Numerical simulations of CSR impedance

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and

Acknowledgements: K. Ohmi, T. Agoh, Y. Cai, K. Yokoya, M. Kikuchi, K. Shibata, H. Ikeda

CSR mini-workshop, KEK, Nov. 08, 2010

Outline

Introduction

Motivations:

1. To find out the unknown source of longitudinal impedance which drive the microwave instability (MWI) in the KEKB LER
2. To work out a reliable impedance model for SuperKEKB DR
3. CSR in wigglers, with interference, or with resistive wall

Existing publications on numerical calculations of CSR impedance:

1. T. Agoh and K. Yokoya, PRST-AB 7, 054403 (2004) and T. Agoh, PhD. Thesis (2004)
2. K. Oide, Presentation at KEKB ARC 2009 and PAC09
3. G. Stupakov and I. Kotelnikov, PRST-AB 12, 104401 (2009)

Algorithms - Fundamental equations

Parabolic equation in curvilinear coordinate system:

$$\frac{\partial \vec{E}_\perp}{\partial s} = \frac{i}{2k}\left(\nabla_\perp^2 \vec{E}_\perp - \mu_0 c^2 \nabla_\perp \rho_0 + \frac{2k^2 x}{\rho(s)}\vec{E}_\perp\right)$$

β=1

Field separation:

$$\vec{E}_\perp = \vec{E}_\perp^r + \vec{E}_\perp^b$$

$$\frac{\partial \vec{E}_\perp^r}{\partial s} = \frac{i}{2k}\left[\nabla_\perp^2 \vec{E}_\perp^r + \frac{2k^2 x}{\rho}(\vec{E}_\perp^r + \vec{E}_\perp^b)\right]$$

Beam field in free space (independent of s):

$$\frac{\partial^2 E_x^b}{\partial x^2} + \frac{\partial^2 E_x^b}{\partial y^2} = \mu_0 c^2 \frac{\partial \rho_0}{\partial x}$$

$$\frac{\partial^2 E_y^b}{\partial x^2} + \frac{\partial^2 E_y^b}{\partial y^2} = \mu_0 c^2 \frac{\partial \rho_0}{\partial y}$$

Ref. T. Agoh and K. Yokoya, PRST-AB 7, 054403 (2004)

Beam field in free space with finite beam sizes

Typical beam size: σ_x=0.5mm, σ_y=0.01mm (bi-gaussian)

$$E_x(x,y) = \frac{\lambda(k)}{2\epsilon_0\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}}\mathrm{Im}[F(x,y)]$$

$$E_y(x,y) = \frac{\lambda(k)}{2\epsilon_0\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}}\mathrm{Re}[F(x,y)]$$

$$F(x,y) = w\left(\frac{x+iy}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right) - e^{-\frac{x^2}{2\sigma_x^2}-\frac{y^2}{2\sigma_y^2}} w\left(\frac{\frac{\sigma_y}{\sigma_x}x + i\frac{\sigma_x}{\sigma_y}y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}}\right)$$

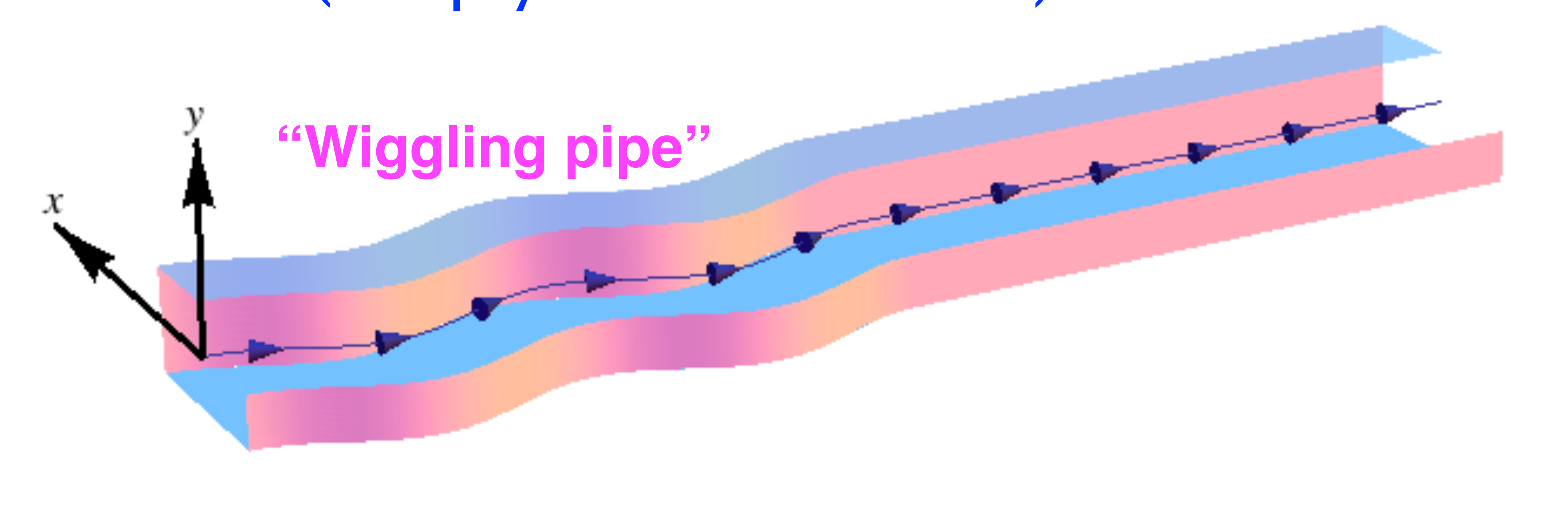
$$w(z) = e^{-z^2}\left(1+\frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right)$$

Ref. T. Agoh, Ph.D. Thesis (2004)

Algorithms - Beam pipe

Model of the beam pipe:

1. The bending radius can be arbitrarily s-dependent, which allows for treating fringe field, wigglers or a series of dipole magnets
2. Uniform rectangular cross section along the beam orbit (simply the calculation)



Field integration along s:

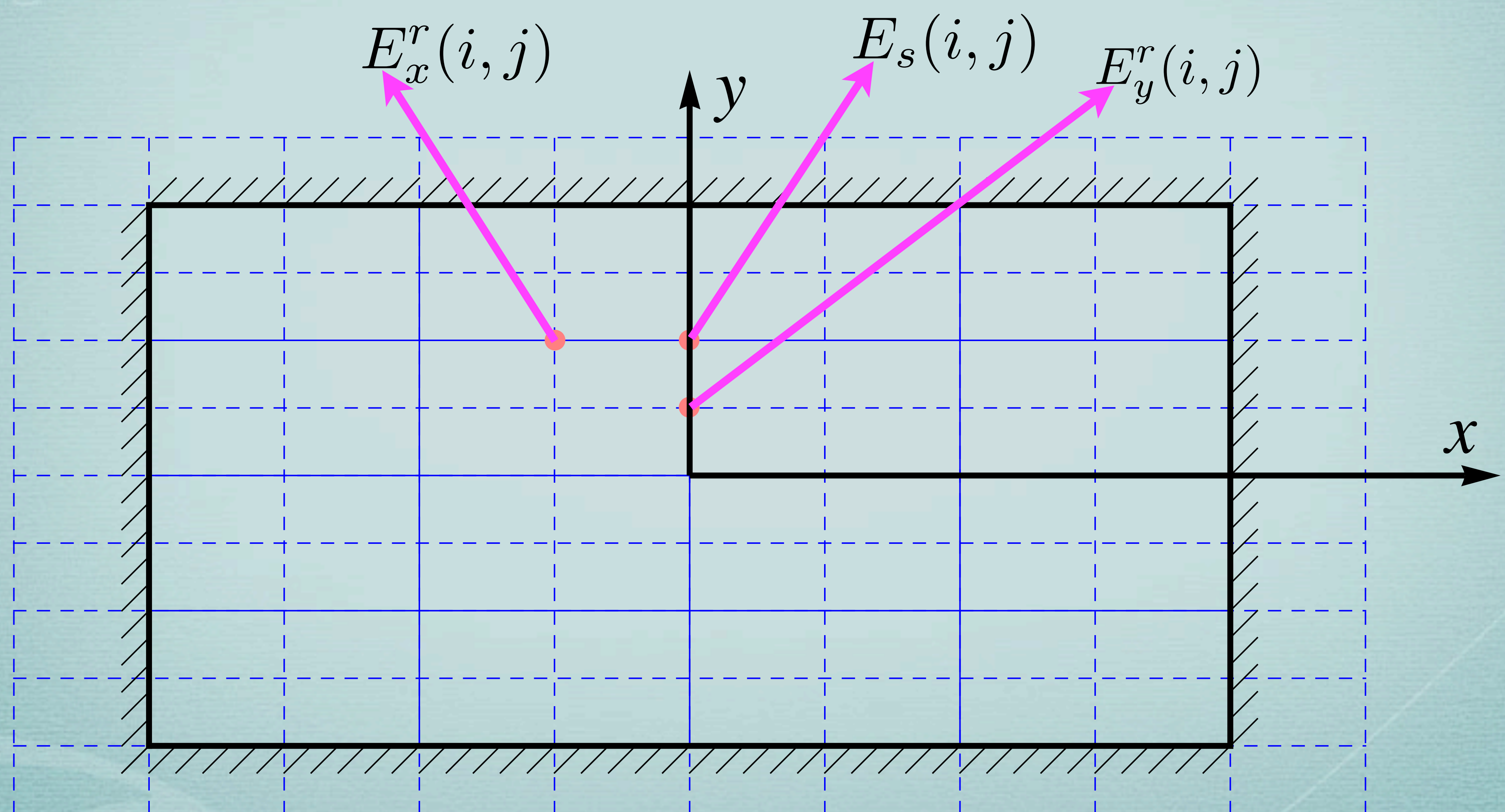
1. Toroidal part: Numerical integration
2. Straight pipe: mode expansion

Ref. T. Agoh, Ph.D. Thesis (2004)
G. Stupakov and I. Kotelnikov, PRST-AB 12, 104401 (2009)

Algorithms - Mesh

Finite-difference discretization:

1. Staggered grid: Central difference
2. Ghost point: Boundary conditions with perfect wall

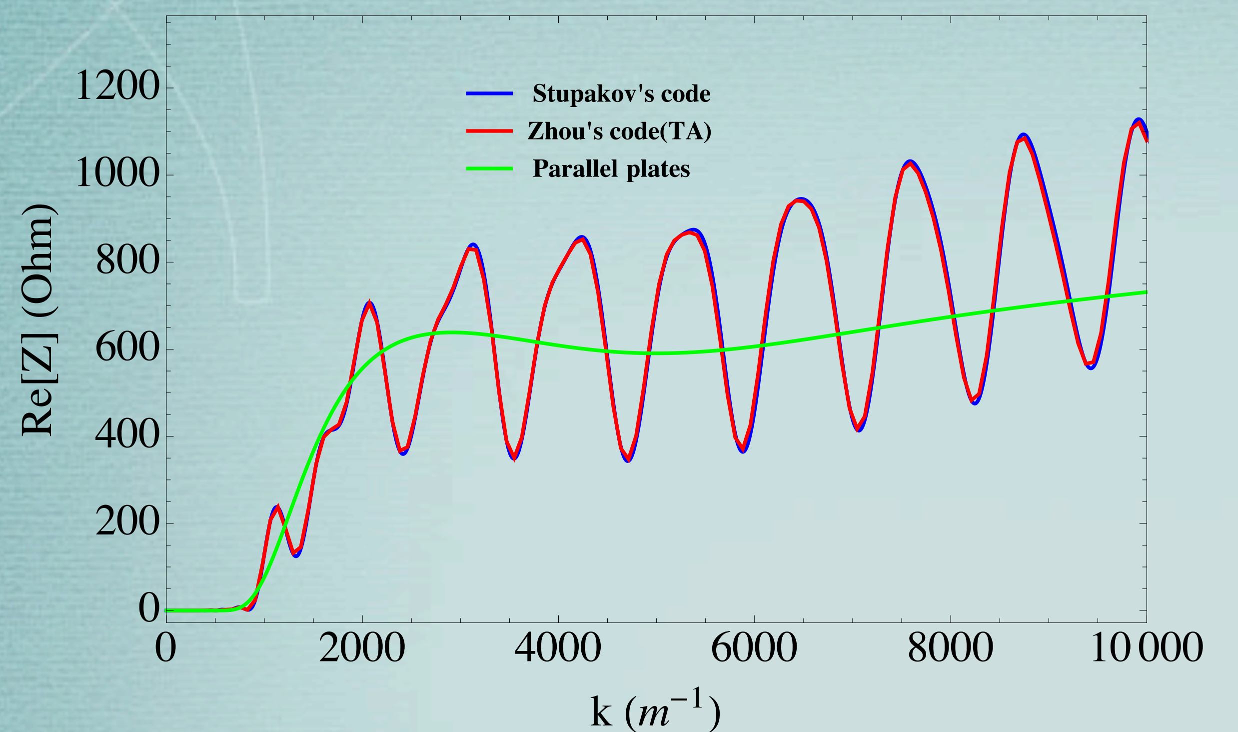


Ref. T. Agoh, Ph.D. Thesis (2004)

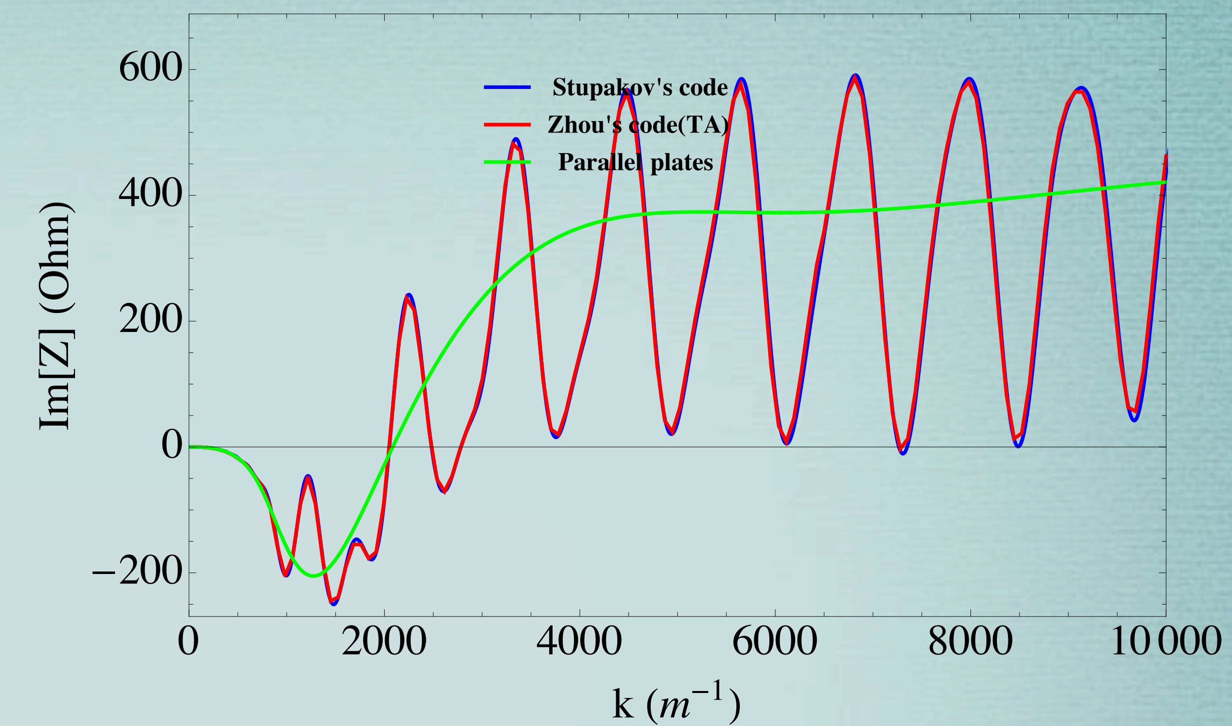
Benchmark results - Single dipole

Collaborate with K. Marit

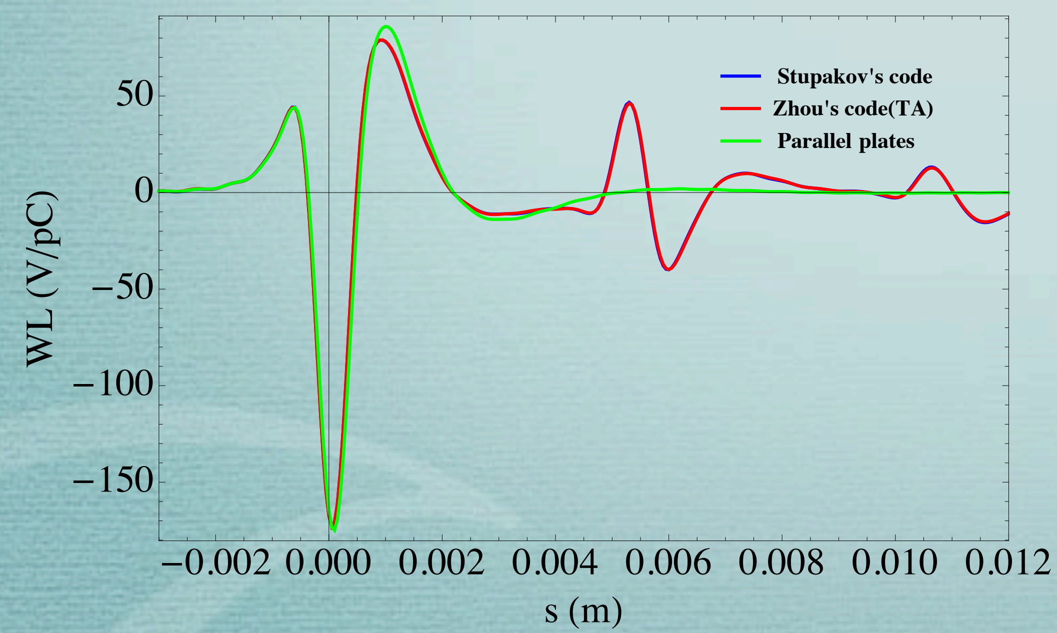
Re.Z_L(k)



Im.Z_L(k)



W_L(s) with σ_z=0.3mm



Fluctuation in impedance is due to reflections of side walls (see Oide's talk)

ANKA

w/h=70/32mm

L_{bend}=2.183m

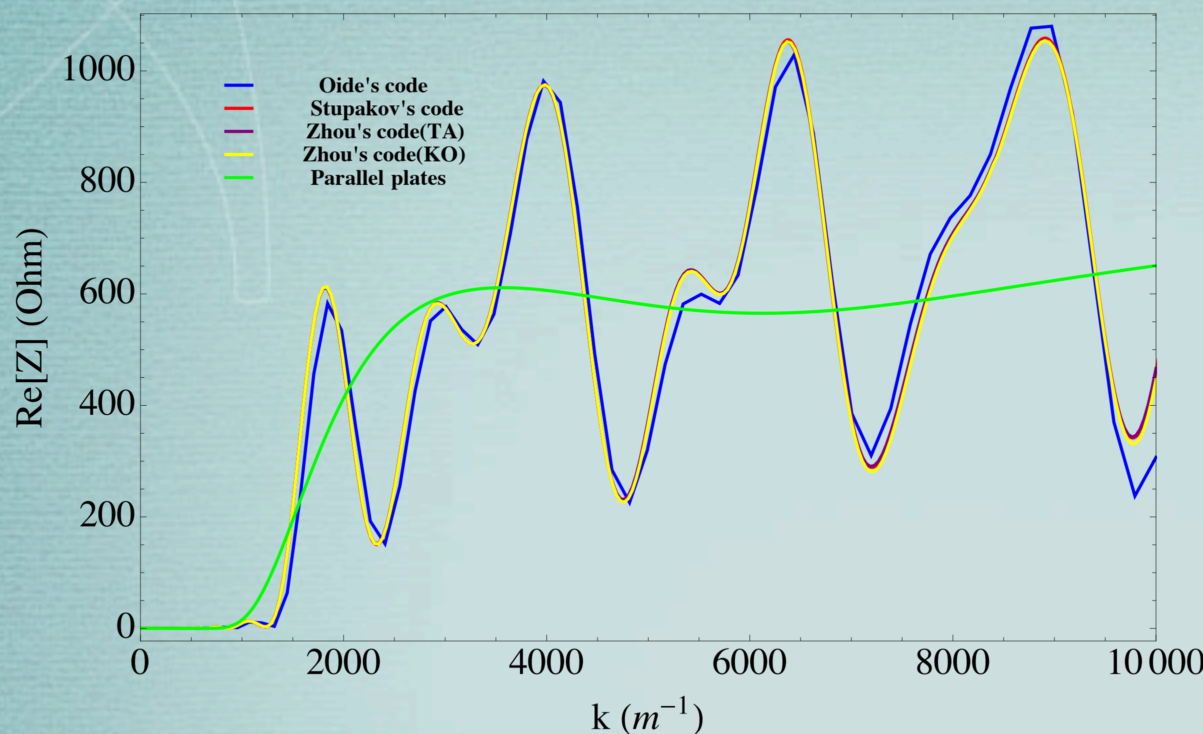
ρ=5.559m

L_{exit}=Infinity (pipe after exit)

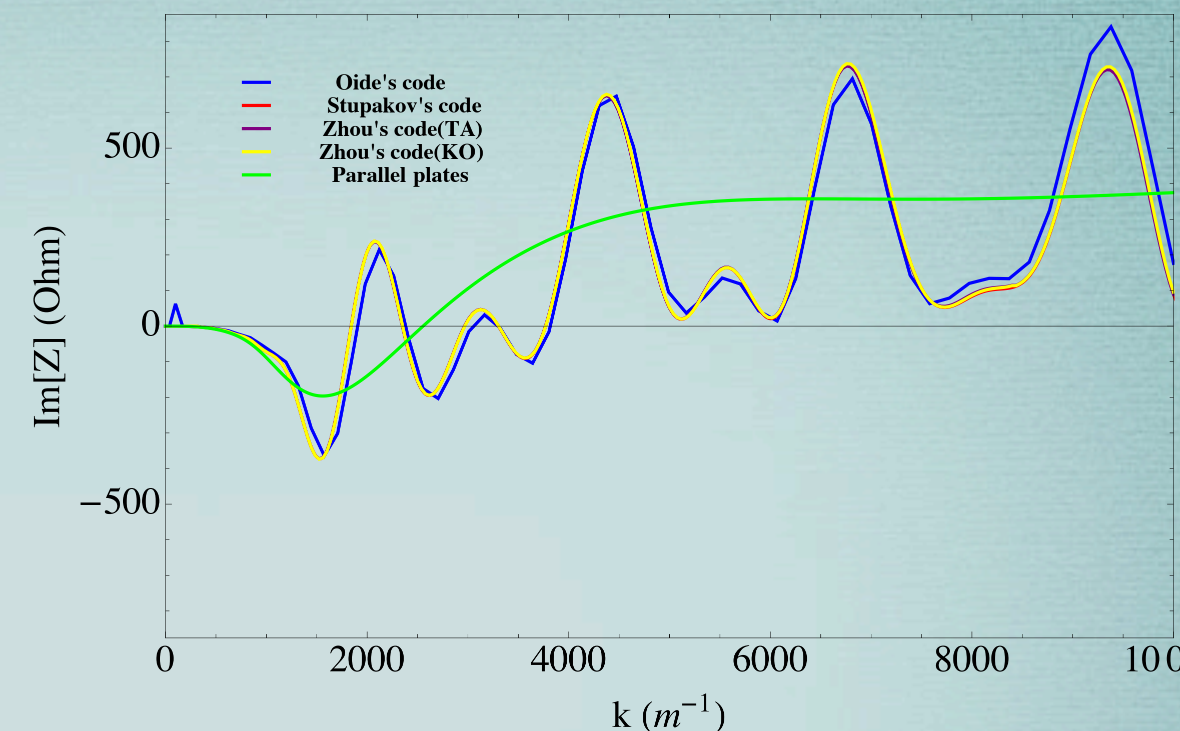
X_{offset}=0mm

Benchmark results - Single dipole

Re.Z_L(k)

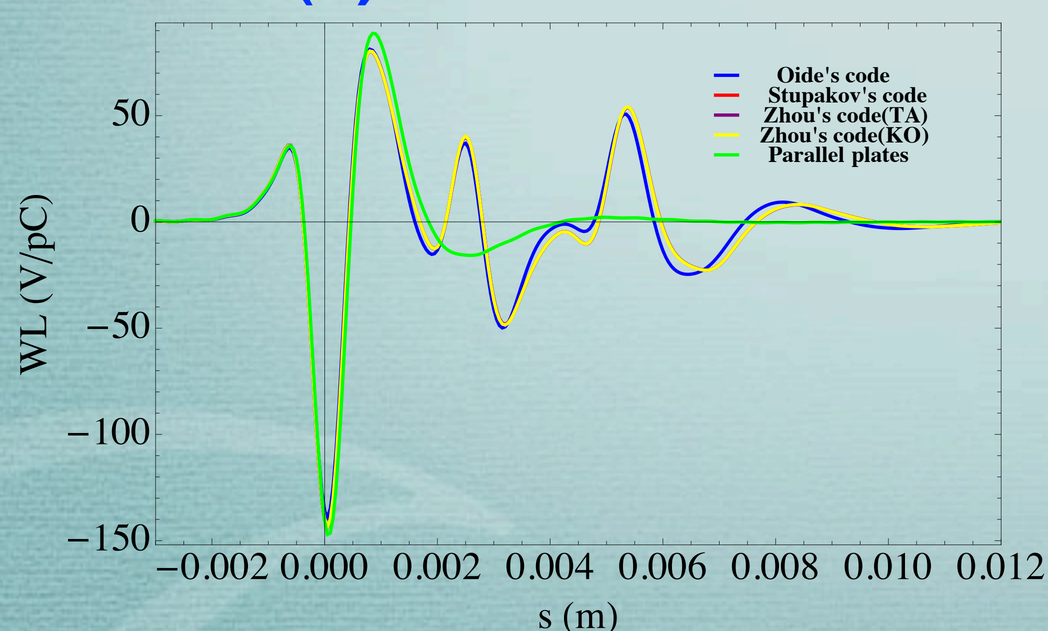


Im.Z_L(k)



TA: Agoh's algorithm
KO: Oide's algorithm

W_L(s) with σ_z=0.3mm



w/h=60/40mm

L_{bend}=4m

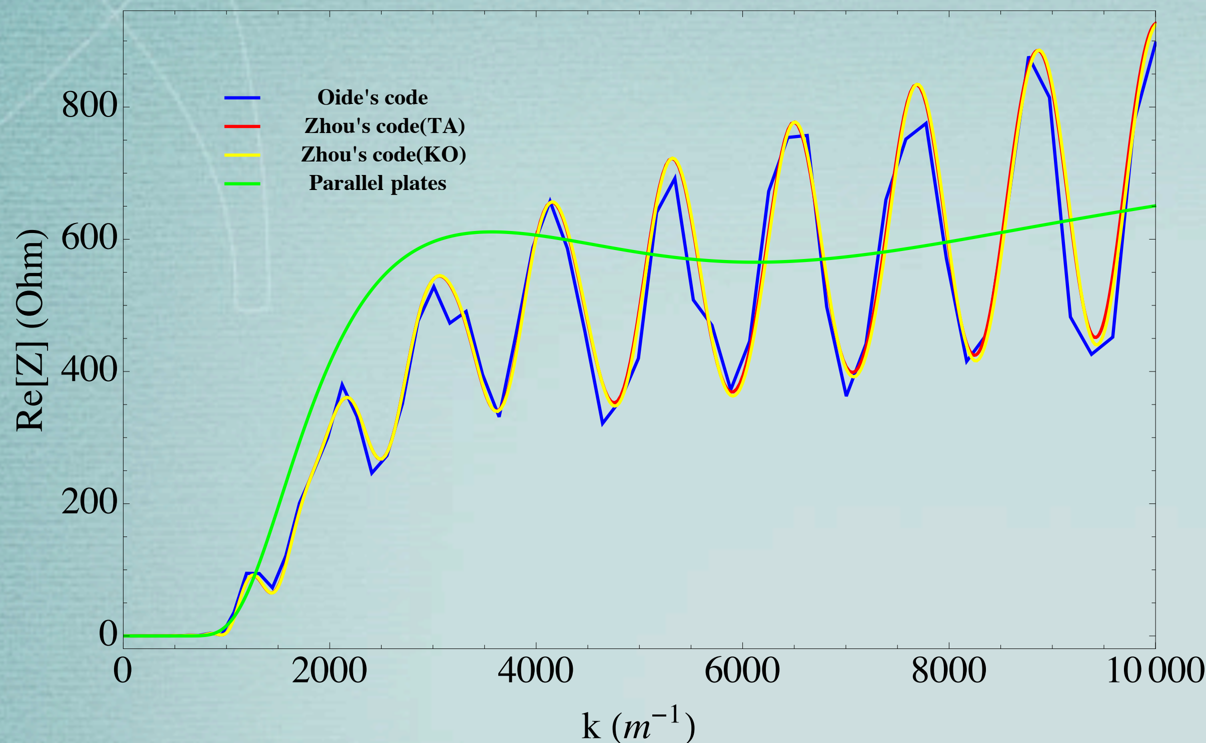
ρ=16.3m

L_{exit}=Infinity (pipe after exit)

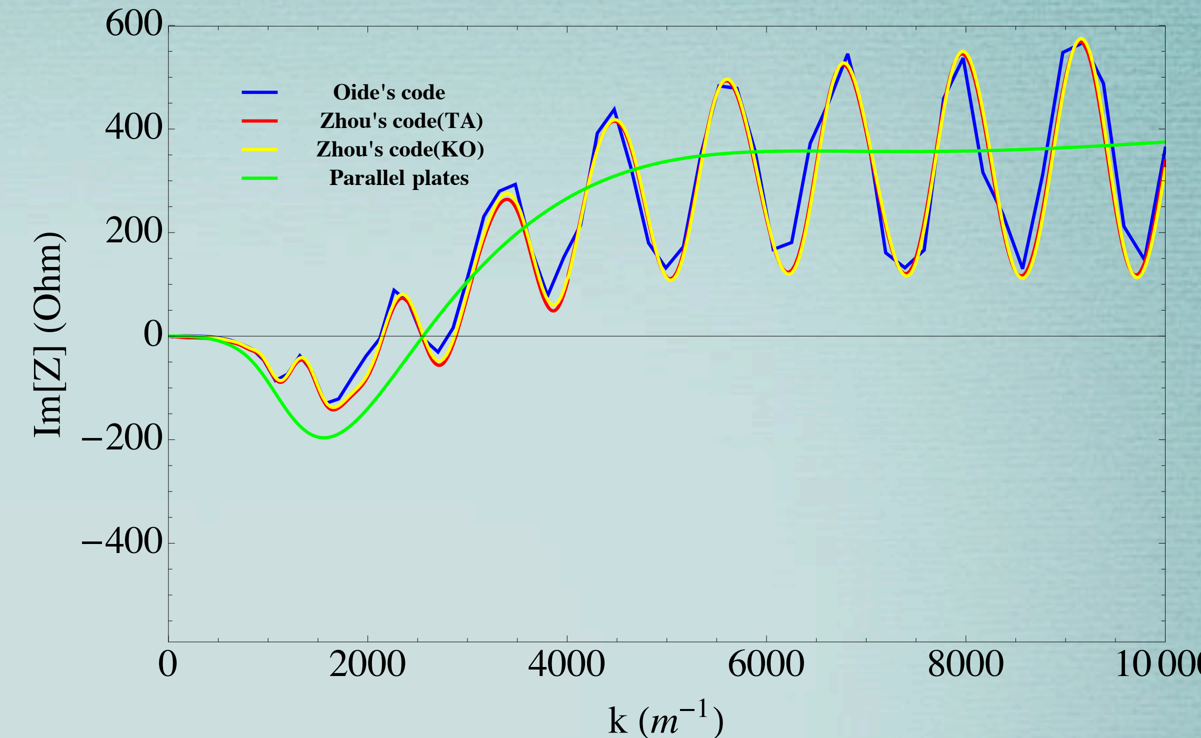
X_{offset}=0mm

Benchmark results - Single dipole

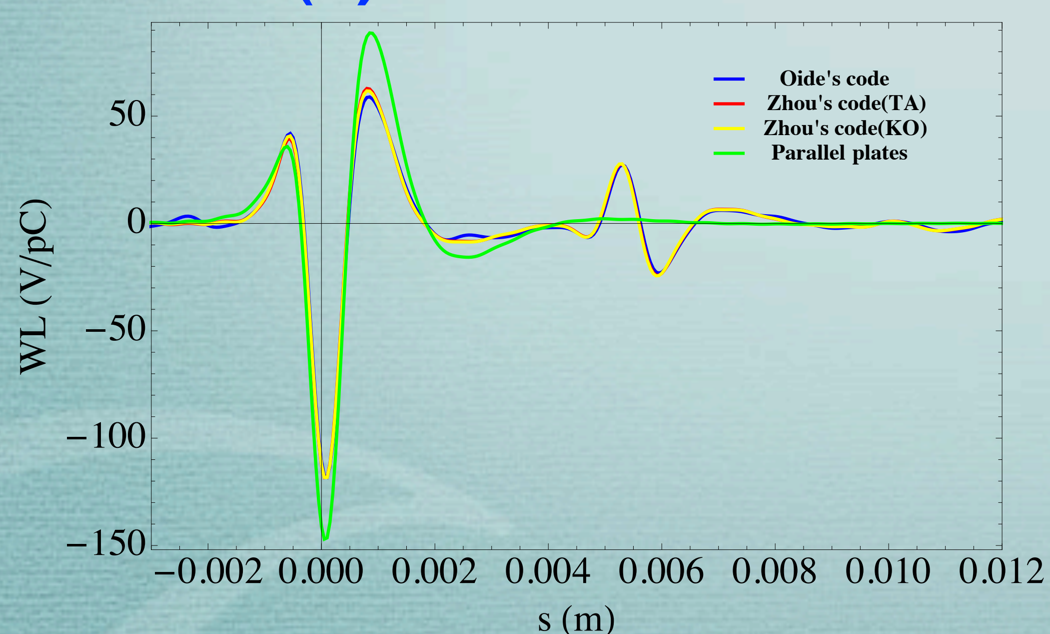
Re.$Z_L(k)$



Im.$Z_L(k)$



$W_L(s)$ with σ_z=0.3mm

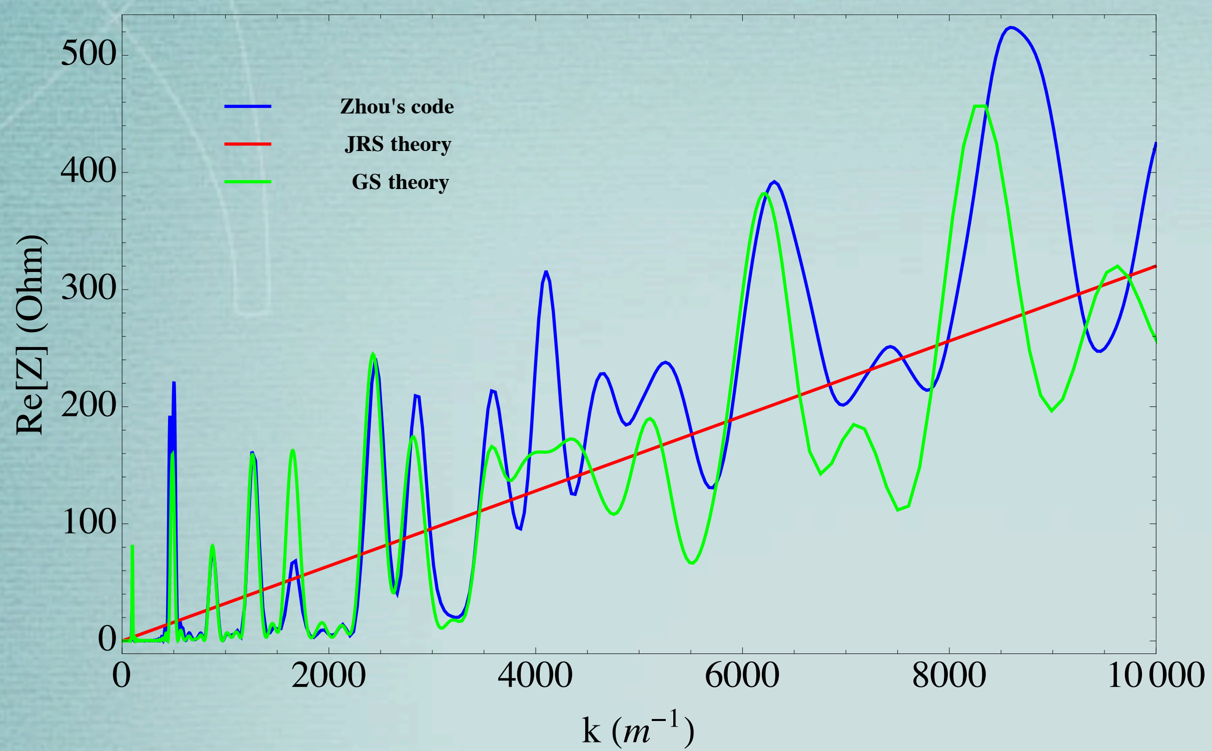


TA: Agoh's algorithm
KO: Oide's algorithm

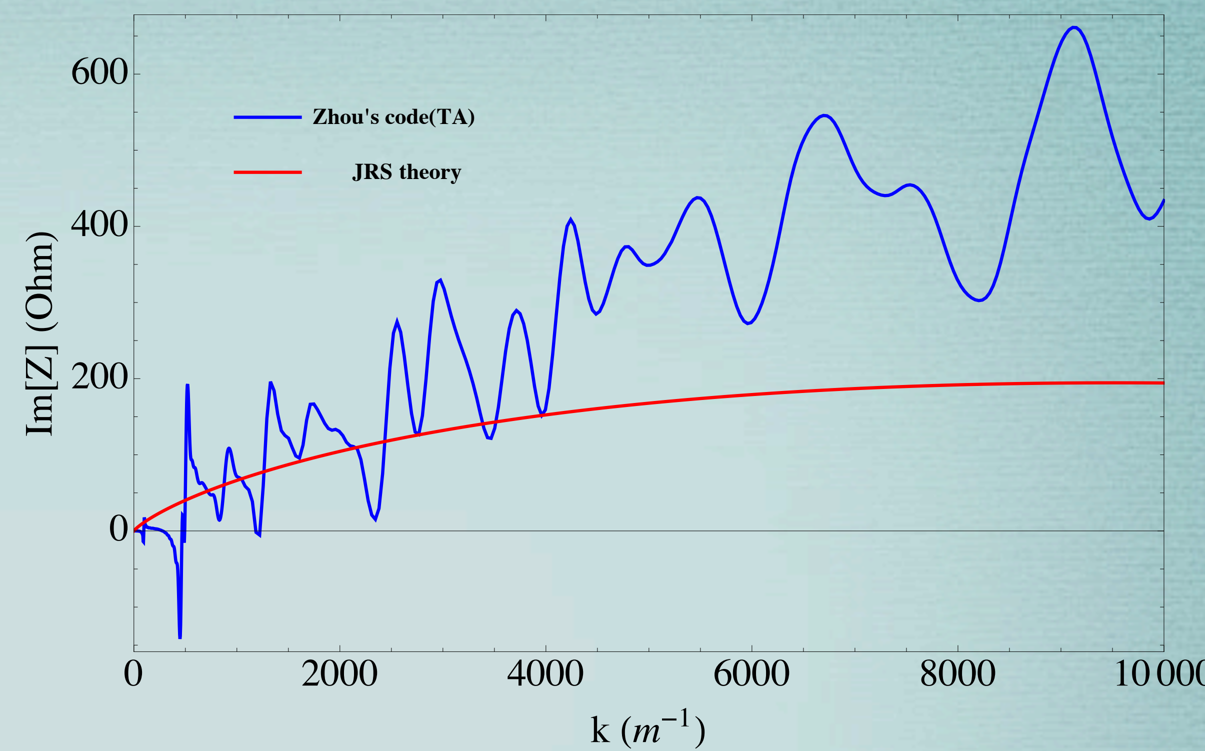
w/h=60/40mm
L_{bend}=4m
ρ=16.3m
L_{exit}=Infinity (pipe after exit)
X_{offset}=-20mm (To inner wall)

Benchmark results - Wiggler

Re.Z_L(k)

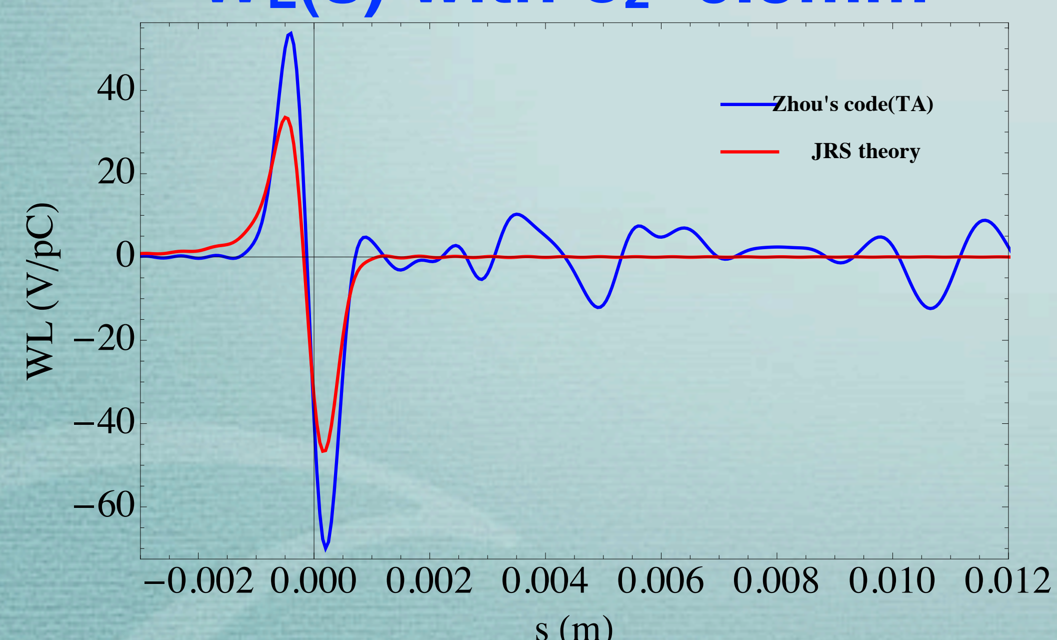


Im.Z_L(k)



JRS: Wu-Stupakov-Raubenheimer theory
J. Wu et al., PRST-AB 6, 040701 (2003)

W_L(s) with σ_z=0.3mm

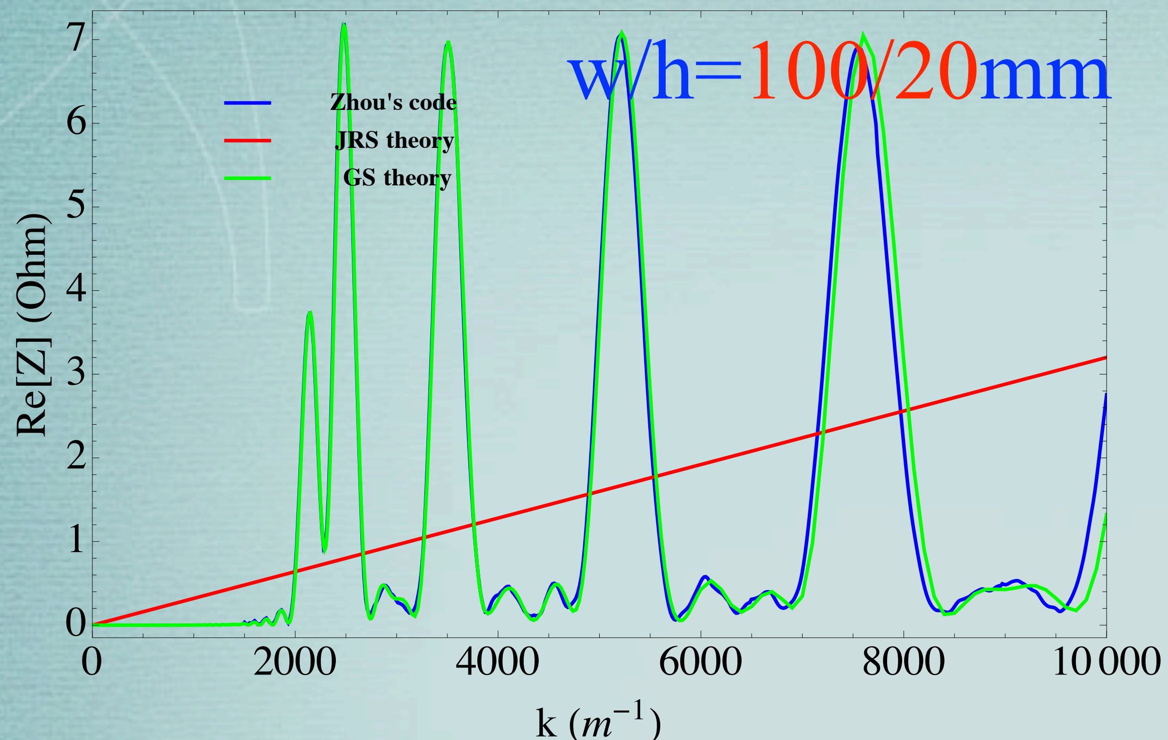


N_{period}=10
w/h=94/94mm
λ_w=1.088m
ρ=15.483m
L_{exit}=Infinity (pipe after exit)
Xoffset=0mm

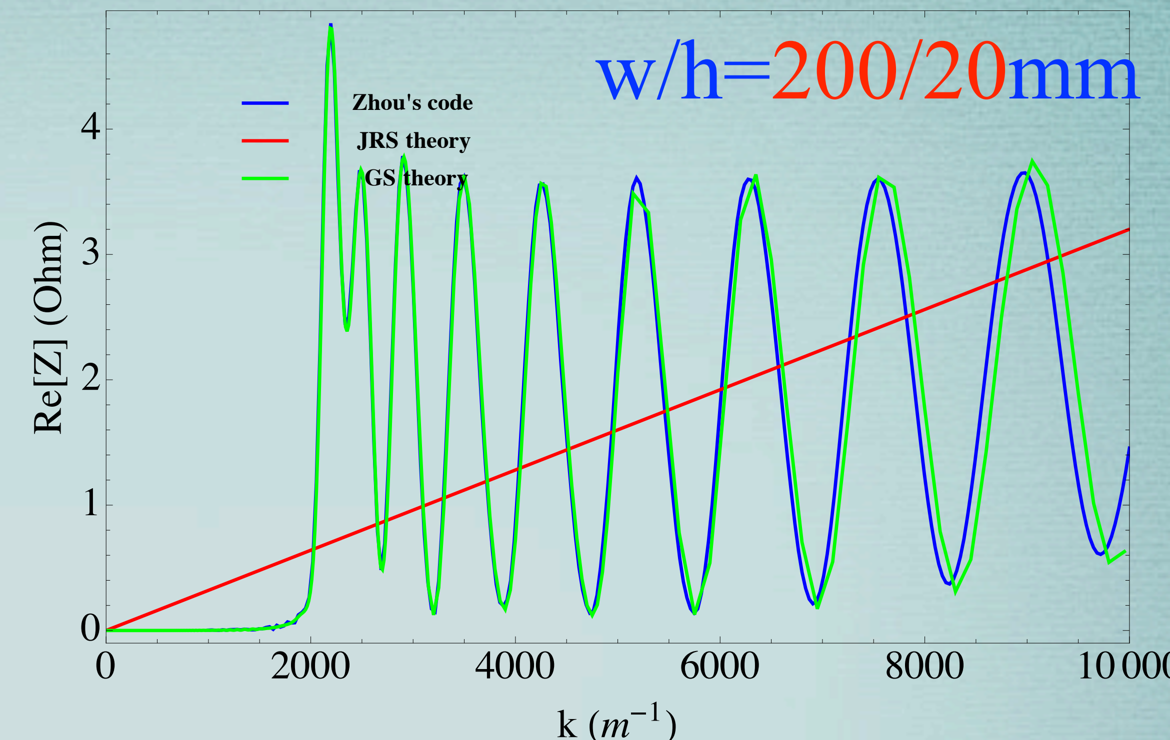
KEKB-LER type

Benchmark results - Wiggler

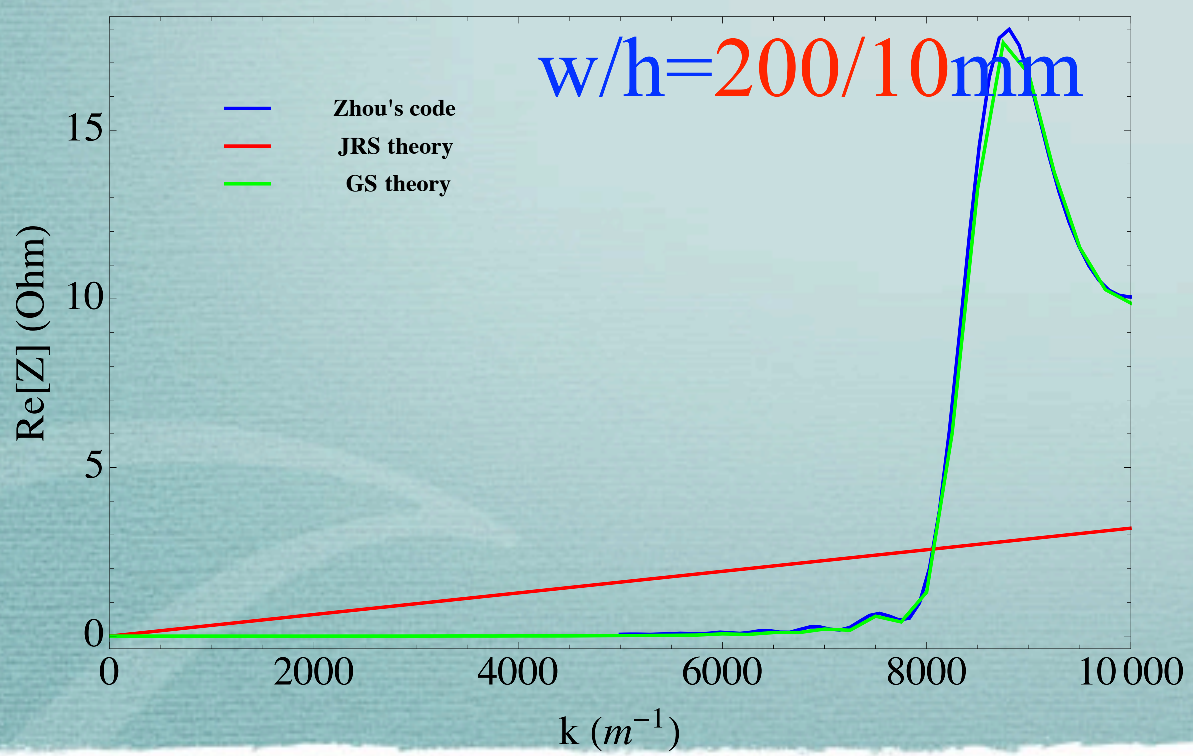
Re.Z_L(k)



Re.Z_L(k)



Re.Z_L(k)



Note: "Wiggling pipe" is not good enough!

N_{period}=10

λ_w=1.088m

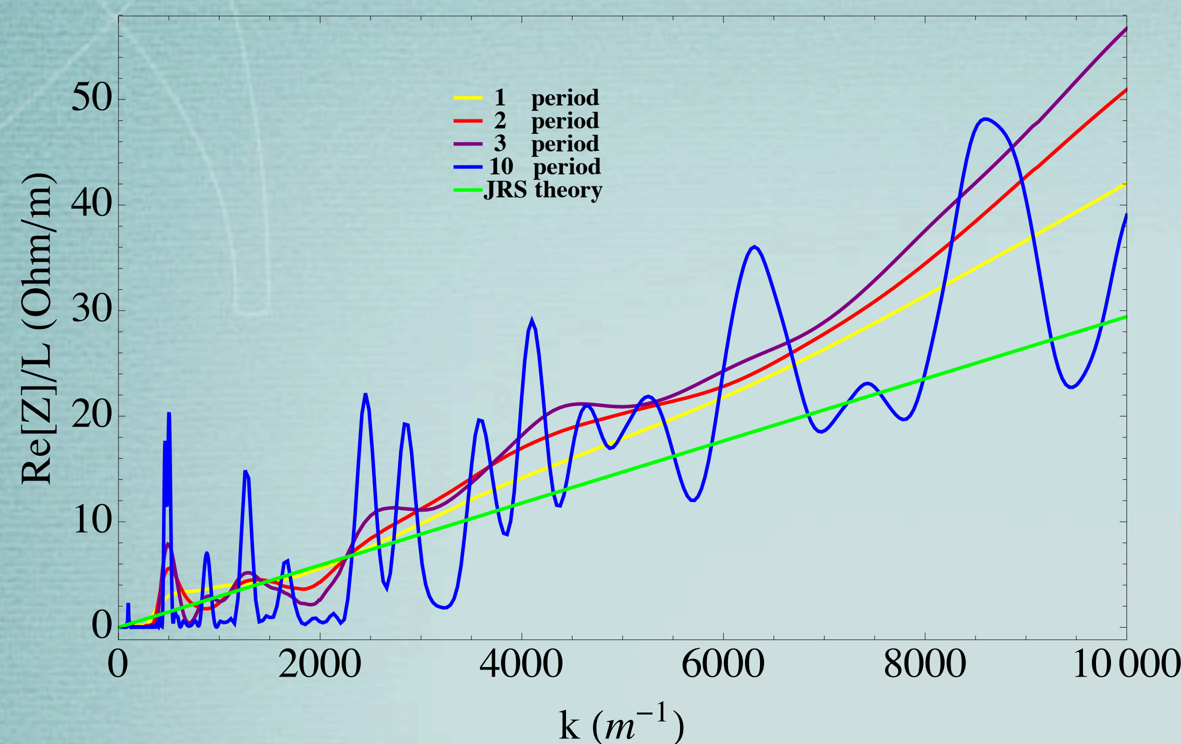
ρ=154.83m

L_{exit}=Infinity (pipe after exit)

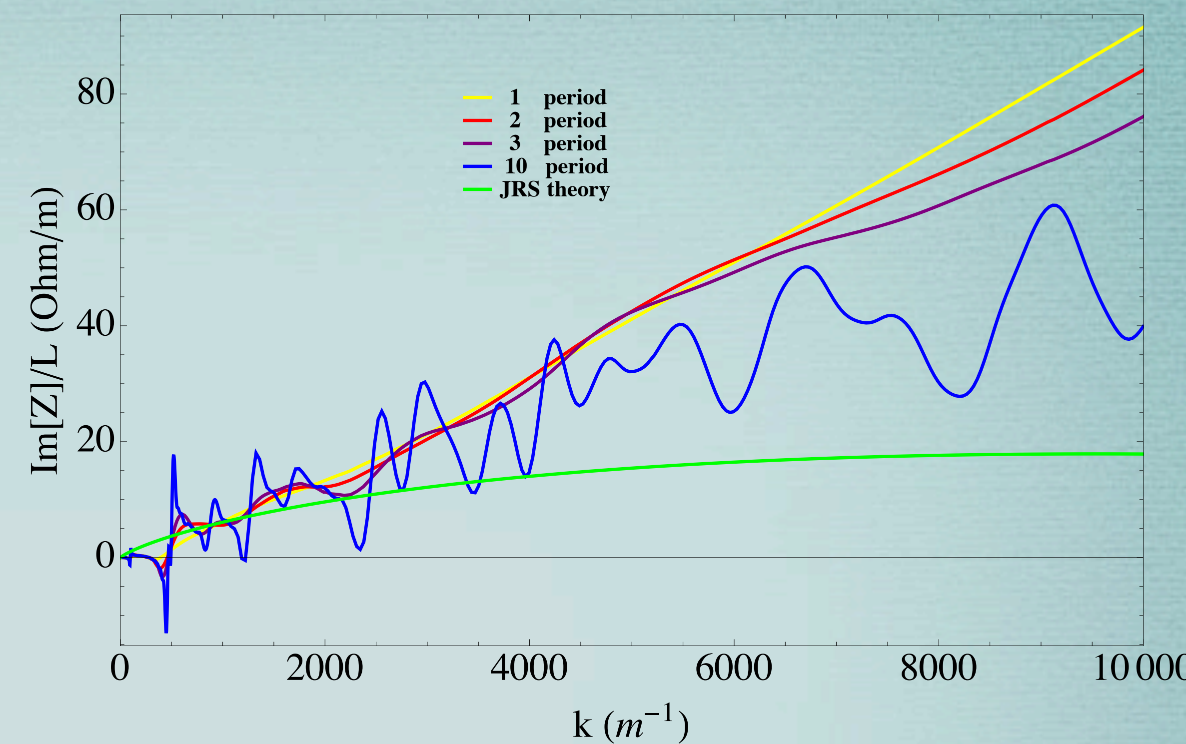
Xoffset=0mm

CSR in wigglers - KEKB LER

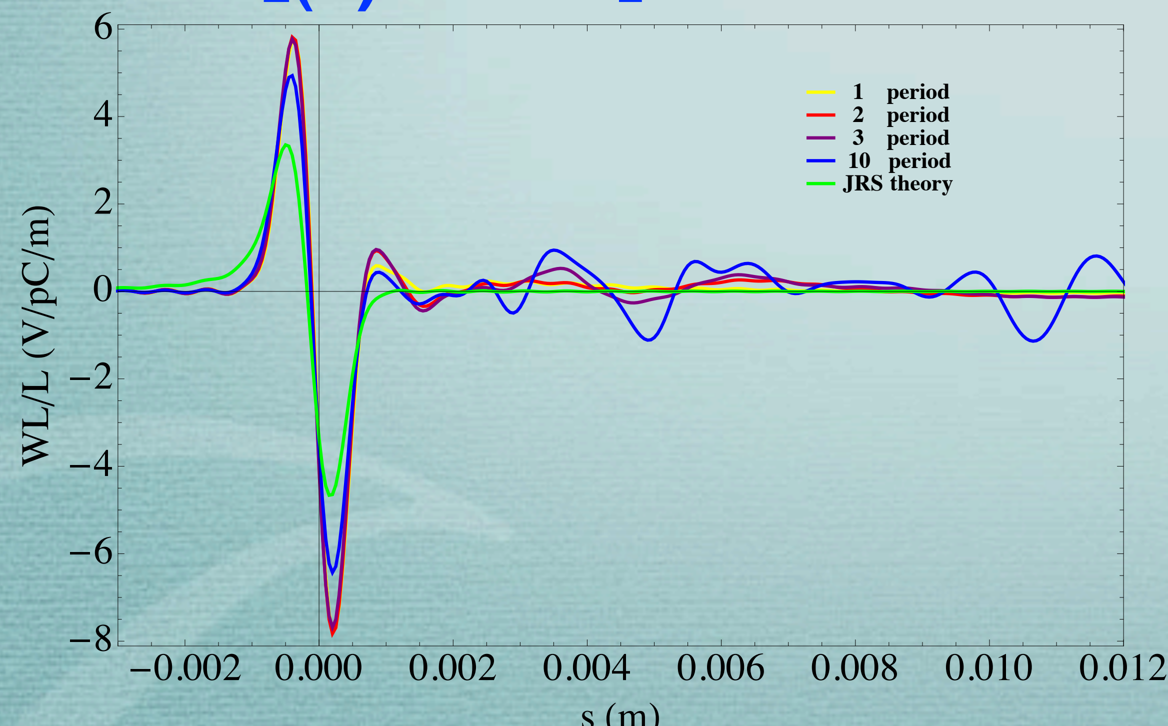
Re.Z_L(k)



Im.Z_L(k)



W_L(s) with σ_z=0.3mm



N_{period}=1/2/3/10

w/h=94/94mm

λ_w=1.088m

ρ=15.483m

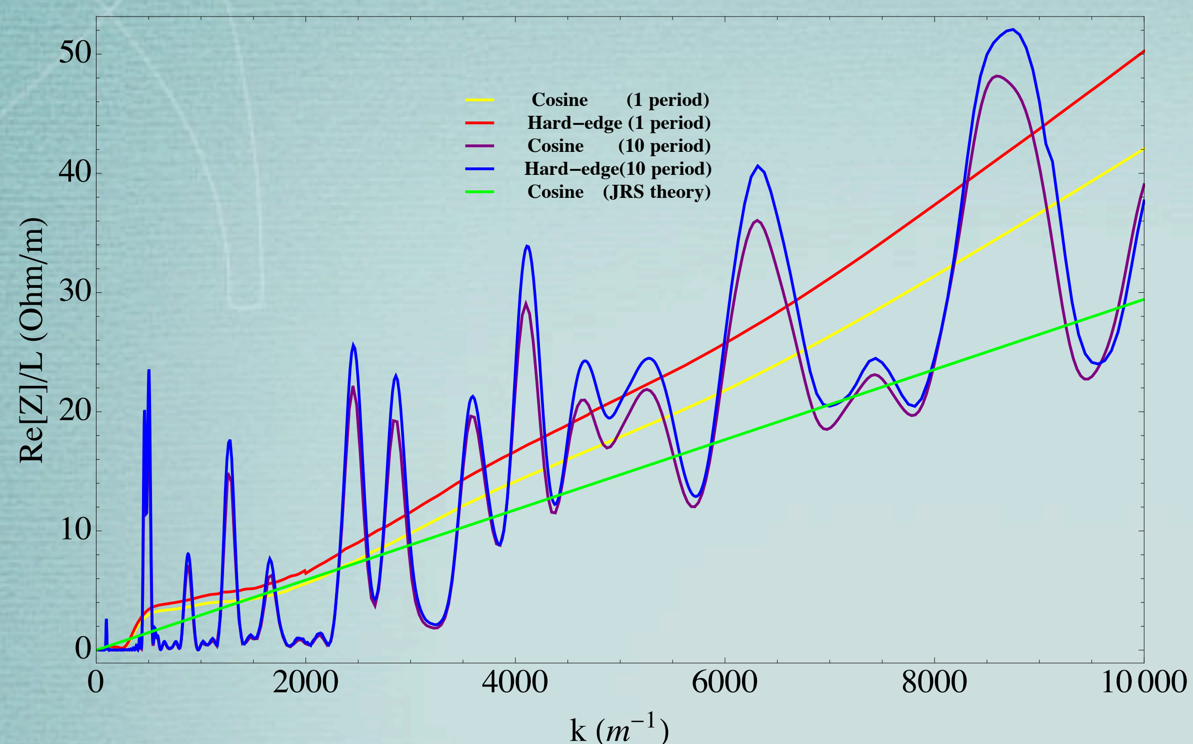
L_{exit}=Infinity (pipe after exit)

Xoffset=0mm

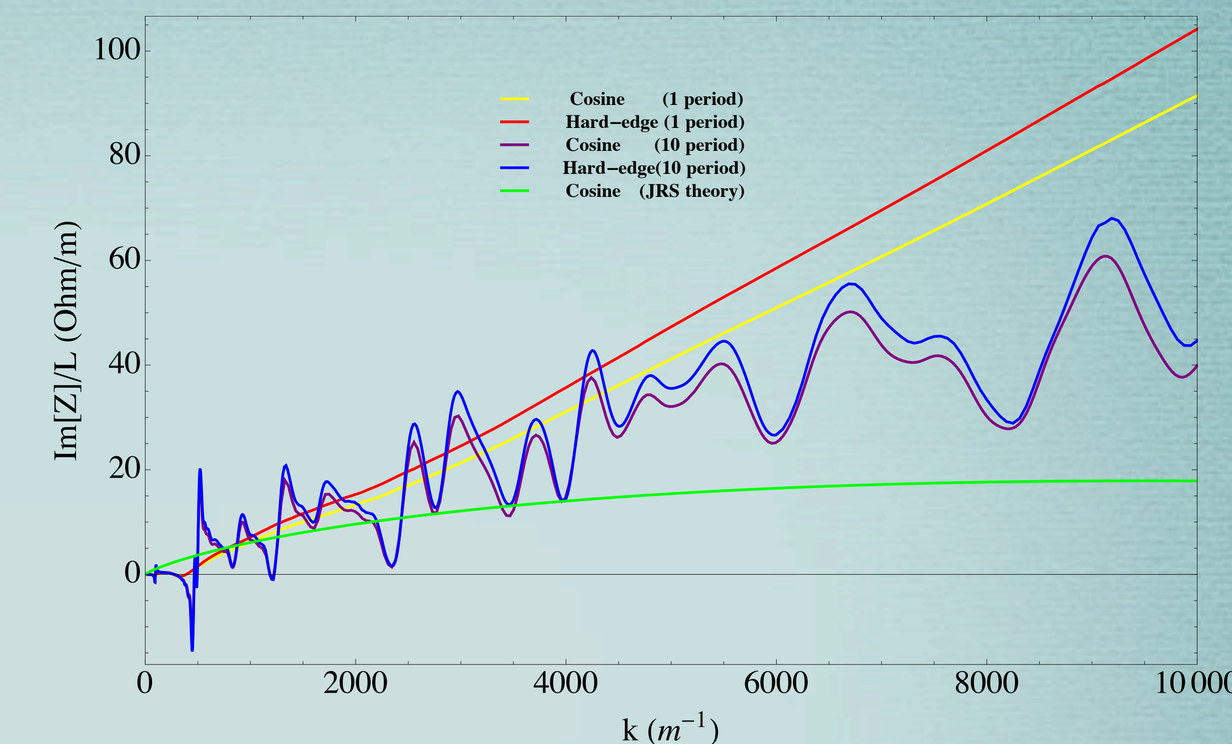
Field distribution: Cosine

CSR in wigglers - Hard-edge approximation

Re.Z_L(k)

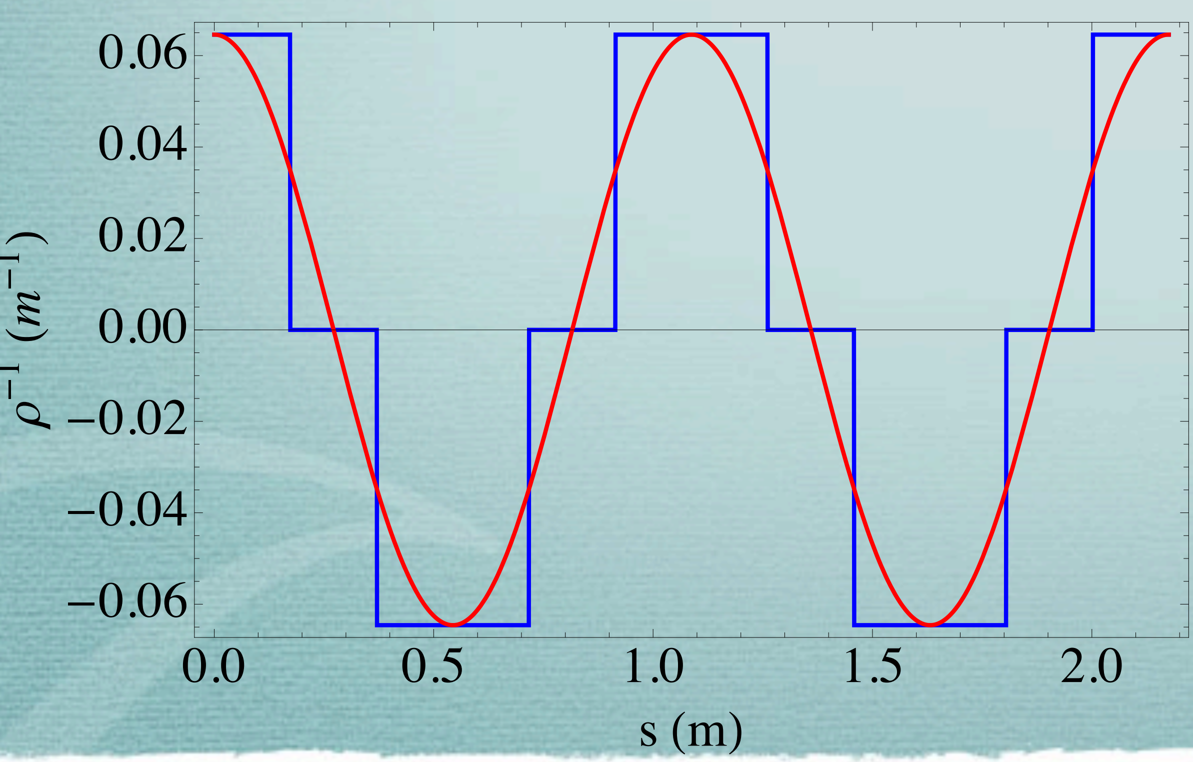


Im.Z_L(k)



H.E. model looks to be good?

Field distribution



N_{period}=1/10

w/h=94/94mm

λ_w=1.088m

ρ=15.483m

L_{exit}=Infinity (pipe after exit)

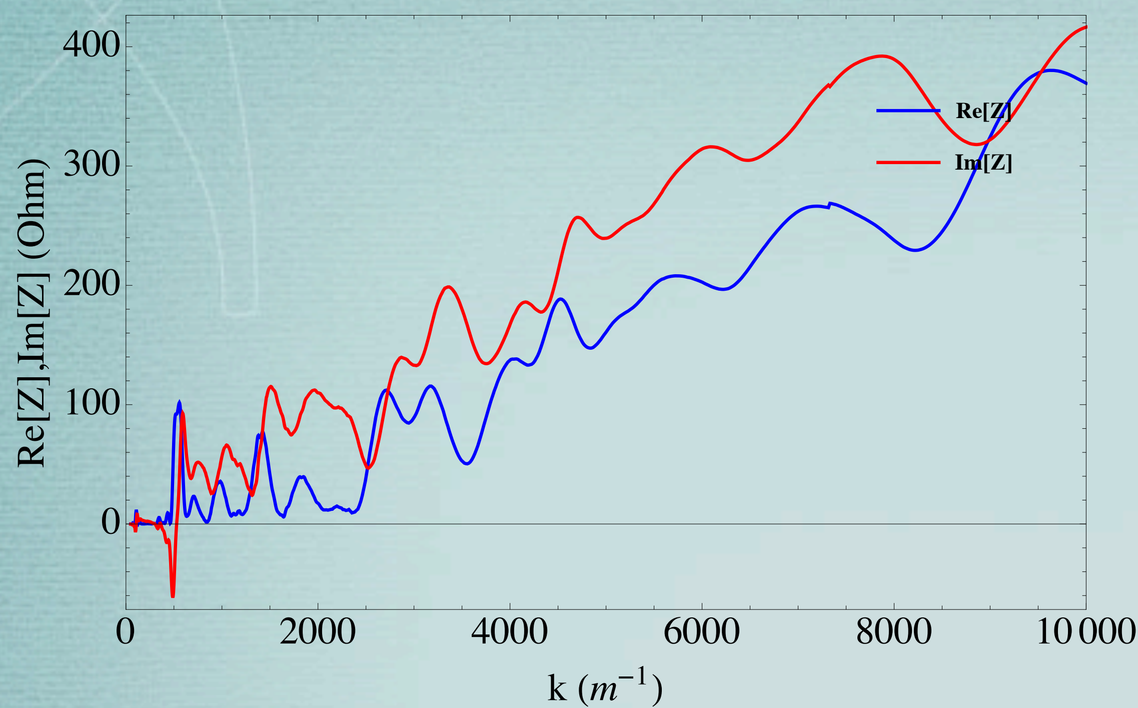
Xoffset=0mm (To inner wall)

Field distribution: Cosine/H.E.

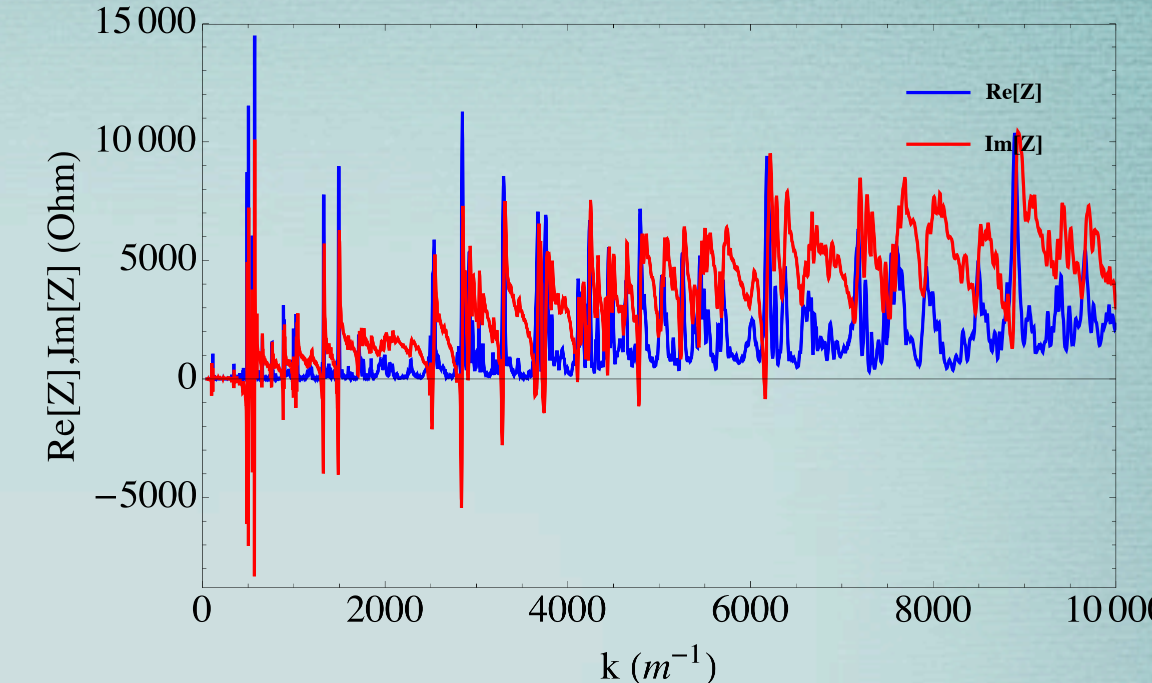
KEKB-LER

CSR in wigglers - SuperKEKB LER

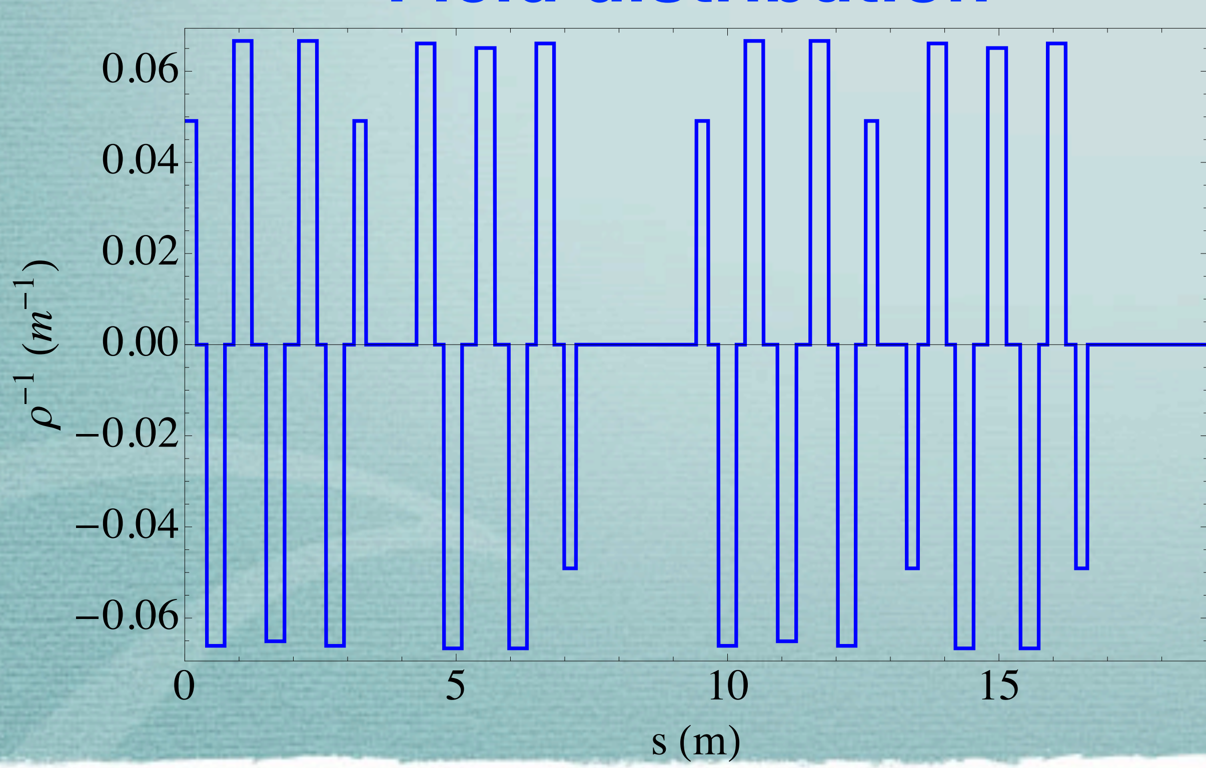
$Z_L(k)$,1 Super-period



$Z_L(k)$,15 Super-periods



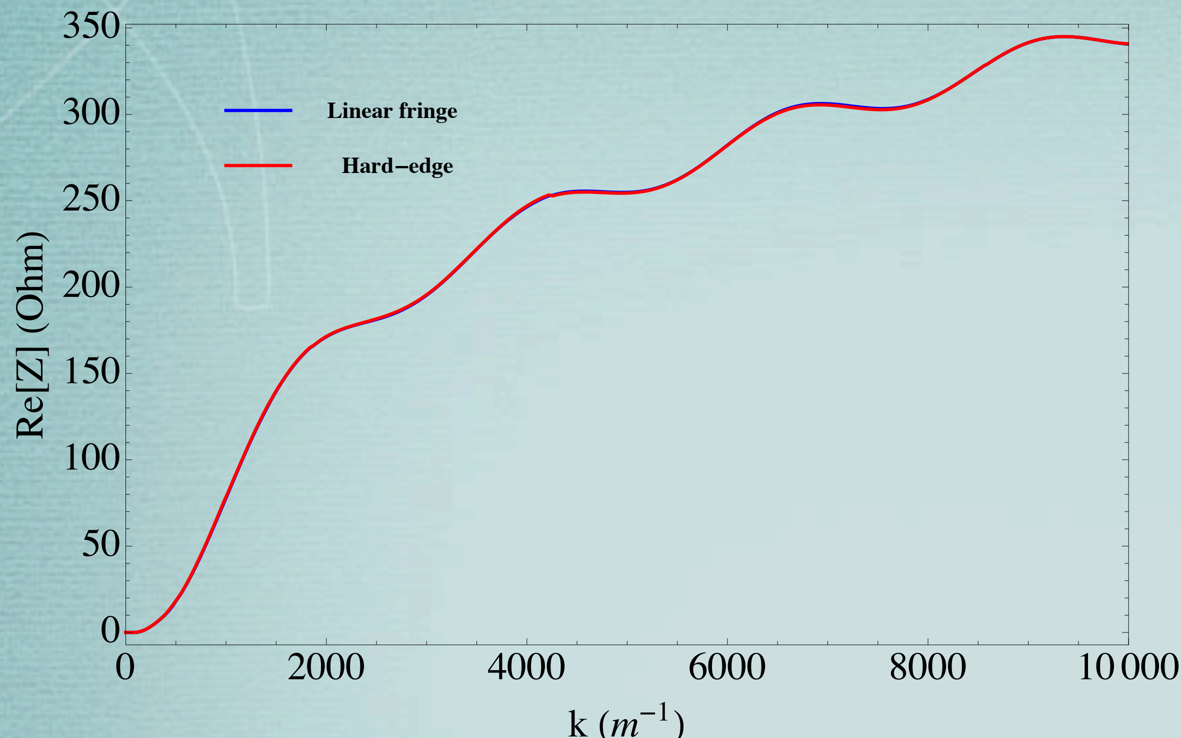
Field distribution



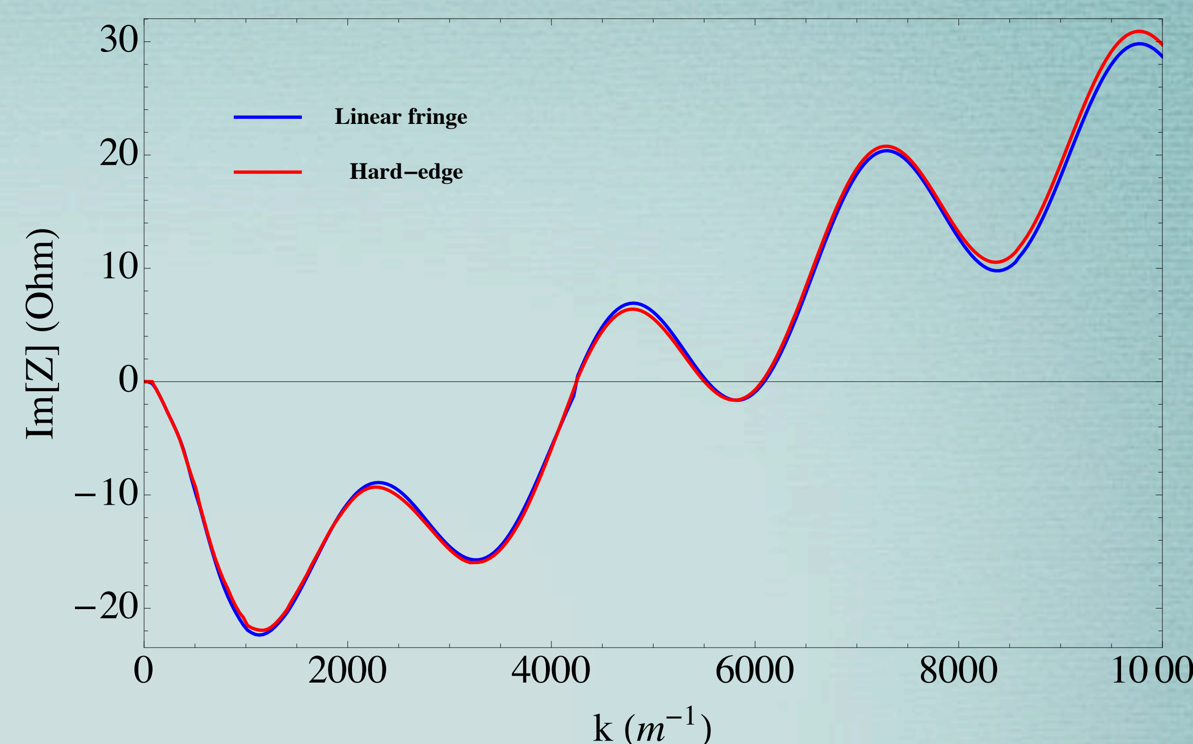
$N_{super\text{-}period}$=1/15
w/h=90/90mm
L_W=140m
ρ≅15m
L_{exit}=Infinity (pipe after exit)
Xoffset=0mm
Field distribution: Hard-edge

Fringe field - KEKB LER

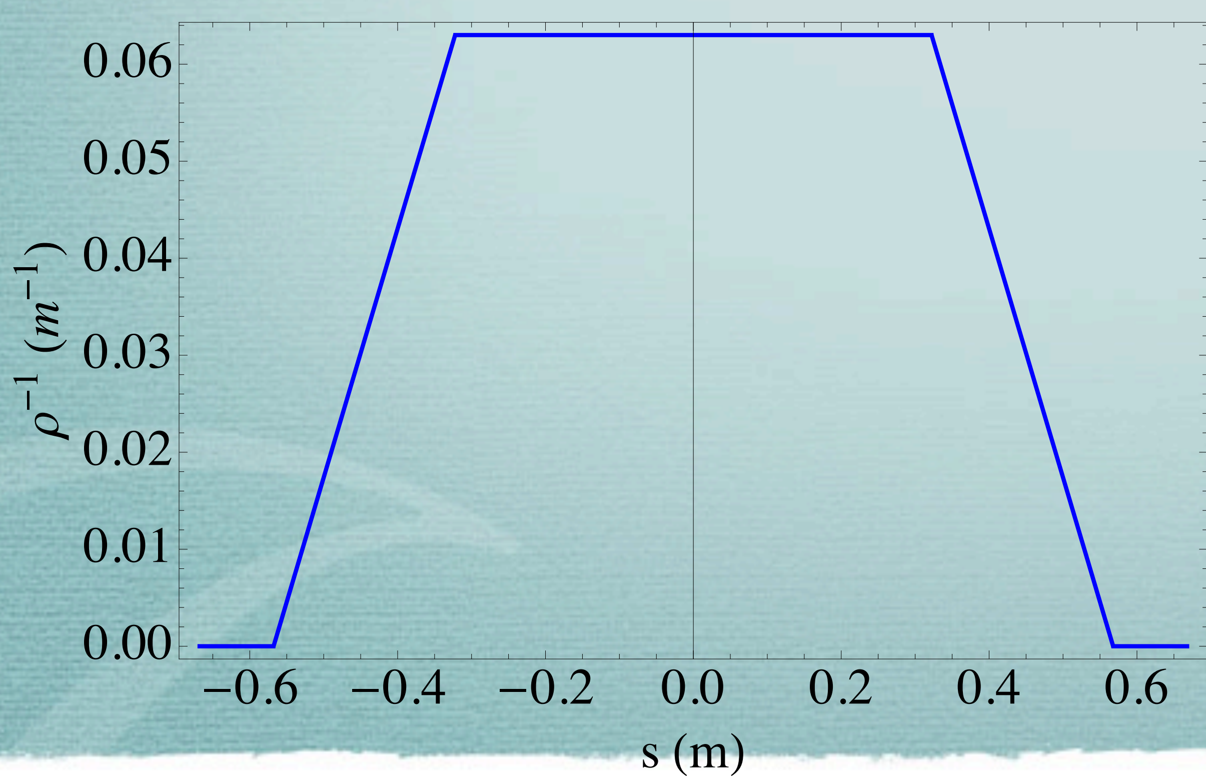
Re.Z$_L$(k)



Im.Z$_L$(k)



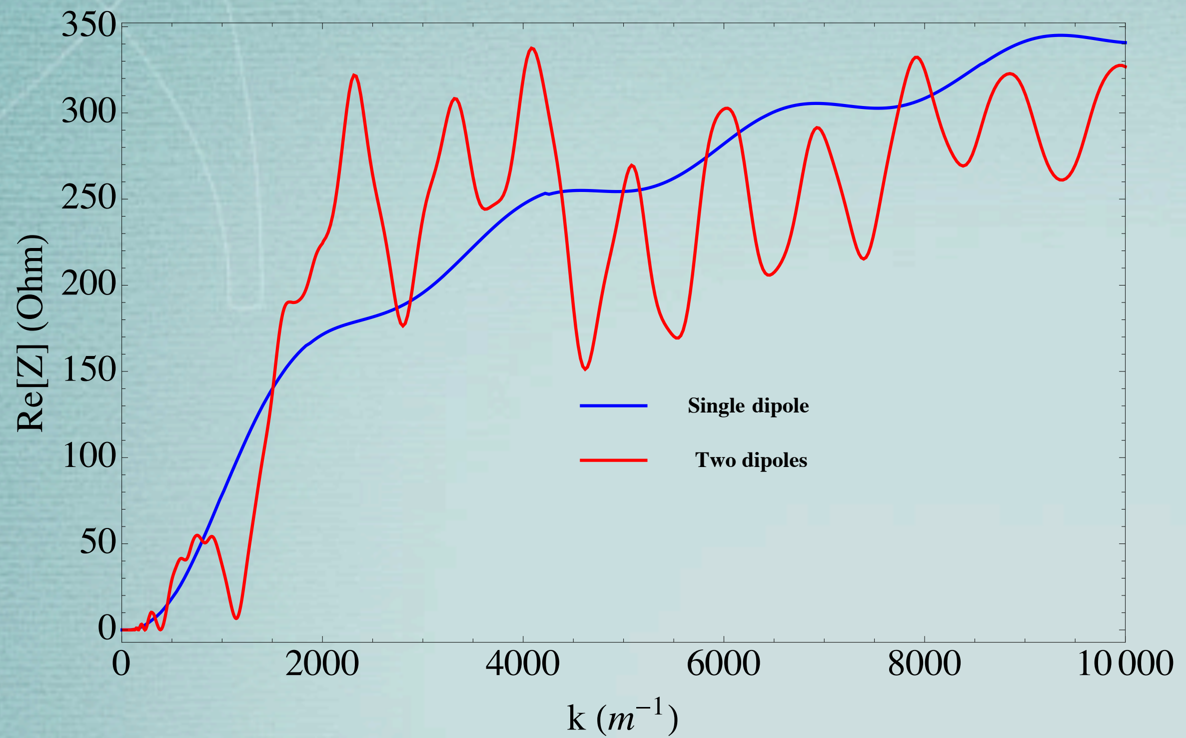
Field distribution



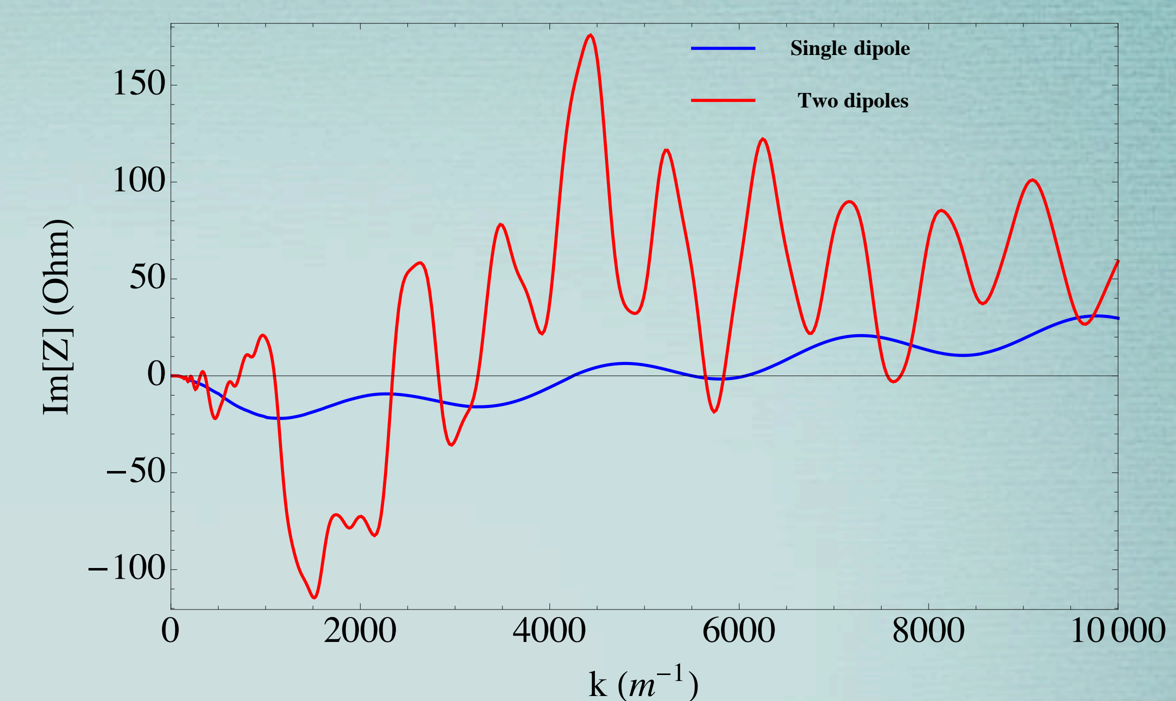
"Fringe effect" is negligible?

Interference - KEKB LER

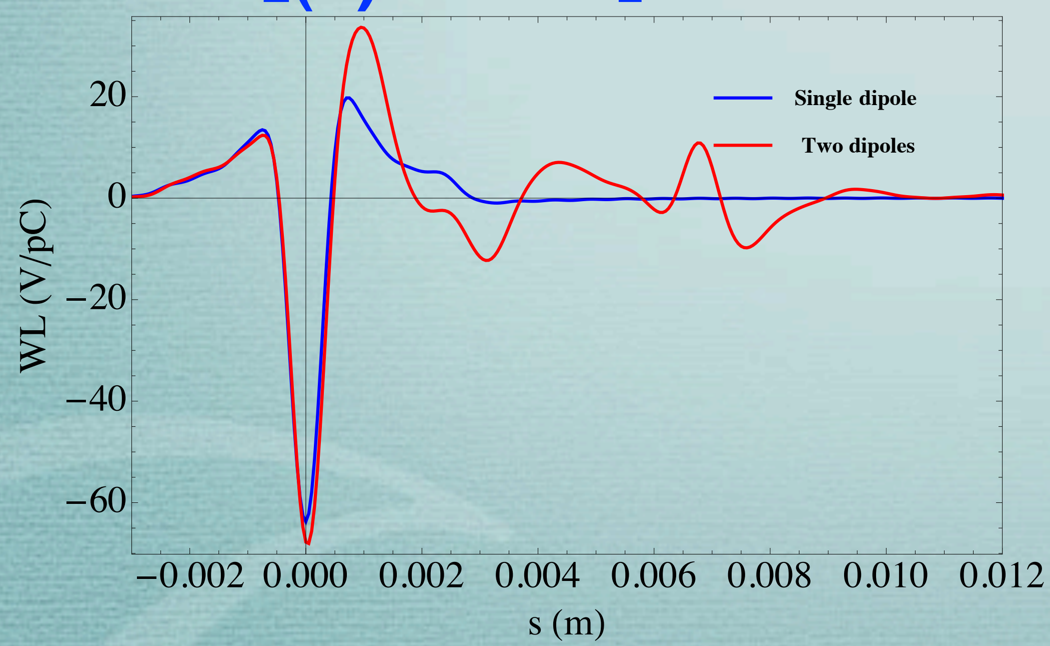
Re.Z_L(k)



Im.Z_L(k)



W_L(s) with σ_z=0.3mm



Two dipoles

w/h=94/94mm

L_{bend}=0.89m

L_{drift}=5.65m

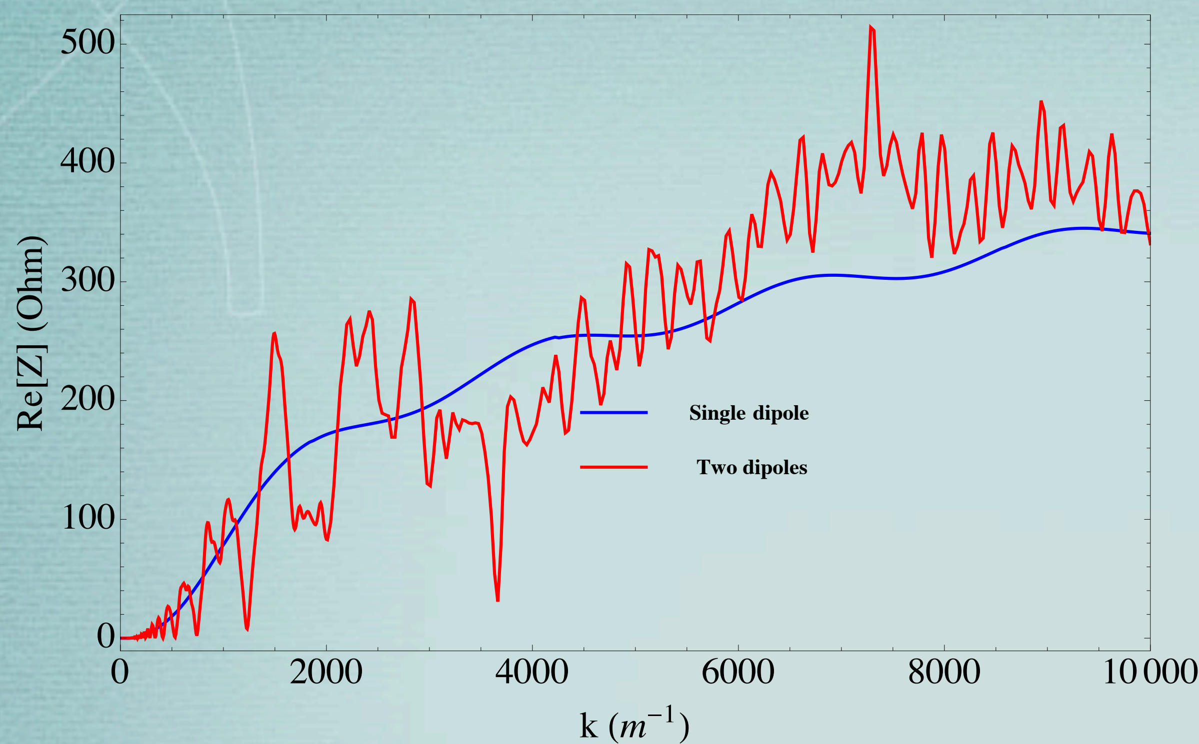
ρ=15.872m

L_{exit}=Infinity (pipe after exit)

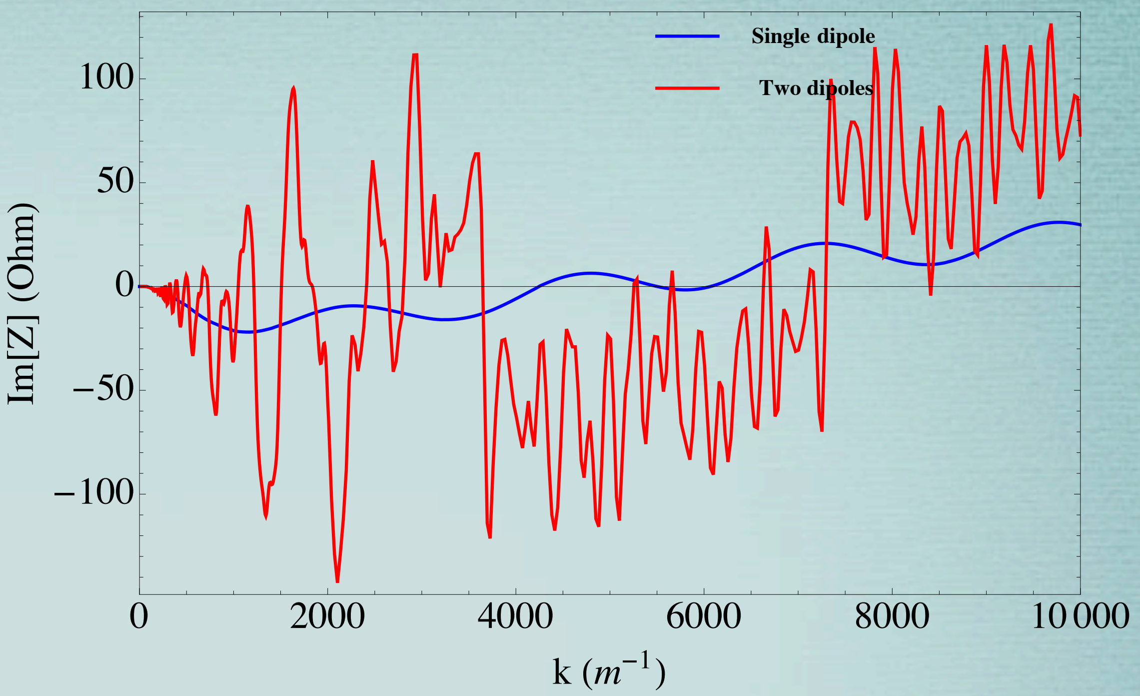
Xoffset=0mm

Interference - KEKB LER

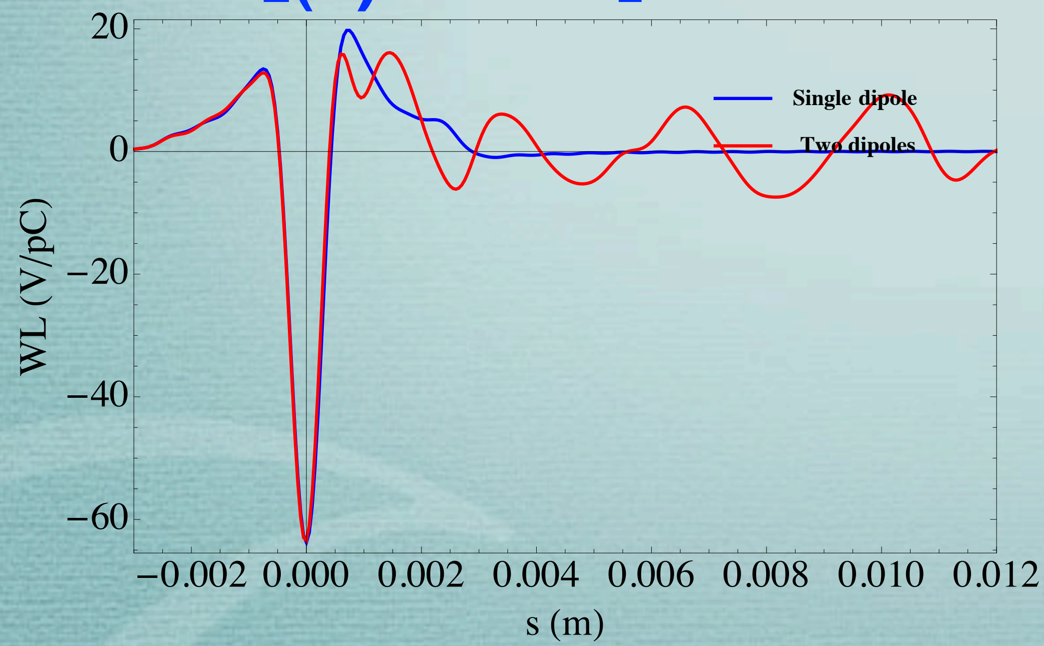
Re.Z_L(k)



Im.Z_L(k)



W_L(s) with σ_z=0.3mm



Two dipoles

w/h=94/94mm

L_{bend}=0.89m

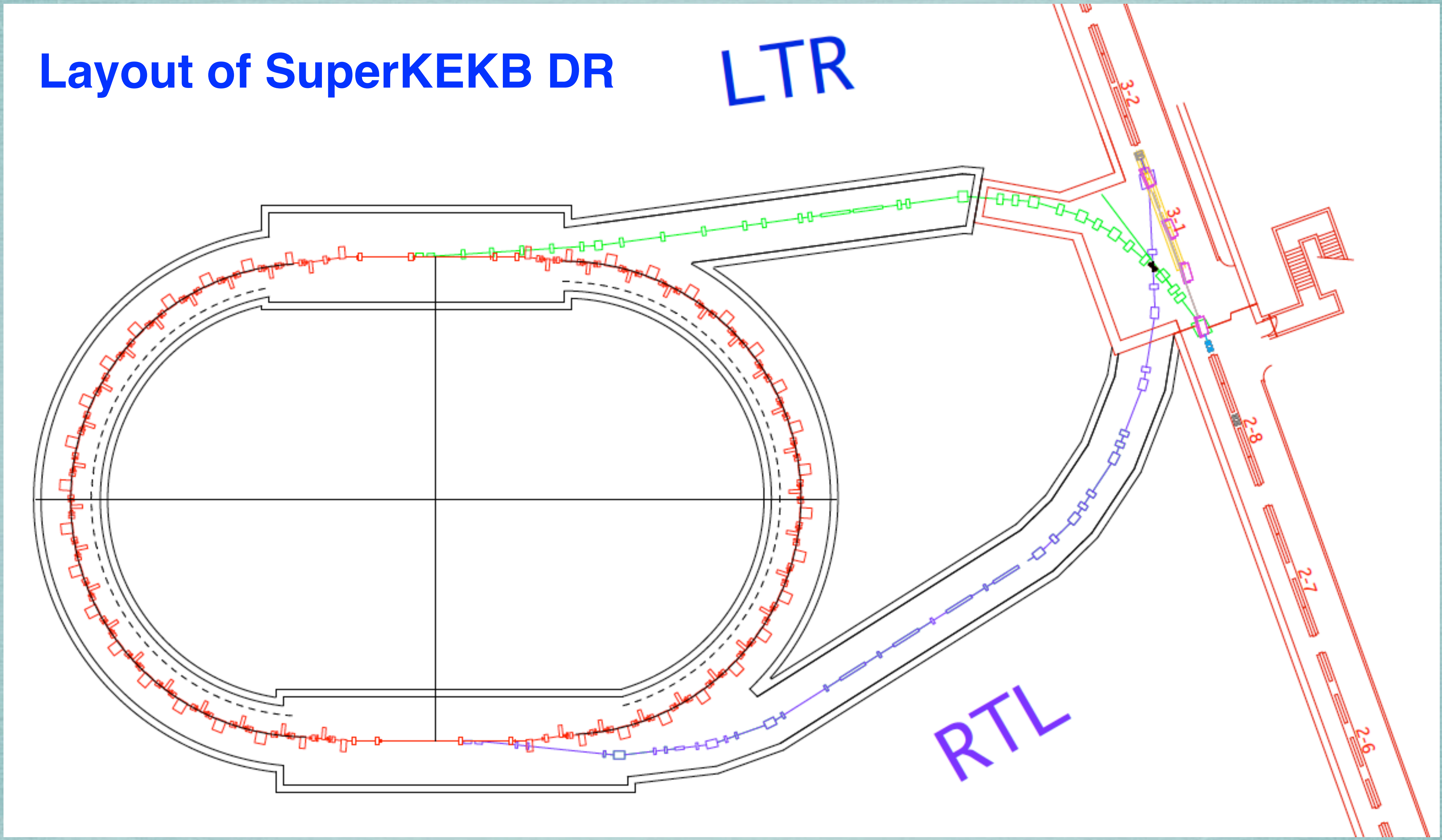
L_{drift}=20m

ρ=15.872m

L_{exit}=Infinity (pipe after exit)

Xoffset=0mm

Interference - SuperKEKB DR



M. Kikuchi

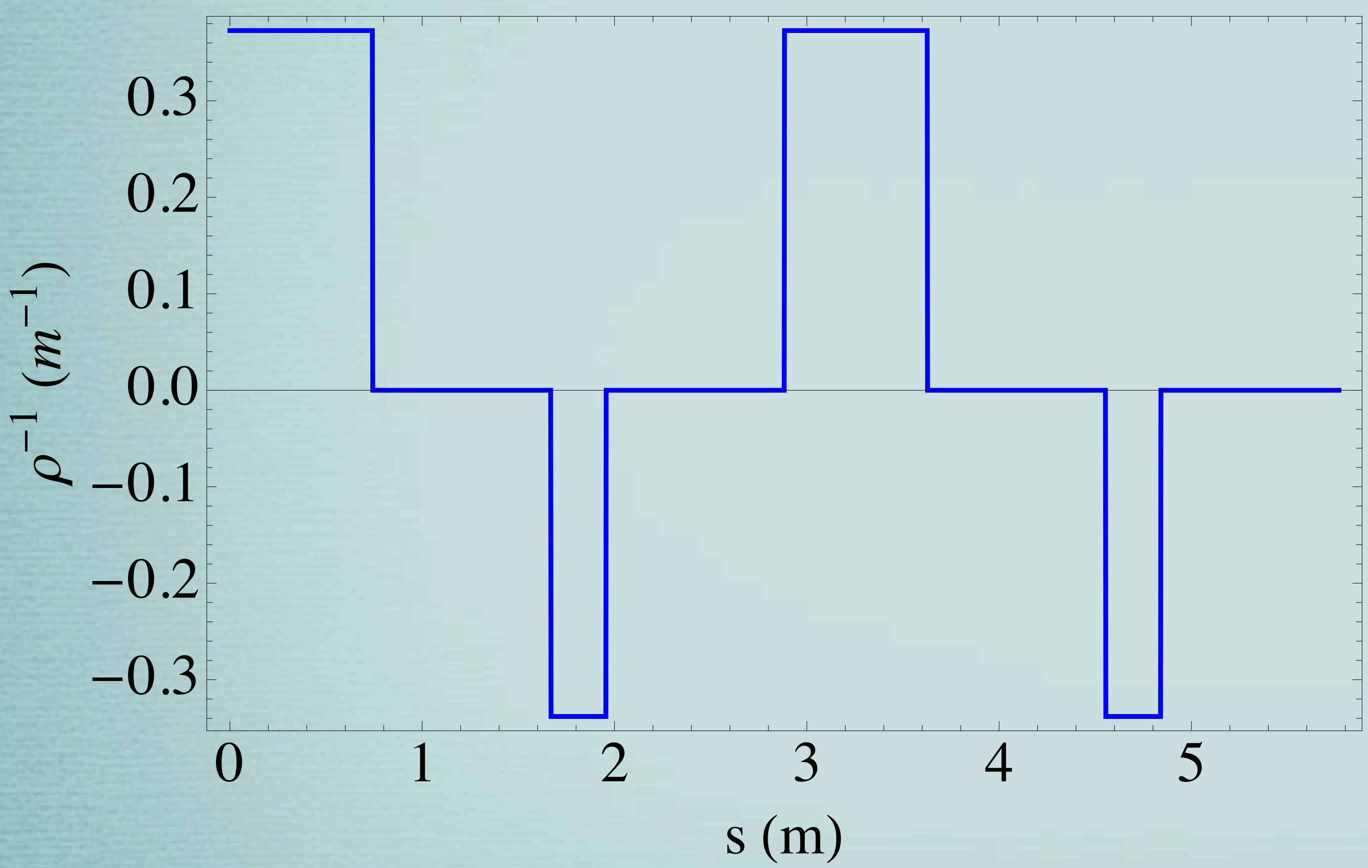
Interference - SuperKEKB DR

SuperKEKB DR parameters

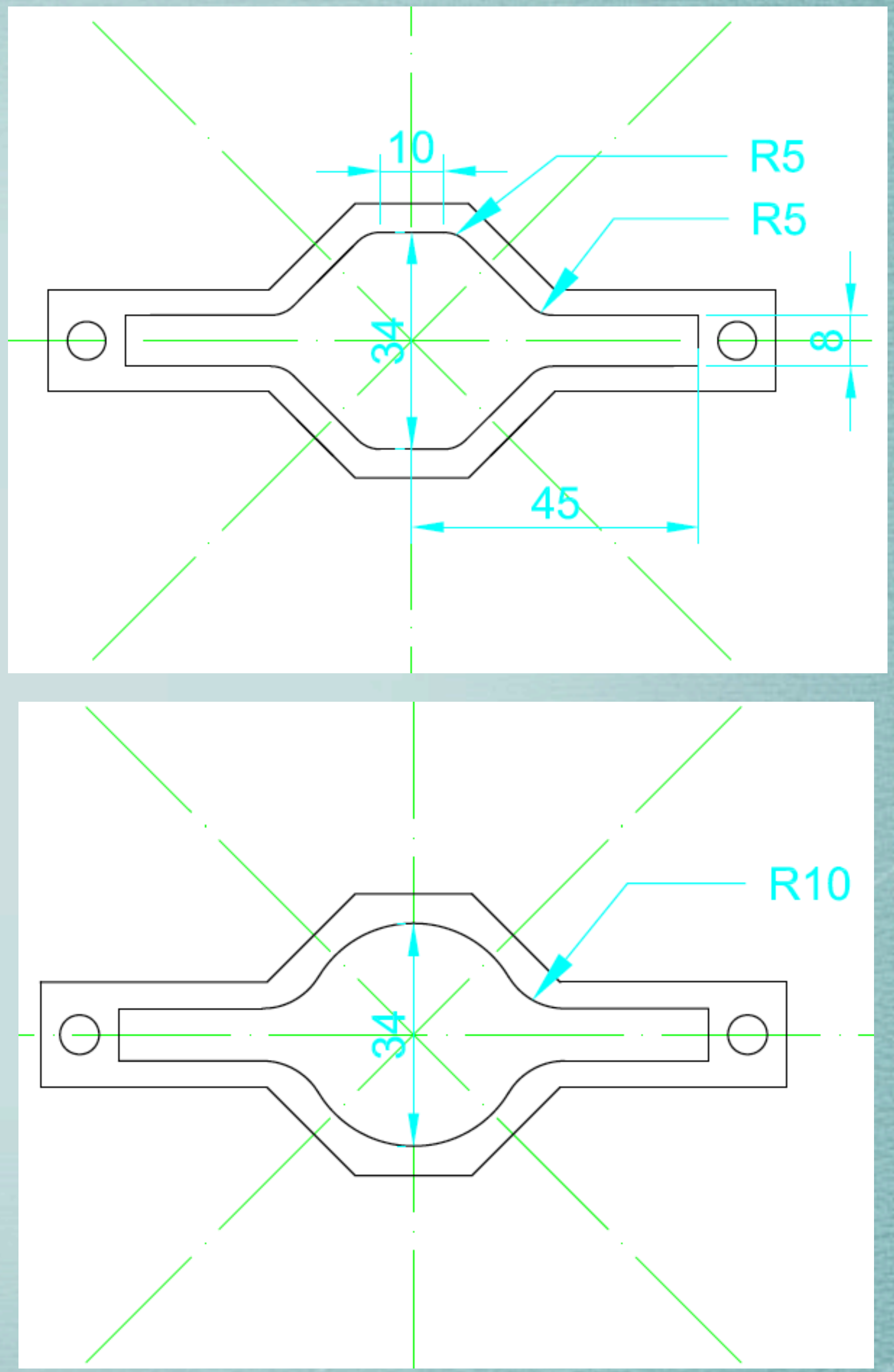
parameter	Value
Beam energy (GeV)	1.1
Circumference (m)	135.502
Bunch Length (mm)	11.1
Rel. Energy spread (10^{-4})	5.53
Beam pipe height in bends (mm)	34
Beam pipe width in bends w/o antechamber (mm)	34
Effective Length of bends (B1/B2/B3/B4)	0.74248/0.28654/0.39208/.47935
Number of bends (B1/B2/B3/B4)	32/38/4/4
Bending radius (m) (B1/B2/B3/B4)	2.68/2.96/3.15/3.15

Interference - SuperKEKB DR

Field distribution (2 cells)



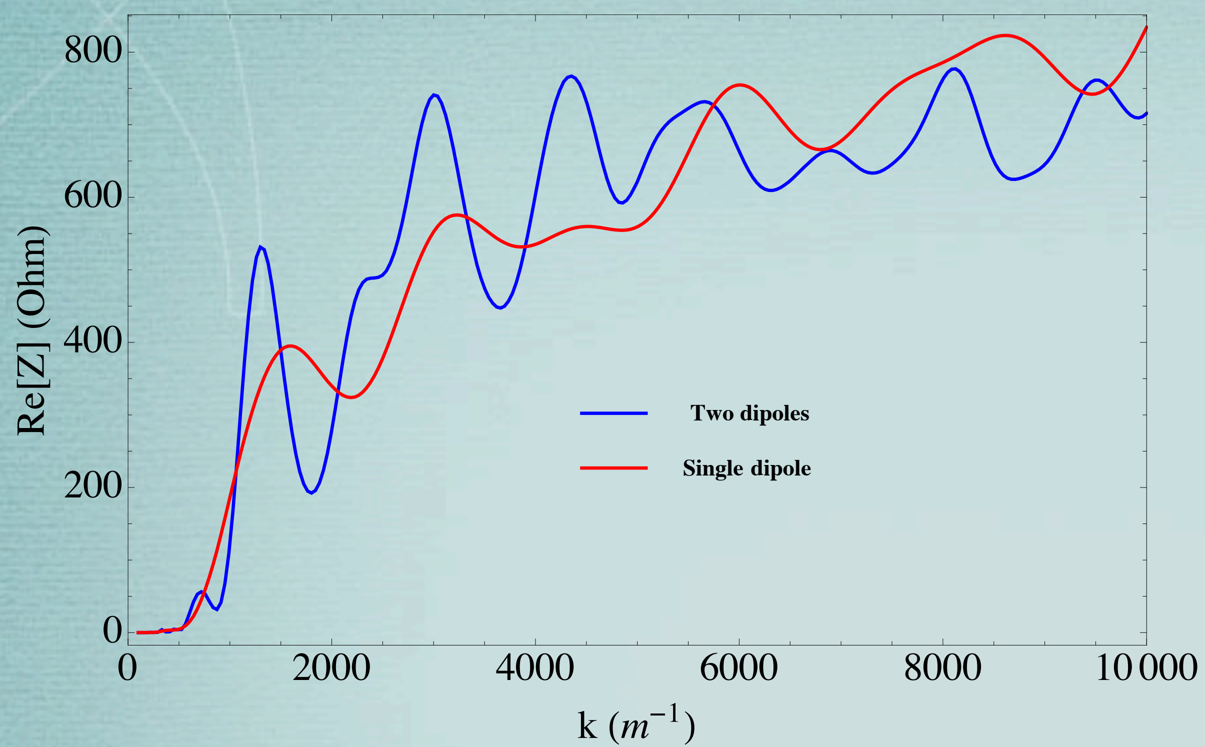
Vacuum chamber (candidates)



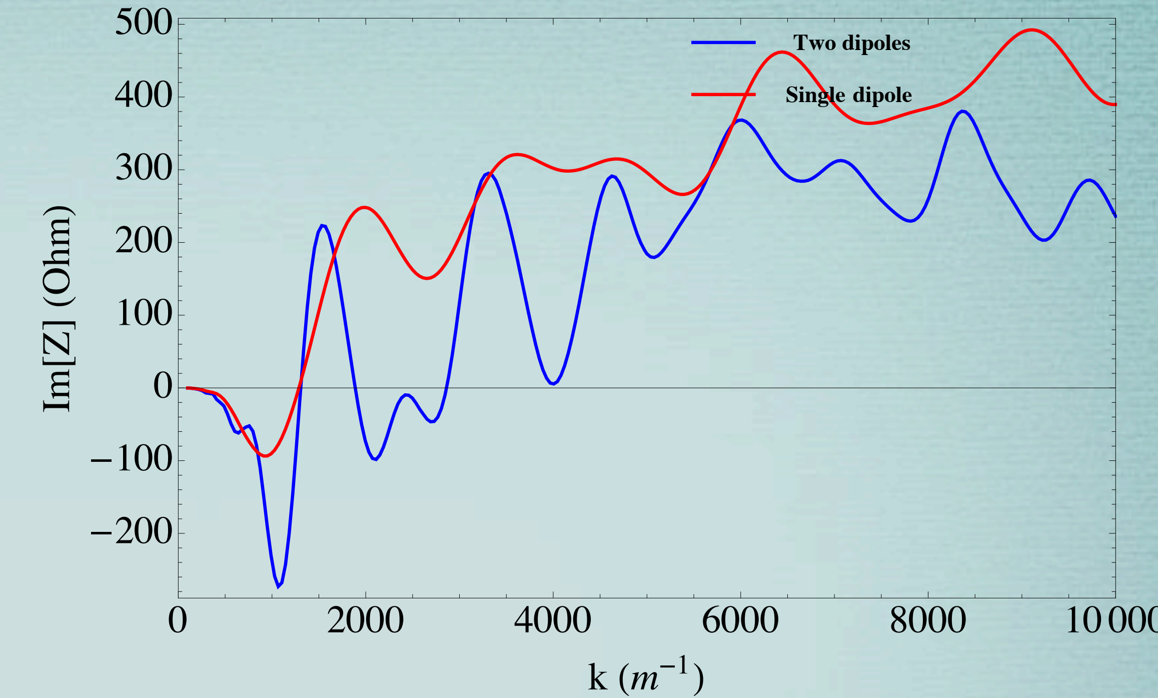
K. Shibata

Interference - SuperKEKB DR

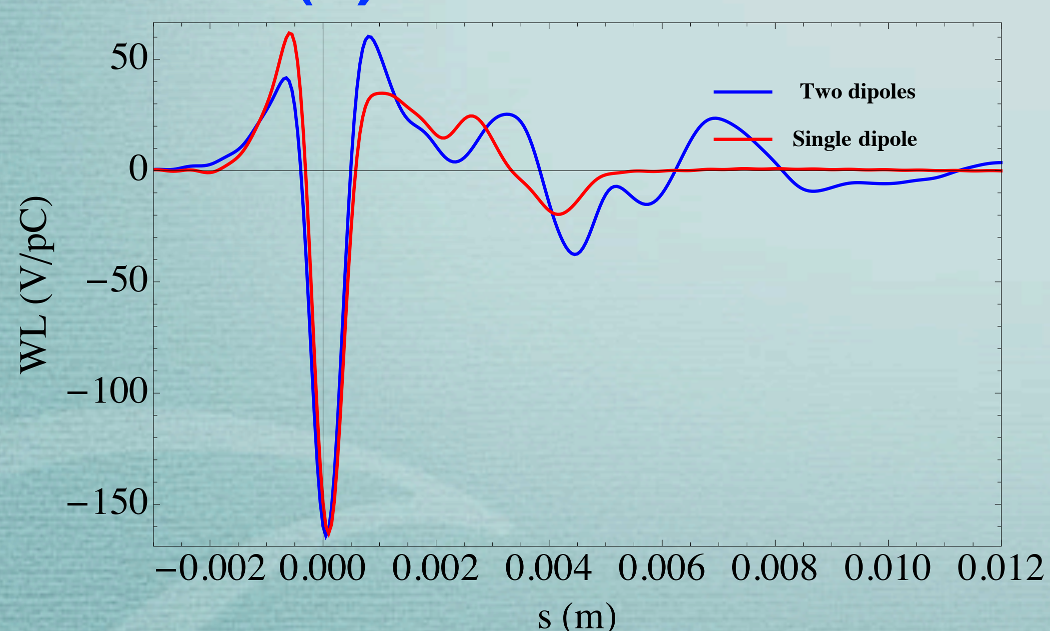
Re.Z_L(k)



Im.Z_L(k)



W_L(s) with σ_z=0.3mm



1 cell

w/h=34/34mm

B1+B2

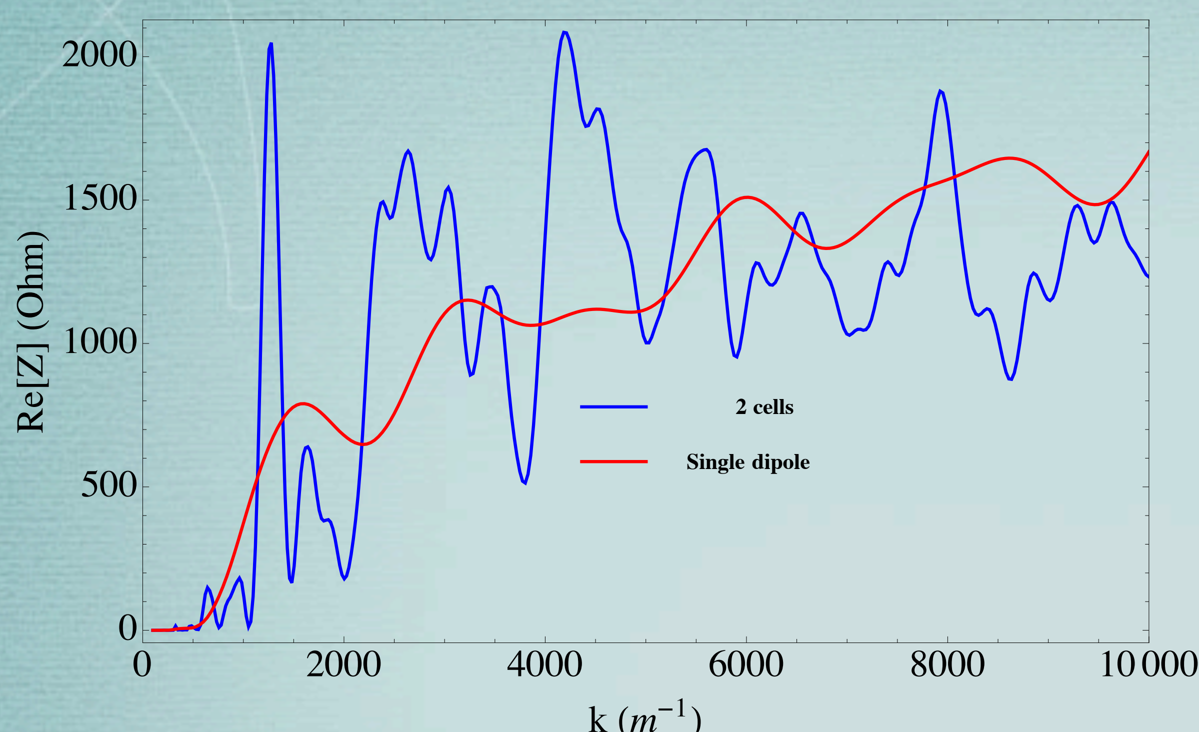
L_{drift}=0.93m

L_{exit}=Infinity (pipe after exit)

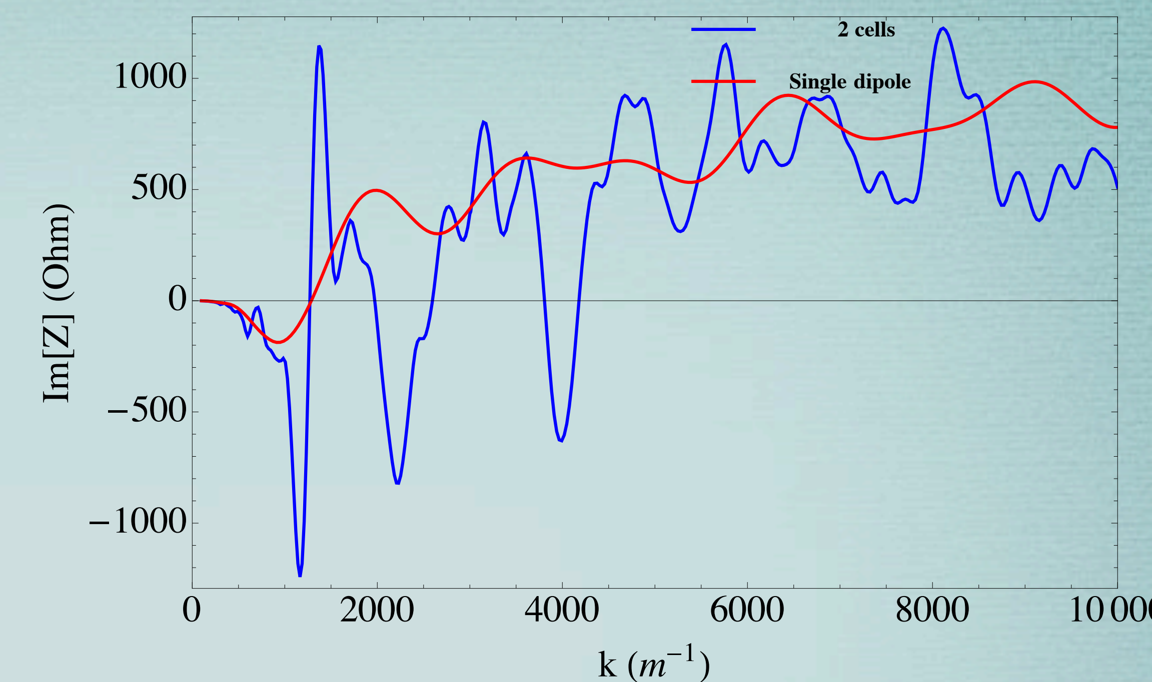
Xoffset=0mm

Interference - SuperKEKB DR

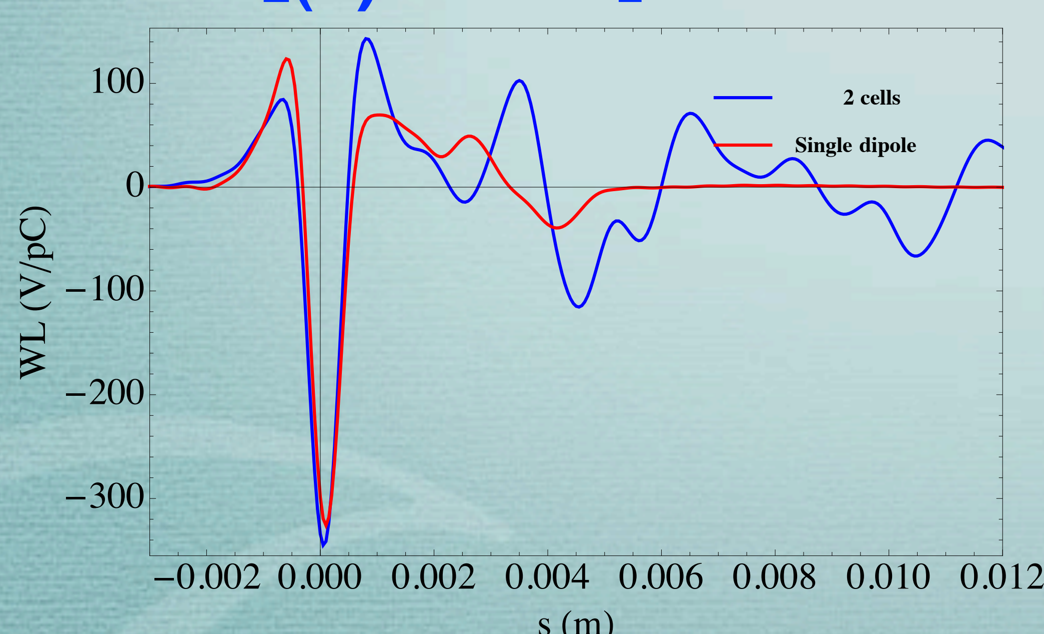
Re.Z_L(k)



Im.Z_L(k)



W_L(s) with σ_z=0.3mm



2 cells

w/h=34/34mm

2×(B1+B2)

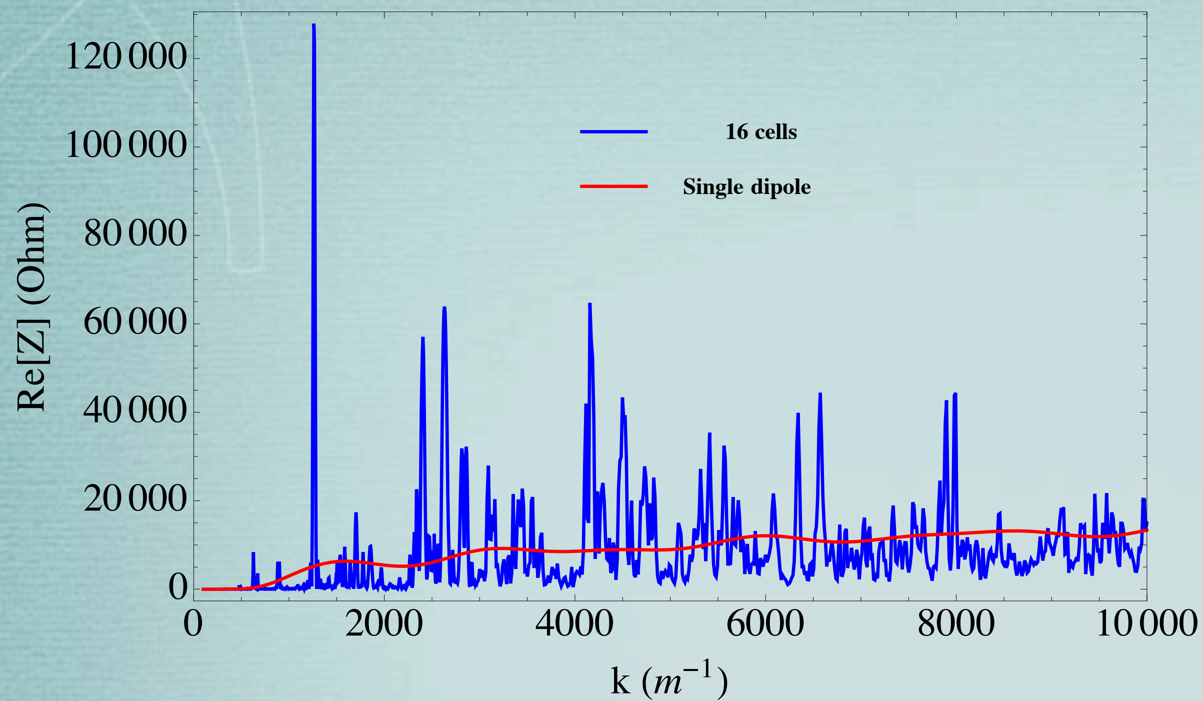
L_{drift}=0.93m

L_{exit}=Infinity (pipe after exit)

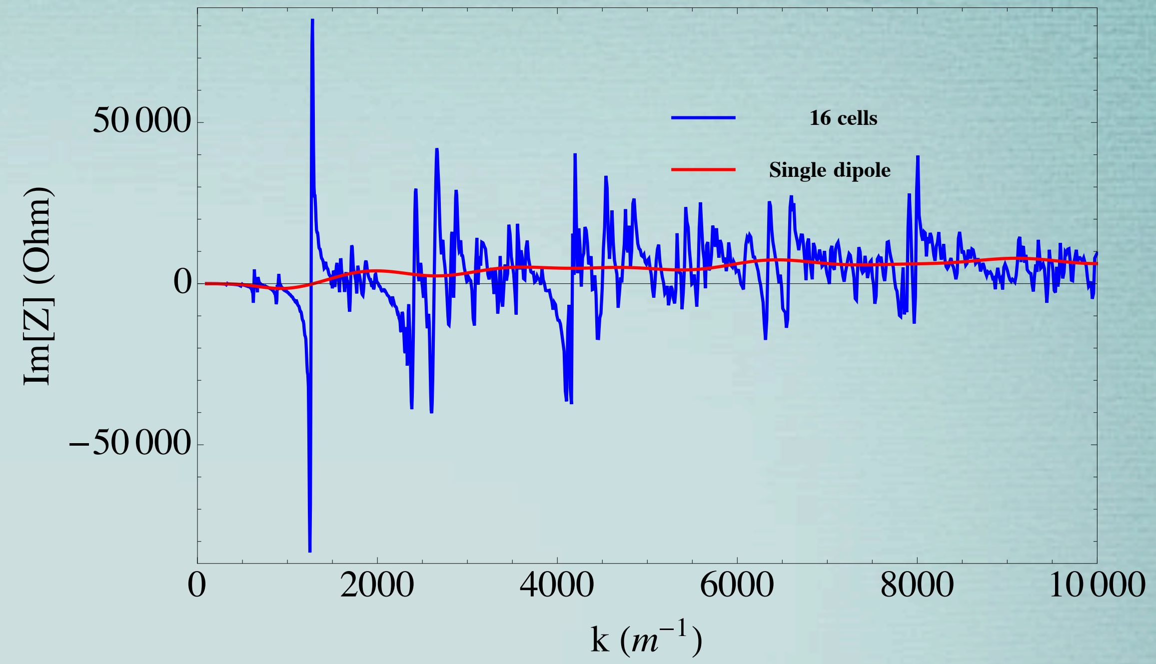
Xoffset=0mm

Interference - SuperKEKB DR

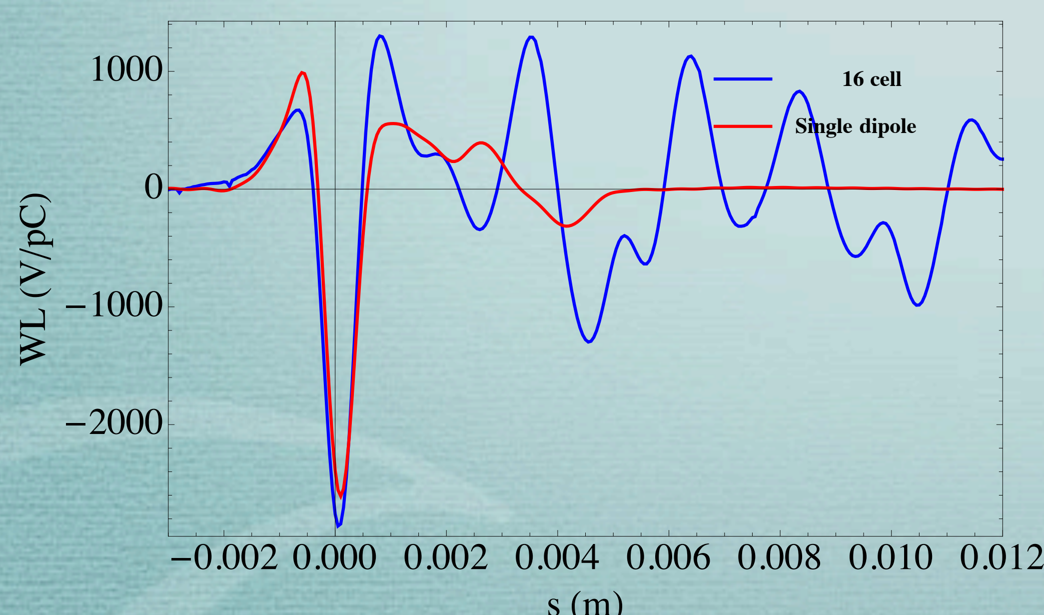
Re.Z_L(k)



Im.Z_L(k)



W_L(s) with σ_z=0.3mm



16 cells

w/h=34/34mm

16×(B1+B2)

L_{drift}=0.93m

L_{exit}=Infinity (pipe after exit)

Xoffset=0mm

Summary

1. Features of the new CSR code (CSRZ):

1.1 Low noise level

1.2 Allow for s-dependent bending radius (fringe field, wigglers, interference between consecutive dipoles)

1.3 Allow for resistive wall (not discussed in this talk and to be benchmarked)

2. Achievements

2.1 Limitations in all three codes (GS, KO, DZ) improved after careful benchmark work

2.2 Narrow-band impedances (spikes) due to CSR of wigglers were observed which are unexpected according to traditional theories

2.3 Interference between consecutive dipoles can be significant and leads to narrow-band CSR impedances (with perfect wall)

2.4 CSR calculation for SuperKEKB project

3. Challenges

3.1 Computing time is not quite acceptable at high freq. or very long components which require refinements in meshes or huge integration steps

3.2 “Wiggling pipe” is not good approximation

3.3 Treating pipe with arbitrary cross section is unavailable