Comparison of resonance driving terms for SuperKEKB before and after optimizations - Updates

Demin Zhou

Acknowledgements:

K. Ohmi, K. Hirosawa, H. Sugimoto, K. Oide, E. Forest, D. Sagan (Cornell)

SuperKEKB mini-optics meeting, Feb. 15, 2018

Outline

- ➢ Introduction
  - Lattice:
    sler_1689.sad
    sler_1689_w_const001.sad (by H. Sugimoto)
- ➢ Theory for resonance driving terms
- ➢ Results by PTC (updates)
  - 3rd and 4th order RDTs
  - Momentum-dependent RDTs
- ➢ Other updates
  - Ohmi-Hirosawa method compared with PTC
  - Luminosity calculation
- ➢ Summary

1. Introduction

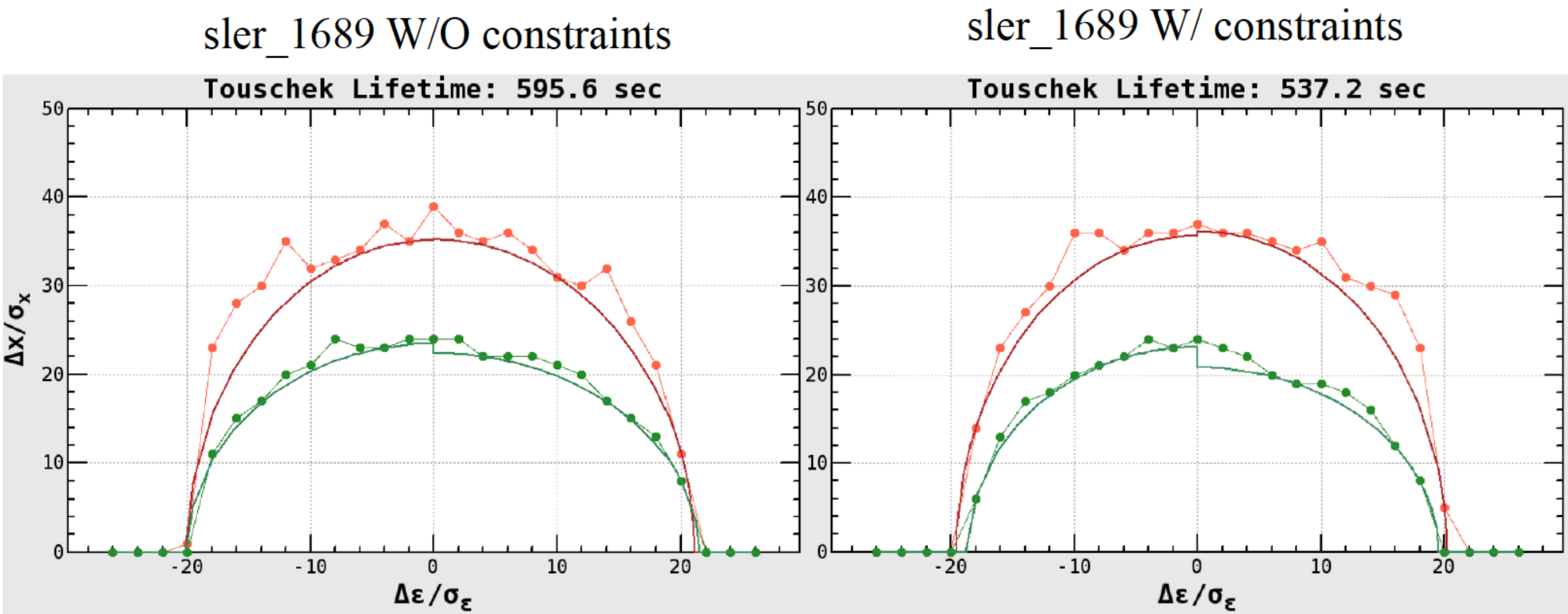
➤ **Relates presentations in past SuperKEKB mini-optics meetings**

- **H. Sugimoto, Sep. 8, 2016**
- **D. Zhou, Dec. 8, 2016 (There were mistakes in my slides, thanks to H. Sugimoto)**
- **H. Sugimoto, Apr. 6, 2017**
- **K. Hirosawa, Jul. 6, 2017**
- **K. Ohmi, Sep. 21, 2017**
- **H. Sugimoto, Oct. 12, 2017**

1. Introduction

➤ **Nonlinear optimization with new constraints [by H. Sugimoto, SuperKEKB mini-optics meeting, Sep. 8, 2016]**

● **It's natural to have smaller DA using additional constraints but with the same number of variables**

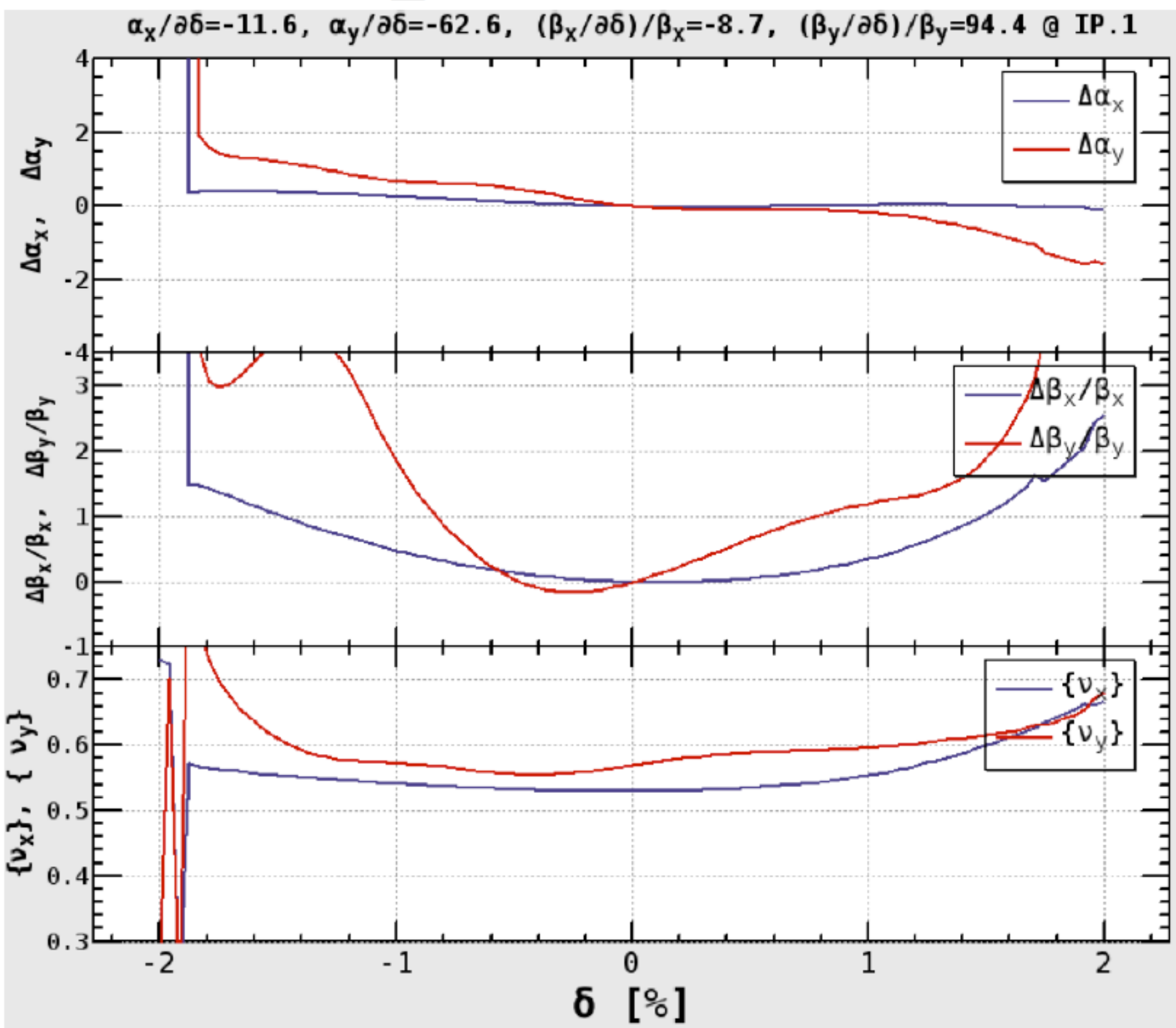


1. Introduction

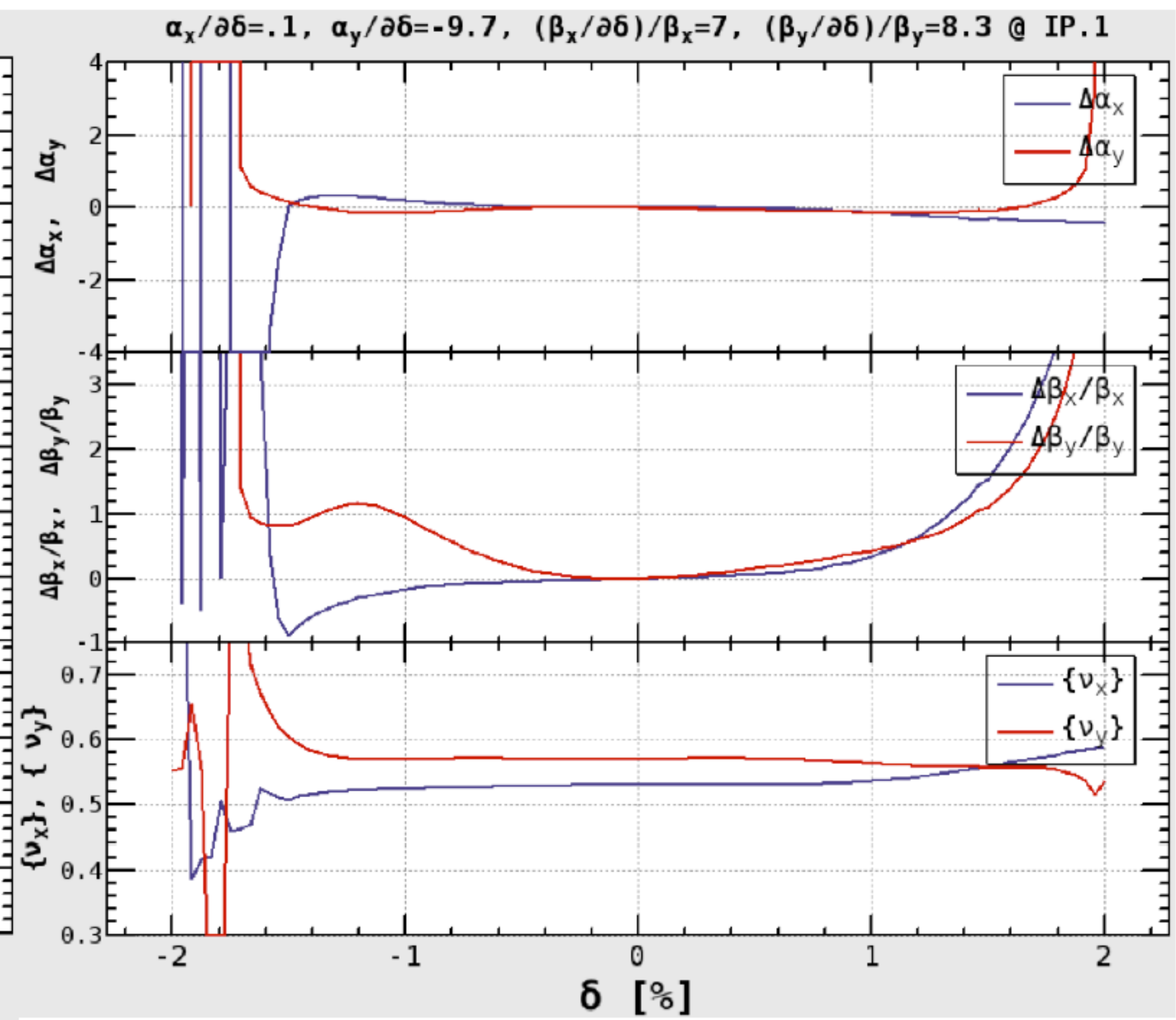
➤ Nonlinear optimization with new constraints [by H. Sugimoto, SuperKEKB mini-optics meeting, Sep. 8, 2016]

- Chromatic $\beta_{x,y}$ and $\nu_{x,y}$ correspond to RDTs of h_{2000e}/h_{0200e} (X), h_{0020e}/h_{0002e} (Y), and h_{1100e} (X), h_{0011e} (Y), respectively

sler_1689 W/O constraints



sler_1689 W/ constraints

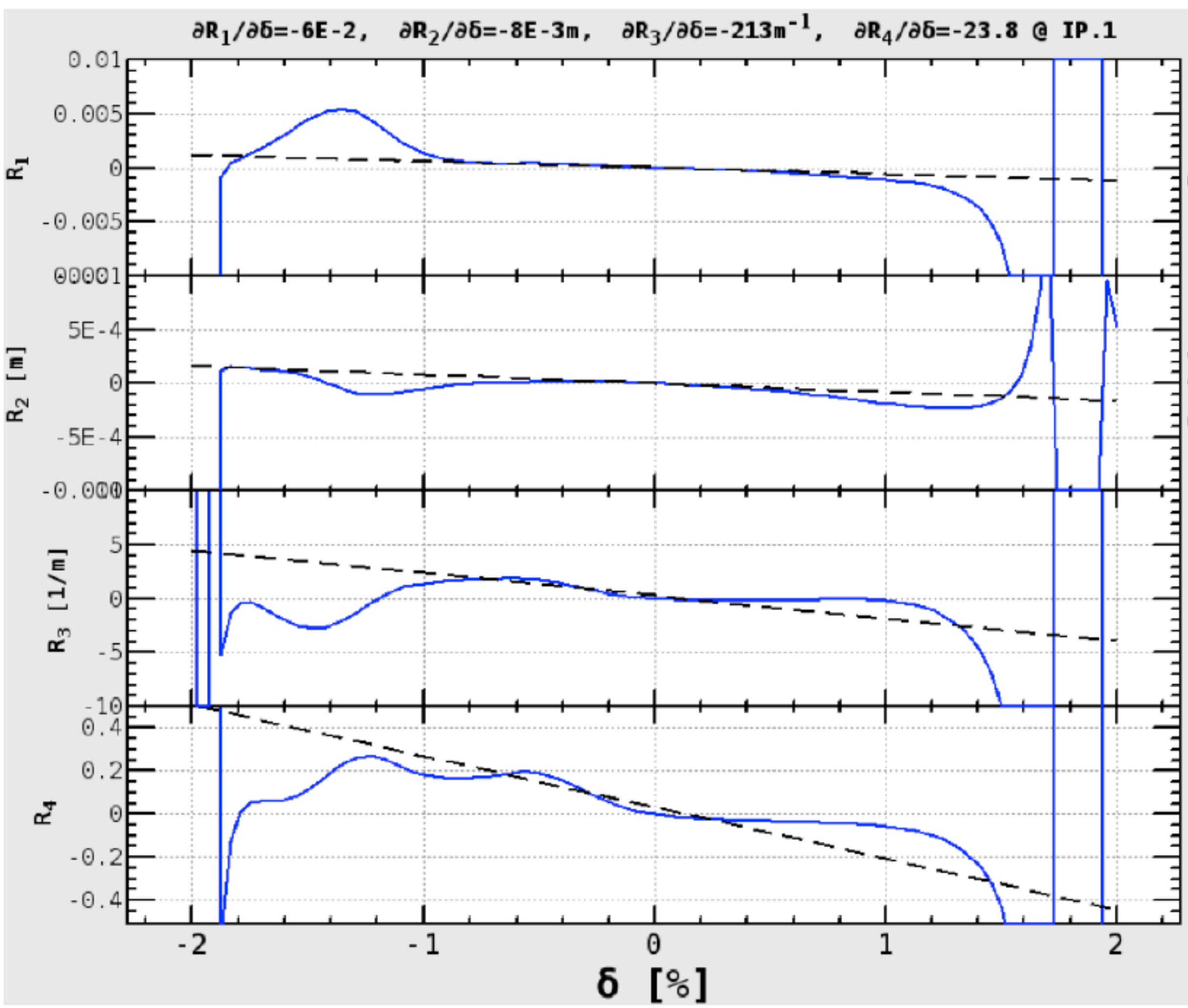


1. Introduction

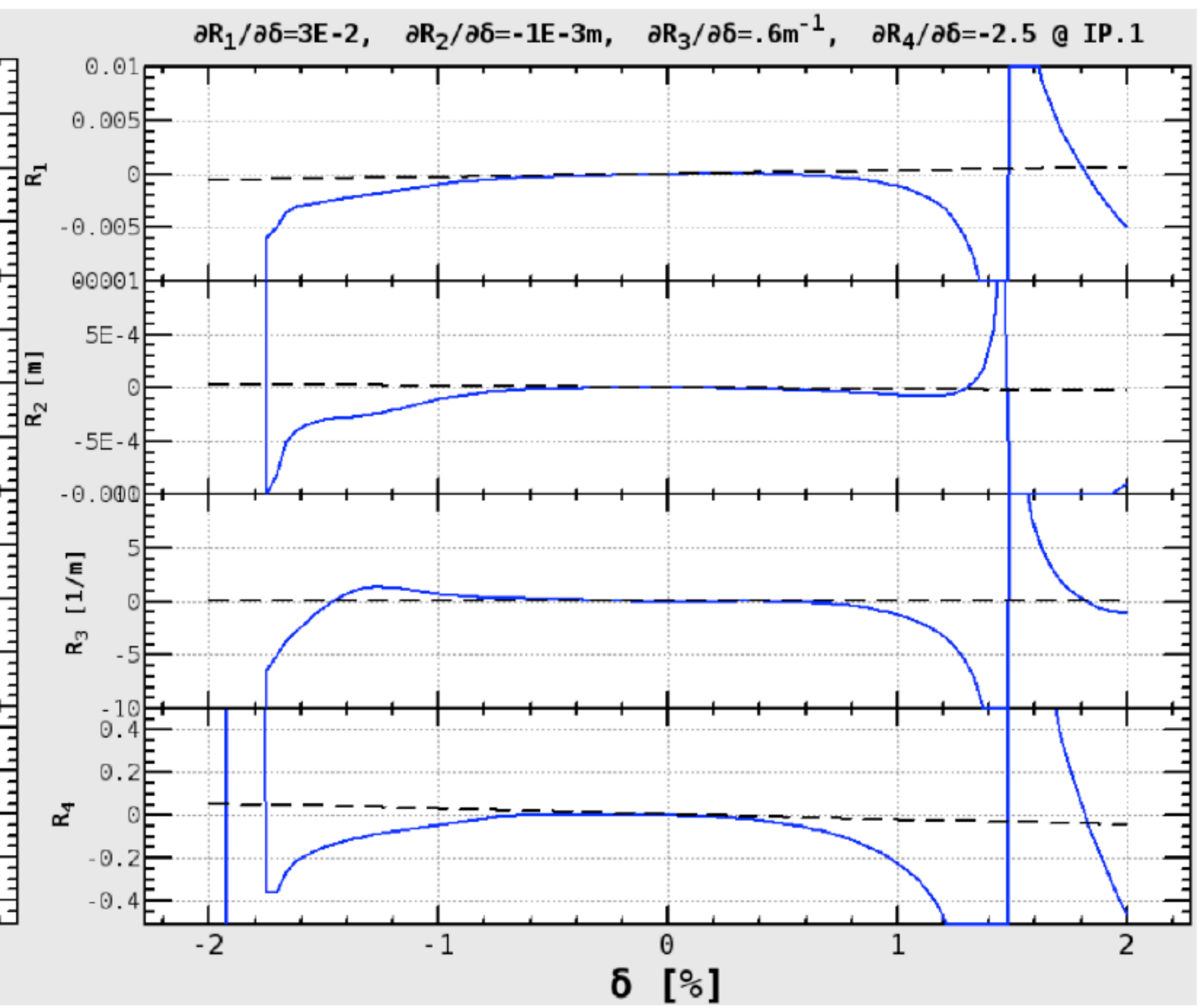
➤ **Nonlinear optimization with new constraints [by H. Sugimoto, SuperKEKB mini-optics meeting, Sep. 8, 2016]**

- **Chromatic X-Y couplings correspond to RDTs of h_{1010e}, h_{0110e}, h_{1001e} and h_{0101e}**

sler_1689 W/O constraints



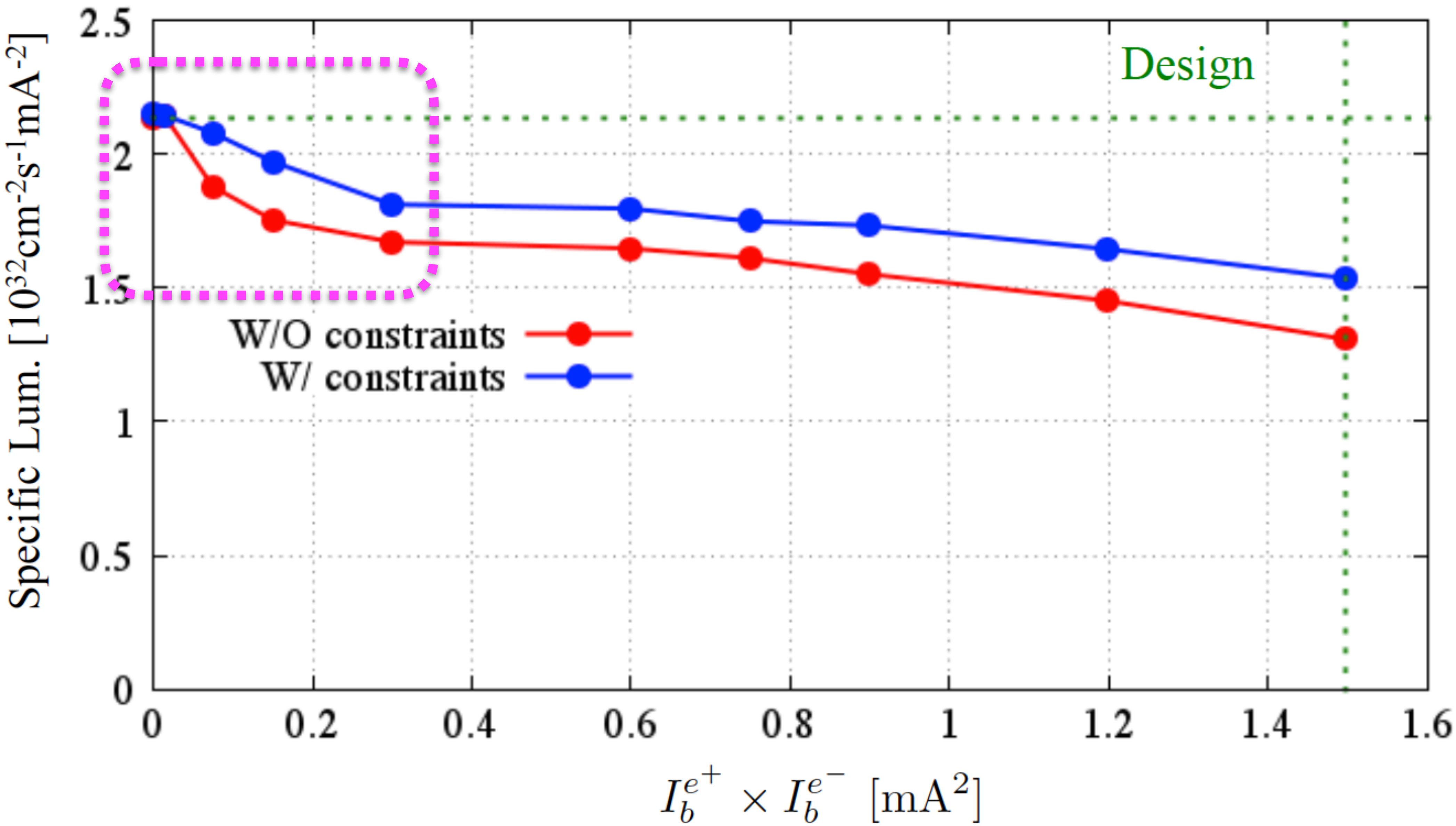
sler_1689 W/ constraints



1. Introduction

➤ **Nonlinear optimization with new constraints [by H. Sugimoto, SuperKEKB mini-optics meeting, Sep. 8, 2016]**

● **Question: Recovery of luminosity is attributed to chromatic effects [because of new constraints]? => Check all RDTs …**



1. Introduction

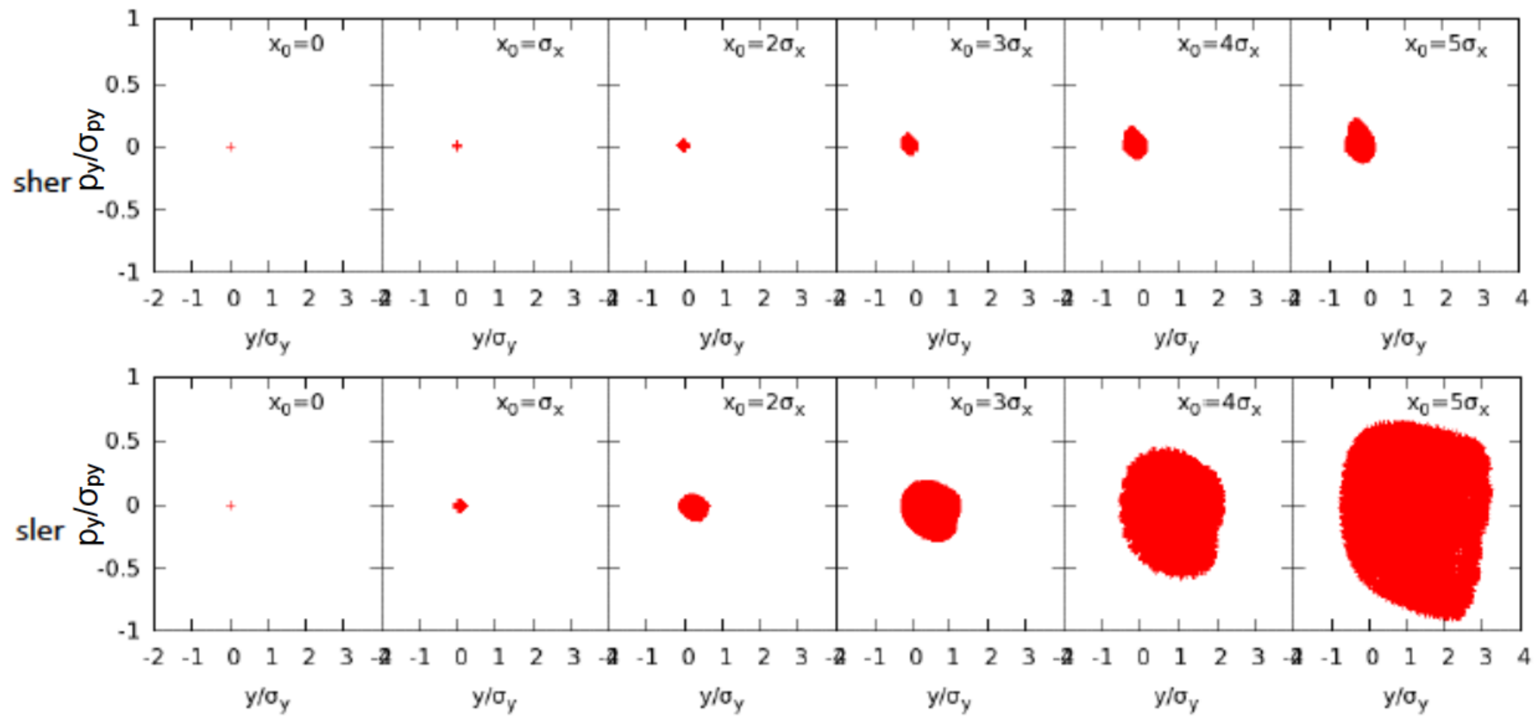
➤ Previous findings

- D. Zhou, 20th KEKB Accelerator Review Committee, Feb. 23, 2015.

2. BB+LN: Nonlin. X-Y coupling

- ➤ Realistic lattice
- ➤ Poincare map in y direction as function of X offset
- ➤ Strong nonlinear X-Y coupling in LER

sher-5767 vs ler-1689 in Y direction



From Y. Zhang

1. Introduction

➤ Previous findings

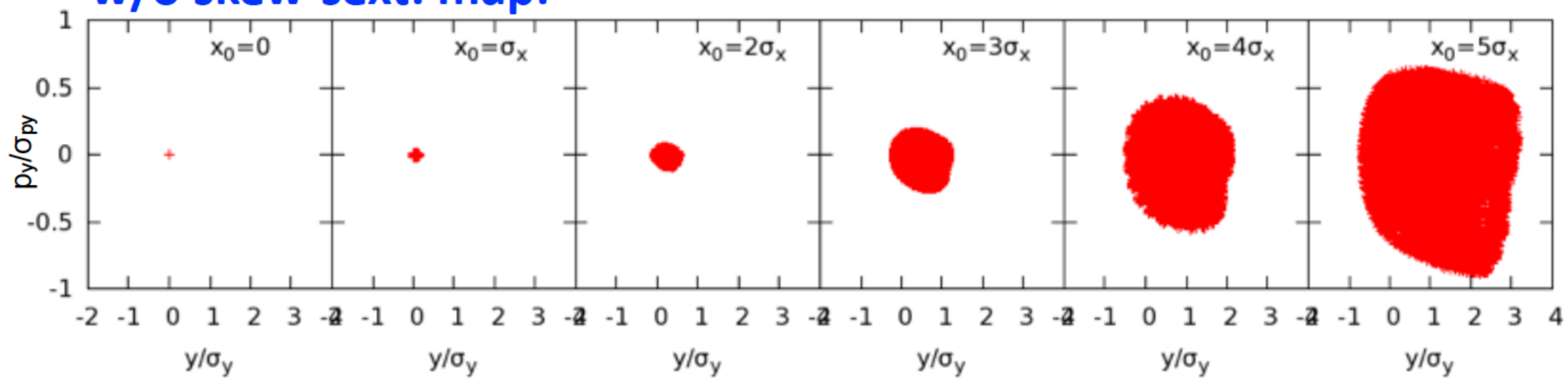
- **D. Zhou, 20th KEKB Accelerator Review Committee, Feb. 23, 2015.**

2. BB+LN: Nonlin. X-Y coupling

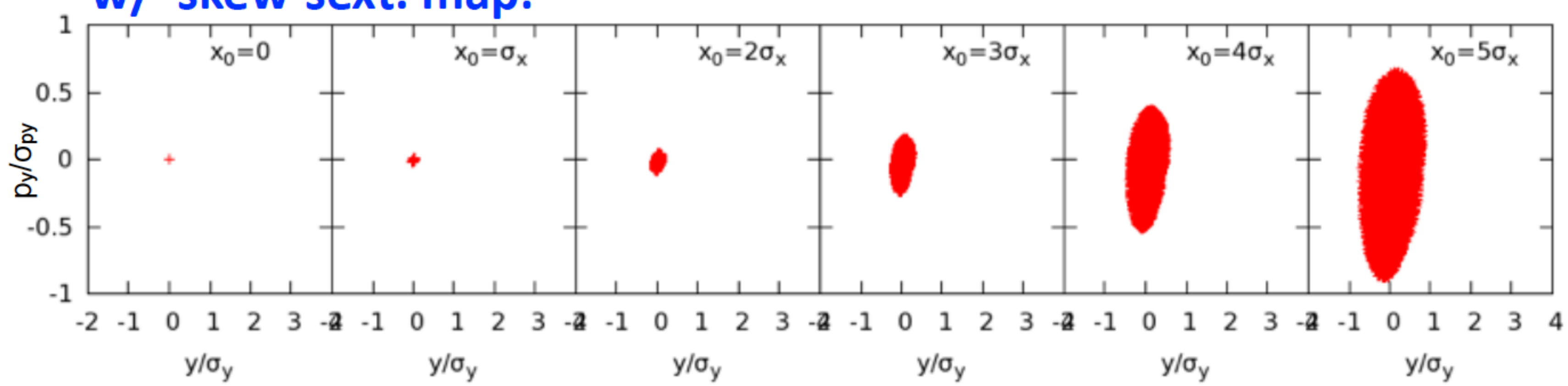
➤ **Test by inserting a map of $H=K*x^2y$ into the LER lattice**

➤ **COD and oscillation amplitude in y are well suppressed as expected**

w/o skew-sext. map:



w/ skew-sext. map:



From Y. Zhang

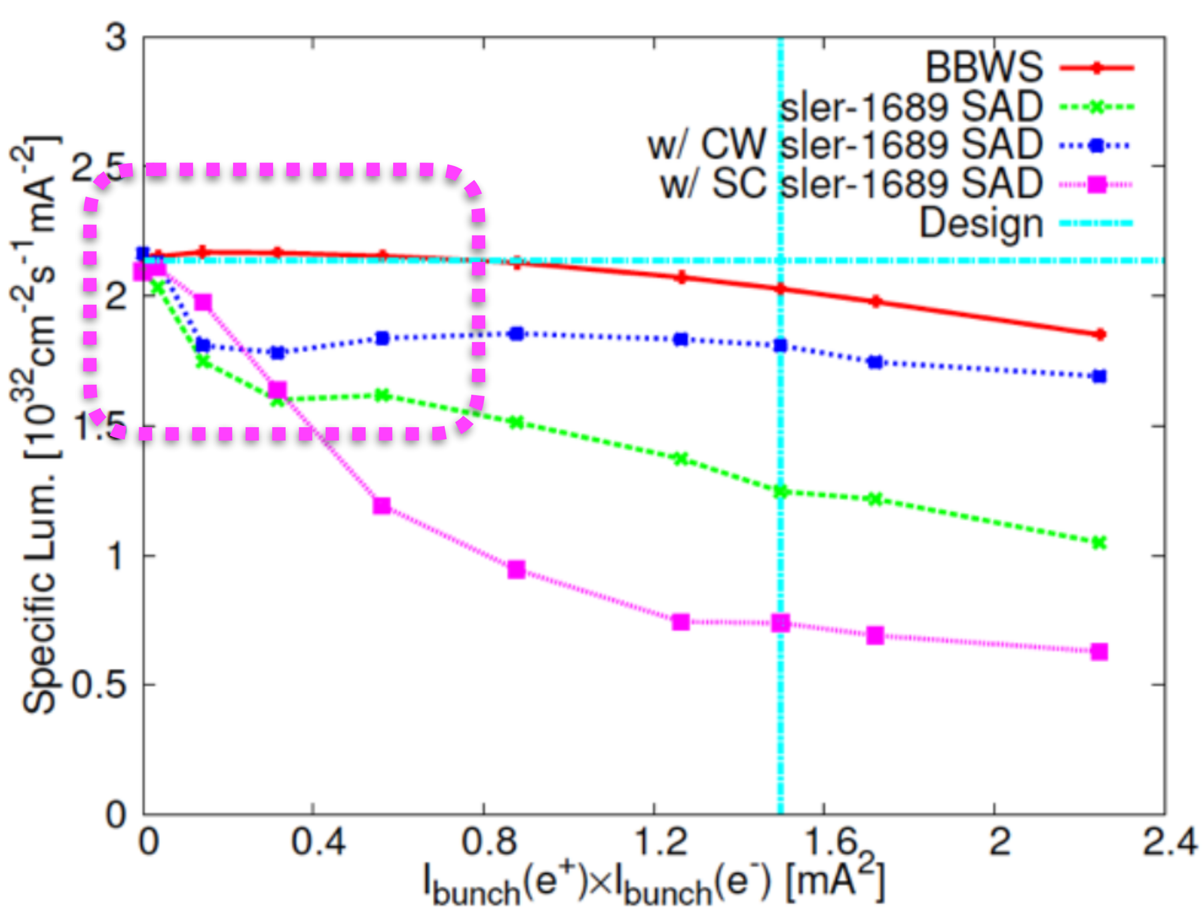
1. Introduction

➤ Previous findings

- D. Zhou, 20th KEKB Accelerator Review Committee, Feb. 23, 2015.

2. BB+LN: Luminosity: LER

- ➤ Realistic lattice: lum. drops at low beam currents
- ➤ Crab-waist:
  - To cancel beam-beam driven resonances
  - Work well at high currents, but not well at low currents



2. Theory for RDTs

➤ Resonance driving terms (RDTs) indicate lattice nonlinearity

The effective Hamiltonian of a ring can be normalized in resonance bases [Ref. E. Forest, *Beam Dynamics – A New Attitude and Framework*, 1998].

For a ring with n elements, one can normalize the one turn map $\mathcal{M}_{1\to n}$ as [Ref. L. Yang *et al.*, Phys. Rev. ST Accel. Beams 14, 054001 (2011)]

$$\mathcal{M}_{1\to n} = \mathcal{A}_1^{-1} e^{:h:} \mathcal{R}_{1\to n} \mathcal{A}_1,$$

with $\mathcal{R}$: rotation, $e^{:h:}$: nonlinear Lie map, $\mathcal{A}_1$: normalizing map. Assume no coupling (the theory can be generalized for nonzero coupling), $\mathcal{A}_i$ in x plane at the ith element can be approximated in perturbation theory as

$$\mathcal{A}_i x = \sqrt{\beta_{x,i}} x + \eta_{x,i} \delta,$$

$$\mathcal{A}_i p_x = \frac{-\alpha_{x,i} x + p_x}{\sqrt{\beta_{x,i}}} + \eta'_{x,i} \delta.$$

2. Theory for RDTs

➤ RDTs indicate lattice nonlinearity

In the resonance basis, using action-angle variables (J, ϕ) one can write

$$h_x^{\pm} \equiv \sqrt{J_x} e^{\pm i\phi_x} = \frac{X \mp iP_x}{\sqrt{2}},$$

$$\mathcal{R}_{i\to j} h_x^{\pm} = \mathcal{R}_{i\to j} \sqrt{J_x} e^{\pm i\phi_x} = e^{\pm i\mu_{i\to j,x}} h_x^{\pm},$$

where $\mu_{i\to j,x}$ is the phase advance of $i \to j$. Consequently, the potential of a multipole magnetic field can be expanded in the resonance bases of h_{abcde} as

$$h = \sum h_{abcde} h_x^{+a} h_x^{-b} h_y^{+c} h_y^{-d} \delta^e.$$

Each h_{abcde} (a complex number in general) drives a certain resonance, and is an explicit function of magnet strengths, beta functions and dispersions.

2. Theory for RDTs

➤ RDTs indicate lattice nonlinearity

The effective Hamiltonian corresponding to chromaticity is

$$h_c = \sum h_{1100e} h_x^{+1} h_x^{-1} \delta^e + \sum h_{0011e} h_y^{+1} h_y^{-1} \delta^e,$$

$$h_c = J_x \sum h_{1100e} \delta^e + J_y \sum h_{0011e} \delta^e.$$

Then the tunes are calculated as

$$\nu_x = -\frac{1}{2\pi} \frac{\partial h_c}{\partial J_x} = -\frac{1}{2\pi} \sum h_{1100e} \delta^e,$$

$$\nu_y = -\frac{1}{2\pi} \frac{\partial h_c}{\partial J_y} = -\frac{1}{2\pi} \sum h_{0011e} \delta^e.$$

Therefore the RDTs of h_{1100e} and h_{0011e} correspond to linear and high-order chromaticity.

2. Theory for RDTs

➤ RDTs indicate lattice nonlinearity

h_{abcde}	Driving effects
h_{11001}, h_{00111}	Linear chromaticity ζ_x, ζ_y
$h_{21000}, h_{12000} \| h_{10110}, h_{01110}$	ν_x $[(J_x)^{3/2}] \| [(J_x)^{1/2}(J_y)]$
$h_{30000}, h_{03000} \| h_{00300}, h_{00030}$	$3\nu_x$ $[(J_x)^{3/2}] \| 3\nu_y$ $[(J_y)^{3/2}]$
$h_{10020}, h_{01200} \| h_{10200}, h_{01020}$	$\nu_x - 2\nu_y \| \nu_x + 2\nu_y$ $[(J_x)^{1/2}(J_y)]$
$h_{20010}, h_{02100} \| h_{20100}, h_{02010}$	$2\nu_x - \nu_y \| 2\nu_x + \nu_y$ $[(J_x)(J_y)^{1/2}]$
$h_{00210}, h_{00120} \| h_{11100}, h_{11010}$	ν_y $[(J_y)^{3/2}] \| [(J_x)(J_y)^{1/2}]$
$h_{22000}, h_{00220}, h_{11110}$	$d\nu_x/dJ_x, d\nu_y/dJ_y, d\nu_{x,y}/dJ_{y,x}$
$h_{40000}, h_{04000} \| h_{00400}, h_{00040}$	$4\nu_x$ $[(J_x)^2] \| 4\nu_y$ $[(J_y)^2]$
$h_{31000}, h_{13000} \| h_{20110}, h_{02110}$	$2\nu_x$ $[(J_x)^2] \| [(J_x)(J_y)]$
$h_{00310}, h_{00130} \| h_{11200}, h_{11020}$	$2\nu_y$ $[(J_y)^2] \| [(J_x)(J_y)]$
$h_{20020}, h_{02200} \| h_{20200}, h_{02020}$	$2\nu_x - 2\nu_y \| 2\nu_x + 2\nu_y$ $[(J_x)(J_y)]$
$h_{30010}, h_{03100} \| h_{30100}, h_{03010}$	$3\nu_x - \nu_y \| 3\nu_x + \nu_y$ $[(J_x)^{3/2}(J_y)^{1/2}]$
$h_{10030}, h_{01300} \| h_{10300}, h_{01030}$	$\nu_x - 3\nu_y \| \nu_x + 3\nu_y$ $[(J_x)^{1/2}(J_y)^{3/2}]$

Table : Low-order driving terms.

2. Theory for RDTs

➤ RDTs indicate lattice nonlinearity: Analytic theories according to J. Bengtsson, SLS Note 9/97

- **Linear chromaticity:**

$$h_{11001} = \frac{1}{4}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{xi} + O\left(\delta^2\right),$$

$$h_{00111} = -\frac{1}{4}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{yi} + O\left(\delta^2\right)$$

- **Chromatic beta functions:**

$$h_{20001} = h^*_{02001} = \frac{1}{8}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{xi}e^{i2\mu_{xi}} + O\left(\delta^2\right),$$

$$h_{00201} = h^*_{00021} = -\frac{1}{8}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{yi}e^{i2\mu_{yi}} + O\left(\delta^2\right),$$

- **Chromatic dispersion:**

$$h_{10002} = h^*_{01002} = \frac{1}{2}\sum_{i=1}^{N}\left[(b_2L)_i - (b_3L)_i\eta_{xi}^{(1)}\right]\eta_{xi}^{(1)}\sqrt{\beta_{xi}}e^{i\mu_{xi}} + O\left(\delta^3\right)$$

2. Theory for RDTs

➤ **RDTs indicate lattice nonlinearity: Analytic theories according to J. Bengtsson, SLS Note 9/97**

- **First order geometric terms (amplitude-dependent):**

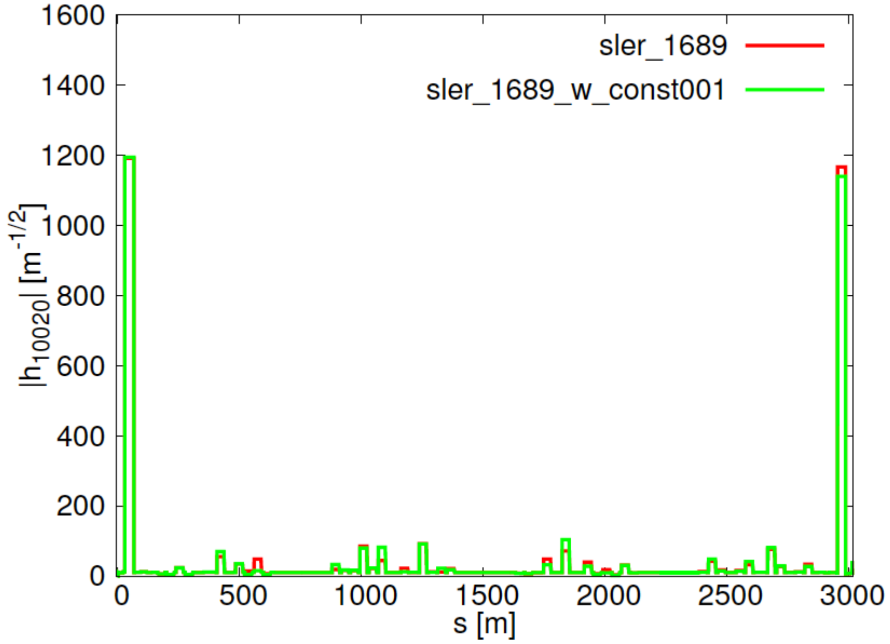
$$
\begin{aligned}
h_{21000} &= h^*_{12000} = -\frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{3/2}e^{i\mu_{xi}}, \\
h_{30000} &= h^*_{03000} = -\frac{1}{24}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{3/2}e^{i3\mu_{xi}}, \\
h_{10110} &= h^*_{01110} = \frac{1}{4}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{1/2}\beta_{yi}e^{i\mu_{xi}}, \\
h_{10020} &= h^*_{01200} = \frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{1/2}\beta_{yi}e^{i(\mu_{xi}-2\mu_{yi})}, \\
h_{10200} &= h^*_{01020} = \frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{1/2}\beta_{yi}e^{i(\mu_{xi}+2\mu_{yi})}
\end{aligned}
$$

3. Results by PTC: 3rd order RDTs

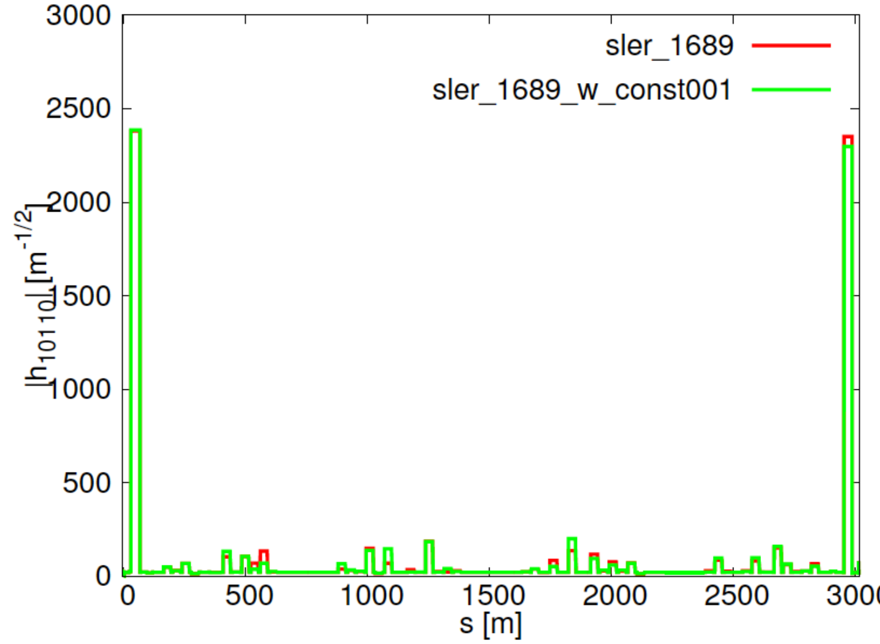
➤ Integration of RDTs along the whole ring

- **Almost perfect cancellation of 3rd order RDTs in the arc sections**

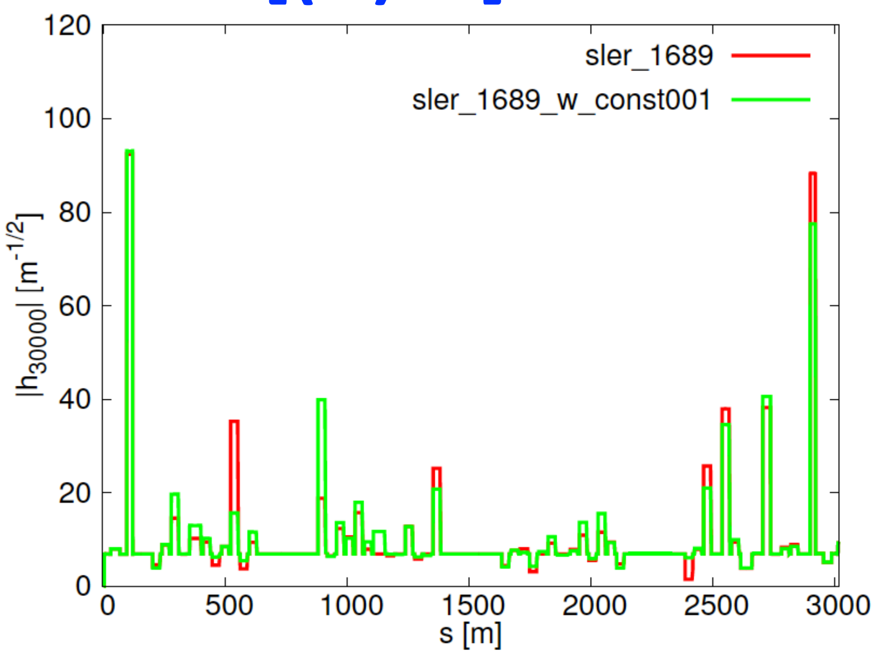
$\nu_x-2\nu_y\ [(J_x)^{1/2}(J_y)]$



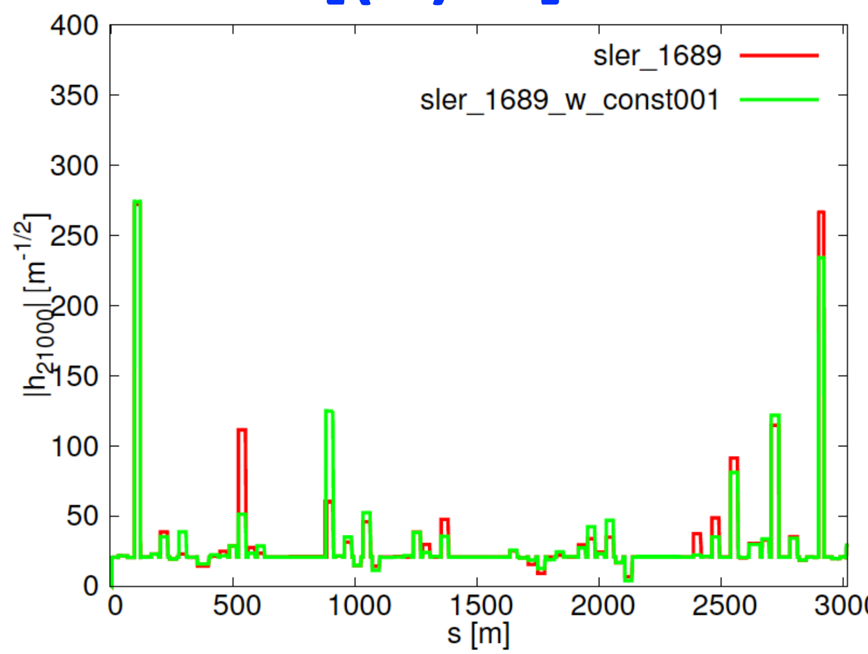
$\nu_x\ [(J_x)^{1/2}(J_y)]$



$3\nu_x\ [(J_x)^{3/2}]$



$\nu_x\ [(J_x)^{3/2}]$

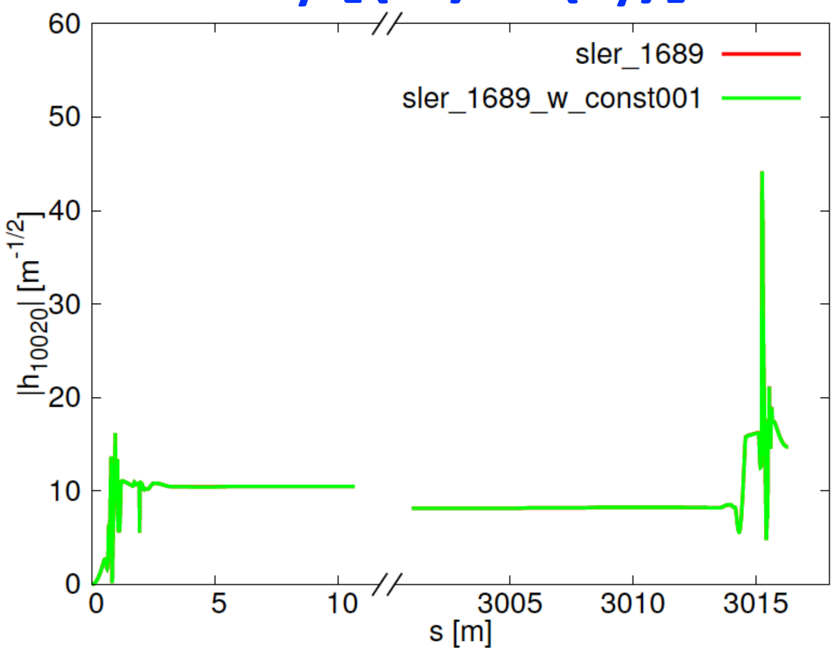


3. Results by PTC: 3rd order RDTs (IR)

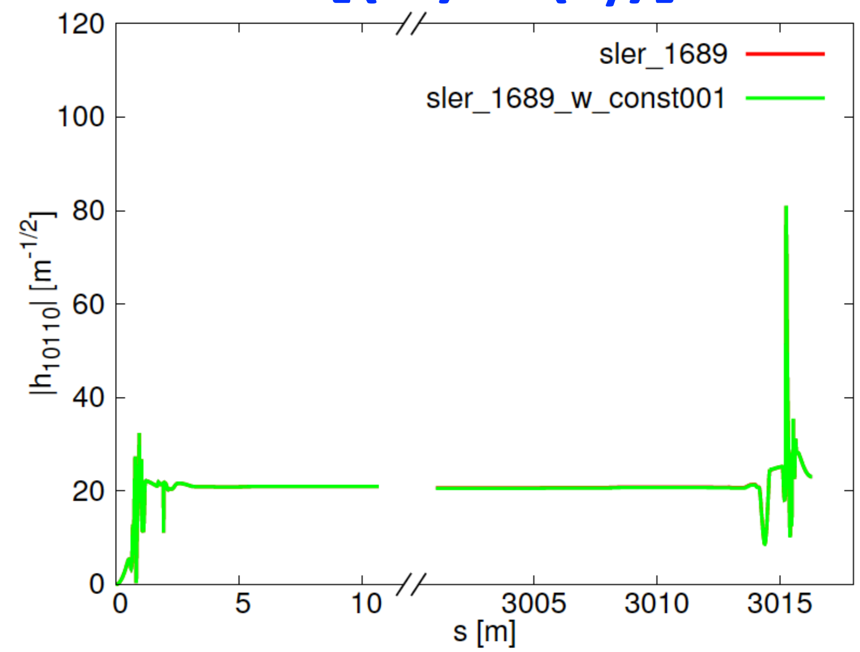
➤ **Integration of RDTs along the whole ring**

- **FFS contributes most of residual RDTs**

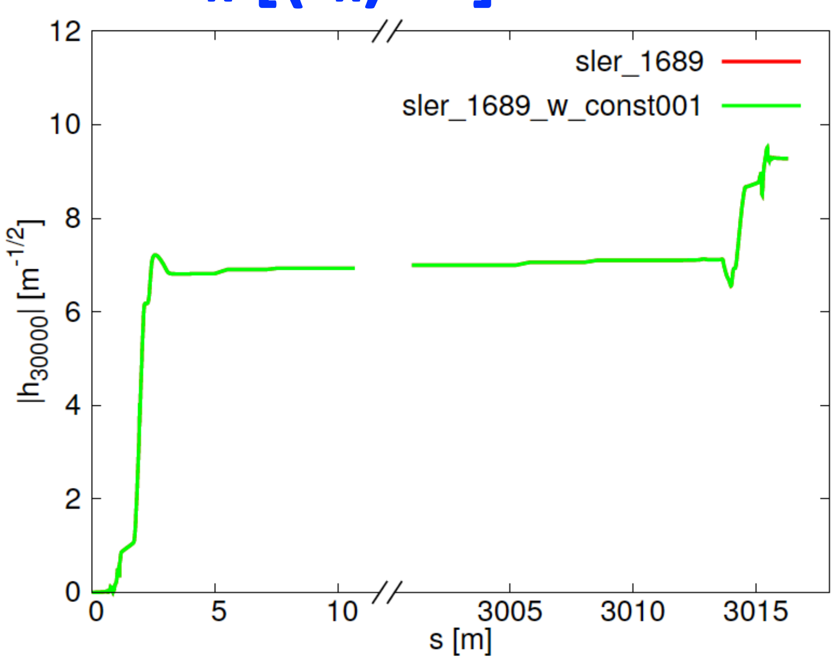
ν_x-$2\nu_y$ $[(J_x)^{1/2}(J_y)]$



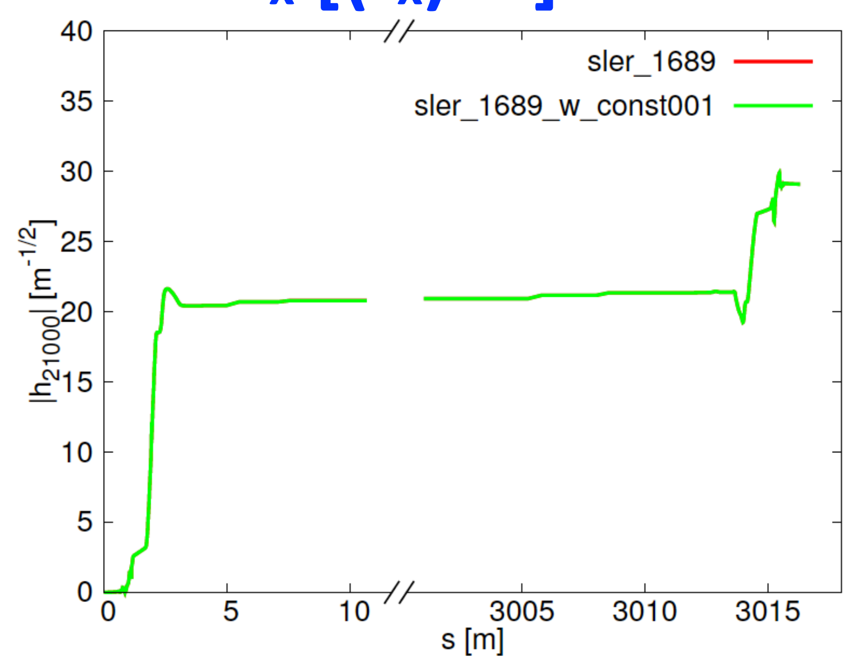
ν_x $[(J_x)^{1/2}(J_y)]$



$3\nu_x$ $[(J_x)^{3/2}]$



ν_x $[(J_x)^{3/2}]$

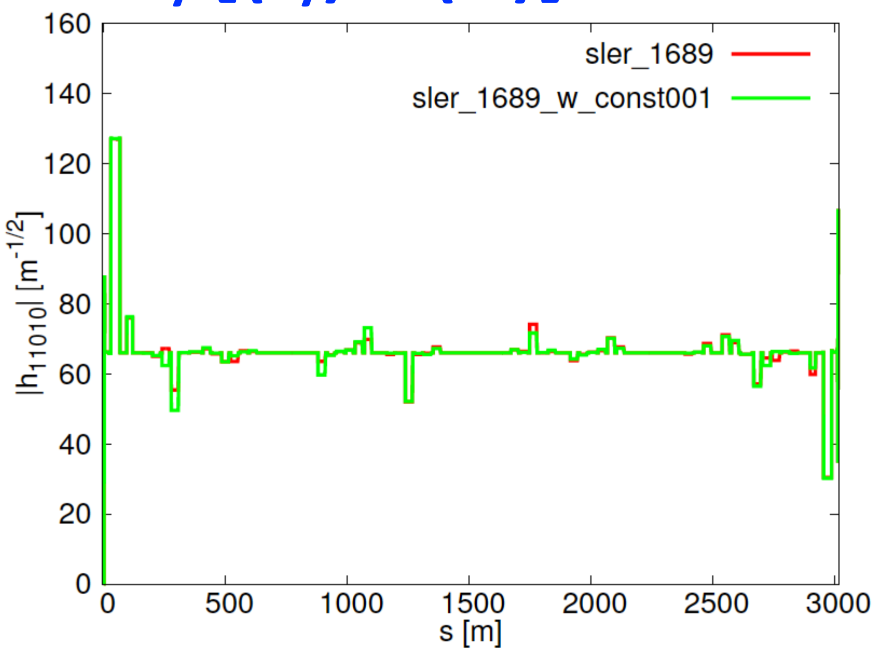


3. Results by PTC: 3rd order RDTs

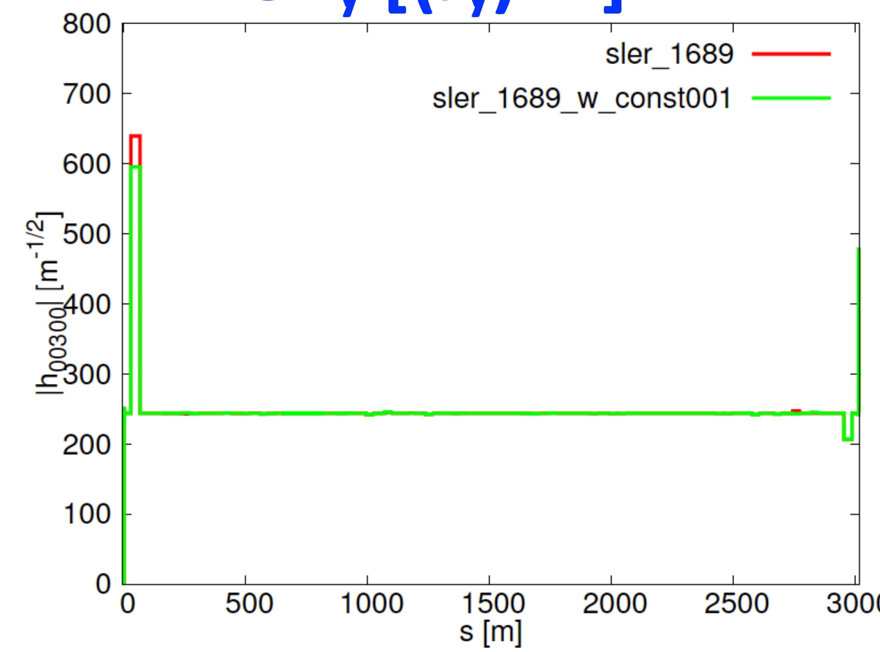
➤ **Integration of RDTs along the whole ring**

- **Almost perfect cancellation of 3rd order RDTs in the arc sections**

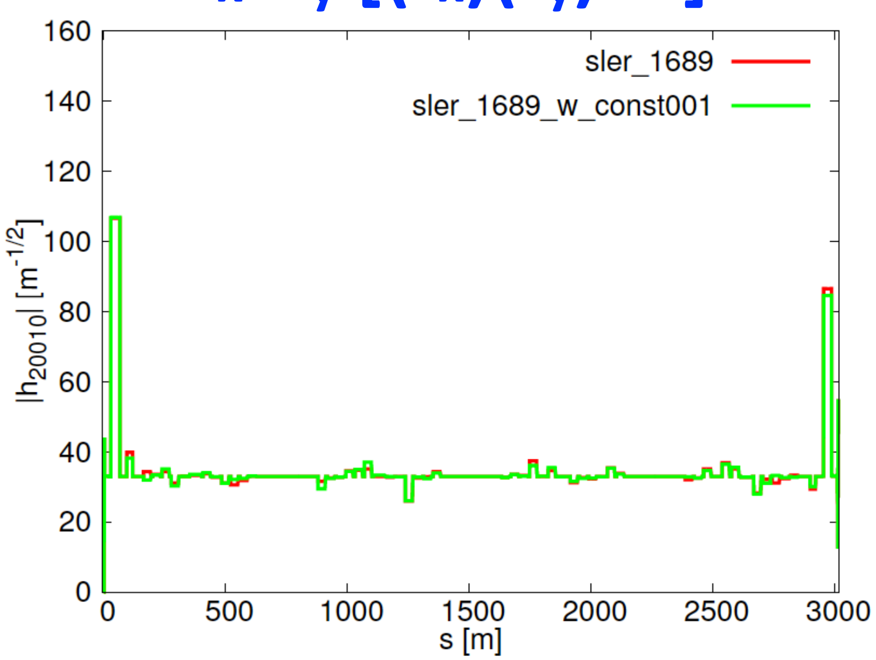
ν_y $[(J_y)^{1/2}(J_x)]$



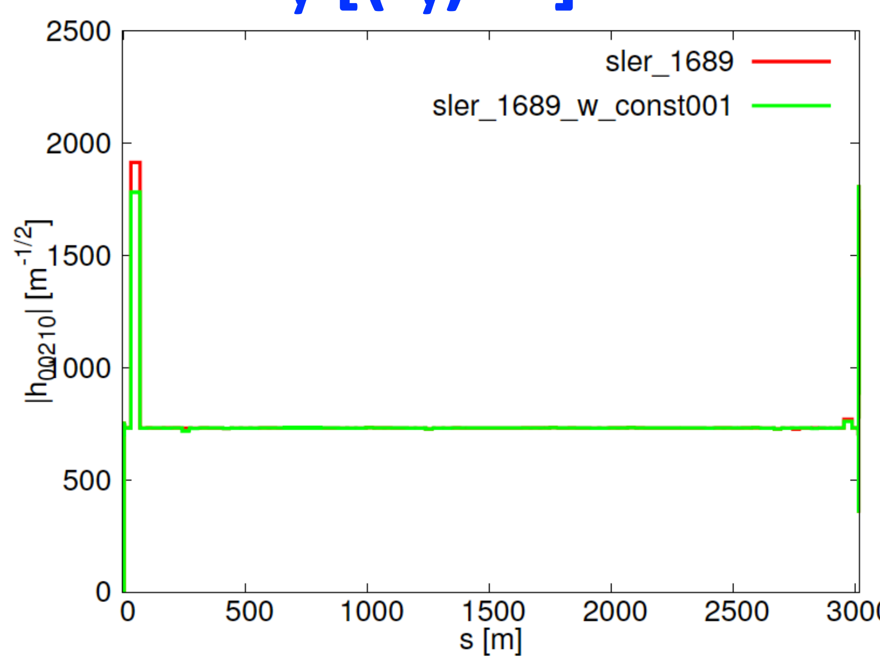
$3\nu_y$ $[(J_y)^{3/2}]$



$2\nu_x$-ν_y $[(J_x)(J_y)^{1/2}]$



ν_y $[(J_y)^{3/2}]$

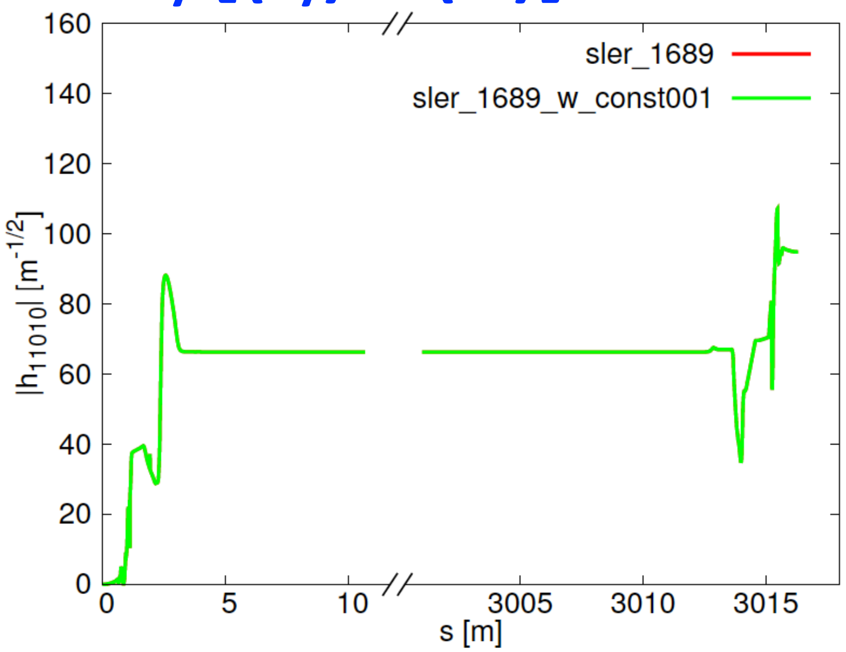


3. Results by PTC: 3rd order RDTs (IR)

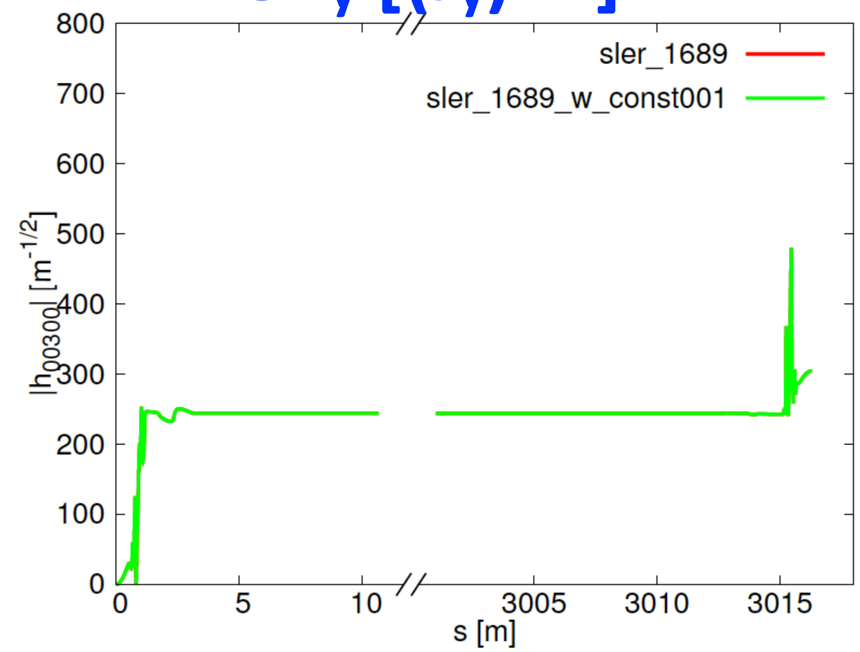
➤ **Integration of RDTs along the whole ring**

- **FFS contributes most of residual RDTs**

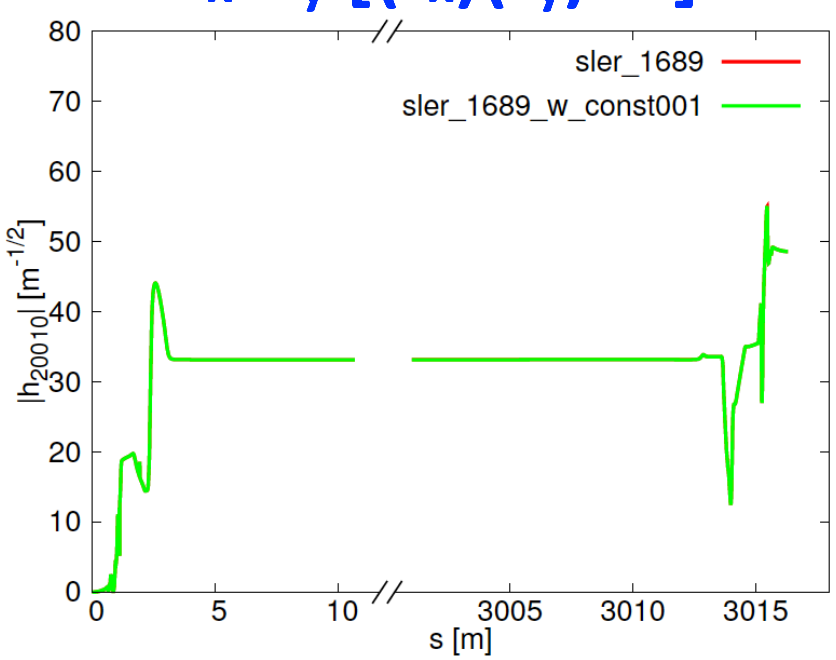
ν_y $[(J_y)^{1/2}(J_x)]$



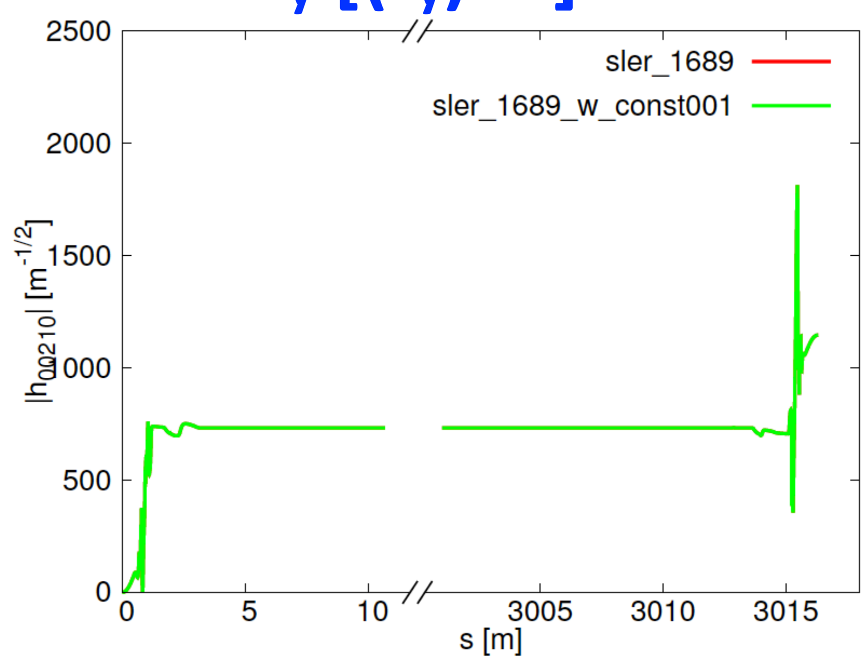
$3\nu_y$ $[(J_y)^{3/2}]$



$2\nu_x-\nu_y$ $[(J_x)(J_y)^{1/2}]$



ν_y $[(J_y)^{3/2}]$

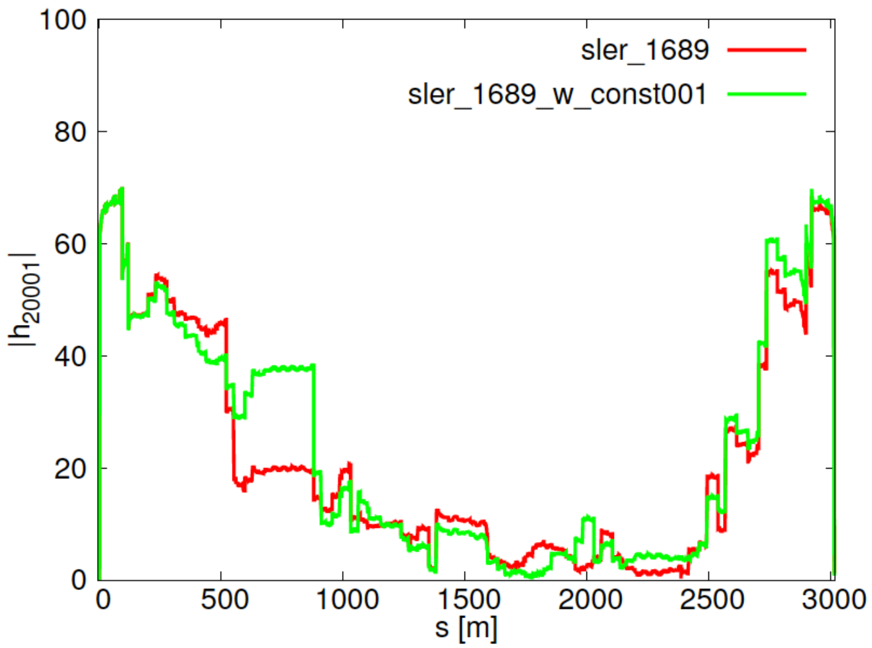


3. Results by PTC: Chromatic β and ν

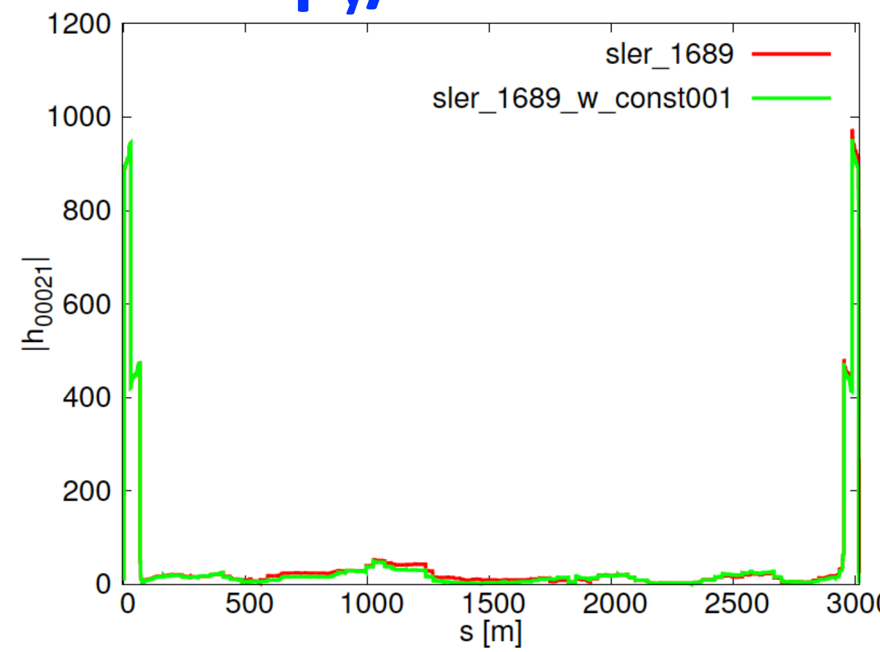
➤ Detuning along the whole ring

- **w/ constraints: chromatic correction**

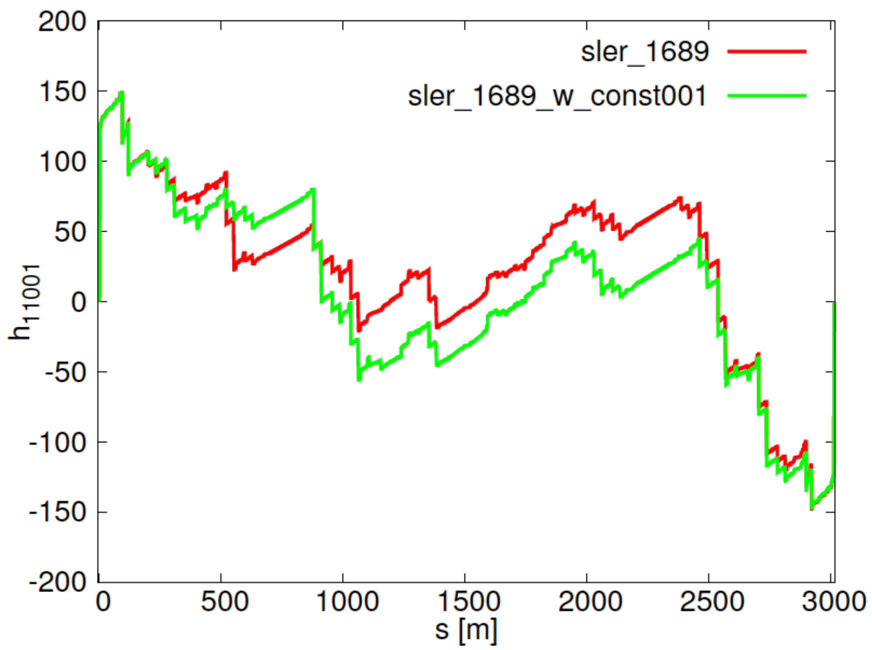
$d\beta_x/d\delta$



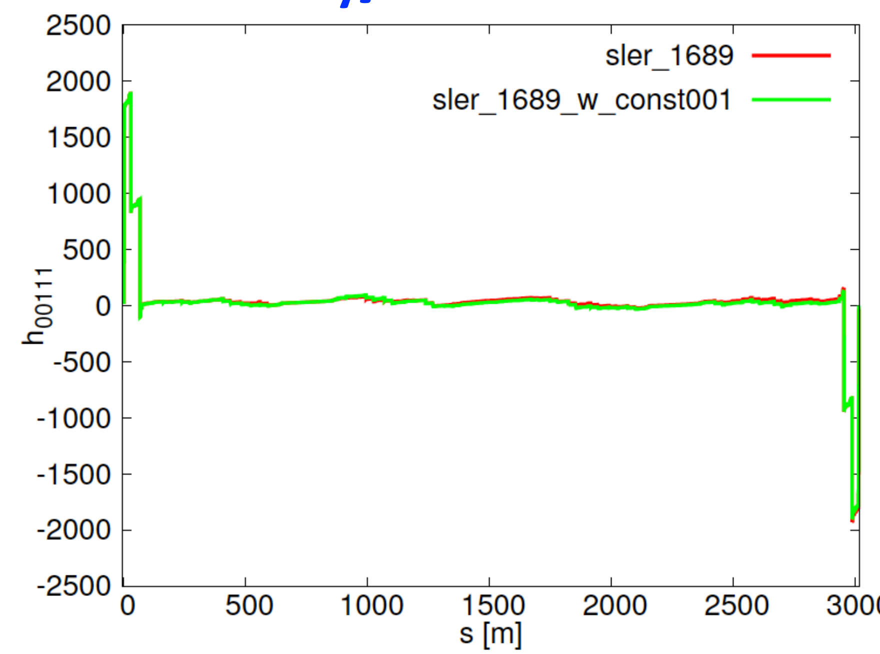
$d\beta_y/d\delta$



$d\nu_x/d\delta$



$d\nu_y/d\delta$

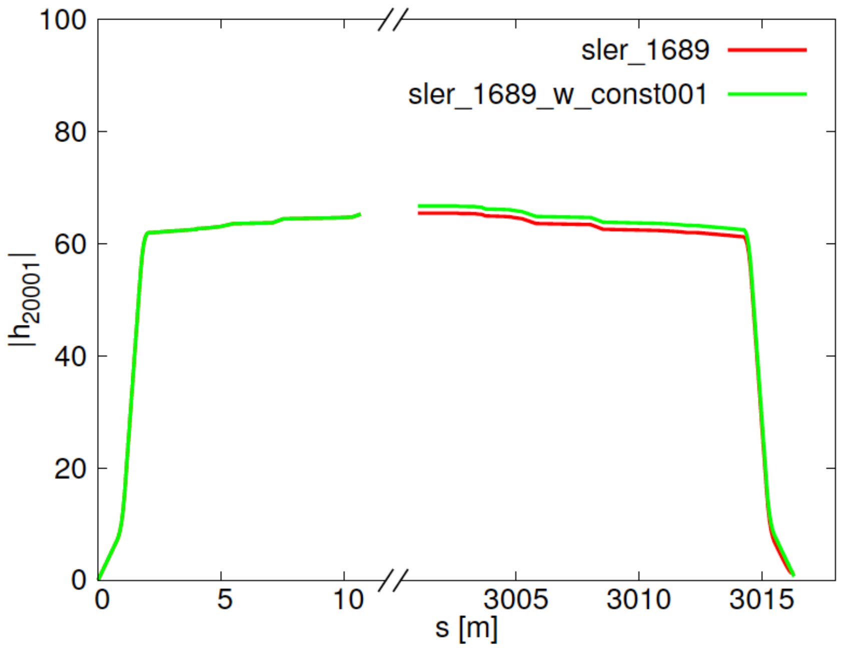


3. Results by PTC: Chromatic β and ν (IR)

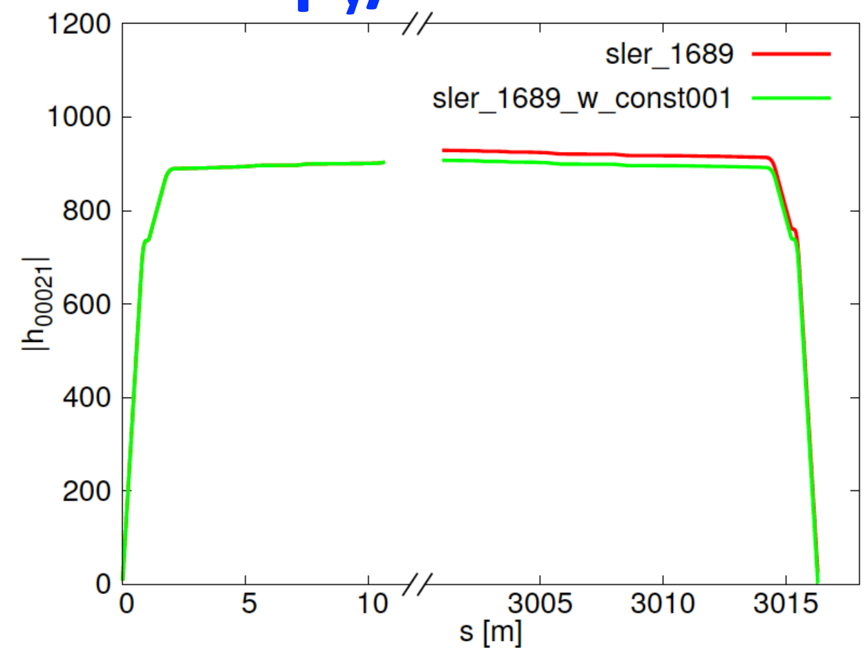
➤ Detuning along the whole ring

- **w/ constraints: chromatic correction**

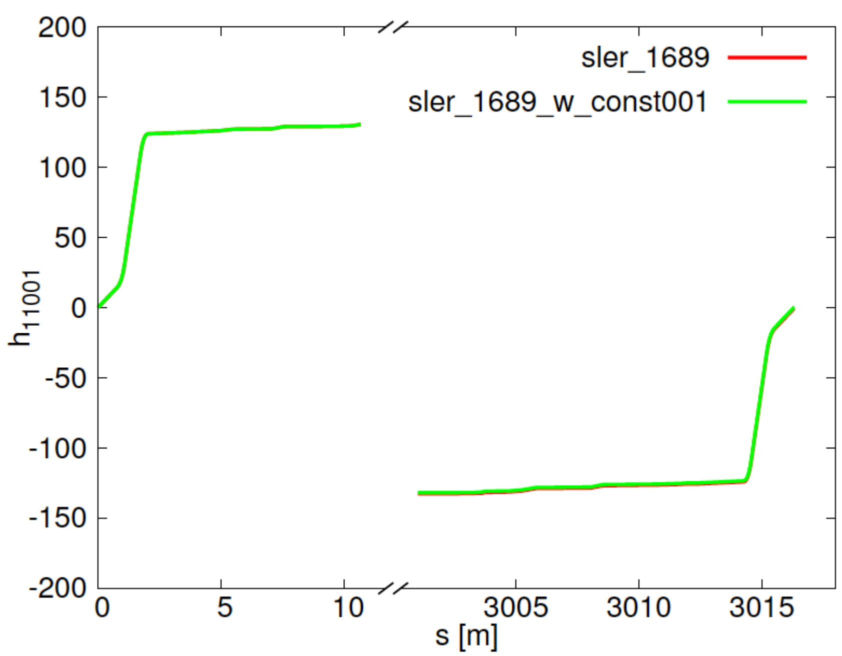
$d\beta_x/d\delta$



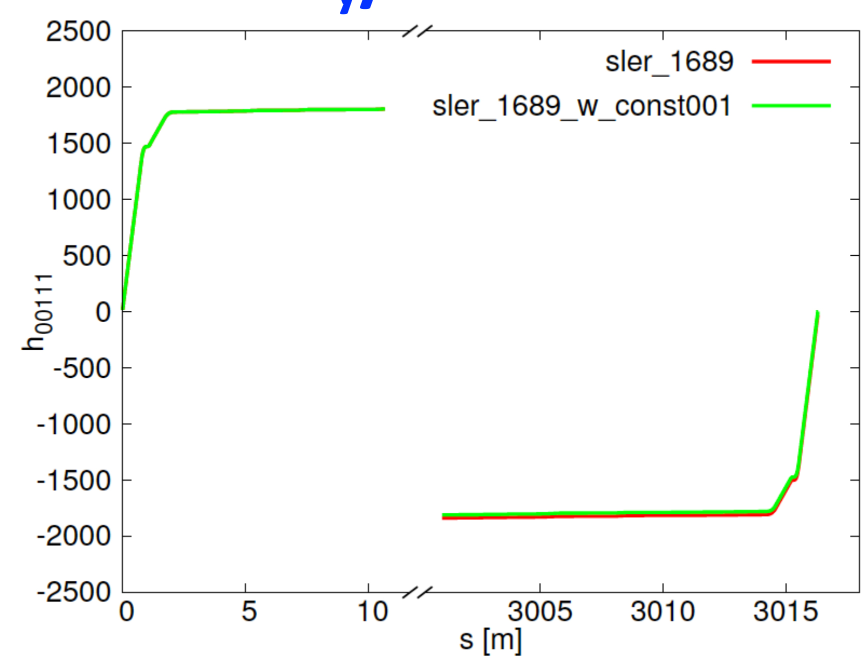
$d\beta_y/d\delta$



$d\nu_x/d\delta$



$d\nu_y/d\delta$

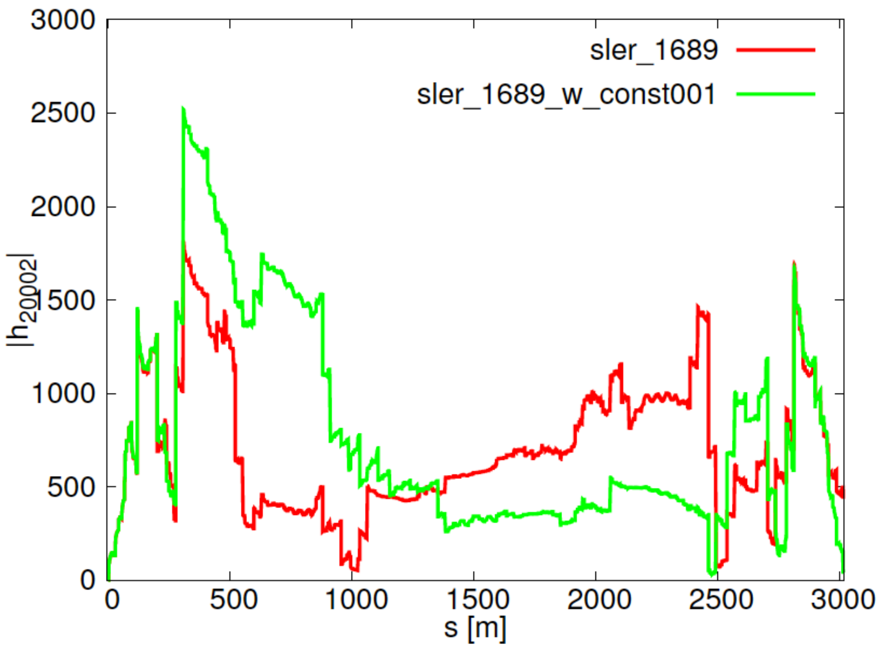


3. Results by PTC: Chromatic β and ν

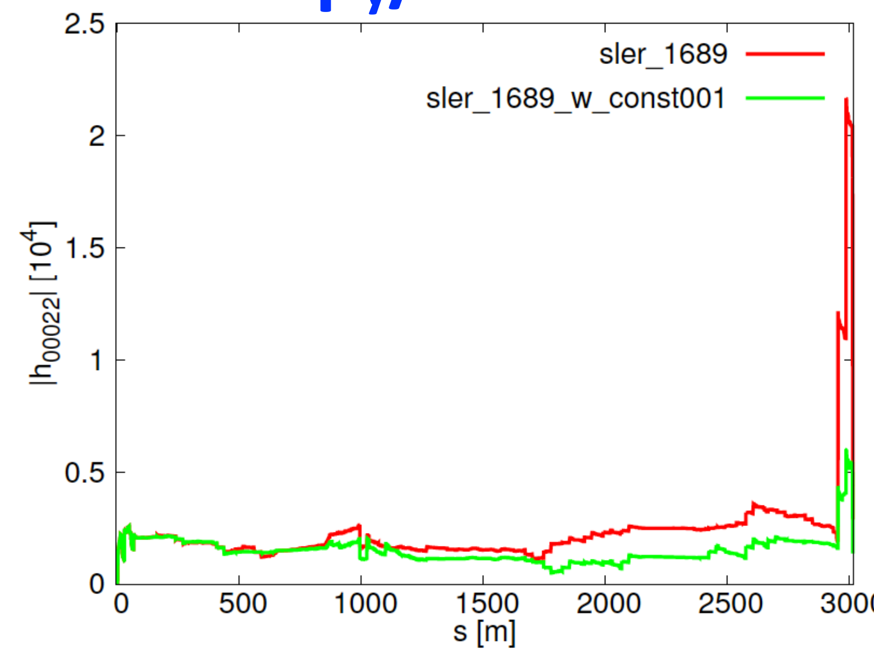
➤ Detuning along the whole ring - second order

• w/ constraints: chromatic correction

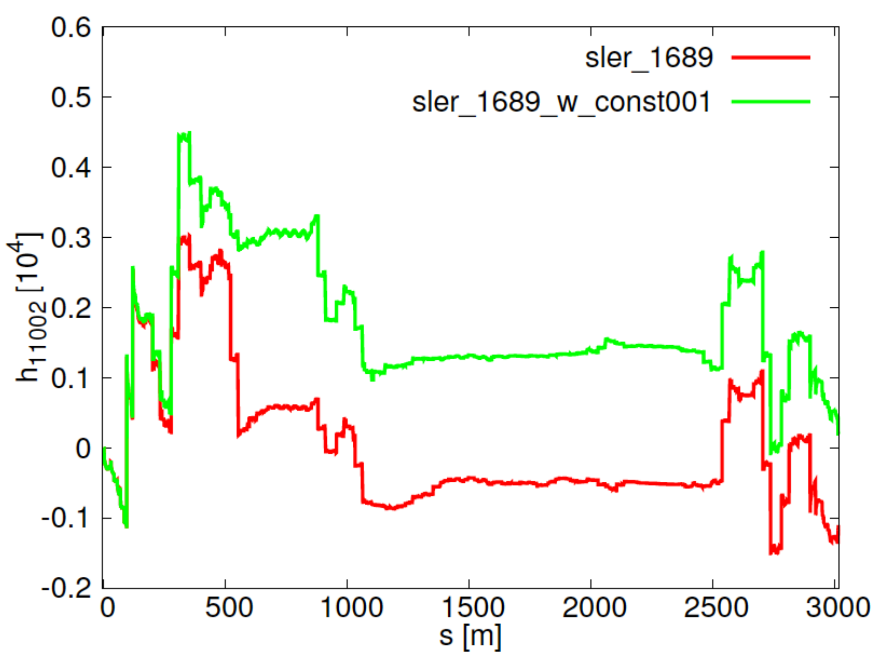
$d^2\beta_x/d\delta^2$



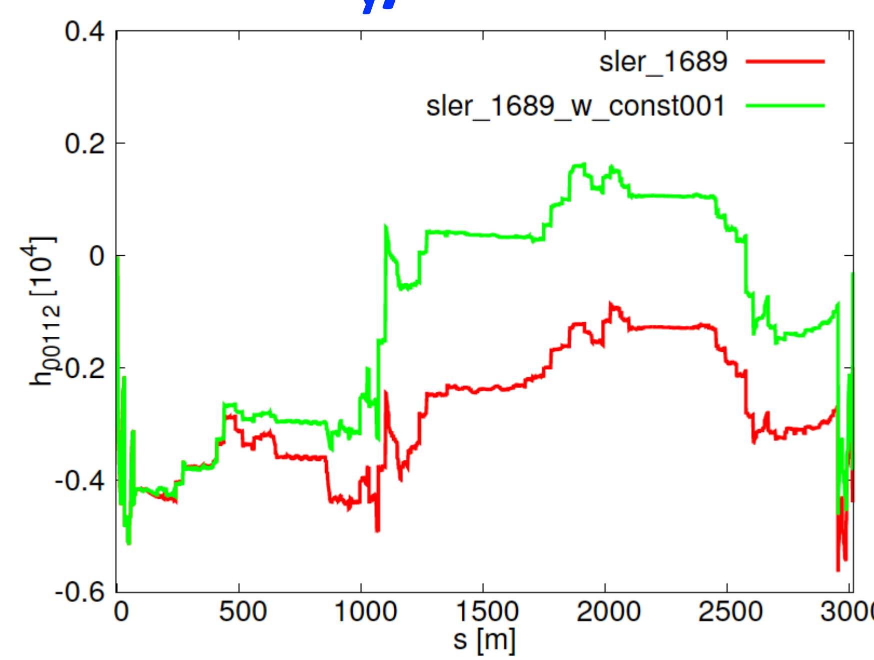
$d^2\beta_y/d\delta^2$



$d^2\nu_x/d\delta^2$



$d^2\nu_y/d\delta^2$

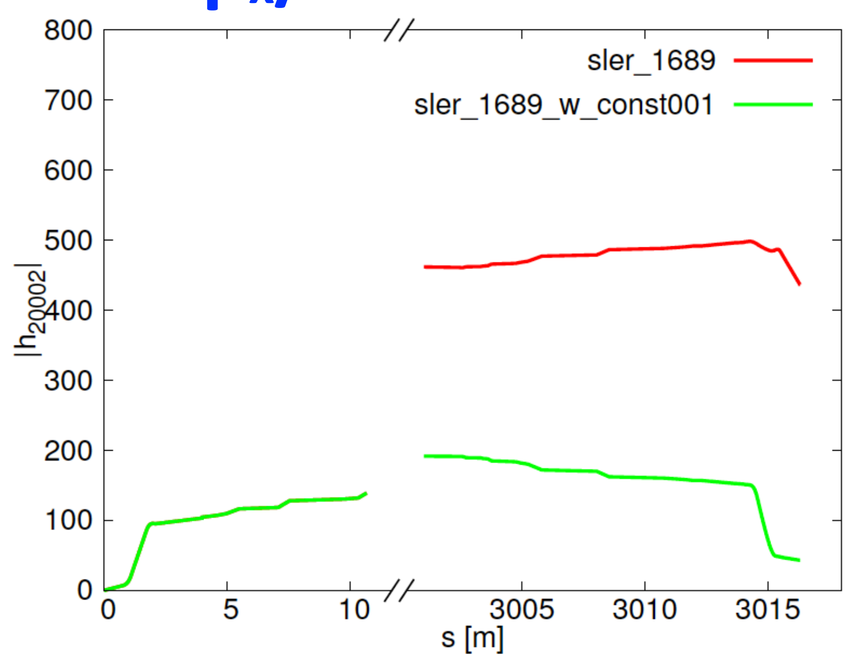


3. Results by PTC: Chromatic β and ν (IR)

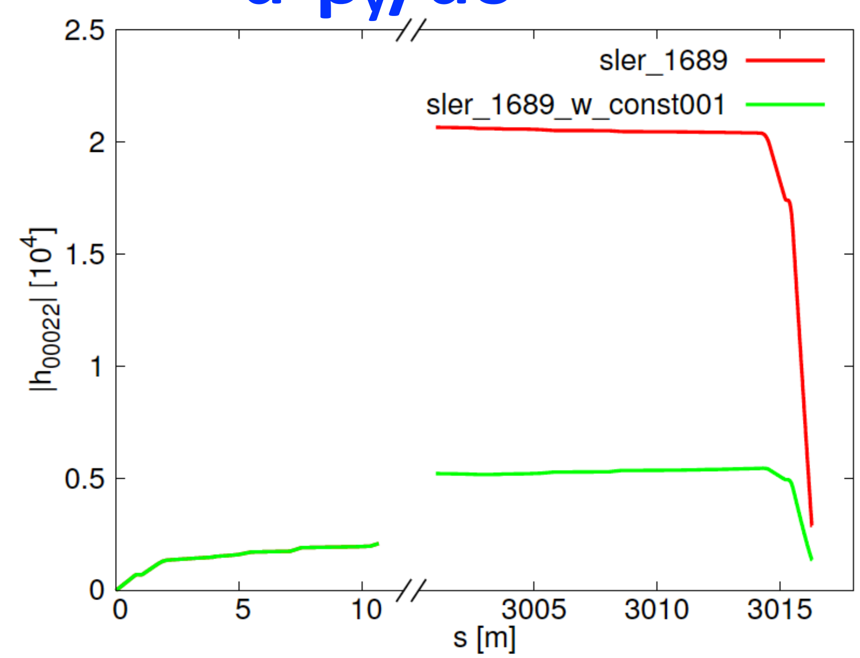
➤ **Detuning along the whole ring - second order**

- **w/ constraints: chromatic correction**

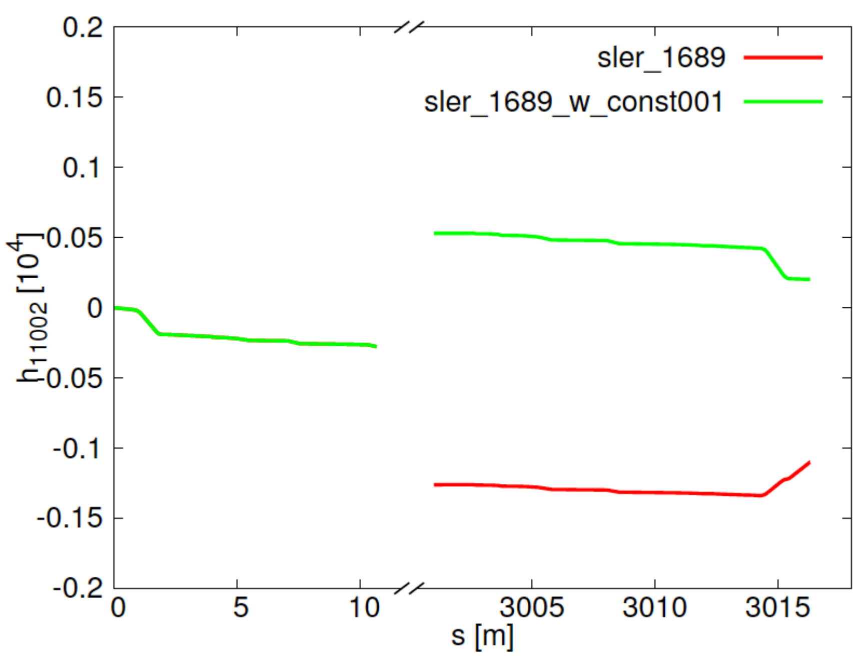
$d^2\beta_x/d\delta^2$



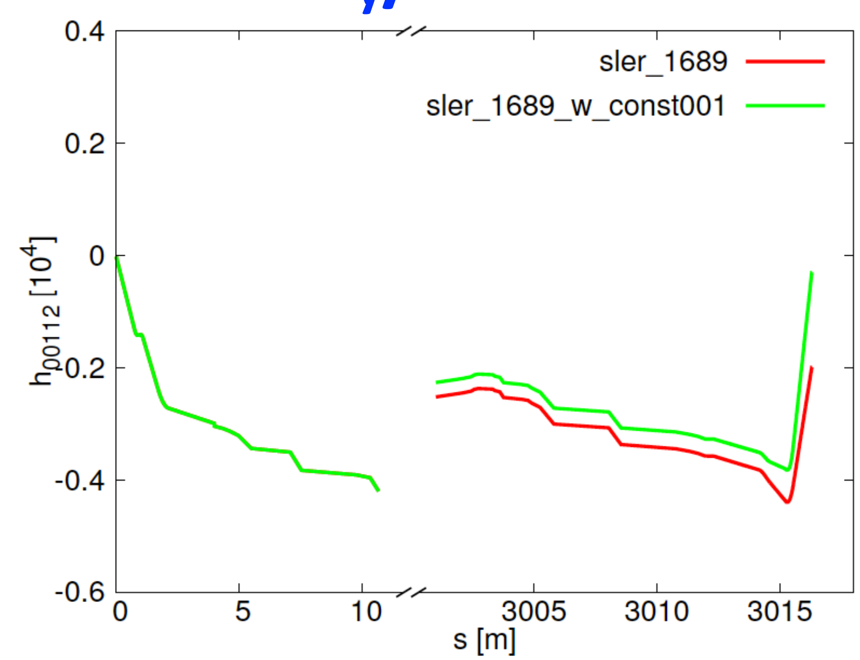
$d^2\beta_y/d\delta^2$



$d^2\nu_x/d\delta^2$



$d^2\nu_y/d\delta^2$

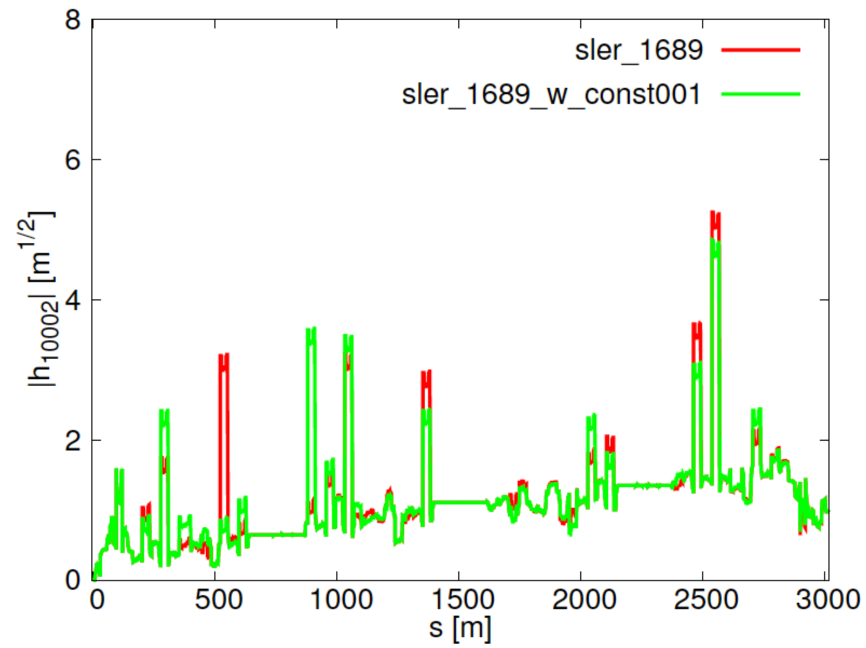


3. Results by PTC: Chromatic dispersion

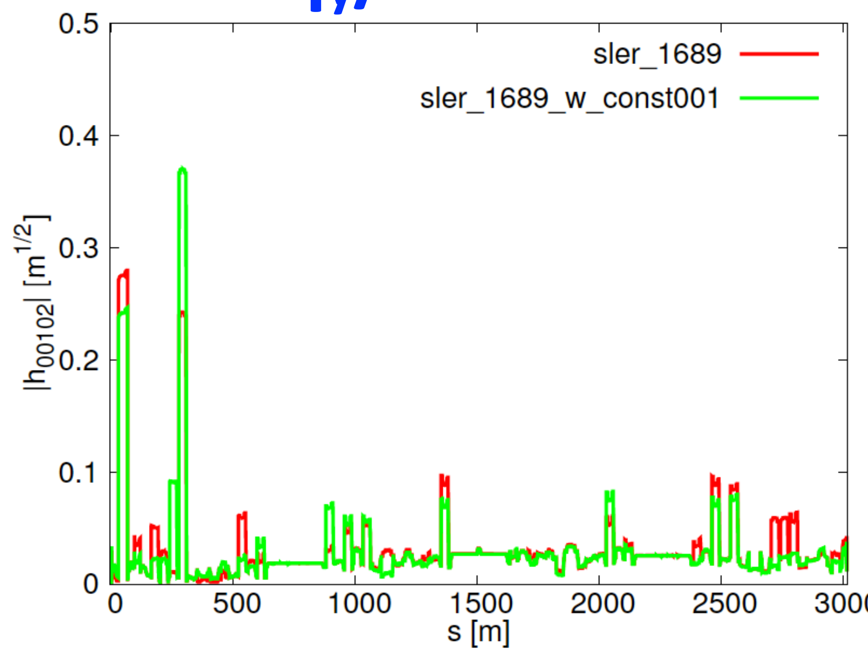
➤ Dispersion along the whole ring

- **w/ constraints: No special control on chromatic dispersions?**

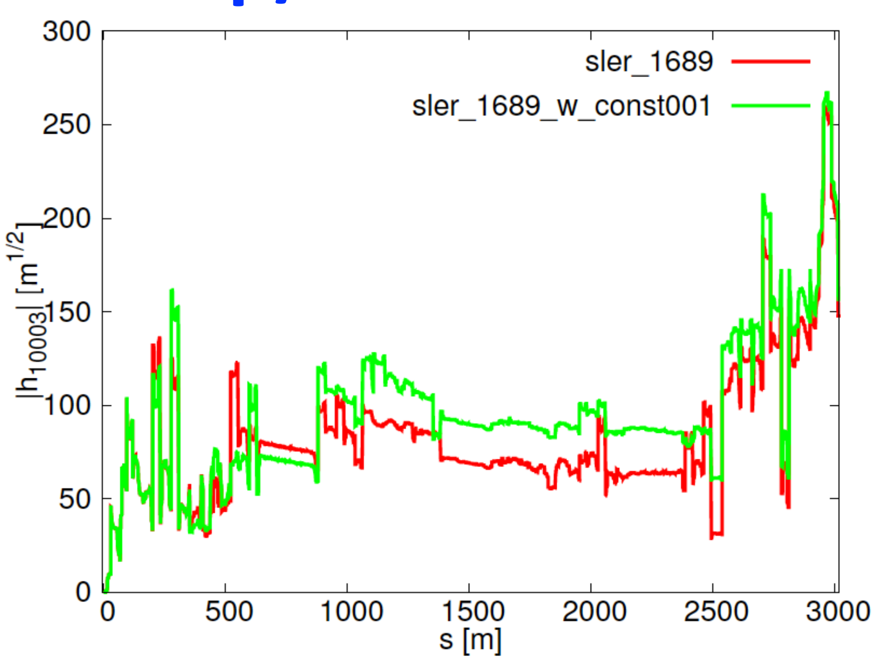
$d\eta_x/d\delta$



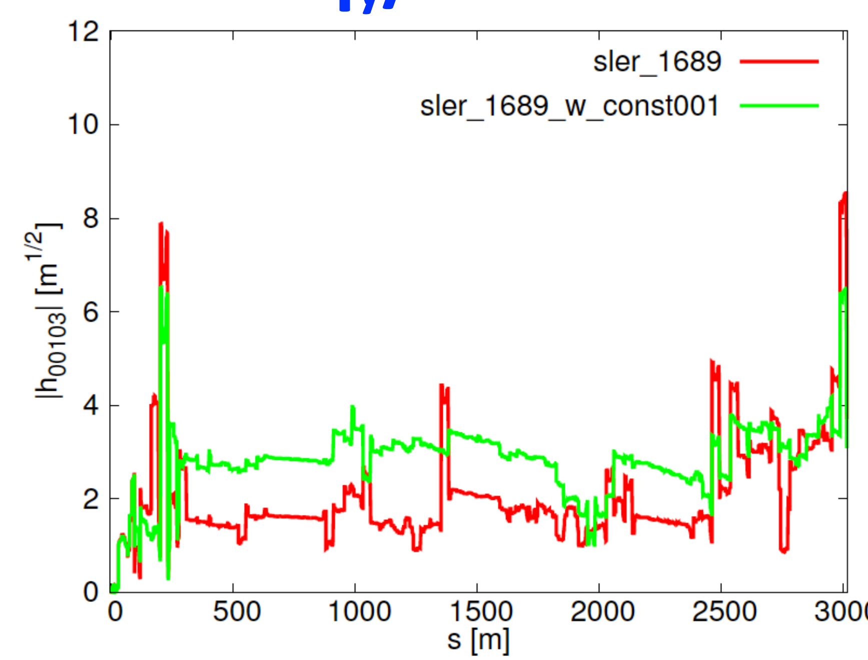
$d\eta_y/d\delta$



$d^2\eta_x/d\delta^2$



$d^2\eta_y/d\delta^2$

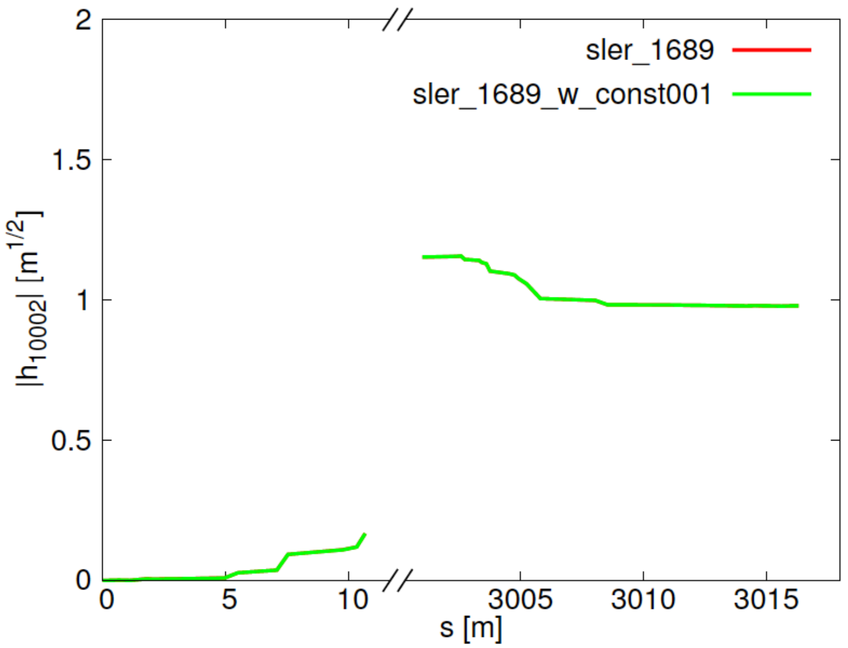


3. Results by PTC: Chromatic dispersion (IR)

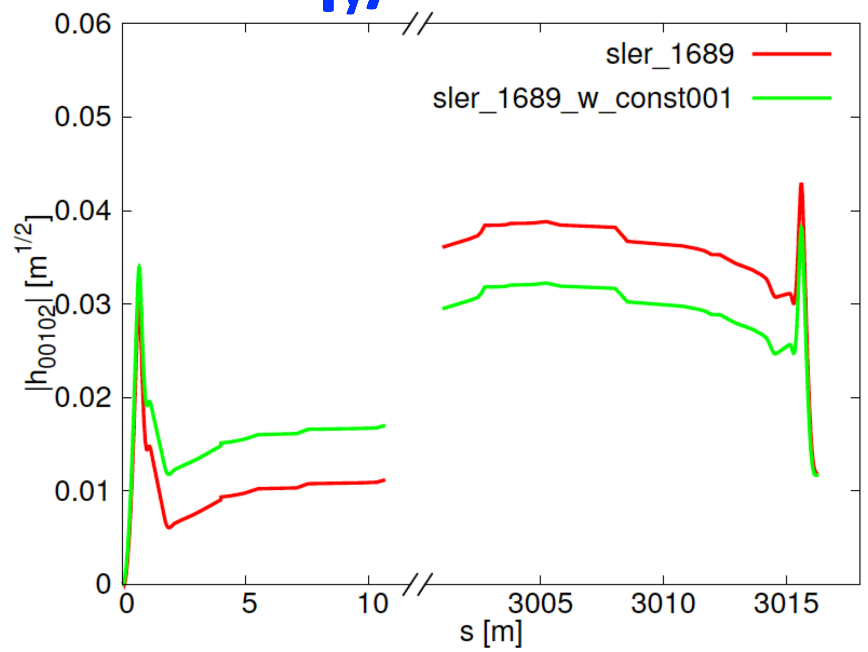
➤ Dispersion along the whole ring

- **w/ constraints: No special control on chromatic dispersions?**

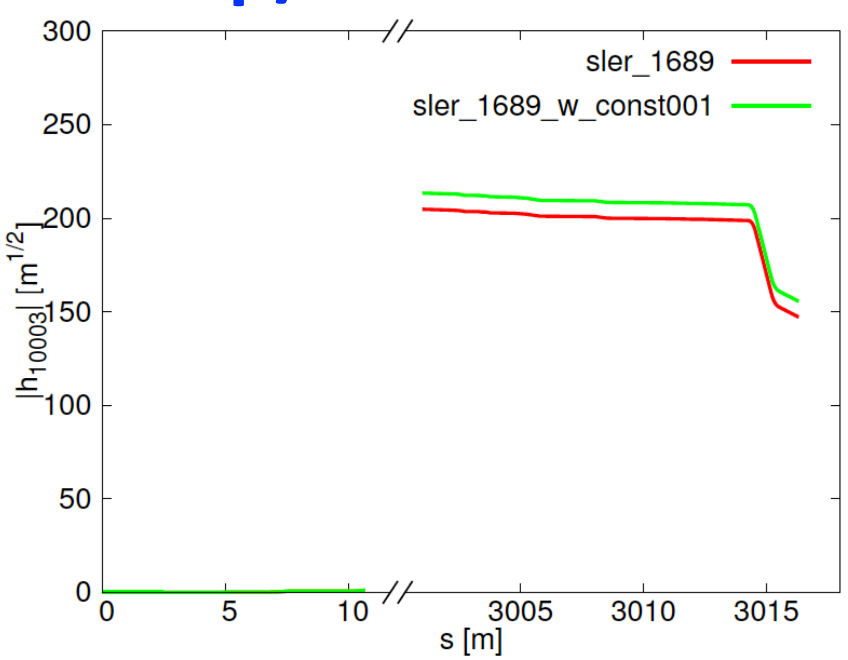
$d\eta_x/d\delta$



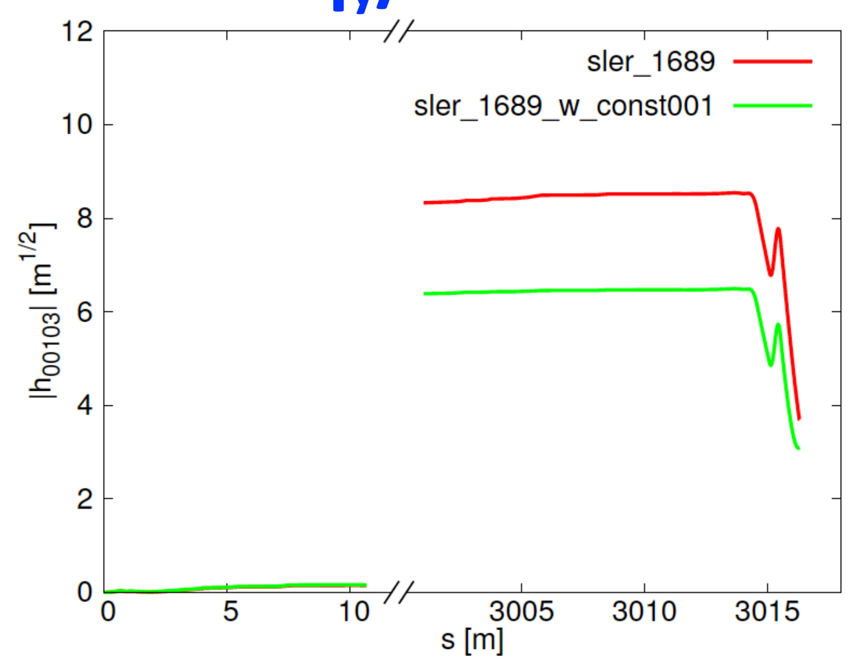
$d\eta_y/d\delta$



$d^2\eta_x/d\delta^2$



$d^2\eta_y/d\delta^2$

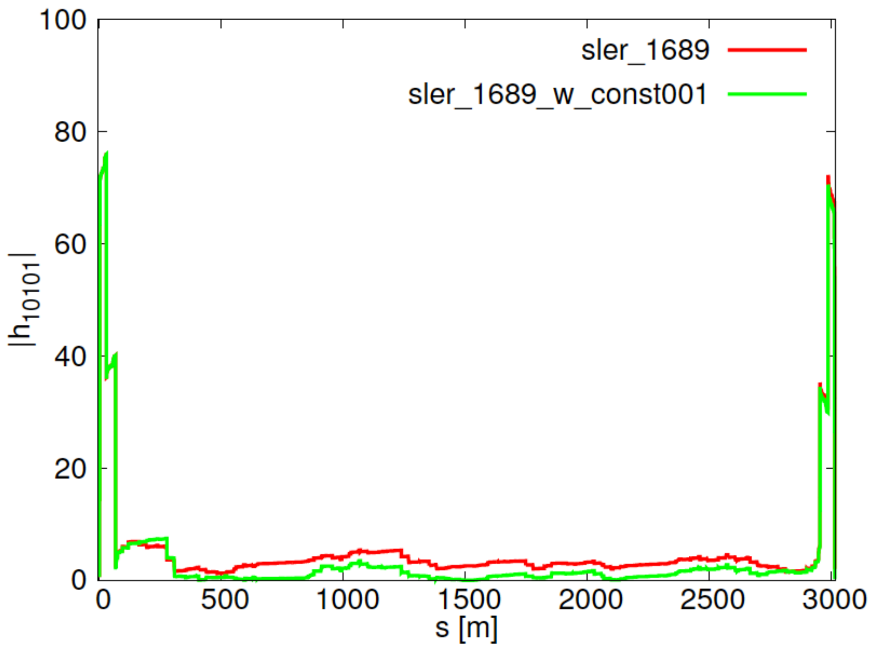


3. Results by PTC: Chromatic coupling

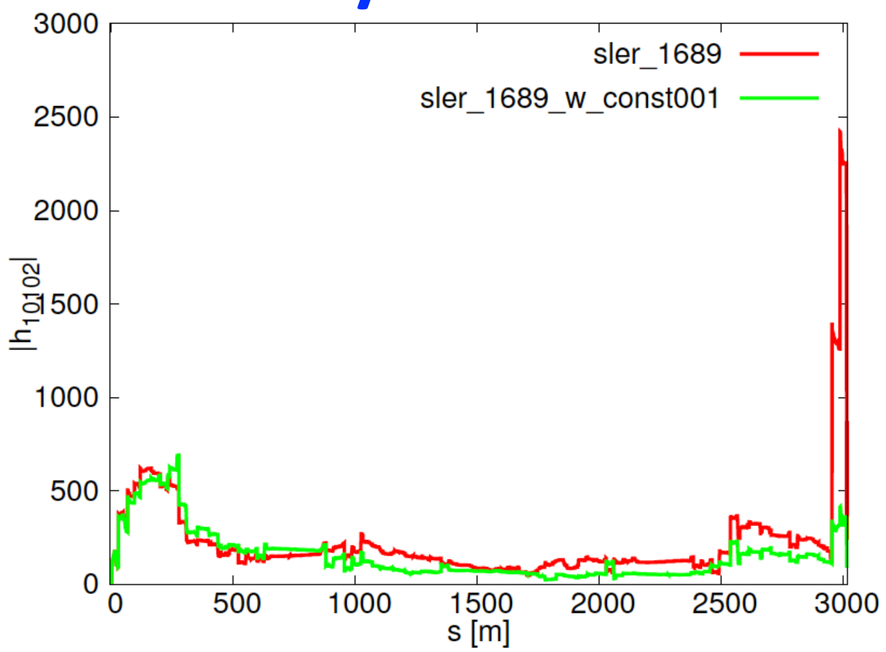
➤ **Chromatic coupling along the whole ring**

- **w/ constraints: Chromatic coupling controlled**

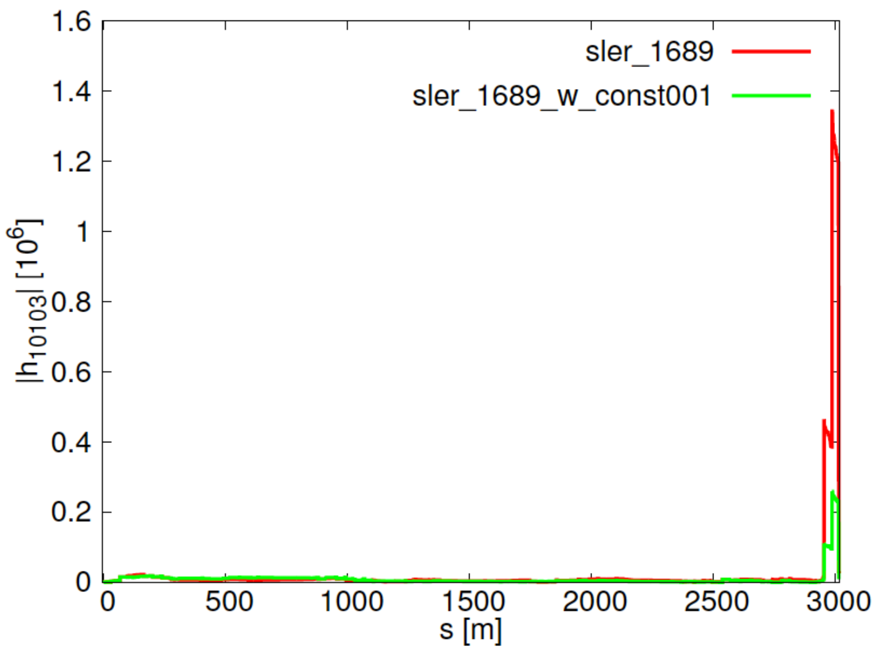
dR/dδ



d²R/dδ²



d³R/dδ³

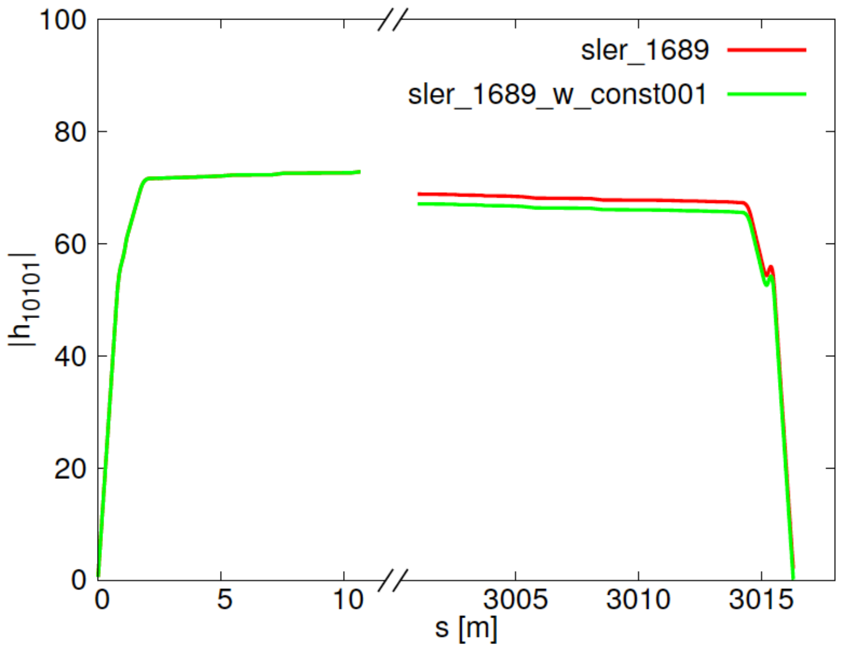


3. Results by PTC: Chromatic coupling (IR)

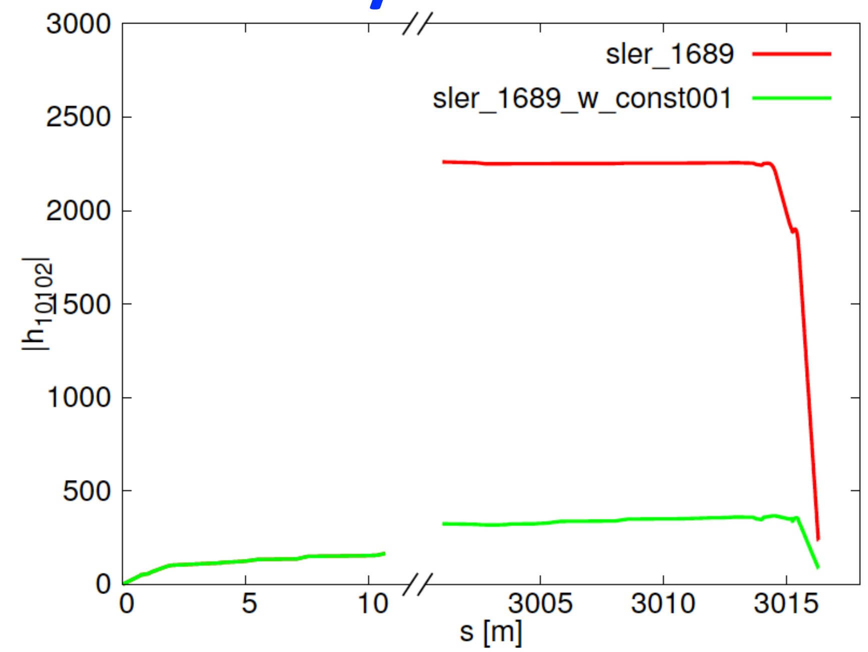
➤ **Chromatic coupling along the whole ring**

- **w/ constraints: Chromatic coupling controlled**

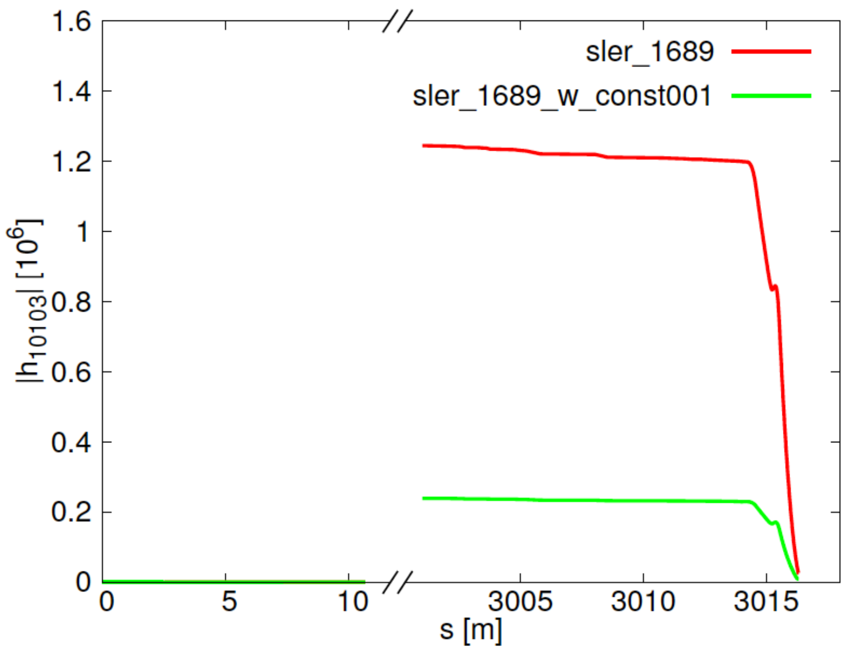
dR/dδ



$d^2R/d\delta^2$



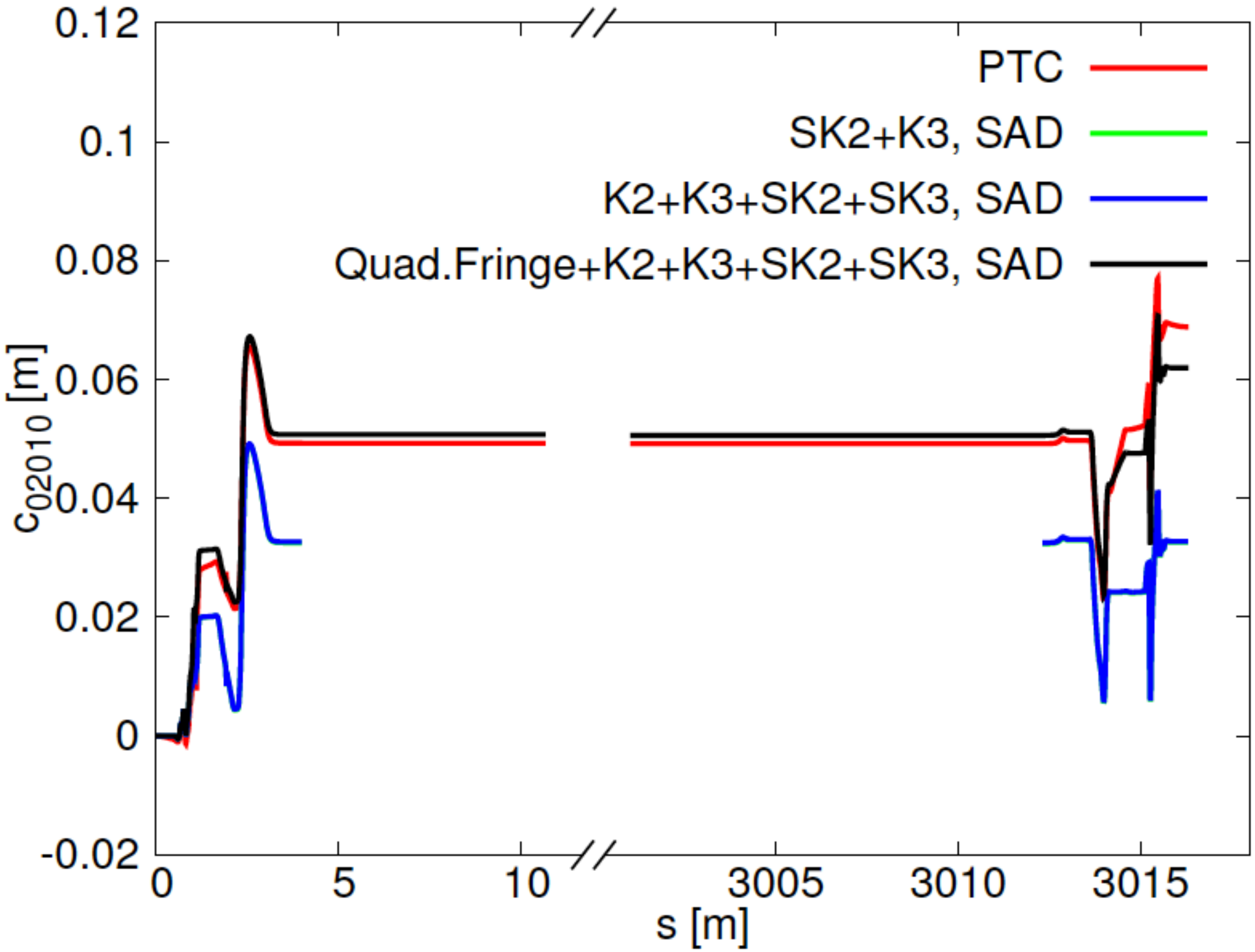
$d^3R/d\delta^3$



4. Compare with SAD

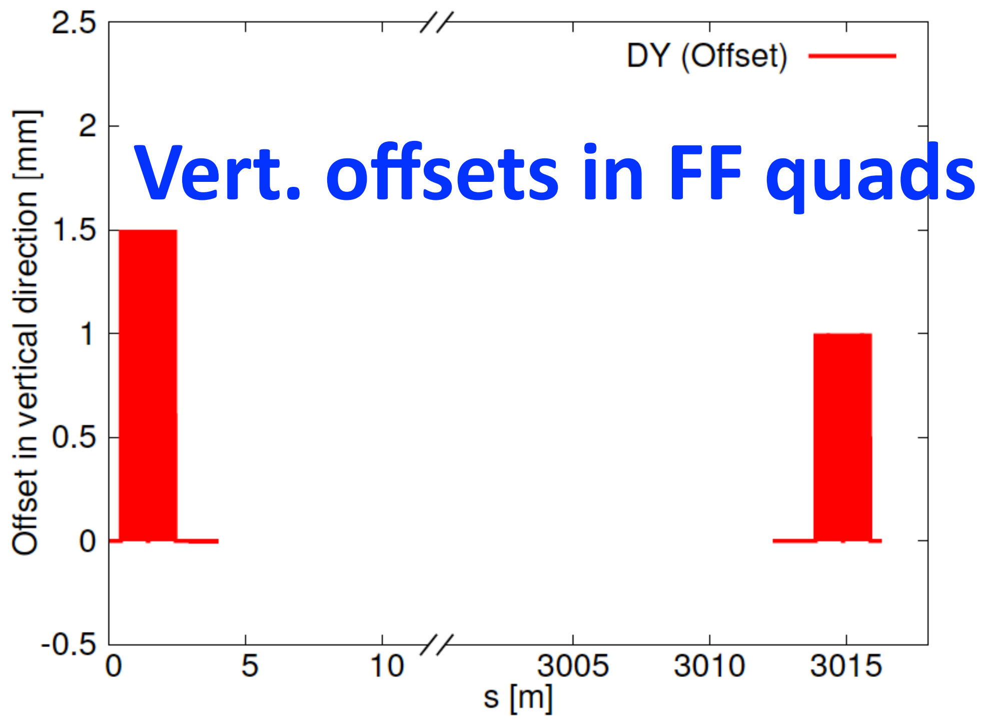
➤ **$p_x^2 p_y$ term**

- **Hard-edge fringe fields of final focus quads are important sources**



4. Compare with SAD

- ➤ $p_x^2 p_y$ term
  - **How quad. hard-edge fringes contribute?**



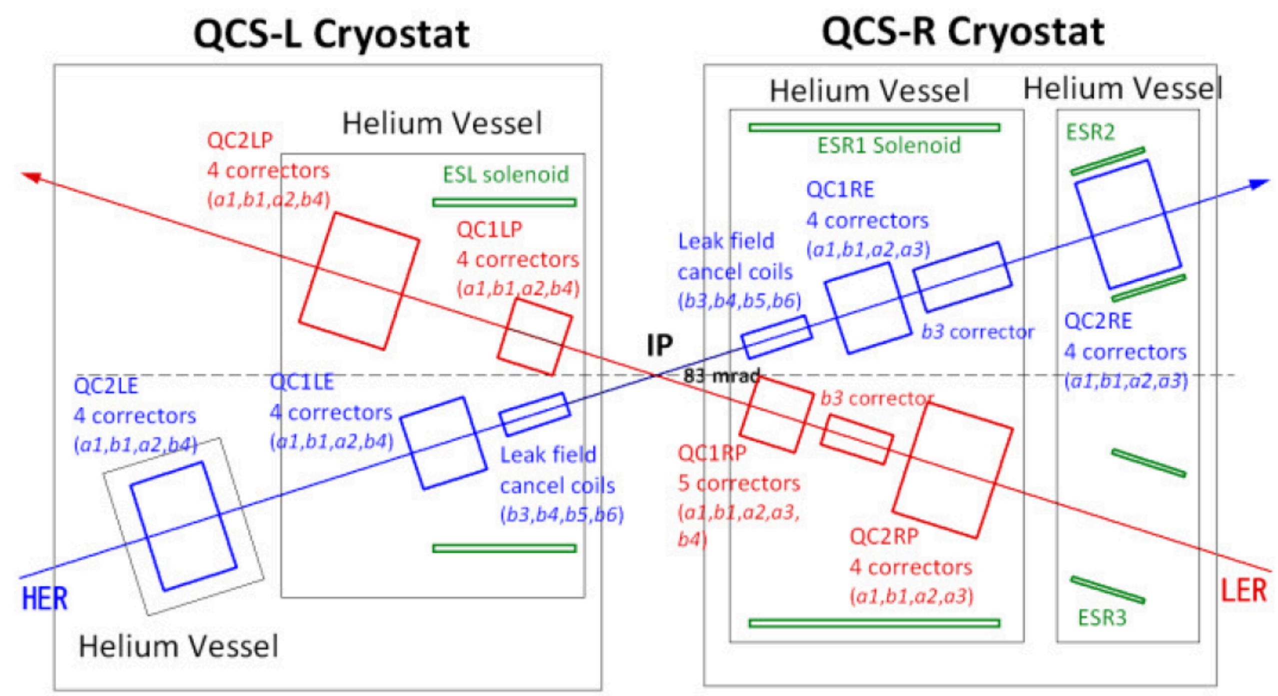
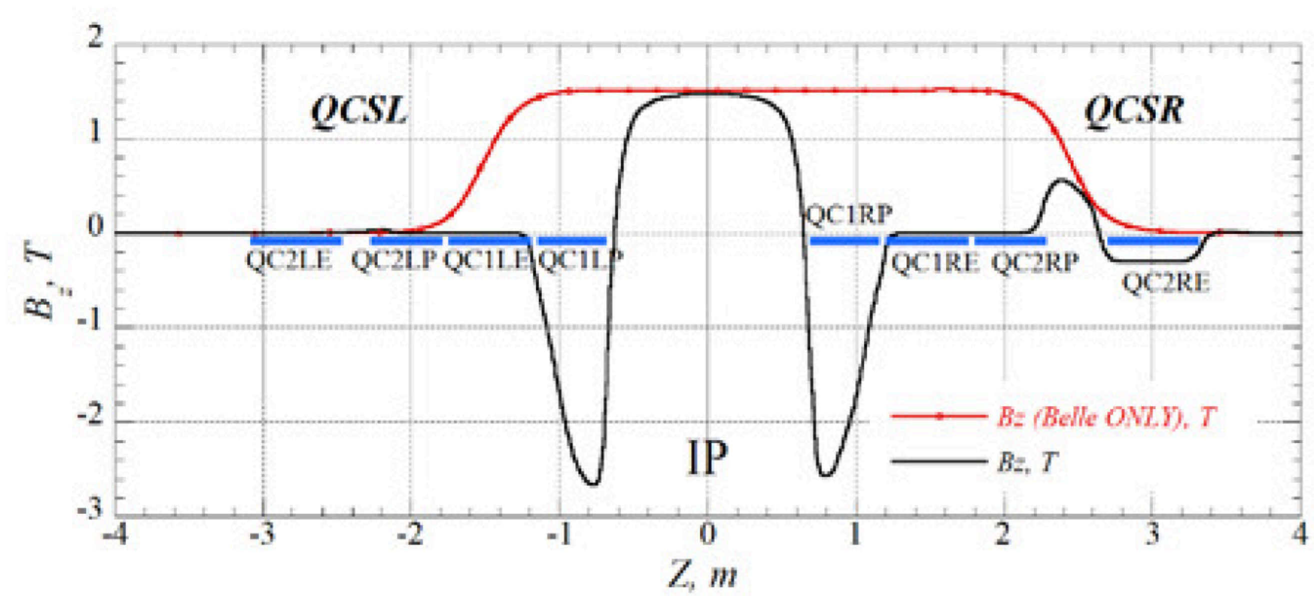


Table 2: Shift Amount of Magnet Axis

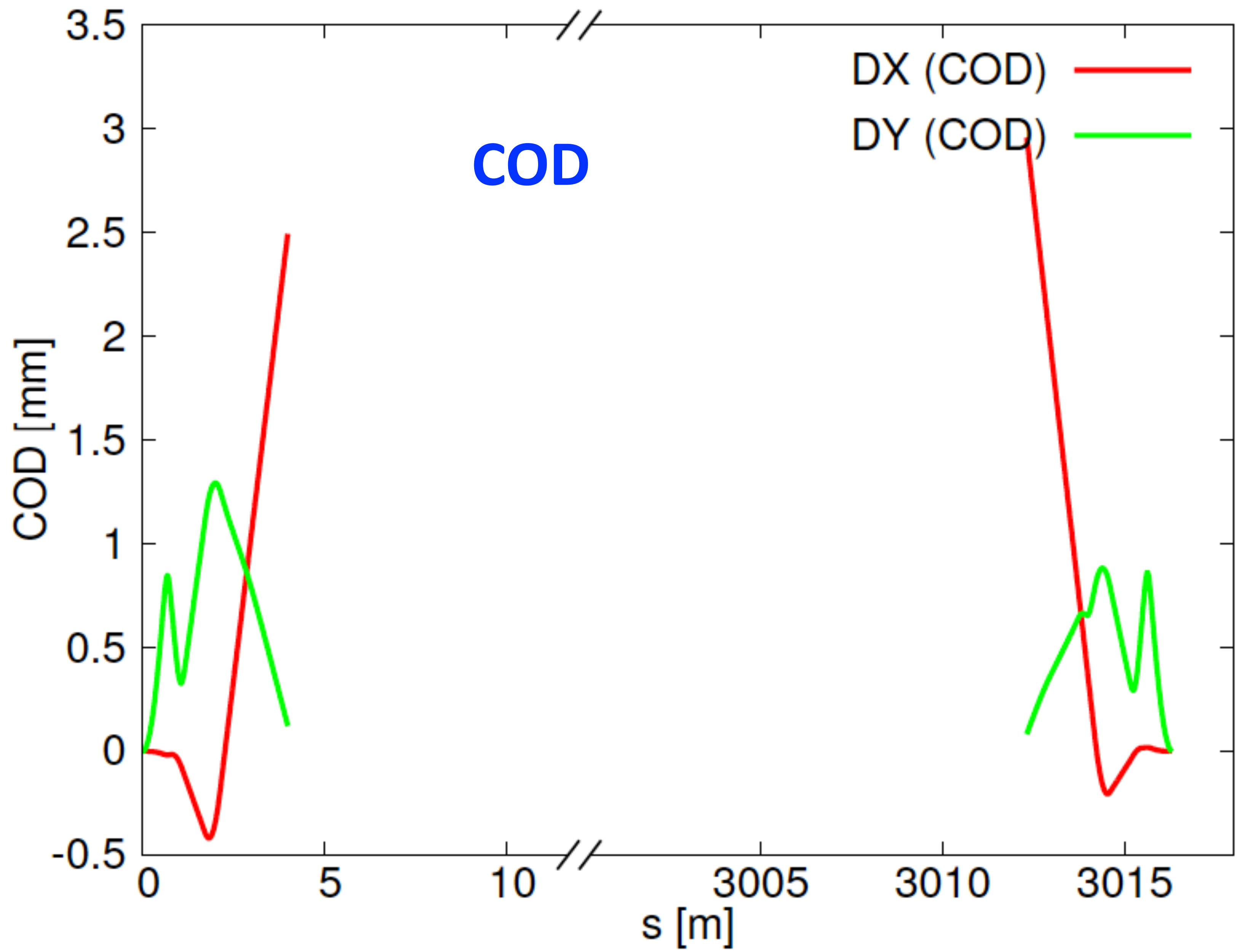
Magnet	ΔY	Magnet	ΔX
QC1RP	-1.0 mm	QC1RE	-0.7 mm
QC2RP	-1.0 mm	QC2RE	-0.7 mm
QC1LP	-1.5 mm	QC1LE	+0.7 mm
QC2LP	-1.5 mm	QC2LE	+0.7 mm



N. Ohuchi et al., IPAC'13

4. Compare with SAD

- **$p_x^2 p_y$ term**
  - **How quad. hard-edge fringes contribute?**



4. Compare with SAD

- $p_x^2 p_y$ term
  - How quad. hard-edge fringes contribute?
    + Magnet offsets + COD => 3rd geometric terms

```
In[1]:= (*f1=K1/(12(1+δ)L)*)
        HQfr = f1*((x^3 + 3 x*y^2) px - (y^3 + 3 x^2 y) py);
        D[HQfr, x]*ΔX
        D[HQfr, px]*ΔPX
        D[HQfr, y]*ΔY
        D[HQfr, py]*ΔPY
```

Out[2]= $f1 \left(-6\,py\,x\,y + px\left(3x^2 + 3y^2\right)\right)\Delta X$

Out[3]= $f1 \left(x^3 + 3\,x\,y^2\right)\Delta PX$

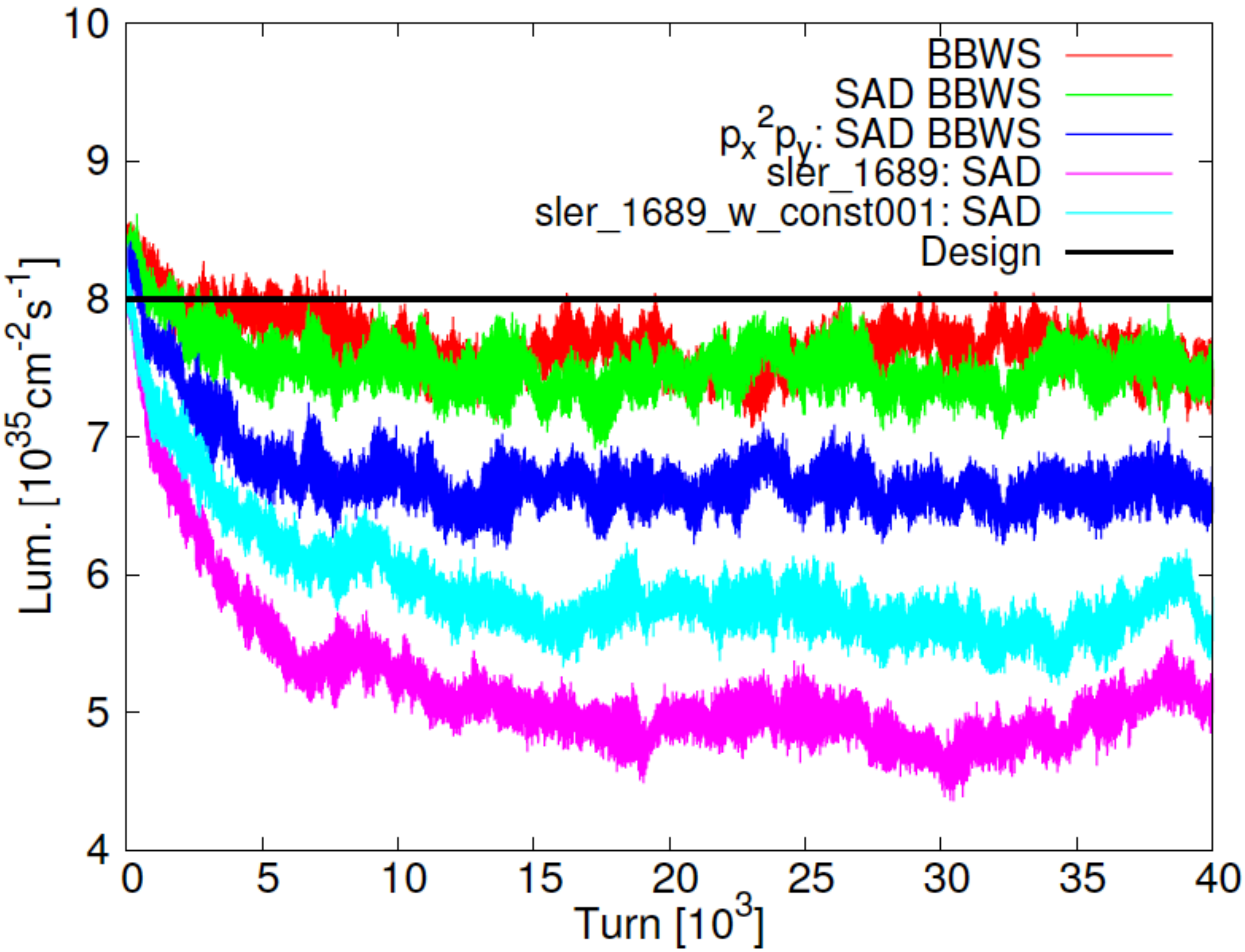
Out[4]= $f1 \left(6\,px\,x\,y - py\left(3x^2 + 3y^2\right)\right)\Delta Y$

Out[5]= $f1 \left(-3x^2 y - y^3\right)\Delta PY$

4. Compare with SAD

➤ Luminosity calculations

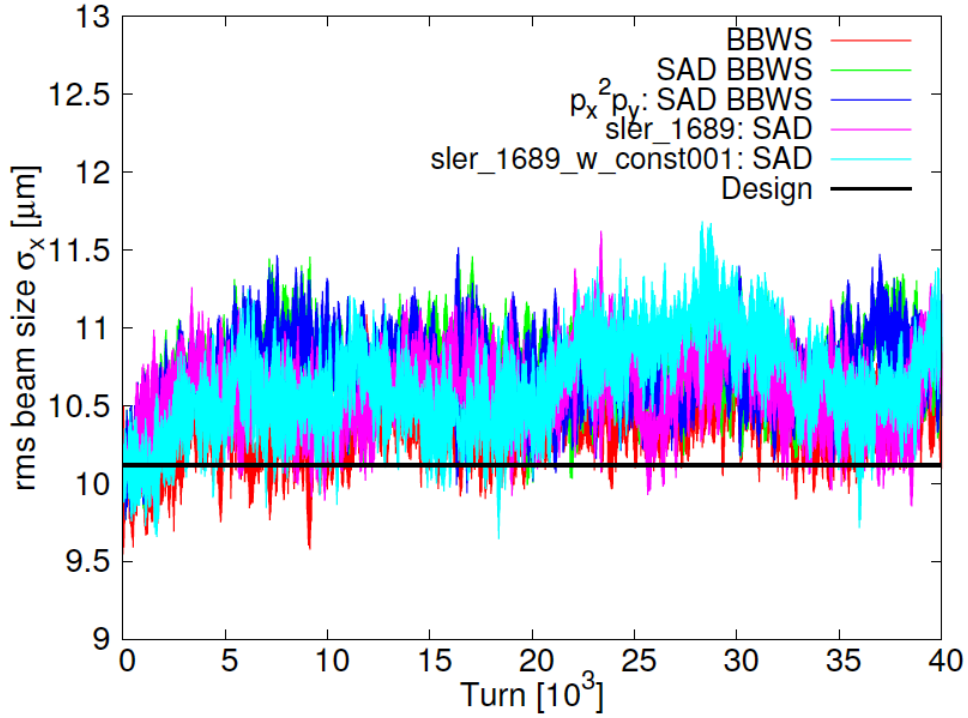
- ~1/3 caused by chromatic effects
- ~1/3 caused by $p_x^2 py$ term (from FFS, strength calculated by PTC)
- ~1/3 unknown sources of lum. loss

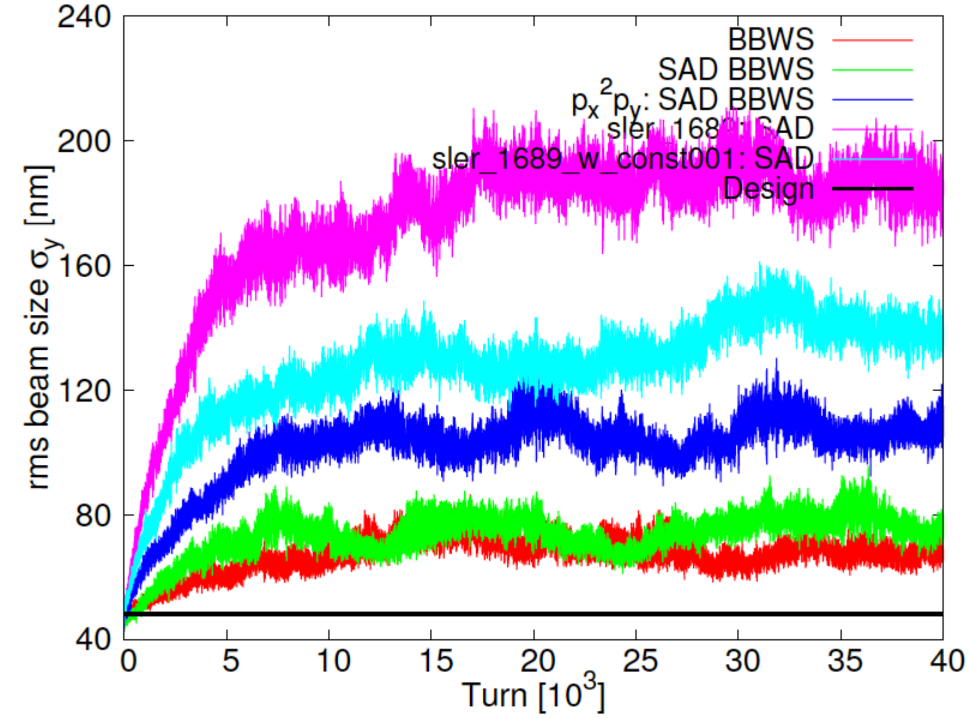


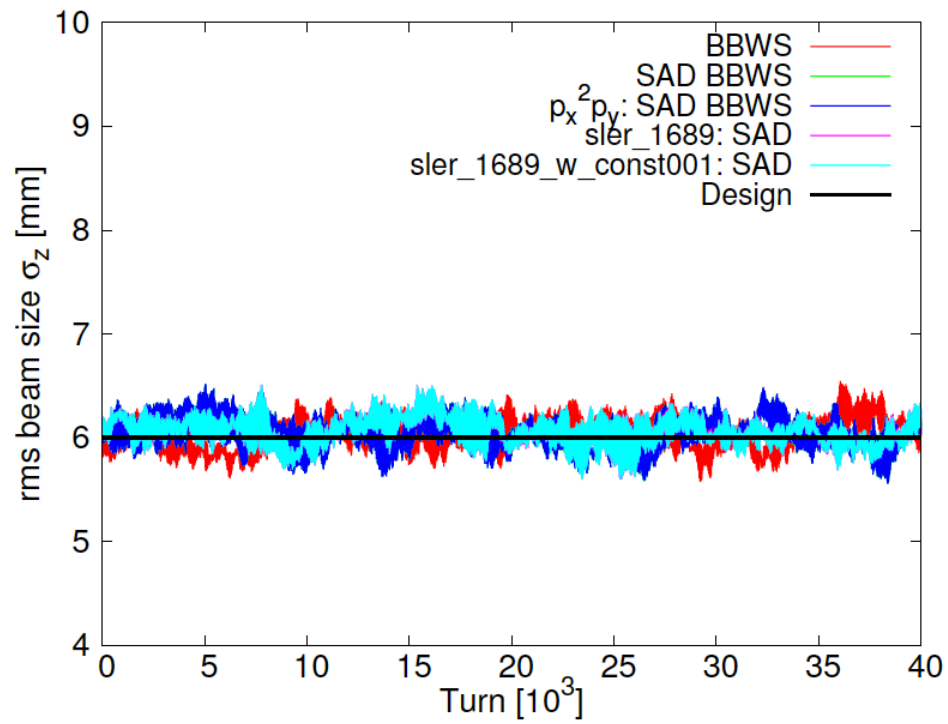
4. Compare with SAD

➤ Luminosity calculations

- **Luminosity is sensitive to vertical beam size**







5. Summary

- **Previous findings**
  - BB + Lattice nonlinearity cause luminosity loss in SuperKEKB
  - Lum. drop happens at low beam current
  - Related to amplitude-dependent latt. nonlin.
- **DA optimization w/ new constraints [by H. Sugimoto]**
  - Small loss of DA and lifetime (reasonable)
  - Nonlinearity in chromatic beta, alpha, tune, and coupling functions [related to RDTs] suppressed successfully
  - Lum. gain achieved at low current
- **Calculation of RDTs using PTC**
  - Suppression of chromatic RDTs observed
- **Compare PTC and SAD in nonlinear terms (3rd order)**
  - Good agreement
  - Source (almost) well understood

5. Summary

- ➤ **Luminosity calculation**
  - Sources of luminosity loss (almost) well understood
  - Calculations for latest lattices to be done
- ➤ **Nonlinear optimization scheme**
  - Use the knowledge of PTC and SAD calculation
  - Use available correctors for correction
  - Consider strategy of simultaneous optimization of DA and luminosity