Translation among SAD, MAD-X and Bmad

Demin Zhou

Acknowledgements:

M. Biagini, L. Deniau, R. De Maria, E. Forest, E. Gianfelice-Wendt, M. Giovannozzi, H. Koiso, M. Lückhof, S.M. Liuzzo, A. Morita, K. Ohmi, Y. Ohnishi, K. Oide, L. van Riesen-Haupt, D. Sagan, P. Skowronski, P. Thrane, I. Tecker, Y. Wang, A. Wegscheider, Y. Zhang, F. Zimmermann

Workshop SAD2019, KEK, Sep. 19, 2019

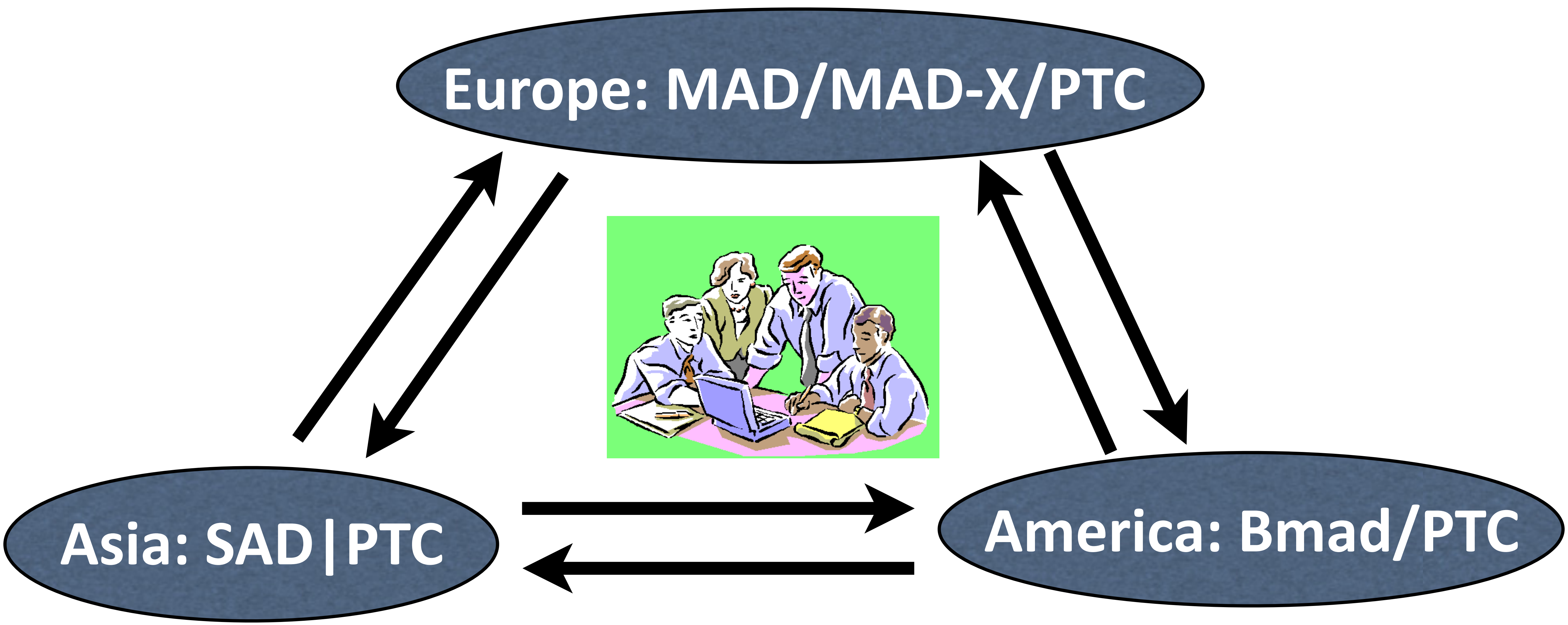
Outline

- Introduction
- Current status
  - Actually not the latest
- Cases of Benchmark studies
- Some applications
- Symplectic tracking in SAD
- Summary and future plan

1. Introduction

➤ Motivation: To improve communications

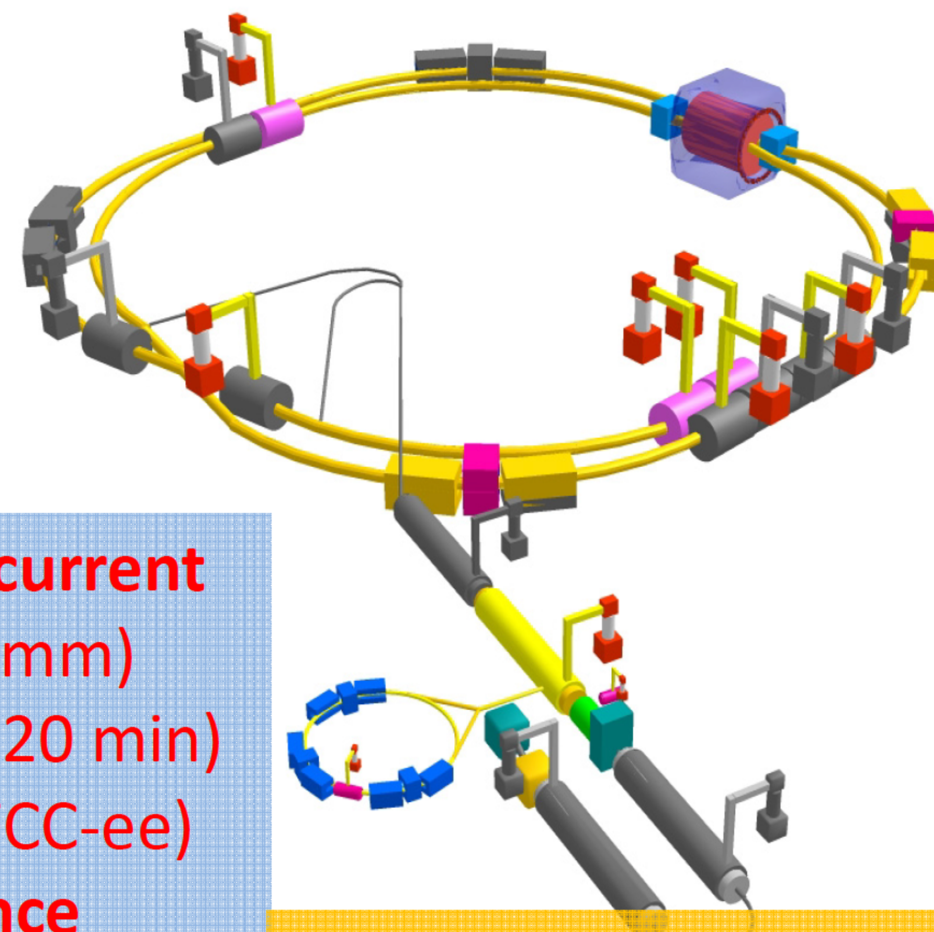
- SAD: TRISTAN, KEKB, J-PARC, SuperKEKB, FCC-ee, …
- Bmad/PTC: CESR, ERL, …
- MAD/MAD-X/PTC: PS, LEP, LHC, FCCs, …



1. Introduction

➤ Motivation: To improve collaborations on projects like SuperKEKB and FCCs

SuperKEKB = *FCC-ee* demonstrator



top up injection at high current
β_y^* =300 μm (FCC-ee: 1 mm)
lifetime 5 min (FCC-ee: ≥20 min)
$\varepsilon_y/\varepsilon_x$ =0.25% (similar to FCC-ee)
off momentum acceptance
(±1.5%, similar to FCC-ee)
e$^+$ **production rate** (2.5x10^{12}/s,
FCC-ee: <1.5x10^{12}/s (*Z* cr.waist)

***SuperKEKB* goes beyond *FCC-ee*, testing all concepts**

Ref. F. Zimmermann et al., IPAC'14, MOXAA01.

1. Introduction

➤ References for this topic

- D. Zhou et al., "*Lattice translation between accelerator simulation codes for SuperKEKB*", in Proceedings of IPAC'16, Busan, Korea, May. 08-13, 2016.
- D. Zhou, "*SuperKEKB lattice translation*", MAD-X meeting, CERN, Sep. 22, 2016, https://indico.cern.ch/event/565330/.
- D. Zhou, "*Recent progress on lattice translation from SAD to MAD-X for FCC-ee*", 47th FCC-ee design meeting, Feb. 24, 2017.
- D. Zhou, "*SAD-MAD converter survey and needs*", FCC-ee design meeting, Mar. 03, 2017.
- D. Sagan, "The Bmad Reference Manual", https://www.classe.cornell.edu/~dcs/bmad/manual.html.
- SAD Home Page, http://acc-physics.kek.jp/SAD/.
- MAD-X user's guide, http://mad.web.cern.ch/mad/
- SAD manual under edition, http://research.kek.jp/people/dmzhou/SAD/Manual/

2. Current status

➤ Efforts for lattice translations: SAD <-> MAD/MAD-X

- MAD -> SAD: H. Koiso(KEK), Y. Wang(IHEP), et al.
- MAD-X -> SAD: A. Morita(KEK, 2008)
- SAD -> MAD: Y. Wang(IHEP), et al.
- SAD -> MAD-X: A. Morita(KEK), K. Oide(KEK), Y. Wang(IHEP), et al.
- … …

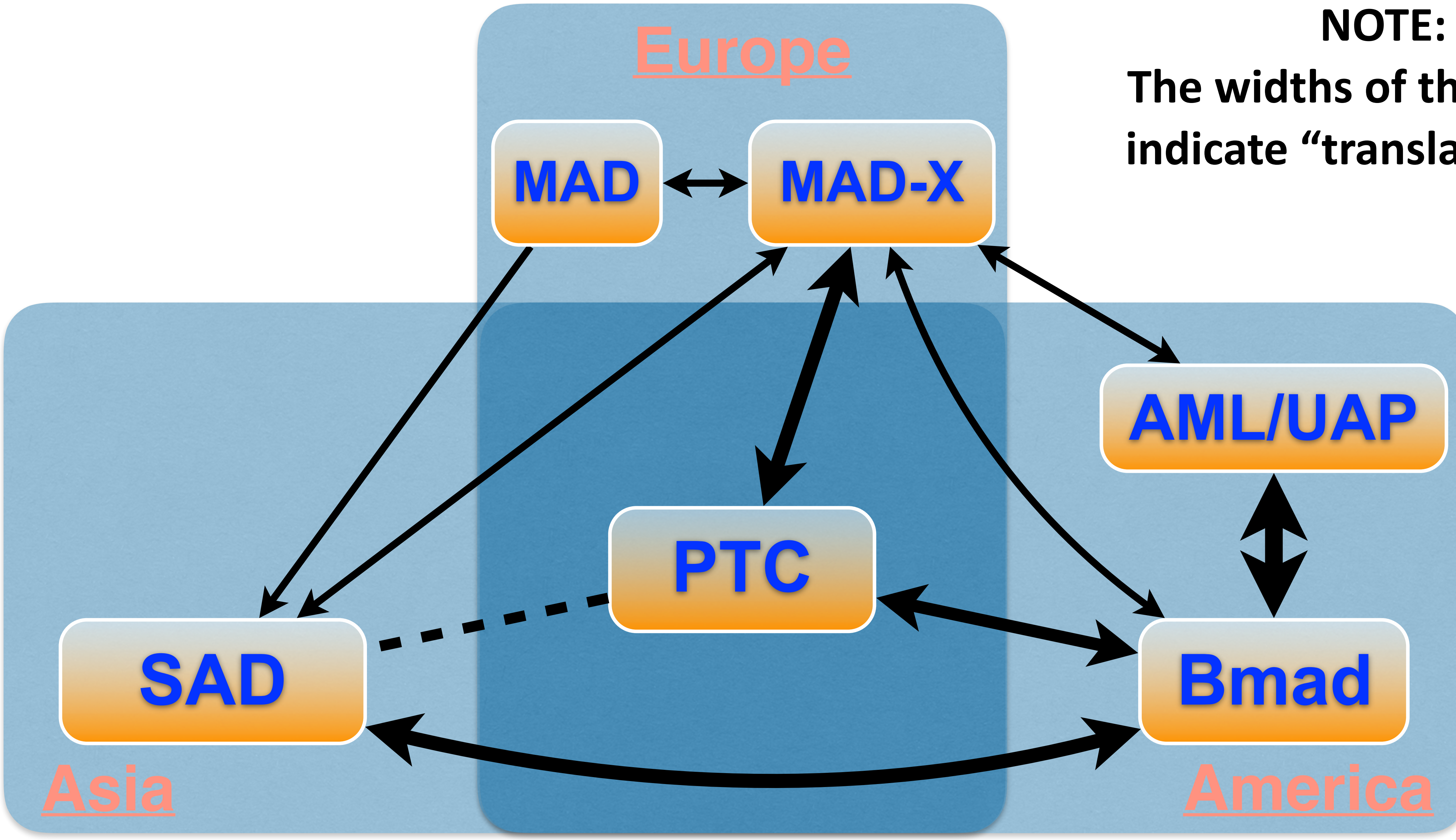
➤ Efforts for lattice translations: Others

- SAD <-> Bmad: D. Sagan(Cornell), E. Forest(KEK), et al.
- Bmad <-> UAP <-> MAD-X: D. Sagan(Cornell), et al.
- Bmad <-> MAD/MAD-X, Bmad <-> PTC, MAD-X <-> PTC: D. Sagan(Cornell), E. Forest(KEK), P. Skowronski, et al.
- SAD -> AT: N. Carmignani, S.M. Liuzzo(ESRF), et al.
- … …

2. Current status

➤ Efforts for lattice translations

- SAD and PTC: developed at KEK, many shared features (transfer maps, symplectic integrator, …)
- PTC integrated into MAD-X and Bmad

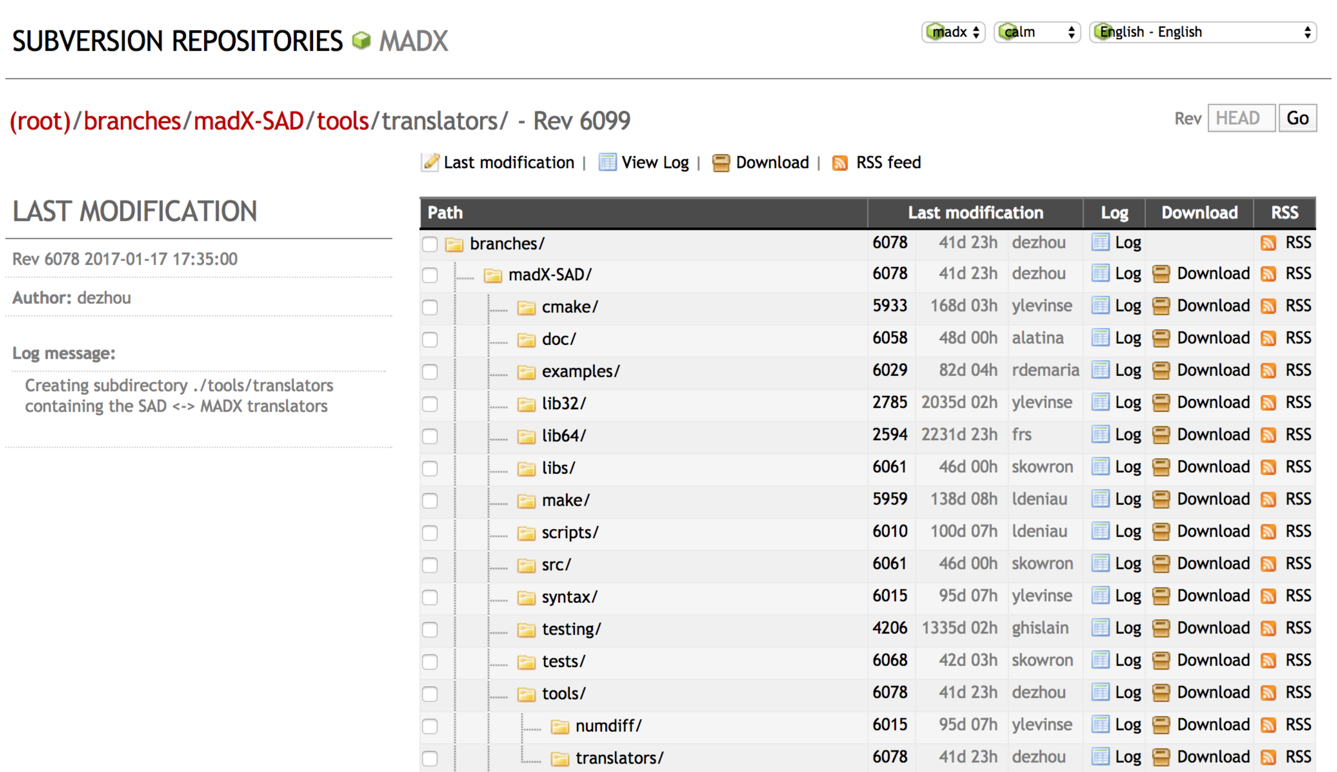


NOTE:
The widths of the arrows indicate “translatability”.

2. Current status

➤ Archive for examples of lattice translation

- MAD-X svn repository: http://svnweb.cern.ch/world/wsvn/madx/branches/madX-SAD/tools/translators/ [Thanks to L. Deniau and I. Tecker]
- Alternative: http://research.kek.jp/people/dmzhou/SAD/Translator/
- Classified by routes of translations
- Used programs: MAD-X/PTC, SAD, Bmad/PTC, UAP
- Sample jobs prepared for demonstration and benchmarks



SUBVERSION REPOSITORIES MADX

madx | calm | English - English

(root)/branches/madX-SAD/tools/translators/ - Rev 6099

Rev HEAD Go

Last modification | View Log | Download | RSS feed

LAST MODIFICATION

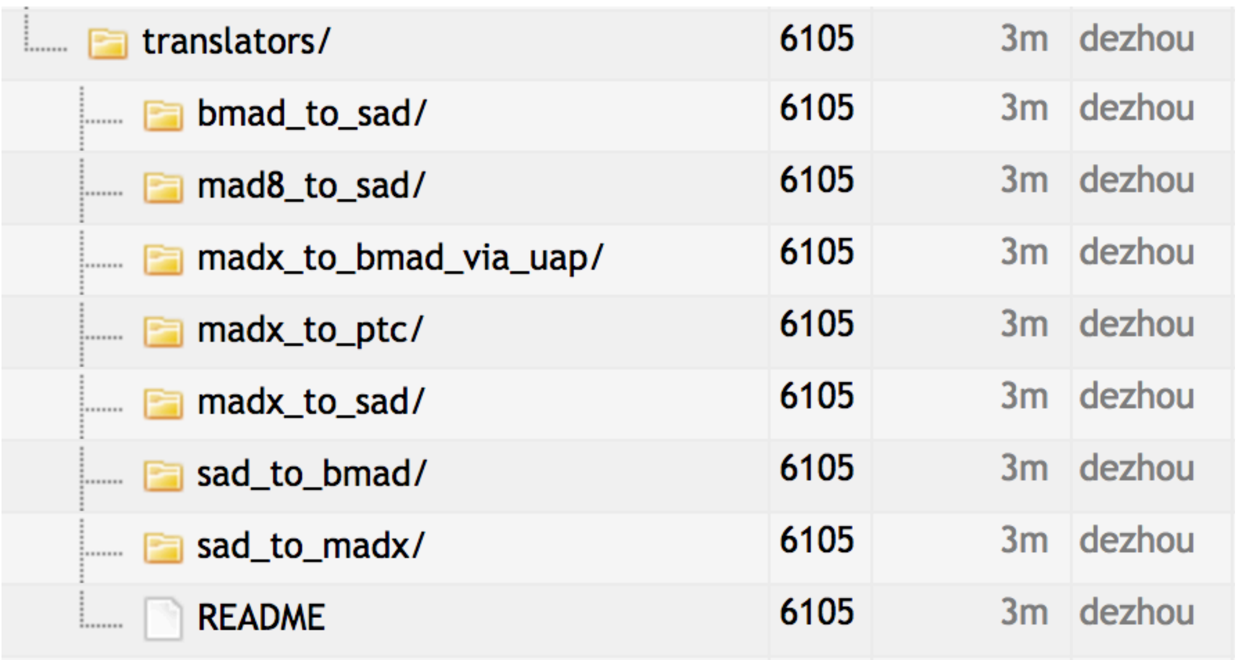
Rev 6078 2017-01-17 17:35:00

Author: dezhou

Log message:

Creating subdirectory ./tools/translators containing the SAD <-> MADX translators

Path	Last modification			Log	Download	RSS
branches/	6078	41d 23h	dezhou	Log		RSS
madX-SAD/	6078	41d 23h	dezhou	Log	Download	RSS
cmake/	5933	168d 03h	ylevinse	Log	Download	RSS
doc/	6058	48d 00h	alatina	Log	Download	RSS
examples/	6029	82d 04h	rdemaria	Log	Download	RSS
lib32/	2785	2035d 02h	ylevinse	Log	Download	RSS
lib64/	2594	2231d 23h	frs	Log	Download	RSS
libs/	6061	46d 00h	skowron	Log	Download	RSS
make/	5959	138d 08h	ldeniau	Log	Download	RSS
scripts/	6010	100d 07h	ldeniau	Log	Download	RSS
src/	6061	46d 00h	skowron	Log	Download	RSS
syntax/	6015	95d 07h	ylevinse	Log	Download	RSS
testing/	4206	1335d 02h	ghislain	Log	Download	RSS
tests/	6068	42d 03h	skowron	Log	Download	RSS
tools/	6078	41d 23h	dezhou	Log	Download	RSS
numdiff/	6015	95d 07h	ylevinse	Log	Download	RSS
translators/	6078	41d 23h	dezhou	Log	Download	RSS



translators/	6105	3m	dezhou
bmad_to_sad/	6105	3m	dezhou
mad8_to_sad/	6105	3m	dezhou
madx_to_bmad_via_uap/	6105	3m	dezhou
madx_to_ptc/	6105	3m	dezhou
madx_to_sad/	6105	3m	dezhou
sad_to_bmad/	6105	3m	dezhou
sad_to_madx/	6105	3m	dezhou
README	6105	3m	dezhou

NOTE: Please inform me IF there are possible copyright violations!

2. Current status

➤ Archive for examples of lattice translation

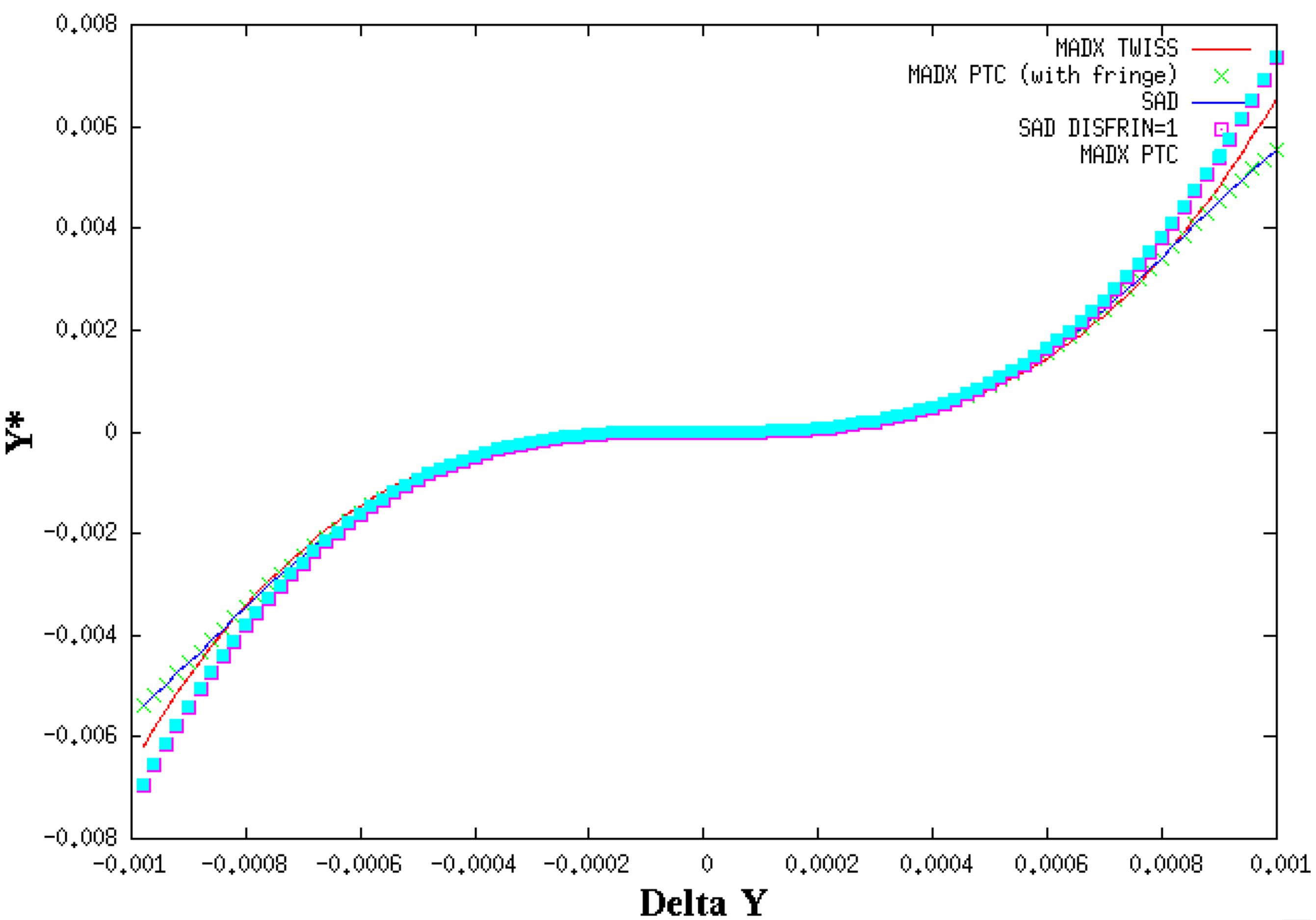
- **Sub-directories for SAD to MAD-X translation:**

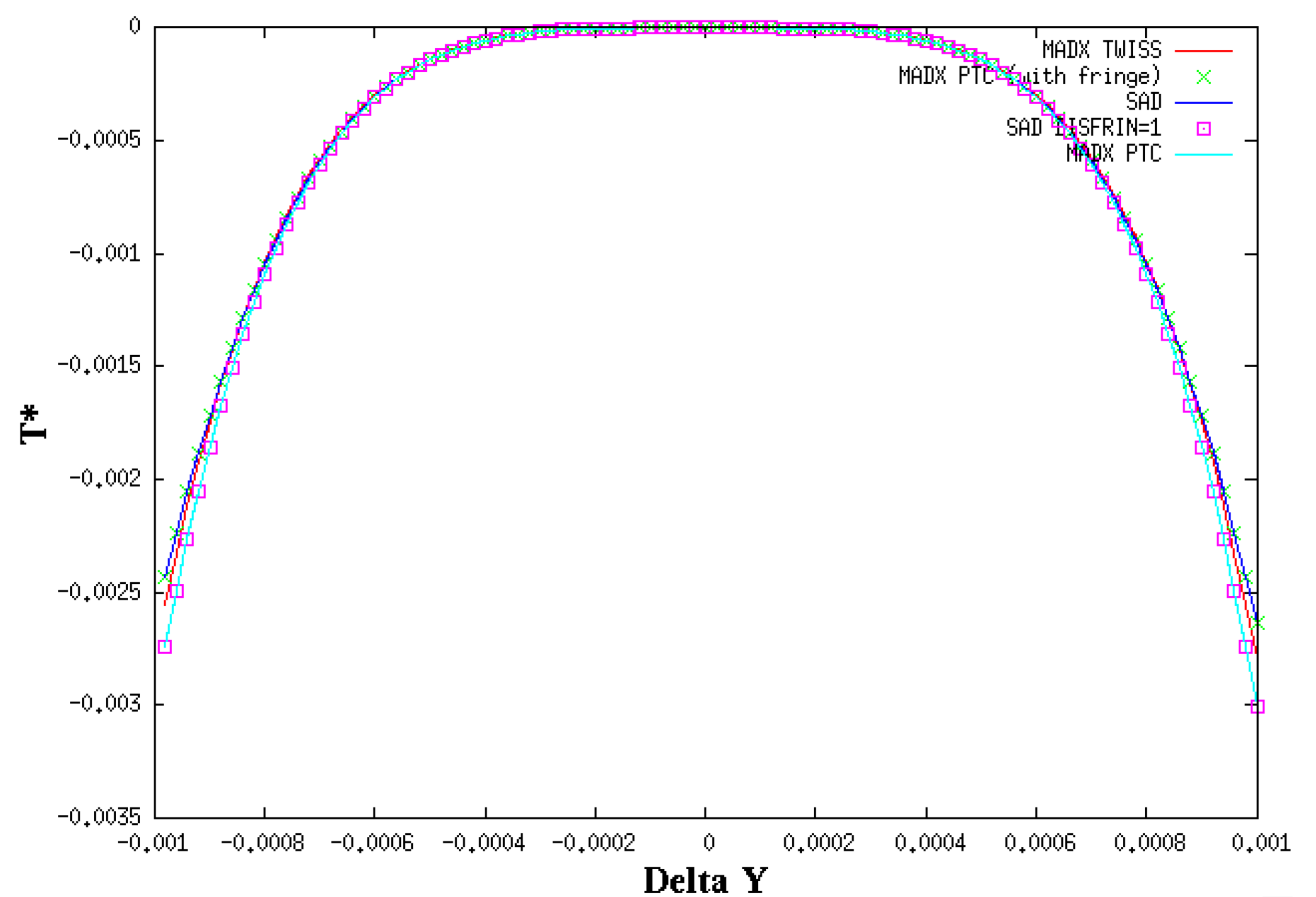
sad_to_madx/	6105
FCC-ee/	6105
FCCee_t_85_by2_nosol/	6105
Oide_method/	6105
Benchmark/	6105
._job.sad	6105
._job_sample.madx	6105
._README	6105
._slice.madx	6105
job.sad	6105
job_sample.madx	6105
log.job.sad	6105
log.job_sample	6105
matrix_sad.txt	6105
orbit_sad.txt	6105
README	6105
slice.madx	6105
survey_fccee_t_85_by2_nosol.tfs	6105
tracking_sad.txt	6105
twiss.tfs	6105
twiss_sad.txt	6105
src/	6105
SuperKEKB/	6105

3. Cases of benchmark studies

➤ A benchmark of MAD-X and SAD: CLIC FFS

- **Conditions:**
  - * Track "orbit" for a test particle
  - * No soft edge fringe, no solenoid
- **Hard edge fringe: SAD = PTC ≈ MAD-X**
- **For longitudinal transformation: SAD = PTC ≈ MAD-X (Settings for PTC: ICASE=6, TIME=true)**





Courtesy of Paul Thrane

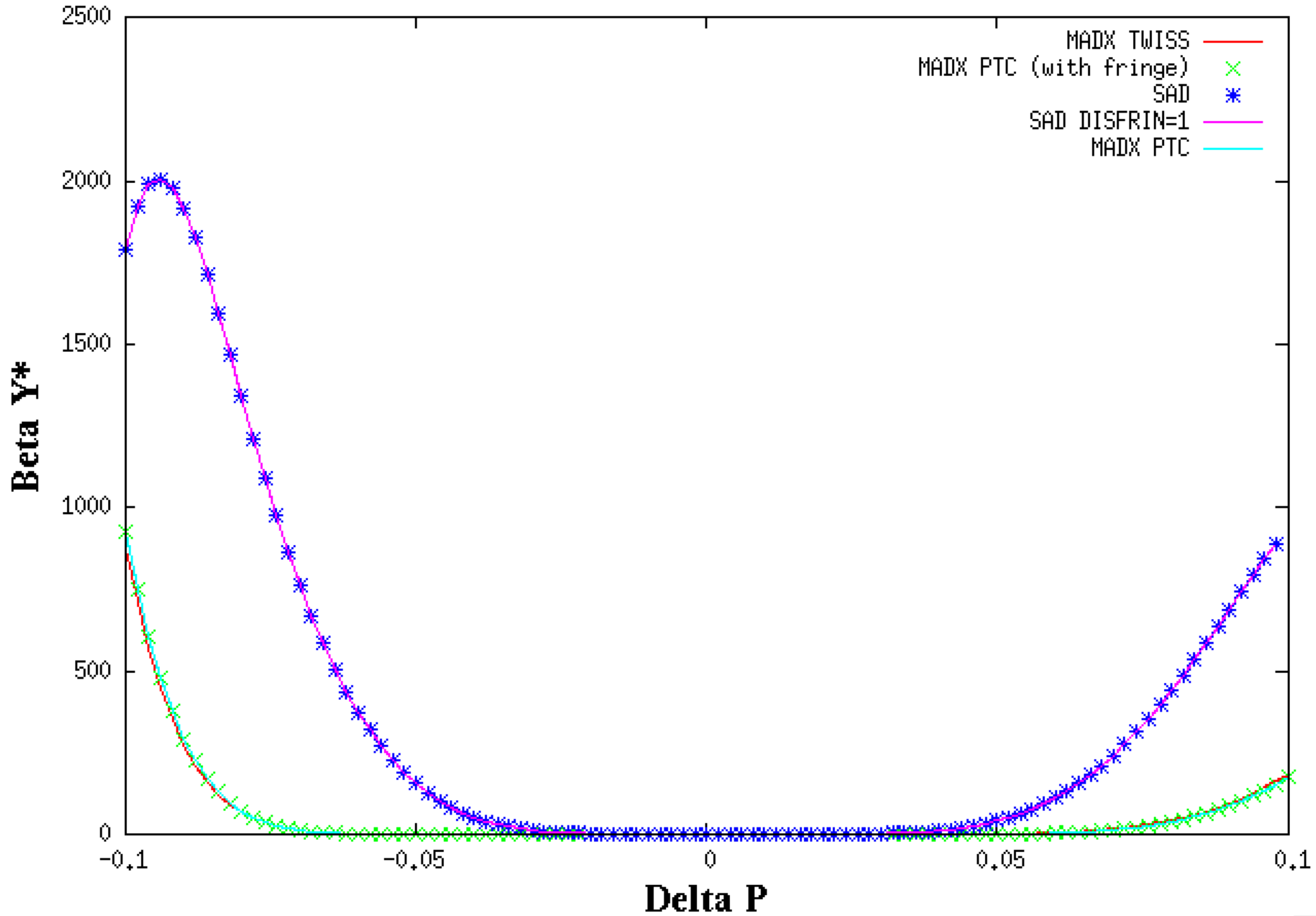
3. Cases of benchmark studies

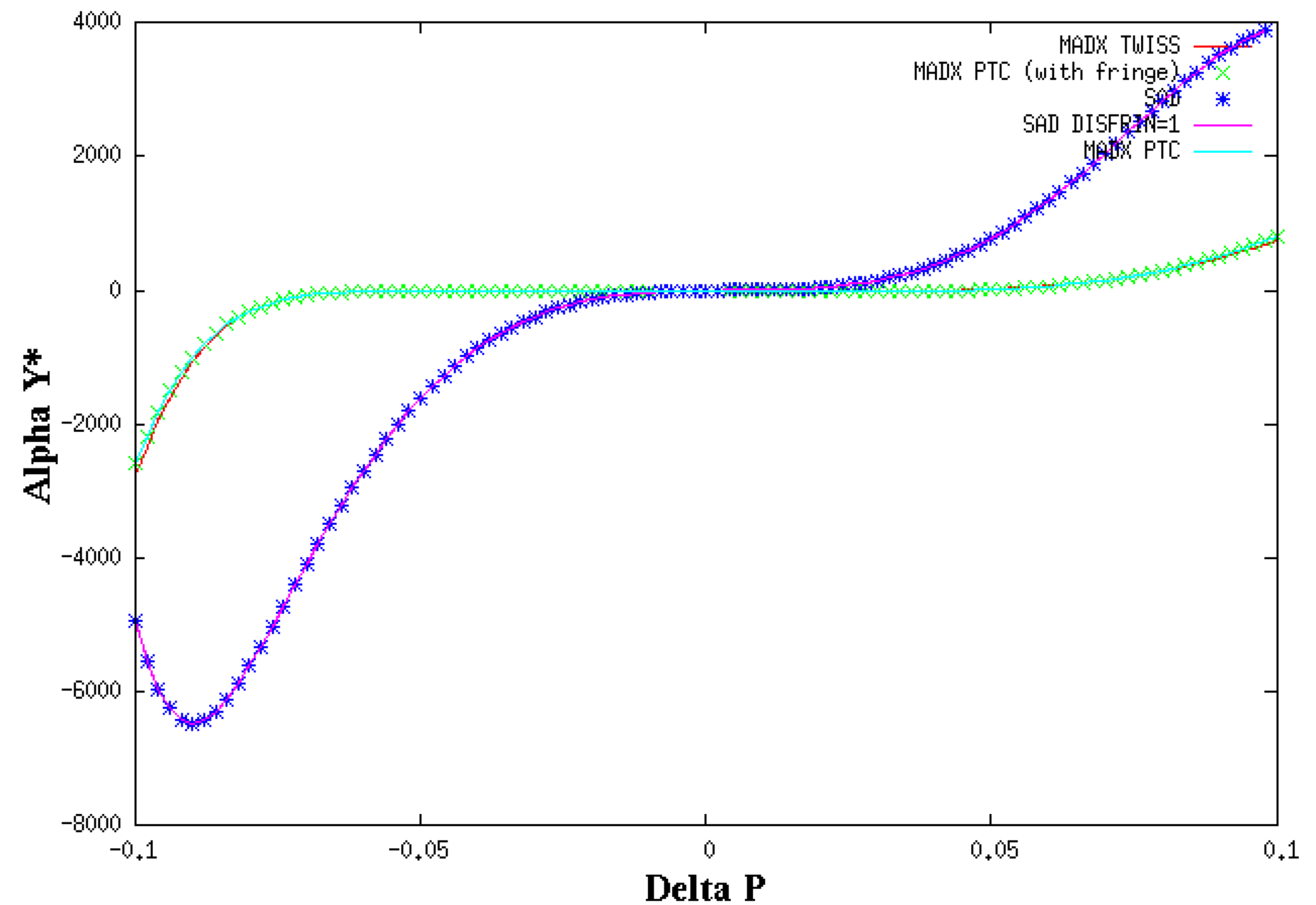
➤ A benchmark of MAD-X and SAD: CLIC FFS

- **Conditions:**
  - *** Twiss with momentum offset**
  - *** No soft edge fringe, no solenoid**
- **Hard edge fringe: Negligible**
- **SAD: High-order chromaticity due to DRIFT**
- **MAD-X/PTC: ICASE=5 (Need proper settings?)**

For a DRIFT:

$$\beta(\delta, s) = \beta_0 + \frac{s^2}{\beta_0(1+\delta)^2}$$



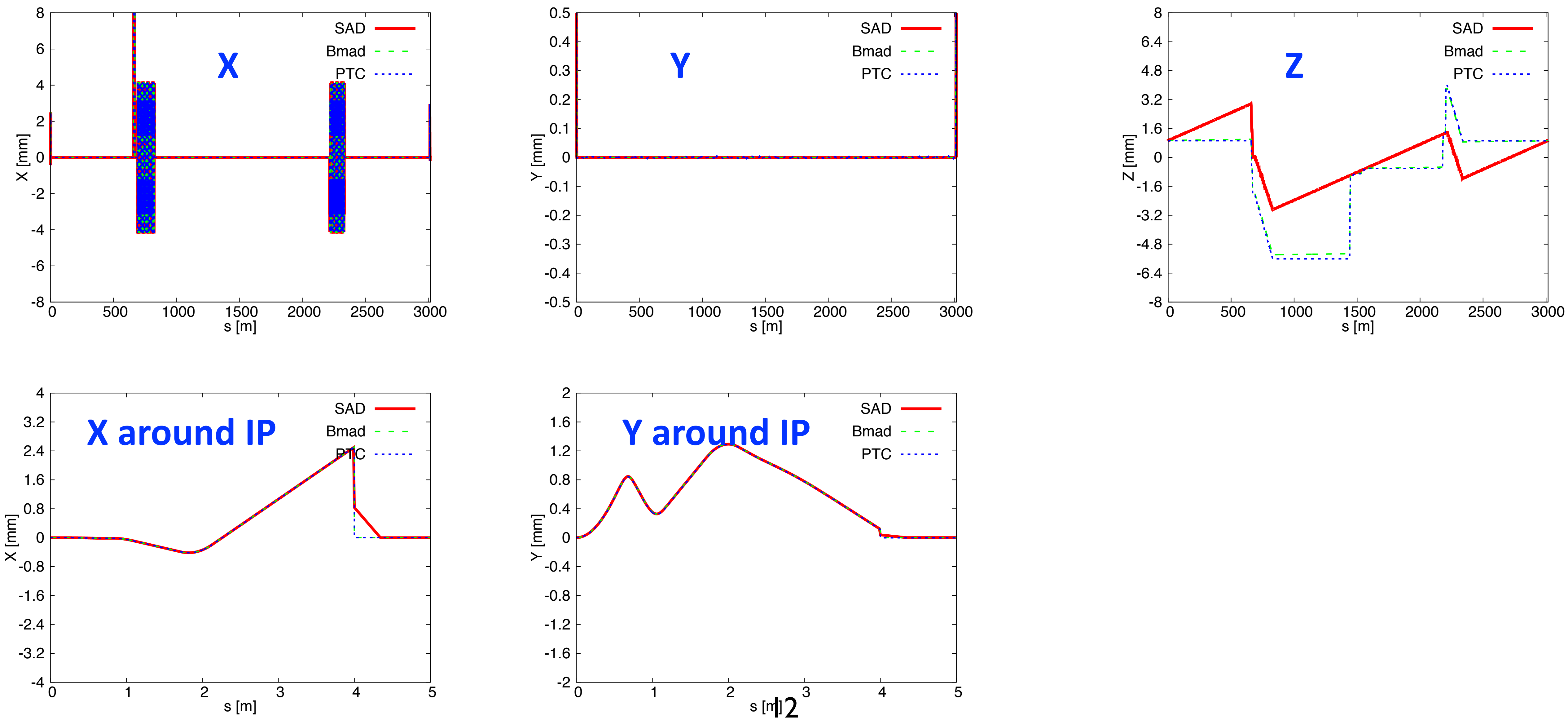


Courtesy of Paul Thrane

3. Cases of benchmark studies

➤ A benchmark of Bmad, PTC and SAD: SuperKEKB sler_1689

- Benchmark done in 2015
- Great agreement in closed orbit, even in the complicated IR
- “Time patching” was treated differently
- Closed orbit (or “fixed point”) is the first thing to compare

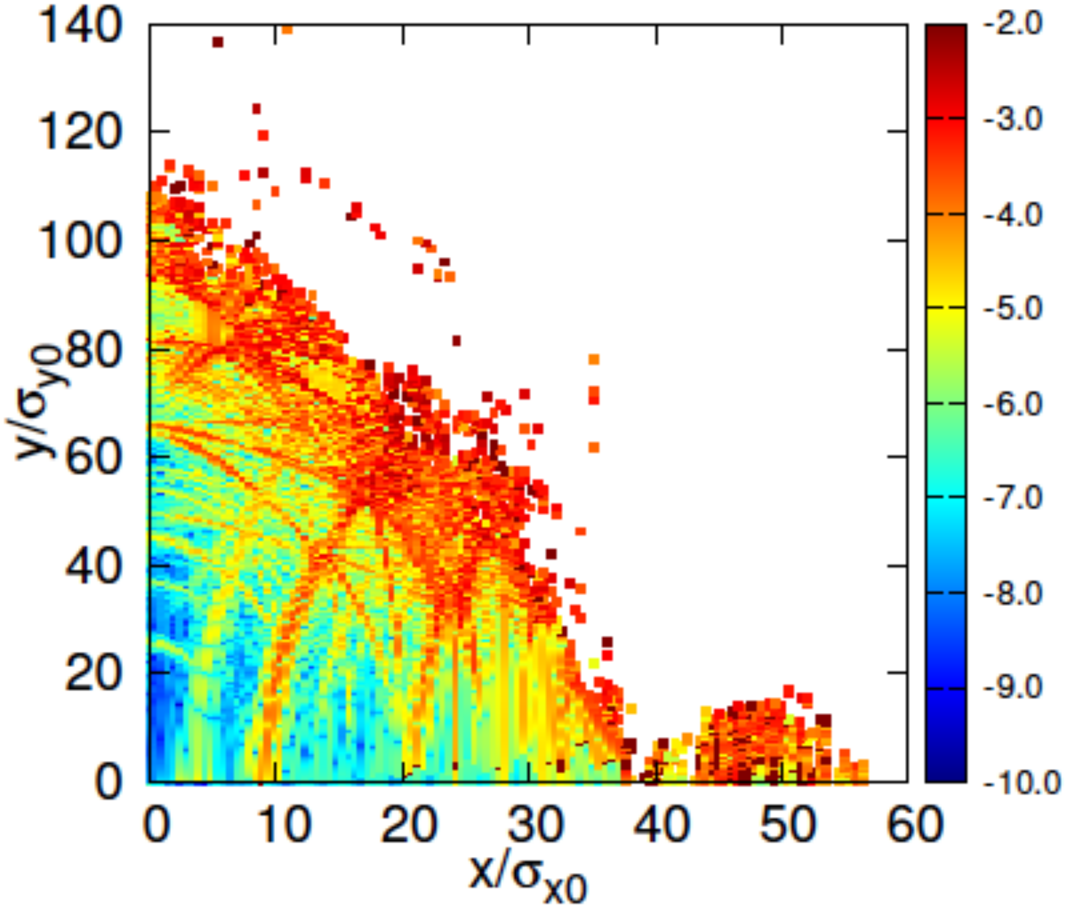


3. Cases of benchmark studies

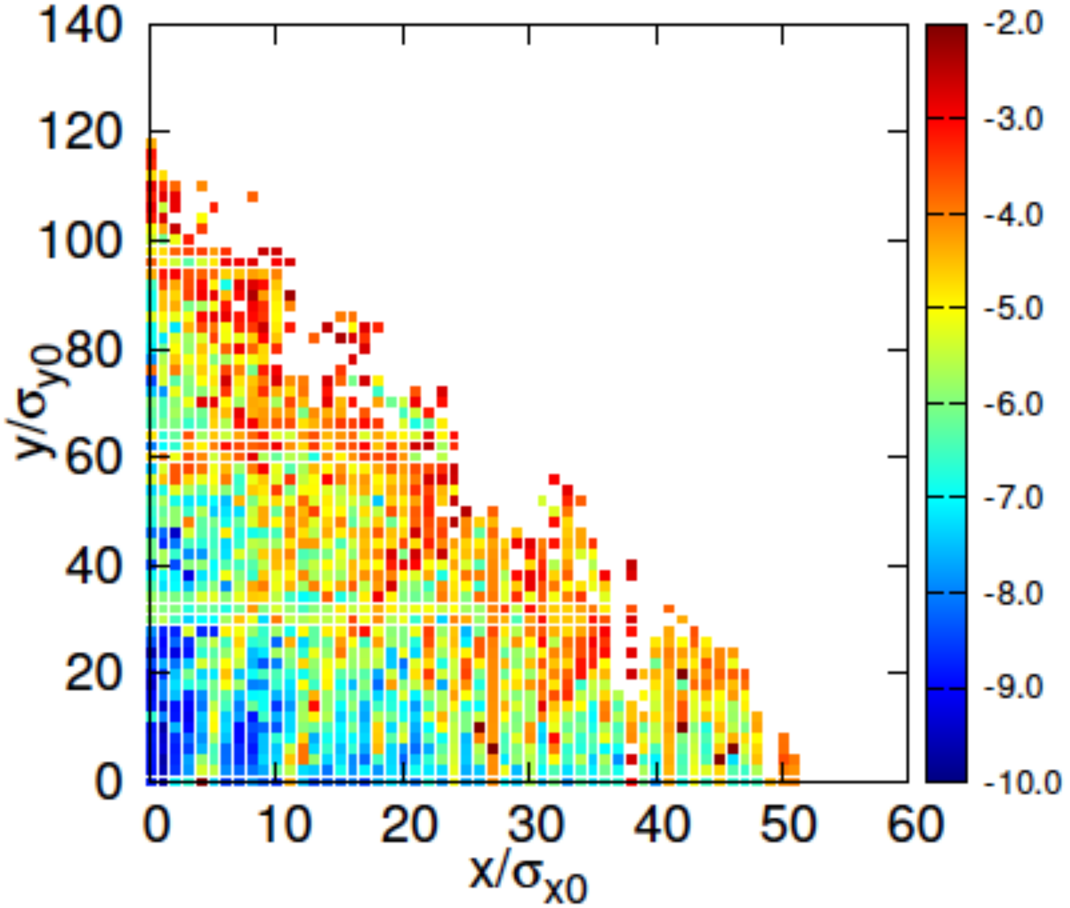
➤ A benchmark of Bmad and SAD: SuperKEKB sler_1684

- **Benchmark done in 2014**
- **Similar size of dynamic aperture**
- **Details are different, showing minor differences for nonlinear maps**

Bmad



SAD

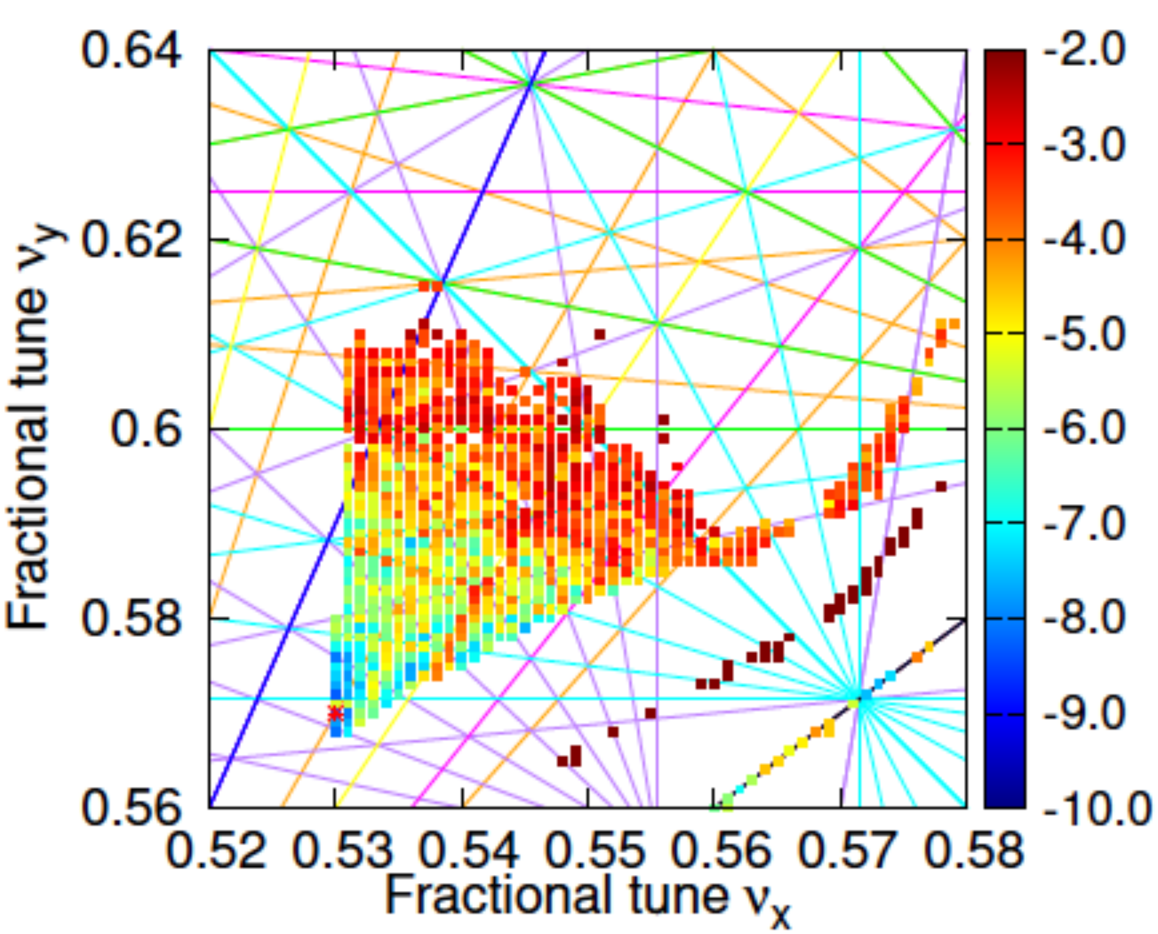


3. Cases of benchmark studies

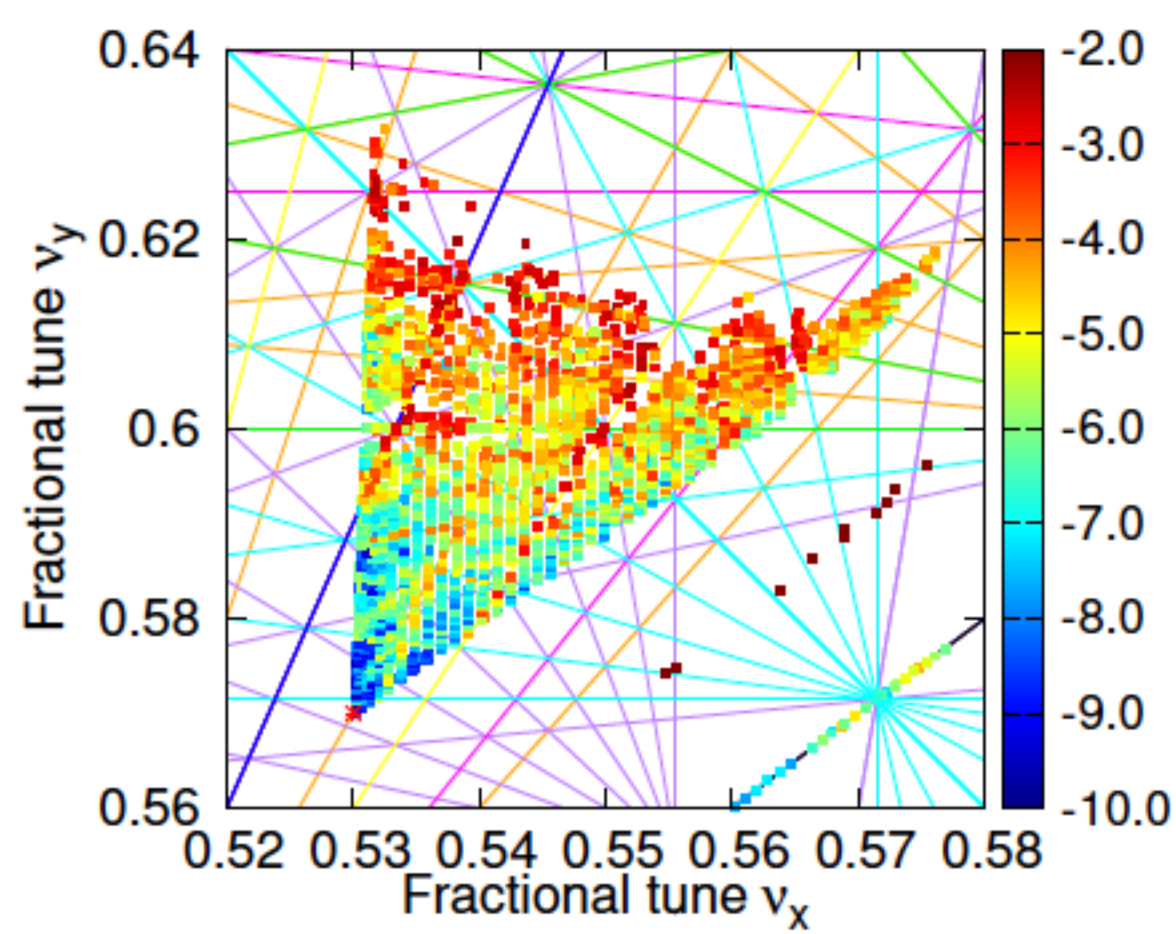
➤ A benchmark of Bmad and SAD: SuperKEKB sler_1684

- Benchmark done in 2014
- Similar footprint of tune spread
- Details are different, showing minor differences for nonlinear maps

Bmad



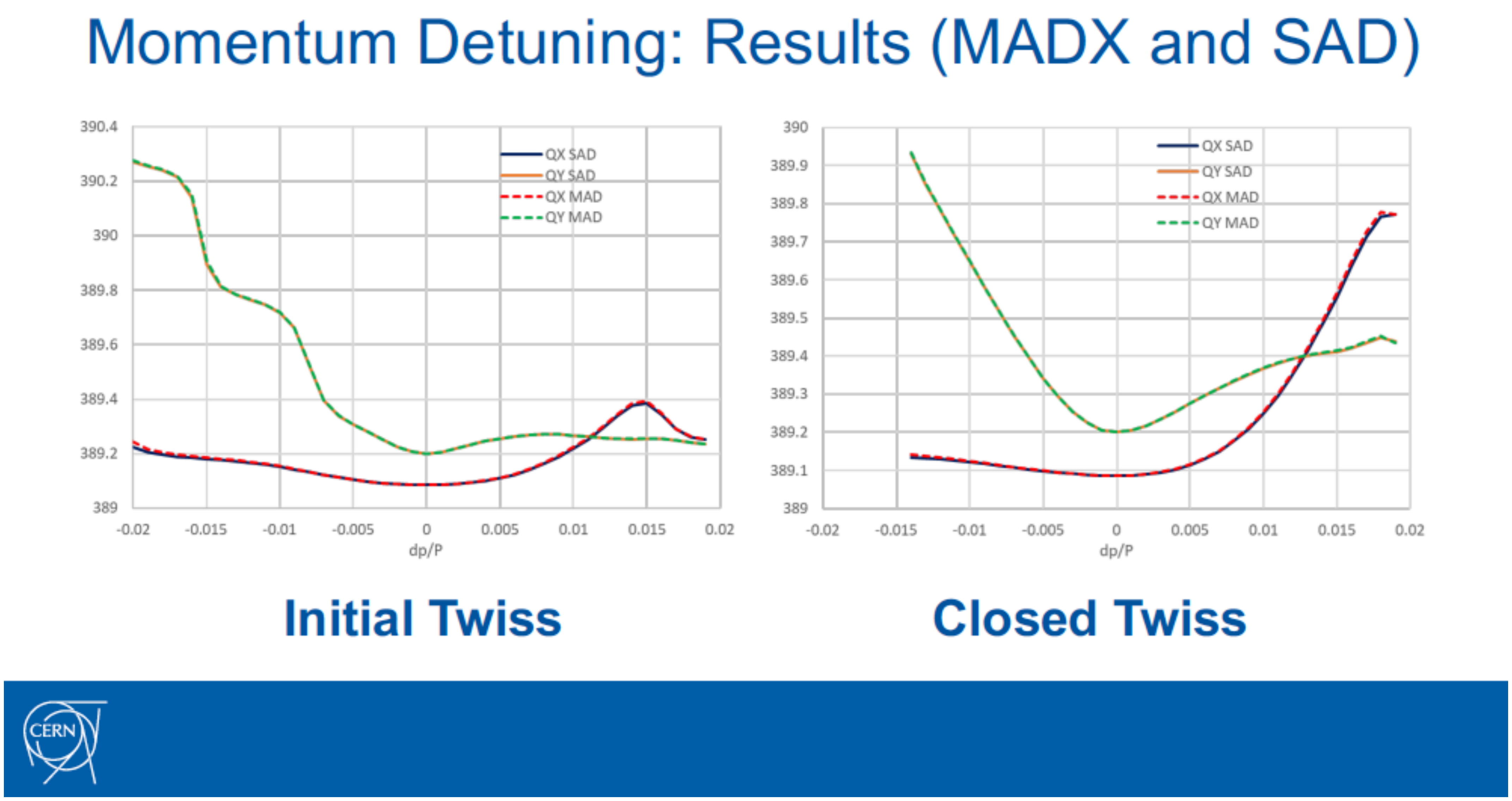
SAD



3. Cases of benchmark studies

➤ A benchmark of Bmad and SAD: FCC-ee

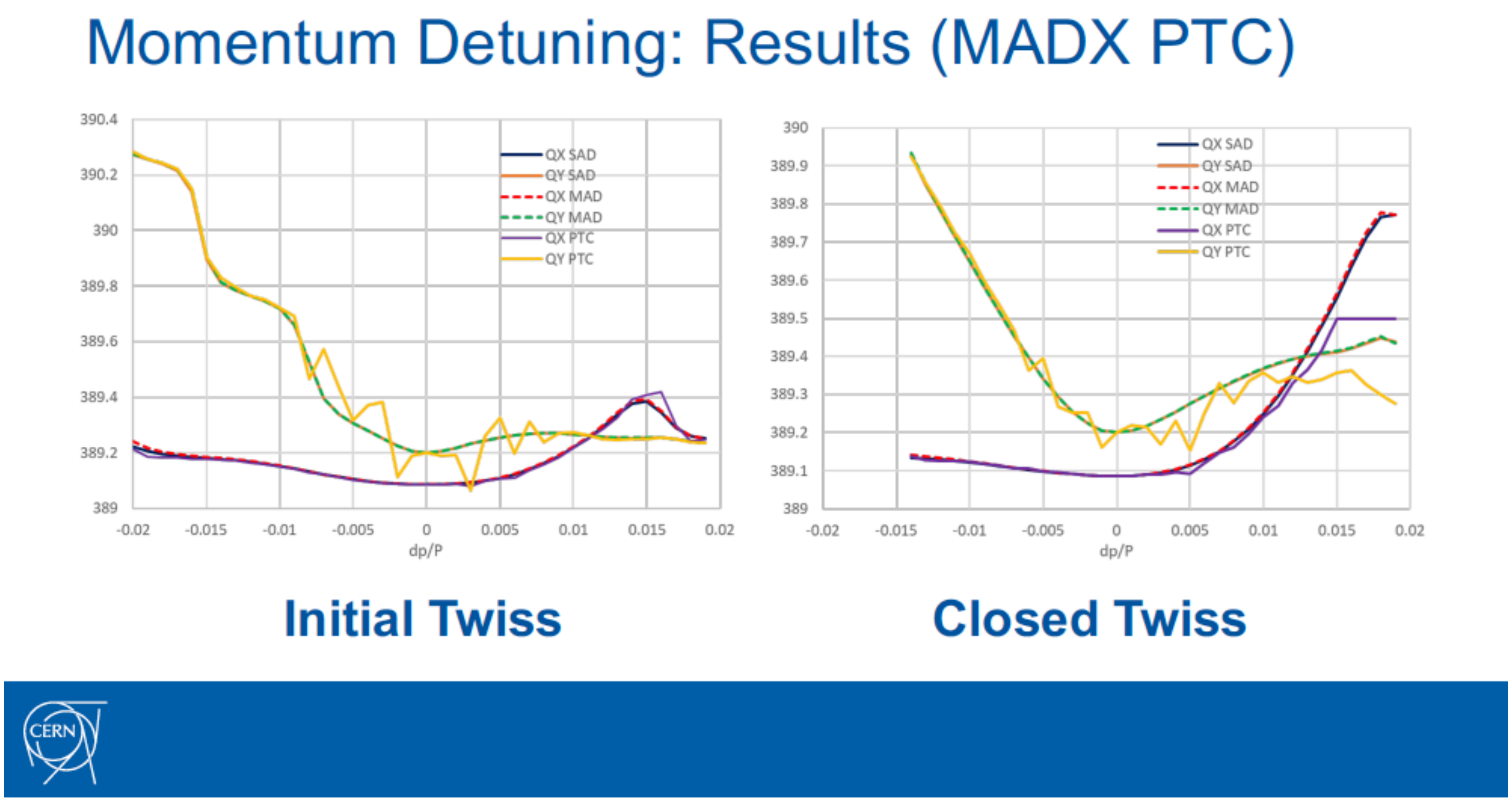
- Recent study by L. van Riesen-Haupt
- Great agreement between MAD-X and SAD



3. Cases of benchmark studies

➤ A benchmark of Bmad and SAD: FCC-ee

- **Recent study by L. van Riesen-Haupt**
- **Discrepancy against PTC might be due to slicing (and/or integration scheme)?**



4. Some applications

➤ Old findings

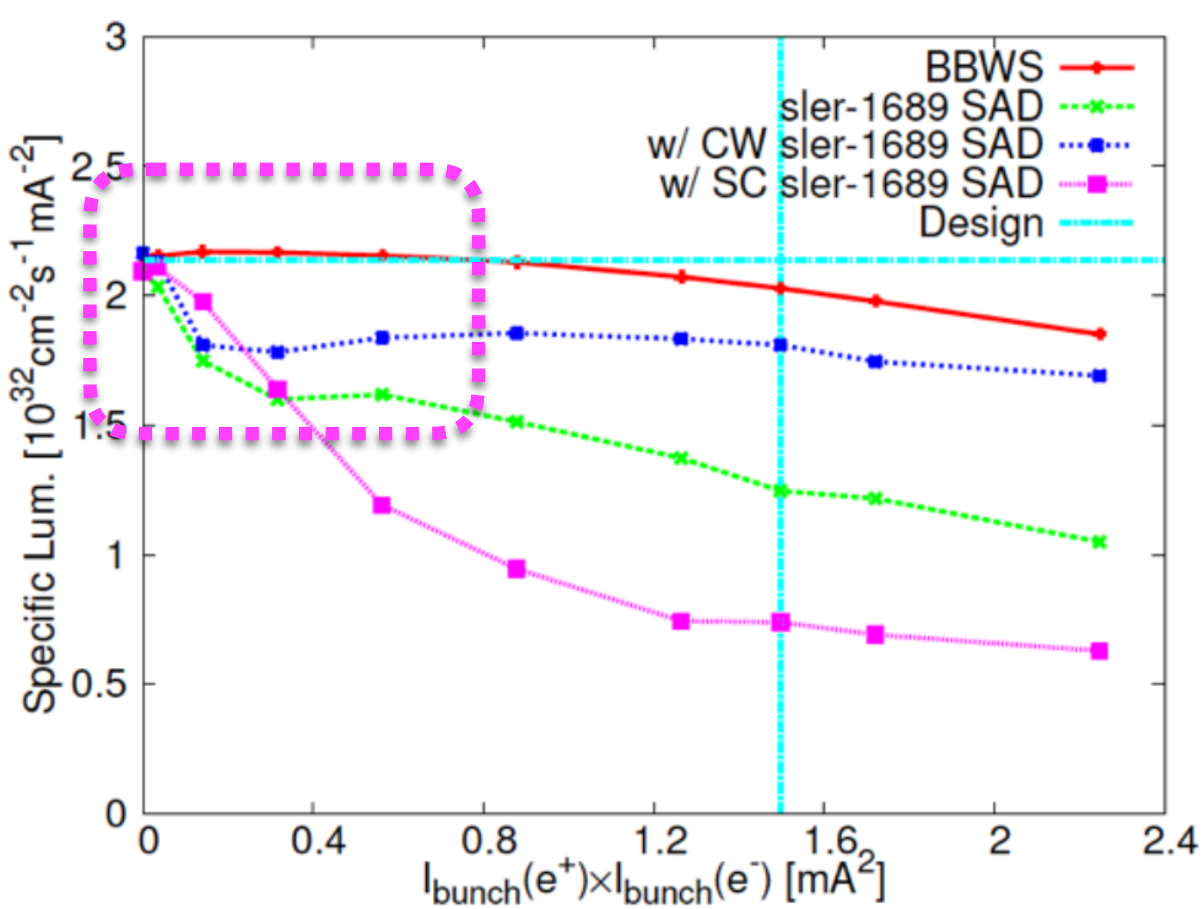
● **My talk to 20th KEKB Accelerator Review Committee, Feb. 23, 2015.**

2. BB+LN: Luminosity: LER

➤ **Realistic lattice: lum. drops at low beam currents**

➤ **Crab-waist:**

- **To cancel beam-beam driven resonances**
- **Work well at high currents, but not well at low currents**



4. Some applications

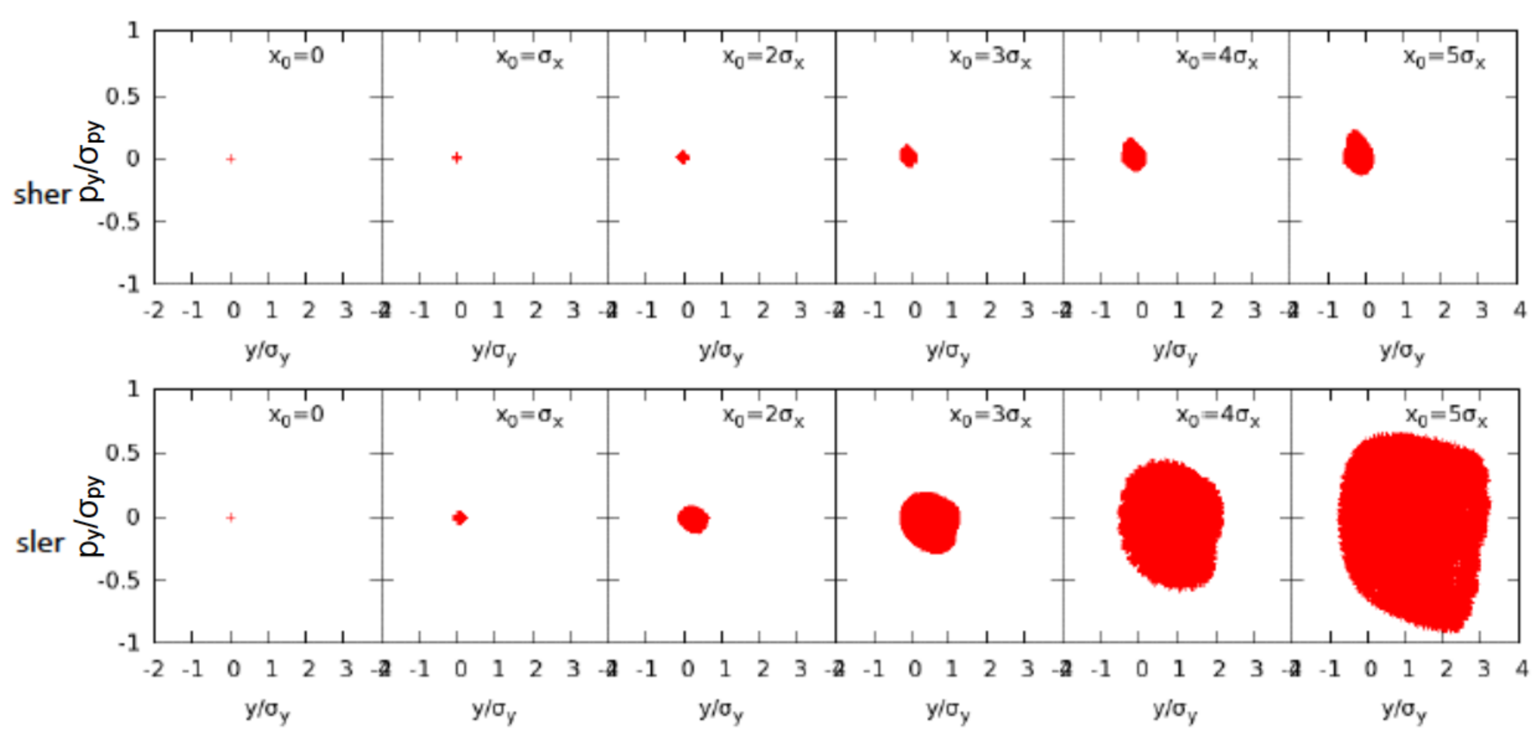
➤ Old findings

- **My talk to 20th KEKB Accelerator Review Committee, Feb. 23, 2015.**

2. BB+LN: Nonlin. X-Y coupling

- ➤ **Realistic lattice**
- ➤ **Poincare map in y direction as function of X offset**
- ➤ **Strong nonlinear X-Y coupling in LER**

sher-5767 vs ler-1689 in Y direction



From Y. Zhang

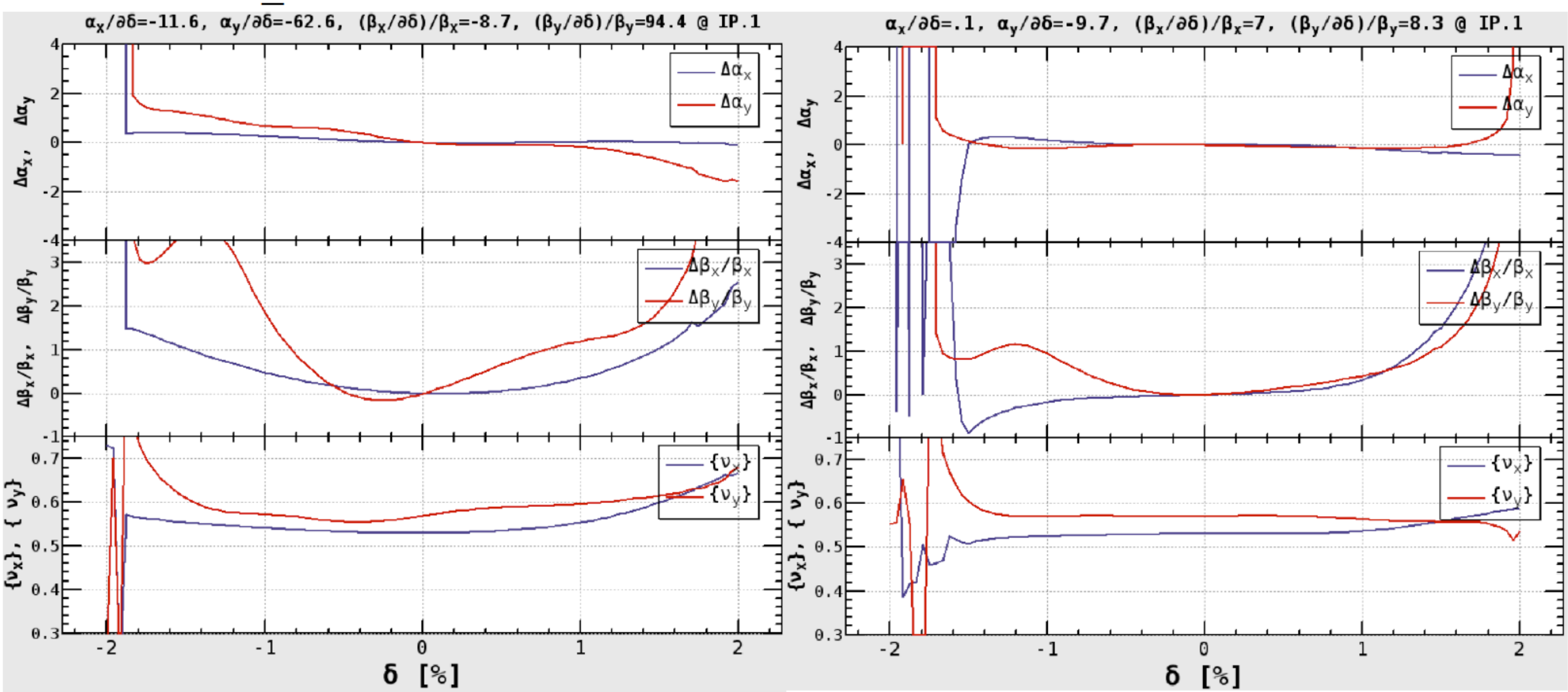
4. Some applications

➤ Nonlinear optimization with chromatic constraints [by H. Sugimoto, SuperKEKB mini-optics meeting, Sep. 8, 2016]

- **Chromatic $\beta_{x,y}$ and $\nu_{x,y}$ correspond to RDTs of h_{2000e}/h_{0200e} (X), h_{0020e}/h_{0002e} (Y), and h_{1100e} (X), h_{0011e} (Y), respectively**

sler_1689 W/O constraints

sler_1689 W/ constraints



4. Some applications

➤ Resonance driving terms (RDTs) indicate lattice nonlinearity

The effective Hamiltonian of a ring can be normalized in resonance bases [Ref. E. Forest, *Beam Dynamics – A New Attitude and Framework*, 1998].

For a ring with n elements, one can normalize the one turn map $\mathcal{M}_{1\to n}$ as [Ref. L. Yang *et al.*, Phys. Rev. ST Accel. Beams 14, 054001 (2011)]

$$\mathcal{M}_{1\to n} = \mathcal{A}_1^{-1} e^{:h:} \mathcal{R}_{1\to n} \mathcal{A}_1,$$

with $\mathcal{R}$: rotation, $e^{:h:}$: nonlinear Lie map, $\mathcal{A}_1$: normalizing map. Assume no coupling (the theory can be generalized for nonzero coupling), $\mathcal{A}_i$ in x plane at the ith element can be approximated in perturbation theory as

$$\mathcal{A}_i x = \sqrt{\beta_{x,i}}\, x + \eta_{x,i}\delta,$$

$$\mathcal{A}_i p_x = \frac{-\alpha_{x,i} x + p_x}{\sqrt{\beta_{x,i}}} + \eta'_{x,i}\delta.$$

4. Some applications

➤ RDTs indicate lattice nonlinearity

In the resonance basis, using action-angle variables (J, ϕ) one can write

$$h_x^{\pm} \equiv \sqrt{J_x} e^{\pm i\phi_x} = \frac{X \mp iP_x}{\sqrt{2}},$$

$$\mathcal{R}_{i\to j} h_x^{\pm} = \mathcal{R}_{i\to j} \sqrt{J_x} e^{\pm i\phi_x} = e^{\pm i\mu_{i\to j,x}} h_x^{\pm},$$

where $\mu_{i\to j,x}$ is the phase advance of $i \to j$. Consequently, the potential of a multipole magnetic field can be expanded in the resonance bases of h_{abcde} as

$$h = \sum h_{abcde} h_x^{+a} h_x^{-b} h_y^{+c} h_y^{-d} \delta^e.$$

Each h_{abcde} (a complex number in general) drives a certain resonance, and is an explicit function of magnet strengths, beta functions and dispersions.

4. Some applications

➤ RDTs indicate lattice nonlinearity

The effective Hamiltonian corresponding to chromaticity is

$$h_c = \sum h_{1100e} h_x^{+1} h_x^{-1} \delta^e + \sum h_{0011e} h_y^{+1} h_y^{-1} \delta^e,$$

$$h_c = J_x \sum h_{1100e} \delta^e + J_y \sum h_{0011e} \delta^e.$$

Then the tunes are calculated as

$$\nu_x = -\frac{1}{2\pi} \frac{\partial h_c}{\partial J_x} = -\frac{1}{2\pi} \sum h_{1100e} \delta^e,$$

$$\nu_y = -\frac{1}{2\pi} \frac{\partial h_c}{\partial J_y} = -\frac{1}{2\pi} \sum h_{0011e} \delta^e.$$

Therefore the RDTs of h_{1100e} and h_{0011e} correspond to linear and high-order chromaticity.

4. Some applications

➤ RDTs indicate lattice nonlinearity

h_{abcde}	Driving effects
h_{11001}, h_{00111}	Linear chromaticity ζ_x, ζ_y
$h_{21000}, h_{12000} \| h_{10110}, h_{01110}$	ν_x $[(J_x)^{3/2}] \| [(J_x)^{1/2}(J_y)]$
$h_{30000}, h_{03000} \| h_{00300}, h_{00030}$	$3\nu_x$ $[(J_x)^{3/2}] \| 3\nu_y$ $[(J_y)^{3/2}]$
$h_{10020}, h_{01200} \| h_{10200}, h_{01020}$	$\nu_x - 2\nu_y \| \nu_x + 2\nu_y$ $[(J_x)^{1/2}(J_y)]$
$h_{20010}, h_{02100} \| h_{20100}, h_{02010}$	$2\nu_x - \nu_y \| 2\nu_x + \nu_y$ $[(J_x)(J_y)^{1/2}]$
$h_{00210}, h_{00120} \| h_{11100}, h_{11010}$	ν_y $[(J_y)^{3/2}] \| [(J_x)(J_y)^{1/2}]$
$h_{22000}, h_{00220}, h_{11110}$	$d\nu_x/dJ_x, d\nu_y/dJ_y, d\nu_{x,y}/dJ_{y,x}$
$h_{40000}, h_{04000} \| h_{00400}, h_{00040}$	$4\nu_x$ $[(J_x)^2] \| 4\nu_y$ $[(J_y)^2]$
$h_{31000}, h_{13000} \| h_{20110}, h_{02110}$	$2\nu_x$ $[(J_x)^2] \| [(J_x)(J_y)]$
$h_{00310}, h_{00130} \| h_{11200}, h_{11020}$	$2\nu_y$ $[(J_y)^2] \| [(J_x)(J_y)]$
$h_{20020}, h_{02200} \| h_{20200}, h_{02020}$	$2\nu_x - 2\nu_y \| 2\nu_x + 2\nu_y$ $[(J_x)(J_y)]$
$h_{30010}, h_{03100} \| h_{30100}, h_{03010}$	$3\nu_x - \nu_y \| 3\nu_x + \nu_y$ $[(J_x)^{3/2}(J_y)^{1/2}]$
$h_{10030}, h_{01300} \| h_{10300}, h_{01030}$	$\nu_x - 3\nu_y \| \nu_x + 3\nu_y$ $[(J_x)^{1/2}(J_y)^{3/2}]$

Table : Low-order driving terms.

4. Some applications

➤ **RDTs indicate lattice nonlinearity: Analytic theories according to J. Bengtsson, SLS Note 9/97**

- **Linear chromaticity:**

$$h_{11001} = \frac{1}{4}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{xi} + O\left(\delta^2\right),$$

$$h_{00111} = -\frac{1}{4}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{yi} + O\left(\delta^2\right)$$

- **Chromatic beta functions:**

$$h_{20001} = h_{02001}^{*} = \frac{1}{8}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{xi}e^{i2\mu_{xi}} + O\left(\delta^2\right),$$

$$h_{00201} = h_{00021}^{*} = -\frac{1}{8}\sum_{i=1}^{N}\left[(b_2L)_i - 2(b_3L)_i\eta_{xi}^{(1)}\right]\beta_{yi}e^{i2\mu_{yi}} + O\left(\delta^2\right),$$

- **Chromatic dispersion:**

$$h_{10002} = h_{01002}^{*} = \frac{1}{2}\sum_{i=1}^{N}\left[(b_2L)_i - (b_3L)_i\eta_{xi}^{(1)}\right]\eta_{xi}^{(1)}\sqrt{\beta_{xi}}e^{i\mu_{xi}} + O\left(\delta^3\right)$$

4. Some applications

➤ **RDTs indicate lattice nonlinearity: Analytic theories according to J. Bengtsson, SLS Note 9/97**

- **First order geometric terms (amplitude-dependent):**

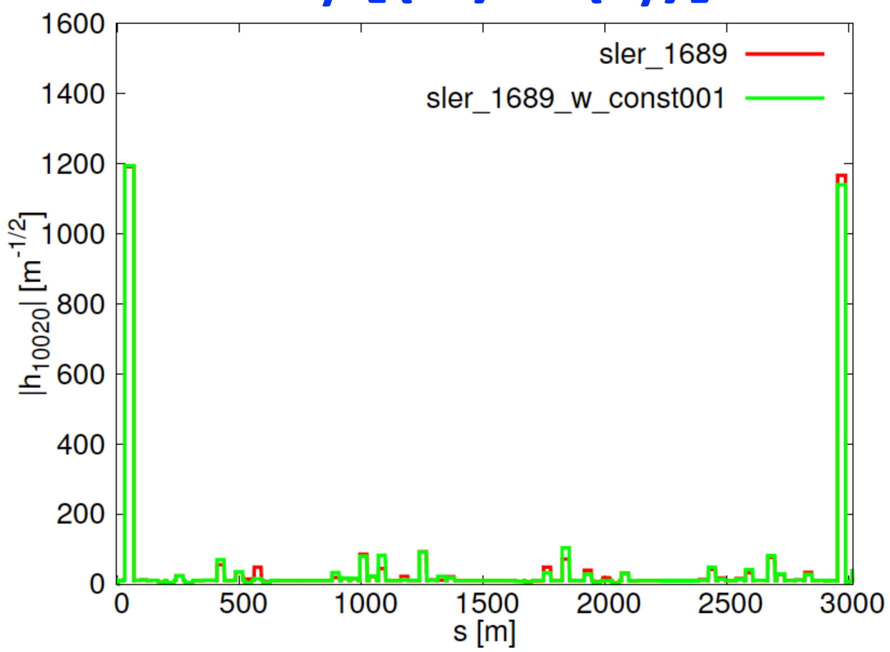
$$
\begin{aligned}
h_{21000} &= h^*_{12000} = -\frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{3/2}e^{i\mu_{xi}}, \\
h_{30000} &= h^*_{03000} = -\frac{1}{24}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{3/2}e^{i3\mu_{xi}}, \\
h_{10110} &= h^*_{01110} = \frac{1}{4}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{1/2}\beta_{yi}e^{i\mu_{xi}}, \\
h_{10020} &= h^*_{01200} = \frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{1/2}\beta_{yi}e^{i(\mu_{xi}-2\mu_{yi})}, \\
h_{10200} &= h^*_{01020} = \frac{1}{8}\sum_{i=1}^{N}(b_{3i}L)\,\beta_{xi}^{1/2}\beta_{yi}e^{i(\mu_{xi}+2\mu_{yi})}
\end{aligned}
$$

4. Some applications

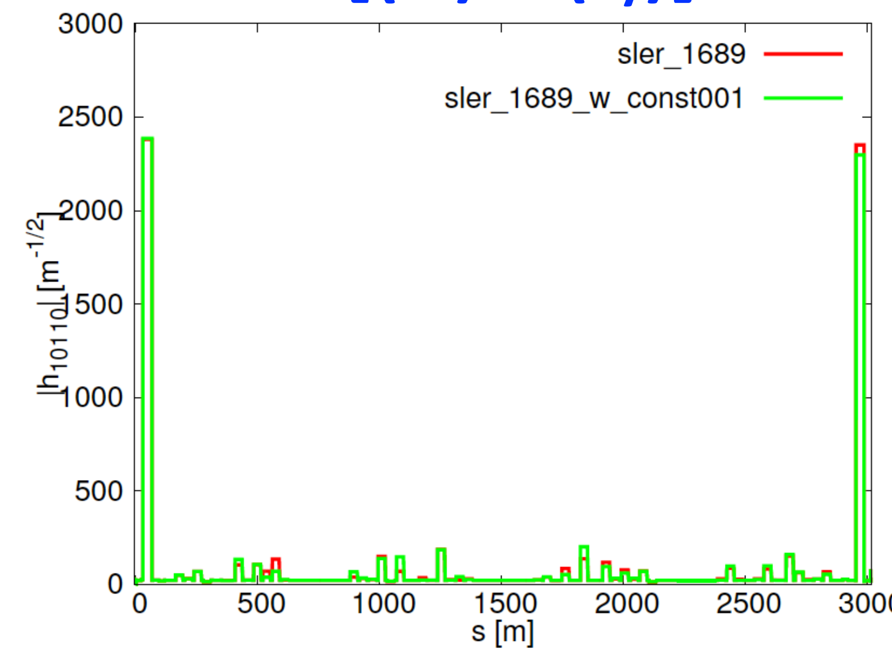
➤ Integration of RDTs along the whole ring

- **Almost perfect cancellation of 3rd order RDTs in the arc sections**

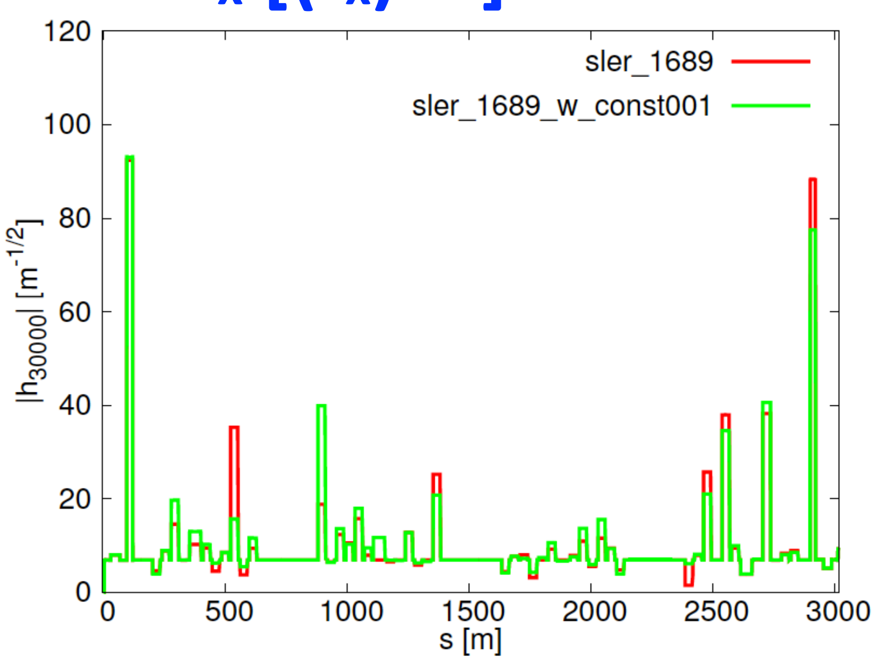
$\nu_x-2\nu_y$ $[(J_x)^{1/2}(J_y)]$



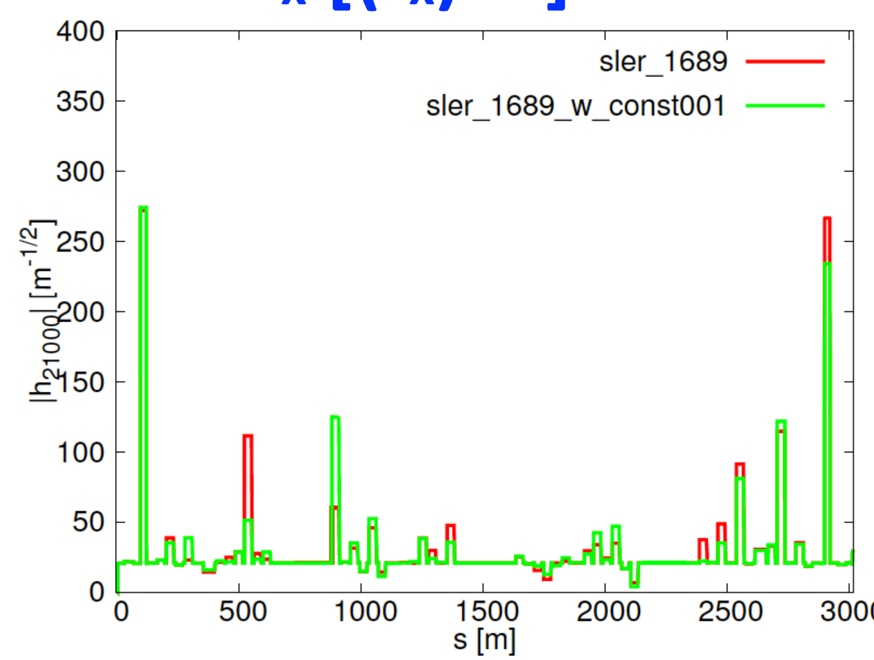
ν_x $[(J_x)^{1/2}(J_y)]$



$3\nu_x$ $[(J_x)^{3/2}]$



ν_x $[(J_x)^{3/2}]$

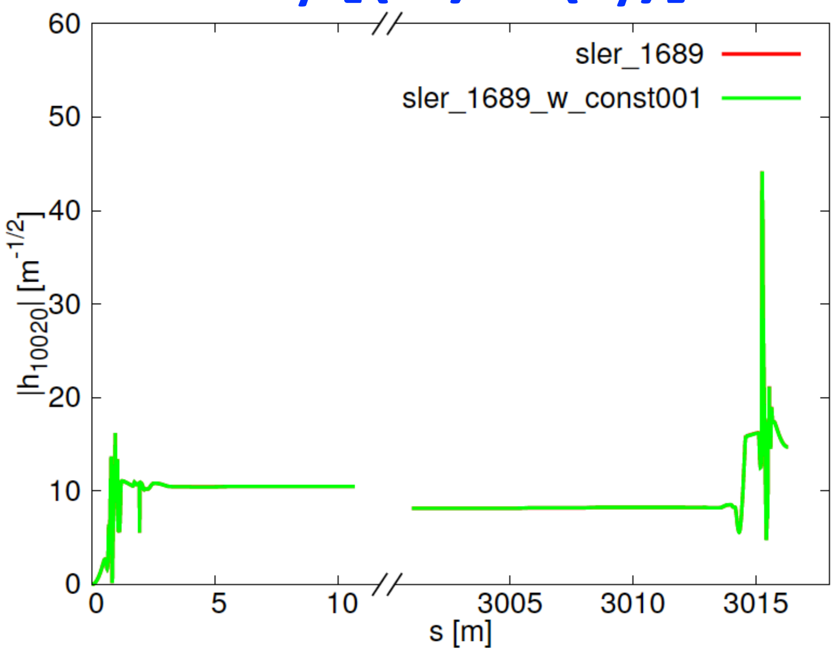


4. Some applications

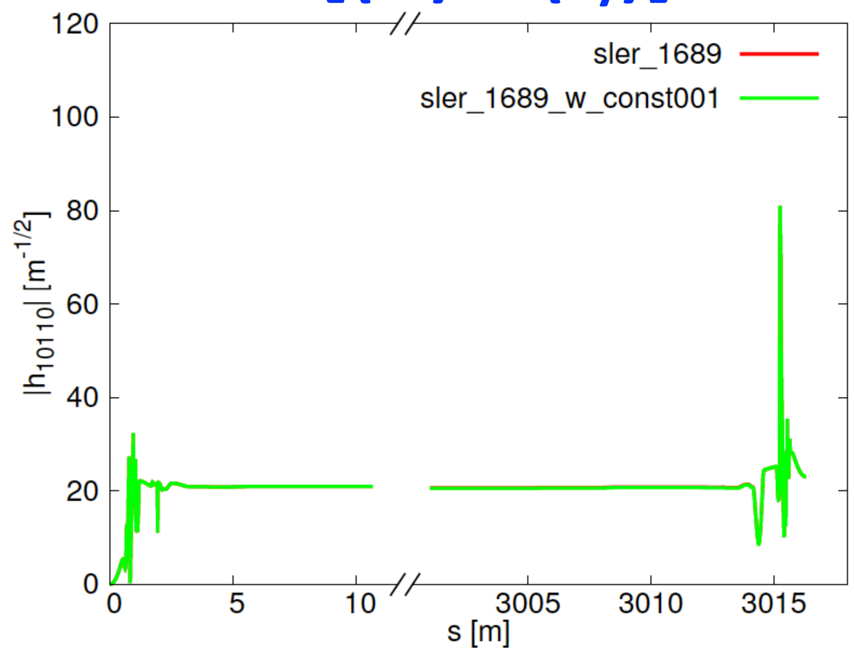
➤ Integration of RDTs along the whole ring

- **FFS contributes most of residual RDTs**

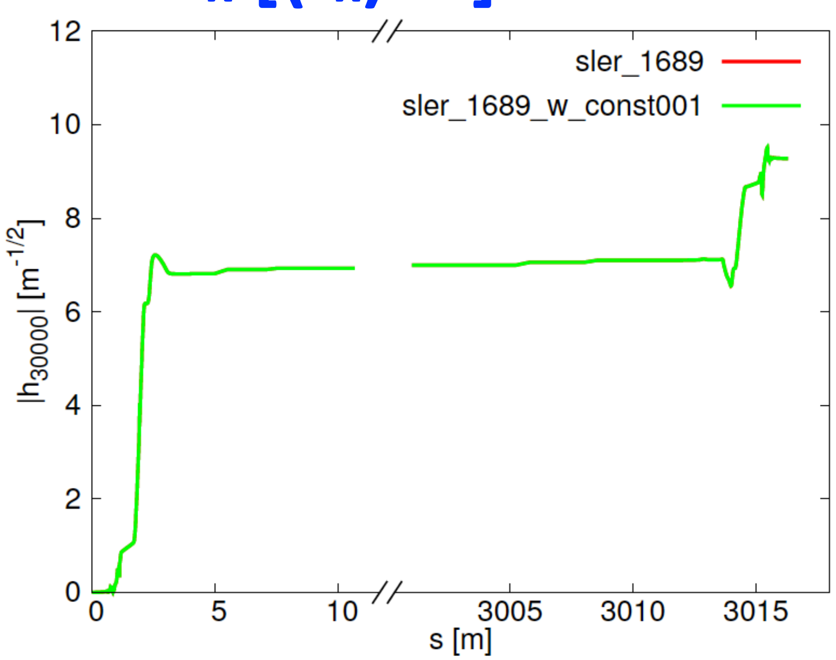
ν_x-2ν_y [$(J_x)^{1/2}(J_y)$]



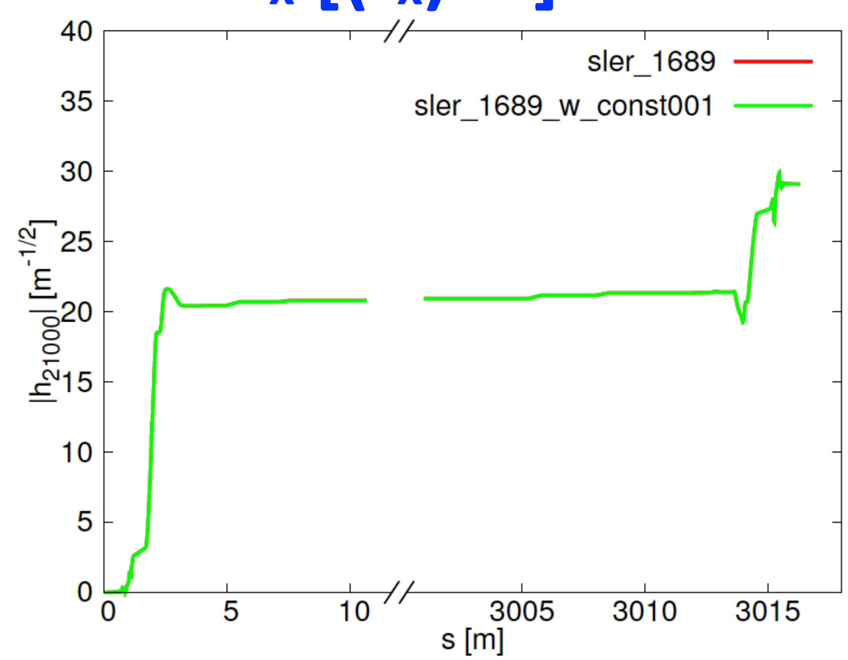
ν_x [$(J_x)^{1/2}(J_y)$]



3ν_x [$(J_x)^{3/2}$]



ν_x [$(J_x)^{3/2}$]

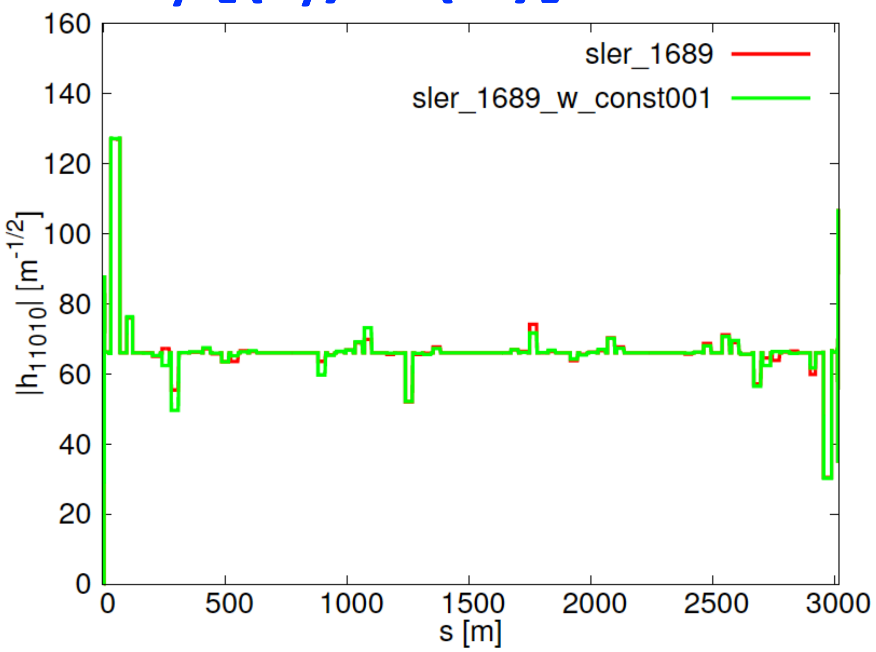


4. Some applications

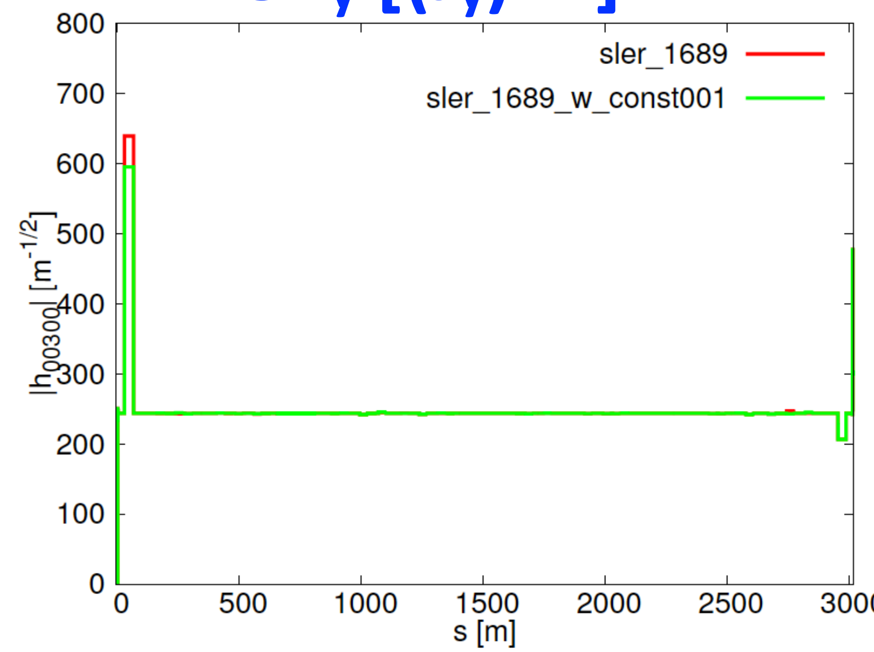
➤ Integration of RDTs along the whole ring

- **Almost perfect cancellation of 3rd order RDTs in the arc sections**

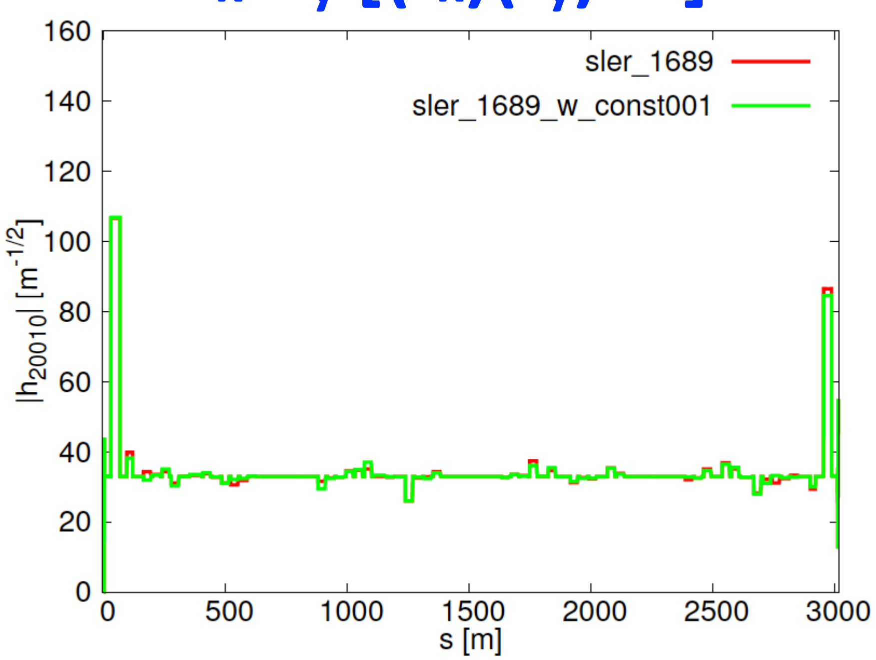
ν_y $[(J_y)^{1/2}(J_x)]$



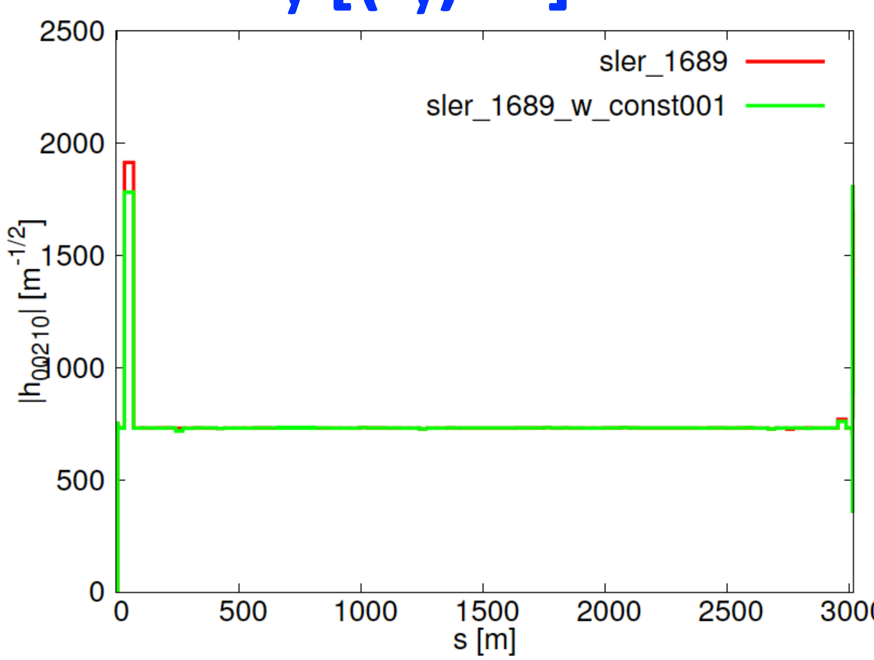
$3\nu_y$ $[(J_y)^{3/2}]$



$2\nu_x-\nu_y$ $[(J_x)(J_y)^{1/2}]$



ν_y $[(J_y)^{3/2}]$

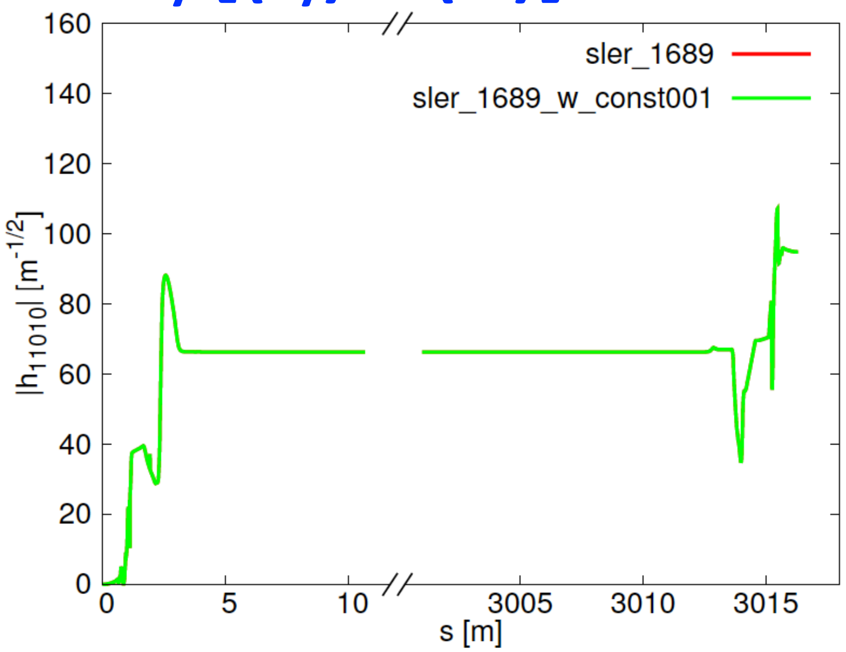


4. Some applications

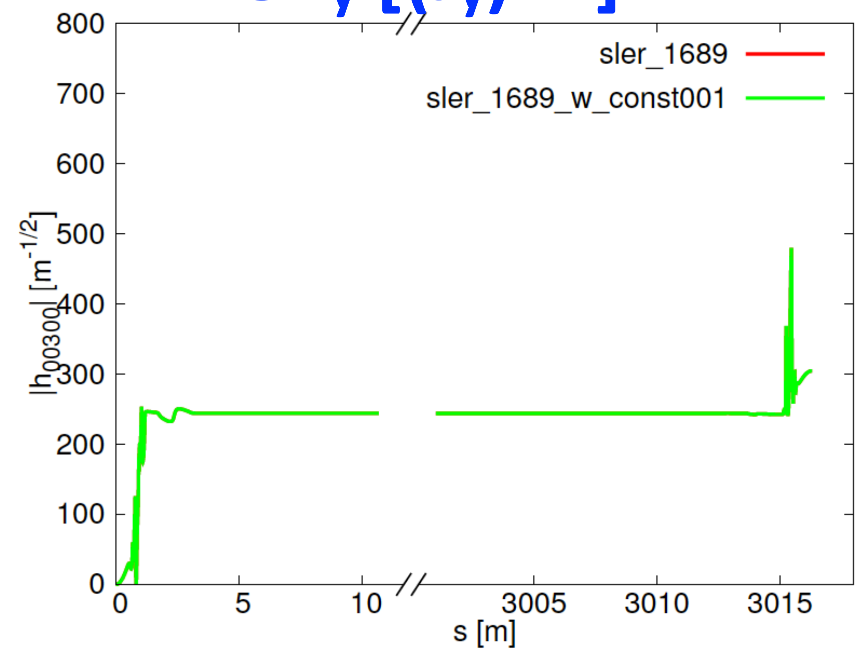
➤ Integration of RDTs along the whole ring

- **FFS contributes most of residual RDTs**

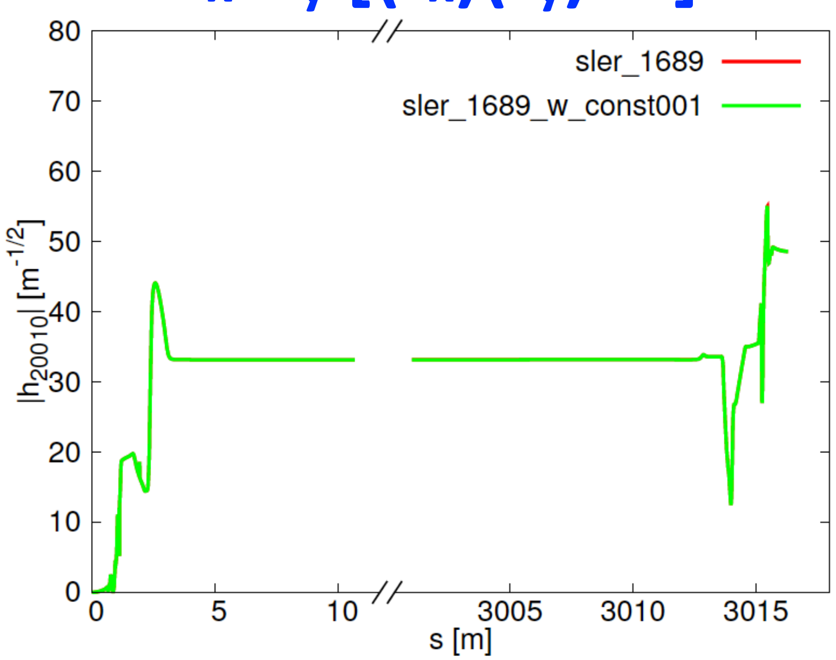
ν_y $[(J_y)^{1/2}(J_x)]$



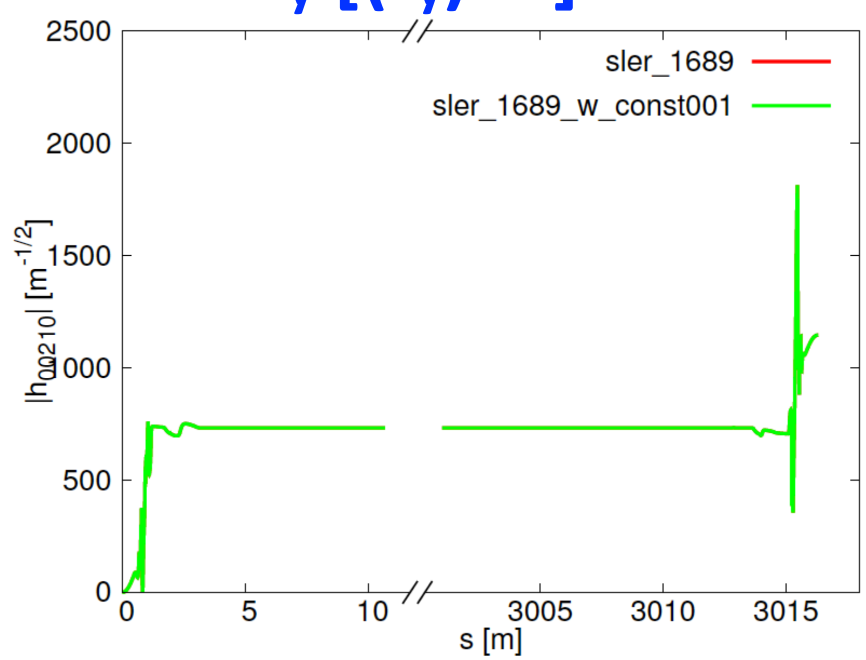
$3\nu_y$ $[(J_y)^{3/2}]$



$2\nu_x-\nu_y$ $[(J_x)(J_y)^{1/2}]$



ν_y $[(J_y)^{3/2}]$

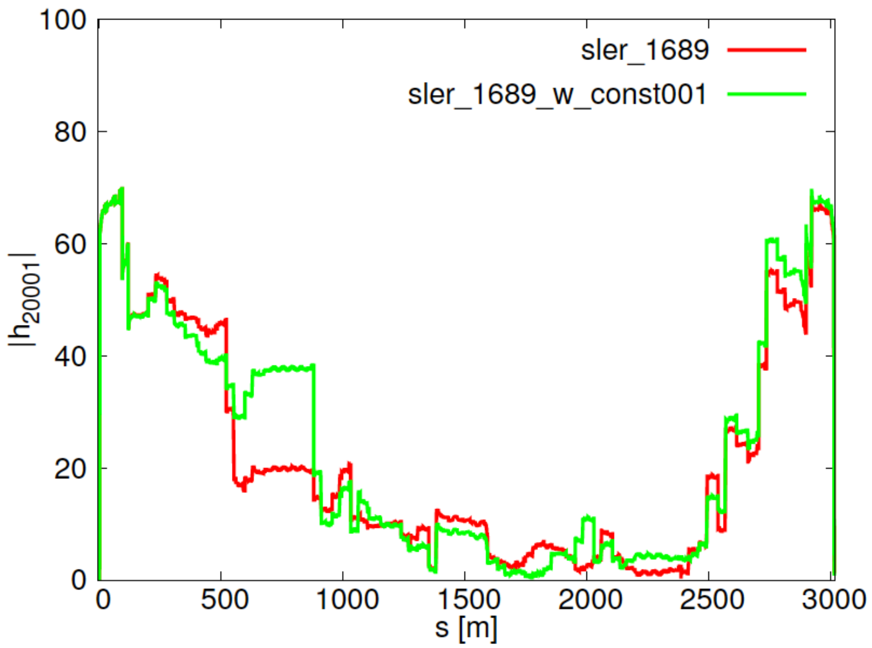


4. Some applications

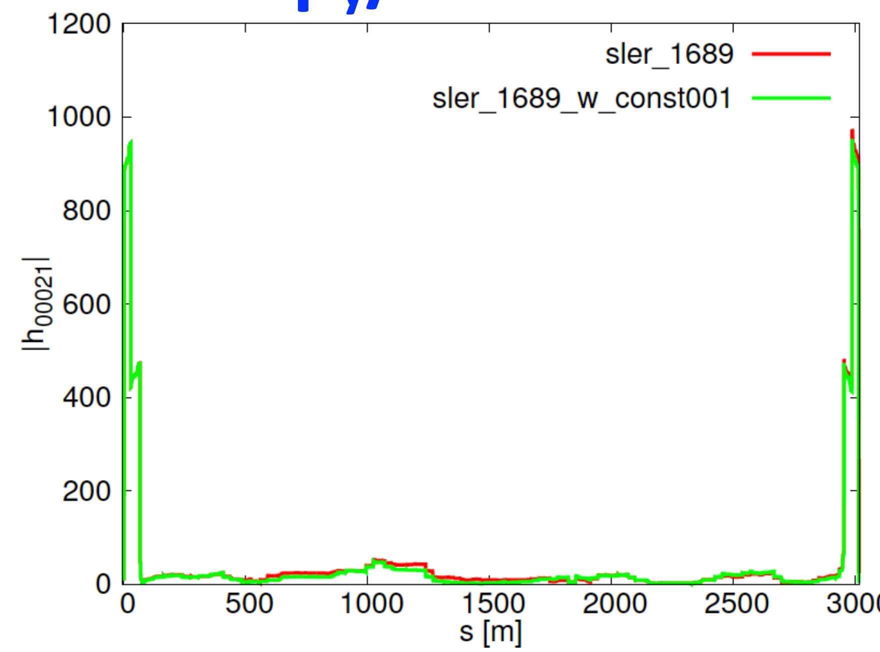
➤ Detuning along the whole ring

• w/ constraints: chromatic correction

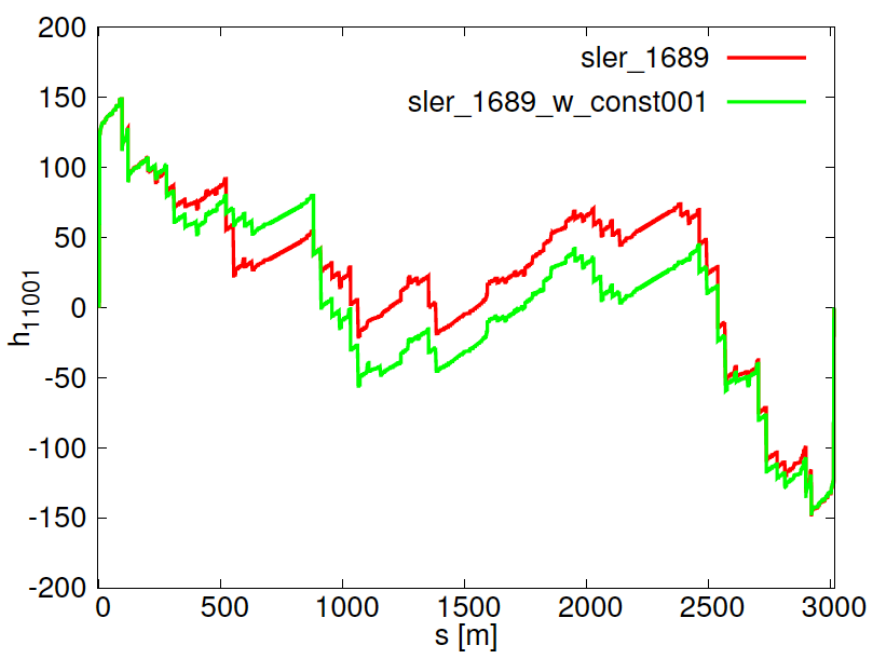
$d\beta_x/d\delta$



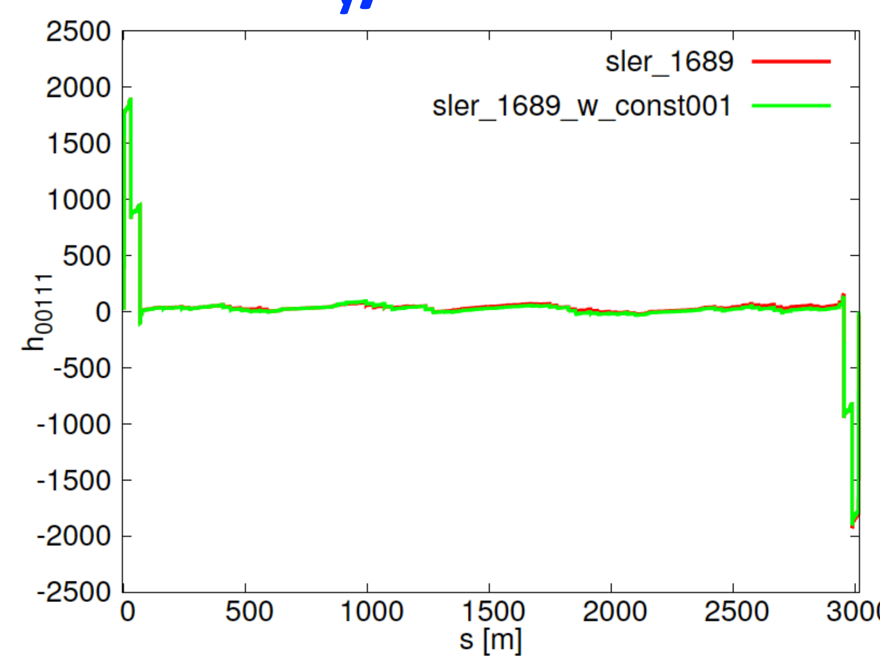
$d\beta_y/d\delta$



$d\nu_x/d\delta$



$d\nu_y/d\delta$

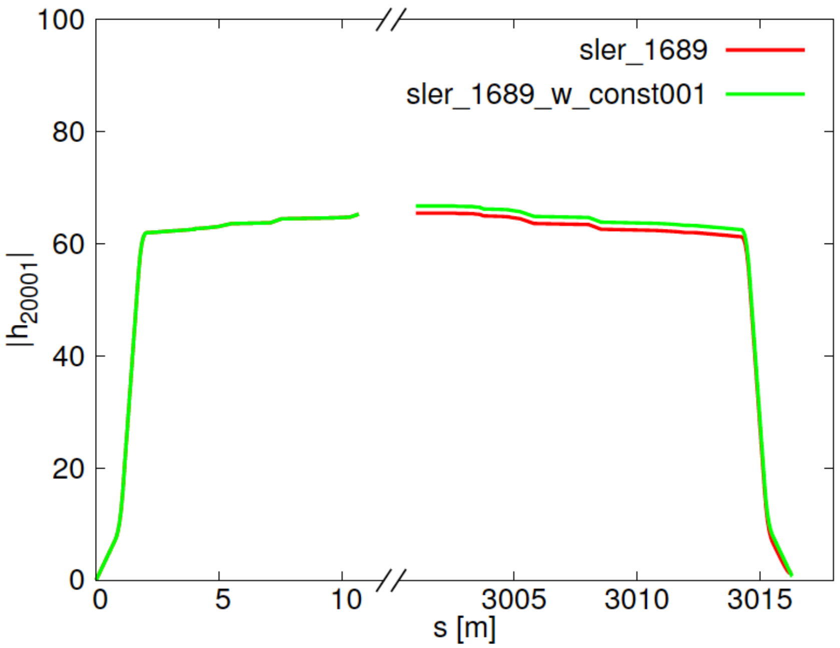


4. Some applications

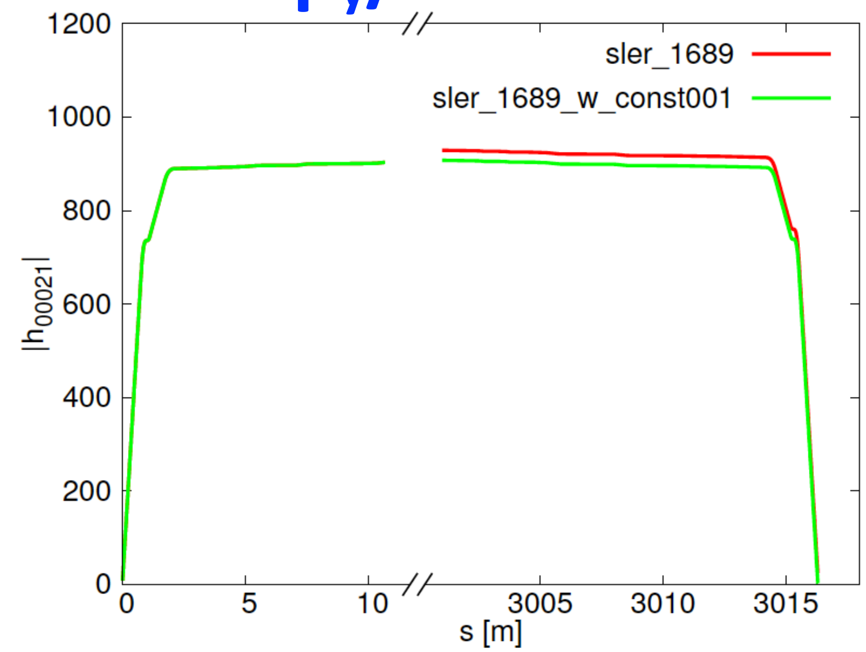
➤ Detuning along the whole ring

• w/ constraints: chromatic correction

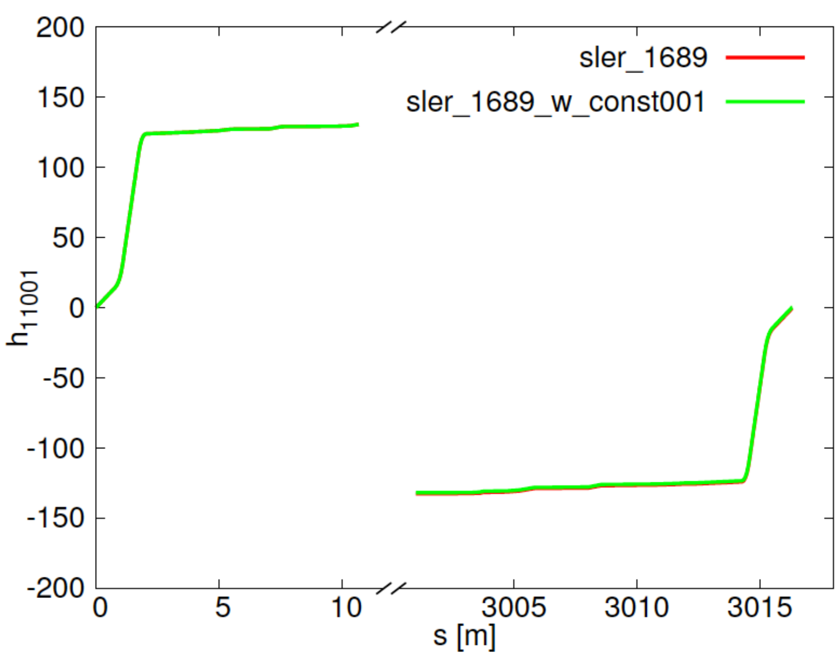
$d\beta_x/d\delta$



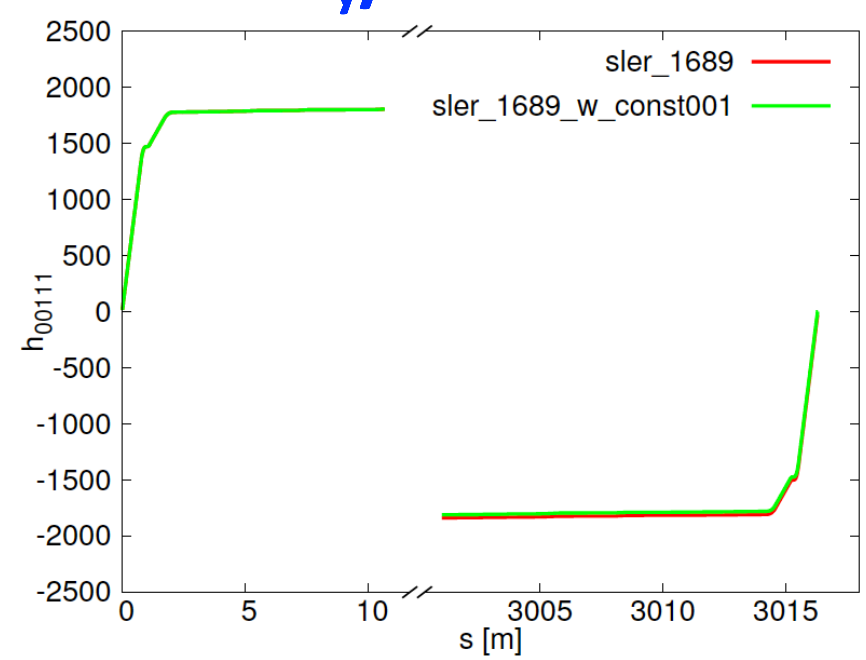
$d\beta_y/d\delta$



$d\nu_x/d\delta$



$d\nu_y/d\delta$

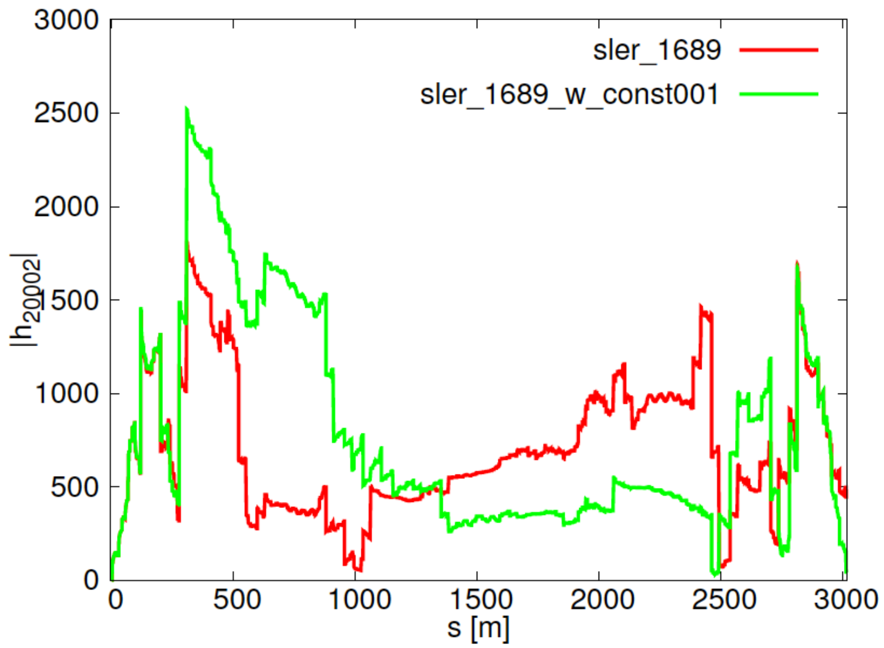


4. Some applications

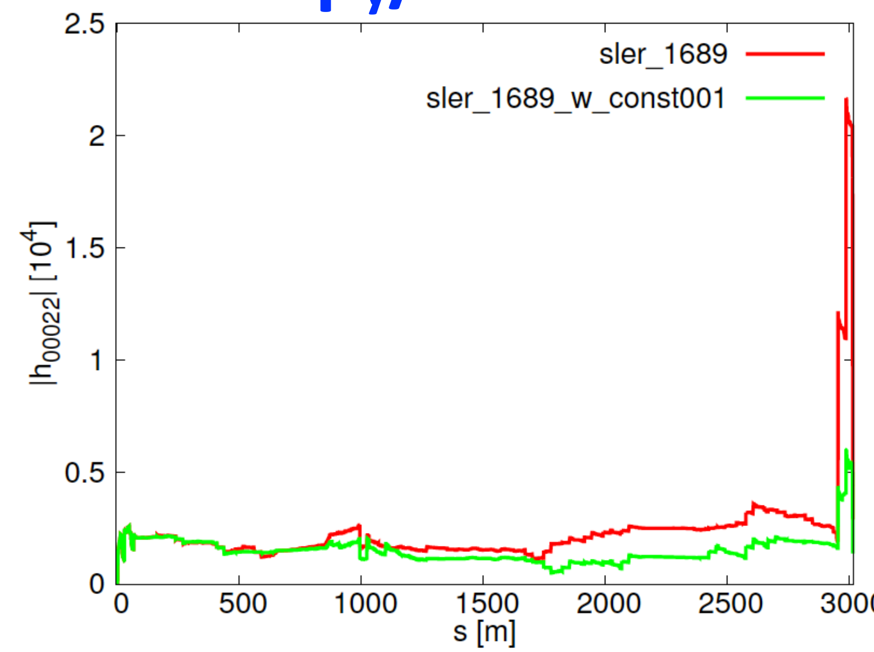
➤ **Detuning along the whole ring - second order**

- **w/ constraints: chromatic correction**

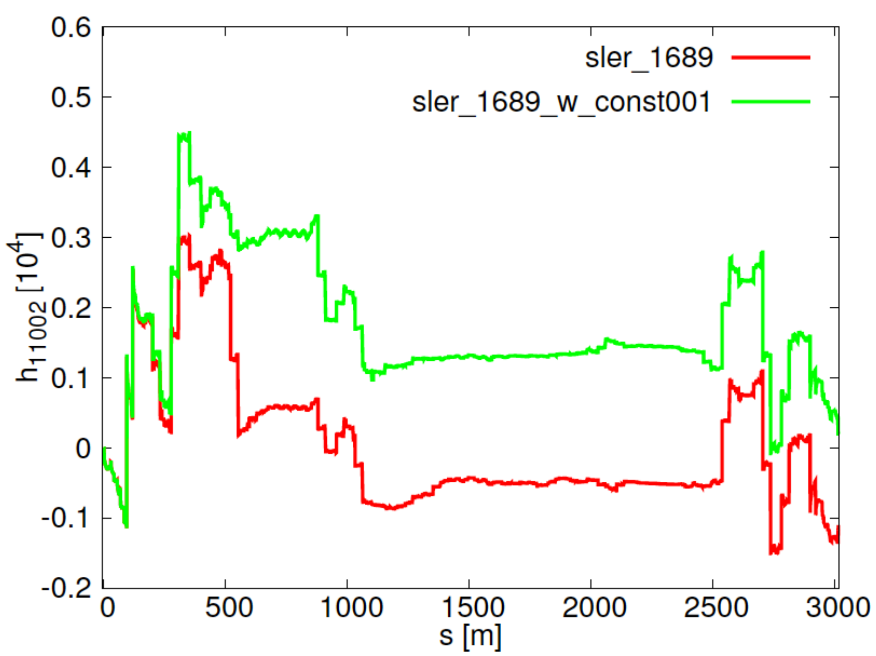
$d^2\beta_x/d\delta^2$



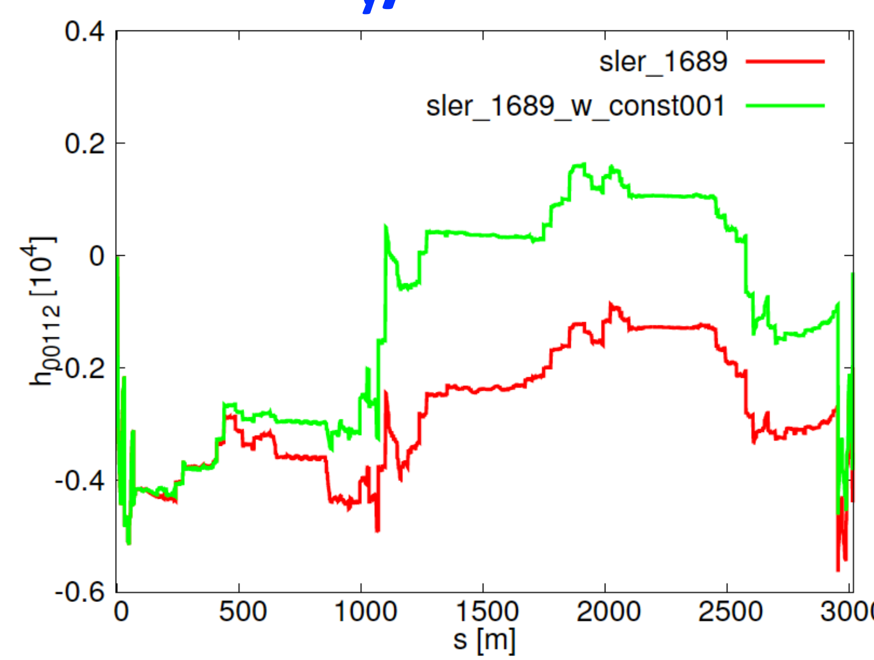
$d^2\beta_y/d\delta^2$



$d^2\nu_x/d\delta^2$



$d^2\nu_y/d\delta^2$

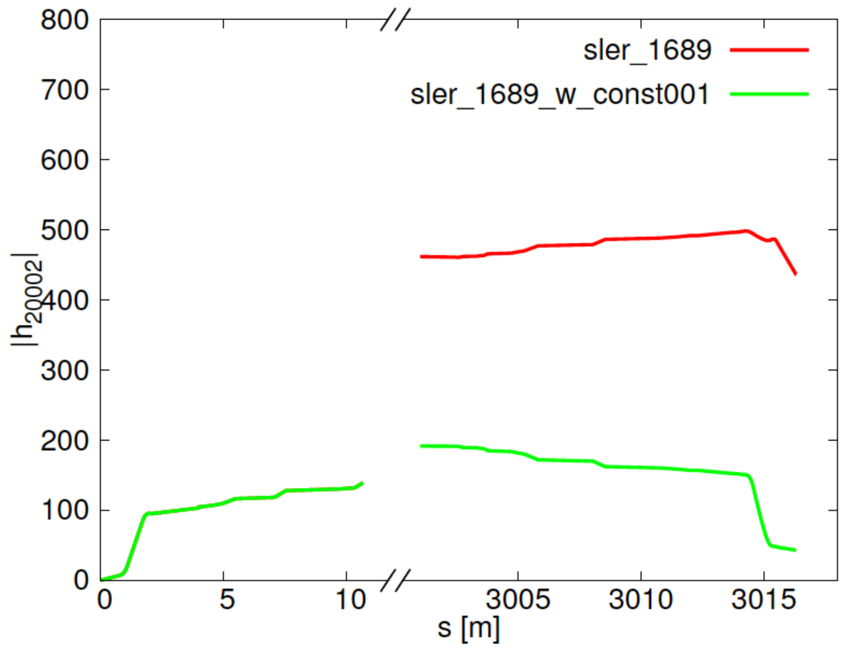


4. Some applications

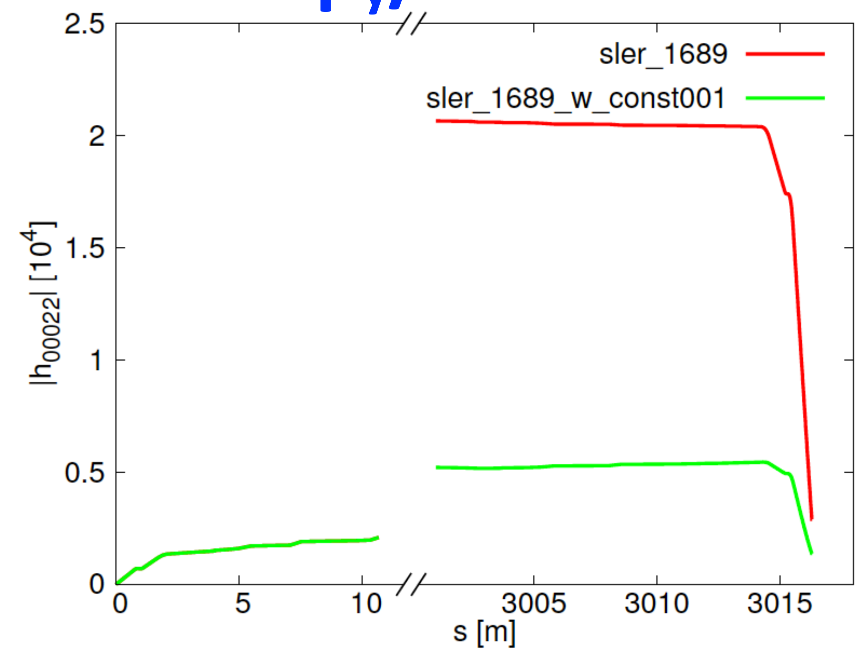
➤ Detuning along the whole ring - second order

- **w/ constraints: chromatic correction**

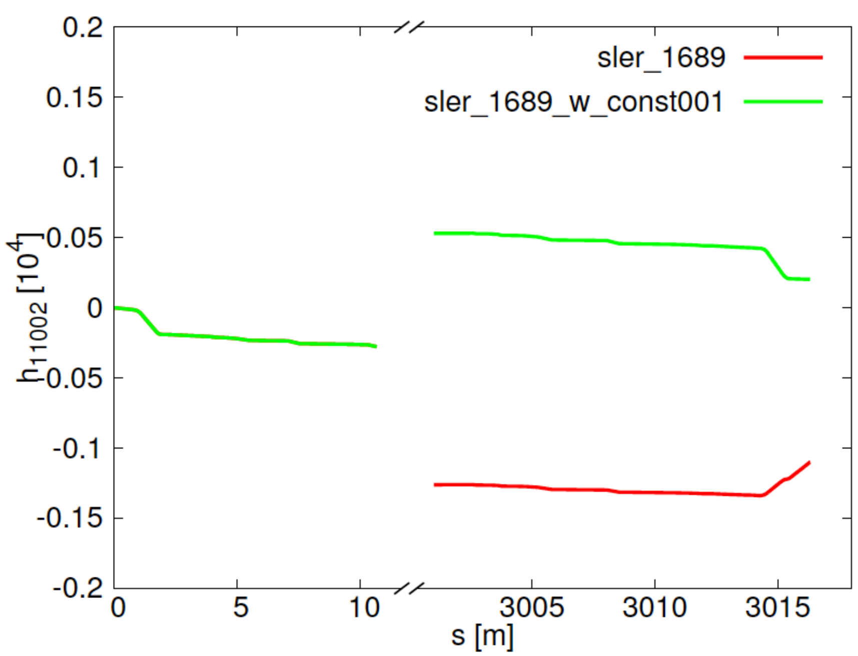
$d^2\beta_x/d\delta^2$



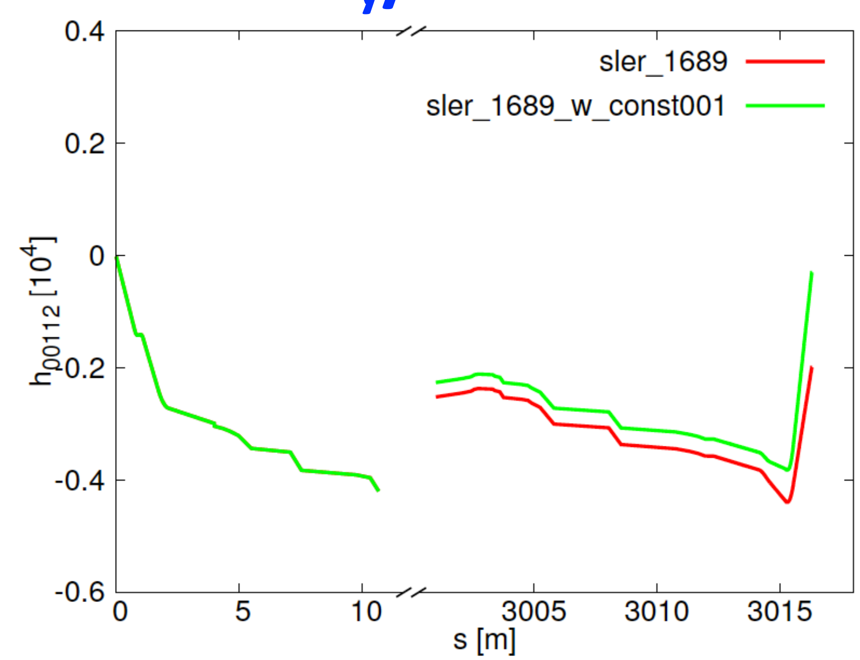
$d^2\beta_y/d\delta^2$



$d^2\nu_x/d\delta^2$



$d^2\nu_y/d\delta^2$

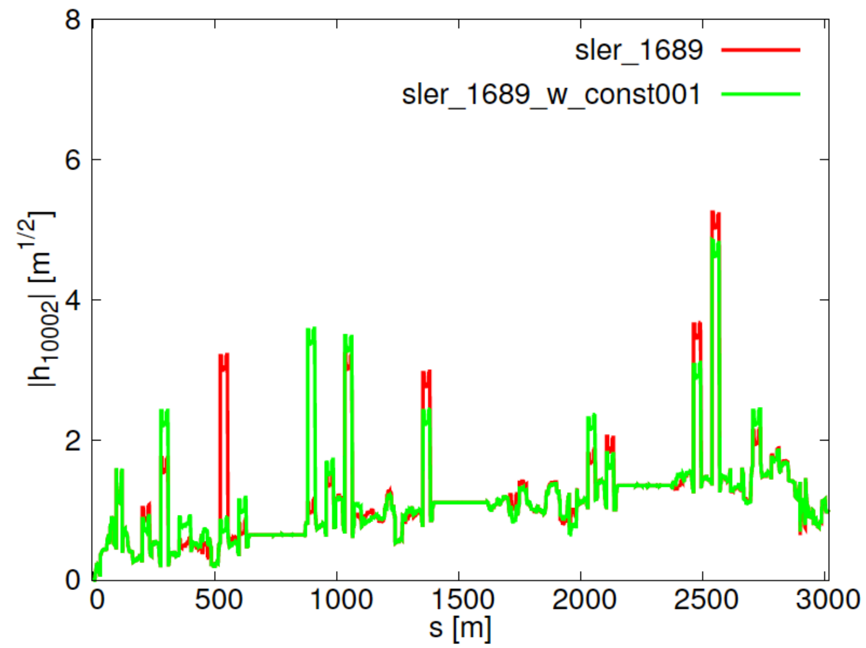


4. Some applications

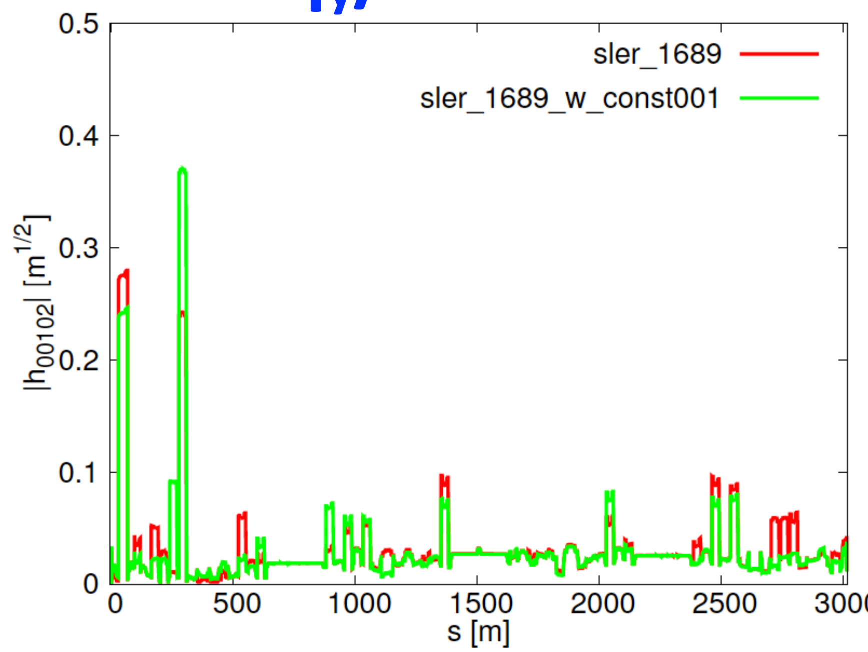
➤ Dispersion along the whole ring

- **w/ constraints: No special control on chromatic dispersions?**

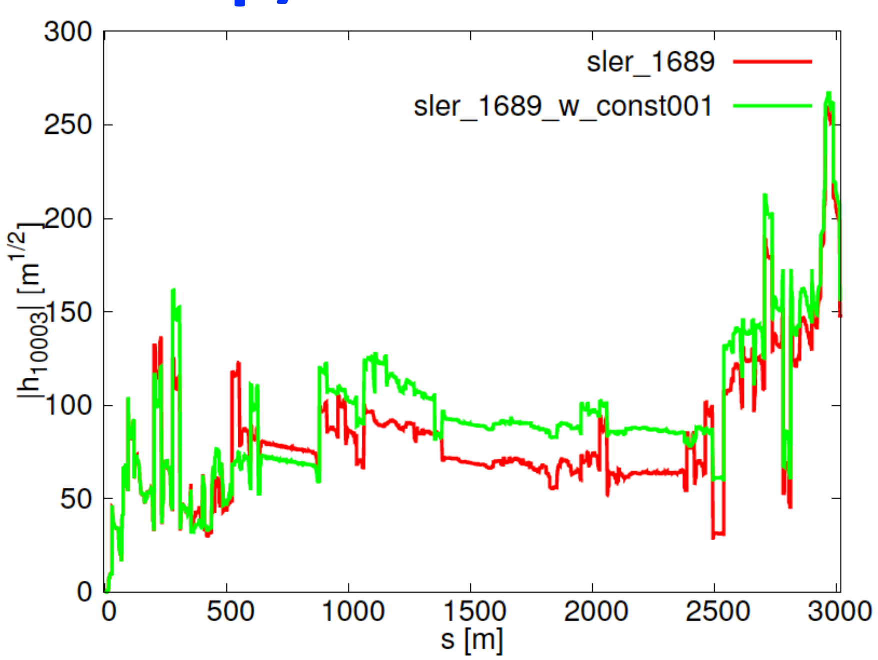
$d\eta_x/d\delta$



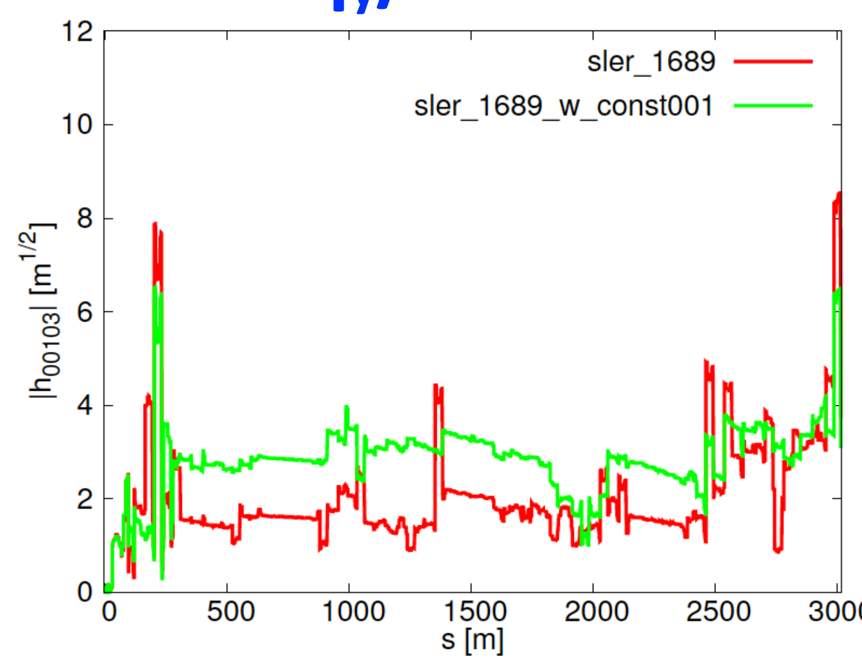
$d\eta_y/d\delta$



$d^2\eta_x/d\delta^2$



$d^2\eta_y/d\delta^2$

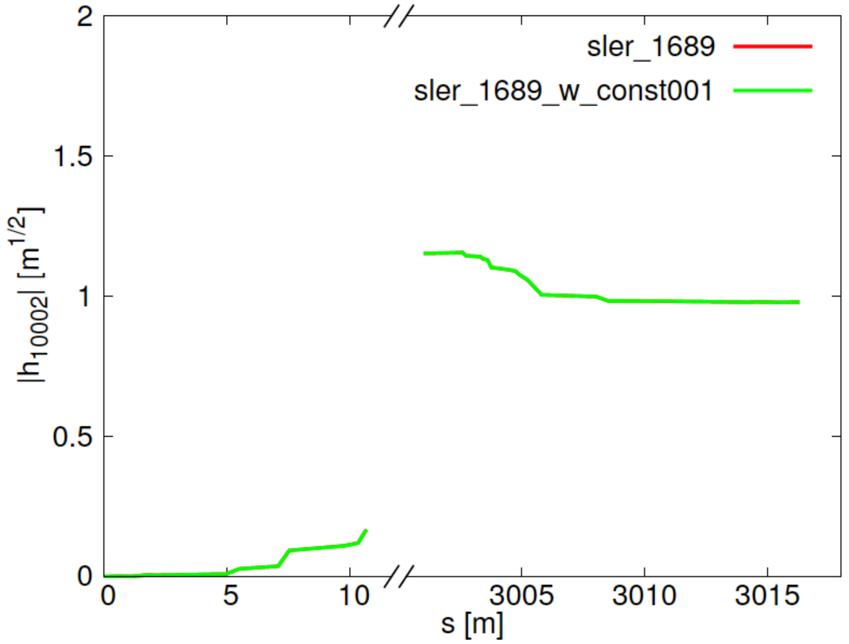


4. Some applications

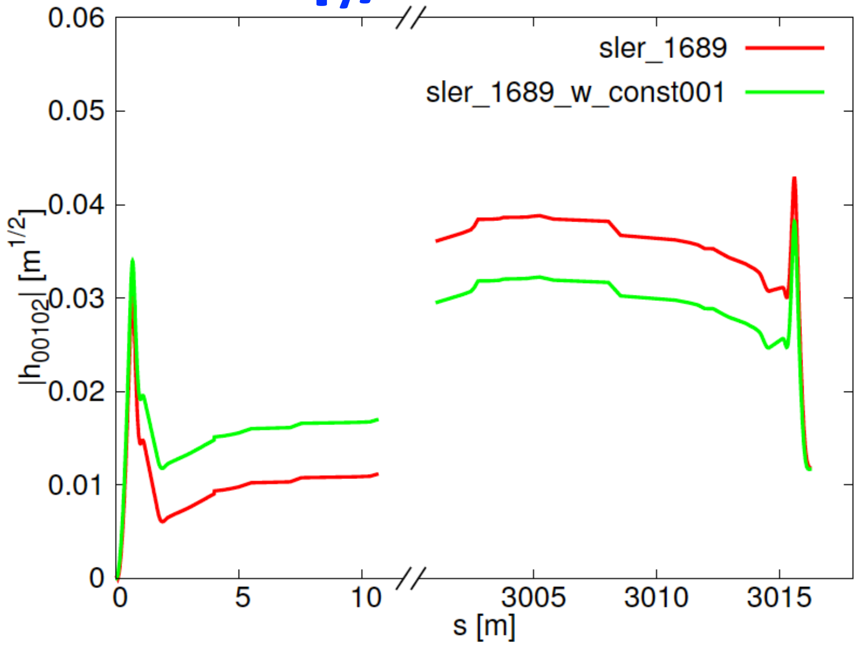
➤ Dispersion along the whole ring

• w/ constraints: No special control on chromatic dispersions?

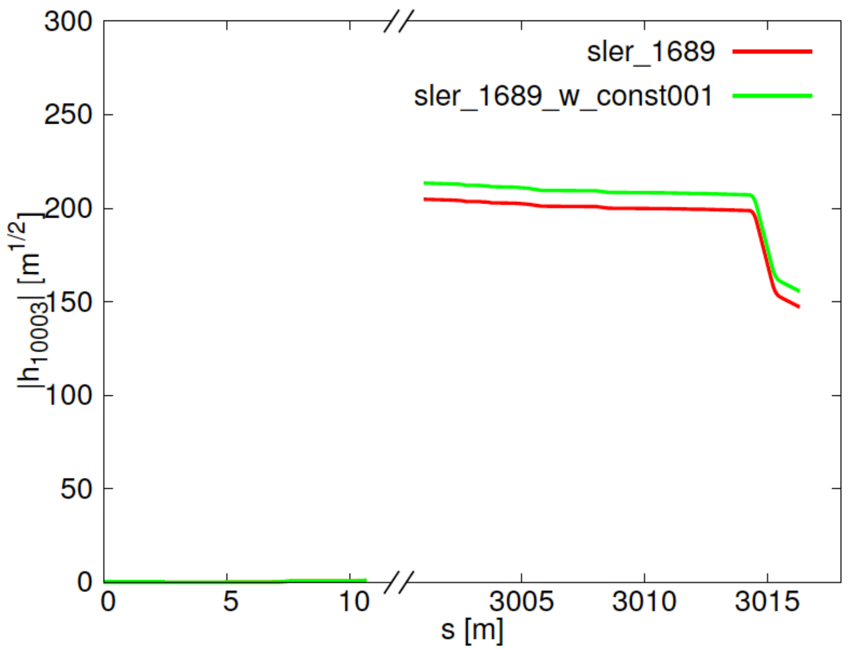
$d\eta_x/d\delta$



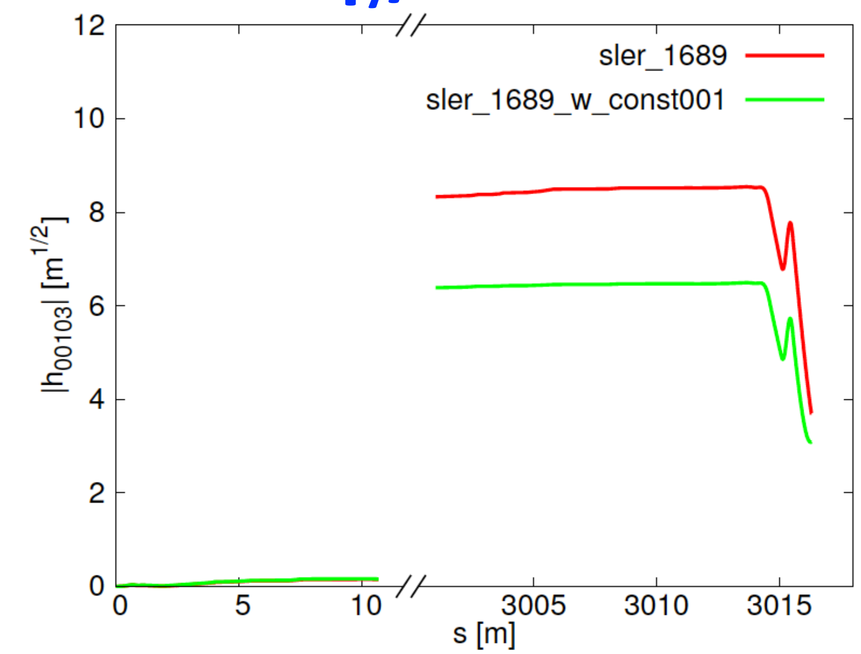
$d\eta_y/d\delta$



$d^2\eta_x/d\delta^2$



$d^2\eta_y/d\delta^2$

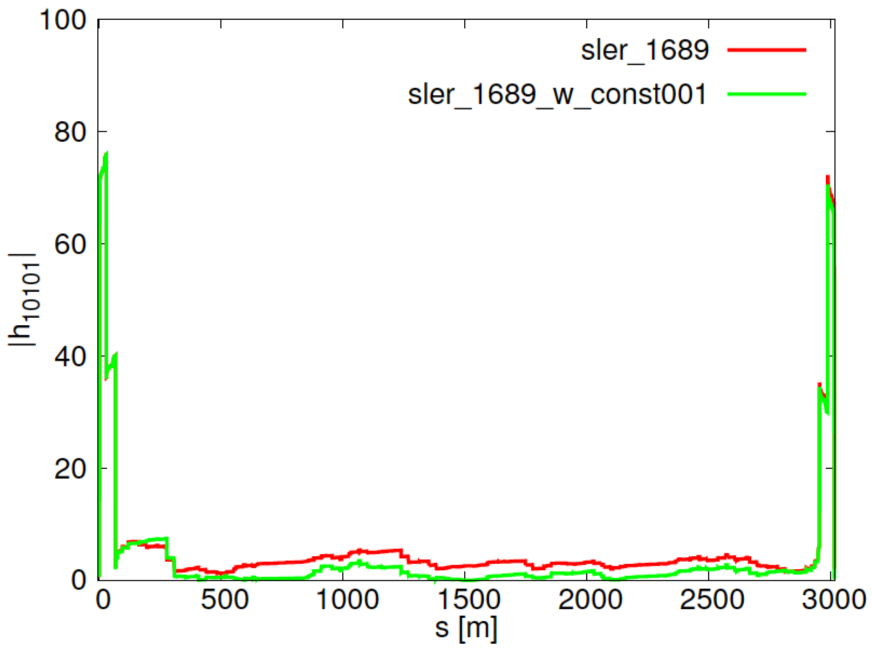


4. Some applications

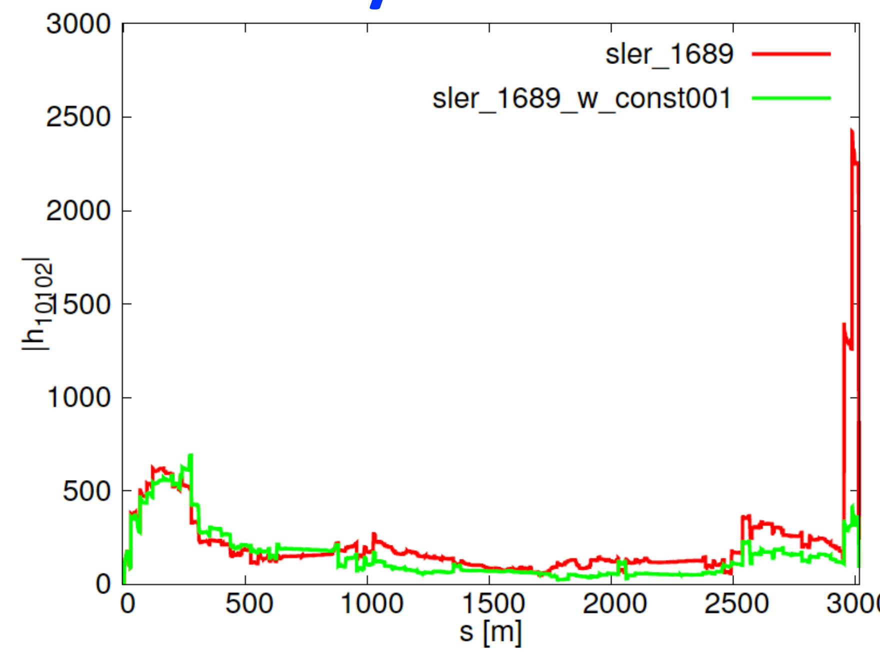
➤ Chromatic coupling along the whole ring

- **w/ constraints: Chromatic coupling controlled**

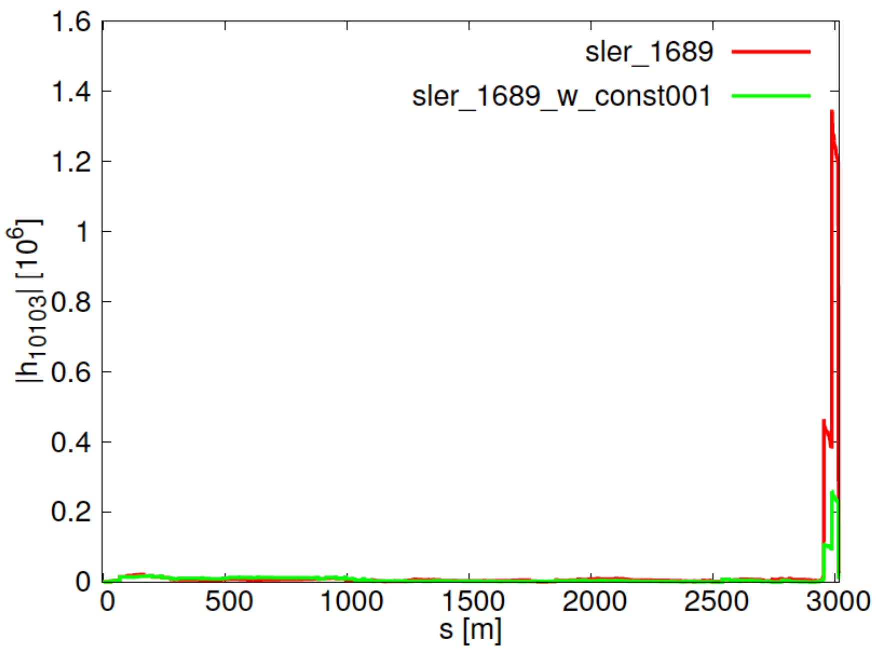
dR/dδ



d²R/dδ²



d³R/dδ³

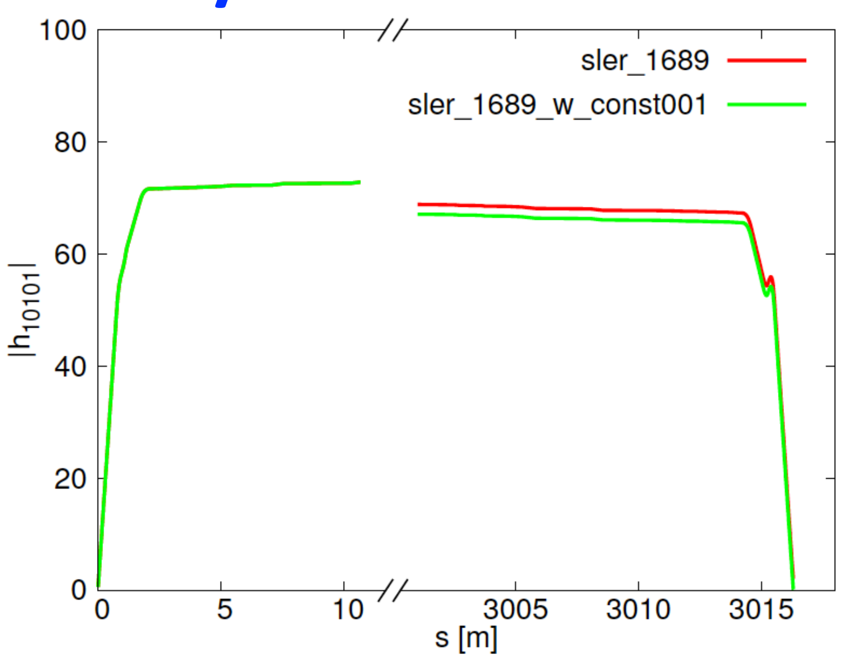


4. Some applications

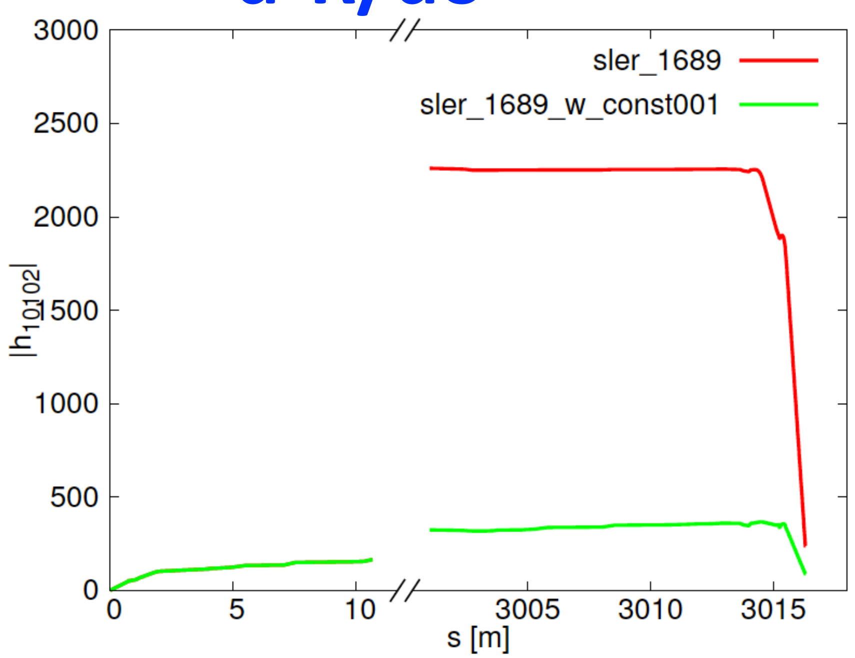
➤ **Chromatic coupling along the whole ring**

- **w/ constraints: Chromatic coupling controlled**

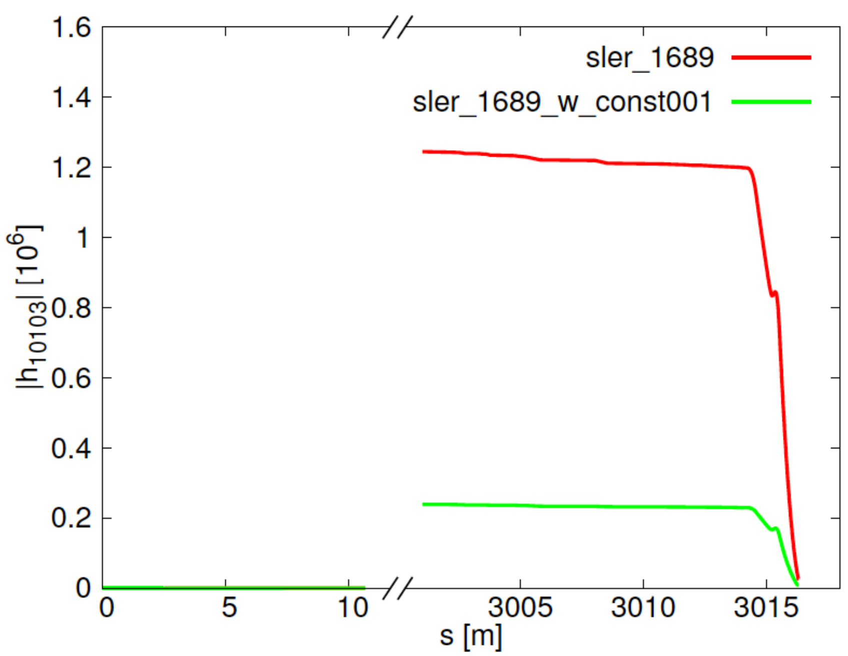
dR/dδ



d²R/dδ²



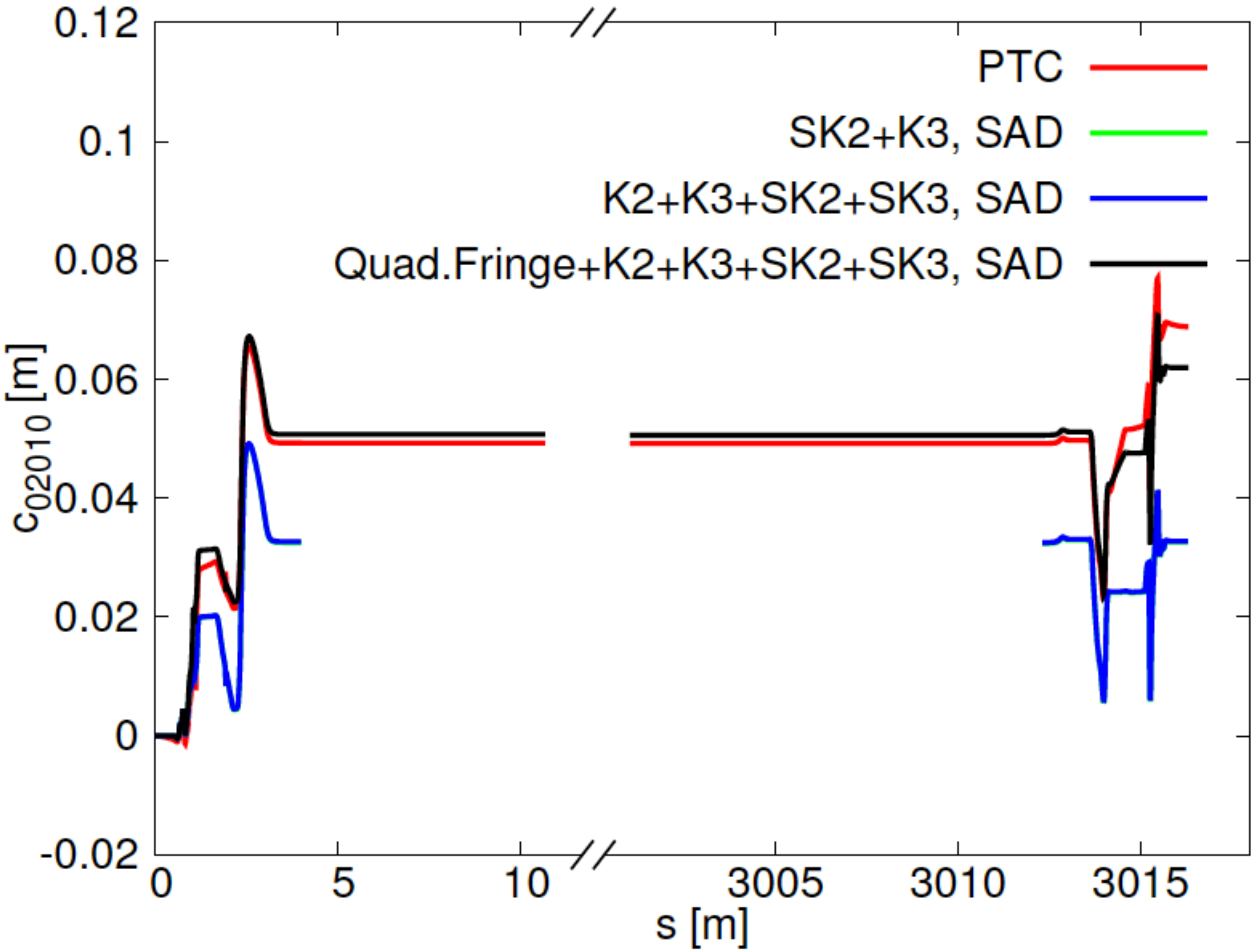
d³R/dδ³



4. Some applications

➤ $p_x^2 p_y$ term: Compare SAD

- Hard-edge fringe fields of final focus quads are important sources

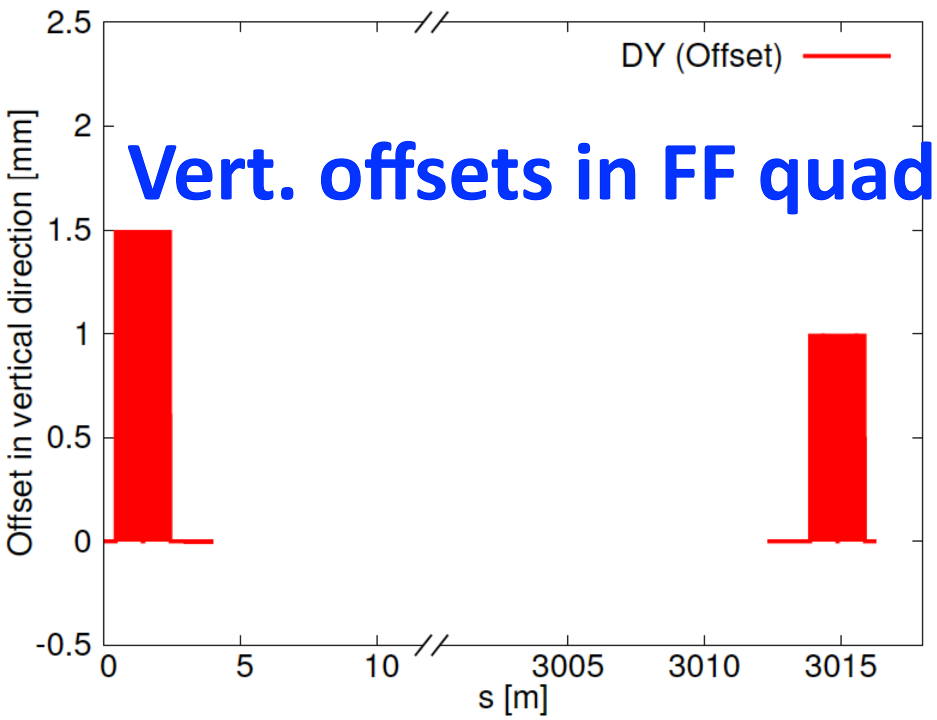


4. Some applications

➤ $p_x^2 p_y$ term

- **How quad. hard-edge fringes contribute?**

Vert. offsets in FF quads



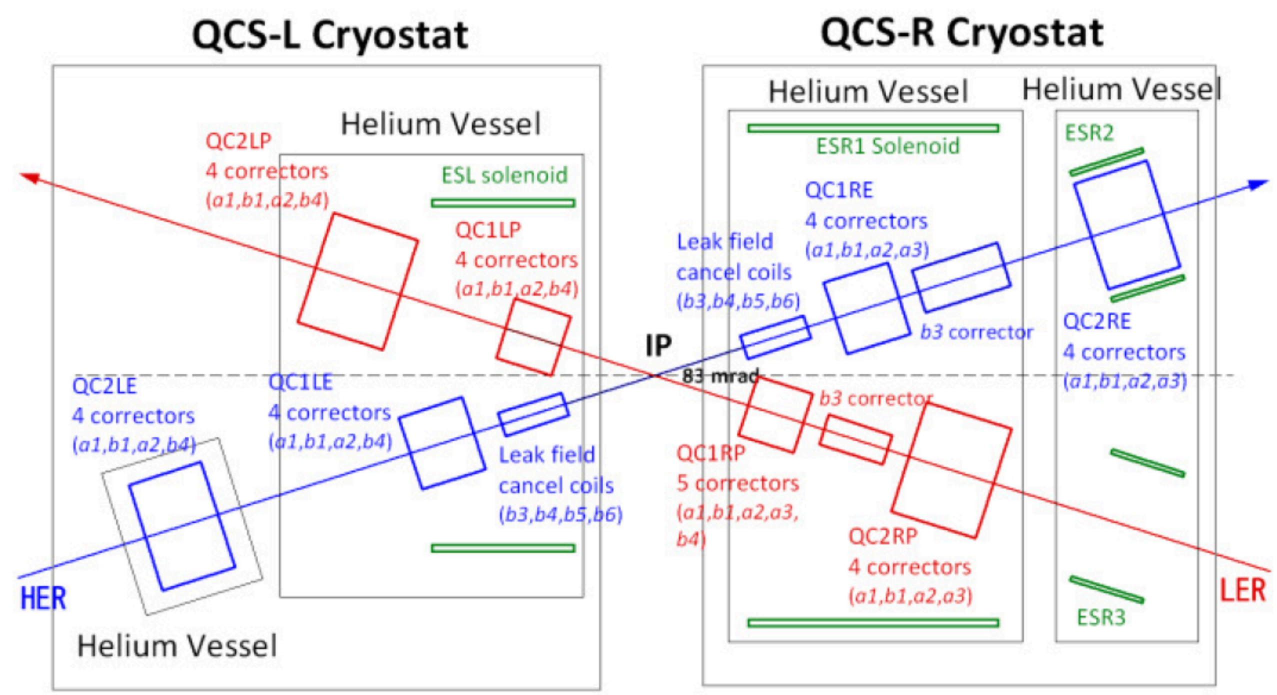
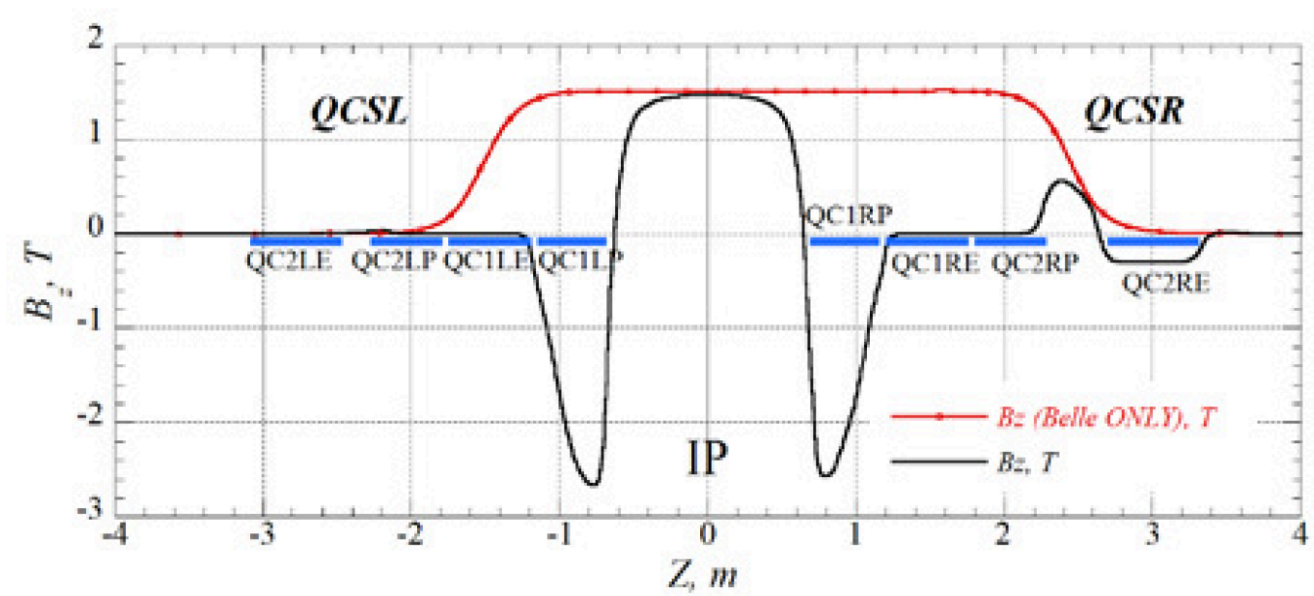


Table 2: Shift Amount of Magnet Axis

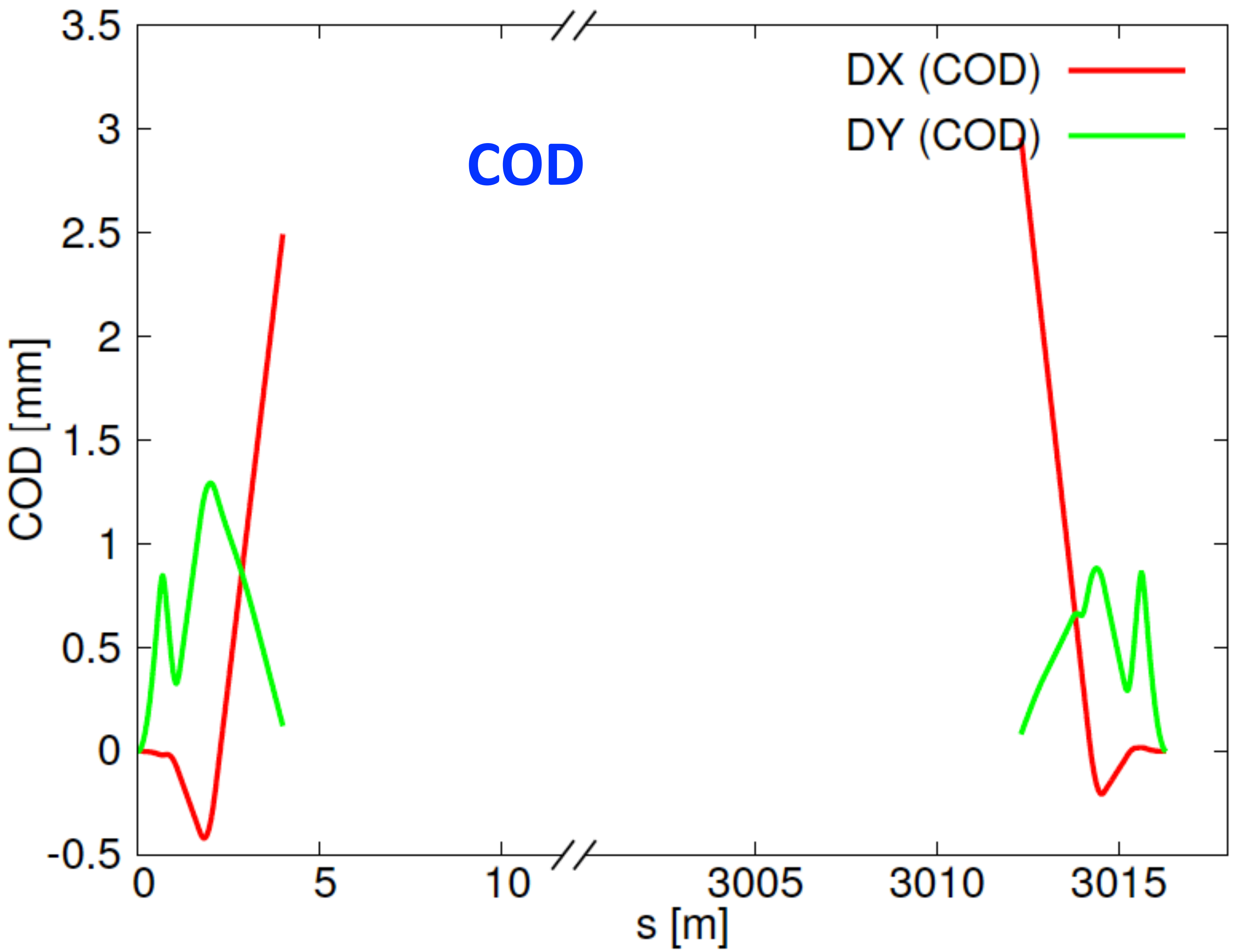
Magnet	ΔY	Magnet	ΔX
QC1RP	-1.0 mm	QC1RE	-0.7 mm
QC2RP	-1.0 mm	QC2RE	-0.7 mm
QC1LP	-1.5 mm	QC1LE	+0.7 mm
QC2LP	-1.5 mm	QC2LE	+0.7 mm



N. Ohuchi et al., IPAC'13

4. Some applications

- **$p_x^2 p_y$ term**
  - **How quad. hard-edge fringes contribute?**



4. Some applications

➤ $p_x^2 p_y$ term

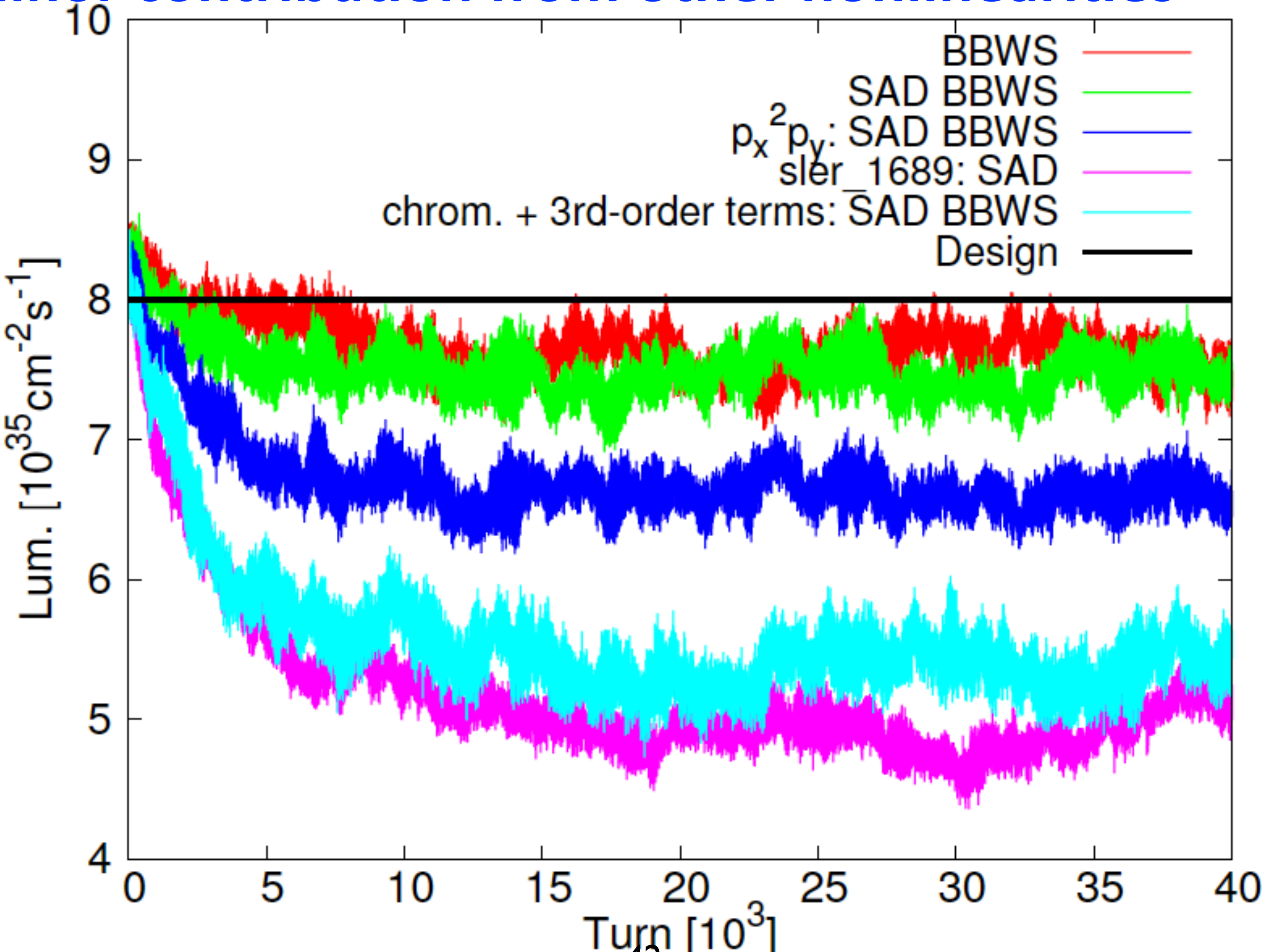
- **How quad. hard-edge fringes contribute?**
  + Magnet offsets + COD => 3rd geometric terms

```
In[1]:= (*f1=K1/(12(1+δ)L)*)
        HQfr = f1*((x^3 + 3 x*y^2) px - (y^3 + 3 x^2 y) py);
        D[HQfr, x]*ΔX
        D[HQfr, px]*ΔPX
        D[HQfr, y]*ΔY
        D[HQfr, py]*ΔPY
Out[2]= f1 (-6 py x y + px (3 x^2 + 3 y^2)) ΔX
Out[3]= f1 (x^3 + 3 x y^2) ΔPX
Out[4]= f1 (6 px x y - py (3 x^2 + 3 y^2)) ΔY
Out[5]= f1 (-3 x^2 y - y^3) ΔPY
```

4. Some applications

➤ **Luminosity calculations**

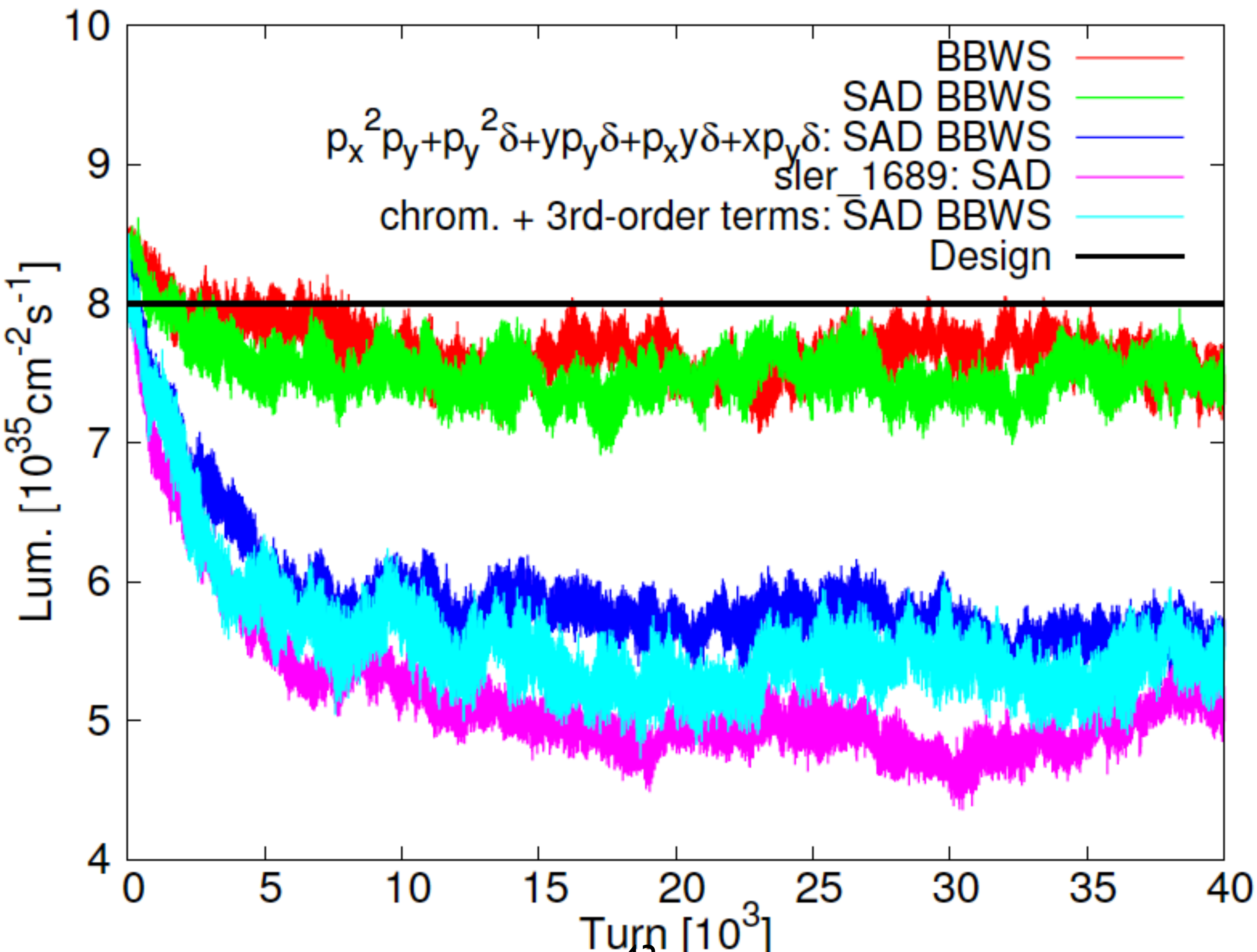
- **~1/3 caused by p_x^2py term (from FFS, strength calculated by PTC)**
- **~1/2 caused by chromatic effects (including interplay with geometric nonlinearities?)**
- **~1/6 minor contribution from other nonlinearities**



4. Some applications

➤ **Luminosity calculations**

- **Important chromatic nonlinear terms (specific to sler_1689.sad): $p_y^2\delta$, $yp_y\delta$, $p_xy\delta$, $xp_y\delta$**
- **Basically SuperKEKB is sensitive to Y-motion coupled to X- and Z-directions**



5. Symplectic tracking in SAD

➤ Hamiltonian

- **Hamiltonian for a relativistic particle in an electromagnetic field in Cartesian coordinate system:**

$$H = \sqrt{(\vec{p} - q\vec{A})^2 c^2 + m_0^2 c^4} + q\phi$$

- **Hamiltonian used in SAD:**

$$H(x, p_x, y, p_y, z, \delta) = \frac{E}{P_0 v_0} - (1 + \frac{x}{\rho_x} + \frac{y}{\rho_y})\sqrt{(1+\delta)^2 - (p_x - \hat{A}_x)^2 - (p_y - \hat{A}_y)^2} - (1 + \frac{x}{\rho_x} + \frac{y}{\rho_y})\hat{A}_s$$

Reference synchronous particle: $P_0 = \gamma_0 m_0 v_0$

$$p = P/P_0 = 1 + \delta \qquad p_x = P_x/P_0, \quad p_y = P_y/P_0$$

$$\hat{A}_x = \frac{qA_x}{P_0} = \frac{A_x}{B\rho}, \hat{A}_y = \frac{qA_y}{P_0} = \frac{A_y}{B\rho}, \hat{A}_s = \frac{qA_s}{P_0} = \frac{A_s}{B\rho}$$

Quadrupole: $\vec{A} \equiv (A_x, A_y, A_s) = (0, 0, \frac{1}{2}B_1(y^2 - x^2))$

$$B_1 = \partial B_y / \partial x$$

$$K_1 = \frac{B_1}{B_0\rho} = \frac{eB_1}{P_0}$$

Solenoid: $\vec{B} = (0, 0, B_s)$

$$\vec{A} \equiv (A_x, A_y, A_s) = (-\frac{1}{2}B_s y, \frac{1}{2}B_s x, 0)$$

5. Symplectic tracking in SAD

➤ A DRIFT (L) is nonlinear …

- **Hamiltonian for a DRIFT:**

$$H(x,p_x,y,p_y,z,\delta)=\frac{1}{v_0}\sqrt{p^2c^2+(\frac{m_0c^2}{P_0})^2}-\sqrt{p^2-p_x^2-p_y^2}$$

- **Symplectic transformation (exact solution):**

$$x_2=x_1+\frac{p_{x1}}{\sqrt{p^2-p_{x1}^2-p_{y1}^2}}L,$$
$$p_{x2}=p_{x1},$$
$$y_2=y_1+\frac{p_{y1}}{\sqrt{p^2-p_{x1}^2-p_{y1}^2}}L,$$
$$p_{y2}=p_{y1},$$
$$z_2=z_1-(\frac{p}{\sqrt{p^2-p_{x1}^2-p_{y1}^2}}-\frac{v}{v_0})L=z_1+(1-\frac{p}{\sqrt{p^2-p_{x1}^2-p_{y1}^2}})L-\frac{v_0-v}{v_0}L$$

5. Symplectic tracking in SAD

➤ A DRIFT with solenoid field (L, BZ)

- **Hamiltonian for a DRIFT + solenoid field:**

$$H(x,p_x,y,p_y,z,\delta) = \frac{E}{P_0 v_0} - \sqrt{p^2 - (p_x + \tfrac{1}{2}b_z y)^2 - (p_y - \tfrac{1}{2}b_z x)^2} \qquad b_z = \frac{eB_z}{P_0}$$

- **Symplectic transformation (exact solution):**

$$\begin{aligned}
x_2 &= x_1 + \frac{(1+\delta)\sin\phi}{b_z} p_{xi} + \frac{(1+\delta)(1-\cos\phi)}{b_z} p_{yi},\\
y_2 &= y_1 - \frac{(1+\delta)(1-\cos\phi)}{b_z} p_{xi} + \frac{(1+\delta)\sin\phi}{b_z} p_{yi},\\
p_{x2} &= p_{xi}\cos\phi + p_{yi}\sin\phi - \frac{b_z}{2(1+\delta)} y_2,\\
p_{y2} &= -p_{xi}\sin\phi + p_{yi}\cos\phi + \frac{b_z}{2(1+\delta)} x_2,\\
z_2 &= z_1 + \left[\frac{\sqrt{1-p_{xi}^2-p_{yi}^2}-1}{\sqrt{1-p_{xi}^2-p_{yi}^2}} - \Delta v\right] L
\end{aligned}$$

$$\begin{aligned}
\phi &= \frac{b_z L}{(1+\delta)\sqrt{1-p_{xi}^2-p_{yi}^2}},\\
p_{xi} &= p_{x1} + \frac{b_z y_1}{2(1+\delta)},\\
p_{yi} &= p_{y1} - \frac{b_z x_1}{2(1+\delta)},\\
\Delta v &= \frac{v_0 - v}{v_0}
\end{aligned}$$

- **The SOL element in SAD is special: NO attribute of L**
- **The next case: L≠0, BZ≠0, K0≠0, SK0≠0 [Solvable]**

5. Symplectic tracking in SAD

➤ Fringe fields: Bend soft edge fringe (From Bmad manual)

Bmad defines the bend soft edge map in terms of the field integral F_{H1} for the entrance end and F_{H2} for the exit end given by (see Eq. (3.5))

$$F_{H1} \equiv F_{int}\, H_{gap} = \int_{pole} ds\, \frac{B_y(s)\,(B_{y0} - B_y(s))}{2\, B_{y0}^2} \tag{19.46}$$

With a similar equation for F_{H2}. The soft edge map is then

$$\begin{aligned} x_2 &= x_1 + c_1\, p_z \\ p_{y2} &= p_{y1} + c_2\, y_1 - c_3\, y_1^3 \\ z_2 &= z_1 + \frac{1}{1+p_{z1}} \left(c_1\, p_{x1} + \frac{1}{2}\, c_2\, y_1^2 - \frac{1}{4}\, c_3\, y_1^4 \right) \end{aligned} \tag{19.47}$$

For the entrance face:

$$c_1 = \frac{g_{\rm tot}\, F_{H1}^2}{2\,(1+p_z)}, \qquad c_2 = \frac{2\, g_{\rm tot}^2\, F_{H1}}{1+p_z}, \qquad c_3 = 0 \tag{19.48}$$

with **g_tot** is the total bending strength

$$g_{\rm tot} = \mathrm{g} + \mathrm{g\_err} \tag{19.49}$$

g being the reference bend strength and **g_err** being bend the difference between the actual and reference bend strengths (§3.5).

For the exit face, the subsitution is made

$$\begin{aligned} F_{H1} &\rightarrow F_{H2} \\ g_{\rm tot} &\rightarrow -g_{\rm tot} \end{aligned} \tag{19.50}$$

When the SAD bend soft edge map is used (§4.20), the map is the same except that the value of c_3 is

$$c_3 = \frac{8\, g_{\rm tot}^2}{F_{H1}\,(1+p_z)} \tag{19.51}$$

5. Symplectic tracking in SAD

➤ Fringe fields: Quad. soft edge fringe (From Bmad manual)

Only the quadrupole soft edge fringe is modeled in *Bmad*. The model is adapted from SAD[SAD]. The fringe map is:

$$
\begin{aligned}
x_2 &= x_1\, e^{g_1} + g_2\, p_{x1} \\
p_{x2} &= p_{x1} e^{-g_1} \\
y_2 &= y_1\, e^{-g_1} - g_2\, p_{y1} \\
p_{y2} &= p_{y1} e^{g_1} \\
z_2 &= z_1 - \left[g_1\, x_1\, p_{x1} + g_2 \left(1 + \frac{g_1}{2}\right) e^{-g_1}\, p_{x1}^2\right] + \left[g_1\, y_1\, p_{y1} + g_2 \left(1 - \frac{g_1}{2}\right) e^{g_1}\, p_{y1}^2\right]
\end{aligned}
\tag{19.53}
$$

where

$$
\begin{aligned}
g_1 &= K_1\, \mathrm{fq1} \\
g_2 &= K_1\, \mathrm{fq2}
\end{aligned}
\tag{19.54}
$$

K_1 is the quadrupole strength, and `fq1` and `fq2` are the fringe quadrupole parameters. These parameters are related to the field integral I_n via

$$
\begin{aligned}
\mathrm{fq1} &= I_1 - \frac{1}{2}\, I_0^2 \\
\mathrm{fq2} &= I_2 - \frac{1}{3}\, I_0^3
\end{aligned}
\tag{19.55}
$$

where I_n is defined by

$$
I_n = \frac{1}{K_1} \int_{-\infty}^{\infty} \left(K_1(s) - H(s - s_0)\, K_1\right) (s - s_0)^n \, ds \tag{19.56}
$$

and $H(s)$ is the step function

$$
H(s) = \begin{cases} 1 & s > 0 \\ 0 & s < 0 \end{cases} \tag{19.57}
$$

and it is assumed that the quadrupole edge is at s_0 and the interior is in the region $s > s_0$.

5. Symplectic tracking in SAD

➤ Fringe fields: Hard edge fringe (From Bmad manual)

The magnetic multipole hard edge fringe field is modeled using the method shown in Forest[Forest98]. For the m^{th} order multipole the Lee transform is (Forest Eq. (13.29):

$$
\begin{aligned}
f_{\pm} &= \mp \Re \left[\frac{(b_m + i\, a_m)\,(x + i\, y)^{m+1}}{4\,(m+2)\,(1+\delta)} \left\{ x\, p_x + y\, p_y + i \frac{m+3}{m+1} (x\, p_x - y\, p_y) \right\} \right] \\
&\equiv \frac{p_x\, f^x + p_y\, f^y}{1+\delta}
\end{aligned}
\tag{19.58}
$$

The multipole strengths a_m and b_m are given by (14.9) and the second equation defines f^x and f^y. On the right had side of the first equation, the minus sign is appropriate for particles entering the magnet and the plus sign is for particle leaving the magnet. Notice that here the multipole order m is equivalent to $n-1$ in Forest's notation.

With this, the implicit multipole map is (Forest Eq. (13.31))

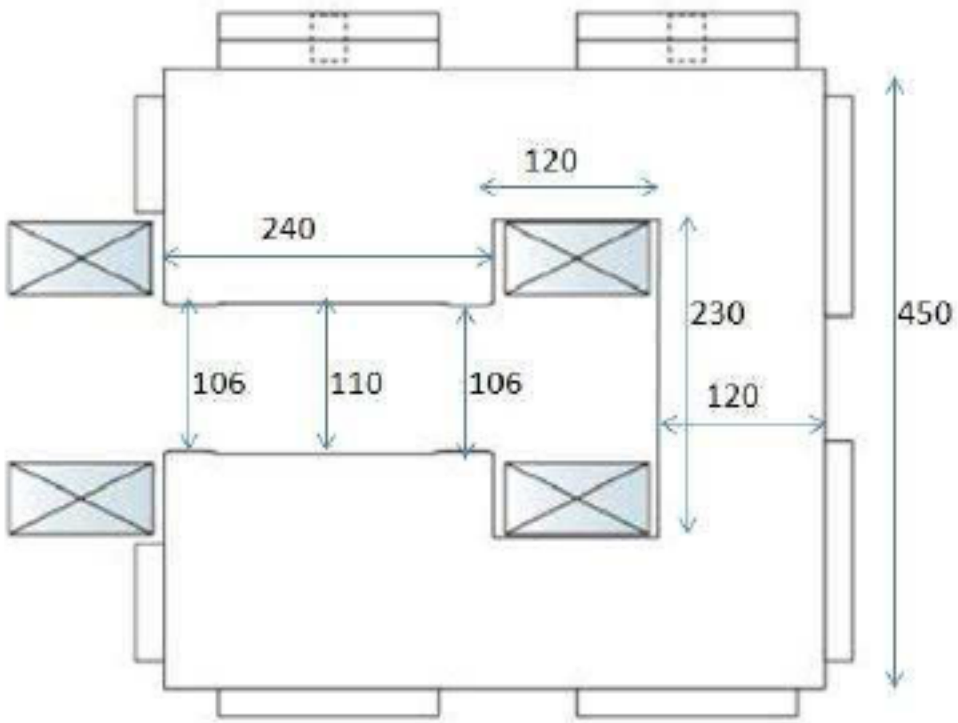
$$
\begin{aligned}
x^f &= x - \frac{f^x}{1+\delta} \\
p_x &= p_x^f - \frac{p_x^f\, \partial_x f^x + p_y^f\, \partial_x f^y}{1+\delta} \\
y^f &= y - \frac{f_y}{1+\delta} \\
p_y &= p_y^f - \frac{p_x^f\, \partial_y f^x + p_y^f\, \partial_y f^y}{1+\delta} \\
\delta^f &= \delta \\
z^f &= \frac{p_x^f\, f^x + p_y^f\, f^y}{(1+\delta)^2}
\end{aligned}
\tag{19.59}
$$

Note:
This equation is general, applying for BEND, QUAD, SEXT, … to arbitrary order. But BEND is special!

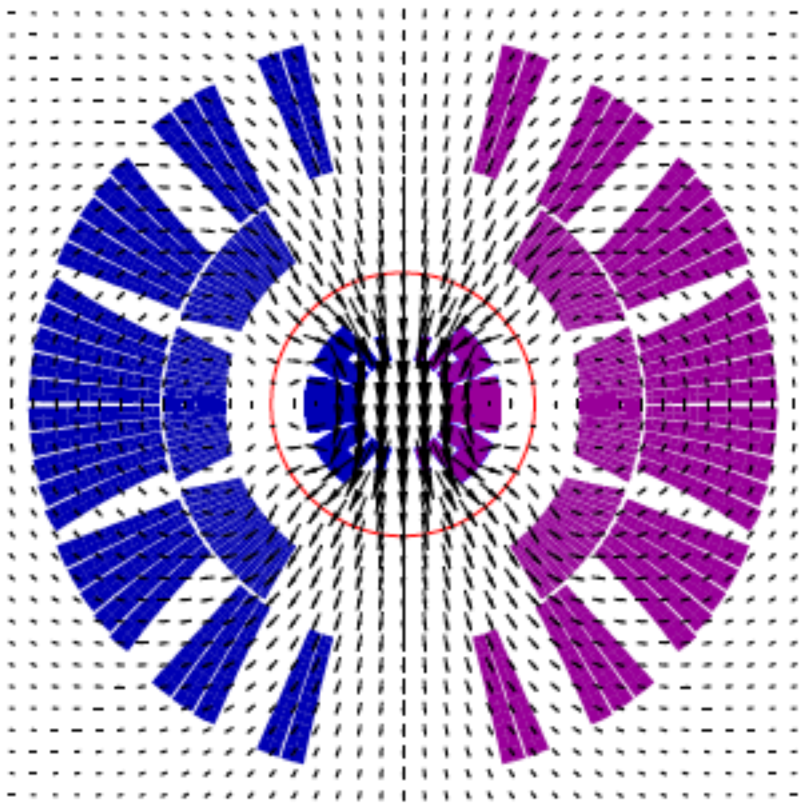
5. Symplectic tracking in SAD

➤ Fringe fields: Hard edge fringe for BEND

- Two models found for hard-edge fringe
  - * E. Forest: "Parallel-plate" shape (popular theory)
  - * Y. Cai: Round shape (SLAC-PUB-11181, apply for SC magnets?)



Usual case
(From SuperKEKB TDR)

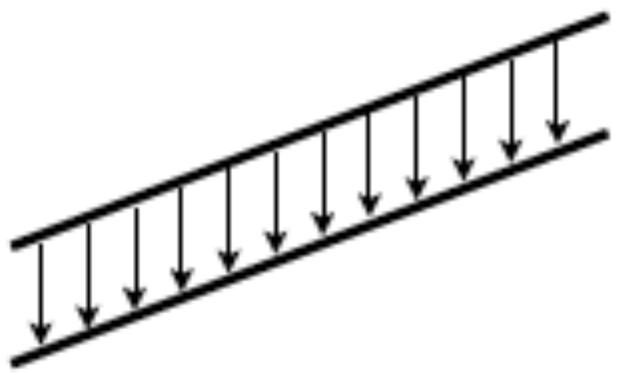


SC magnet
(From S. Russenschuck's textbook, 2010)

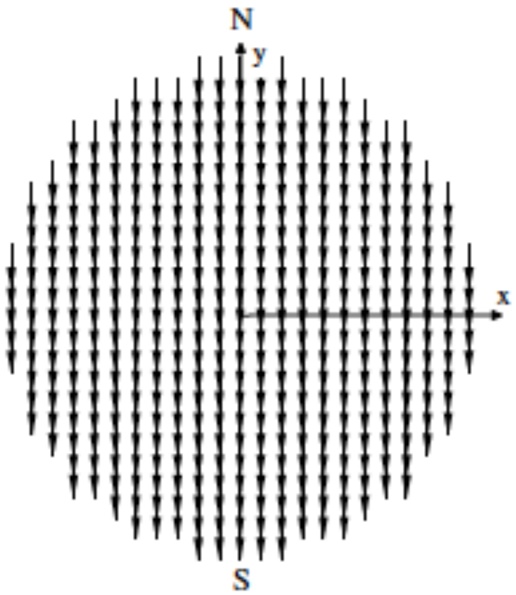
5. Symplectic tracking in SAD

➤ Fringe fields: Hard edge fringe for BEND

- **Maxwellian solution for hard-edge dipole field**
  - *** G. Lee-Whiting et al. => E. Forest et al.**
  - *** S. Caspi et al. => M. Bassetti et al. => Y. Cai et al.**



The model for wide magnet.
The filed is confined at region of
$-b<y<b$ and $-\infty<x<\infty$



The model for harmonics expansion.
The field is confined inside a circle
with $r<r_0$
(From S. Russenschuck's textbook, 2010)

5. Symplectic tracking in SAD

➤ Fringe fields: Hard edge fringe for BEND

- **Maxwellian solution for hard-edge dipole field**
  - * G. Lee-Whiting et al. => E. Forest et al.
  - * S. Caspi et al. => M. Bassetti et al. => Y. Cai et al.

$$A_s = -xB(s) = -xB_0\theta(s).$$

$$\vec{A} = (A_x, 0, A_s)$$

$$\nabla \times \nabla \times \vec{A} = 0$$

$$A_x = B_0 \sum_{n=1}^{\infty} \frac{(-1)^n \theta^{(2n-1)}(s)}{(2n)!} y^{2n}$$

$$A_y = 0$$

$$A_x = \frac{1}{2}(x^2 - y^2) \sum_{p=0}^{\infty} \frac{1}{2+p} G_{1,2p+1}(s)(x^2+y^2)^p,$$

$$A_y = xy \sum_{p=0}^{\infty} \frac{1}{2+p} G_{1,2p+1}(s)(x^2+y^2)^p,$$

$$A_s = -x \sum_{p=0}^{\infty} G_{1,2p}(s)(x^2+y^2)^p.$$

$$G_{n,2p}(s) = (-1)^p \frac{n!}{4^p(n+p)!p!} \frac{d^{2p} G_{n,0}(s)}{ds^{2p}},$$

$$G_{n,2p+1}(s) = \frac{dG_{n,2p}(s)}{ds},$$

$$A_y \neq 0$$

5. Symplectic tracking in SAD

➤ Fringe fields: Hard edge fringe for BEND

- **Maxwellian solution for hard-edge dipole field**
  - * G. Lee-Whiting et al. => E. Forest et al.
  - * S. Caspi et al. => M. Bassetti et al. => Y. Cai et al.

Field distribution with hard-edge:

$$B_x = 0,$$

$$B_y(y, s) = B_0 \sum_{n=1}^{\infty} \frac{(-1)^n y^{2n} \theta^{(2n)}(s)}{(2n)!},$$

$$B_s(y, s) = -2B_0 \sum_{n=1}^{\infty} \frac{(-1)^n n y^{2n-1} \theta^{(2n-1)}(s)}{(2n)!}.$$

Field distribution with hard-edge:

$$B_x(x, y, s) = -\frac{1}{2} B_0 xy \sum_{p=0}^{\infty} \frac{(-1)^p (x^2 + p^2)^p \theta^{(2p+2)}(s)}{4^p p!(p+2)!}, \tag{20}$$

$$B_y(x, y, s) = B_0 \theta(s) + B_0 \sum_{p=0}^{\infty} \frac{(-1)^{p+1} (x^2 + y^2)^p \theta^{(2p+2)}(s)}{4^{p+1} (p+1)!(p+2)!} \left[x^2 + (2p+3)y^2\right], \tag{21}$$

$$B_s(x, y, s) = B_0 y \sum_{n=1}^{\infty} \frac{(-1)^p (x^2 + y^2)^p \theta^{(2p+1)}(s)}{4^p p!(p+1)!}. \tag{22}$$

5. Symplectic tracking in SAD

➤ Fringe fields: Hard edge fringe for BEND

- **Maxwellian solution for hard-edge dipole field**
  - * G. Lee-Whiting et al. => E. Forest et al.
  - * S. Caspi et al. => M. Bassetti et al. => Y. Cai et al.

$$f = -V_1 = -\frac{1}{2\rho(1+\delta)} p_x y^2$$

Implemented in SAD:

$$x_2 = x_1 - \frac{1}{\rho(1+\delta)} y_1^2,$$

$$p_{y2} = p_{y1} + \frac{1}{\rho(1+\delta)} y p_{x1},$$

$$z_2 = z_1 + \frac{y_1^2}{2\rho(1+\delta)^2} p_{x2}.$$

Apply for LHC and FCCs?:

$$f = \frac{1}{8\rho(1+\delta)}(-p_x x^2 + 2p_y xy - 3p_x y^2)$$

$$x_2 = x_1 - \frac{1}{8\rho(1+\delta)}(x_1^2 + 3y_1^2),$$

$$y_2 = y_1 + \frac{1}{4\rho(1+\delta)} x_1 y_1,$$

$$p_{x2} = \frac{1}{d}\left[p_{x1} - \frac{1}{4\rho(1+\delta)}(y_1 p_{y1} - x_1 p_{x1})\right],$$

$$p_{y2} = \frac{1}{d}\left[p_{y1} - \frac{1}{4\rho(1+\delta)}(x_1 p_{y1} - 3y_1 p_{x1})\right],$$

$$z_2 = z_1 + \frac{x_1^2 + 3y_1^2}{8\rho(1+\delta)^2} p_{x2} - \frac{x_1 y_1}{4\rho(1+\delta)^2} p_{y2},$$

$$d = 1 + \frac{3y_1^2 - x_1^2}{16\rho^2(1+\delta)^2}.$$

5. Symplectic tracking in SAD

➤ Solenoid region

- **The most complicated part in SAD**
- **SAD uses GEO and BOUND to define a solenoid region**
- **Acceptable elements inside solenoid region: DRIFT, BEND(ANGLE=0), QUAD and MULT**
- **To simplify the transformation: In a SOL region, the coordinate is shifted on the axis of the solenoid, no matter how the design orbit bends there.**

5. Symplectic tracking in SAD

➤ Solenoid region

- DRIFT with BZ≠0
- BEND with BZ≠0: L≠0, K0≠0, SK0≠0, ANGLE=0 [Solvable]
- QUAD with BZ≠0: L≠0, K1≠0, SK1≠0 [Solvable?]
- The general case: MULT with BZ≠0 [Need multi-step integration]
  * Step 1: Solenoid fringe at the entrance
  * Step 2: Rotation of coordinate to cancel SK1
  * Step 3: Calculate the number slices for tracking
  * Step 4: Nonlinear Maxwellian fringe map at the entrance
  * Step 5: Linear soft edge fringe at the entrance
  * Step 6: Body part using "drift-kick-drift" integration
  * Step 7-11: Maps at exit

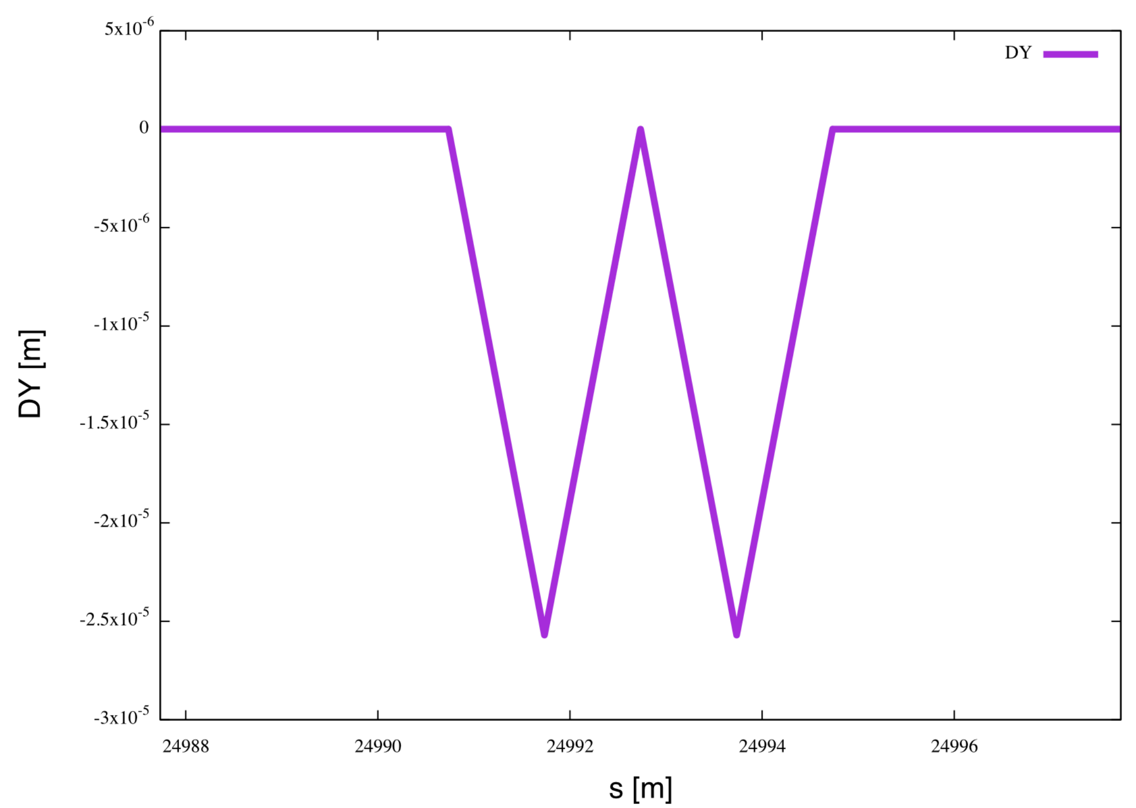
5. Symplectic tracking in SAD

➤ Tilted solenoid: FCC-ee as an example

- **SAD: Orbit patching**

```
SOL     ES3L    =(BZ =0   DX =-.03000219149072553  DY =3.394659937371677e-14     DZ =.0002250210956802542 BOUND =1
           CHI1 =.01499991193234515 CHI2 =-1.69694146889447e-14   CHI3 =-2.545603126440053e-16  F1 =.3 )
        ES2L    =(BZ =2   F1 =.1 )
        ES1L    =(BZ =-2  DPX =-.015     BOUND =1  CHI1 =-.015     GEO =1 )
        ES1R    =(BZ =2   DPX =-.015     BOUND =1  CHI1 =.015      CHI2 =-1.388395193188980 8e-28 CHI3 =-2.070759876205156e-30
           GEO =1 )
        ES2R    =(BZ =-2  F1 =.1 )
        ES3R    =(BZ =0   DX =.03000219149072553   DY =3.394659937371677e-14     DZ =.0002250210956802542 BOUND =1
           CHI1 =-.01499991193234515     CHI2 =-1.6971323927087947e-14 CHI3 =1.4946246979225722e-21  F1 =.3 )
```

- **Beam line: (-ES3L -LX2 -ES2L -LX1 -ES1L -IP IP ES1R LX1 ES2R LX2 ES3R)**



6. Summary

➤ **Lattice translation**

- **Translators collected**
- **Examples uploaded to MAD-X svn repository and my webpage**
- **Benchmark of SAD, Bmad and MAD-X/PTC for several projects**

➤ **Applications**

- **Synchrotron radiation simulation using Bmad (Synrad3D)**
- **RDT calculations using PTC**
- **Analysis of lattice nonlinearity in SuperKEKB and simulations of beam-beam with nonlinearity**

➤ **Future plan**

- **Translators to be improved (joint efforts)**
- **Accelerator design/simulations: Applications**