CSR calculation in ERL merger section

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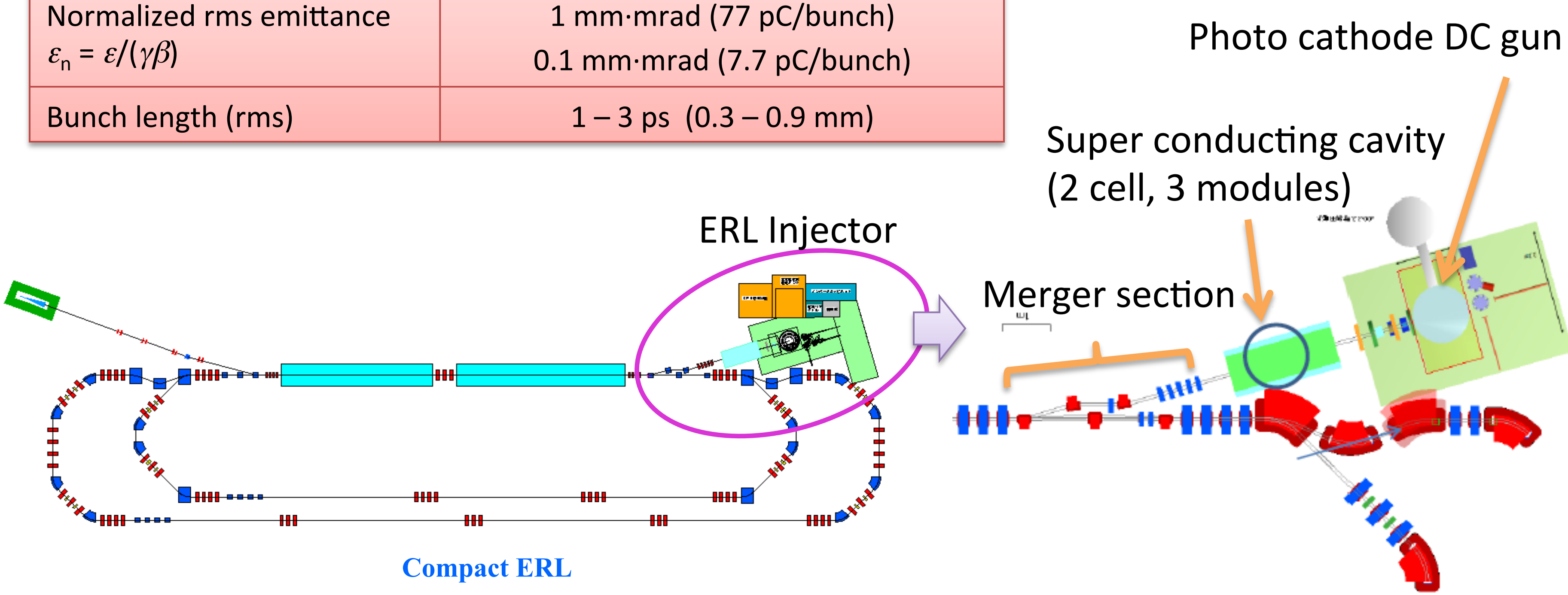
Contents

ERL injector

- ERL injector: to generate electron beam with lower emittance and shorter bunch length

Parameters of the Compact ERL Injector

Beam energy	5 – 10 MeV
Beam current	10 – 100 mA
Normalized rms emittance $\varepsilon_n = \varepsilon/(\gamma\beta)$	1 mm·mrad (77 pC/bunch) 0.1 mm·mrad (7.7 pC/bunch)
Bunch length (rms)	1 – 3 ps (0.3 – 0.9 mm)

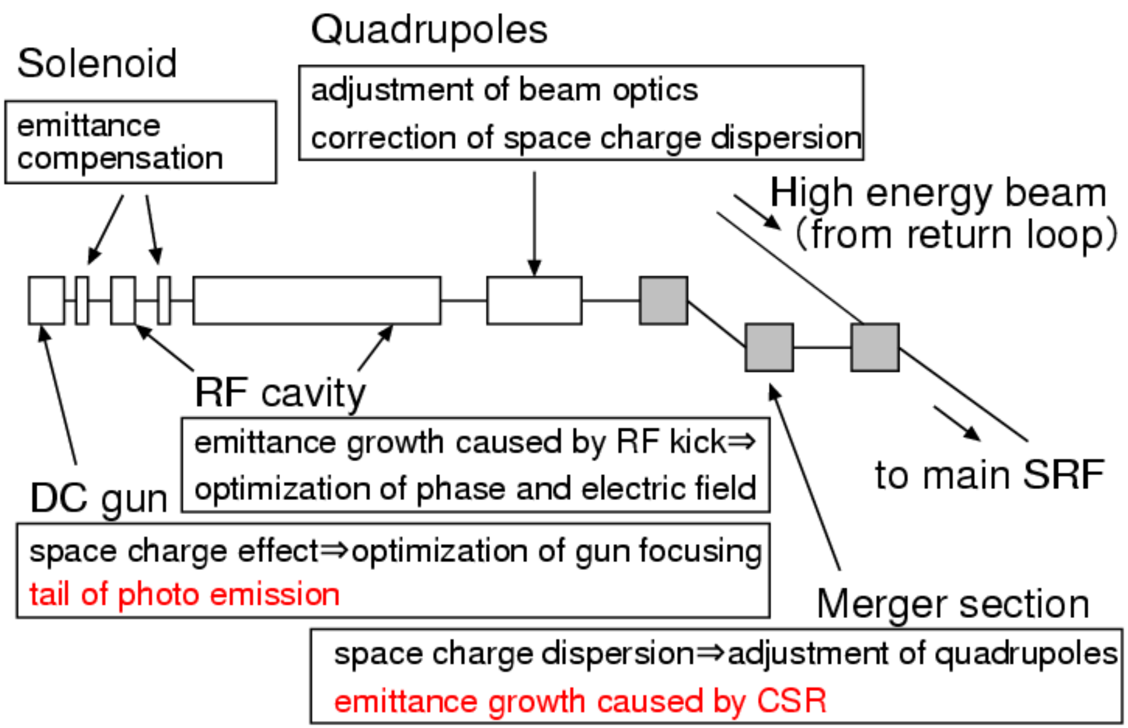


Compact ERL

Physics in ERL injector

(1) Space charge effect（Coulomb force between electrons)
(2) Solenoid focusing（Emittance compensetion）
(3) RF kick in RF cavity
(4) Coherent Synchrotron Radiation (CSR) in merger section
(5) Response time of photo cathode（It generates tail of emission.）

These effects combine in the ERL injector.



To obtain high quality beam at the exit of merger, optimization of beamline parameters is required.

Method to research the beam dynamics:
Macro particle tracking simulation with space charge effect is used.

The simulation code have to include
(1) External electric and magnetic field,
(2) Space charge effect (3D space charge).

CSR calculation in ERL merger section

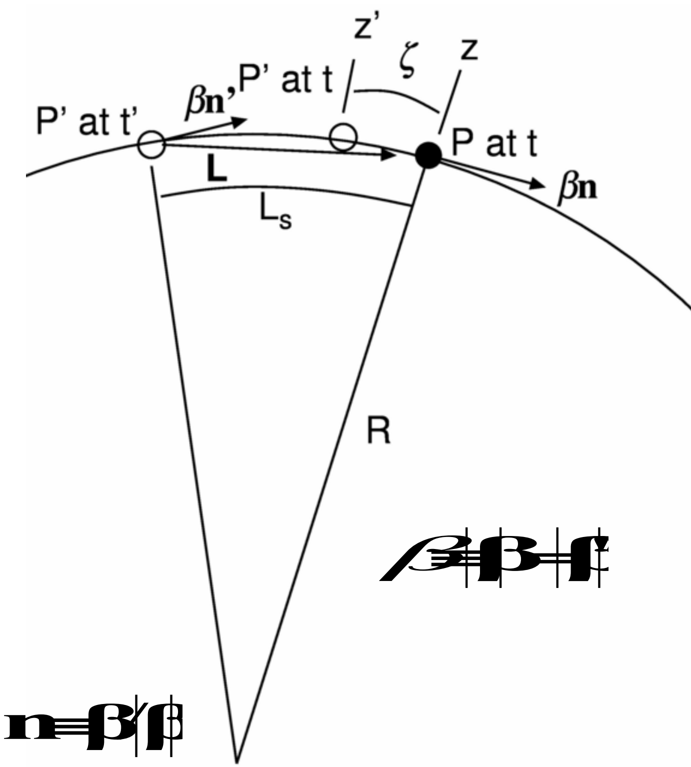
- In order to study CSR effect in ERL merger section, we developed a 1D CSR routine, which is effective for lower beam energy, e.g. 10 MeV.
- 1D CSR wake calculation in GPT using D. Sagan's formula.
  - General Particle Tracer (GPT) is a particle tracking code, which includes 3D space charge effect based on a nonequidistant multigrid Poisson solver or a point-to-point method.
  - The routine can calculate 1D-wake functions for arbitrary beam trajectories as well as CSR shielding effect.
  - In particular, the CSR routine does not assume ultrarelativistic electron beam and is therefore applicable at low beam energies in the injector.
- I. V. Bazarov and T. Miyajima, "Calculation of Coherent Synchrotron Radiation in General Particle Tracer", Proc of EPAC 2008, MOPC024
- D. Sagan, "AN EFFICIENT FORMALISM FOR SIMULATING THE LONGITUDINAL KICK FROM COHERENT SYNCHROTRON RADIATION", Proc of EPAC 2006, THPCH024

Sagan's formula
Two particle interaction

Two particle interaction

- The source particle at point P'.
- An electric field E(P) at the position of the kicked particle at point P and time due to the source particle at point P' and retarded time t'.
- The Lienard-Wiechert formula

[illegible]



- The CSR term

$$\mathbf{E}_{CSR} = \mathbf{E} - \mathbf{E}_S$$

- Here, the space charge term is

$$\mathbf{E}_S(\zeta) = \frac{e\,\mathrm{sig}(\zeta)}{4\pi\varepsilon_0 \gamma^2 \zeta^2}$$

$$\beta = |\boldsymbol{\beta}|$$

- The rate of energy change is given by

[illegible]

$$\mathbf{n} = \boldsymbol{\beta}/|\boldsymbol{\beta}|$$

Space charge term

- The space charge term

$$E_s(\zeta) = \frac{e\,\mathrm{sign}(\zeta)}{4\pi\varepsilon_0 \gamma^2 \zeta^2}$$

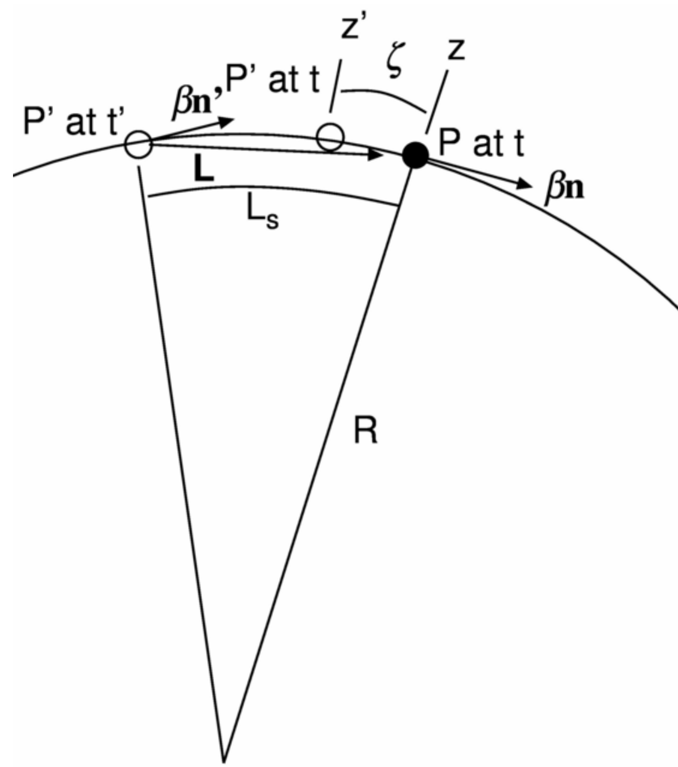
$$\zeta = z - z'$$

- The longitudinal distance ζ is required to calculate the space charge term.
- The change of the longitudinal position of the source particle is

$$L_s = c\beta(t - t')$$

- The longitudinal distance between P' and P at time t is

$$\zeta = L_s - \beta L$$

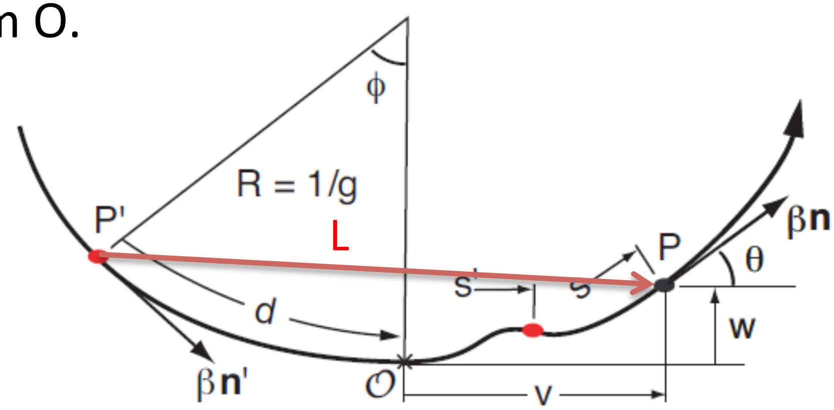


In next step, retarded time t' is calculated from saved orbit data.

Calculation of retarded time t' with ζ on arbitrary orbit

- The orbit is divided into N elements from O.

save
- ϕ_i: bend angle
- $g_i = 1/R_i$: bend strength
- $\psi_i = \sum_{k=1}^{i-1} \phi_k$: orientation angle
- d_i: path length



- The path length: $L_s = c t_n$
- v and w components of the vector **L**:

$$L_v = v + R\sin\phi$$
$$L_w = w - R(1-\cos\phi)$$
$$L = \sqrt{L_v^2 + L_w^2}$$

$$v = \nu_1 - \nu_3$$
$$w = \omega_2$$

$$L_v = [\nu_1 + d] - \left[\nu_3 + \frac{g^2 d^3}{6}\right]$$
$$L_w = \omega_2 - \frac{g d^2}{2}$$
$$L = [\nu_1 + d] - \left[\nu_3 + \frac{g^2 d^3}{6} - \frac{1}{8}\frac{(2\omega_2 - g d^2)^2}{\nu_1 + d}\right]$$

In the simulation, the orbit parameters are saved every time step.

$$\nu_1 = \sum_{i=1}^{N} d_i$$
$$\nu_3 = \sum_{i=1}^{N} d_i\left(\frac{1}{2}\psi_i^2 + \frac{1}{2}\psi_i g_i d_i + \frac{1}{6} g_i^2 d_i^2\right)$$
$$\omega_2 = \sum_{i=1}^{N} d_i\left(\psi_i + \frac{1}{2} g_i d_i\right)$$

Lv, Lw, L can be calculated from ν_1, ω_2, ν_3.

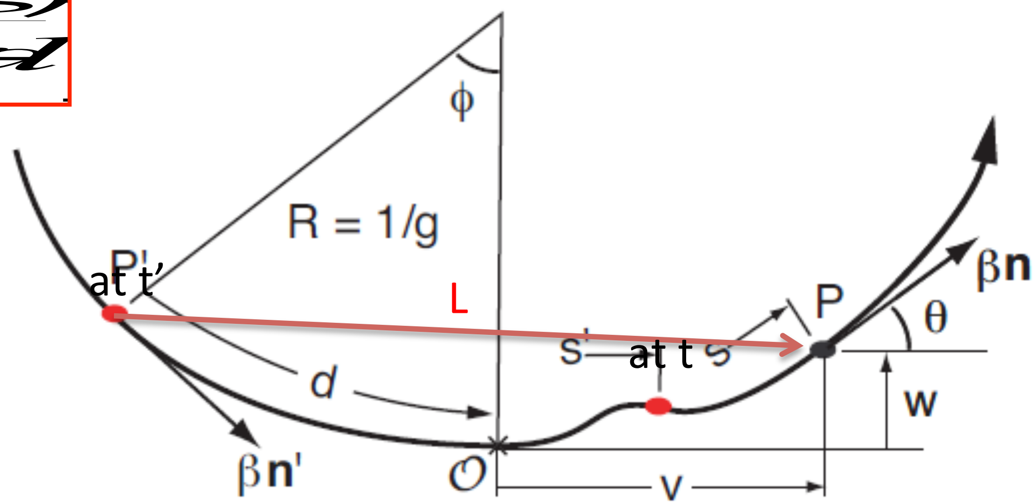
- The distance, ζ: $E_s(\zeta)=\dfrac{e\,\mathrm{sign}(\zeta)}{4\pi\varepsilon_0\gamma^2\zeta^2}$

$$\zeta = L_s - \beta L$$

$$\begin{cases} L_s = d + \nu_1 \\ L_v = [\nu_1 + d] - \left[\nu_3 + \dfrac{g^2 d^3}{6}\right] \\ L_w = \omega_2 - \dfrac{g d^2}{2} \\ L = [\nu_1 + d] - \left[\nu_3 + \dfrac{g^2 d^3}{6} - \dfrac{1}{8}\dfrac{(2\omega_2 - g d^2)^2}{\nu_1 + d}\right] \end{cases}$$

$$\boxed{\zeta = \frac{\nu_1 + d}{2\gamma^2} + \left[\nu_3 + \frac{g^2 d^3}{6} - \frac{1}{8}\frac{(2\omega_2 - g d^2)^2}{\nu_1 + d}\right]}$$

Using this equation, we can calculate retarded time t’ from saved orbit parameters, ν_1, ω_2, ν_3 .



Calculation of CSR kick on arbitrary orbit

- CSR kick:

[illegible]

$$\frac{1}{(L - \mathbf{L}\cdot\beta\mathbf{n})^3} = \frac{8\gamma^9\tau^3}{(\tau^2+\alpha^2)^3}$$

$$\mathbf{n}\left(\mathbf{L} - L\beta\mathbf{n}/\gamma^2\right) = \frac{\tau^2 - \alpha^2 + 2\tau\alpha\kappa}{2\gamma^5\tau}$$

$$\mathbf{n}\left(L\times\left[(\mathbf{L} - L\beta\mathbf{n})\times\mathbf{a}\right]\right)/c^2 = \frac{8(\tau^2-\alpha^2)(\alpha-\tau\kappa)}{2\gamma^5\tau}$$

$$\tau = \gamma(d+\nu_1)$$

$$\alpha = \gamma^2\left(\omega_2 + g d \nu_1 + \frac{1}{2} g d^2\right)$$

$$\kappa = \gamma(\theta + g d)$$

[illegible]

At time t’

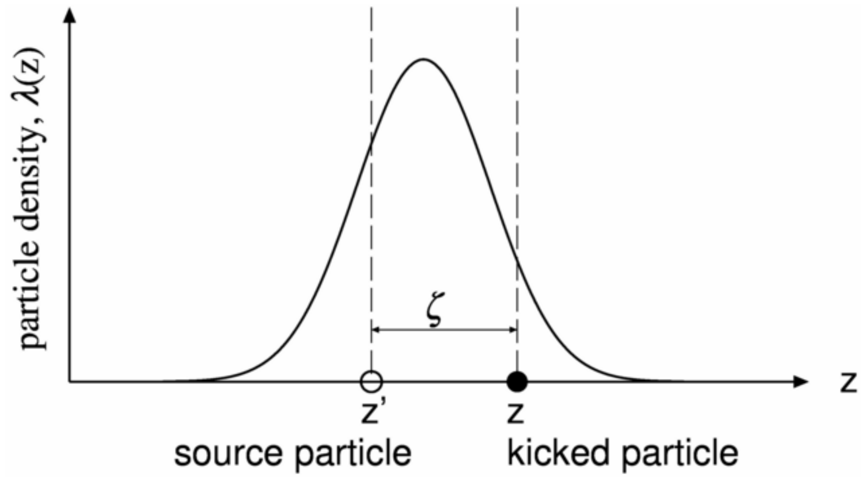
At time t

CSR kick, Kcsr can be calculated from ν_1, ω_2, ν_3 with respect to t’ and ζ.

[illegible]

1D longitudinal particle distribution

Longitudinal particle density



We consider that the bunch has 1D longitudinal particle density, $\lambda(z)$.

$$\tau = \gamma(d + \nu_1)$$

$$\alpha = \gamma^2\left(\omega_2 + g d \nu_1 + \frac{1}{2} g^2 d^2\right)$$

$$\kappa = \gamma(\theta + g d)$$

CSR kick at z is calculated from the following equation,

$$\left(\frac{d\varepsilon}{ds}\right)_{CSR} = \int_{-\infty}^{\infty} dz' \lambda(z') K_{CSR}(z - z')$$

$$= \int_{-\infty}^{\infty} dz' \frac{d\lambda(z')}{dz'} I_{CSR}(z - z')$$

where

$$I_{CSR}(z - z') = \int_{-\infty}^{z'} dz'' K_{CSR}(z - z'')$$

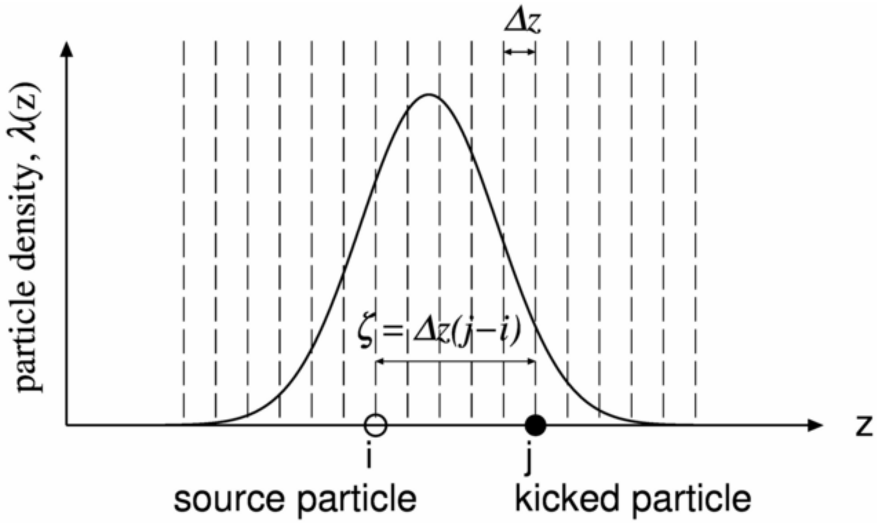
(Integration by parts)

Icsr can be calculated from the saved orbit parameters, ν_1, ω_2, ν_3 and ζ.

$$I_{CSR}(z - z') = \ldots$$

CSR calculation in numerical simulation

Procedure of CSR calculation



1. Save particle orbit (ν_1, ω_2, ν_3) every time step Δt.
2. Calculate longitudinal particle density λ(z).
3. Calculate retarded time t', which satisfies ζ = Δz(i-j).
4. Calculate (ν_1, ω_2, ν_3) with respect to retarded time, t'.
5. Calculate CSR kick, Icsr(j), and energy change,

$$\tau=\gamma(d+\nu_1)$$

$$\alpha=\gamma^2\left(\omega_2+g d\nu_1+\frac{1}{2}g d^2\right)$$

$$\kappa=\gamma(\theta+g d)$$

6. Repeat 3. to 5.

CSR calculation in GPT

Commands of GPT/CSR

- Command name
  - csr1Dwakexz();
- Assumption
  - It is assumed that the particles move on x-z plane. Namely, the vertical component of the average velocity is zero.
- Options
  - The GPT/CSR has 16 options.

Options of GPT/CSR

1. CSRTimestep (double) (s)
2. CSRCalcTstep (double) (s)
3. CSRMeshNbin (long)
4. CSRBGTolerance (double)
5. CSRMeshBoxSize (double)
6. CSRMeshNbfac (double)
7. CSRMeshStep (double) (m)
8. CSRTriangleWidth (double) (m)
9. CSRSign (double)
10. CSRHshield (double) (m)
11. CSRNimage (int)
12. CSRDriftLength (double) (m)
13. CSRCalcArea (double) (m)
14. CSRArcRadius (double) (m)
15. CSRArcAngle (double) (rad)
16. CSROutputWake (double) (m)

```
#-----------------------------
# example of CSR calculation
#-----------------------------
csr_dt     = 10.0e-12;
csr_tstep = 0.0;
csr_Nb     = 0;
csr_bgtol  = 1.0e-2;
csr_nstd   = 20.0;
csr_mNbfac = 0.1;
csr_mdl    = 0.06e-3;
csr_dtri   = 0.6e-3;
csr_sign   = -1.0;
csr_h      = 1.0;
csr_Nh     = 0;
csr_inids  = 10.0;
csr_xin    = -10.0;
csr_xout   = 10.0;
csr_zin    = -10.0;
csr_zout   = 10.0;
csr_arcr   = 0.0;
csr_arcang = 0.0;
csr_wfrom  = 0.0;
csr_wto    = 0.0;
csr_wstep  = 0.0;
#------------------------------------------
# please comment out the following line
# for calculation without CSR
#------------------------------------------
csr1Dwakexz("CSRTimestep", csr_dt, "CSRCalcTstep", csr_tstep, "CSRMeshNbin", csr_Nb,
"CSRBGTolerance", csr_bgtol, "CSRMeshBoxSize", csr_nstd, "CSRMeshNbfac", csr_mNbfac,
"CSRMeshStep", csr_mdl, "CSRTriangleWidth", csr_dtri, "CSRSign", csr_sign,
"CSRHshield", csr_h, "CSRNimage", csr_Nh, "CSRDriftLength", csr_inids,
"CSRCalcArea", csr_xin, csr_xout, csr_zin, csr_zout,
"CSRArcRadius", csr_arcr, "CSRArcAngle", csr_arcang,
"CSROutputWake", csr_wfrom, csr_wto, csr_wstep);
```

Energy Loss and Spread (1)

- The steady-state energy loss and spread for various beam energies are compared as calculated by GPT/CSR, elegant, and analytical expression for a circular orbit.

•The CSR routine in elegant includes the assumption of ultrarelativistic beam.

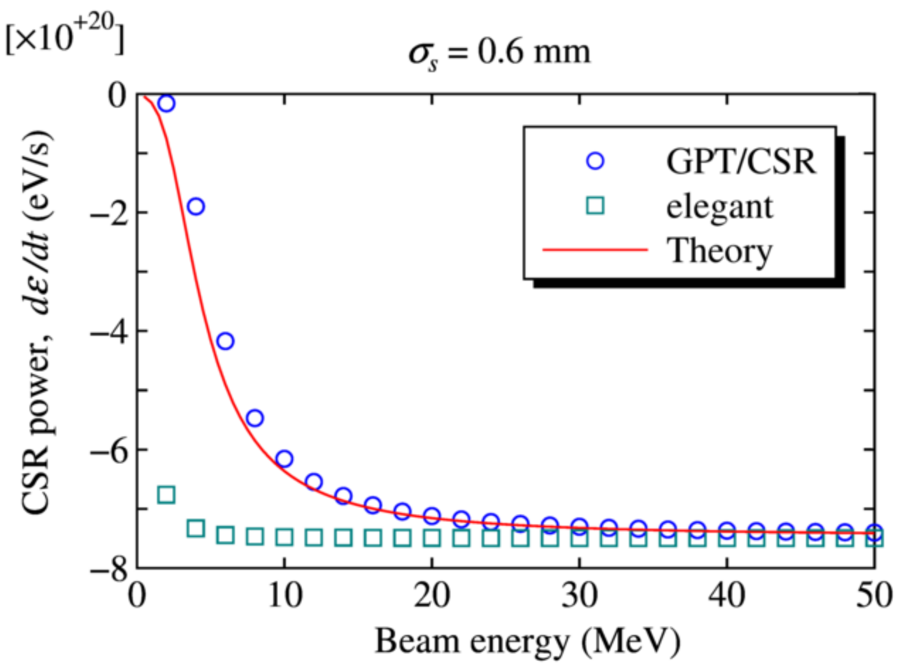
•GPT/CSR reproduces the analytical result accurately.

•Bending radius: $\rho = 1.0$ m
•Bunch length: $\sigma_s = 0.6$ mm
•Initial distribution: Gaussian
•Bunch charge: $Q = 80$ pC.

Analytical expression derived by C. Mayes

[illegible]

$K_{5/6}(x)$: the modified Bessel function
N : the number of election in the bunch
r_e : the classical electron radius



Energy loss and spread (2)

- The results of GPT/CSR and elegant both reproduce well the analytical result for higher beam energy, $E_0 > 40\mathrm{MeV}$.
- The results of elegant and the theory diverge to infinity for $E_0 \to 0$.
- The result of GPT/CSR approaches zero as expected.

Analytical expression with the assumption of $\gamma >> (r/\sigma_s)^{1/3}$ [1,2]

$$\frac{d\sigma_\delta}{dt} \approx 0.22\frac{N r_e c}{\gamma \rho^{2/3}\sigma_s^{4/3}}$$

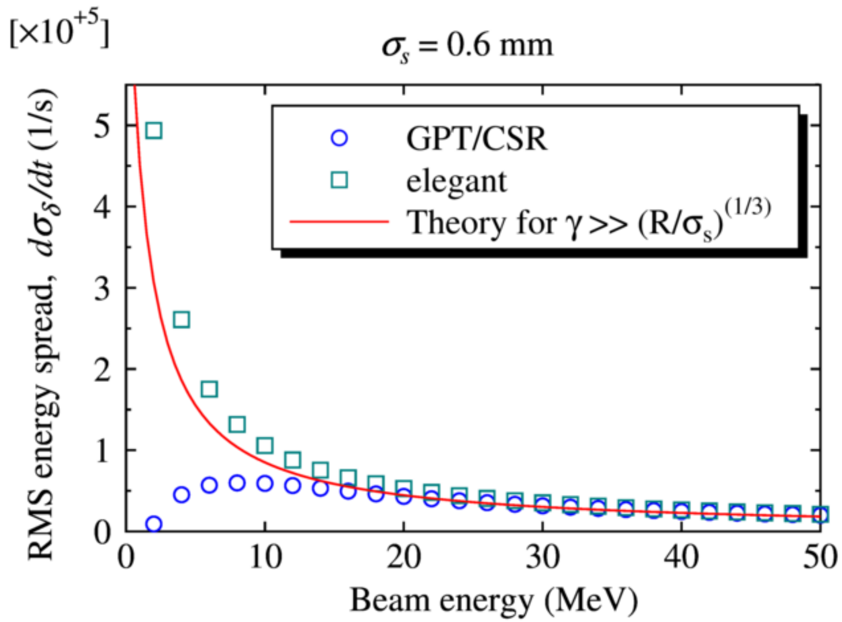
c : the speed of light

γ : the Lorentz energy factor

[illegible]

[1] P. Emma and R. Brinkmann, Proceedings of PAC97, Vancouver, B.C., Canada, 1997, pp. 1679-1681.

[2] Ya. S. Derbenev. et.al., TESLA FEL-Report 1995-05.

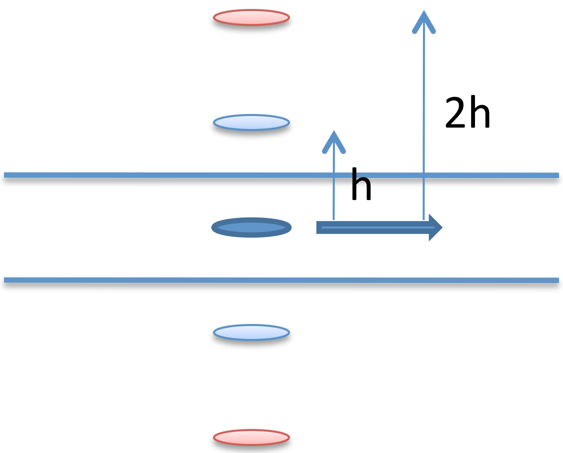


These results show that the GPT/CSR is effective for wide range of beam energies, and can be used to investigate beam dynamics in ERL and FEL photoinjectors.

CSR shielding effect

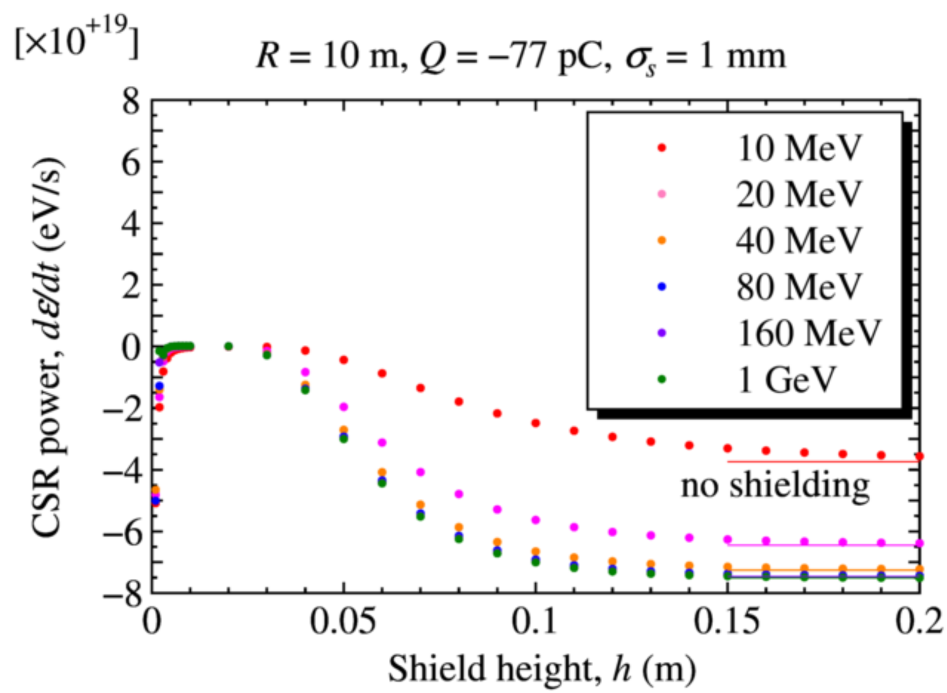
- Image charge layer

Chamber height, h



The effect of CSR shielding is calculated by GPT/CSR for a circular orbit.

- Bending radius: $\rho = 10.0$ m
- Bunch length: $\sigma_s = 1.0$ mm
- Initial distribution: Gaussian
- Bunch charge: $Q = 80$ pC.
- Number of image charge layers: 32

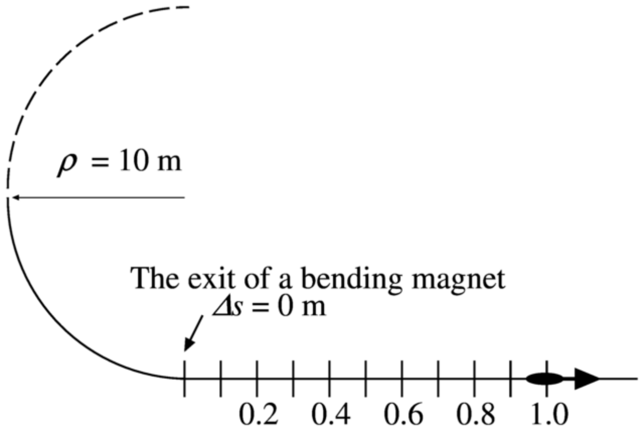


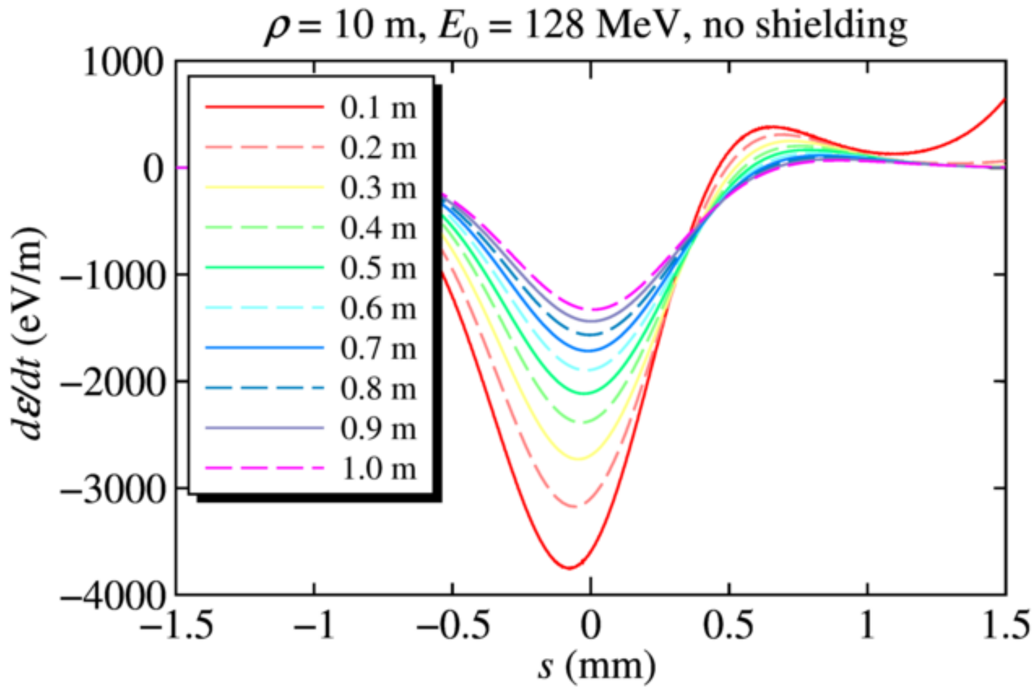
As the shielding height increases, the energy loss approaches to the analytical value.

CSR in transient state without shielding

- As an example of CSR effect in a transient state, the CSR wake form is calculated by GPT/CSR after the exit of a bending magnet.

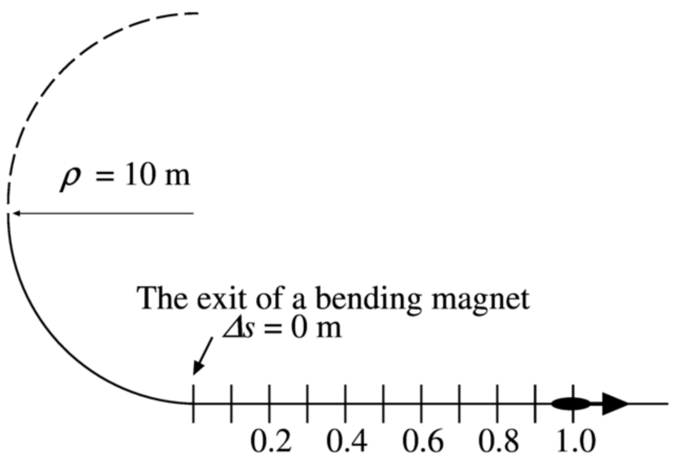
- Beam energy: 128 MeV
- Bending radius: $\rho = 10.0$ m
- Bunch length: $\sigma_s = 0.3$ mm
- Initial distribution: Gaussian
- Bunch charge: $Q = 80$ pC
- Shielding chamber height: $h = \infty$
- Number of image charge layers: 32

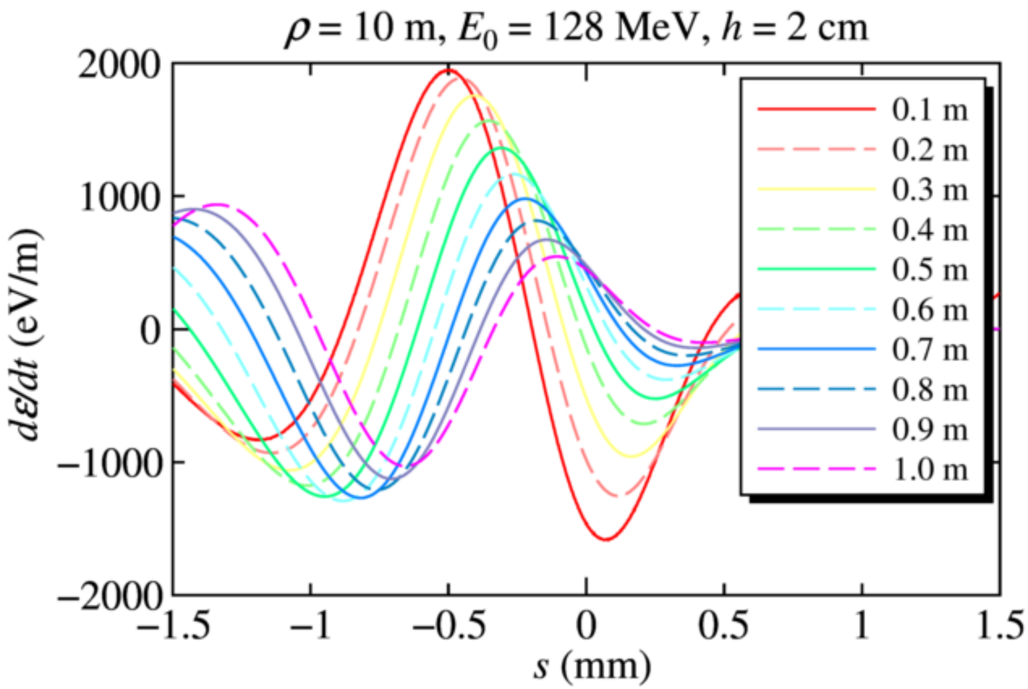




CSR in transient state with shielding

- Beam energy: 128 MeV
- Bending radius: $\rho = 10.0$ m
- Bunch length: $\sigma_s = 0.3$ mm
- Initial distribution: Gaussian
- Bunch charge: $Q = 80$ pC
- Shielding chamber height: $h = 2$ cm
- Number of image charge layers: 32



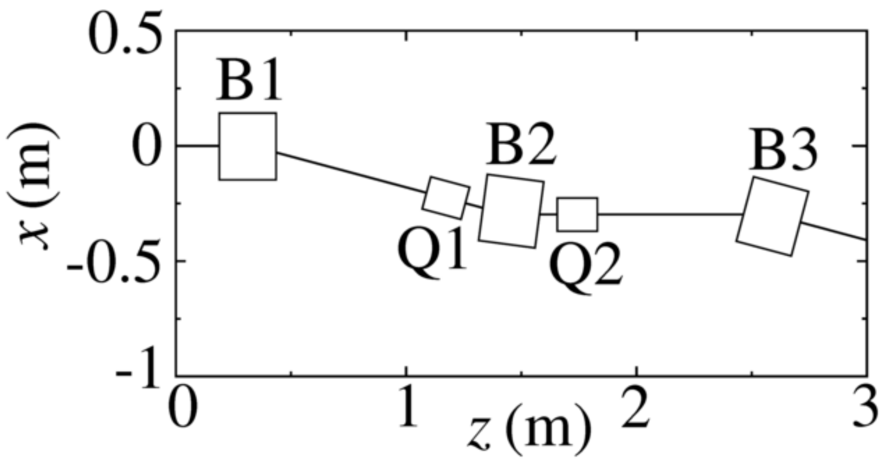


The figures show that the CSR wake reduces as the distance from the exit of the bending magnet increases as expected.

CSR calculation in ERL merger section

CSR in ERL merger section

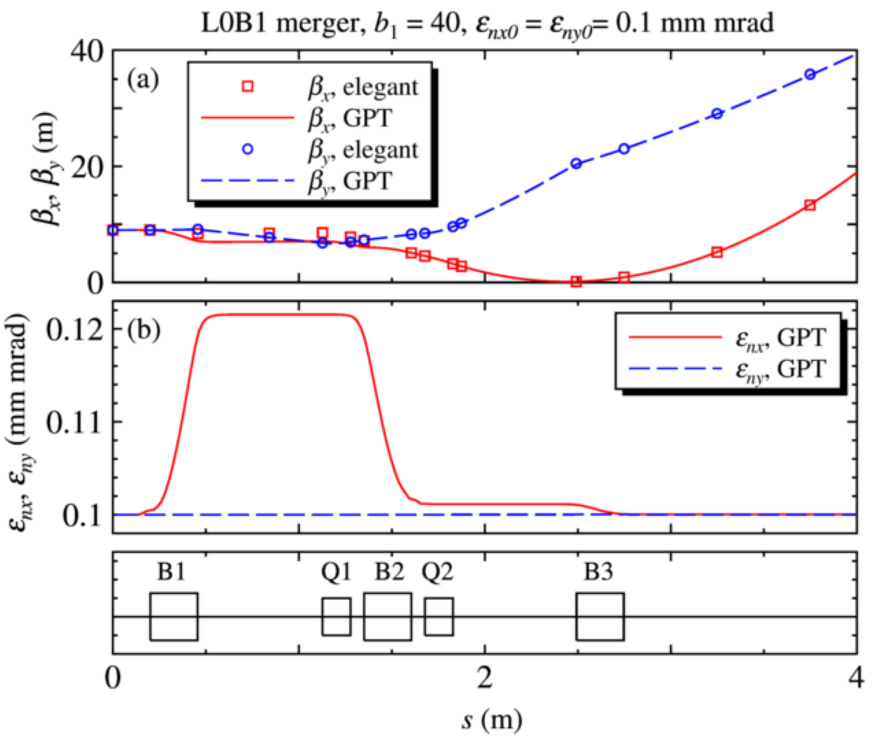
- As an example, the transverse emittance in a 3-dipole merger of ERL project at Cornell University is calculated by GPT/CSR and elegant for two different conditions:
- (a) $p_0 = 10$ MeV/c and (b) $p_0 = 500$ MeV/c.

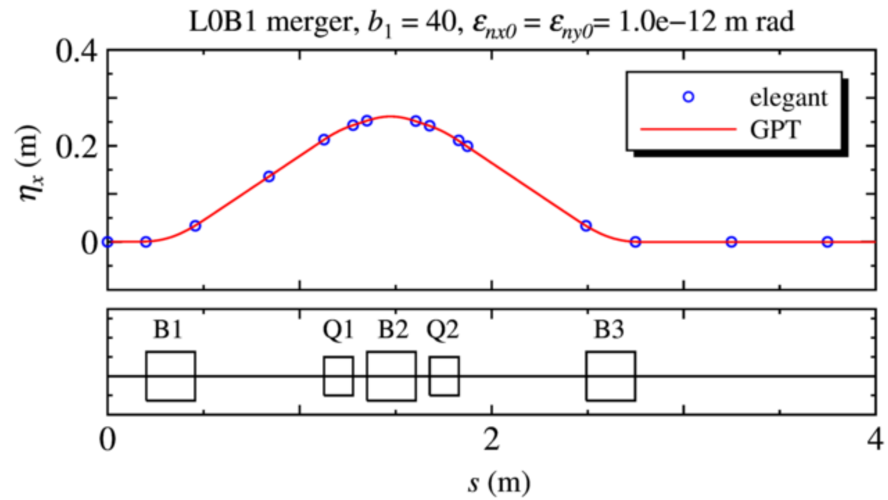


- Bunch length: $\sigma_s = 0.3$ mm
- Initial distribution: Gaussian
- Bunch charge: $Q = 80$ pC
- Initial emittance : $\varepsilon_{nx} = 1\times10^{-12}$ m rad
- Initial betatron function : $\beta_x = \beta_y = 9$ m
- Without shielding and space charge

CS parameters

Dispersion function

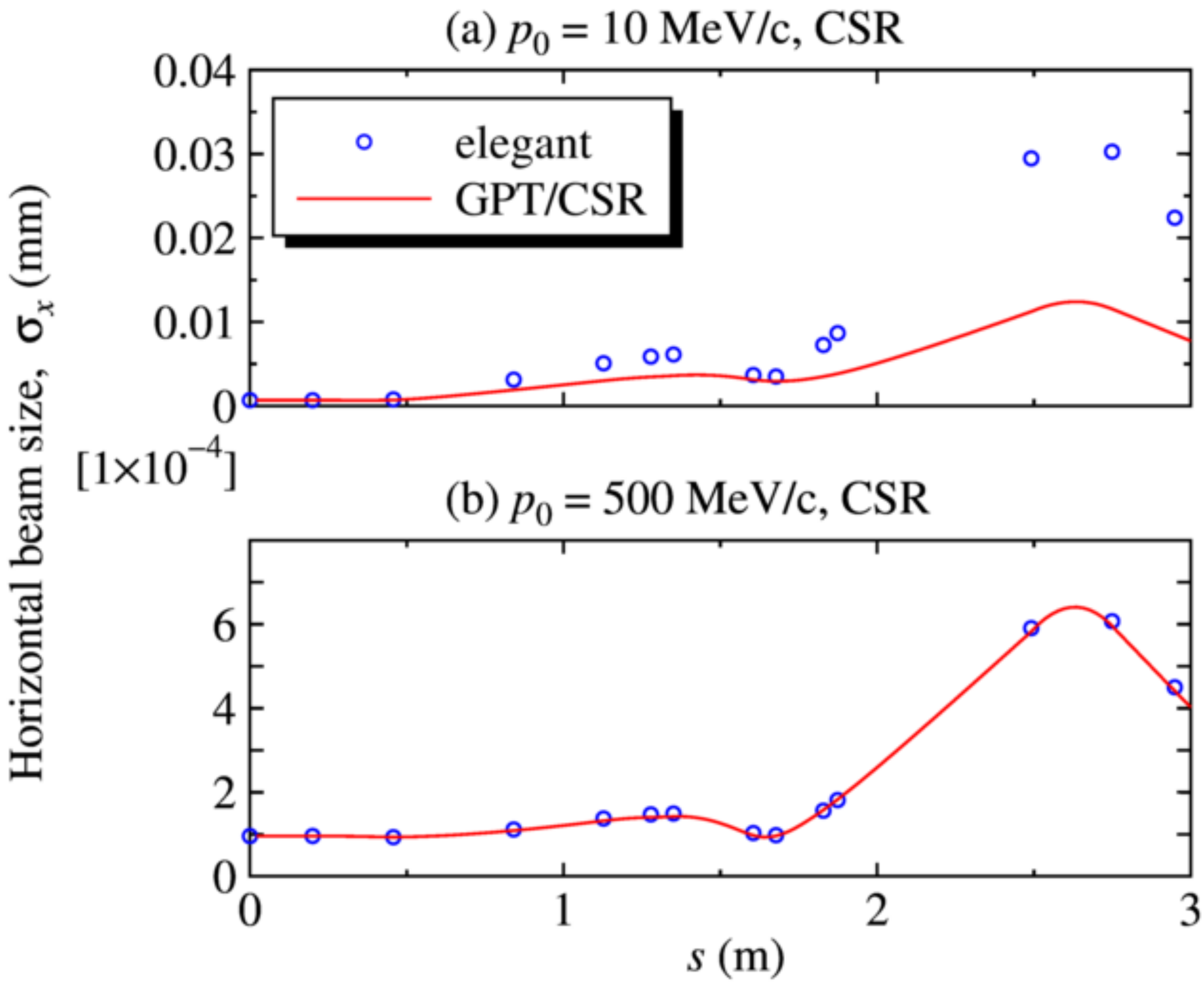




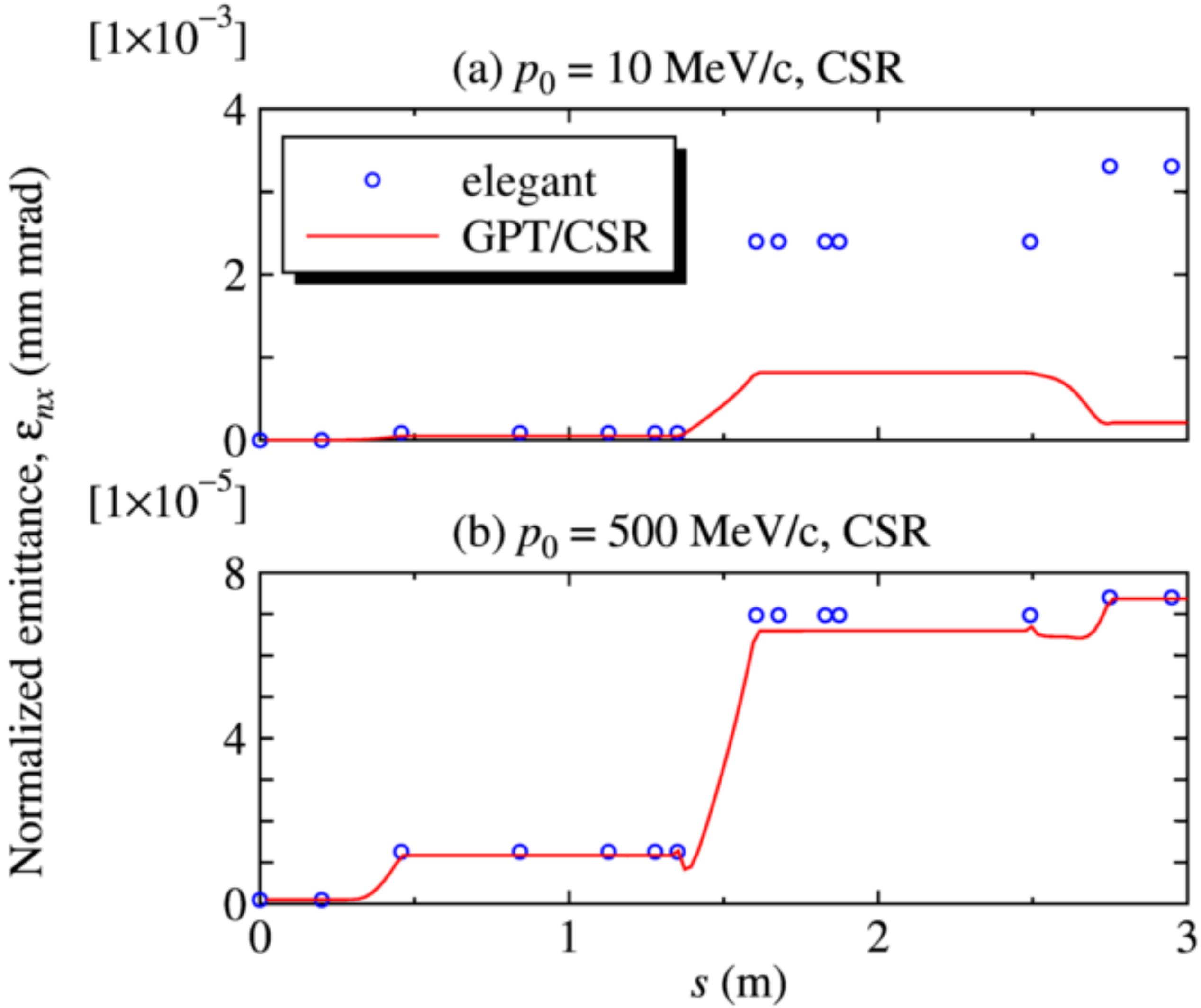
Normalized emittances are calculated by particle distirubion using the following equations,

$$\epsilon_{nx} = \langle\gamma\rangle\sqrt{\langle x_c^2\rangle\langle\beta_{xc}^2\rangle - \langle x_c\beta_{xc}\rangle^2}$$

$$\epsilon_{ny} = \langle\gamma\rangle\sqrt{\langle y_c^2\rangle\langle\beta_{yc}^2\rangle - \langle y_c\beta_{yc}\rangle^2}$$



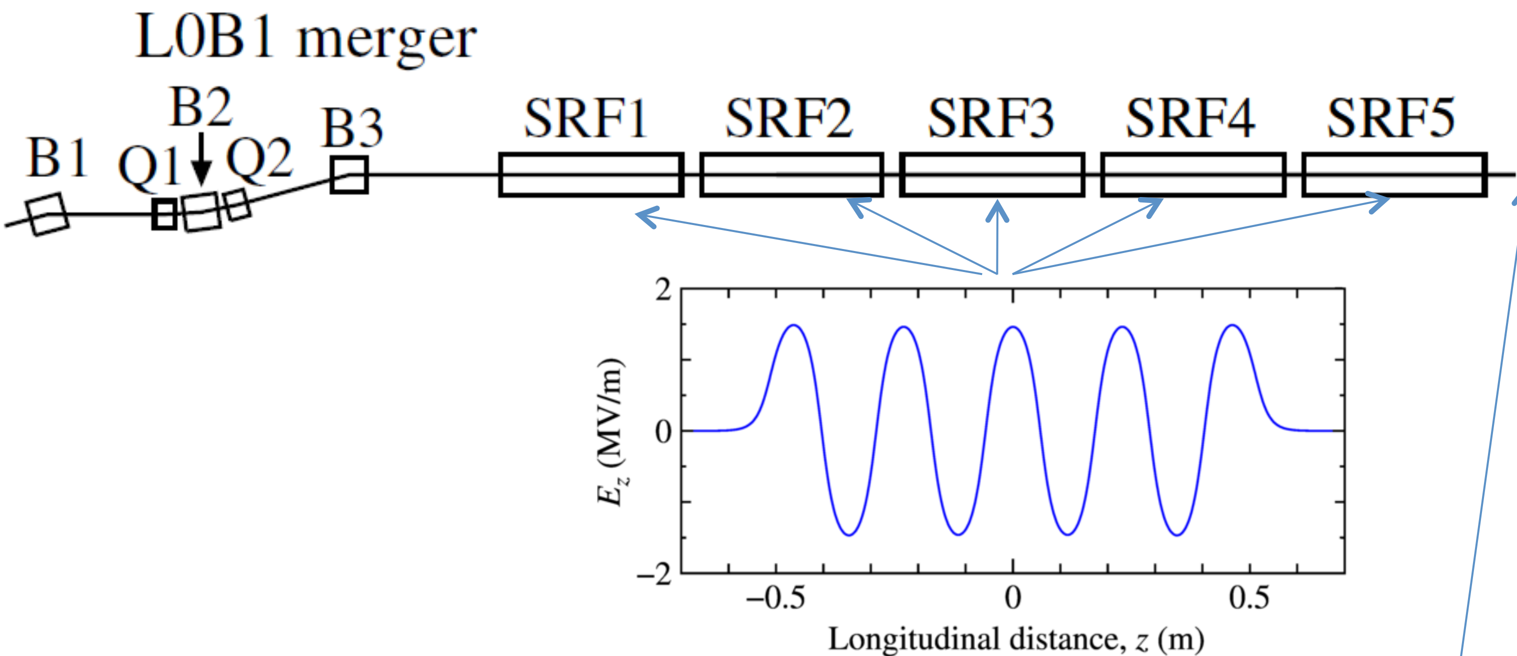
(a) p_0 = 10 MeV/c, CSR
0.04
0.03
0.02
0.01
0
elegant
GPT/CSR
Horizontal beam size, σ_x (mm)
[1×10^{-4}]
(b) p_0 = 500 MeV/c, CSR
6
4
2
0
0
1
2
3
s (m)



•For (a) $p_0 = 10$ MeV/c, the GPT/CSR and elegant results disagree.
•For (b) $p_0 = 500$ MeV/c, the agreement is good demonstrating that GPT/CSR reproduces elegant CSR calculations at higher beam energies as expected.

CSR and Space charge effects in ERL merger section

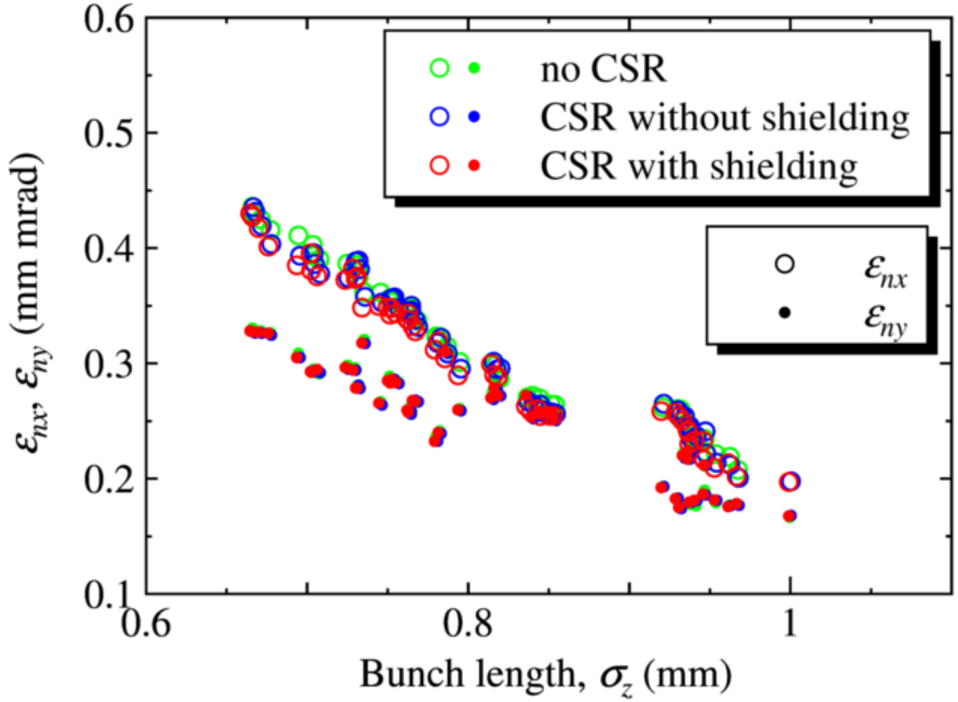
- CSR and Space charge effects in ERL merger section were calculated by the GPT/CSR.
- The beam line consists of 3 dipoles merger and SRF cavities.

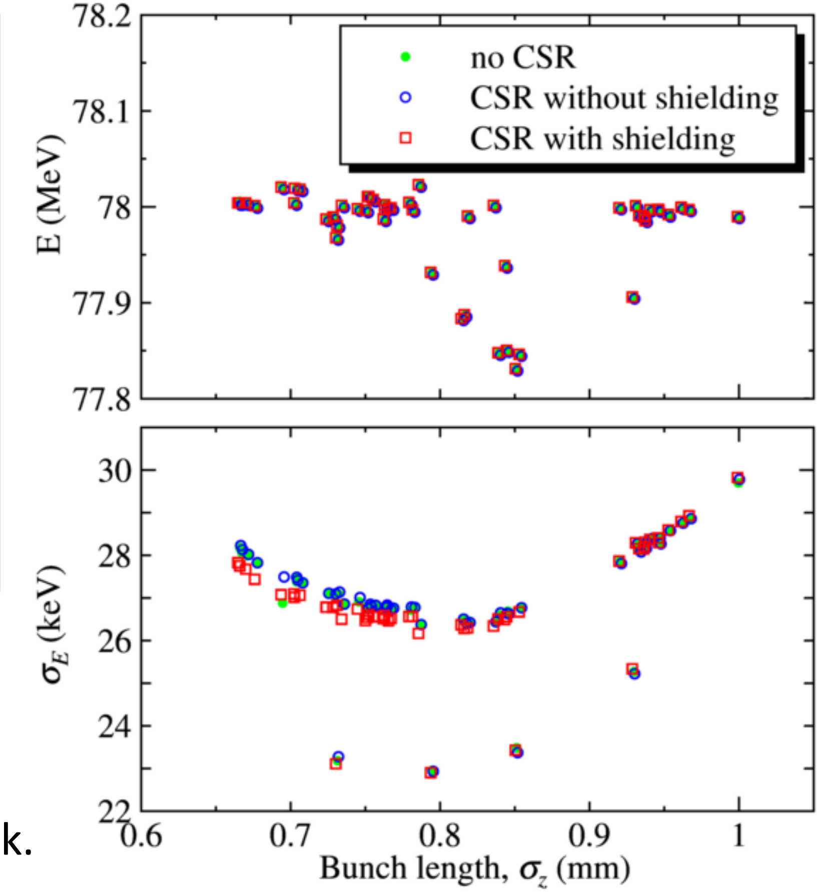


The beam parameters were calculated at the eixt of SRF5.

Minimizing emittance and bunch length

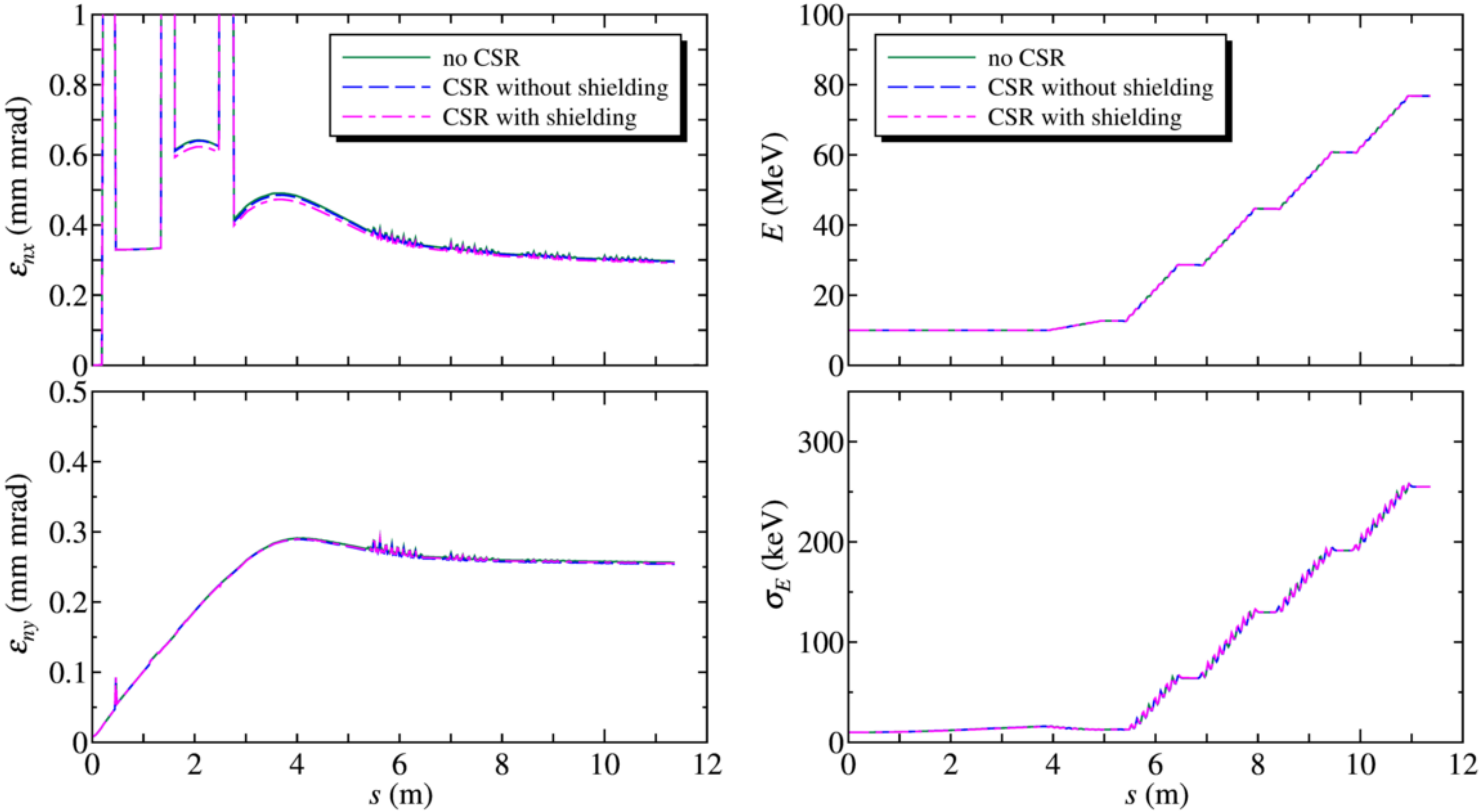
- The beam line parameters were optimized to minimize emittance and bunch length at the exit of beam line with and without CSR effect.
- Initial beam energy and bunch charge are 10 MeV and 80 pC/bunch.





The results shows that the effect of CSR is weak.

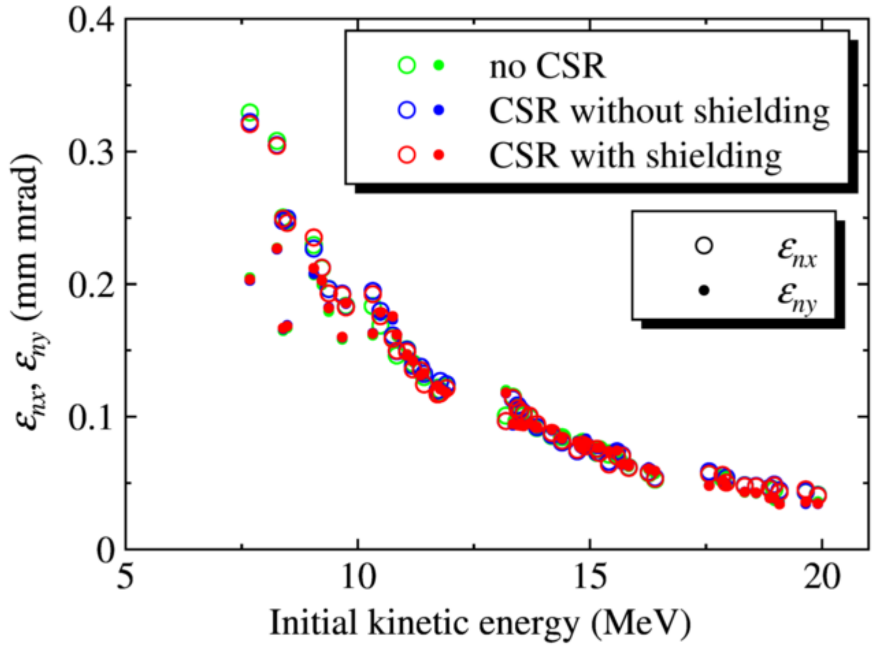
- Time evolutions with the bunch length of 0.8 mm were calculated.

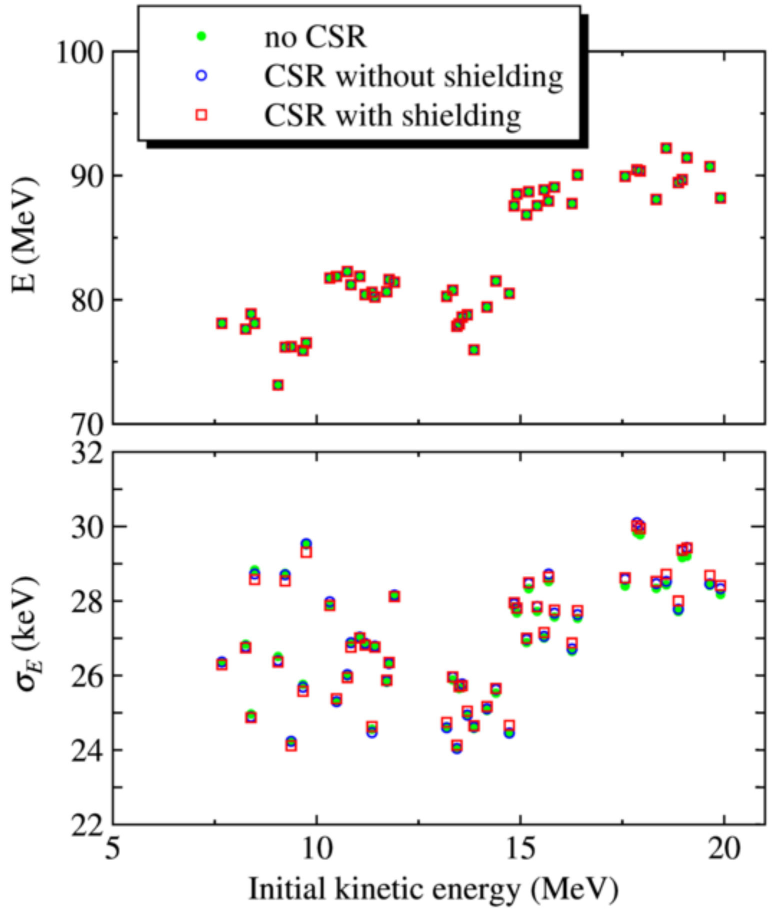


In this case, CSR effect is negligible.

Minimizing emittance and kinetic energy

- The beam line parameters were optimized to minimize emittance and kinetic energy at the exit of beam line with and without CSR effect.
- Initial bunch length and bunch charger are 0.9 mm and 80 pC/bunch.





CSR effect is negligible for emittance calculation.

Summary

- We have developed a CSR routine for GPT in order to investigate beam dynamics in ERL and FEL injectors.
- To check GPT/CSR, energy loss and energy spread are calculated by GPT/CSR, elegant and analytical expression.
- The results show GPT/CSR to be effective in a wide range of beam energies.

- We calculated CSR effect in ERL merger section using the GPT/CSR.
- The results shows the CSR effect in the ERL merger section is negligible.

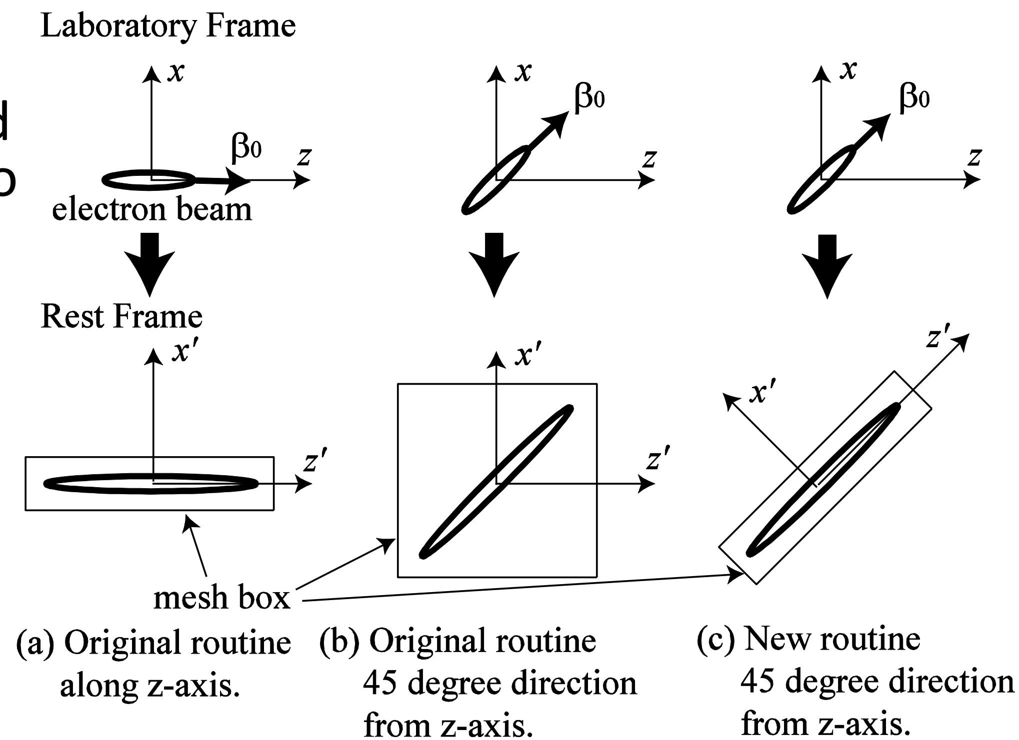
Enhanced 3D Space Charge Routine in GPT

Enhanced 3D Space Charge Routine in GPT

- To calculate the space charge field in the 3D mesh-based routine in GPT, the particle coordinates are transformed from the laboratory frame to the rest frame according to

  relative to the direction of motion.

- When the bunch does not move along the z-axis, the bounding box ends up improperly oriented.



In this case, for example, the transverse emittance incorrectly depends on the angle relative to the z-axis in a straight trajectory.

To fix this problem, we have added a transformation of rotation in the rest frame in the space charge routine.