch length, as driving beam) including CSR. Then we use this wake model in tracking codes or Vlasov solver to detect CSR effects [14].
  - For the nominal bunch length, CSR wake looks to be weak, but can be a non-negligible source affecting MWI threshold (see results of MWI simulations in next page).

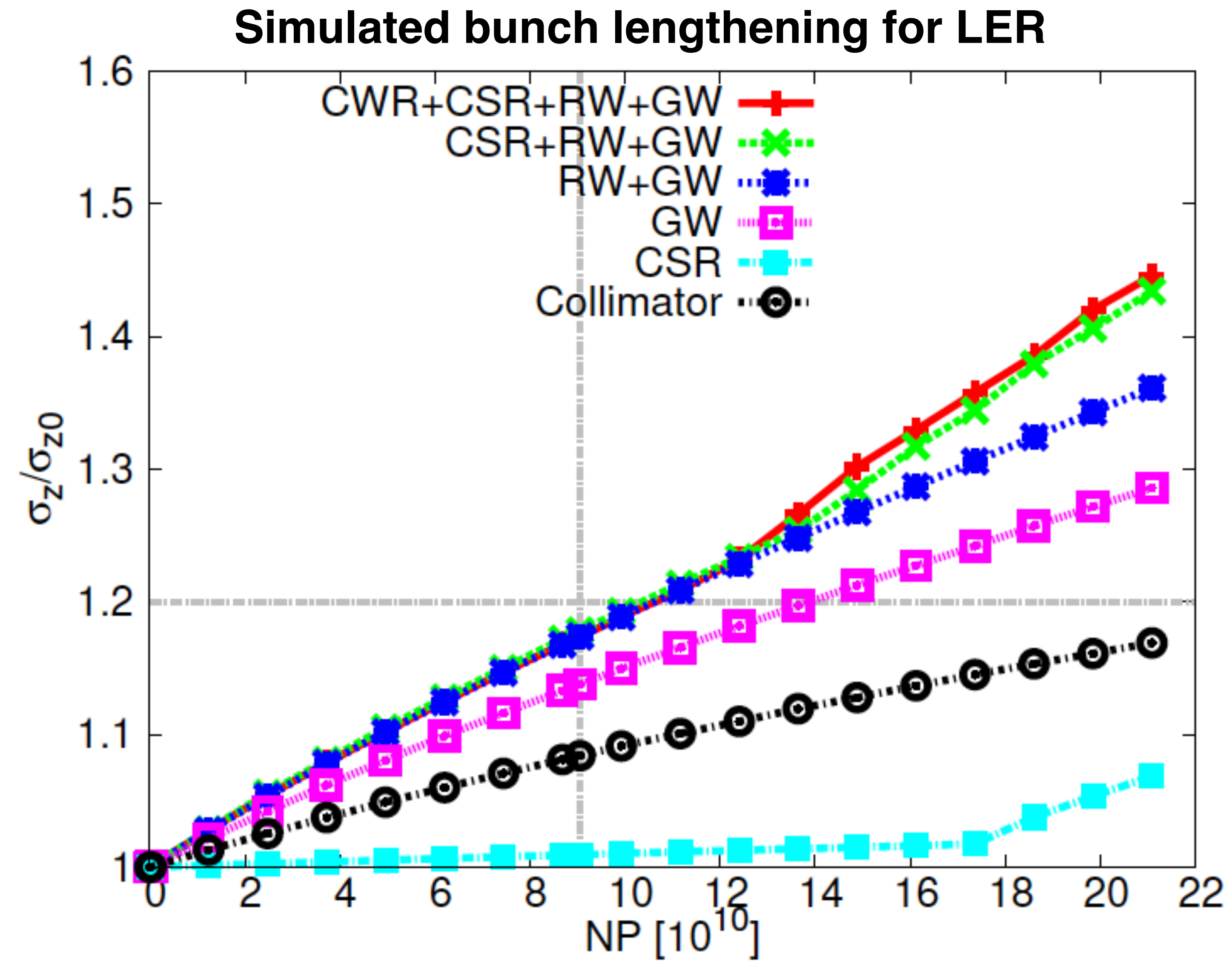


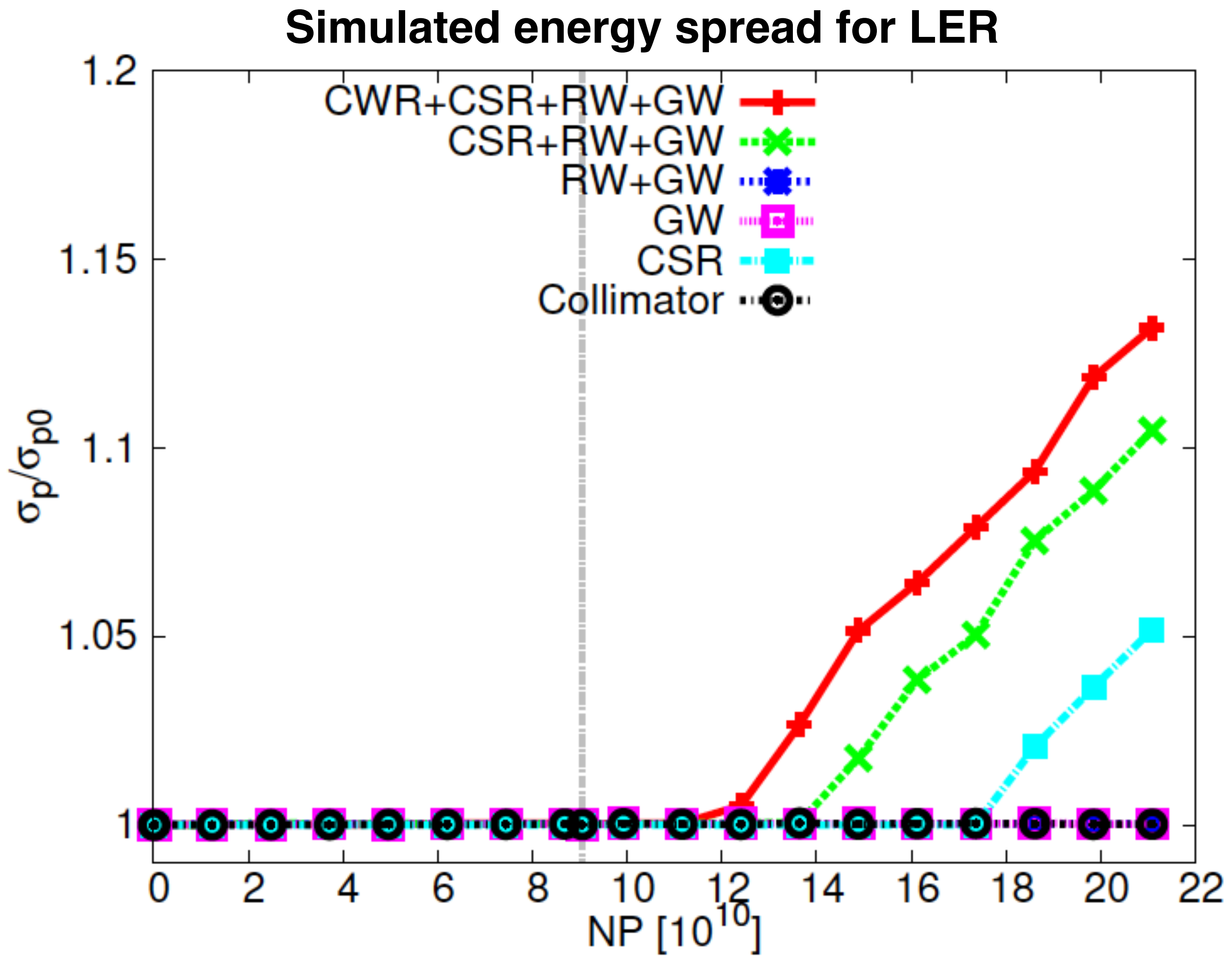




[14] D. Zhou, Beam dynamics issues in SuperKEKB, Talk presented at the 20th KEKB Accelerator Review Committee, KEK, Feb. 23, 2015.

# Prediction of CSR effects using tracking codes or Vlasov solver (cont'd)

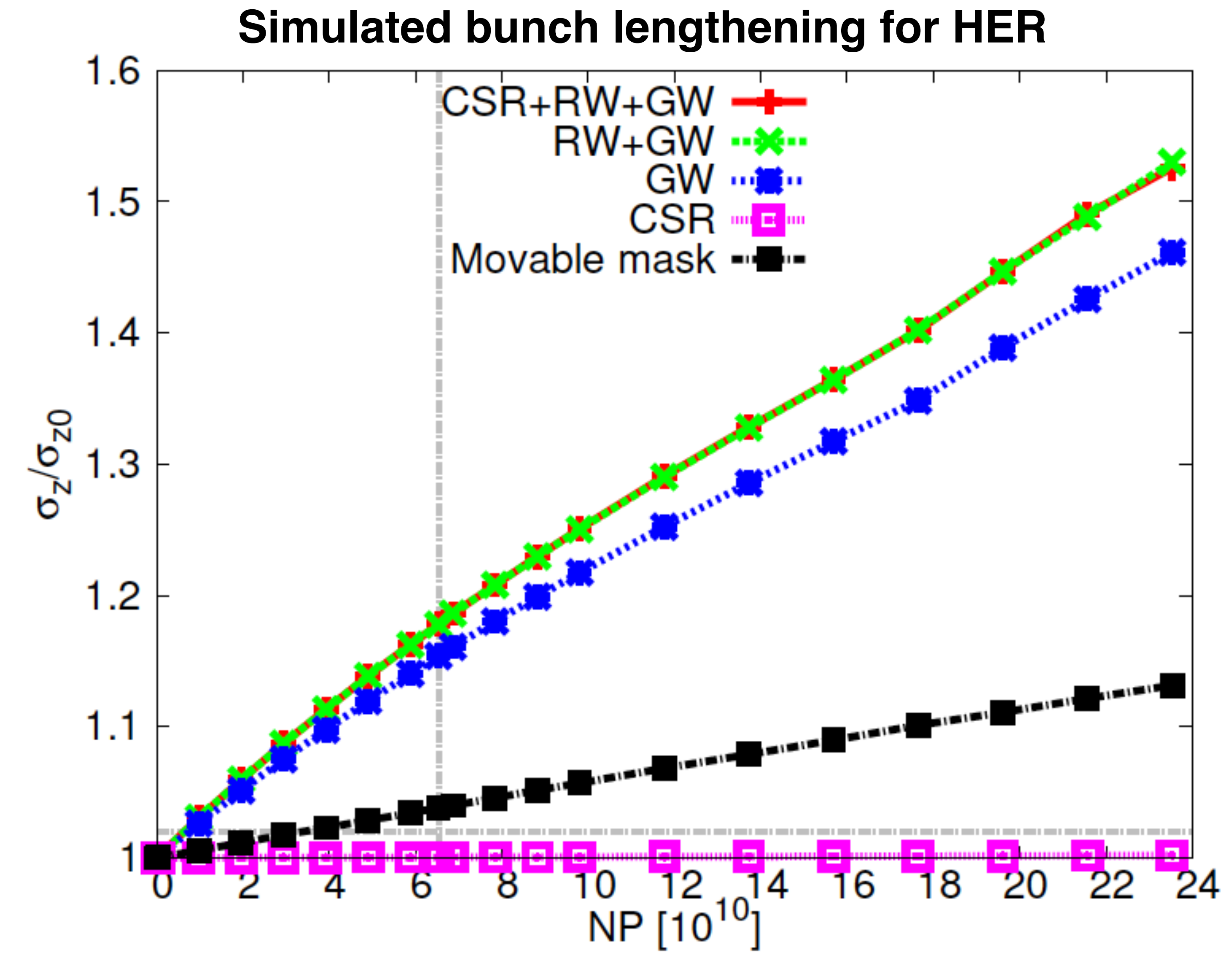
- MWI simulations including CSR
  - Using Pseudo-Green function wake ($\sigma_z = 0.5$ mm) for CSR is questionable: It means cutoff of high-frequency CSR impedance and consequently the high-frequency CSR is damped.
  - We can use shorter bunch or even inverse Fourier transform of impedance for CSR effects, but we have to refine the mesh in z-direction in MWI simulations (typically mesh size $\Delta z \ll 1/k_{\text{max}}$ with $k_{\text{max}}$ the maximum wavenumber for CSR impedance taken into account). Especially, for tracking codes, we have to take care of the interaction between CSR and numerical noise in the histogram of charge density due to statistical error with finite number of macroparticles (typically $N_{\text{particle}}$=1E6).

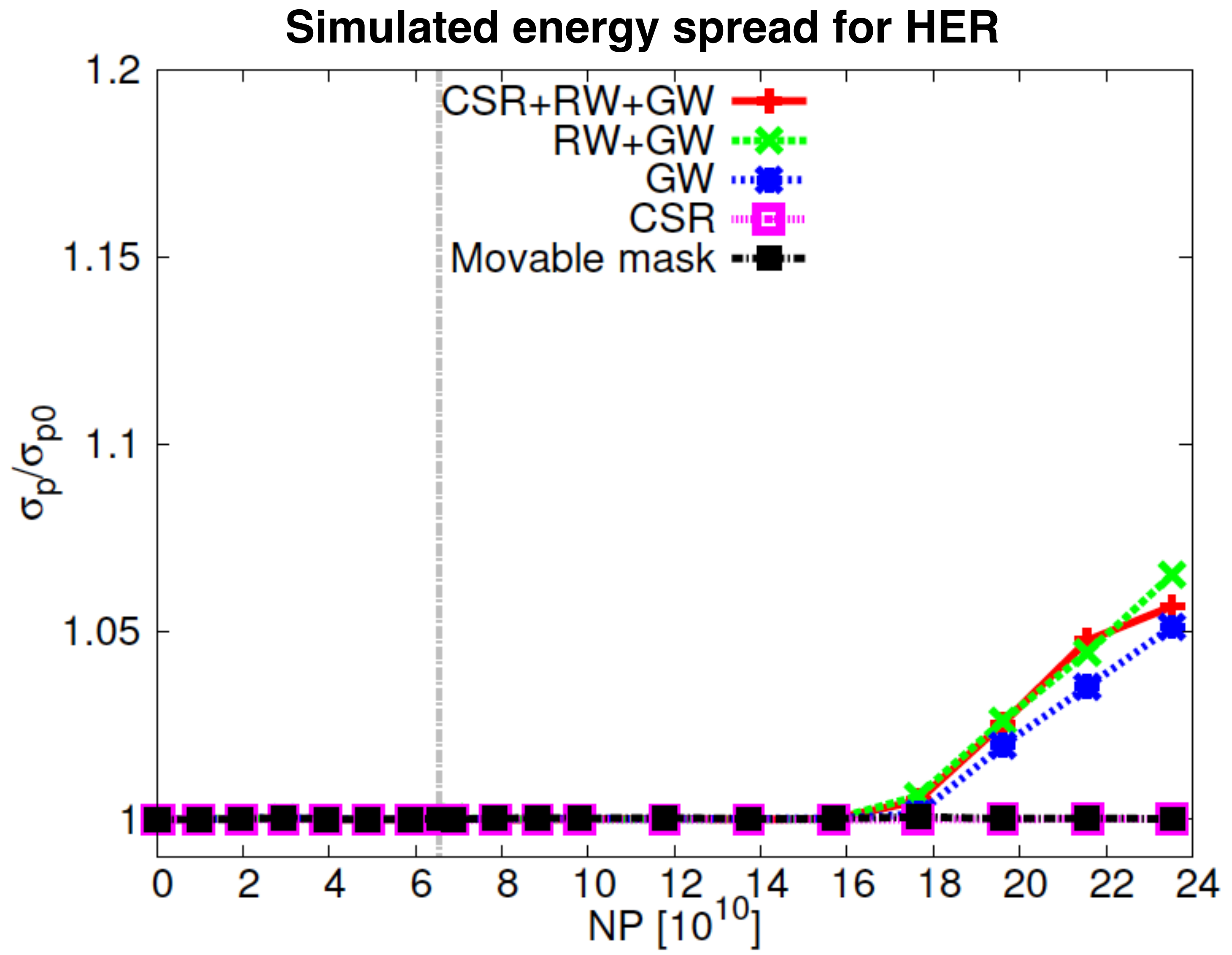


# Prediction of CSR effects using tracking codes or Vlasov solver (cont'd)

- MWI simulations including CSR
  - In storage rings, CSR wakes are distributed in the arcs. Consequently, in MWI simulations with CSR, it is necessary to distribute the wake along the ring. It is not good to lump all CSR wakes at one point (one wake kick per turn). The rule of thumb is $k_{\text{max}}\alpha_p\sigma_p\Delta s \ll 1$ [15]. This is derived from the rigid beam assumption of wakefield theory: After a travel length of $\Delta s$, the slip distance (proportional to $\alpha_p\sigma_p\Delta s$) of a particle inside the bunch should be much smaller than the wavelength of the wakefield under consideration.
  - For SuperKEKB LER, $\alpha_p = 3.2 \times 10^{-4}$ is the momentum compaction factor, $\sigma_p = 7.5 \times 10^{-4}$ is the energy spread. For CSR wake with $\sigma_z = 0.5$ mm, we take $k_{\text{max}} = 2\pi/\sigma_z = 12566\ \text{m}^{-1}$. Then the step of distance for tracking or Vlasov solver should be $\Delta s \ll 330$ m, comparing ring circumference of $C = 3016$ m. In practice, we use >20 kicks per turn for MWI simulations with CSR.
  - For tracking with CSR, see Ref.[16] as an example.





[15] Private communication with K. Ohmi.

[16] H. Ikeda et al., in Proceedings of IPAC'13, Shanghai, China.

# Summary

- Analytic theories (such as S-H theory) can be used to check if CSR is or not important in a storage ring.
- Simulation code like CSRZ can be used to calculate CSR and CWR/CUR impedance of storage rings.
- Cares should be taken in simulations of MWI with CSR impedance.