Comparison of resonance driving terms for SuperKEKB before and after optimizations

Demin Zhou

Acknowledgements:

H. Sugimoto, K. Oide, E. Forest, D. Sagan (Cornell)

SuperKEKB mini-optics meeting, Dec. 08, 2016

Outline

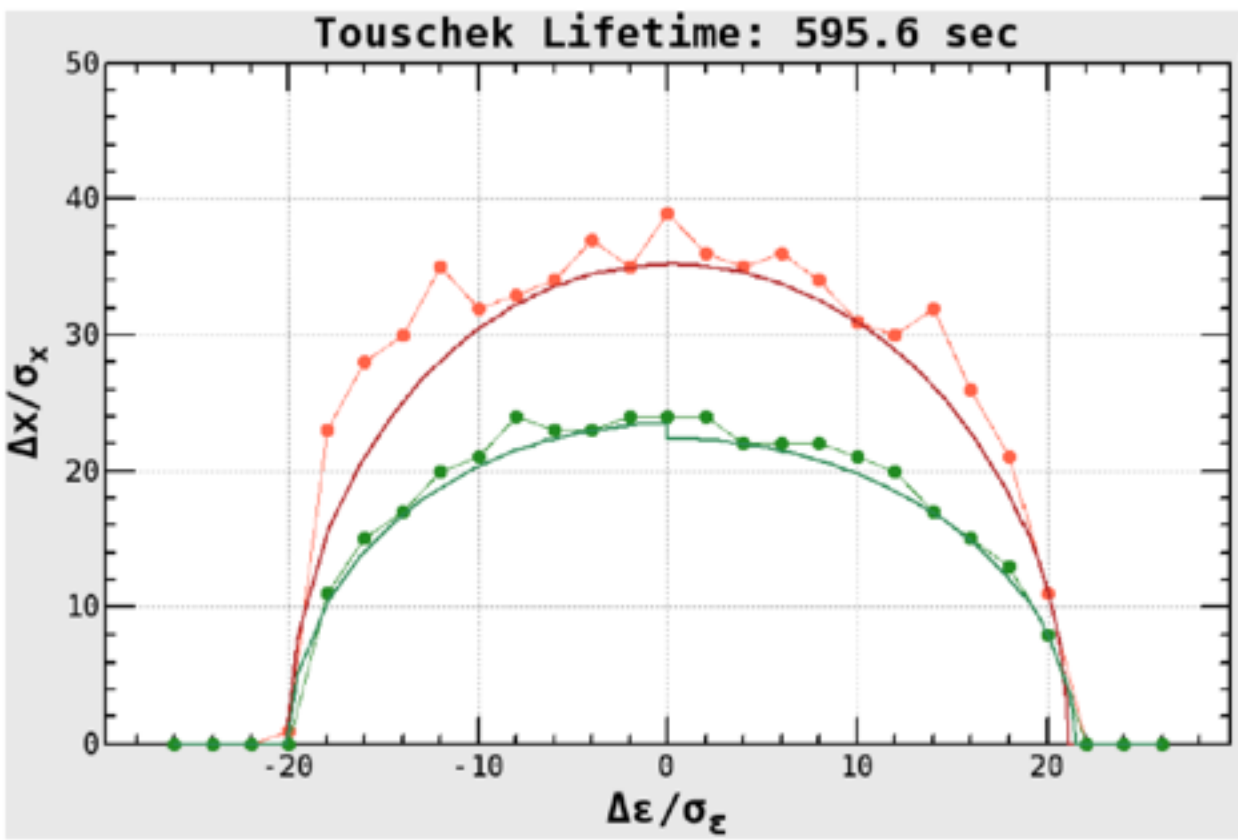
- **Introduction**
  - **Lattice:**
    sler_1689.sad
    sler_1689_w_const001.sad (provided by H. Sugimoto)
- **Theory for resonance driving terms**
- **Results by PTC**
  - **3rd and 4th order RDTs**
  - **Momentum-dependent RDTs**
- **Summary**

1. Introduction

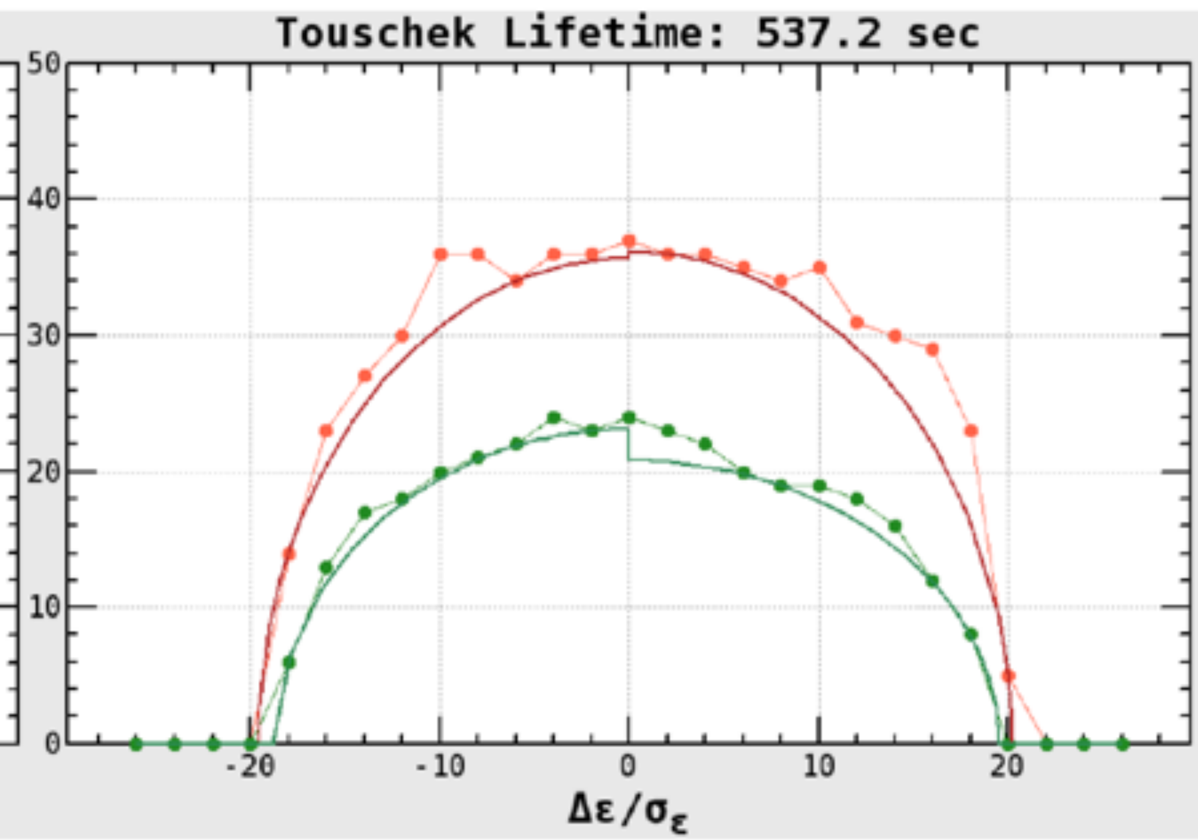
➤ **Nonlinear optimization with new constraints [by H. Sugimoto, SuperKEKB mini-optics meeting, Sep. 8, 2016]**

● **It's natural to have smaller DA using additional constraints but with the same number of variables**

sler_1689 W/O constraints



sler_1689 W/ constraints

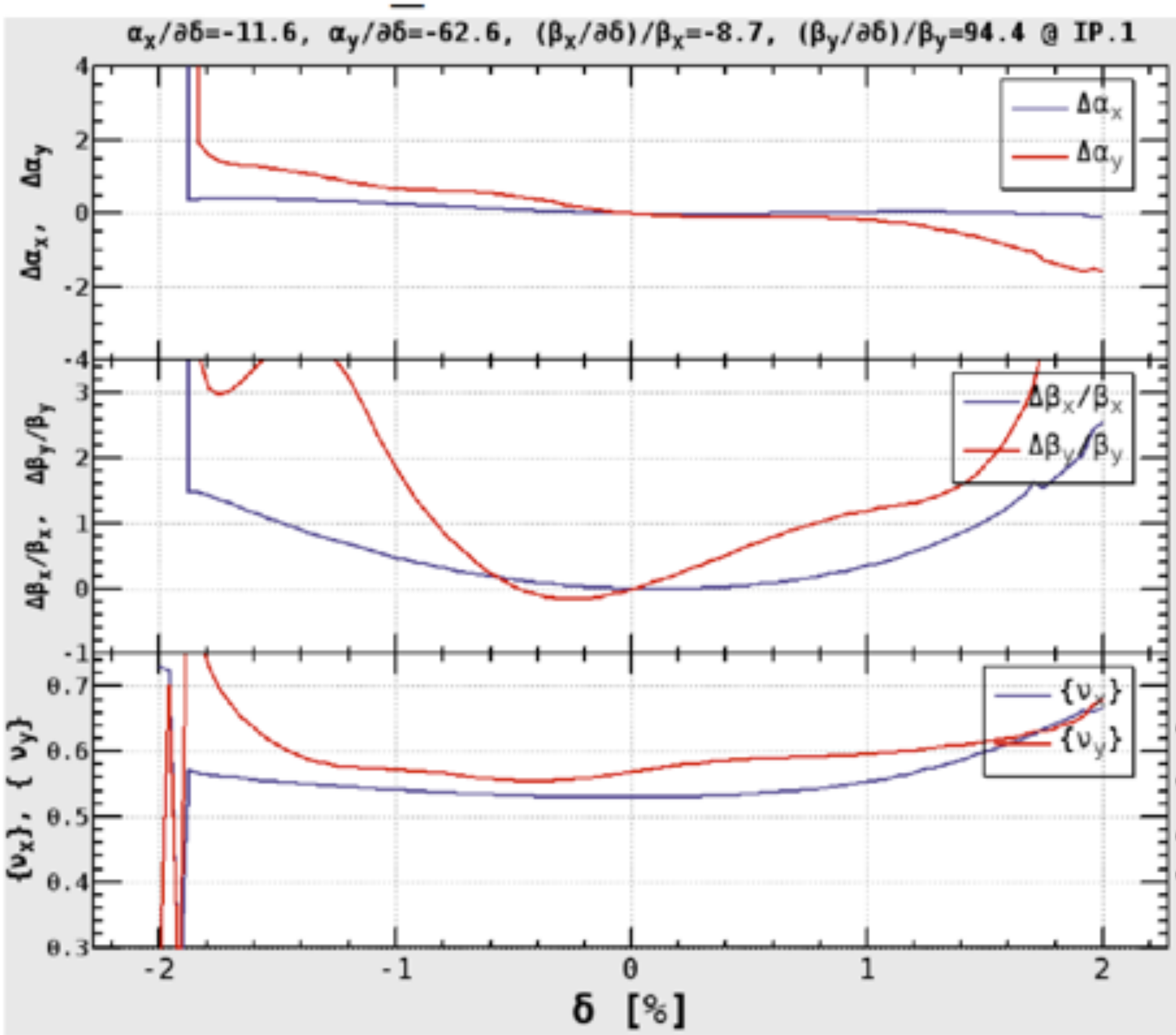


1. Introduction

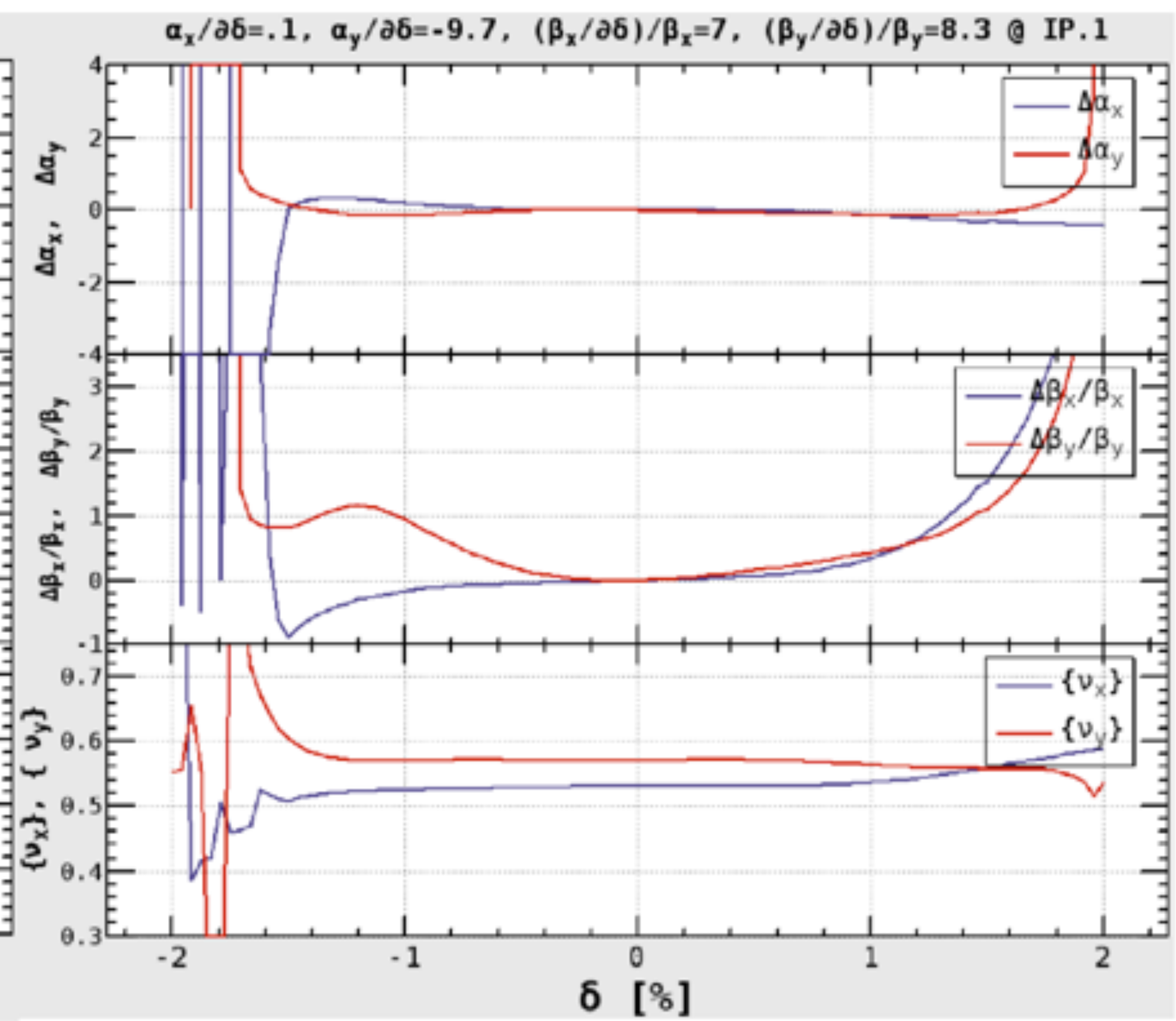
➤ Nonlinear optimization with new constraints [by H. Sugimoto, SuperKEKB mini-optics meeting, Sep. 8, 2016]

- **Chromatic $\beta_{x,y}$ and $\nu_{x,y}$ correspond to RDTs of h_{2000e}/h_{0200e} (X), h_{0020e}/h_{0002e} (Y), and h_{1100e} (X), h_{0011e} (Y), respectively**

sler_1689 W/O constraints



sler_1689 W/ constraints

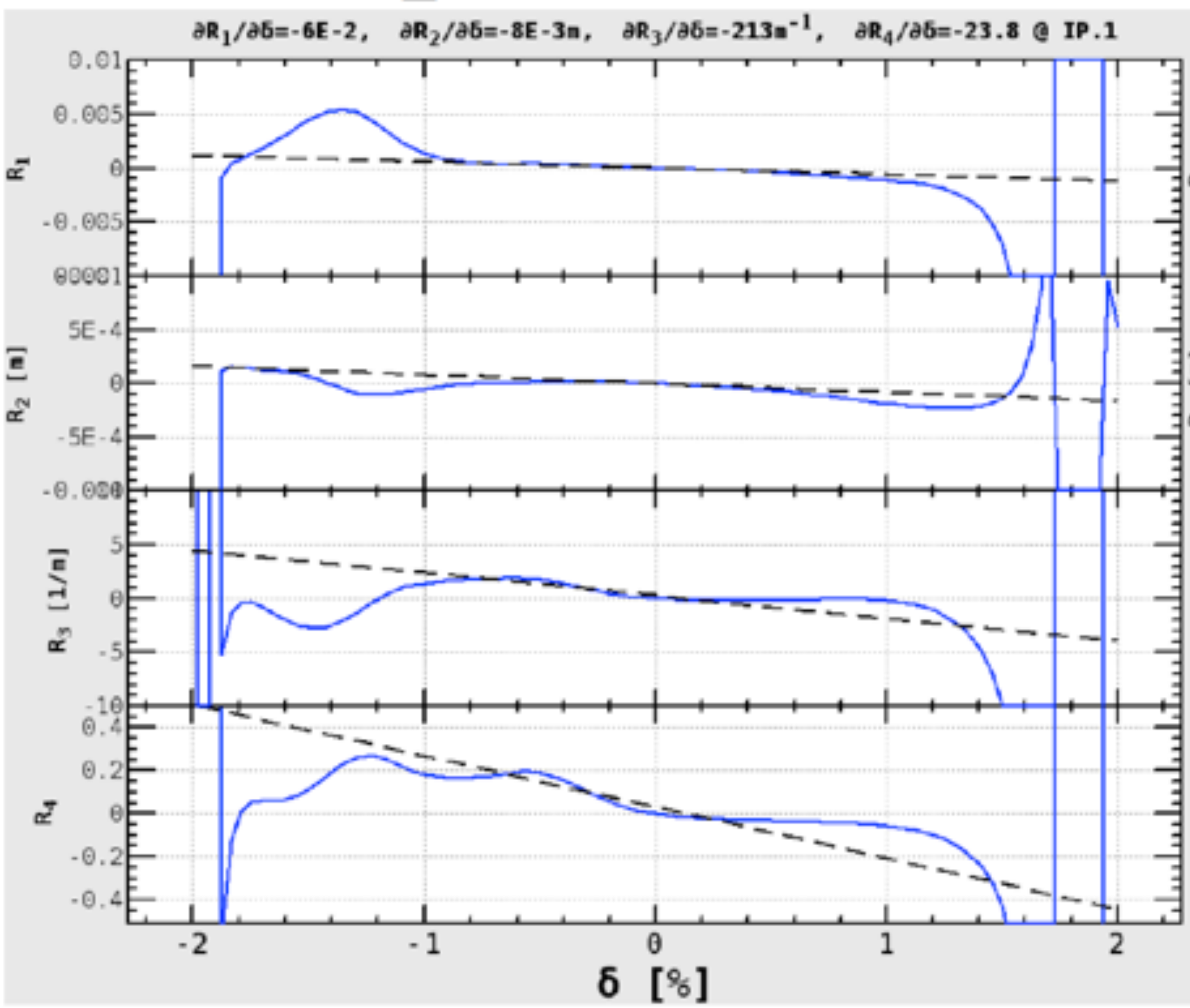


1. Introduction

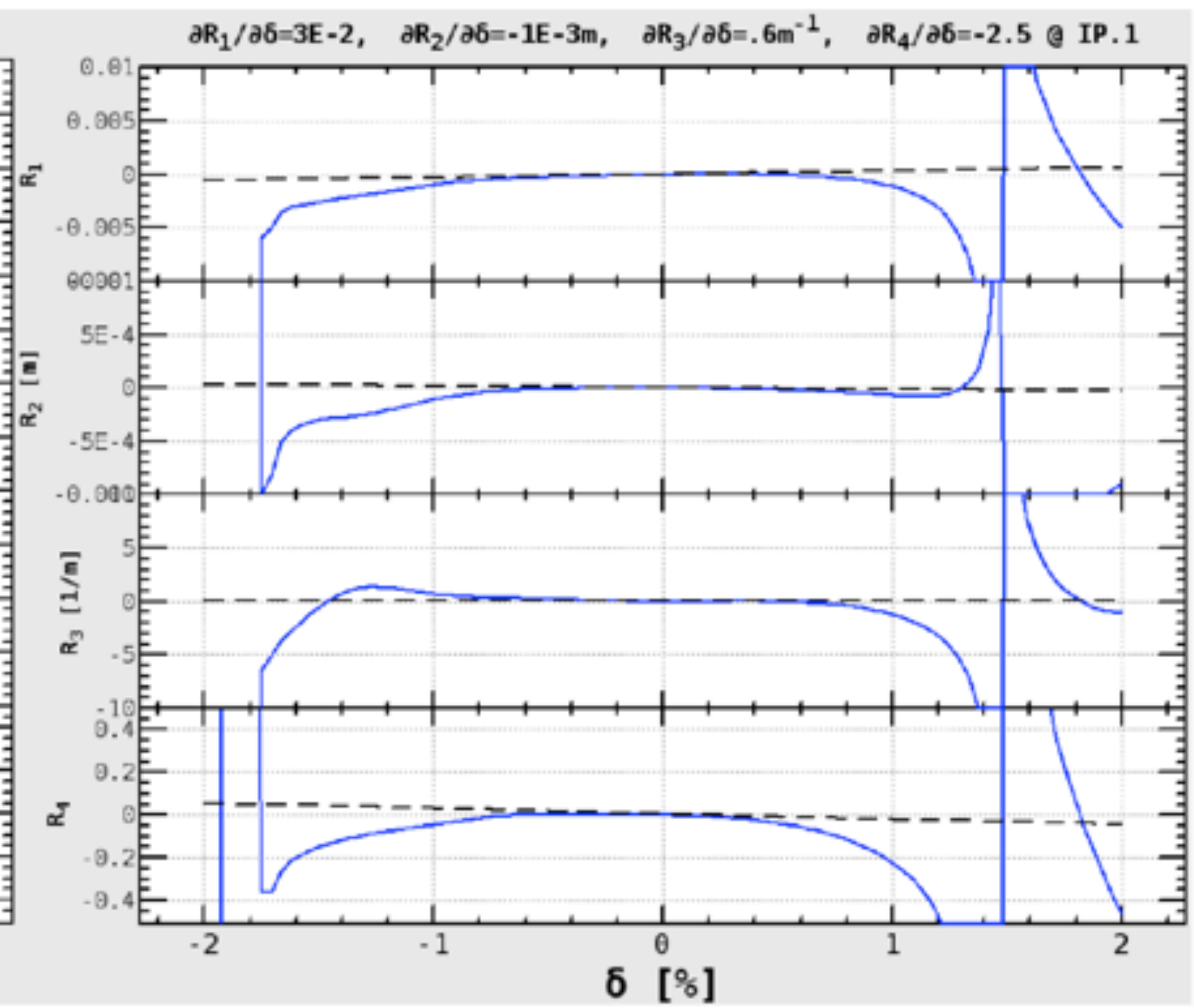
➤ **Nonlinear optimization with new constraints [by H. Sugimoto, SuperKEKB mini-optics meeting, Sep. 8, 2016]**

- **Chromatic X-Y couplings correspond to RDTs of h_{1010e}, h_{0110e}, h_{1001e} and h_{0101e}**

sler_1689 W/O constraints



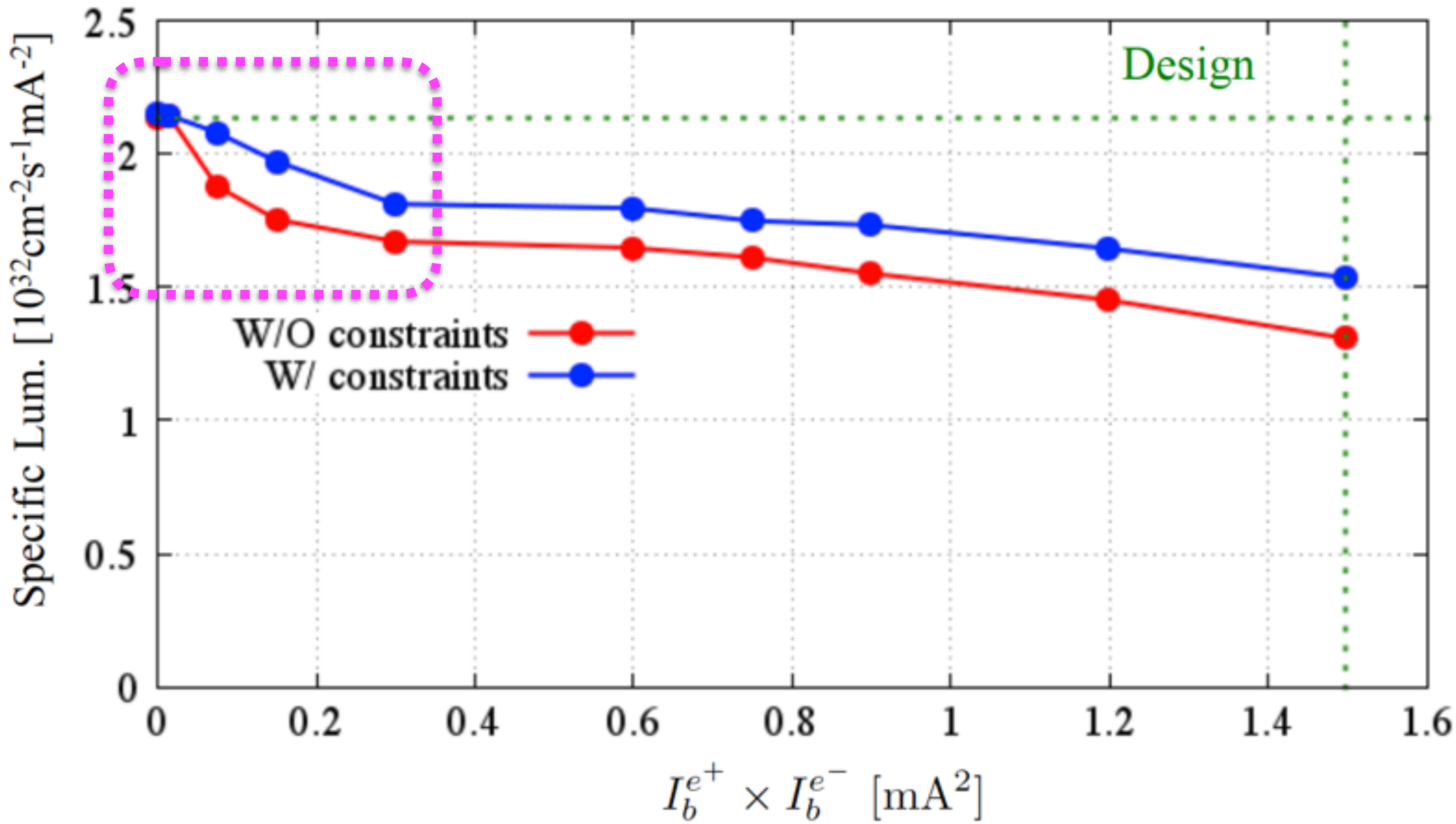
sler_1689 W/ constraints



1. Introduction

➤ **Nonlinear optimization with new constraints [by H. Sugimoto, SuperKEKB mini-optics meeting, Sep. 8, 2016]**

- **Question: Recovery of luminosity is attributed to chromatic effects [because of new constraints]? => Check all RDTs …**



1. Introduction

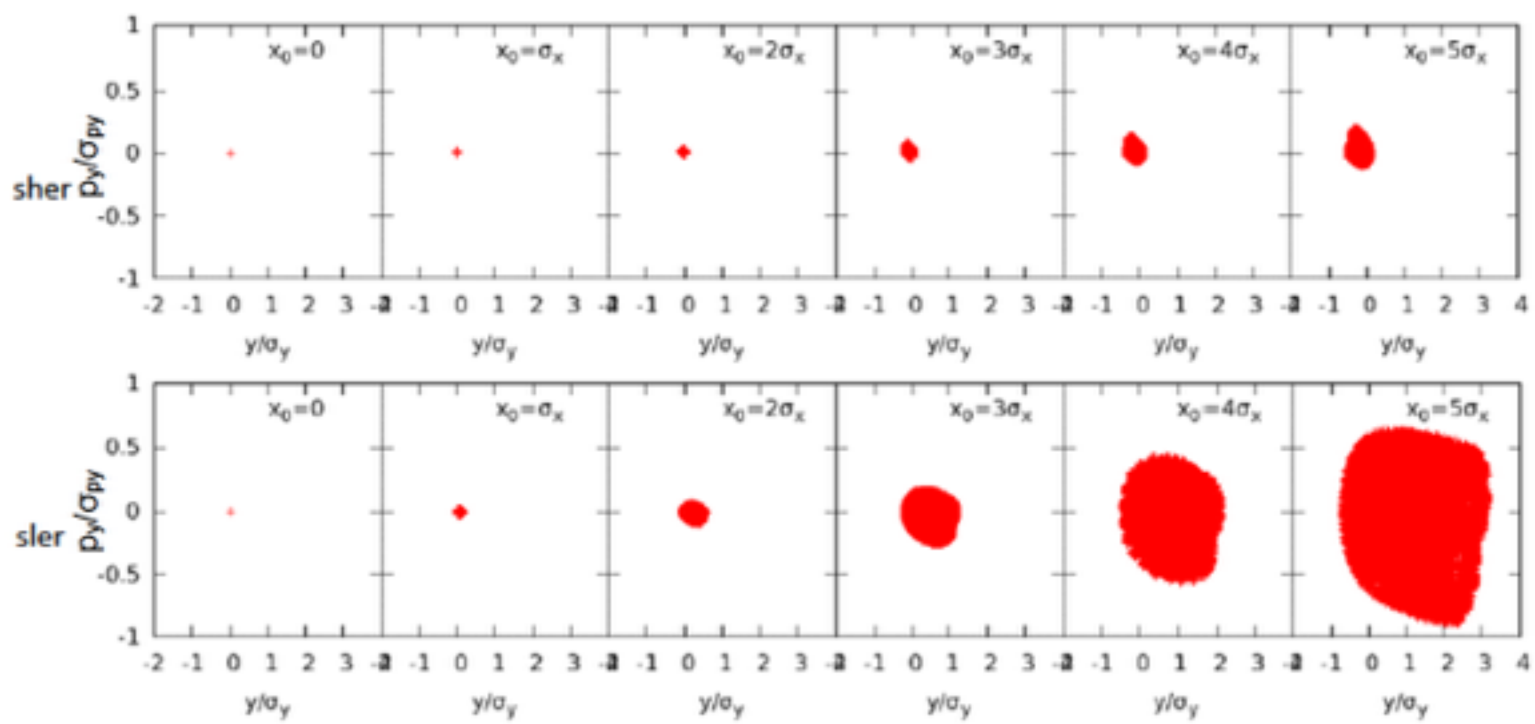
➤ Previous findings

- D. Zhou, 20th KEKB Accelerator Review Committee, Feb. 23, 2015.

2. BB+LN: Nonlin. X-Y coupling

➤ Realistic lattice
➤ Poincare map in y direction as function of X offset
➤ Strong nonlinear X-Y coupling in LER

sher-5767 vs ler-1689 in Y direction



From Y. Zhang

1. Introduction

➤ Previous findings

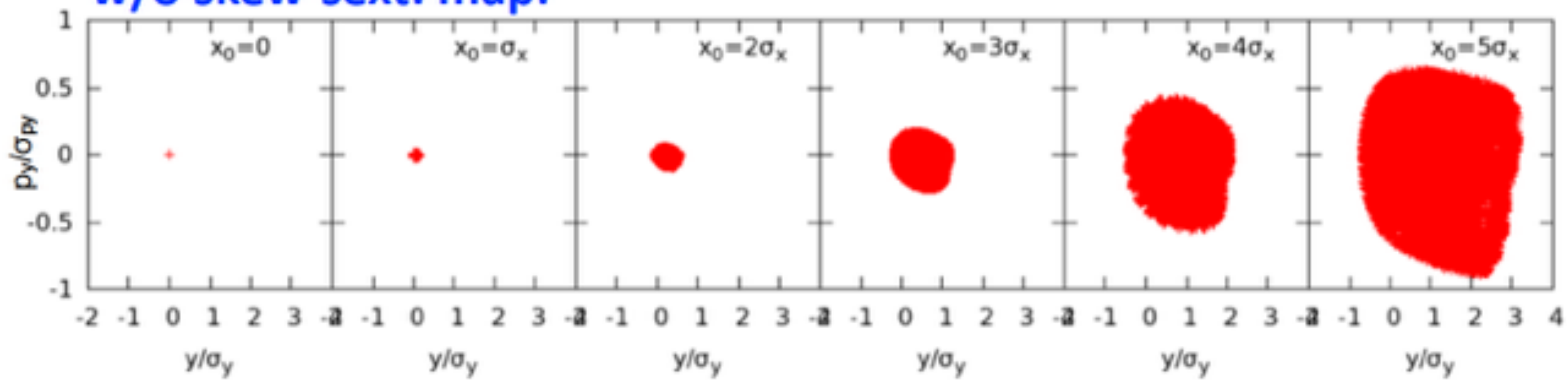
- D. Zhou, 20th KEKB Accelerator Review Committee, Feb. 23, 2015.

2. BB+LN: Nonlin. X-Y coupling

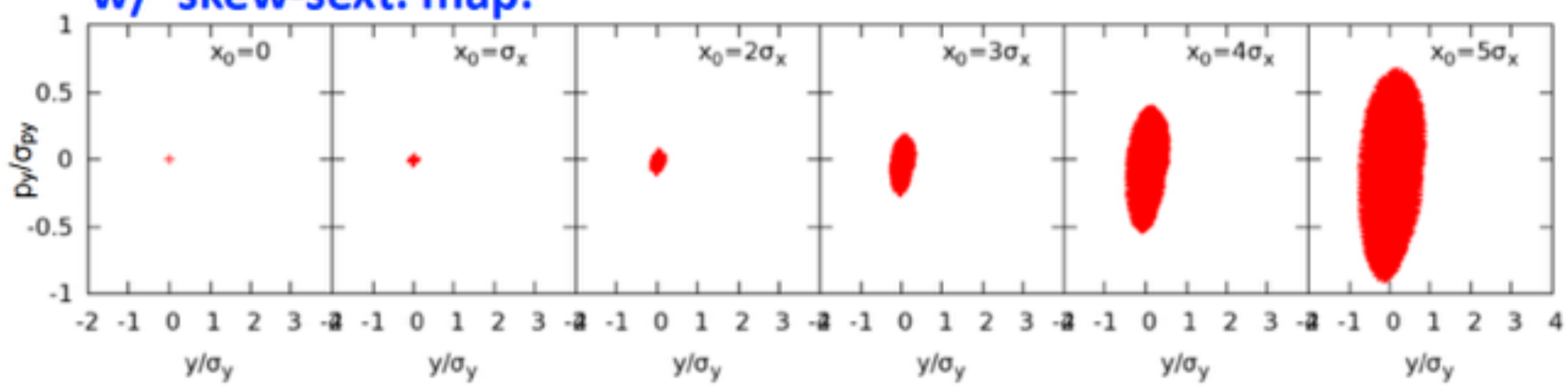
➤ Test by inserting a map of $H=K*x^2y$ into the LER lattice

➤ COD and oscillation amplitude in y are well suppressed as expected

w/o skew-sext. map:



w/ skew-sext. map:



From Y. Zhang

1. Introduction

➤ **Previous findings**

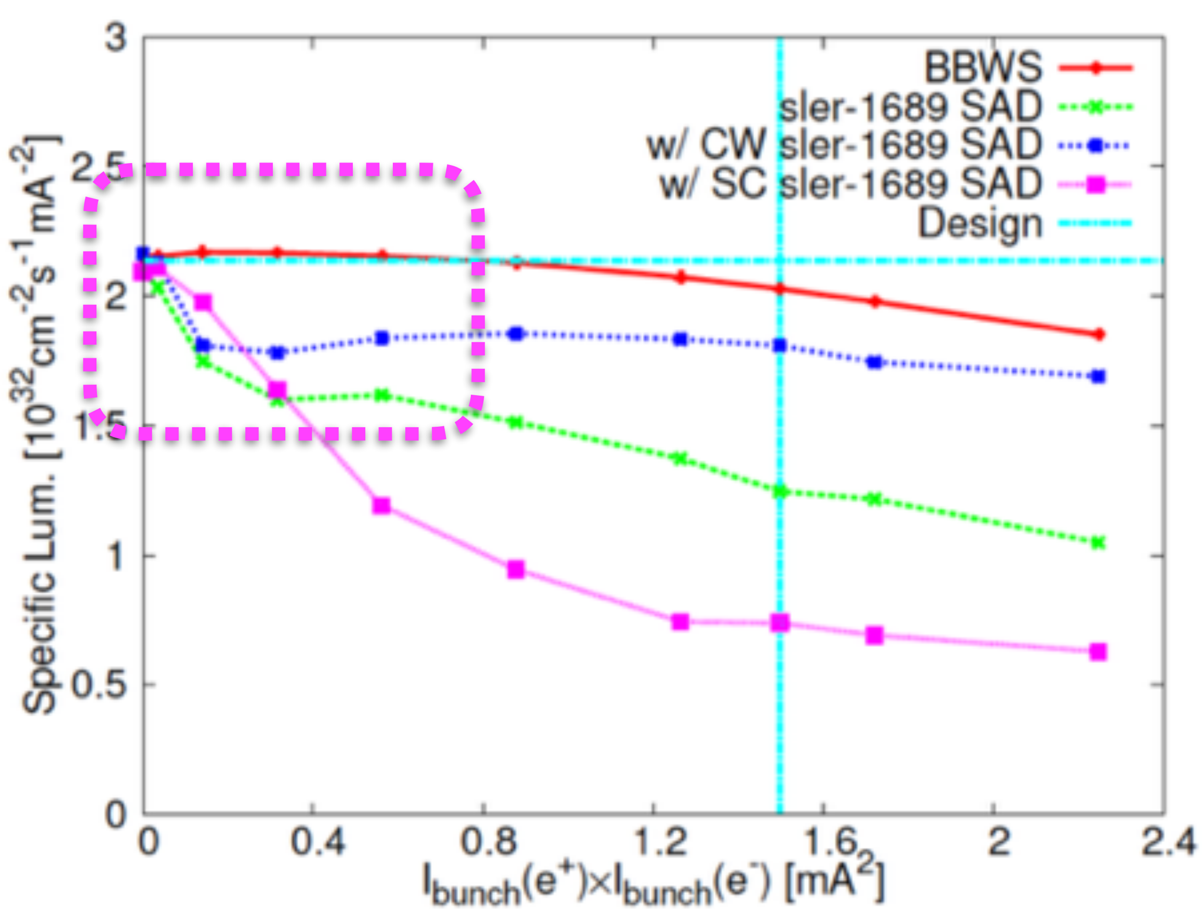
- **D. Zhou, 20th KEKB Accelerator Review Committee, Feb. 23, 2015.**

2. BB+LN: Luminosity: LER

➤ **Realistic lattice: lum. drops at low beam currents**

➤ **Crab-waist:**

- **To cancel beam-beam driven resonances**
- **Work well at high currents, but not well at low currents**



2. Theory for RDTs

➤ Resonance driving terms (RDTs) indicate lattice nonlinearity

The effective Hamiltonian of a ring can be normalized in resonance bases [Ref. E. Forest, *Beam Dynamics – A New Attitude and Framework*, 1998].

For a ring with n elements, one can normalize the one turn map $\mathcal{M}_{1\to n}$ as [Ref. L. Yang *et al.*, Phys. Rev. ST Accel. Beams 14, 054001 (2011)]

$$\mathcal{M}_{1\to n} = \mathcal{A}_1^{-1} e^{:h:} \mathcal{R}_{1\to n} \mathcal{A}_1,$$

with $\mathcal{R}$: rotation, $e^{:h:}$: nonlinear Lie map, $\mathcal{A}_1$: normalizing map. Assume no coupling (the theory can be generalized for nonzero coupling), $\mathcal{A}_i$ in x plane at the ith element can be approximated in perturbation theory as

$$\mathcal{A}_i x = \sqrt{\beta_{x,i}}\, x + \eta_{x,i}\delta,$$

$$\mathcal{A}_i p_x = \frac{-\alpha_{x,i} x + p_x}{\sqrt{\beta_{x,i}}} + \eta'_{x,i}\delta.$$

2. Theory for RDTs

➤ RDTs indicate lattice nonlinearity

In the resonance basis, using action-angle variables (J, ϕ) one can write

$$h_x^{\pm} \equiv \sqrt{J_x} e^{\pm i\phi_x} = \frac{X \mp iP_x}{\sqrt{2}},$$

$$\mathcal{R}_{i\to j} h_x^{\pm} = \mathcal{R}_{i\to j} \sqrt{J_x} e^{\pm i\phi_x} = e^{\pm i\mu_{i\to j,x}} h_x^{\pm},$$

where $\mu_{i\to j,x}$ is the phase advance of $i \to j$. Consequently, the potential of a multipole magnetic field can be expanded in the resonance bases of h_{abcde} as

$$h = \sum h_{abcde} h_x^{+a} h_x^{-b} h_y^{+c} h_y^{-d} \delta^e.$$

Each h_{abcde} (a complex number in general) drives a certain resonance, and is an explicit function of magnet strengths, beta functions and dispersions.

2. Theory for RDTs

➤ RDTs indicate lattice nonlinearity

The effective Hamiltonian corresponding to chromaticity is

$$h_c = \sum h_{1100e} h_x^{+1} h_x^{-1} \delta^e + \sum h_{0011e} h_y^{+1} h_y^{-1} \delta^e,$$

$$h_c = J_x \sum h_{1100e} \delta^e + J_y \sum h_{0011e} \delta^e.$$

Then the tunes are calculated as

$$\nu_x = -\frac{1}{2\pi} \frac{\partial h_c}{\partial J_x} = -\frac{1}{2\pi} \sum h_{1100e} \delta^e,$$

$$\nu_y = -\frac{1}{2\pi} \frac{\partial h_c}{\partial J_y} = -\frac{1}{2\pi} \sum h_{0011e} \delta^e.$$

Therefore the RDTs of h_{1100e} and h_{0011e} correspond to linear and high-order chromaticity.

2. Theory for RDTs

➤ **RDTs indicate lattice nonlinearity**

h_{abcde}	Driving effects
h_{11001}, h_{00111}	Linear chromaticity ζ_x, ζ_y
$h_{21000}, h_{12000} \| h_{10110}, h_{01110}$	ν_x $[(J_x)^{3/2}] \| [(J_x)^{1/2}(J_y)]$
$h_{30000}, h_{03000} \| h_{00300}, h_{00030}$	$3\nu_x$ $[(J_x)^{3/2}] \| 3\nu_y$ $[(J_y)^{3/2}]$
$h_{10020}, h_{01200} \| h_{10200}, h_{01020}$	$\nu_x - 2\nu_y \| \nu_x + 2\nu_y$ $[(J_x)^{1/2}(J_y)]$
$h_{20010}, h_{02100} \| h_{20100}, h_{02010}$	$2\nu_x - \nu_y \| 2\nu_x + \nu_y$ $[(J_x)(J_y)^{1/2}]$
$h_{00210}, h_{00120} \| h_{11100}, h_{11010}$	ν_y $[(J_y)^{3/2}] \| [(J_x)(J_y)^{1/2}]$
$h_{22000}, h_{00220}, h_{11110}$	$d\nu_x/dJ_x, d\nu_y/dJ_y, d\nu_{x,y}/dJ_{y,x}$
$h_{40000}, h_{04000} \| h_{00400}, h_{00040}$	$4\nu_x$ $[(J_x)^2] \| 4\nu_y$ $[(J_y)^2]$
$h_{31000}, h_{13000} \| h_{20110}, h_{02110}$	$2\nu_x$ $[(J_x)^2] \| [(J_x)(J_y)]$
$h_{00310}, h_{00130} \| h_{11200}, h_{11020}$	$2\nu_y$ $[(J_y)^2] \| [(J_x)(J_y)]$
$h_{20020}, h_{02200} \| h_{20200}, h_{02020}$	$2\nu_x - 2\nu_y \| 2\nu_x + 2\nu_y$ $[(J_x)(J_y)]$
$h_{30010}, h_{03100} \| h_{30100}, h_{03010}$	$3\nu_x - \nu_y \| 3\nu_x + \nu_y$ $[(J_x)^{3/2}(J_y)^{1/2}]$
$h_{10030}, h_{01300} \| h_{10300}, h_{01030}$	$\nu_x - 3\nu_y \| \nu_x + 3\nu_y$ $[(J_x)^{1/2}(J_y)^{3/2}]$

Table : Low-order driving terms.

2. Theory for RDTs

➤ RDTs indicate lattice nonlinearity: Analytic theories according to J. Bengtsson, SLS Note 9/97

- **Linear chromaticity:**

$$h_{11001} = \frac{1}{4}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{xi} + O\left(\delta^2\right),$$

$$h_{00111} = -\frac{1}{4}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{yi} + O\left(\delta^2\right)$$

- **Chromatic beta functions:**

$$h_{20001} = h_{02001}^{*} = \frac{1}{8}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{xi}e^{i2\mu_{xi}} + O\left(\delta^2\right),$$

$$h_{00201} = h_{00021}^{*} = -\frac{1}{8}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{yi}e^{i2\mu_{yi}} + O\left(\delta^2\right),$$

- **Chromatic dispersion:**

$$h_{10002} = h_{01002}^{*} = \frac{1}{2}\sum_{i=1}^{N}\left[(b_2L)_i - (b_3L)_i\eta_{xi}^{(1)}\right]\eta_{xi}^{(1)}\sqrt{\beta_{xi}}e^{i\mu_{xi}} + O\left(\delta^3\right)$$

2. Theory for RDTs

➤ **RDTs indicate lattice nonlinearity: Analytic theories according to J. Bengtsson, SLS Note 9/97**

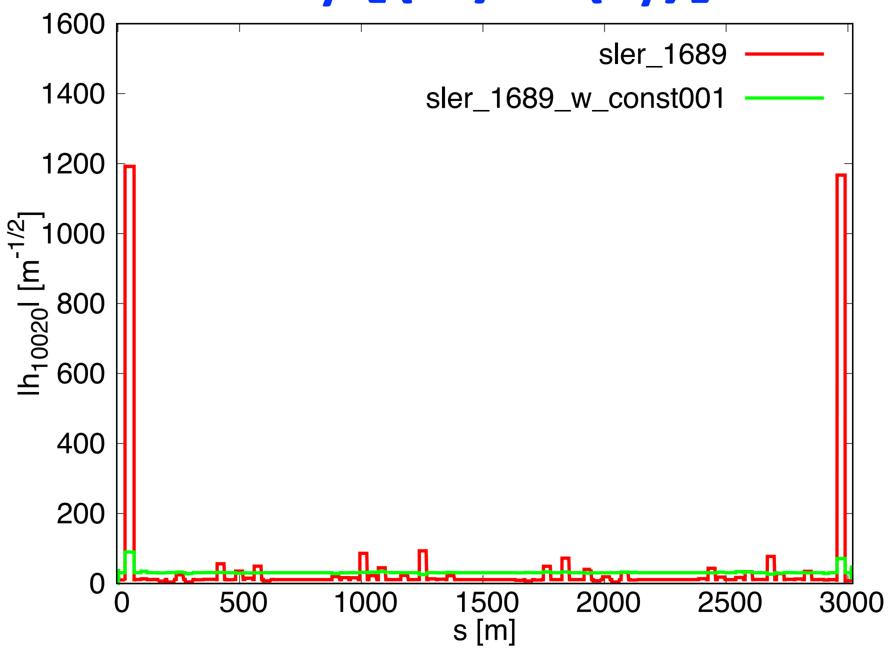
- **First order geometric terms (amplitude-dependent):**

$$
\begin{aligned}
h_{21000} &= h^*_{12000} = -\frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{3/2}e^{i\mu_{xi}},\\
h_{30000} &= h^*_{03000} = -\frac{1}{24}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{3/2}e^{i3\mu_{xi}},\\
h_{10110} &= h^*_{01110} = \frac{1}{4}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{1/2}\beta_{yi}e^{i\mu_{xi}},\\
h_{10020} &= h^*_{01200} = \frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{1/2}\beta_{yi}e^{i(\mu_{xi}-2\mu_{yi})},\\
h_{10200} &= h^*_{01020} = \frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{1/2}\beta_{yi}e^{i(\mu_{xi}+2\mu_{yi})}
\end{aligned}
$$

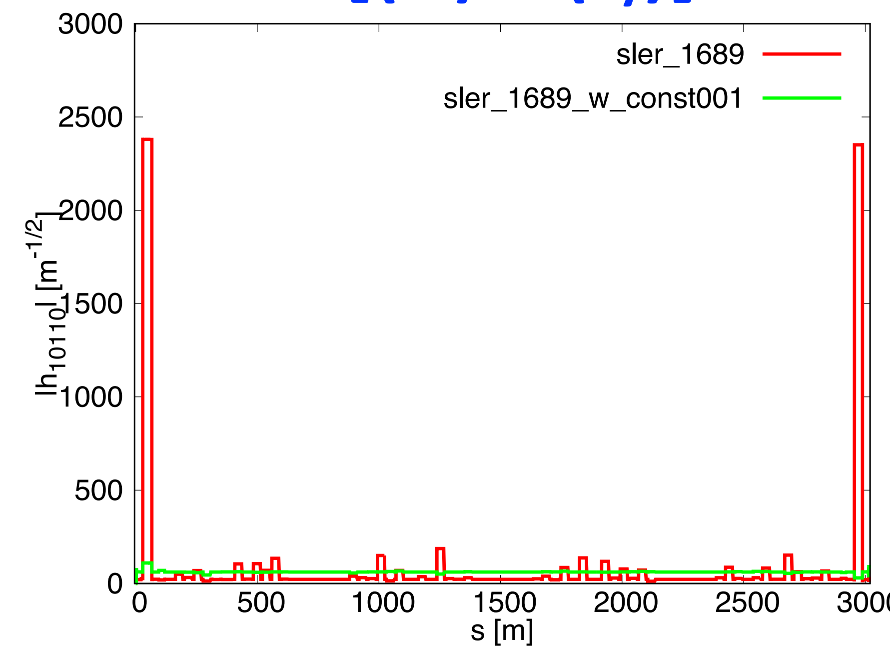
3. Results by PTC: 3rd order RDTs

- **Integration of RDTs along the whole ring**
  - **Almost perfect cancellation of 3rd order RDTs in the arc sections**

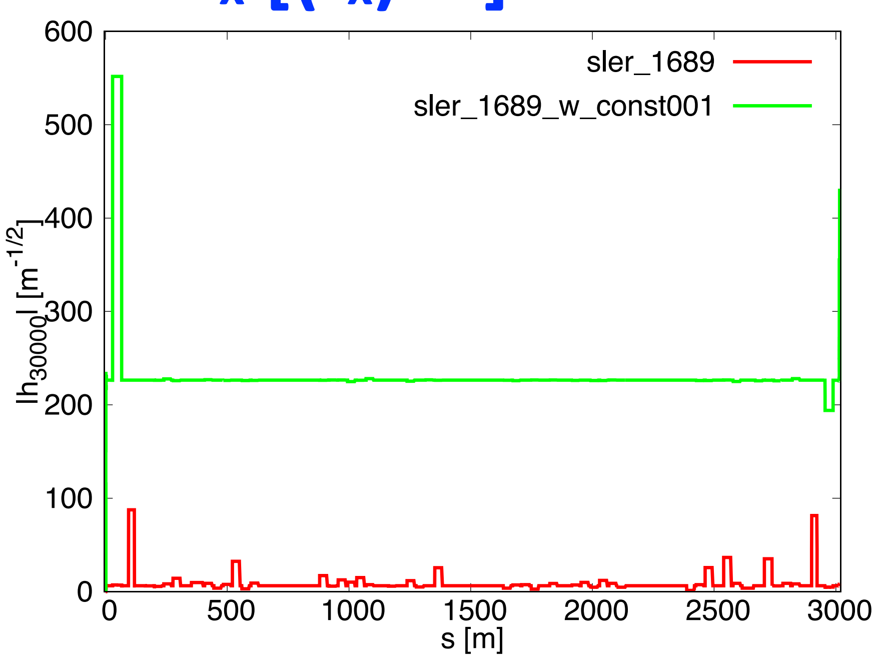
ν_x-2ν_y [$(J_x)^{1/2}(J_y)$]



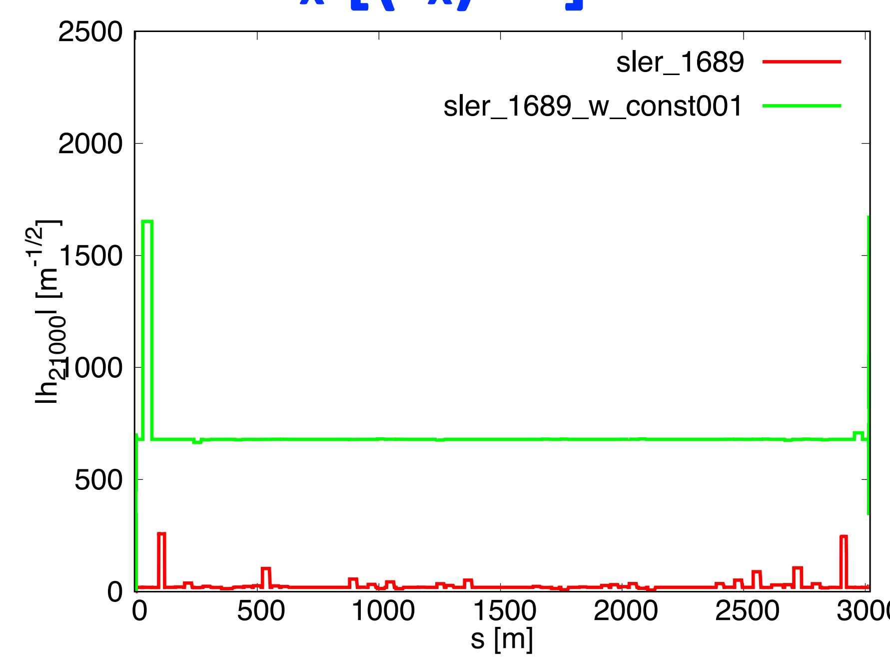
ν_x [$(J_x)^{1/2}(J_y)$]



3ν_x [$(J_x)^{3/2}$]



ν_x [$(J_x)^{3/2}$]

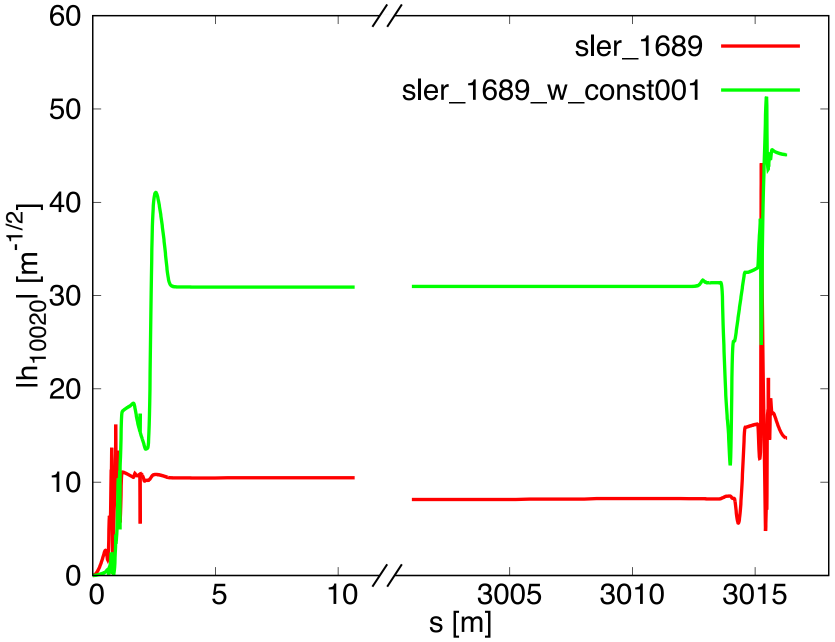


3. Results by PTC: 3rd order RDTs (IR)

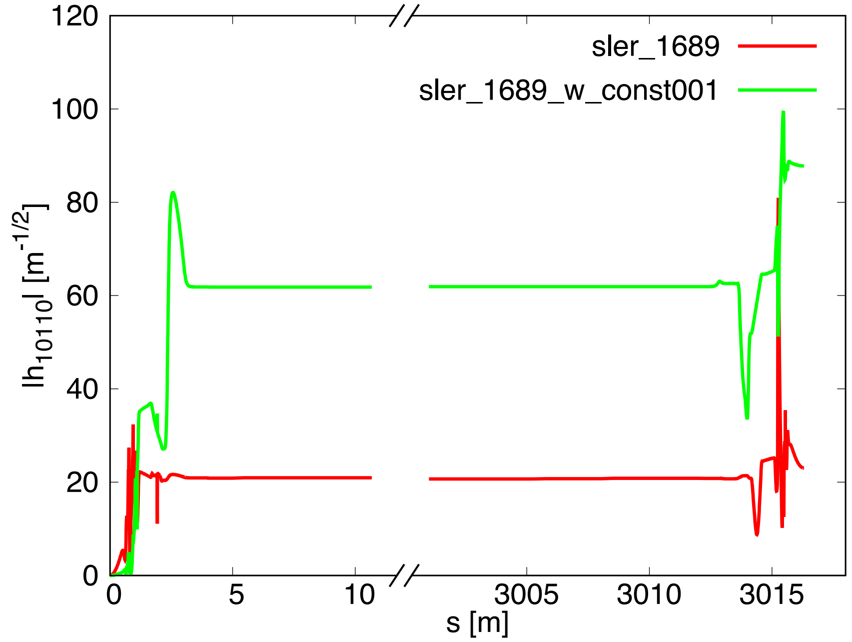
➤ **Integration of RDTs along the whole ring**

- **FFS contributes most of residual RDTs**

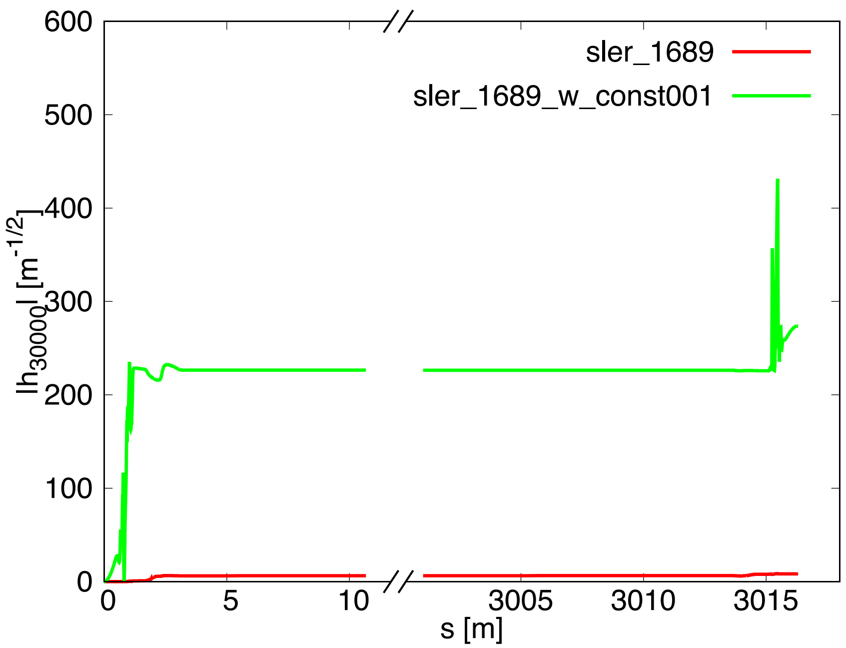
ν_x-2ν_y [$(J_x)^{1/2}(J_y)$]



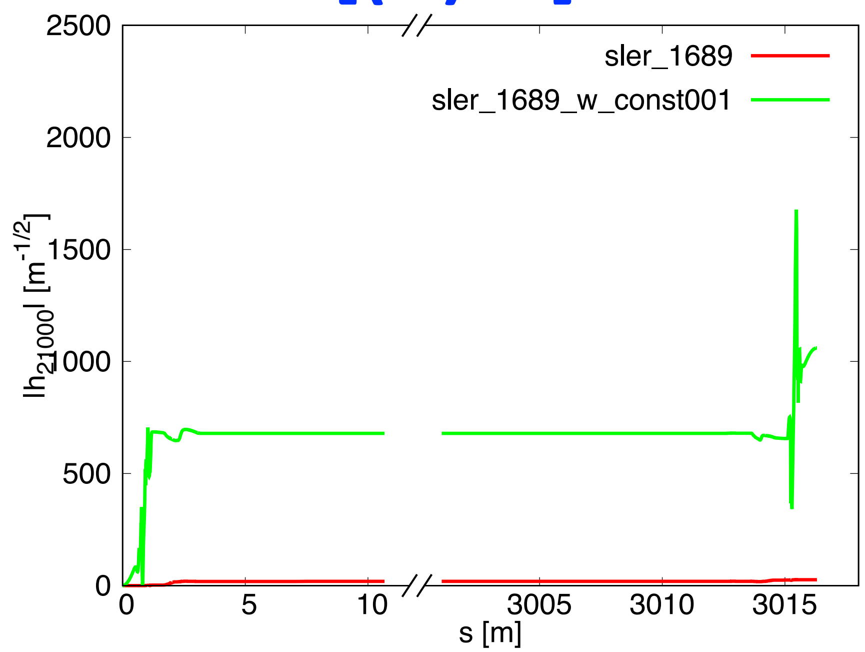
ν_x [$(J_x)^{1/2}(J_y)$]



3ν_x [$(J_x)^{3/2}$]



ν_x [$(J_x)^{3/2}$]

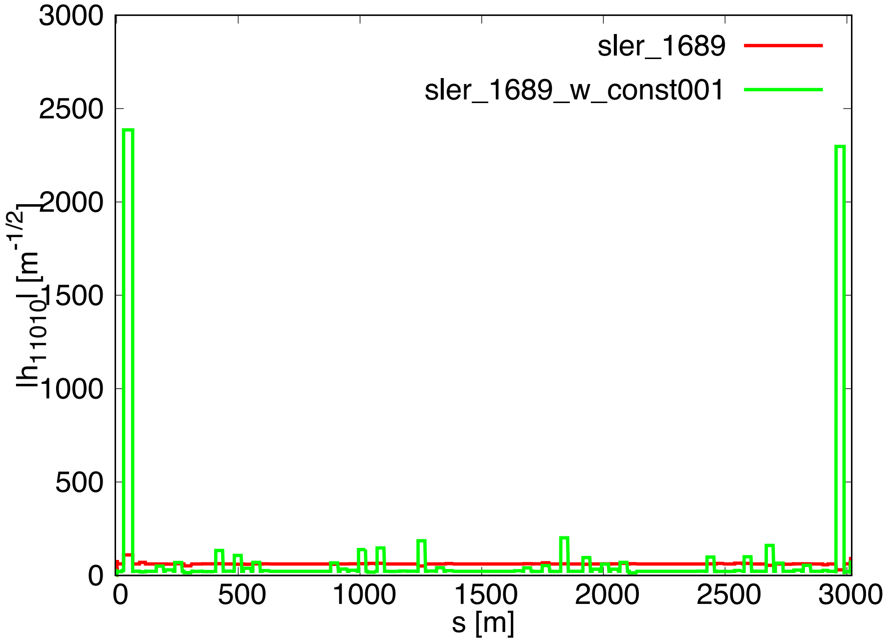


3. Results by PTC: 3rd order RDTs

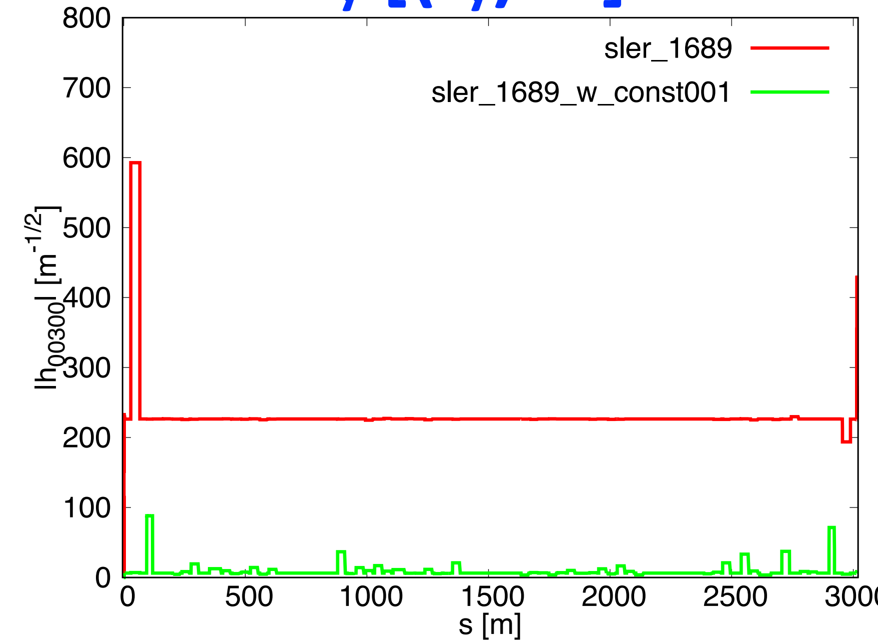
➤ Integration of RDTs along the whole ring

- **Almost perfect cancellation of 3rd order RDTs in the arc sections**

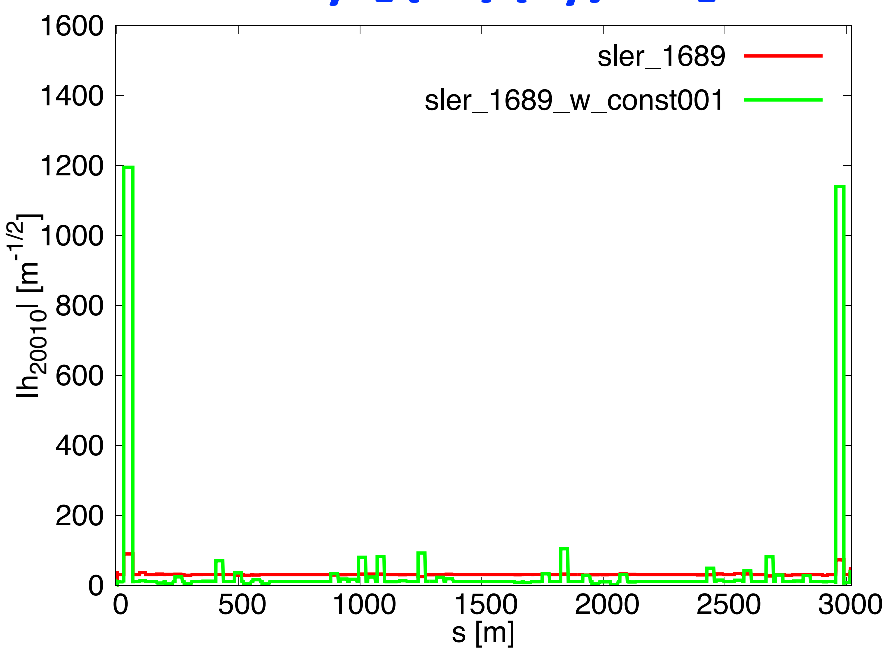
ν_y $[(J_y)^{1/2}(J_x)]$



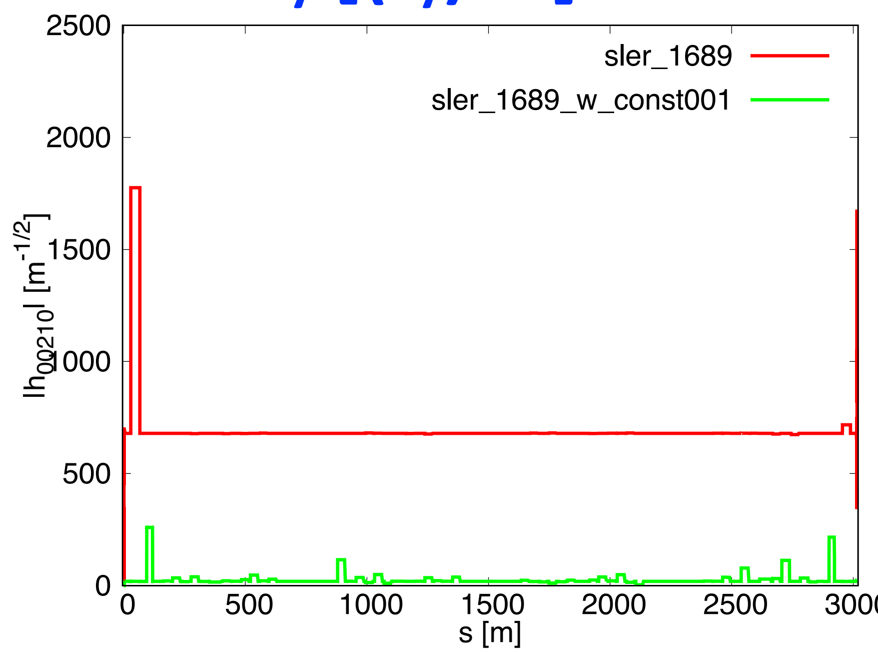
$3\nu_y$ $[(J_y)^{3/2}]$



$2\nu_x-\nu_y$ $[(J_x)(J_y)^{1/2}]$



ν_y $[(J_y)^{3/2}]$

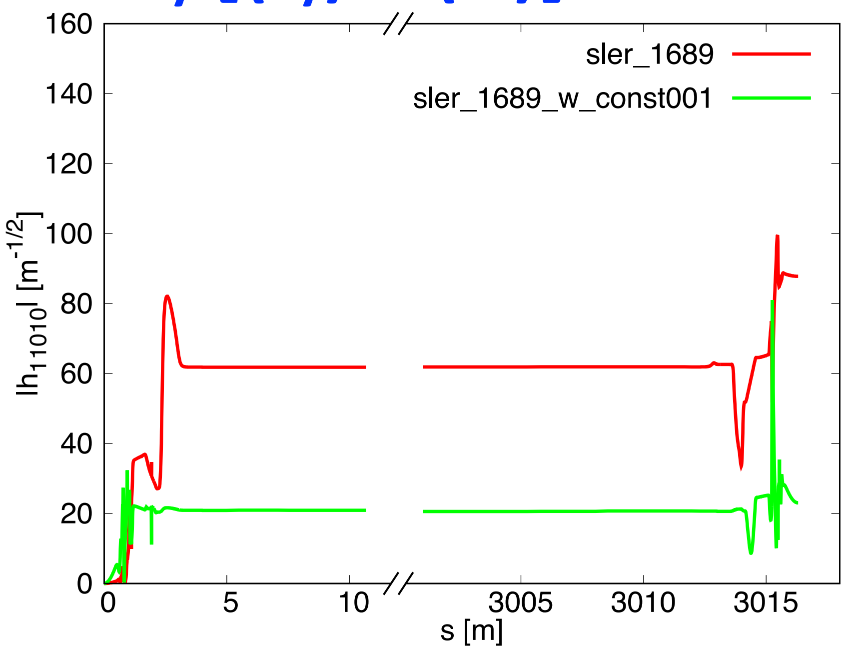


3. Results by PTC: 3rd order RDTs (IR)

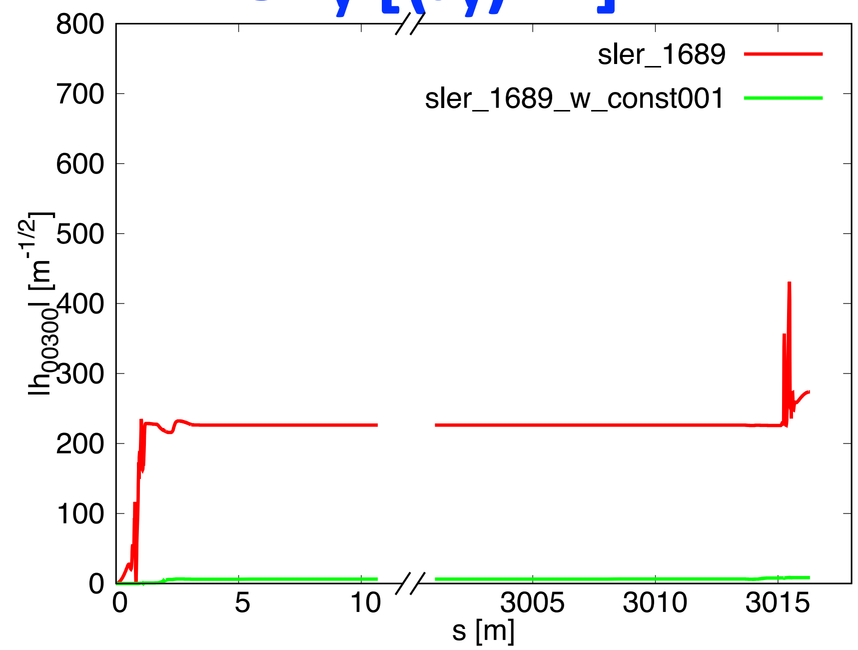
➤ **Integration of RDTs along the whole ring**

- **FFS contributes most of residual RDTs**

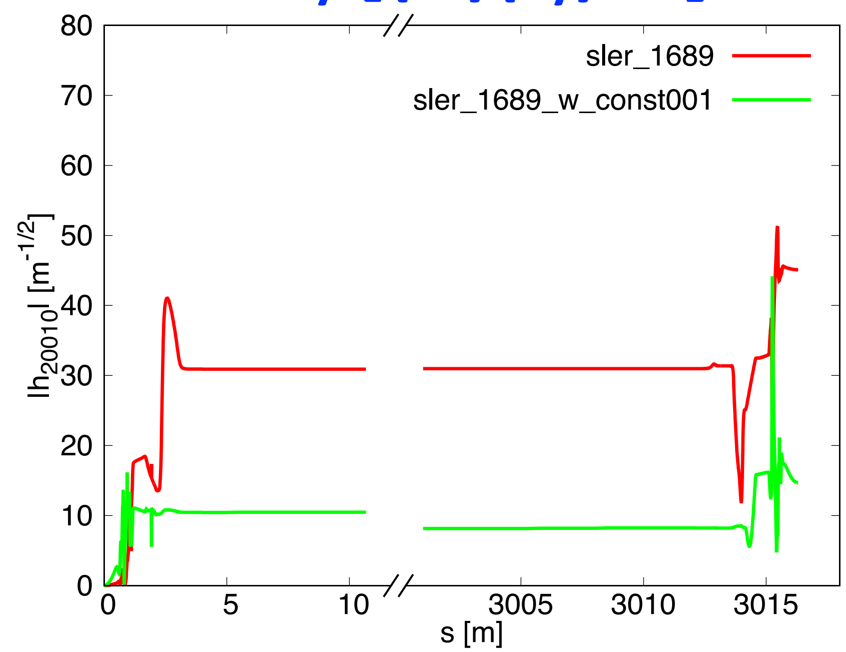
ν_y $[(J_y)^{1/2}(J_x)]$



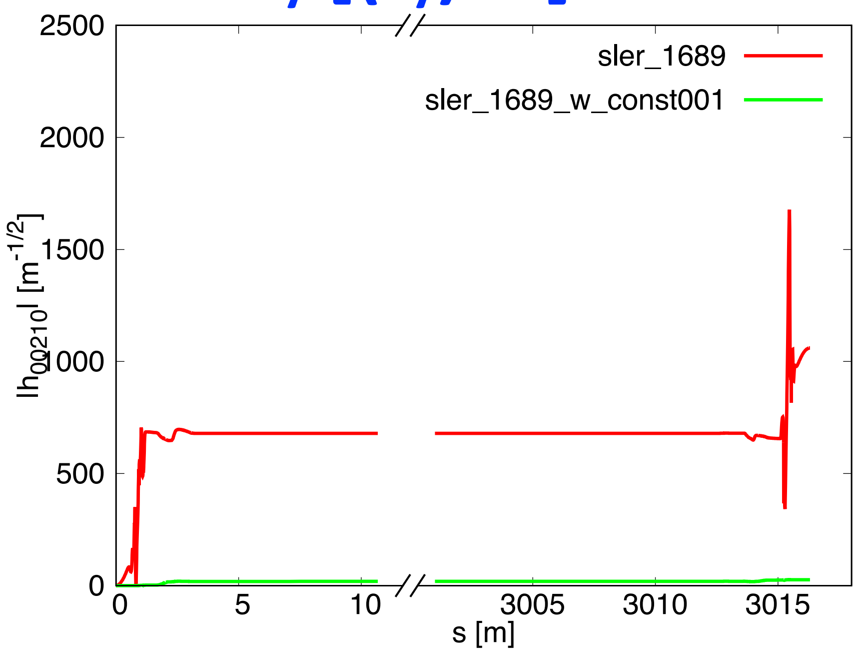
$3\nu_y$ $[(J_y)^{3/2}]$



$2\nu_x-\nu_y$ $[(J_x)(J_y)^{1/2}]$



ν_y $[(J_y)^{3/2}]$



3. Results by PTC: 3rd order RDTs

➤ **Final strengths of 3rd order RDTs (one-turn)**

- **Main terms in Y-direction suppressed, but main terms in X-direction enlarged**
- **Need additional magnets to achieve suppression of RDTs in both X- and Y-directions?**

Correlated?

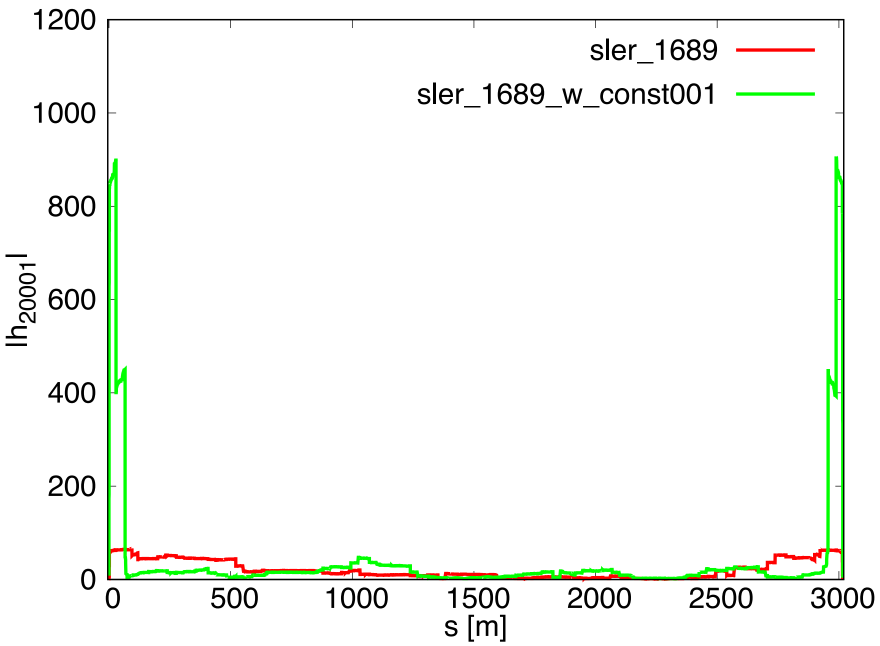
3rd RDTs	Amplitude [m^{-1/2}]	
	sler_1689	sler_1689_w_const001
|h_10020|	1.473E+01	4.510E+01
|h_10110|	2.304E+01	8.778E+01
|h_30000|	8.324E+00	2.738E+02
|h_21000|	2.623E+01	1.060E+03
|h_11010|	8.778E+01	2.306E+01
|h_00300|	2.738E+02	8.325E+00
|h_20010|	4.510E+01	1.470E+01
|h_00210|	1.060E+03	2.623E+01

3. Results by PTC: Chromatic β and ν

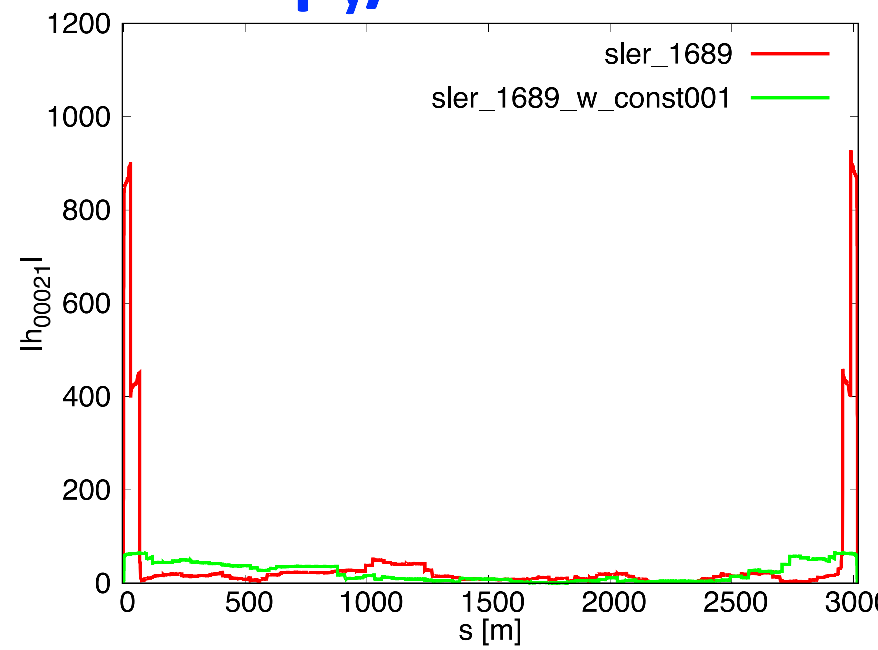
➤ Detuning along the whole ring

- **w/ constraints: control Y-direction but relax X-direction?**

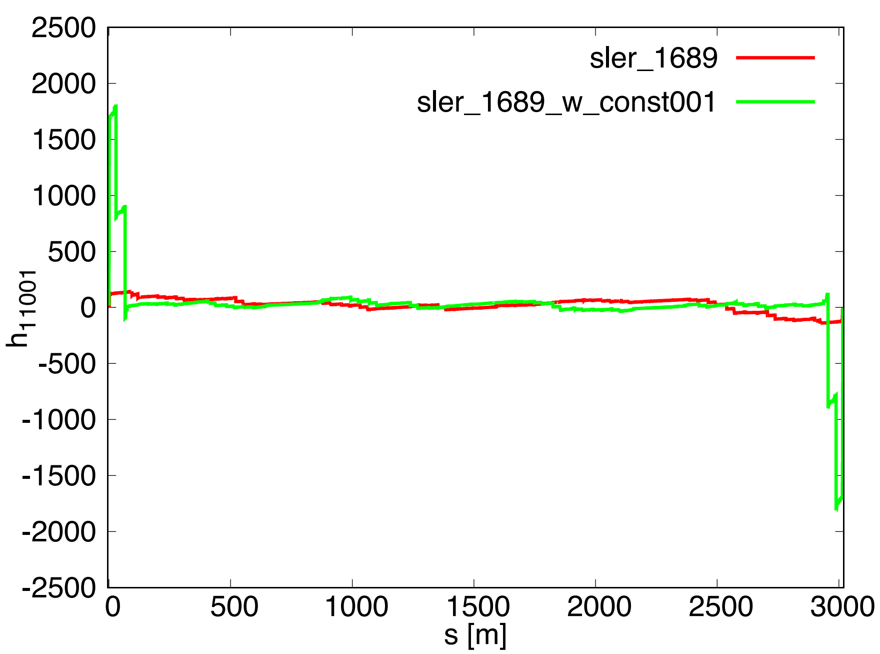
$d\beta_x/d\delta$



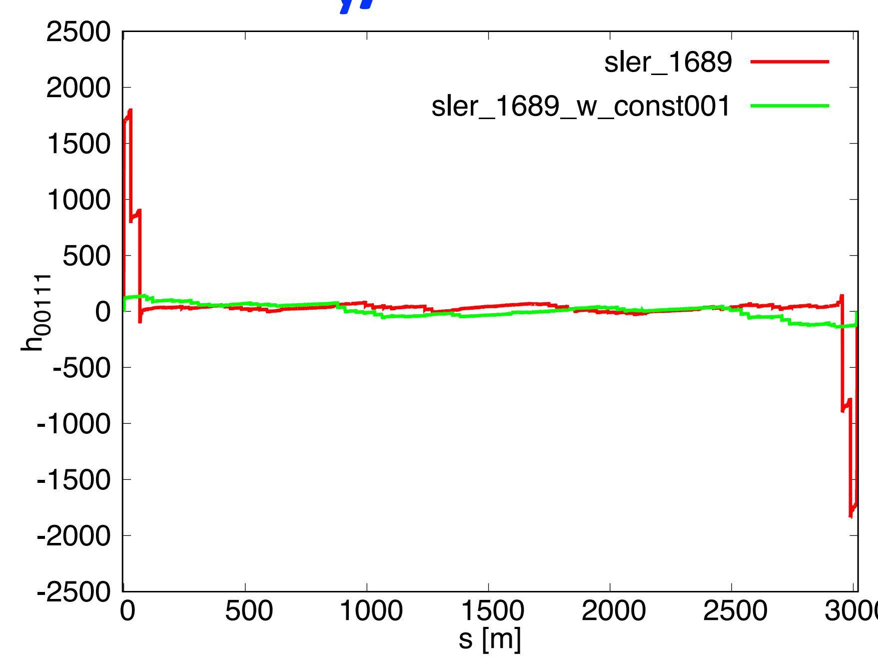
$d\beta_y/d\delta$



$d\nu_x/d\delta$



$d\nu_y/d\delta$

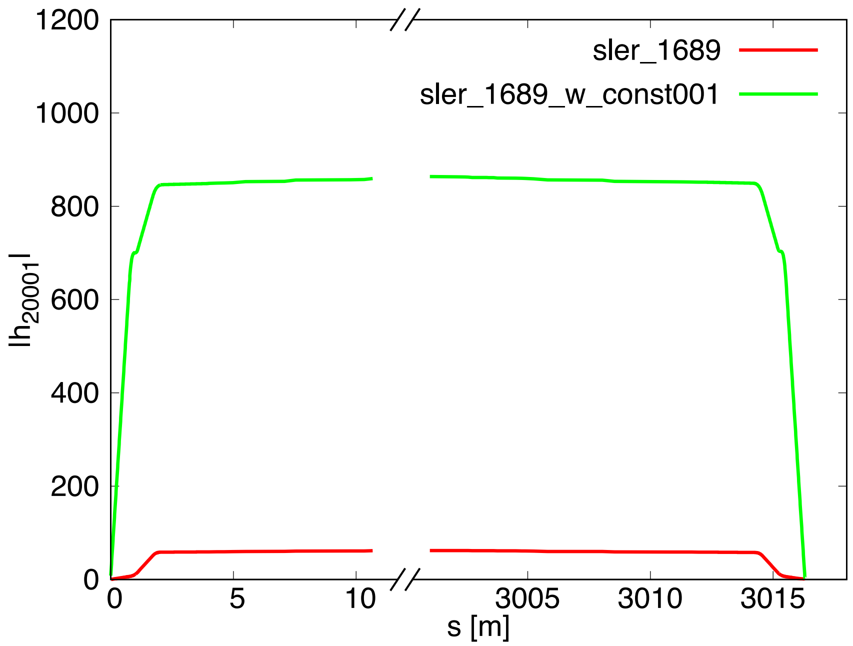


3. Results by PTC: Chromatic β and ν (IR)

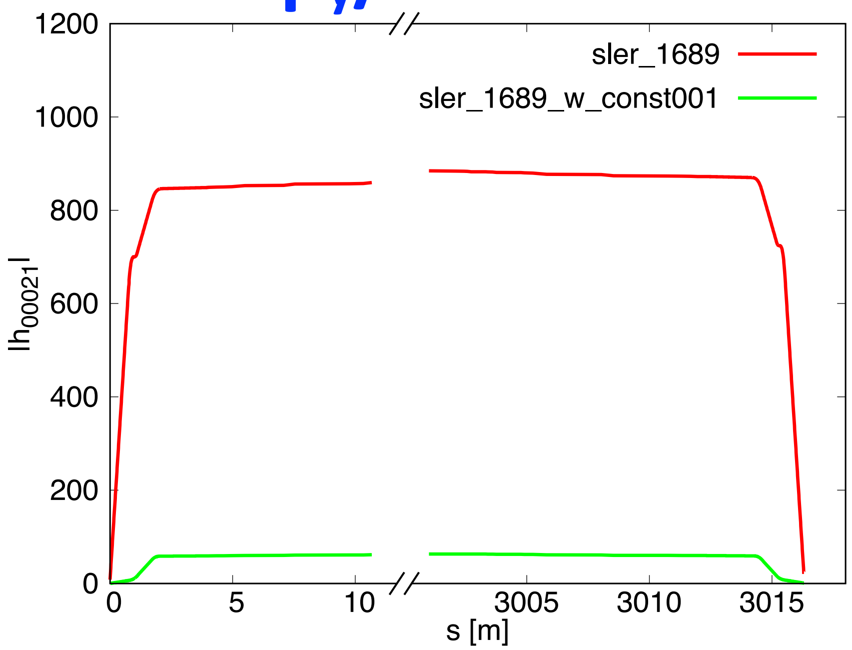
➤ Detuning along the whole ring

- **w/ constraints: control Y-direction but relax X-direction?**

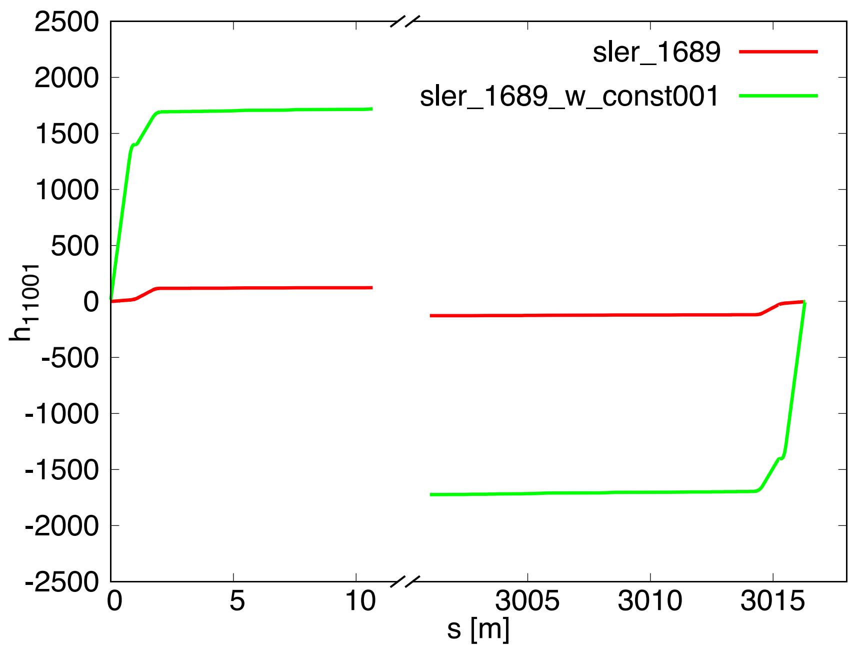
$d\beta_x/d\delta$



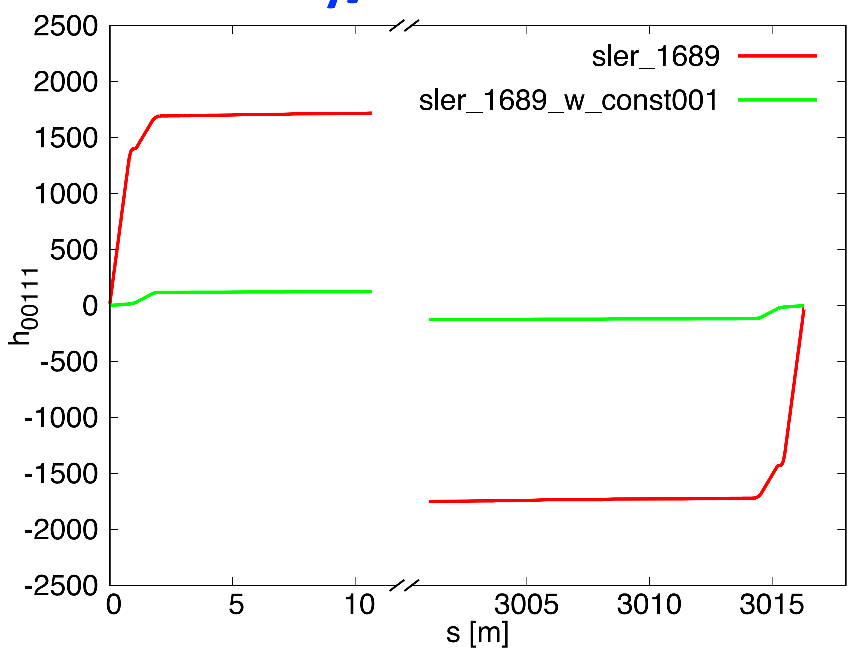
$d\beta_y/d\delta$



$d\nu_x/d\delta$



$d\nu_y/d\delta$

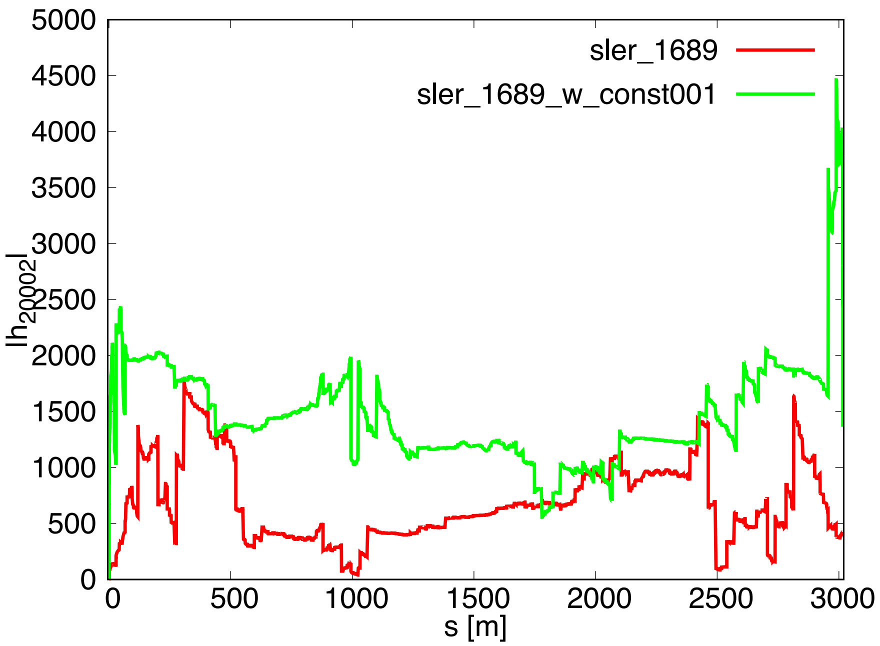


3. Results by PTC: Chromatic β and ν

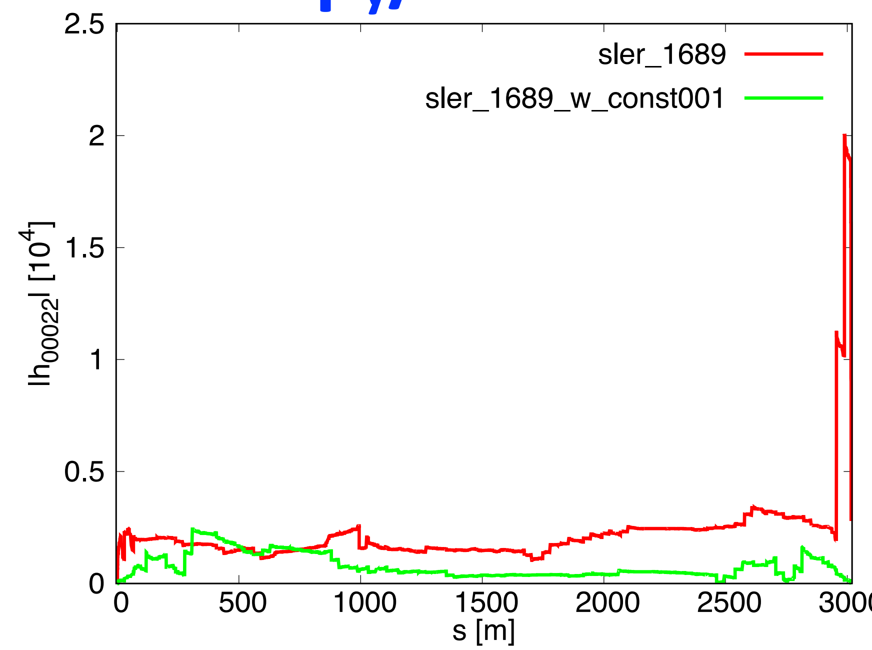
➤ Detuning along the whole ring - second order

- **w/ constraints: control Y-direction but relax X-direction?**

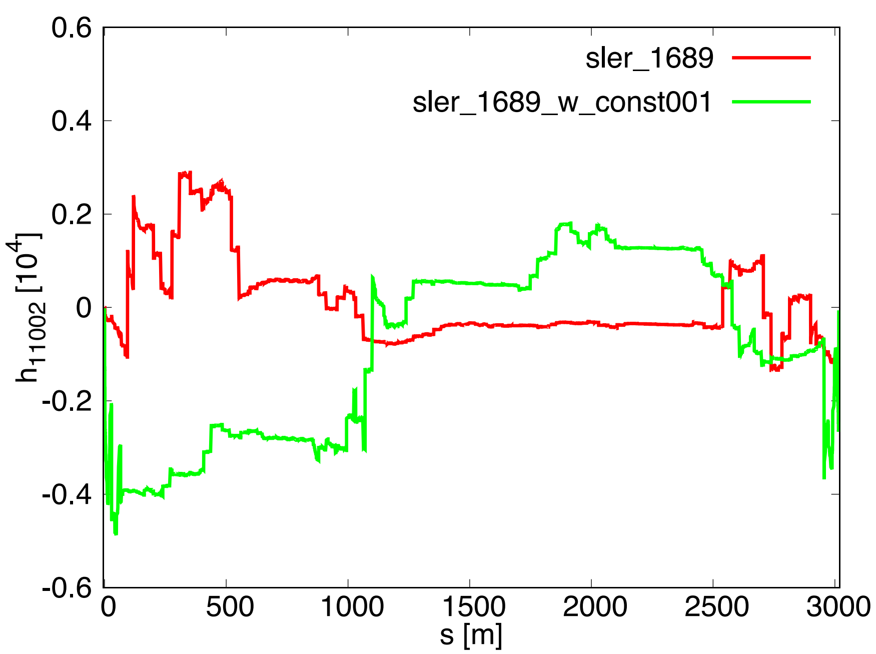
$d^2\beta_x/d\delta^2$



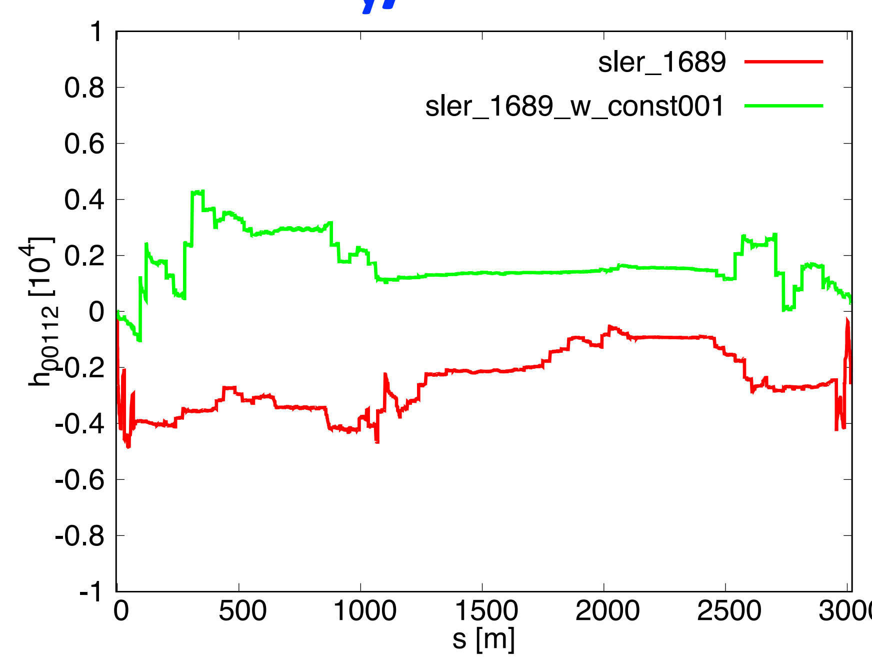
$d^2\beta_y/d\delta^2$



$d^2\nu_x/d\delta^2$



$d^2\nu_y/d\delta^2$

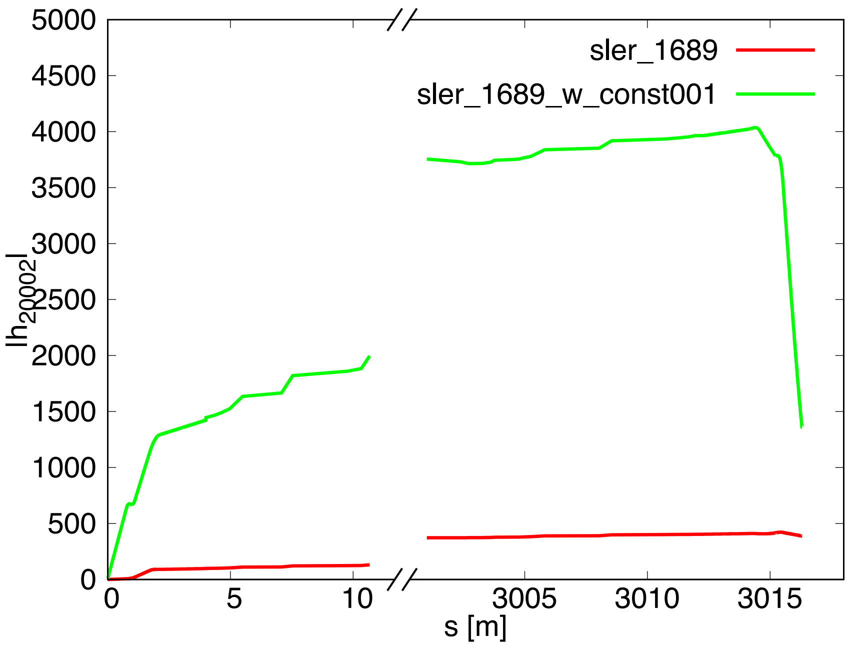


3. Results by PTC: Chromatic β and ν (IR)

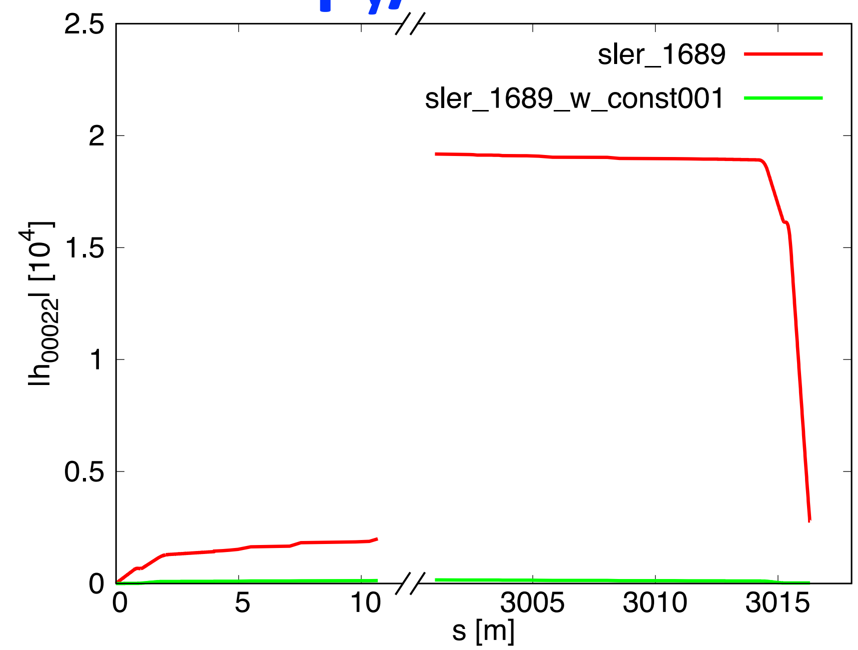
➤ **Detuning along the whole ring - second order**

- **w/ constraints: control Y-direction but relax X-direction?**

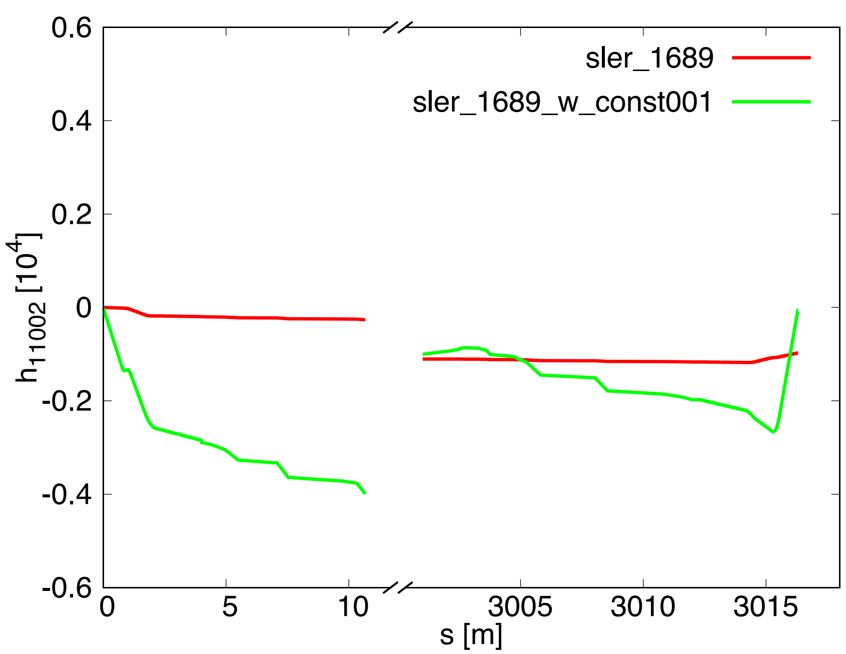
$d^2\beta_x/d\delta^2$



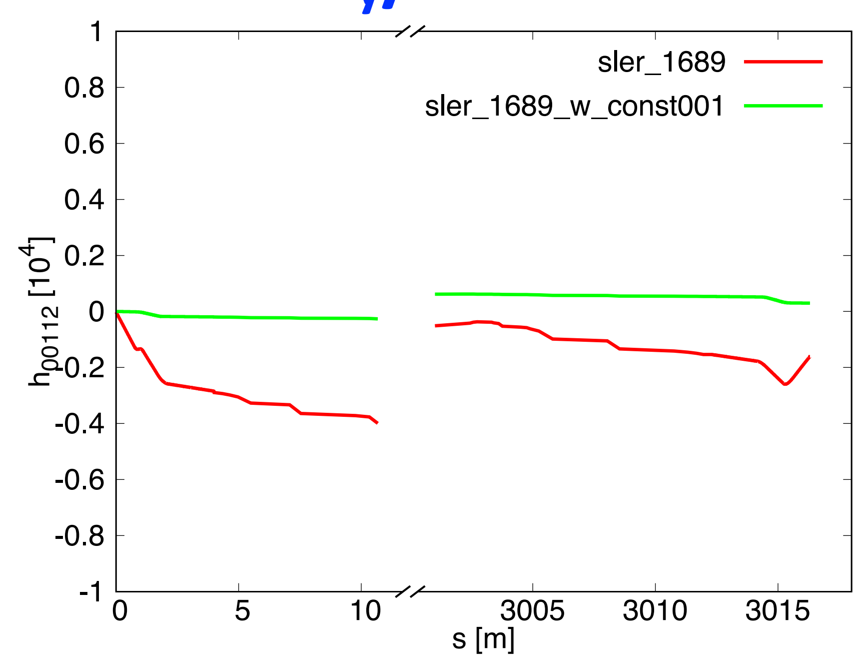
$d^2\beta_y/d\delta^2$



$d^2\nu_x/d\delta^2$



$d^2\nu_y/d\delta^2$

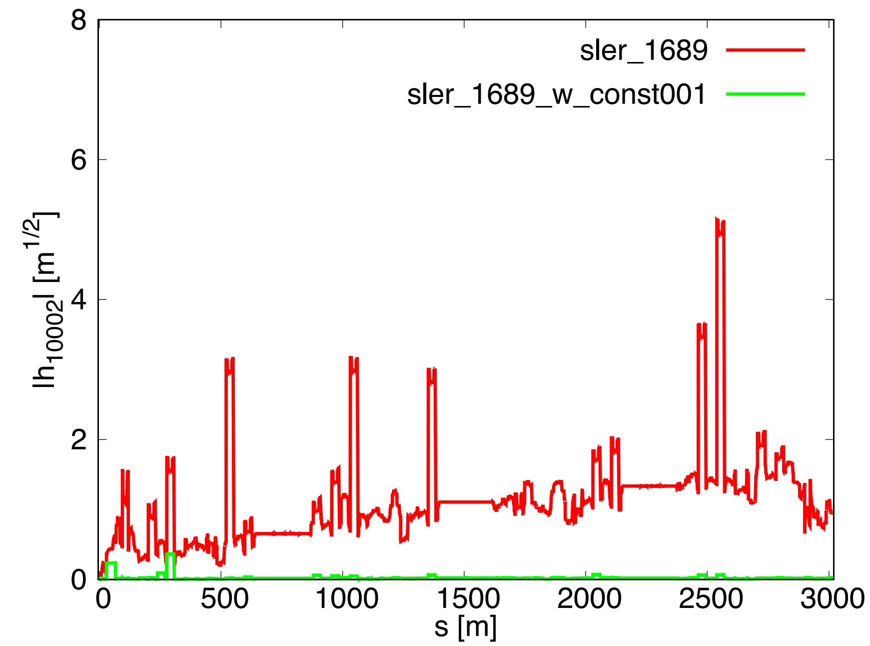


3. Results by PTC: Chromatic dispersion

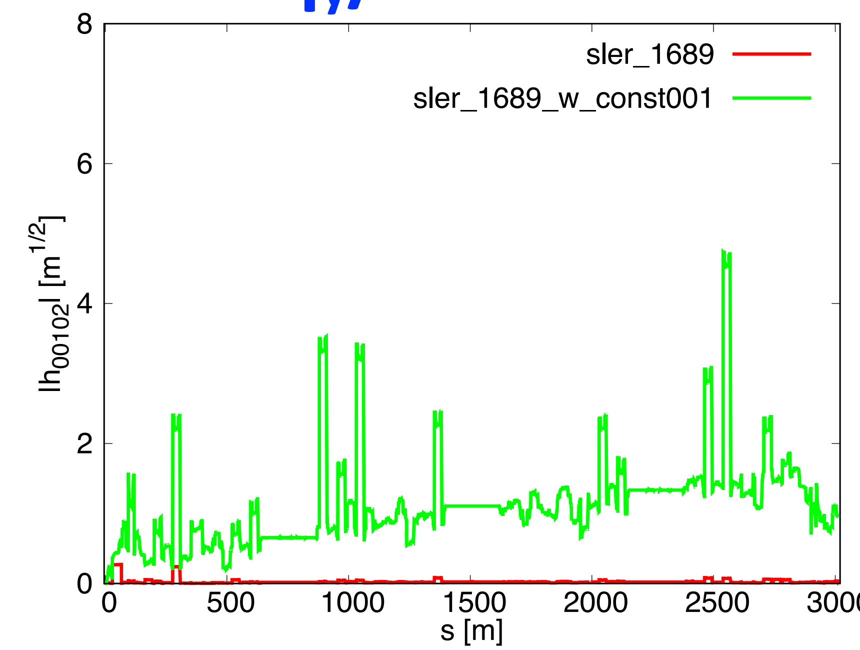
➤ Dispersion along the whole ring

- **w/ constraints: control X-direction but relax Y-direction?**

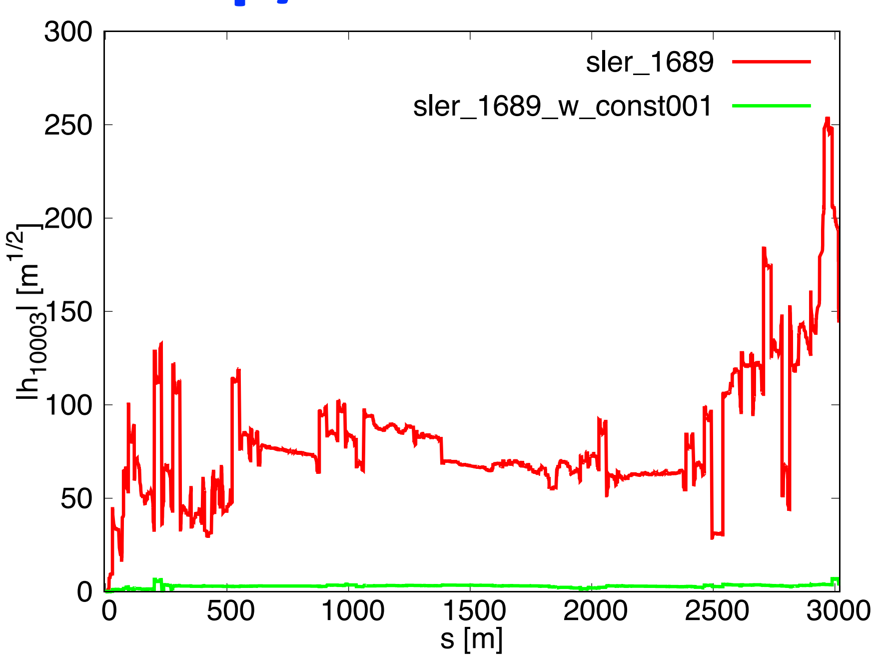
$d\eta_x/d\delta$



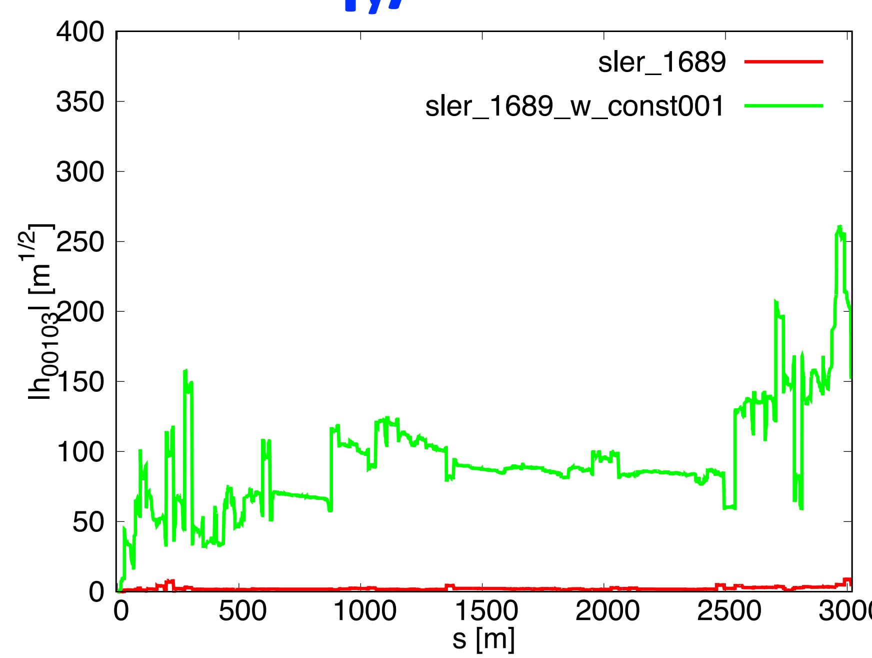
$d\eta_y/d\delta$



$d^2\eta_x/d\delta^2$



$d^2\eta_y/d\delta^2$

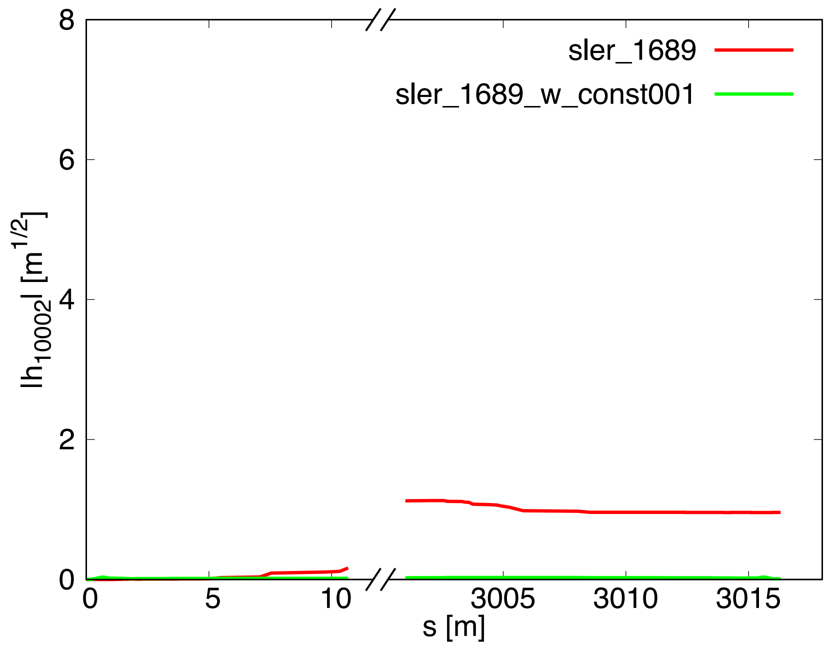


3. Results by PTC: Chromatic dispersion (IR)

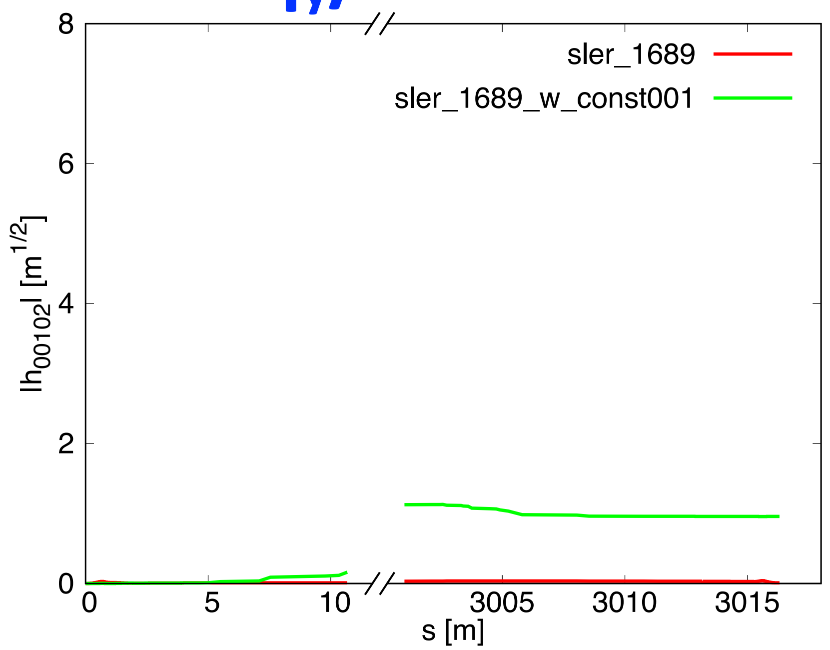
➤ **Dispersion along the whole ring**

- **w/ constraints: control X-direction but relax Y-direction?**

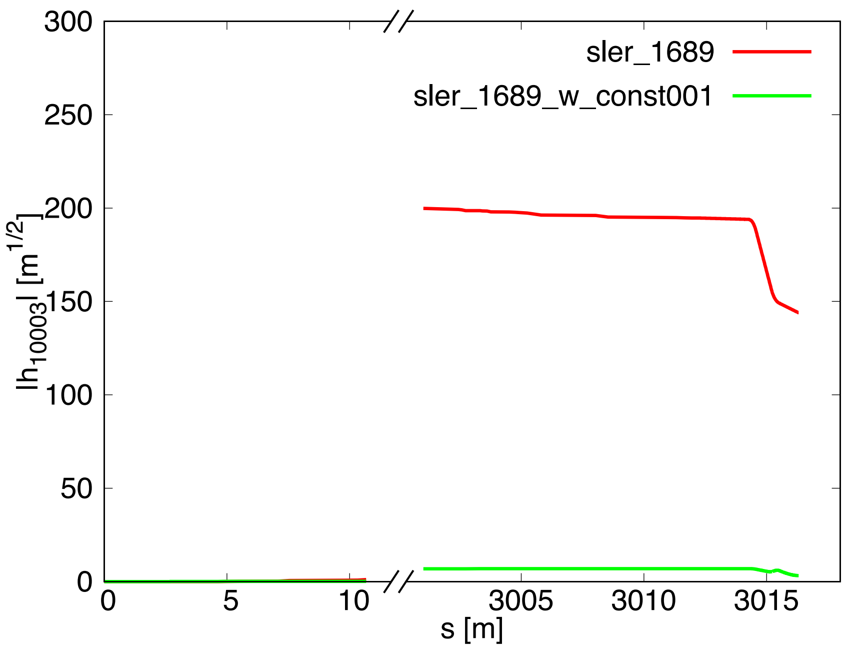
$d\eta_x/d\delta$



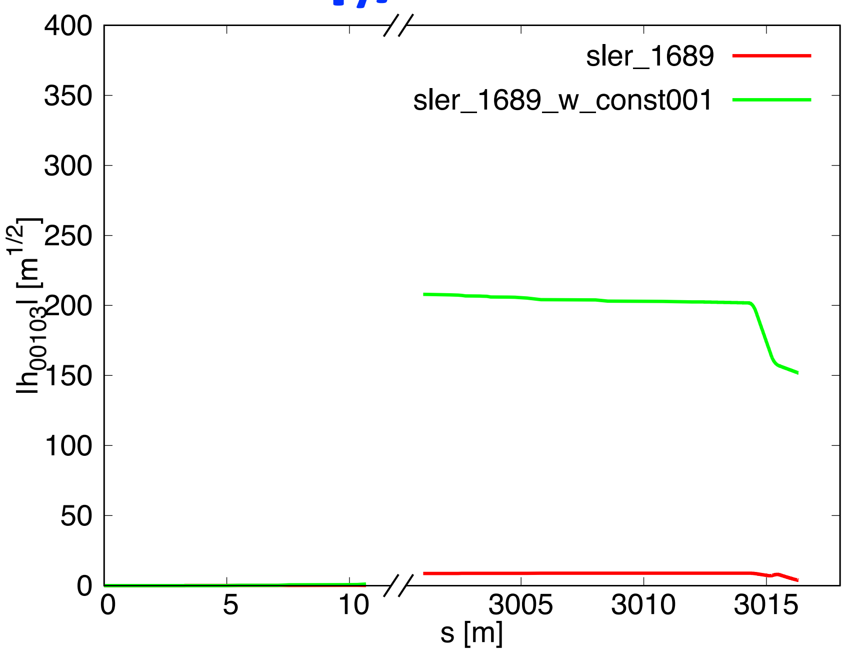
$d\eta_y/d\delta$



$d^2\eta_x/d\delta^2$



$d^2\eta_y/d\delta^2$

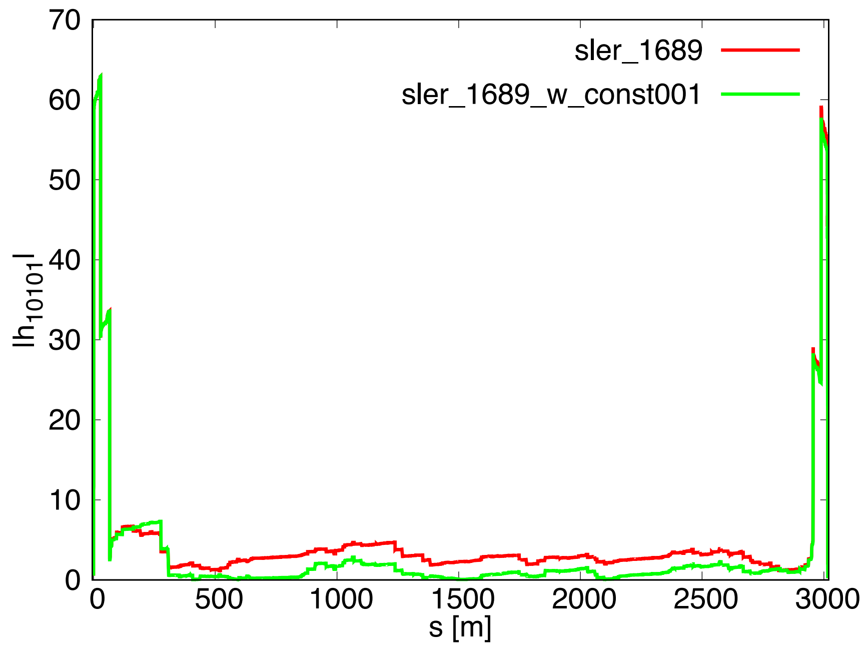


3. Results by PTC: Chromatic coupling

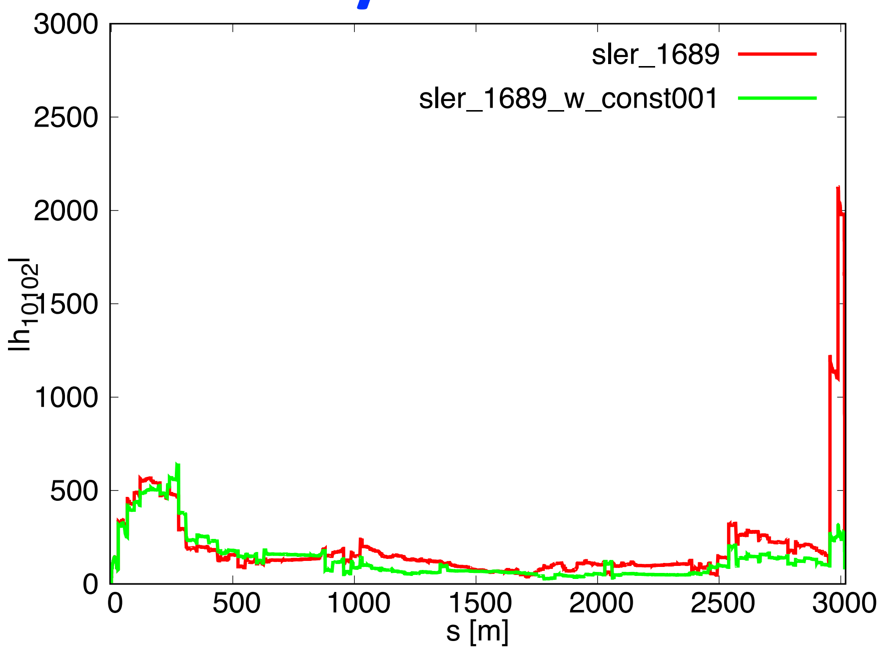
➤ Chromatic coupling along the whole ring

- w/ constraints: Chromatic coupling controlled

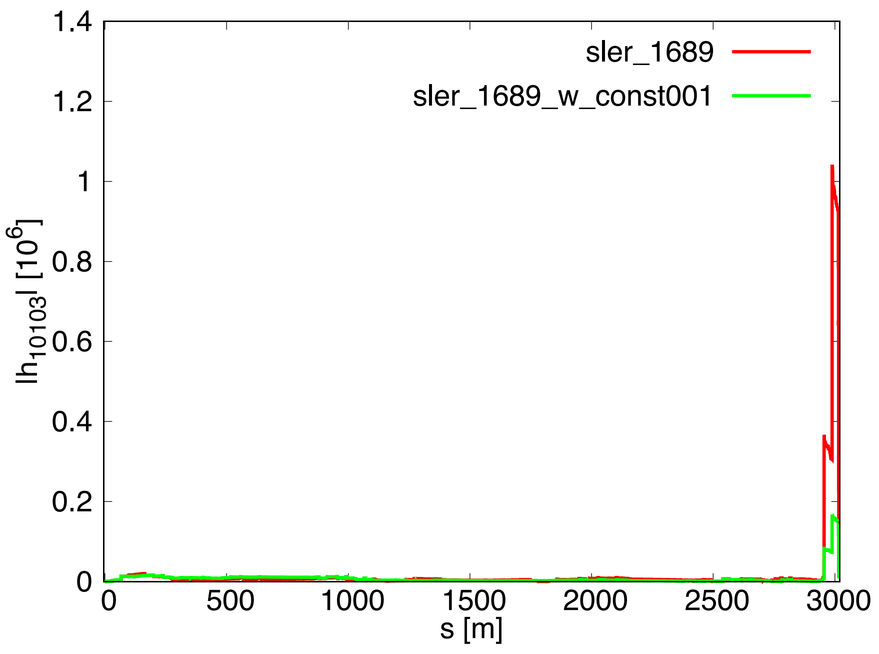
$dR/d\delta$



$d^2R/d\delta^2$



$d^3R/d\delta^3$

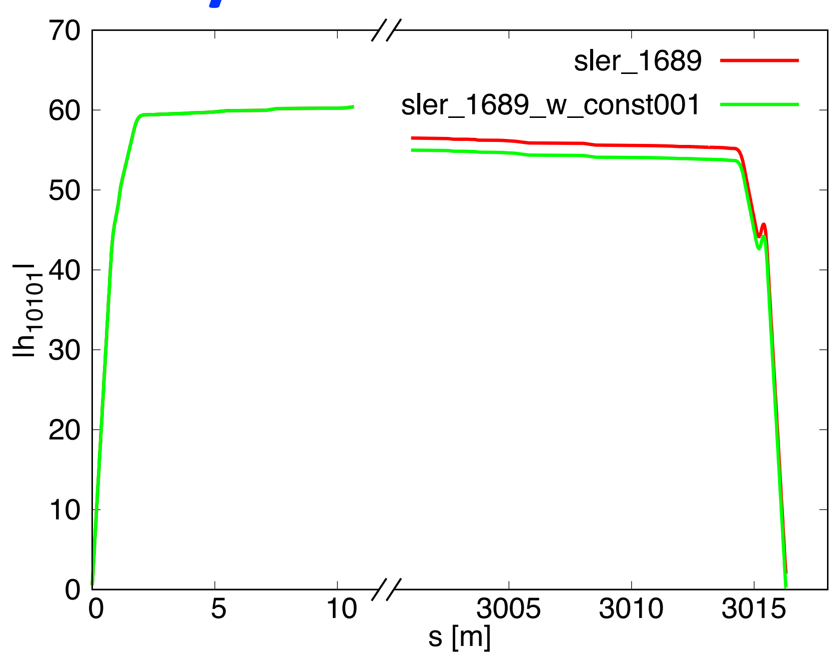


3. Results by PTC: Chromatic coupling (IR)

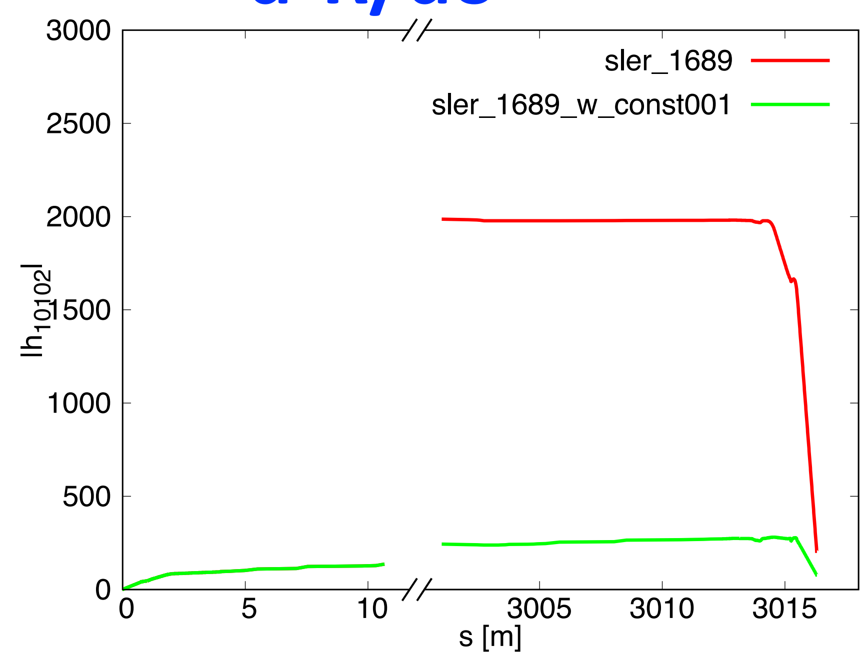
➤ Chromatic coupling along the whole ring

- **w/ constraints: Chromatic coupling controlled**

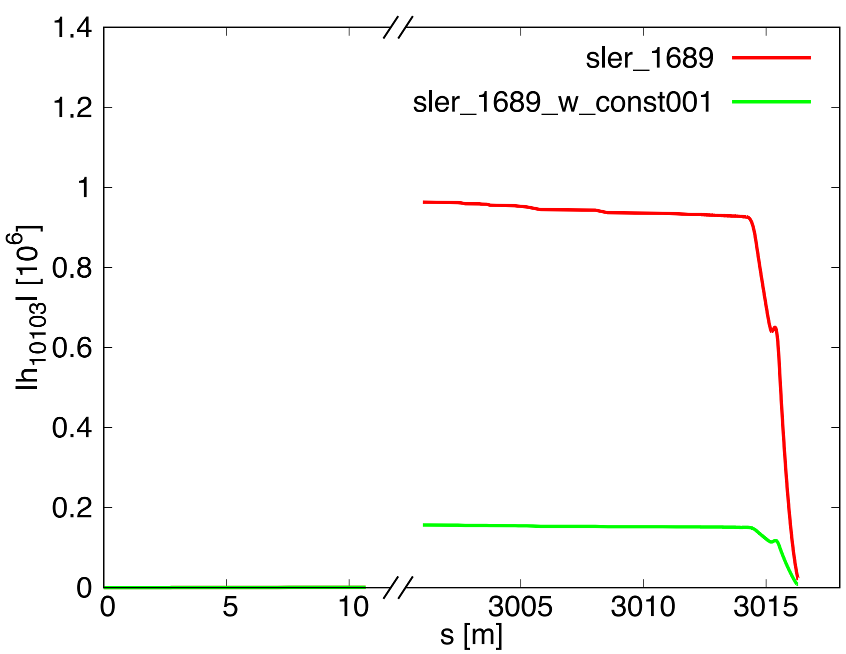
$dR/d\delta$



$d^2R/d\delta^2$



$d^3R/d\delta^3$



3. Results by PTC: Chromatic Twiss functions

➤ **Final strengths of chromatic Twiss functions (one-turn)**

- **Main terms in Y-direction suppressed, but main terms in X-direction enlarged**
- **Need additional magnets to achieve suppression of RDTs in both X- and Y-directions?**

RDTs for chrom. beta & tune	Strength	
	sler_1689	sler_1689_w_const001
|h_20001|	1.379E+00	3.589E+00
|h_20002|	3.884E+02	1.364E+03
h_11001	-1.675E+00	-3.203E+00
h_11002	-9.742E+02	-6.102E+01
|h_00021|	2.409E+01	8.585E-01
|h_00022|	2.791E+03	2.513E+01
h_00111	-2.880E+01	-7.384E-01
h_00112	-1.614E+03	2.970E+02

3. Results by PTC: Chromatic Twiss functions

➤ **Final strengths of chromatic Twiss functions (one-turn)**

- **Chromatic dispersions in X-direction suppressed, but enlarged in Y-direction**
- **Chromatic coupling suppressed w/ new constraints in DA optimization**

RDTs for chrom. dispersion	Strength [m^{1/2}]	
|h_10002|	9.577E-01	9.891E-03
|h_10003|	1.440E+02	3.227E+00
|h_00102|	9.818E-03	9.581E-01
|h_00103|	3.832E+00	1.519E+02

RDTs for chrom. coupling	Strength	
|h_10101|	1.916E+00	2.107E-01
|h_10102|	2.050E+02	7.761E+01
|h_10103|	2.261E+04	8.000E+03

3. Results by PTC: higher-order RDTs

Final values (one-turn): Chromatic 3rd order

Chrom. 3rd RDTs	Amplitude [m^{-1/2}]	
	sler_1689	sler_1689_w_const001
|h_10021|	2.402E+04	1.949E+03
|h_10111|	4.764E+04	3.796E+03
|h_30001|	5.639E+02	2.551E+03
|h_21001|	7.948E+02	9.527E+03
|h_11011|	4.579E+03	4.958E+04
|h_00301|	1.089E+04	6.565E+02
|h_20011|	2.378E+03	2.489E+04
|h_00211|	4.368E+04	6.113E+02

3. Results by PTC: higher-order RDTs

Final values (one-turn): Chromatic 3rd order

Chrom. 3rd RDTs	Amplitude [m^{-1/2}]	
	sler_1689	sler_1689_w_const001
|h_10022|	2.520E+06	1.193E+05
|h_10112|	5.955E+06	2.792E+05
|h_30002|	1.135E+05	4.855E+05
|h_21002|	2.430E+05	1.694E+06
|h_11012|	3.424E+05	5.333E+06
|h_00302|	8.249E+05	1.088E+05
|h_20012|	1.671E+05	2.168E+06
|h_00212|	6.717E+05	2.321E+05

3. Results by PTC: 4th order RDTs

Final values (one-turn): 4th order

4th RDTs	Amplitude [m^{-1}]	
	sler_1689	sler_1689_w_const001
|h_00040|	4.170E+05	3.729E+04
|h_00130|	2.716E+06	1.553E+05
|h_00220|	4.676E+06	2.370E+05
|h_02020|	3.028E+05	3.028E+05
|h_02110|	3.215E+05	4.636E+05
|h_02200|	1.612E+05	1.612E+05
|h_04000|	3.731E+04	4.169E+05
|h_11020|	4.635E+05	3.215E+05
|h_11110|	5.118E+05	5.117E+05
|h_13000|	1.553E+05	2.721E+06
|h_22000|	2.371E+05	4.685E+06

3. Results by PTC: 4th order RDTs

Final values (one-turn): Chromatic 4th order

Chrom. 4th RDTs	Amplitude [m^{-1}]	
	sler_1689	sler_1689_w_const001
|h_00041|	4.845E+07	3.191E+05
|h_00131|	2.676E+08	1.460E+06
|h_00221|	4.590E+08	2.621E+06
|h_02021|	9.383E+06	9.317E+05
|h_02111|	1.002E+07	1.931E+06
|h_02201|	9.569E+06	1.064E+05
|h_04001|	4.454E+05	2.343E+07
|h_11021|	1.782E+07	1.263E+06
|h_11111|	1.324E+07	9.603E+06
|h_13001|	1.742E+06	1.405E+08
|h_22001|	2.916E+06	2.393E+08

3. Results by PTC: 4th order RDTs

Final values (one-turn): Chromatic 4th order

Chrom. 4th RDTs	Amplitude [m^{-1}]	
	sler_1689	sler_1689_w_const001
|h_00042|	5.637E+09	9.360E+06
|h_00132|	8.167E+09	2.882E+07
|h_00222|	4.022E+09	4.948E+07
|h_02022|	2.475E+09	1.192E+09
|h_02112|	3.918E+09	2.353E+09
|h_02202|	2.313E+09	9.442E+08
|h_04002|	3.172E+07	3.057E+09
|h_11022|	4.830E+09	9.228E+08
|h_11112|	7.352E+09	9.039E+08
|h_13002|	7.145E+07	5.951E+09
|h_22002|	7.357E+07	1.343E+10

4. Summary

➤ Previous findings

- BB + Lattice nonlinearity cause luminosity loss in SuperKEKB
- Lum. drop happens at low beam current
- Related to amplitude-dependent latt. nonlin.

➤ DA optimization w/ new constraints [by H. Sugimoto]

- Small loss of DA and lifetime (reasonable?)
- Nonlinearity in chromatic beta, alpha, tune, and coupling functions [related to RDTs] (at IP?) suppressed successfully
- Lum. gain achieved at low current

➤ Calculation of RDTs using PTC

- Suppression of some RDTs (mostly in Y-direction) observed
- Also other terms enlarged rather than suppressed

➤ Open questions

- Need additional correctors (equivalent to more variables) to suppress RDTs?
- Possible to achieve better DA, lifetime and lum. simultaneously?