Impedance modeling and impedance effects

Demin Zhou

Accelerator theory group, Accelerator laboratory, KEK

Acknowledgments

J. Tang, T. Ishibashi

Accelerator physics seminar, Nov. 10, 2023, USTC, Hefei, China

Impedance modeling

- 束流尾场模拟计算简介
- 麦克斯韦方程
- 洛伦兹力
- 尾场函数和阻抗基本定义
- Panofsky-Wenzel定理介绍
- 探讨1: 尾场函数和阻抗的特性
- 探讨2: 尾场函数定义的推广
- 探讨3: Panofsky-Wenzel定理的推广

参考资料

- [1] http://www.gdfidl.de/
- *[2] S. Heights, A. Wagner, B. Zotter, “Generalized Impedance and Wakes in Asymmetric Structures”, SLAC/AP110, Jan. 1998.
- *[3] A. Chao, “Lecture Notes on Topics in Accelerator Physics”, SLAC-PUB-9574, Nov. 2002.
- *[4] A. Chao, “Lectures on Accelerator Physics”, World Scientific, 2020.
- [5] A. Chao et al., “Handbook of Accelerator Physics and Engineering”, World Scientific, 2013.
- [6] D. Zhou and C.-Y. Tsai, “Generalized Panofsky-Wenzel theorem in curvilinear coordinate systems applicable to non-ultrarelativistic beams”, arXiv:2101.04369.
- *[7] A. Wolski, “Beam Dynamics in High Energy Particle Accelerators”, Imperial College Press, 2014.
- [8] T. Agoh, “Dynamics of Coherent Synchrotron Radiation by Paraxial Approximation”, Ph.D. thesis, 2004.

- *推荐阅读

束流尾场模拟计算简介

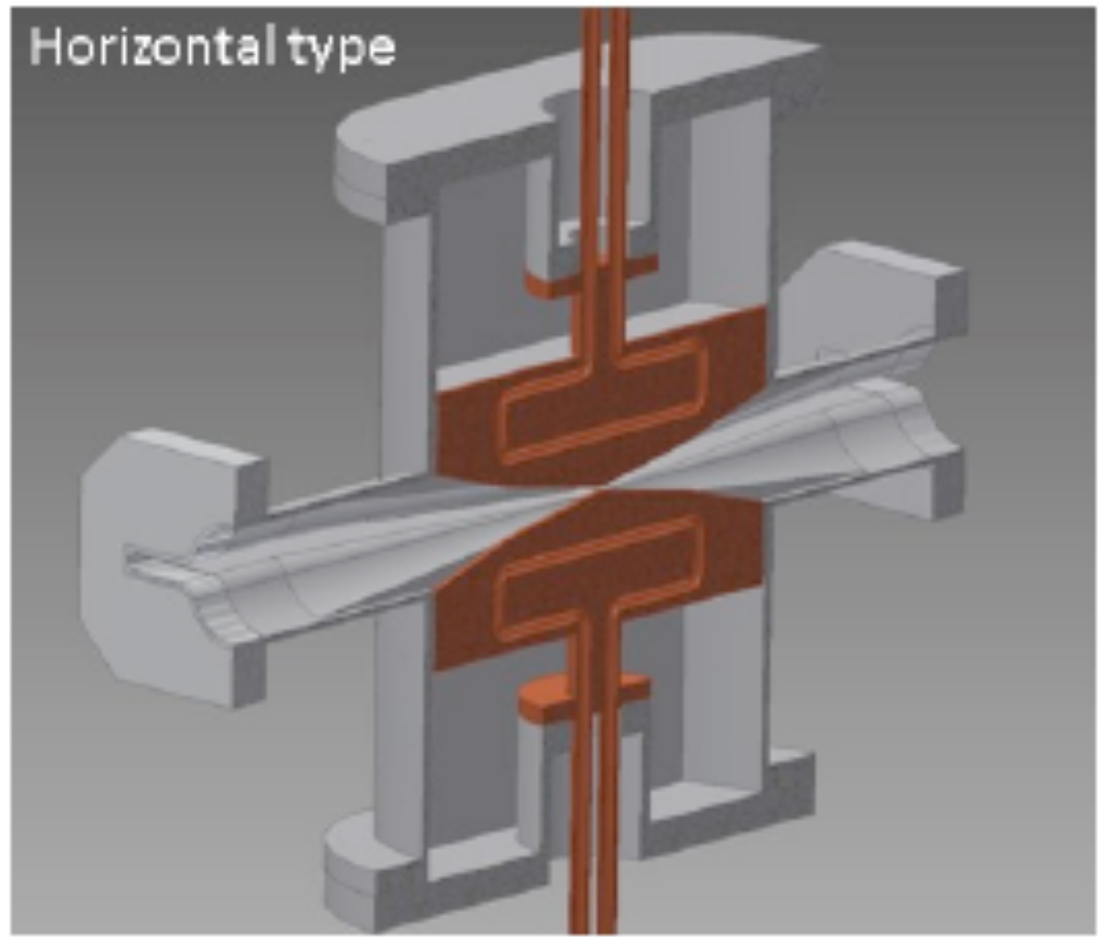
- 基本流程: 驱动束团通过2D或3D结构，记录束流激励产生的电磁场，进而计算尾场势
- 以GdfidL[1]为例，束团通过9-cell TESLA/ILC加速腔时束流尾场:

From http://www.gdfidl.de/

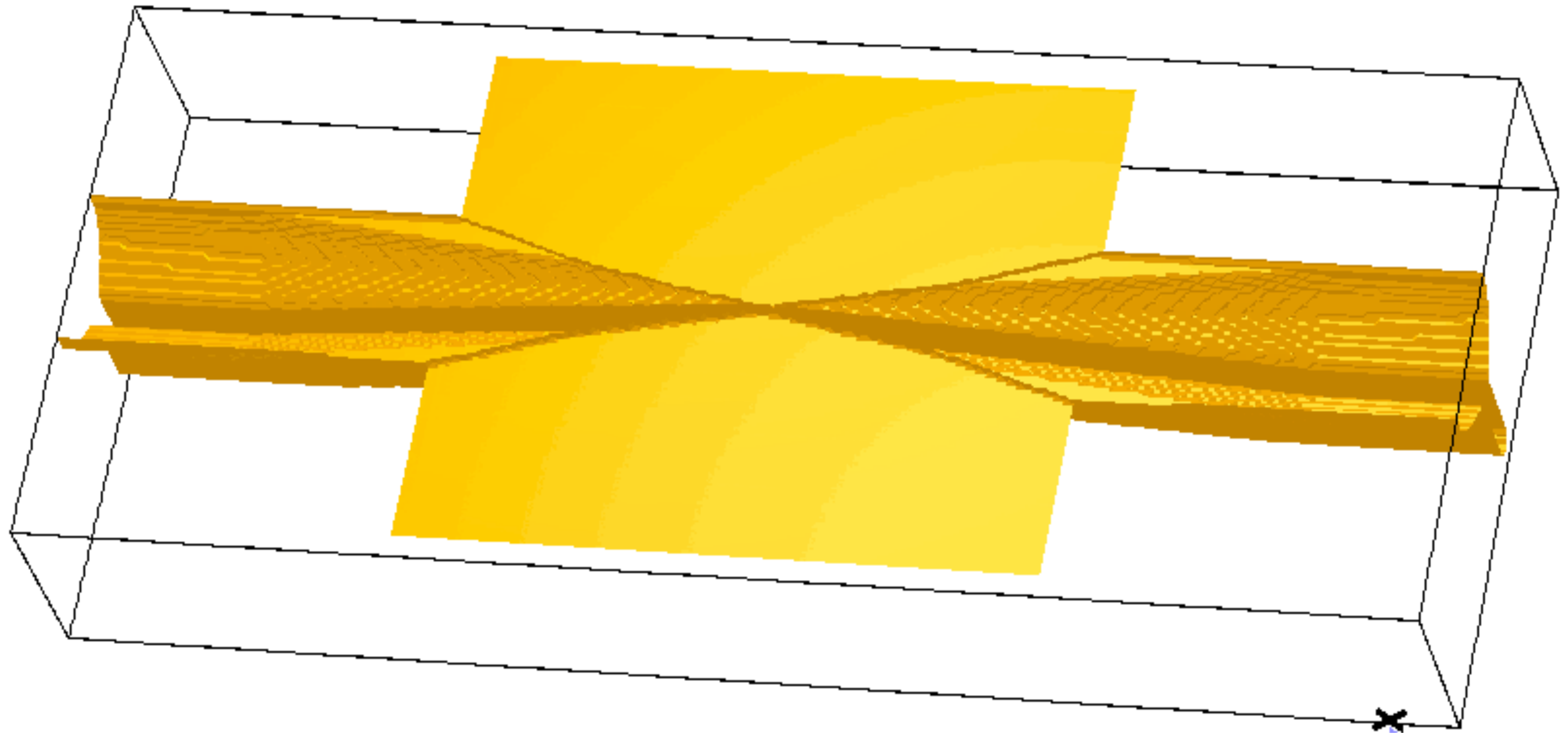
- 一些基本观察：
  - 束流以接近光速通过结构，束流自身电磁场(self-field)紧随束流前行。
  - 3D结构为腔型结构时，结构本身有不同频率的谐振模。部分谐振模被束流激励，形成驻波式电磁场。
  - 高频电磁场不会被局部俘获，能够在波导结构中朝束流行进方向(和反方向)传输。
  - 束流以光速直线行进时，束流激励起的电磁场不会超越束流本身。

束流尾场模拟计算简介

- 使用GdfidL计算尾场势流程
  - 3D设计 -> GdfidL 3D建模 -> 束流参数设置 -> 电磁场数值模拟 -> 后处理(计算尾场势和耦合阻抗)



CAD design of SuperKEKB collimator



GdfidL model of SuperKEKB collimator

Courtesy of T. Ishibashi

束流尾场模拟计算简介

- 以水平方向尾场势为例(定义参考[2]):

$$W_x(s,x,0,x_0,0) \approx W_x(s,0,0,0,0) + \frac{\partial W_x}{\partial x_0}\Delta x_0 + \frac{\partial W_x}{\partial x}\Delta x$$

Monopolar part

Dipolar part

Quadrupolar part

Relative position in moving direction

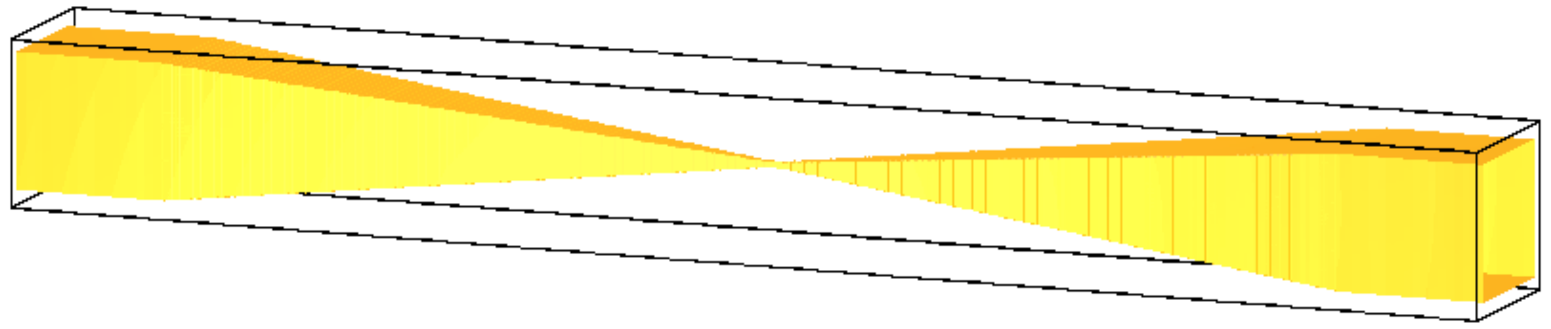
Offset of driving beam

Offset of monitoring position

- GdfidL输入参数设置
  - 驱动束流: -lcharge xposition=0, yposition=0
  - 尾场观测点: -wakes wxatxy=(0.5e-3,0.), wyatxy=(0.,0.5e-3)
- 设置不同的xposition/yposition和wxatxy/wyatxy，做多次计算，然后根据上式即可得到单极(Monopolar)、二极(Dipolar)、和四极(Quadrupolar)尾场势
- 其他尾场计算程序: ABCI(https://abci.kek.jp/), ECHO(https://echo4d.de/), CST Particle Studio

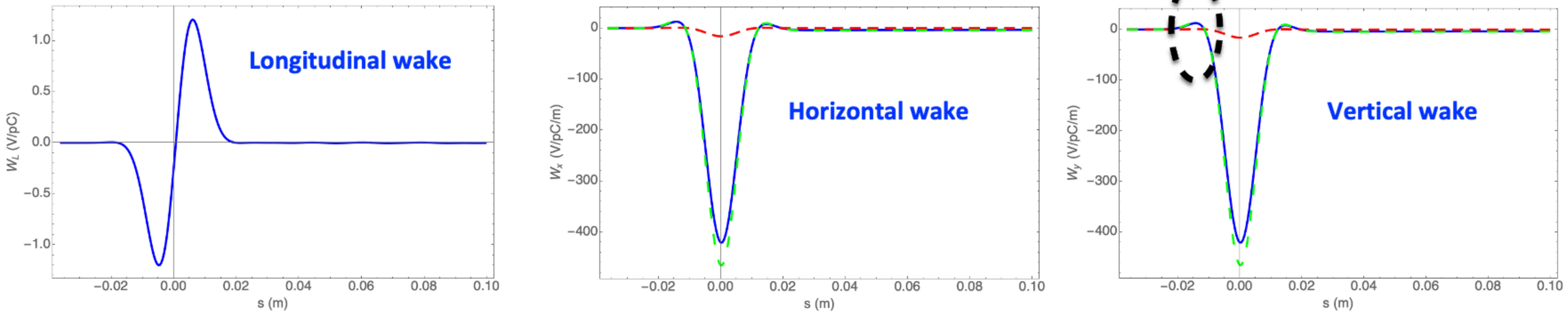
束流尾场模拟计算简介

- GdfidL计算实例: 简化的束流准直器

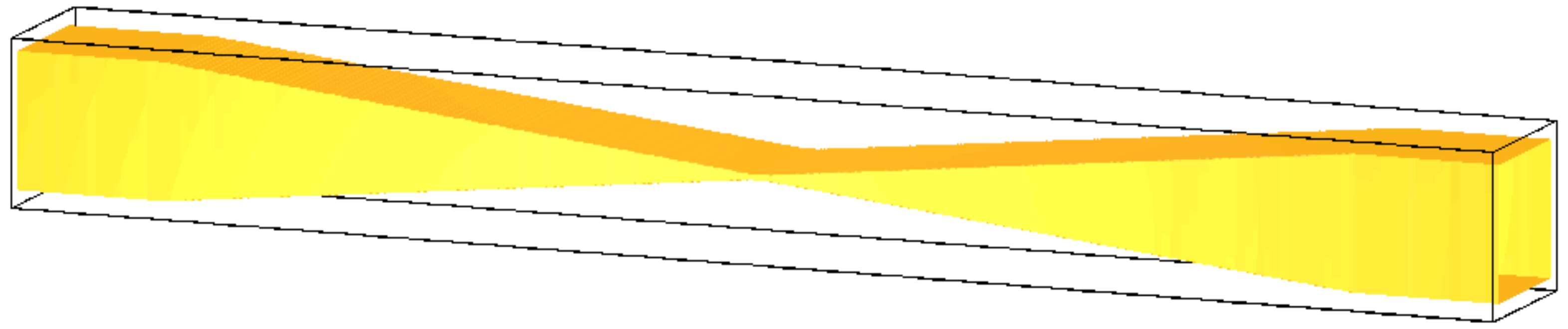


● w/h = 4/4 mm (close to round collimator):

*** For round collimators, dipolar wakes dominates the quadrupolar wakes**

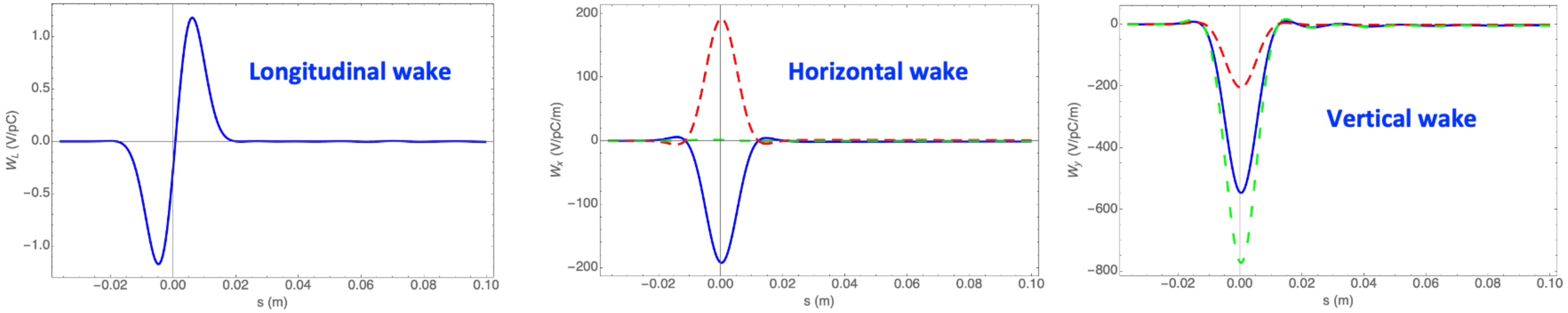


Numerical noise



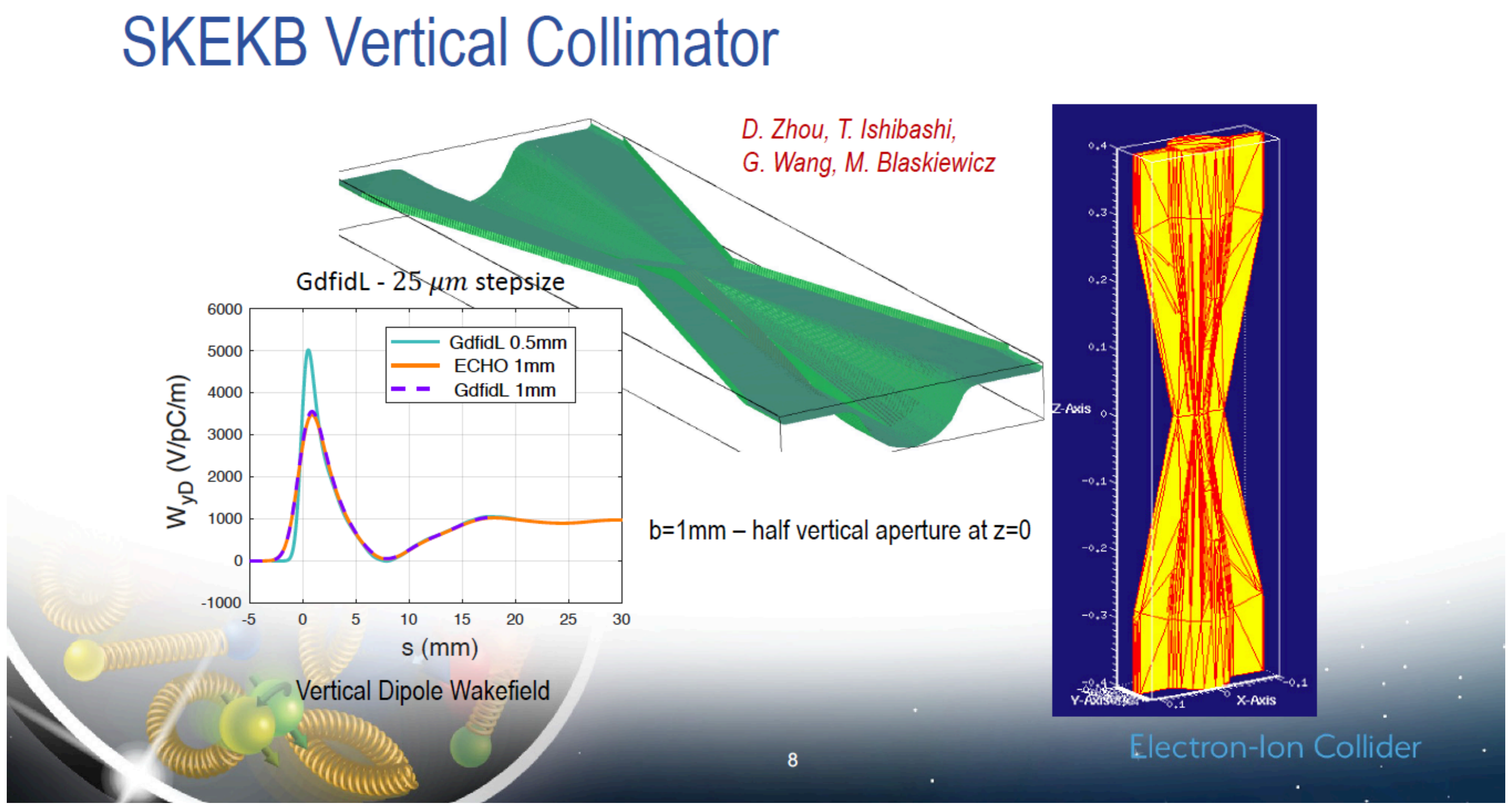
Blue: Dipolar
Red: Quadrupolar
Green: D + Q

● w/h = 10/4 mm:



SuperKEKB-type collimator

- Wake calculation using GdfidL and ECHO3D at BNL
  - BNL (GdfidL): Driving bunch σ_z=0.5/1 mm; Mesh sizes: dx=dy=dz=0.025 mm; Use“-windowwake” method.
  - DESY (ECHO3D): Driving bunch σ_z=0.5 mm; Mesh sizes: dx=dy=0.1 mm, dz=0.25 mm.
  - KEK (GdfidL): Driving bunch σ_z=0.5 mm; Mesh sizes: dx=dy=0.2 mm, dz=0.1 mm (limited by available computing resources); Use standard “-fdtd” method.



SKEKB Vertical Collimator

D. Zhou, T. Ishibashi, G. Wang, M. Blaskiewicz

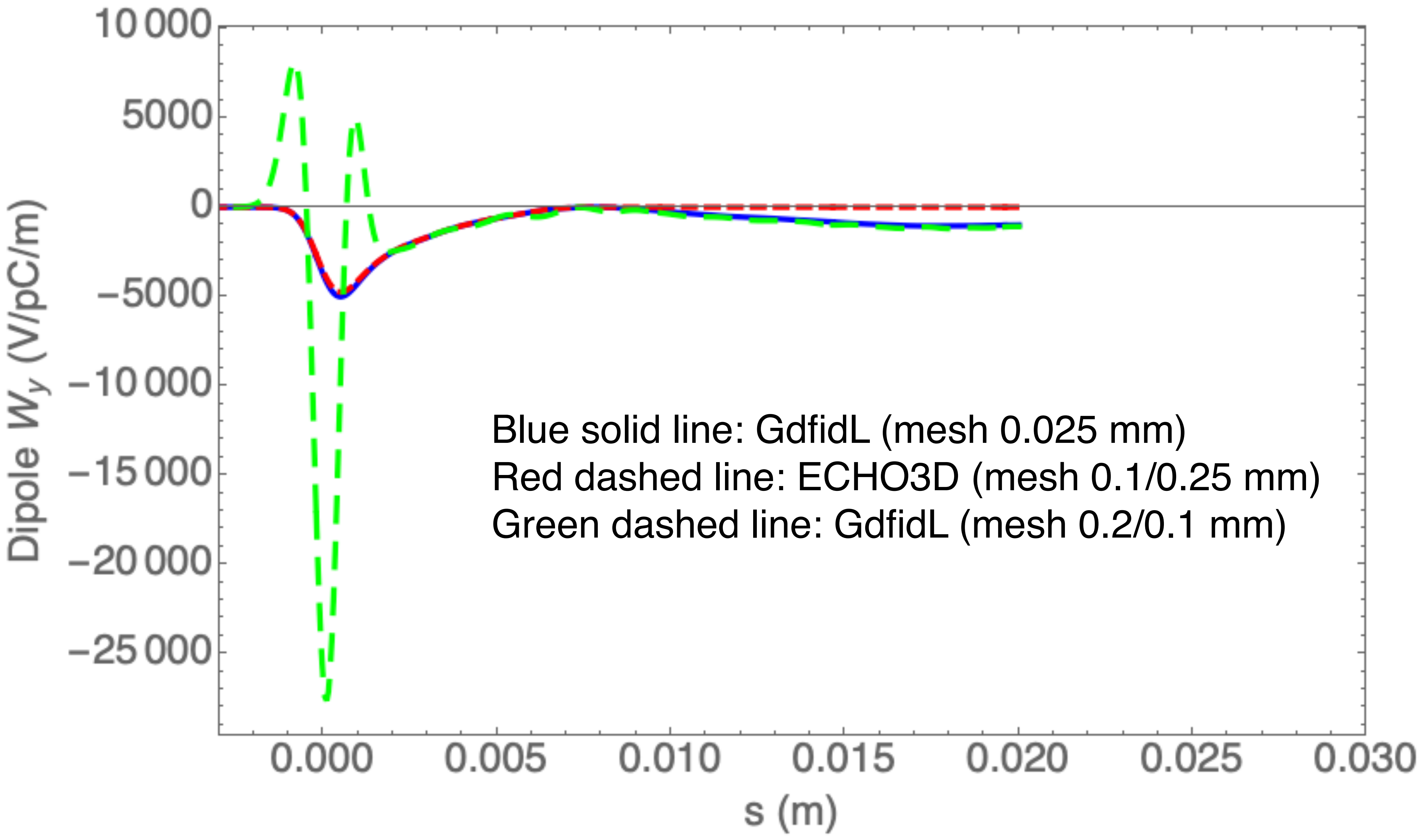
b=1mm – half vertical aperture at z=0

Vertical Dipole Wakefield

Electron-Ion Collider

8

Courtesy of A. Blednykh

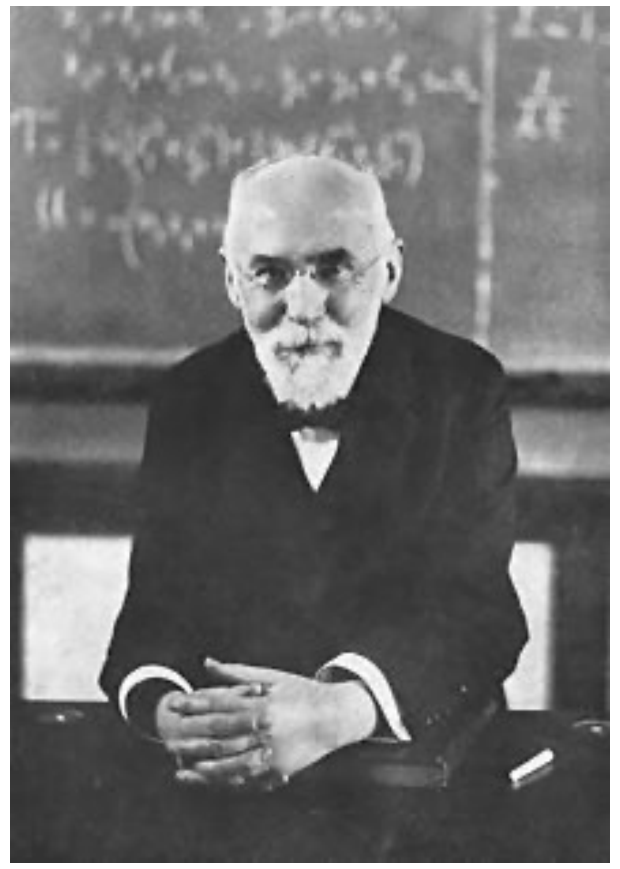


Blue solid line: GdfidL (mesh 0.025 mm)
Red dashed line: ECHO3D (mesh 0.1/0.25 mm)
Green dashed line: GdfidL (mesh 0.2/0.1 mm)

Computing short-bunch wakes of small-collimators is a challenging task (pushing CPU resources)

麦克斯韦方程

- 加速器中研究束流动力学的三个基础[3,4]
  - 麦克斯韦方程: 描述电磁场变化规律
  - 洛伦兹力: 描述电磁场对带电离子的作用规律
  - 弗拉索夫方程: 描述束流在6D相空间中的变化规律



Hendrik Lorentz
(1853-1928)
Dutch
-> Lorentz force

Ludvig Lorenz
(1829-1891)
Danish
-> Lorenz gauge

Photos from Wikipedia

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \longrightarrow \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \Phi$$

$$\nabla \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J},$$

$$\nabla \cdot \vec{B} = 0, \longrightarrow \vec{B} = \nabla \times \vec{A}.$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0},$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}. \longrightarrow \vec{J} = \rho \vec{v}.$$

$$\longrightarrow$$

$$\nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J}$$

$$\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}.$$

Lorenz gauge: $\nabla \cdot \vec{A} = -\frac{1}{c^2} \frac{\partial \Phi}{\partial t}$

麦克斯韦方程

- 加速器中研究束流动力学的三个基础[3,4]
  - 麦克斯韦方程: 描述电磁场变化规律
  - 洛伦兹力: 描述电磁场对带电离子的作用规律
  - 弗拉索夫方程: 描述束流在6D相空间中的变化规律[5]
- 联合方程的解耦[3,4]
  - 刚性束流(Rigid beam)近似
  - 冲击(Impulse)近似

    注: 这两个近似在加速器物理中非常重要, 是很多理论(特别是尾场函数和阻抗理论)的前提条件
- 束流动力学模拟的基本流程
  - “Drift”: 外部场(External fields)作用下束流运动(束团内部粒子间不相互作用->单粒子动力学)
  - “Kick”: 自身场(Self-field)作用(束团内部粒子间相互作用->束流集体效应)
  - “Drift” -> “Kick” -> “Drift” -> “Kick” ...

$$\vec{F} = \underset{\substack{\big\uparrow\\ \text{Charge}}}{q\vec{E}} + \underset{\substack{\big\uparrow\\ \text{Current}}}{\boxed{q\vec{v}}} \times \vec{B}$$

$$\frac{\partial\psi}{\partial t} + \dot{\vec{q}}\cdot\frac{\partial\psi}{\partial\vec{q}} + \dot{\vec{p}}\cdot\frac{\partial\psi}{\partial\vec{p}} = 0$$

$$\frac{\partial\psi}{\partial t} + \vec{v}\cdot\frac{\partial\psi}{\partial\vec{x}} + \frac{1}{m}\vec{F}(\vec{x},t)\cdot\frac{\partial\psi}{\partial\vec{v}} = 0$$

洛伦兹力

- 笛卡尔坐标系下洛伦兹力的旋度和散度[6]
  - 从麦克斯韦方程考察洛伦兹力的特点

$$\nabla \times \vec{F} = q\nabla \times \left[\vec{E} + \vec{v} \times \vec{B}\right]$$

$$\nabla \times (\vec{a} \times \vec{b}) = \vec{a} \cdot (\nabla \cdot \vec{b}) - \vec{b}(\nabla \cdot \vec{a}) + (\vec{b} \cdot \nabla)\vec{a} - (\vec{a} \cdot \nabla)\vec{b},$$

$$\nabla \times (\vec{v} \times \vec{B}) = \vec{v} \cdot (\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{v}) + (\vec{B} \cdot \nabla)\vec{v} - (\vec{v} \cdot \nabla)\vec{B}.$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \cdot \vec{v} = 0, \text{ and } \nabla\vec{v} = 0$$

$$\nabla \times (\vec{v} \times \vec{B}) = -(\vec{v} \cdot \nabla)\vec{B}$$

$$\nabla \times \vec{F} = -q\left[\frac{\partial}{\partial t}\vec{B} + (\vec{v} \cdot \nabla)\vec{B}\right]$$

$$\nabla \cdot \vec{F} = q\nabla \cdot \left[\vec{E} + \vec{v} \times \vec{B}\right]$$

$$\nabla \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\nabla \times \vec{a}) - \vec{a} \cdot (\nabla \times \vec{b}),$$

Vorticity (涡度): $\nabla \times \vec{v} = \vec{\Omega}$

$$\nabla \cdot \vec{F} = q\left[\frac{1}{\gamma^2\epsilon_0}\rho + \vec{\Omega} \cdot \vec{B} - \frac{1}{c^2}\vec{v} \cdot \frac{\partial \vec{E}}{\partial t}\right]$$

$$\nabla \times \vec{F} = -q\frac{d\vec{B}}{dt}$$

尾场函数和阻抗基本定义

Photo from Wikipedia

- 基本概念[5,7]
  - 尾场(Wake field): 束流激励的电磁场(广义定义包含空间电荷场、辐射场和环境相互作用产生的场)
    - 通常束流接近光束前行，其激励的电磁场作用于尾随的粒子，称为尾场。但是束流非光速或沿曲折轨道前行时，尾场能作用于前行离子。
  - 尾场势(Wake potential): 尾场对带电粒子作用力的积分效应。尾场势适用于单粒子或束团
  - 尾场函数(Wake function): 对尾场势进行多级展开，展开式中仅与纵向位置有关的系数项
  - 阻抗(Impedance): 尾场函数的傅里叶变换
- 经典数学表达式[5]

$$\overline{\vec{F}_\perp} = -eI_m \boxed{W_m(z)}\, m r^{m-1}\left(\hat{r}\cos(m\theta) - \hat{\theta}\sin(m\theta)\right)$$

$$\overline{F_\parallel} = -eI_m \boxed{W'_m(z)}\, m r^{m}\cos(m\theta)$$

Wake function

$$\overline{\vec{F}} = \int_{-L/2}^{L/2} ds\vec{F}$$

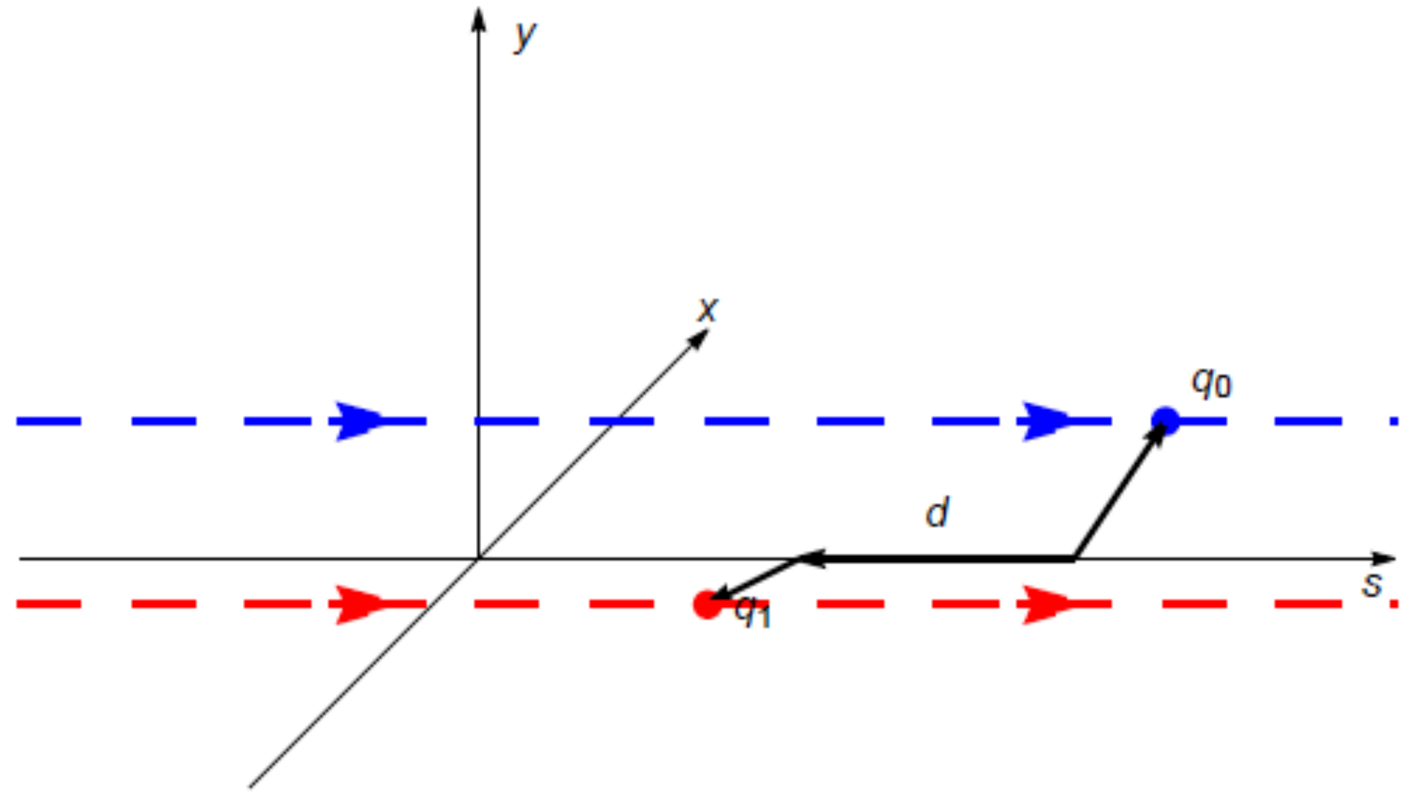
Wake potential

Wake field

Impedance:

$$Z_{\parallel m}(\omega) = \int_{-\infty}^{\infty}\frac{dz}{v}e^{-i\omega z/v}W'_m(z) \qquad Z_{\perp m}(\omega) = \frac{i}{v/c}\int_{-\infty}^{\infty}\frac{dz}{v}e^{-i\omega z/v}W_m(z)$$

尾场函数和阻抗基本定义



- 广义数学表达形式[2,6]
  - 尾场函数: 单位电荷通过环境时对尾随单位电荷作用的积分效应(格林函数定义, 描述环境对束流的响应; 激励电荷可以沿直线匀速行进，也可以沿曲线行进)
  - 尾场势：尾场函数对电荷分布的积分
  - 阻抗：尾场函数的傅里叶变换
- 变量$_{d\,=\,v\tau}$的物理含义: 源粒子和测试粒子的相对位置(非常重要)

$$\rho(\vec{R},t) = q_0\delta(x-x_0)\delta(y-y_0)\delta(z-z_0)$$

$$\vec{J}(\vec{R},t) \;=\; \rho(\vec{R},t)\vec{v}. \qquad \vec{v} = \vec{i}_z v$$

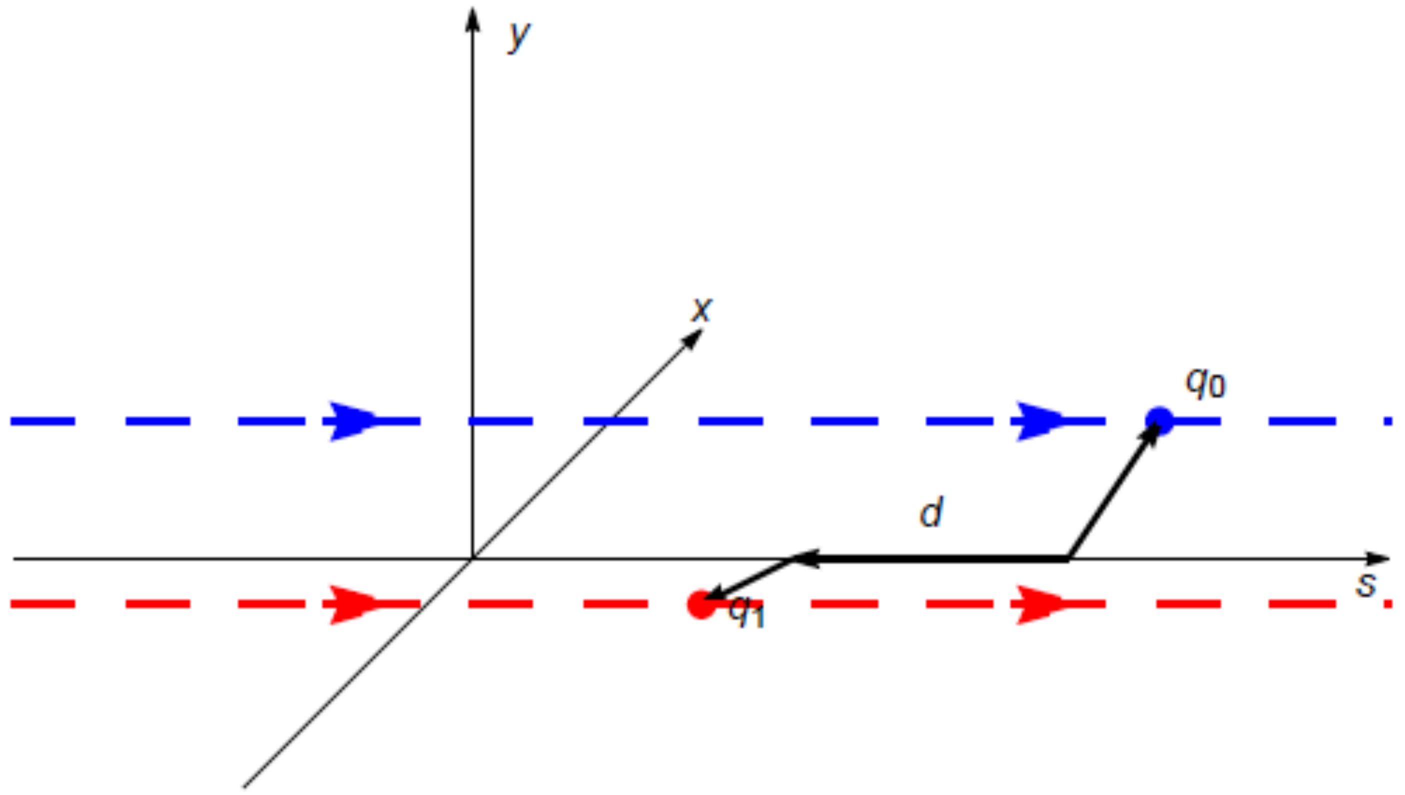
$$z_0 \equiv vt \text{ and } \vec{R}_0 \equiv (x_0, y_0, z_0)$$

$$\vec{F}(\vec{R},\vec{R}_0;t) = q_1\left[\vec{E}(\vec{R},\vec{R}_0;t) + \vec{v}\times\vec{B}(\vec{R},\vec{R}_0;t)\right]$$

$$\tau = d/v = (z_0 - z)/v$$

$$\vec{F}(\underbrace{\vec{r},\vec{r}_0;\tau}_{\text{变量个数: 5}}) = \int_{-\infty}^{\infty} dt\; v\; \vec{F}(\underbrace{\vec{R},\vec{R}_0;t}_{\text{变量个数: 7}})\Big|_{\underbrace{z_0=vt,z=vt-d}_{\text{约束条件}}}.$$

尾场函数和阻抗基本定义



- 广义数学表达形式[2,6]
  - 尾场函数: 单位电荷通过环境时对尾随单位电荷作用的积分效应(格林函数定义, 描述环境对束流的响应; 激励电荷可以沿直线匀速行进， 也可以沿曲线行进)
  - 纵向尾场函数w_z中的负号(-)和$d = v\tau = z_0 - z$相关联(非简单的传统定义， 参考Panofsky-Wenzel定理的讨论)
  - 横向阻抗$z_\perp$和尾场函数$w_\perp$中的κ和传统有关。
    - κ=1对应简单的傅里叶变换定义，但是Panofsky-Wenzel定理的表达形式不够简洁
    - $\kappa = i/\beta$(其中$\beta = v/c$)为通常定义[5]， Panofsky-Wenzel定理形式简洁
    - $\kappa = i$在文献中(特别是1990年代以前的早期文献)也较常见
  - 要特别注意傅里叶变换中$d = v\tau$的对应关系。这里v为束流的绝对速度。$\beta < 1$在质子、重离子加速器中、或低能电子加速器中会出现, 此时阻抗和尾场函数的变化必须考虑$\beta < 1$(阅读文献时要留意)

$$w_z(\vec{r},\vec{r}_0;d) = -\frac{1}{q_0 q_1}\overline{F}_z(\vec{r},\vec{r}_0;\tau),\qquad w_z(\vec{r},\vec{r}_0;d) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ Z_\parallel(\vec{r},\vec{r}_0;\omega)e^{-i\omega\tau},\qquad Z_\parallel(\vec{r},\vec{r}_0;\omega) = \int_{-\infty}^{\infty} d\tau\ w_z(\vec{r},\vec{r}_0;d)e^{i\omega\tau},$$

$$w_\perp(\vec{r},\vec{r}_0;d) = \frac{1}{q_0 q_1}\overline{F}_\perp(\vec{r},\vec{r}_0;\tau),\qquad w_\perp(\vec{r},\vec{r}_0;d) = \frac{1}{2\pi\kappa}\int_{-\infty}^{\infty} d\omega\ Z_\perp(\vec{r},\vec{r}_0;\omega)e^{-i\omega\tau}.\qquad Z_\perp(\vec{r},\vec{r}_0;\omega) = \kappa\int_{-\infty}^{\infty} d\tau\ w_\perp(\vec{r},\vec{r}_0;d)e^{i\omega\tau},$$

尾场函数和阻抗基本定义

- 广义数学表达形式[2,6]
  - 广义的尾场函数和阻抗定义包含五个参数：源粒子和测试粒子的横向坐标、以及纵向相对位置。这个定义不方便使用，因此会做各种简化。
  - 一种简化的方法是将尾场函数或阻抗对源粒子和测试粒子的横向坐标做泰勒展开(参考[2]中的讨论)。这在尾场的数值计算中非常常见(参考“束流尾场模拟计算简介”部分的讨论)
  - 假定尾场函数和阻抗与横向粒子坐标关联性很小，则可以忽略横向坐标。从而将3D的阻抗模型简化为1D，这种处理方法在空间电荷效应(SC)和相干同步辐射(CSR)问题中较常见
  - 另外一种处理办法是对电荷分布进行积分，计算”平均”效应。公式表达如下:

$$\mathcal{Z}_u(\vec{r};k) = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \rho_\perp(x',y') Z_u(\vec{r},\vec{r}';k) \longrightarrow$$

对源电荷的横向分布做平均
这里假定横向分布和纵向位置无关

$$\overline{\mathcal{Z}}_u(k) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \rho_\perp(x,y) \mathcal{Z}_u(\vec{r};k), \longrightarrow$$

对测试电荷的横向分布做平均
这里假定横向分布和纵向位置无关

尾场函数和阻抗基本定义

- 广义数学表达形式[2,6]
  - 广义的尾场函数和阻抗定义包含五个参数：源粒子和测试粒子的横向坐标、以及纵向相对位置。这个定义不方便使用，因此会做各种简化。
  - 一种简化的方法是将尾场函数或阻抗对源粒子和测试粒子的横向坐标做泰勒展开(参考[2]中的讨论)。这在尾场的数值计算中非常常见(参考“束流尾场模拟计算简介”部分的讨论)
  - 假定尾场函数和阻抗与横向粒子坐标关联性很小，则可以忽略横向坐标。从而将3D的阻抗模型简化为1D，这种处理方法在空间电荷效应(SC)和相干同步辐射(CSR)问题中较常见
  - 另外一种处理办法是对电荷分布进行积分，计算”平均”效应。公式表达如下:

$$W_u(\vec{r};d) = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dz' \rho(x',y',z') w_u(\vec{r},\vec{r}';d-z') \longrightarrow$$

对源电荷的横向分布做平均
电荷密度可以是三维分布

$$\overline{W}_u(d) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \rho_\perp(x,y,d) W_u(\vec{r};d). \longrightarrow$$

对测试电荷的横向分布做平均
电荷密度可以是三维分布

Panofsky-Wenzel定理

- 经典表述
  - Panofsky-Wenzel (P-W)定理的经典表述假定$\vec{v}(t)=v\vec{i}_z$ (源粒子和测试粒子以相同速度沿同一直线方向运动)
  - P-W定理是麦克斯韦方程(结合两个近似假定[3,4])的直接推论。
  - P-W定理是尾场函数和阻抗理论体系的基础。

$$\nabla \times \vec{F}(\vec{R},\vec{R}_0;t) = q_1 \nabla \times \left[\vec{E}(\vec{R},\vec{R}_0;t) + \vec{i}_z v \times \vec{B}(\vec{R},\vec{R}_0;t)\right].$$

$$\nabla \times \vec{F}(\vec{R},\vec{R}_0;t) = -q_1 \left[\frac{\partial}{\partial t}\vec{B}(\vec{R},\vec{R}_0;t) + v\frac{\partial}{\partial z}\vec{B}(\vec{R},\vec{R}_0;t)\right].$$

$$\nabla' \times \vec{w}(\vec{r},\vec{r}_0;d) = \frac{v}{q_0 q_1}\int_{-\infty}^{\infty} dt \left[\nabla \times \vec{F}(\vec{R},\vec{R}_0;t)\right]_{z=vt-d}. \quad \longleftarrow \quad \frac{\partial}{\partial d} = -\frac{\partial}{\partial z}$$

$$\nabla' \times \vec{w}(\vec{r},\vec{r}_0;d) = -\frac{v}{q_0}\int_{-\infty}^{\infty} dt \left[\frac{\partial}{\partial t}\vec{B}(\vec{R},\vec{R}_0;t) + v\frac{\partial}{\partial z}\vec{B}(\vec{R},\vec{R}_0;t)\right]_{z=vt-d}.$$

Panofsky-Wenzel定理

- Panofsky-Wenzel (P-W)定理经典表述
  - P-W定理的经典表述假定$\vec{v}(t)=v\vec{i}_z$ (源粒子和测试粒子以相同速度沿同一方向运动)
  - P-W定理是麦克斯韦方程(结合两个近似假定[3,4])的直接推论(不需要边界条件)。
  - P-W定理是尾场函数和阻抗理论体系的基础。
  - P-W定理确定了横向和纵向尾场函数(包括阻抗)的严格关系，与所处理问题的边界条件以及介质等无关。
- 衍生含义
  - 学习阻抗和尾场理论应以P-W定理为起点，同时注意其成立的条件
  - 做阻抗、尾场的相关研究(数值计算、理论推导)或使用相关的理论和数值结果时，须时时考察P-W定理

$$\nabla' \times \vec{w}(\vec{r},\vec{r}_0;d) = -\frac{v}{q_0}\,\vec{B}(\vec{R},\vec{R}_0;t)\Big|_{t=-\infty}^{t=\infty},$$

$$\vec{B}(\vec{R},\vec{R}_0;t)\Big|_{t=-\infty} = \vec{B}(\vec{R},\vec{R}_0;t)\Big|_{t=\infty}.$$

$$\nabla' \times \vec{w}(\vec{r}_1,\vec{r}_0;d) = 0.$$

$$\frac{\partial w_x}{\partial d} = \frac{\partial w_z}{\partial x},$$

$$\frac{\partial w_y}{\partial d} = \frac{\partial w_z}{\partial y},$$

$$\frac{\partial w_x}{\partial y} = \frac{\partial w_y}{\partial x}.$$

$$\frac{\partial \vec{w}_\perp(\vec{r},\vec{r}_0;d)}{\partial d} = \nabla_\perp w_z(\vec{r},\vec{r}_0;d).$$

$$Z_x(\vec{r},\vec{r}_0;k) = \frac{i\beta\kappa}{k}\frac{\partial Z_\parallel(\vec{r},\vec{r}_0;k)}{\partial x}$$

$$Z_y(\vec{r},\vec{r}_0;k) = \frac{i\beta\kappa}{k}\frac{\partial Z_\parallel(\vec{r},\vec{r}_0;k)}{\partial y}$$

探讨1: 尾场函数和阻抗的特性

- 尾场函数为实函数，确定阻抗实部和虚部的关系[6]

$$Z_{\|}(-\omega) = Z_{\|}^{*}(\omega) \text{ and } Z_{\perp}(-\omega) = -Z_{\perp}^{*}(\omega).$$

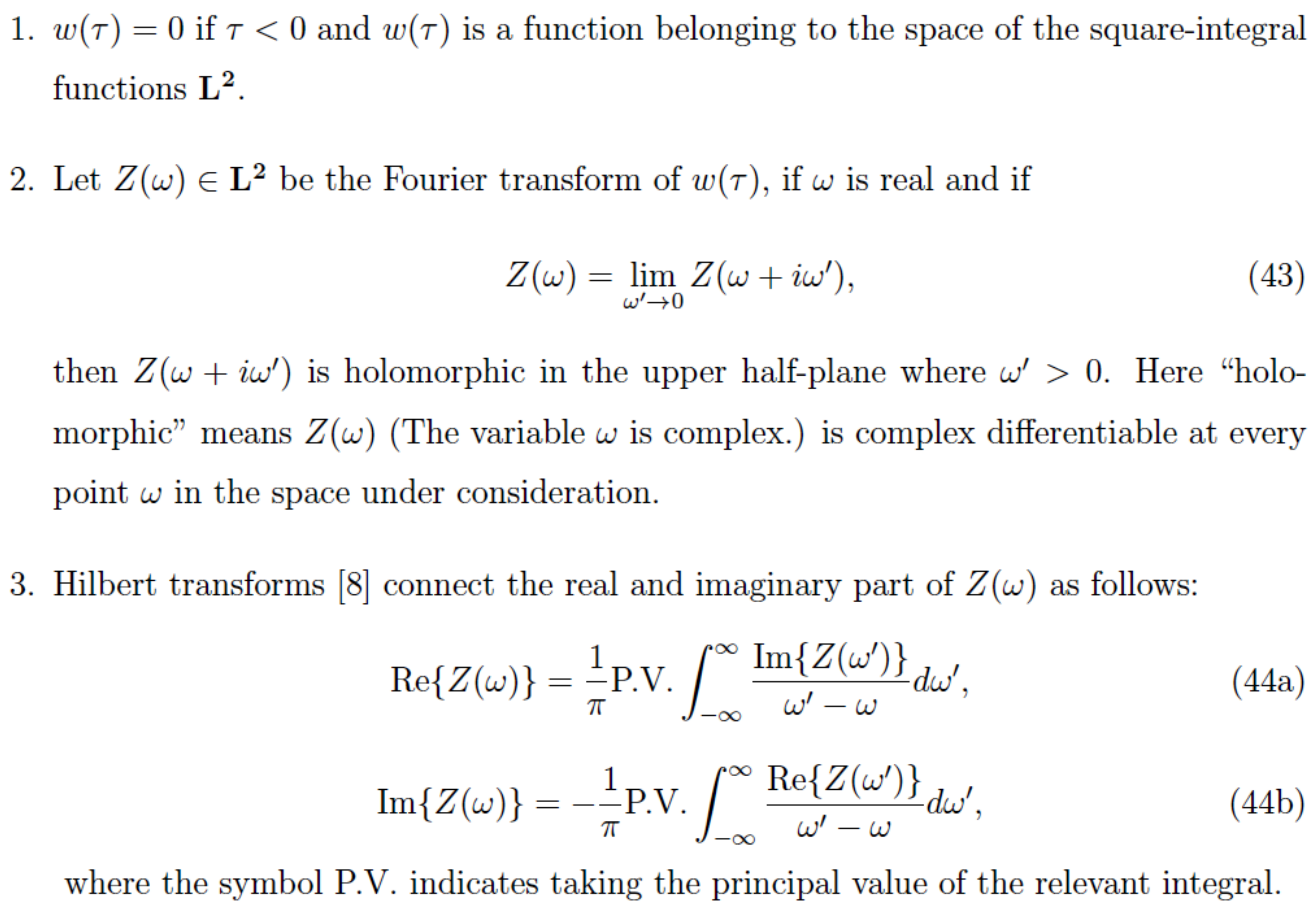
注意: κ的定义影响横向阻抗的特性

问题: κ为实数时，结果如何？

- $v = c$时阻抗特性
  - 因果律(Causality)-> 等价表述:

$$\mathrm{Re}\{Z(\omega)\} = \frac{2}{\pi}\mathrm{P.V.}\int_0^{\infty}\frac{\omega'\mathrm{Im}\{Z(\omega')\}}{\omega'^2-\omega^2}d\omega',$$

$$\mathrm{Im}\{Z(\omega)\} = -\frac{2\omega}{\pi}\mathrm{P.V.}\int_0^{\infty}\frac{\mathrm{Re}\{Z(\omega')\}}{\omega'^2-\omega^2}d\omega'.$$



1. $w(\tau) = 0$ if $\tau < 0$ and $w(\tau)$ is a function belonging to the space of the square-integral functions $\mathbf{L}^2$.

2. Let $Z(\omega) \in \mathbf{L}^2$ be the Fourier transform of $w(\tau)$, if ω is real and if

$$Z(\omega) = \lim_{\omega' \to 0} Z(\omega + i\omega'), \tag{43}$$

   then $Z(\omega + i\omega')$ is holomorphic in the upper half-plane where $\omega' > 0$. Here "holomorphic" means $Z(\omega)$ (The variable ω is complex.) is complex differentiable at every point ω in the space under consideration.

3. Hilbert transforms [8] connect the real and imaginary part of $Z(\omega)$ as follows:

$$\mathrm{Re}\{Z(\omega)\} = \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{\infty}\frac{\mathrm{Im}\{Z(\omega')\}}{\omega'-\omega}d\omega', \tag{44a}$$

$$\mathrm{Im}\{Z(\omega)\} = -\frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{\infty}\frac{\mathrm{Re}\{Z(\omega')\}}{\omega'-\omega}d\omega', \tag{44b}$$

   where the symbol P.V. indicates taking the principal value of the relevant integral.

注1: 可以在实际问题应用这些关系做辅助计算

注2: 这些关系不适用于包含SC和CSR的问题

探讨2: 尾场函数定义的推广

- 当粒子沿曲线运动时，尾场函数和阻抗的定义需要推广

$$\vec{F}(\vec{R}_1,\vec{R}_0;t)=\int\int\int dV\ \rho_1(\vec{R},\vec{R}_1,t-\tau)\left[\vec{E}(\vec{R},\vec{R}_0;t)+\vec{v}\times\vec{B}(\vec{R},\vec{R}_0;t)\right]$$

- 纵向尾场势表示为洛伦兹力对测试粒子的做功(更妥当)[6]
  - SC和CSR问题探讨中必须使用推广的尾场函数定义

$$\overline{F}_{\|}(\vec{r},\vec{r}_0;\tau)=\int_{-\infty}^{\infty}dt\ \vec{v}\cdot\vec{F}(\vec{R},\vec{R}_0;t).$$

$$\overline{F}_{\|}(\vec{r}_1,\vec{r}_0;\tau)=\int_{-\infty}^{\infty}dt\ \int\int\int dV\ \rho_1(\vec{R},\vec{R}_1,t-\tau)\vec{v}\cdot\vec{E}(\vec{R},\vec{R}_0;t).$$

$$\rho_1(\vec{R},\vec{R}_1,t-\tau)\vec{v}\ =\ \vec{J}_1(\vec{R},\vec{R}_1,t-\tau) \qquad w_z(\vec{r}_1,\vec{r}_0;\tau)=-\frac{1}{q_0q_1}\int_{-\infty}^{\infty}dt\ \int\int\int dV\ \vec{J}_1(\vec{R},\vec{R}_1,t)\cdot\vec{E}(\vec{R},\vec{R}_0;t+\tau).$$

探讨3: Panofsky-Wenzel定理的推广

- 广义的P-W定理[3,4,6]
  - P-W定理的关键是麦克斯韦发出和洛伦兹力的特点，描述电磁场冲量的自身特性

$$\nabla \times \vec{F}(\vec{R},\vec{R}_0;t) = -q_1 \frac{d\vec{B}(\vec{R},\vec{R}_0;t)}{dt}.$$

$$\int_{-\infty}^{\infty} dt \nabla \times \vec{F}(\vec{R},\vec{R}_0;t) = \boxed{-q_1 \, \vec{B}(\vec{R},\vec{R}_0;t)\Big|_{t=-\infty}^{t=\infty}}$$

← **注: 此项能否为零, 需要具体问题具体分析**

$$\int_{-\infty}^{\infty} dt \nabla \times \vec{F}(\vec{R},\vec{R}_0;t) = 0.$$

← **广义的P-W定理**

$$\int_{-\infty}^{\infty} dt \nabla \cdot \vec{F}(\vec{R},\vec{R}_0;t) = G(\vec{R},\vec{R}_0).$$

← **另一个关系**

探讨3: Panofsky-Wenzel定理的推广

- 广义的P-W定理[3,4,6]
  - $\vec{v}(t)=v\vec{i}_z$作为特例
  - $\vec{v}(t)=c\vec{i}_z$是排除空间电荷效应的特例

$$\nabla\cdot\vec{F}(\vec{R},\vec{R}_0;t)=q_1\left[\frac{1}{\gamma^2\epsilon_0}\rho-\frac{1}{c^2}\vec{v}\cdot\frac{\partial\vec{E}}{\partial t}\right] \qquad \frac{\partial\vec{E}}{\partial t}+(\vec{v}\cdot\nabla)\vec{E}=\frac{d\vec{E}}{dt}.$$

$$\nabla\cdot\vec{F}(\vec{R},\vec{R}_0;t)=q_1\left[\frac{1}{\gamma^2\epsilon_0}\rho-\frac{1}{c^2}\vec{v}\cdot\left(\frac{d}{dt}\vec{E}(\vec{R},\vec{R}_0;t)-(\vec{v}\cdot\nabla)\vec{E}(\vec{R},\vec{R}_0;t)\right)\right]$$

$$\nabla'\cdot\vec{w}(\vec{r},\vec{r}_0;d)=\frac{v}{q_0q_1}\int_{\infty}^{\infty}dt\left[\nabla\cdot\vec{F}(\vec{R}_1,\vec{R}_0;t)\right]_{z=vt-d}=\frac{v}{q_0q_1}G(\vec{r},\vec{r}_0;d).$$

非常重要的关系

$$\frac{\partial w_x}{\partial x}+\frac{\partial w_y}{\partial y}+\frac{1}{\gamma^2}\frac{\partial w_z}{\partial d}=\frac{v}{\gamma^2q_0\epsilon_0}\int_{-\infty}^{\infty}dt\rho. \qquad \gamma\to\infty \qquad \boxed{\frac{\partial w_x}{\partial x}+\frac{\partial w_y}{\partial y}=0.}$$

探讨3: Panofsky-Wenzel定理的推广

- 广义的P-W定理[8]
  - 同步辐射(SR)和扭摆磁铁辐射(Undulator radiation, UR)不能用经典P-W定理
  - P-W定理的自然推广是源粒子$\vec{v}(t)$为任意函数。如何确定冲量矢量的三个分量之间的关系是关键

$$\int_{-\infty}^{\infty} dt \nabla \times \vec{F}(\vec{R}, \vec{R}_0; t) = 0.$$

  - 问题: 刚性束流近似如何沿用或推广？这个问题比较复杂。可以假定$\vec{v}(t)$不是空间坐标的函数(即空间内任意位置的电荷运动速度相同)
  - 以CSR问题为例，可以选择Frenet-Serret坐标系。此时测试粒子的速度为空间位置的函数

$$\vec{v}(t) = v\vec{e}_s$$

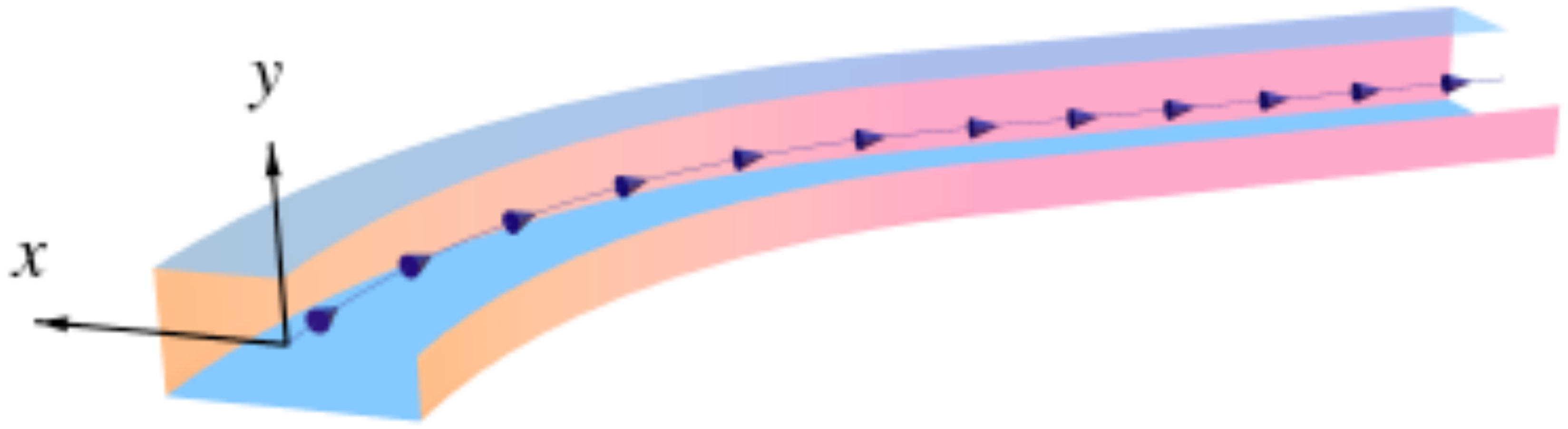
$$\frac{\vec{e}_x}{ds} = \frac{1}{\rho_x}\vec{e}_s,$$

$$\frac{\vec{e}_y}{ds} = 0,$$

$$\frac{\vec{e}_s}{ds} = -\frac{1}{\rho_x}\vec{e}_x,$$

$$\frac{d\vec{r}}{dt} = (\beta_x\vec{e}_x + \beta_y\vec{e}_y + g\vec{e}_s)\frac{ds}{dt},$$

$$\beta_x = dx/ds,\ \beta_y = dy/ds, \text{ and } g = 1 + x/\rho$$

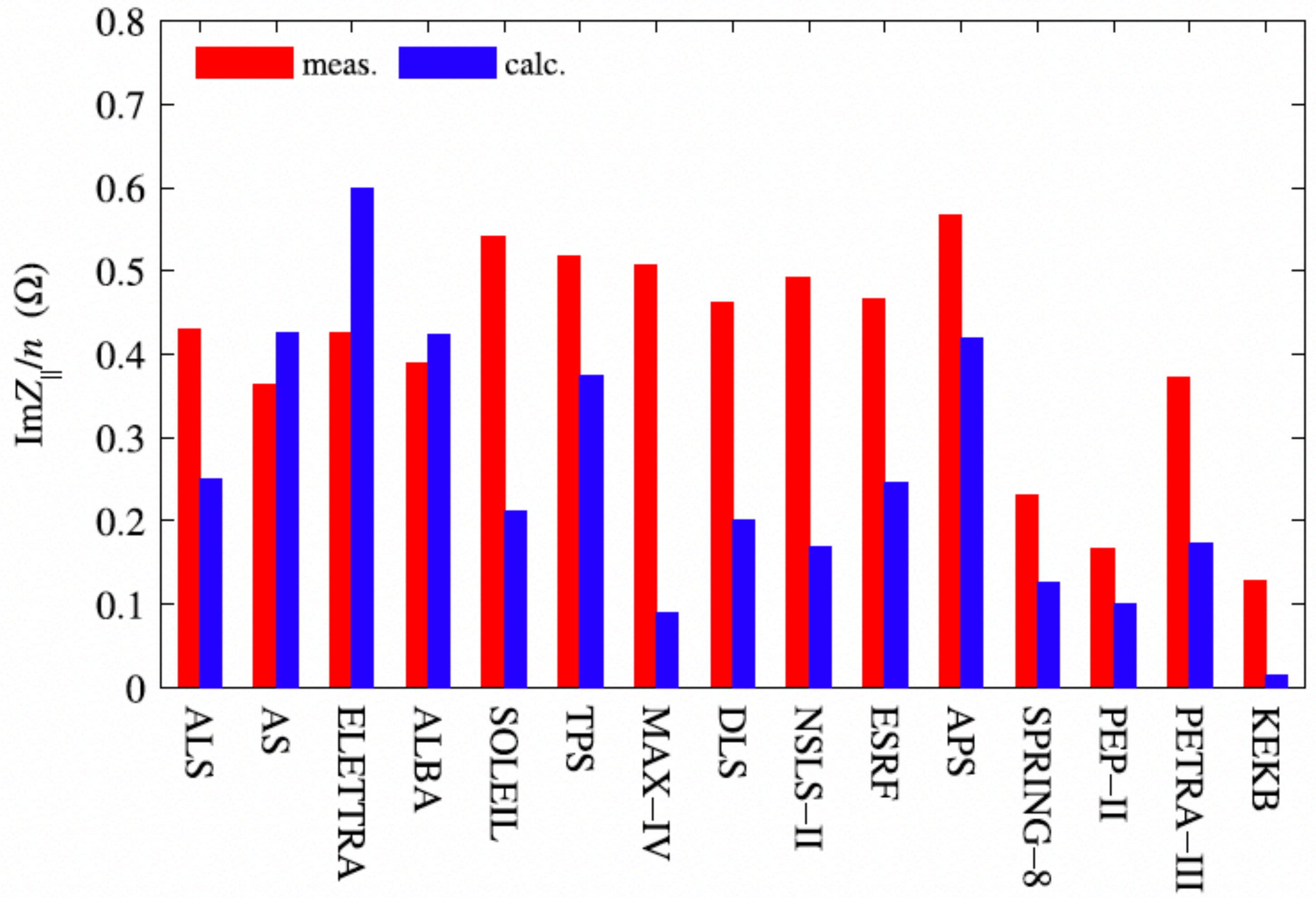


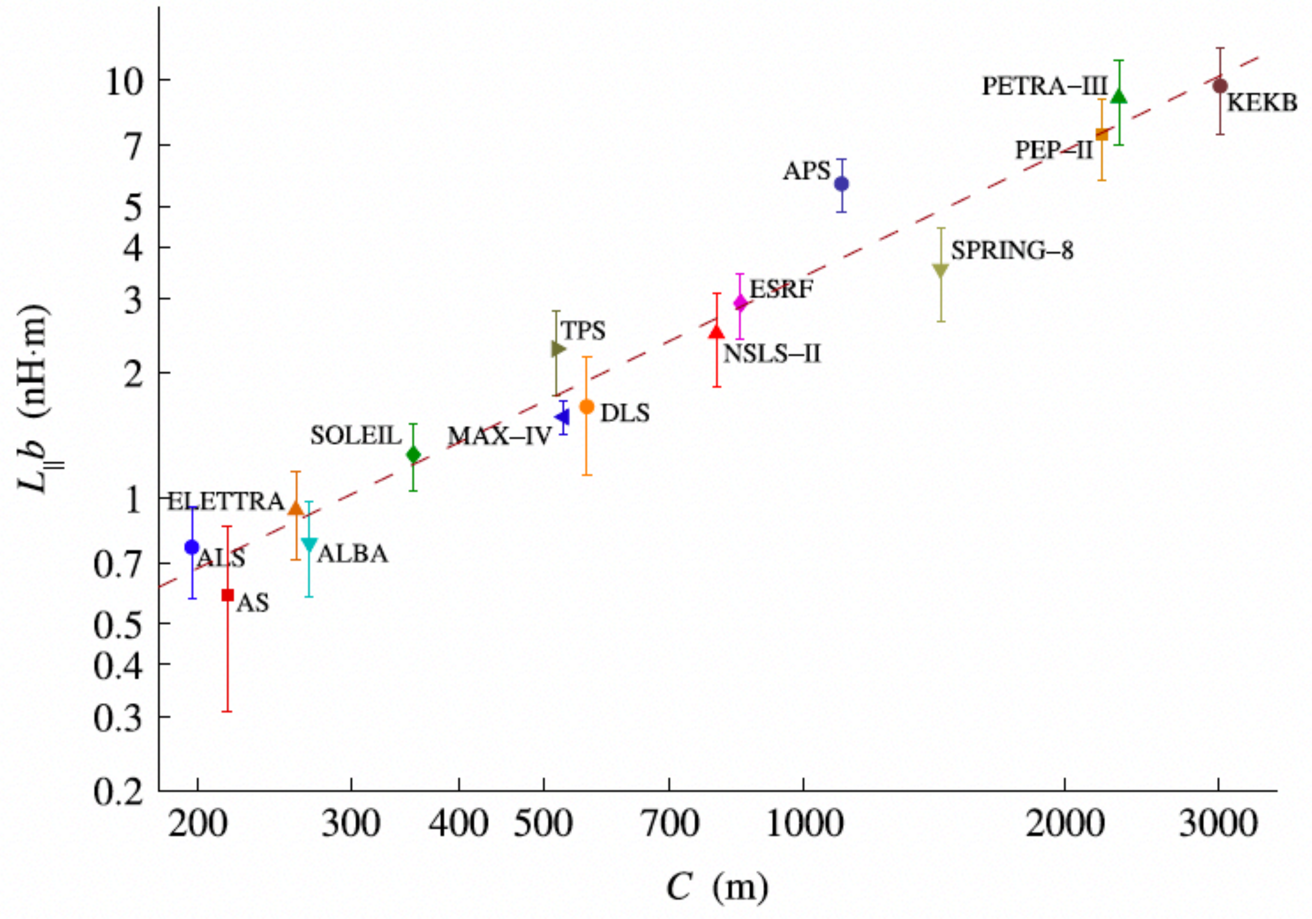
Impedance effects (mainly in e+e- circular colliders)

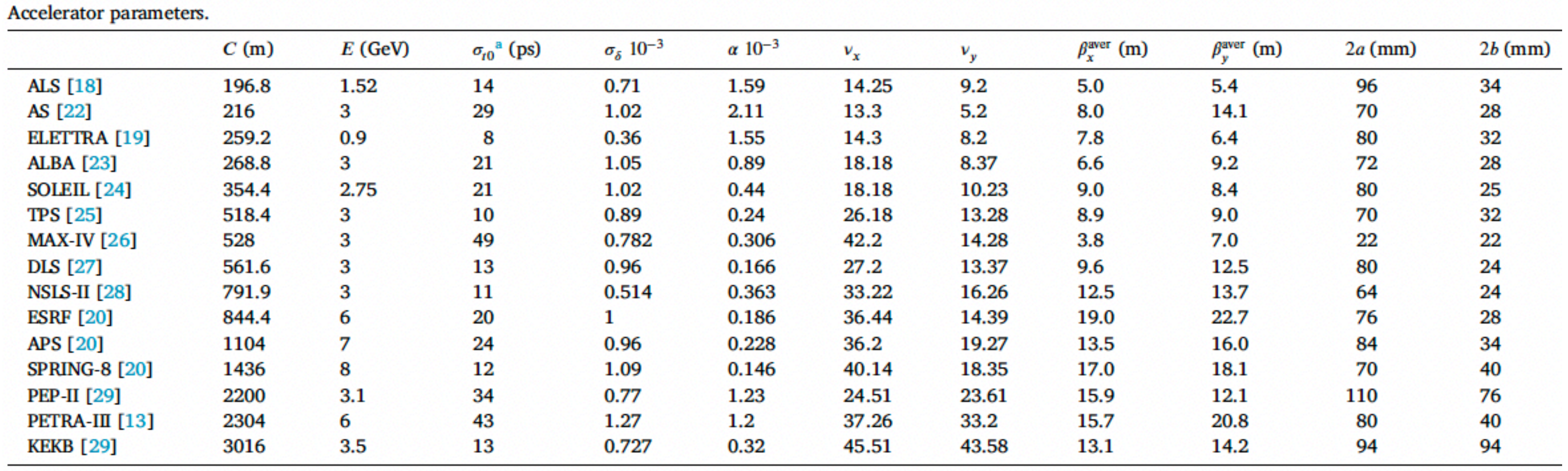
- Longitudinal single-bunch effects
    - Potential-well distortion and synchrotron-tune shift/spread
    - Microwave instability
- Longitudinal multi-bunch effects
    - Stationary distribution
    - Longitudinal coupled-bunch instability
- Transverse single-bunch effects
    - Beam tilt (=transverse potential-well distortion)
    - Betatron-tune shift/spread
    - Transverse mode-coupling instability (TMCI)
- Transverse multi-bunch effects
    - Transverse coupled-bunch instability
- Interplay between impedance effects and others (beam-beam, lattice, feed-back, etc.)
    - Wanted: Damping
    - Unwanted: Emittance blowup and instabilities

Potential-well bunch lengthening

- Discrepancy between impedance calculations and beam-based measurements
  - For several decades, theories and numerical tools for impedance calculations have been “well established”.
  - Techniques for experimental observations of impedance effects have also matured in parallel.
  - However, discrepancies remain in each accelerator project, to varying degrees [1].






Accelerator parameters.

	C (m)	E (GeV)	$\sigma_{t0}{}^{a}$ (ps)	σ_δ 10^{-3}	α 10^{-3}	ν_x	ν_y	β_x^{aver} (m)	β_y^{aver} (m)	$2a$ (mm)	$2b$ (mm)
ALS [18]	196.8	1.52	14	0.71	1.59	14.25	9.2	5.0	5.4	96	34
AS [22]	216	3	29	1.02	2.11	13.3	5.2	8.0	14.1	70	28
ELETTRA [19]	259.2	0.9	8	0.36	1.55	14.3	8.2	7.8	6.4	80	32
ALBA [23]	268.8	3	21	1.05	0.89	18.18	8.37	6.6	9.2	72	28
SOLEIL [24]	354.4	2.75	21	1.02	0.44	18.18	10.23	9.0	8.4	80	25
TPS [25]	518.4	3	10	0.89	0.24	26.18	13.28	8.9	9.0	70	32
MAX-IV [26]	528	3	49	0.782	0.306	42.2	14.28	3.8	7.0	22	22
DLS [27]	561.6	3	13	0.96	0.166	27.2	13.37	9.6	12.5	80	24
NSLS-II [28]	791.9	3	11	0.514	0.363	33.22	16.26	12.5	13.7	64	24
ESRF [20]	844.4	6	20	1	0.186	36.44	14.39	19.0	22.7	76	28
APS [20]	1104	7	24	0.96	0.228	36.2	19.27	13.5	16.0	84	34
SPRING-8 [20]	1436	8	12	1.09	0.146	40.14	18.35	17.0	18.1	70	40
PEP-II [29]	2200	3.1	34	0.77	1.23	24.51	23.61	15.9	12.1	110	76
PETRA-III [13]	2304	6	43	1.27	1.2	37.26	33.2	15.7	20.8	80	40
KEKB [29]	3016	3.5	13	0.727	0.32	45.51	43.58	13.1	14.2	94	94

Zotter’s equation [2]:

$$x^3 - x - \frac{cI_b}{\kappa\eta\omega_0\sigma_{z0}\sigma_{\delta0}^2(E/e)}\mathbf{Im}\left(\frac{Z_\parallel}{n}\right)_{eff}^{m=1} = 0$$

We must be cautious:
The model can be a source of discrepancies if its assumptions are violated.

[1] V. Smaluk, NIMA 888, 22 (2018). [2] B. W. Zotter, CERN-SPS-81-14-DI (1981).

Potential-well bunch lengthening

Haissinski equation: $$\lambda_0(z) = Ae^{-\frac{z^2}{2\sigma_{z0}^2}-\frac{I}{\sigma_{z0}}\int_z^\infty dz'\int_{-\infty}^\infty W_\parallel(z'-z'')\lambda_0(z'')dz''}$$

$$I = \frac{I_b\sigma_{z0}}{c\eta\sigma_{\delta0}^2(E/e)}$$

- rms bunch length σ_z

Definition: $$\sigma_z^2 = \int_{-\infty}^{\infty}(z-z_c)^2\lambda_0(z)dz = \int_{-\infty}^{\infty}z^2\lambda_0(z)dz - z_c^2$$

Wake potential: $$\mathbb{W}_\parallel(z) = \int_{-\infty}^{\infty}W_\parallel(z-z')\lambda_0(z')dz'$$

$$\boxed{\frac{d\lambda_0(z)}{dz} + \left[\frac{z}{\sigma_{z0}^2} - \frac{1}{\eta\sigma_\delta^2}F_0(z)\right]\lambda_0(z) = 0} \longrightarrow \boxed{x^2 - 1 - \frac{cI}{2\pi\sigma_{z0}}Z_\parallel^{eff}(x) = 0}$$

Trick: Multiply by z and then integrate this equation over z

1) Exact prediction by Haissinski equation
2) A generalized version of Zotter's equation

Normalized current: $$I = \frac{I_b\sigma_{z0}}{c\eta\sigma_{\delta0}^2(E/e)}$$

$$x = \sigma_z/\sigma_{z0}$$

"Effective impedance" for bunch lengthening:

$$Z_\parallel^{eff} = \frac{2\pi}{c}\int_{-\infty}^{\infty}\boxed{dz(z-z_c)\lambda_0(z)\mathbb{W}_\parallel(z)}$$

Stretching force average over charge density

$$Z_\parallel^{eff} = -\int_{-\infty}^{\infty}\boxed{dkZ_\parallel(k)\tilde{\lambda}_0(k)\left[i\frac{d}{dk}\tilde{\lambda}_0^*(k) + z_c\tilde{\lambda}_0^*(k)\right]}$$

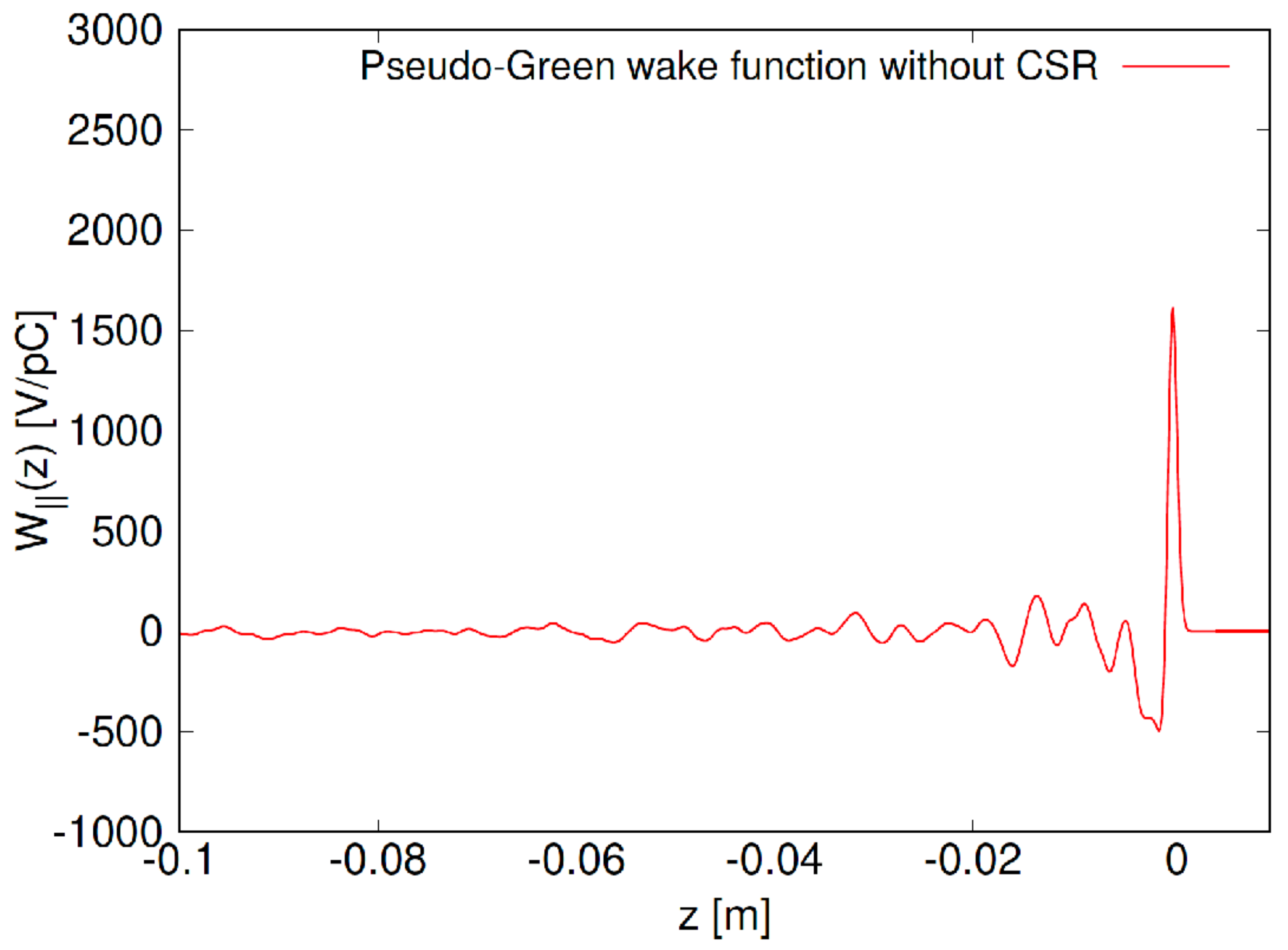
Both real and imaginary parts of impedance contribute to bunch lengthening if the bunch is deformed

- σ_z is sensitive to imaginary part of impedance
- If real part of impedance is large, it also contributes to bunch lengthening

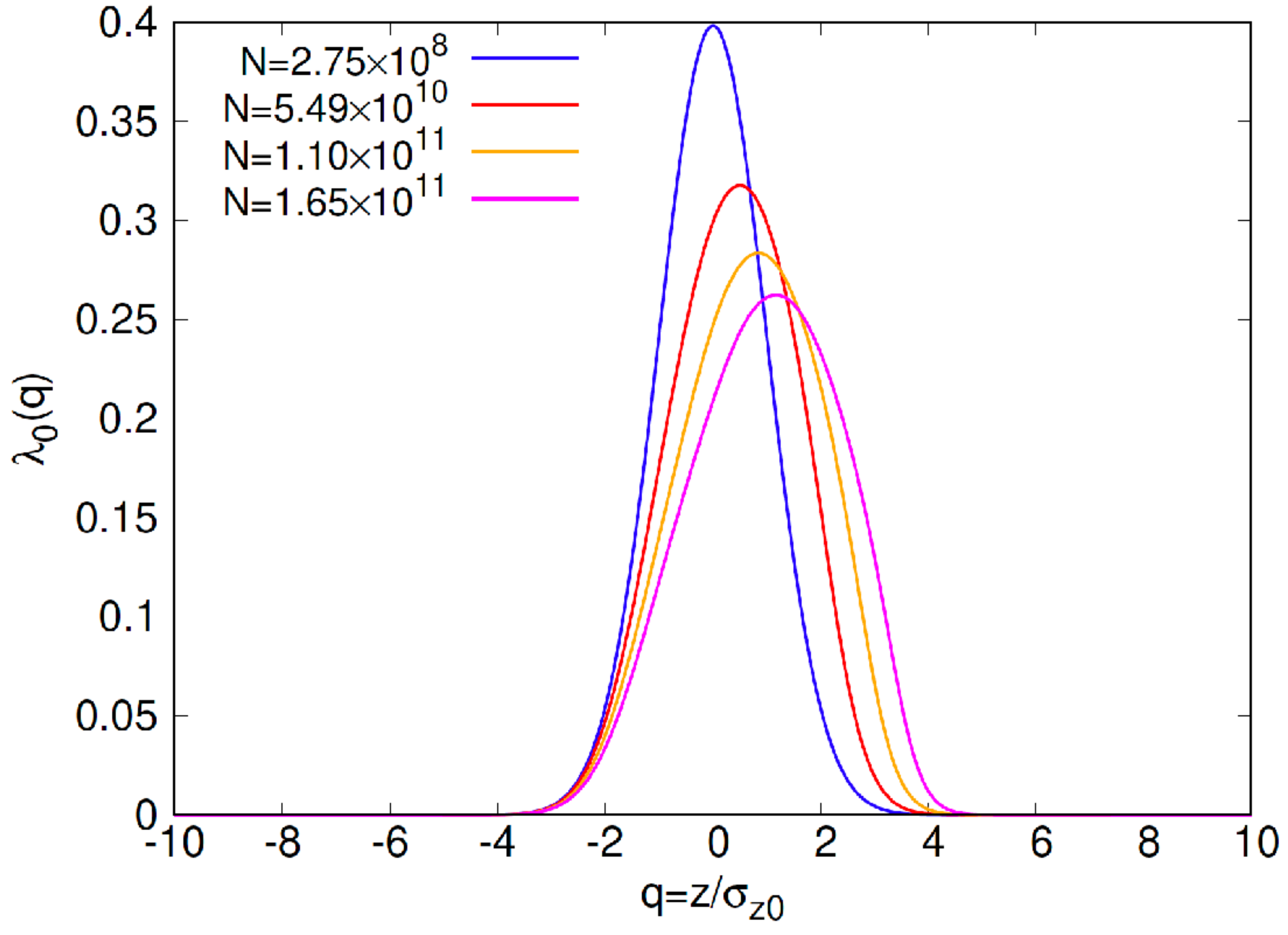
Example 1: SuperKEKB LER

$$x^2 - 1 - \frac{cI}{2\pi\sigma_{z0}} Z_{\parallel}^{eff}(x) = 0$$

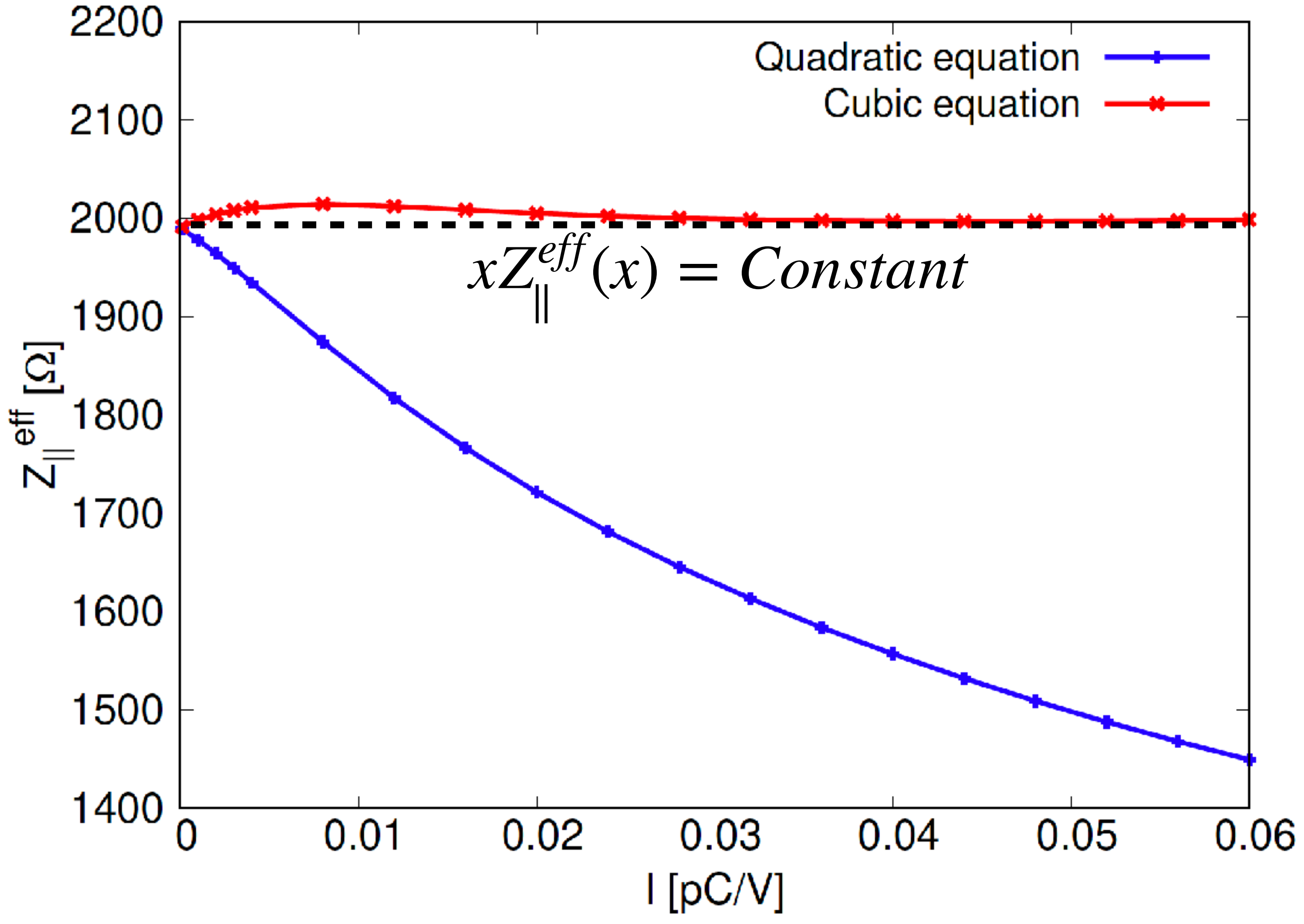
$$x^3 - x - \frac{cI_b}{\kappa\eta\omega_0\sigma_{z0}\sigma_{\delta0}^2(E/e)} \mathbf{Im}\left(\frac{Z_{\parallel}}{n}\right)_{eff}^{m=1} = 0$$

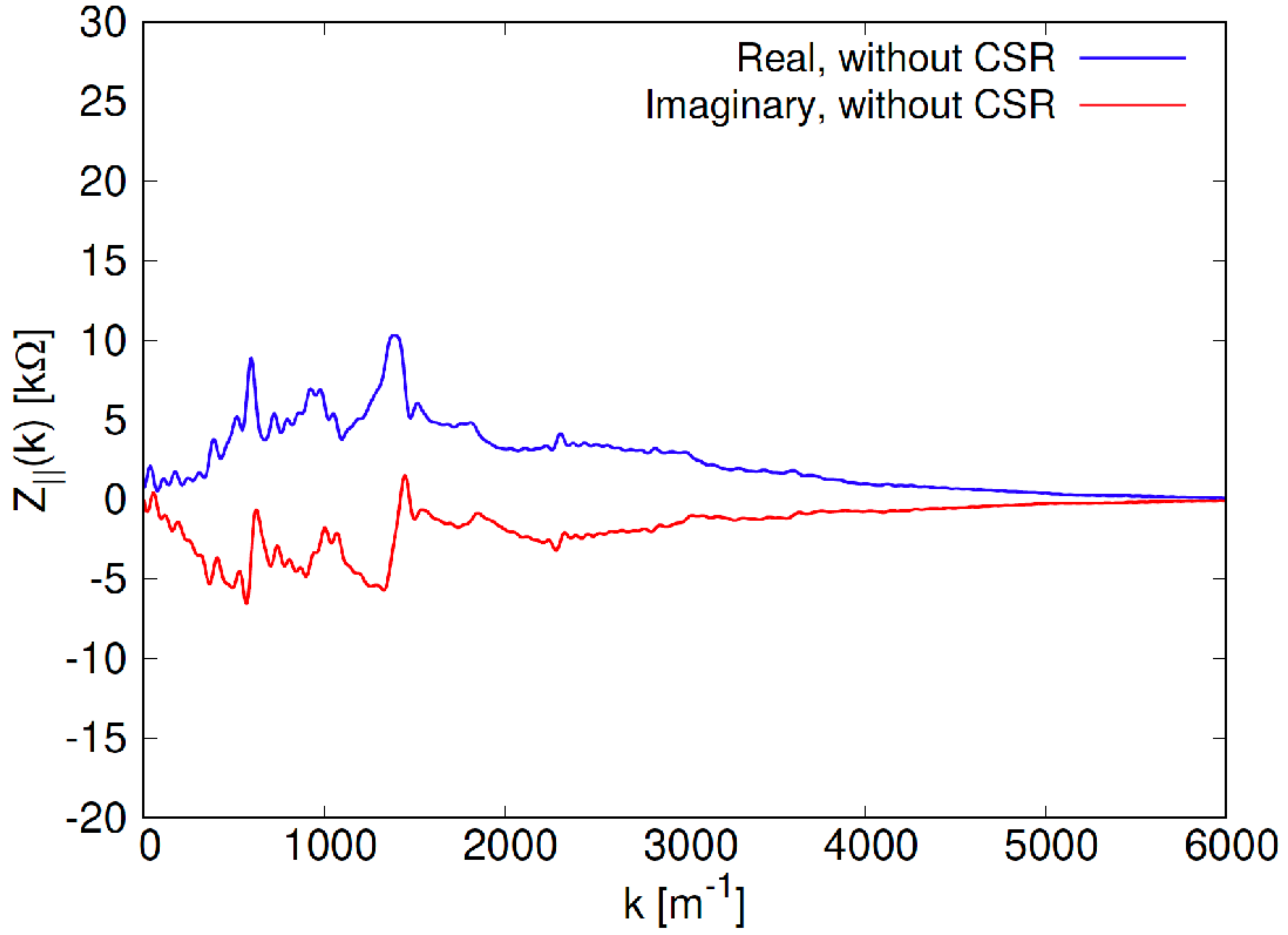


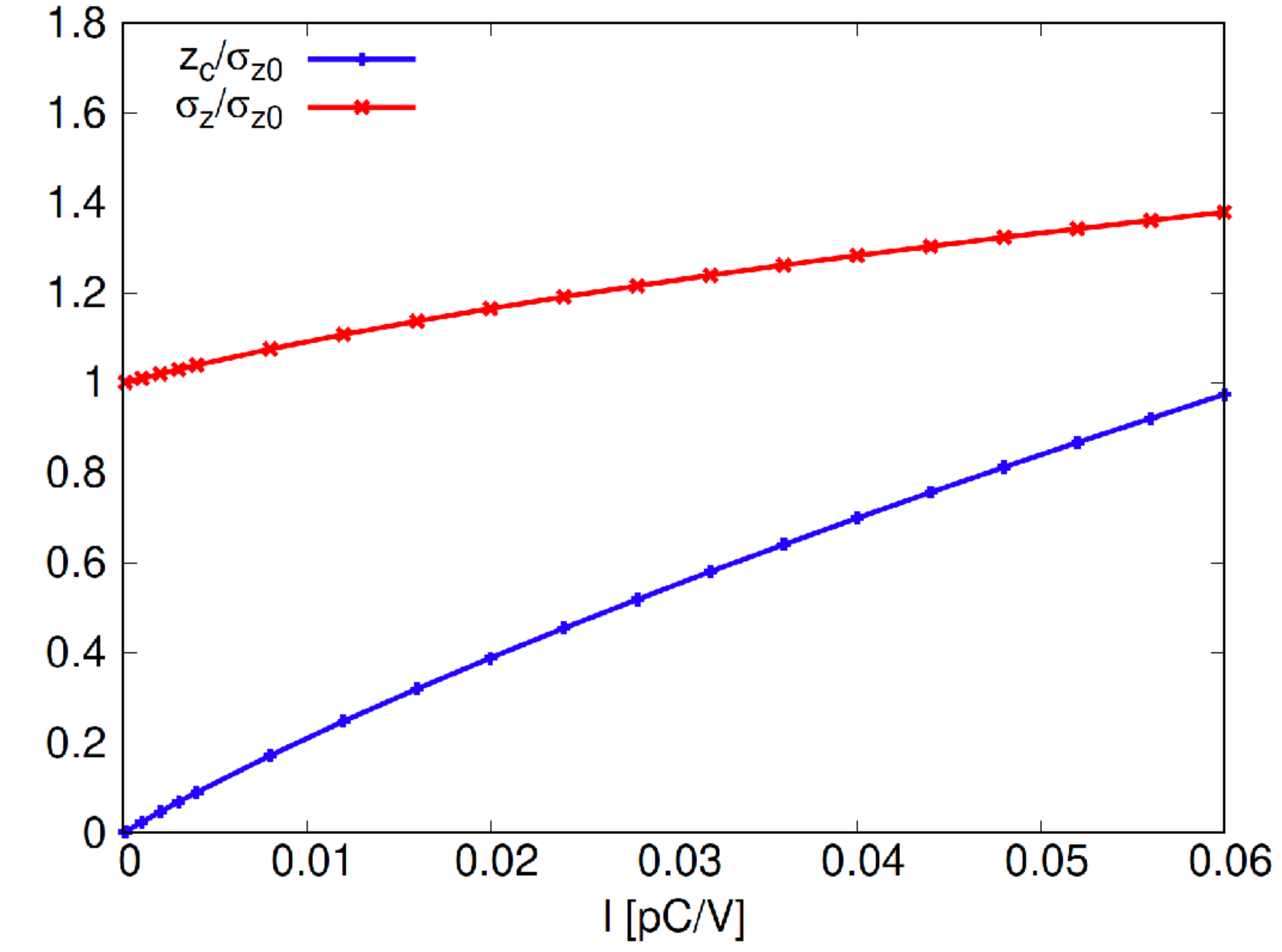
Haissinski distribution

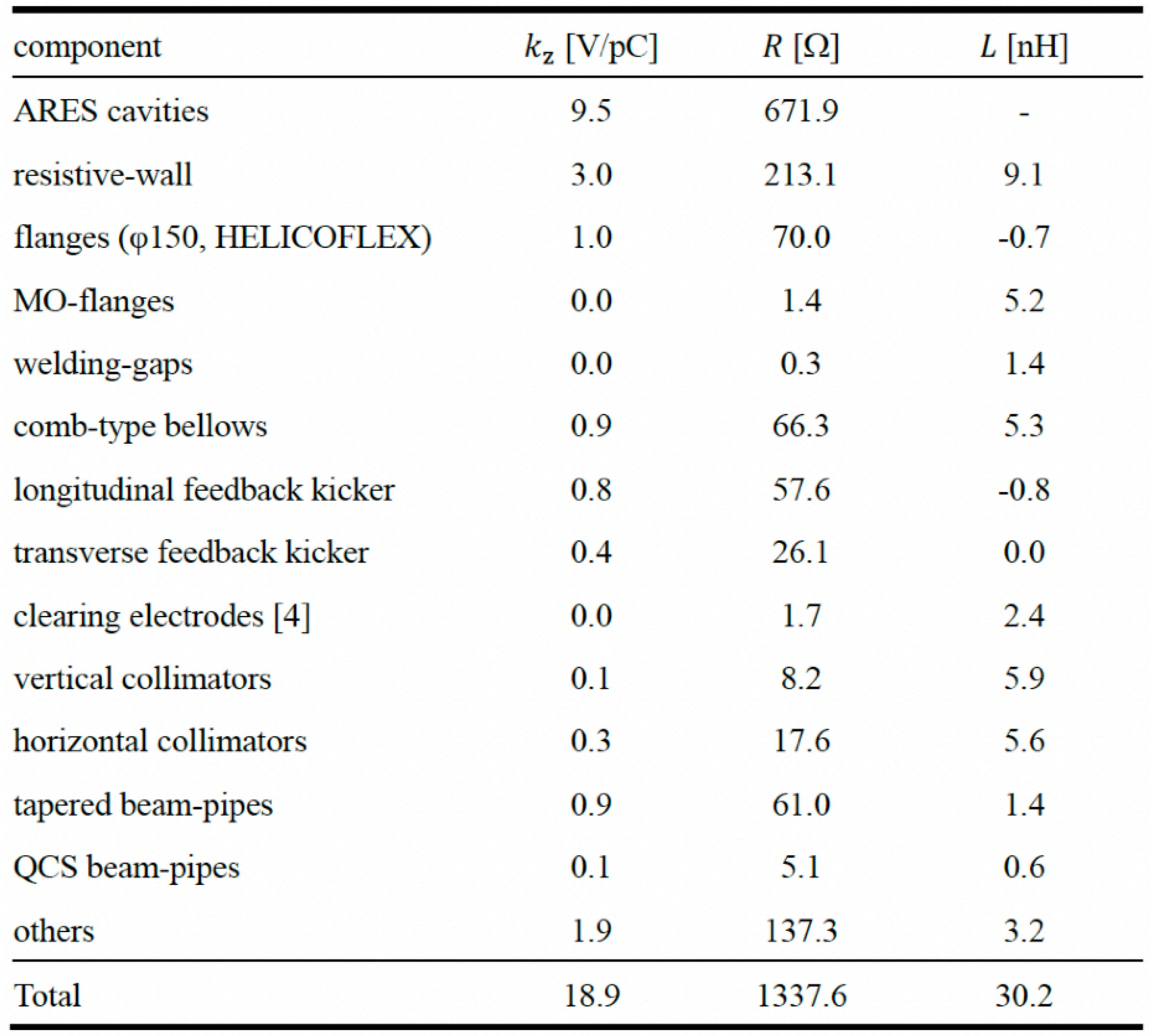


Effective impedance









component	k_z [V/pC]	R [Ω]	L [nH]
ARES cavities	9.5	671.9	-
resistive-wall	3.0	213.1	9.1
flanges (φ150, HELICOFLEX)	1.0	70.0	-0.7
MO-flanges	0.0	1.4	5.2
welding-gaps	0.0	0.3	1.4
comb-type bellows	0.9	66.3	5.3
longitudinal feedback kicker	0.8	57.6	-0.8
transverse feedback kicker	0.4	26.1	0.0
clearing electrodes [4]	0.0	1.7	2.4
vertical collimators	0.1	8.2	5.9
horizontal collimators	0.3	17.6	5.6
tapered beam-pipes	0.9	61.0	1.4
QCS beam-pipes	0.1	5.1	0.6
others	1.9	137.3	3.2
Total	18.9	1337.6	30.2

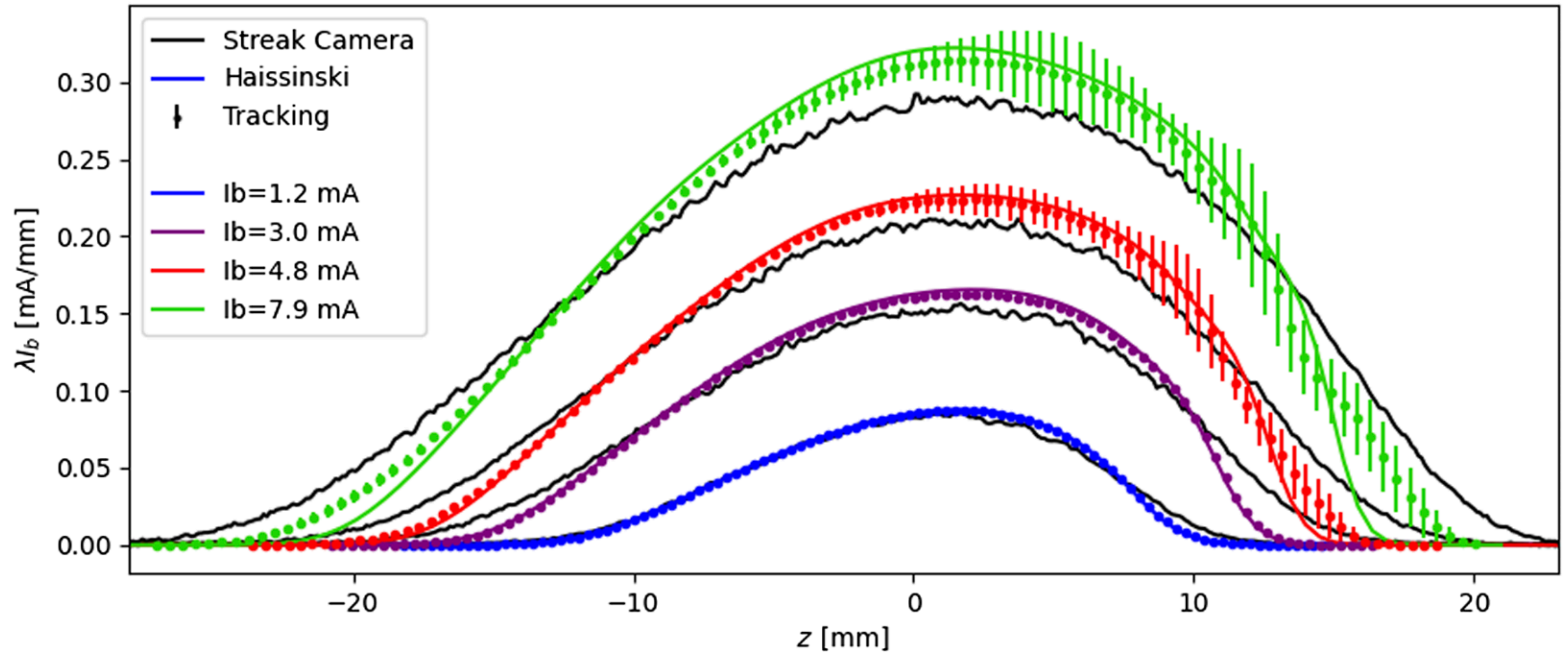
T. Ishibashi et al., ICFA Newsletter #85, 2023

Example 1: SuperKEKB LER

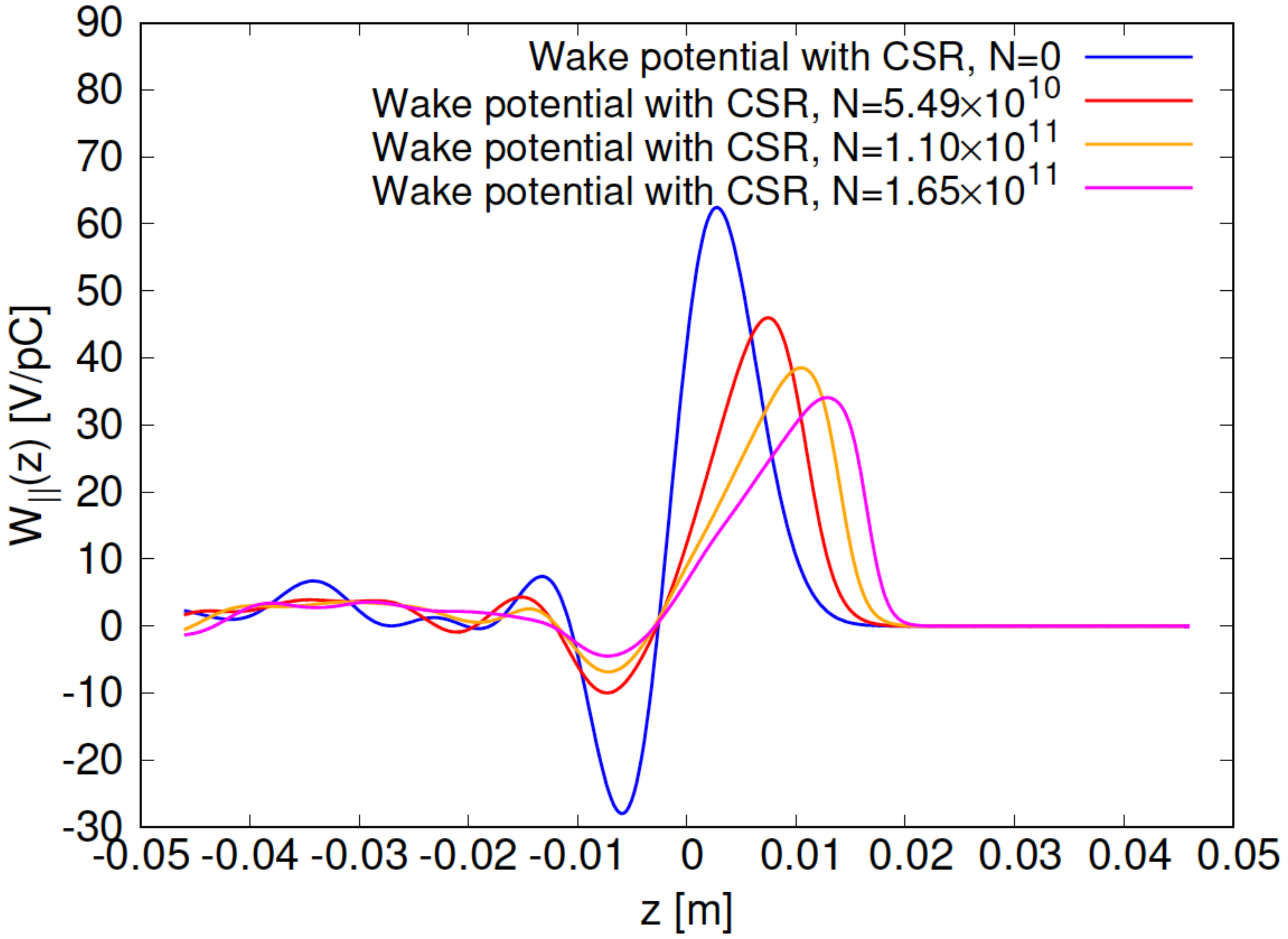
- Inverse problem of Haissinski equation [1]

$$\lambda_0(z) = Ae^{-\frac{z^2}{2\sigma_{z0}^2}-\frac{I}{\sigma_{z0}}\int_z^\infty dz'\int_{-\infty}^\infty W_\parallel(z'-z'')\lambda_0(z'')dz''}$$

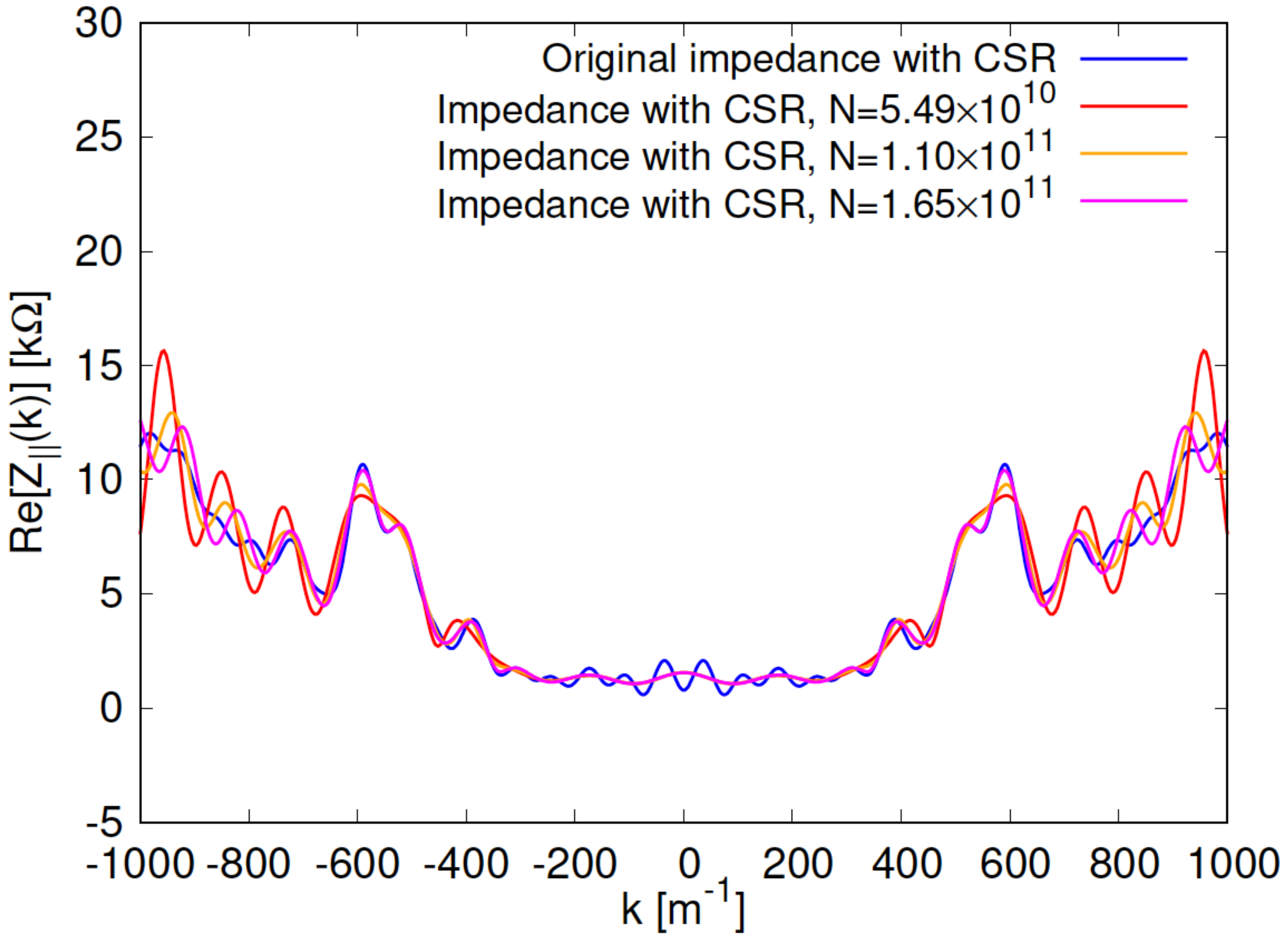
$$Z_\parallel(k)=\frac{\sigma_{z0}}{Ic^2\tilde{\lambda}_0(k)}\int_{-\infty}^{\infty}\left[\frac{d\ln\lambda_0(z)}{dz}+\frac{z}{\sigma_{z0}^2}\right]e^{-ikz}dz\,.$$



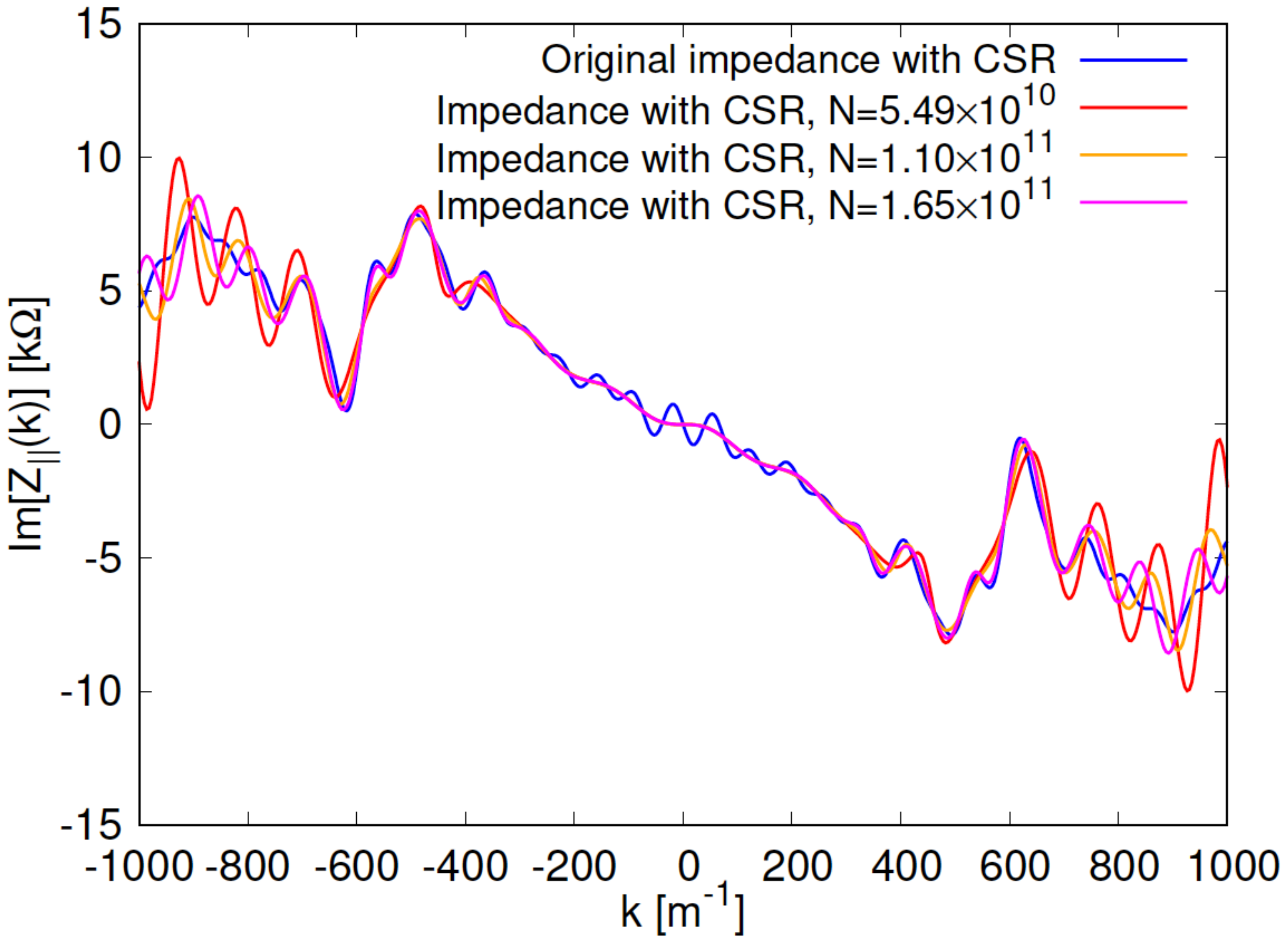
L. Carver et al., PRAB 26, 044402 (2023).



Wake potential with different bunch profiles



Real part of impedance extracted from Haissinski solutions

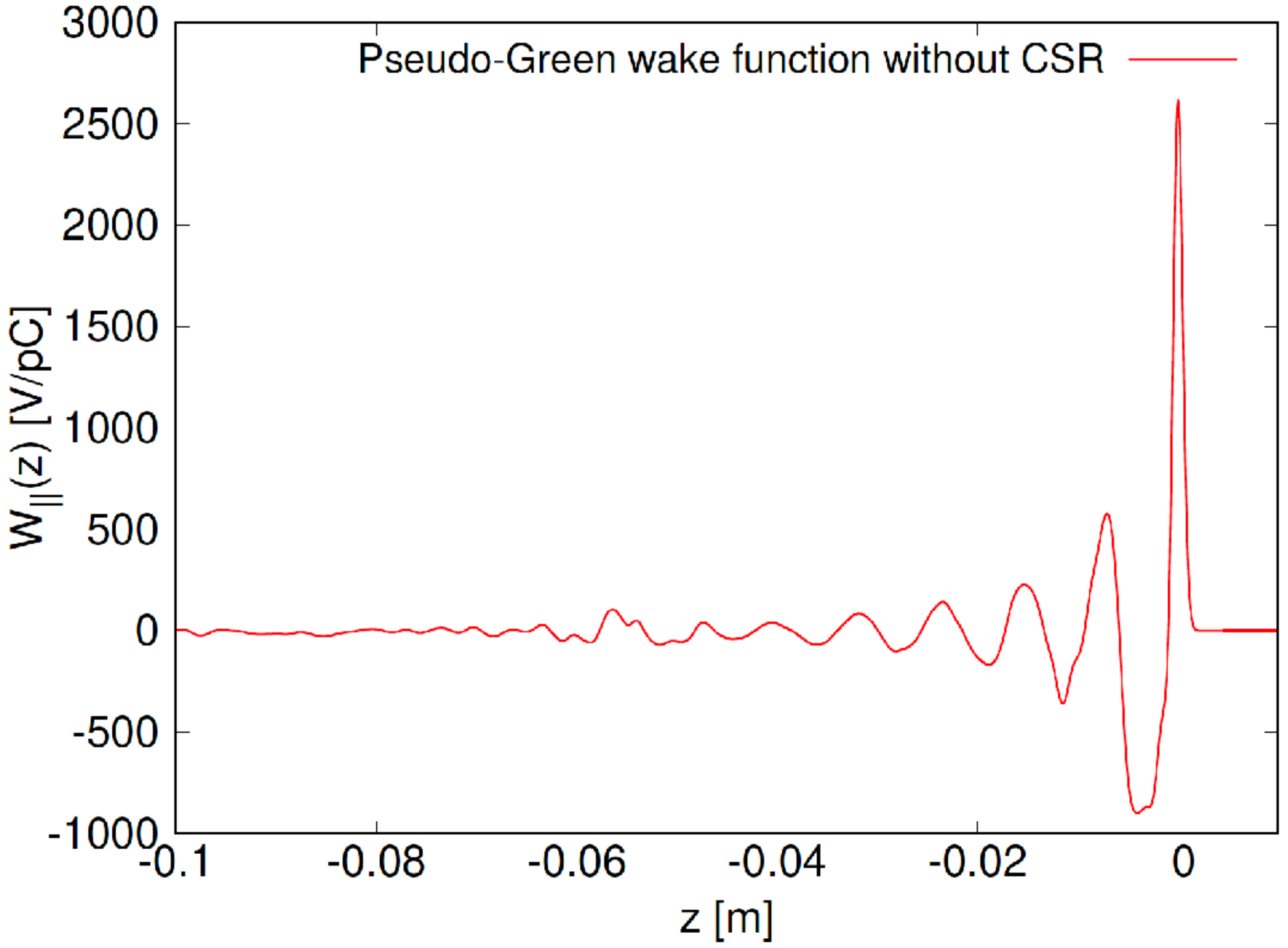


Imaginary part of impedance extracted from Haissinski solutions

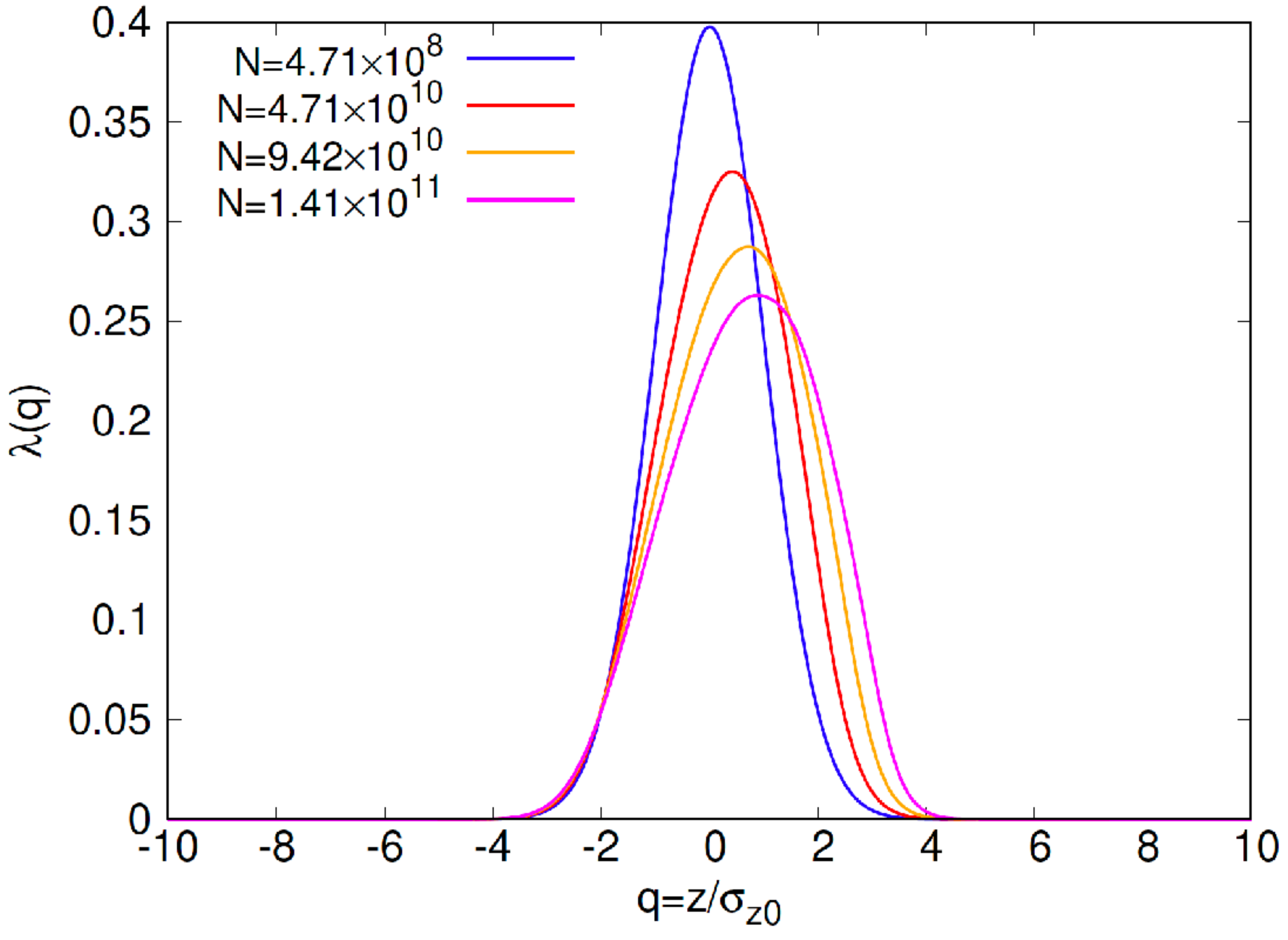
[1] A. Chao, "Lectures on Accelerator Physics", World Scientific, 2020.

Example 2: SuperKEKB HER

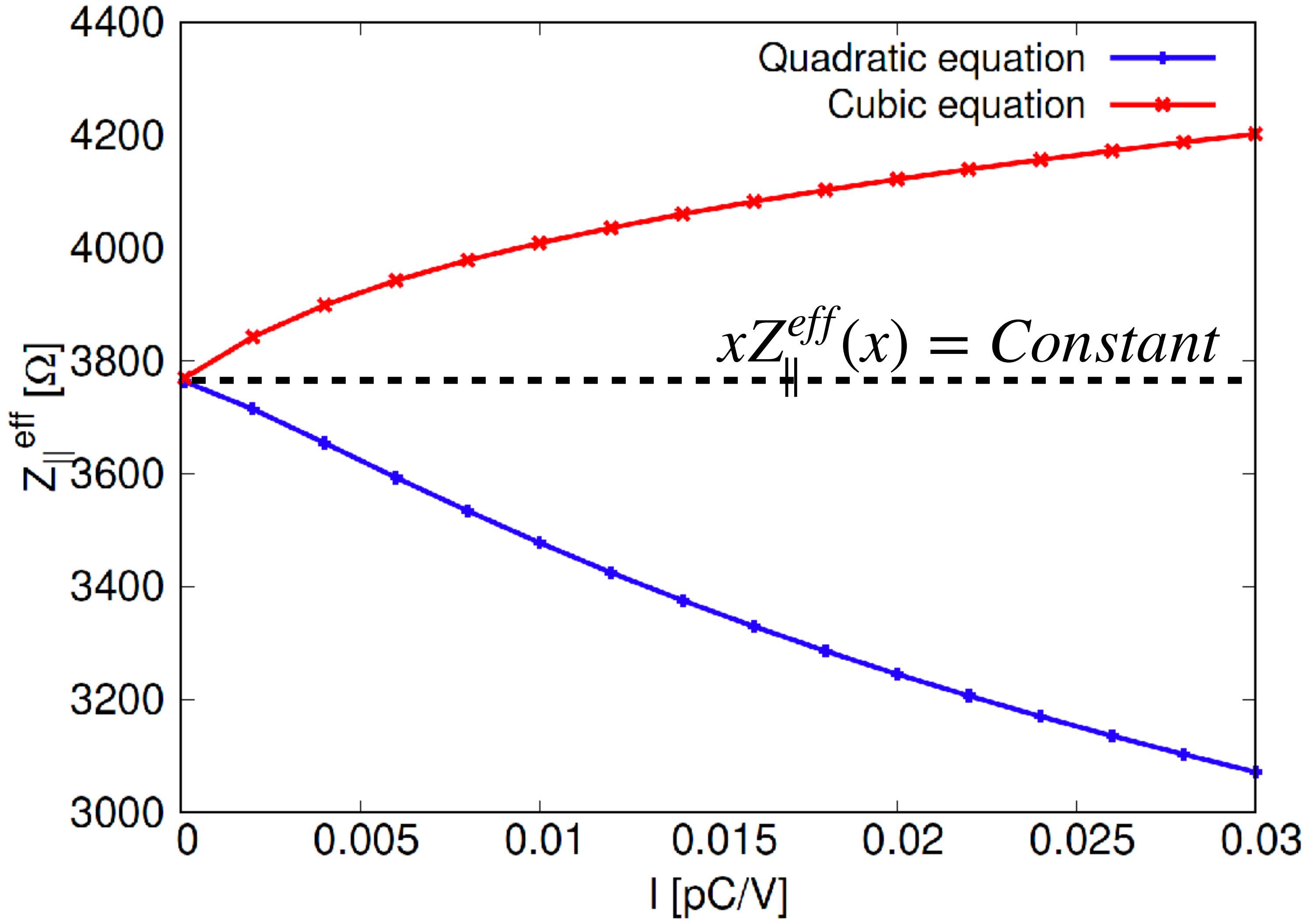
$$x^2 - 1 - \frac{cI}{2\pi\sigma_{z0}} Z_{\parallel}^{eff}(x) = 0 \qquad x^3 - x - \frac{cI_b}{\kappa\eta\omega_0\sigma_{z0}\sigma_{\delta0}^2(E/e)} \mathbf{Im}\left(\frac{Z_{\parallel}}{n}\right)_{eff}^{m=1} = 0$$

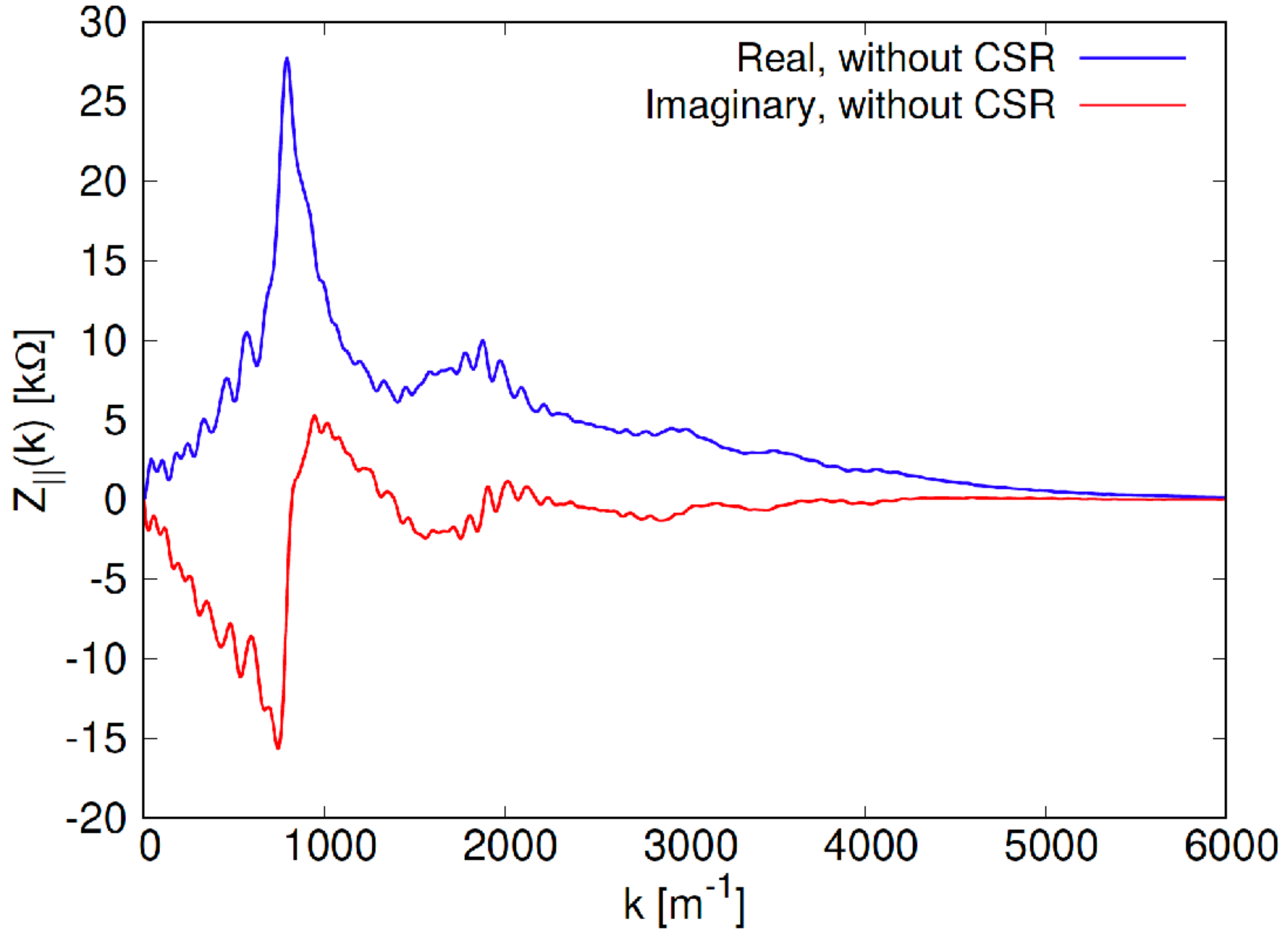


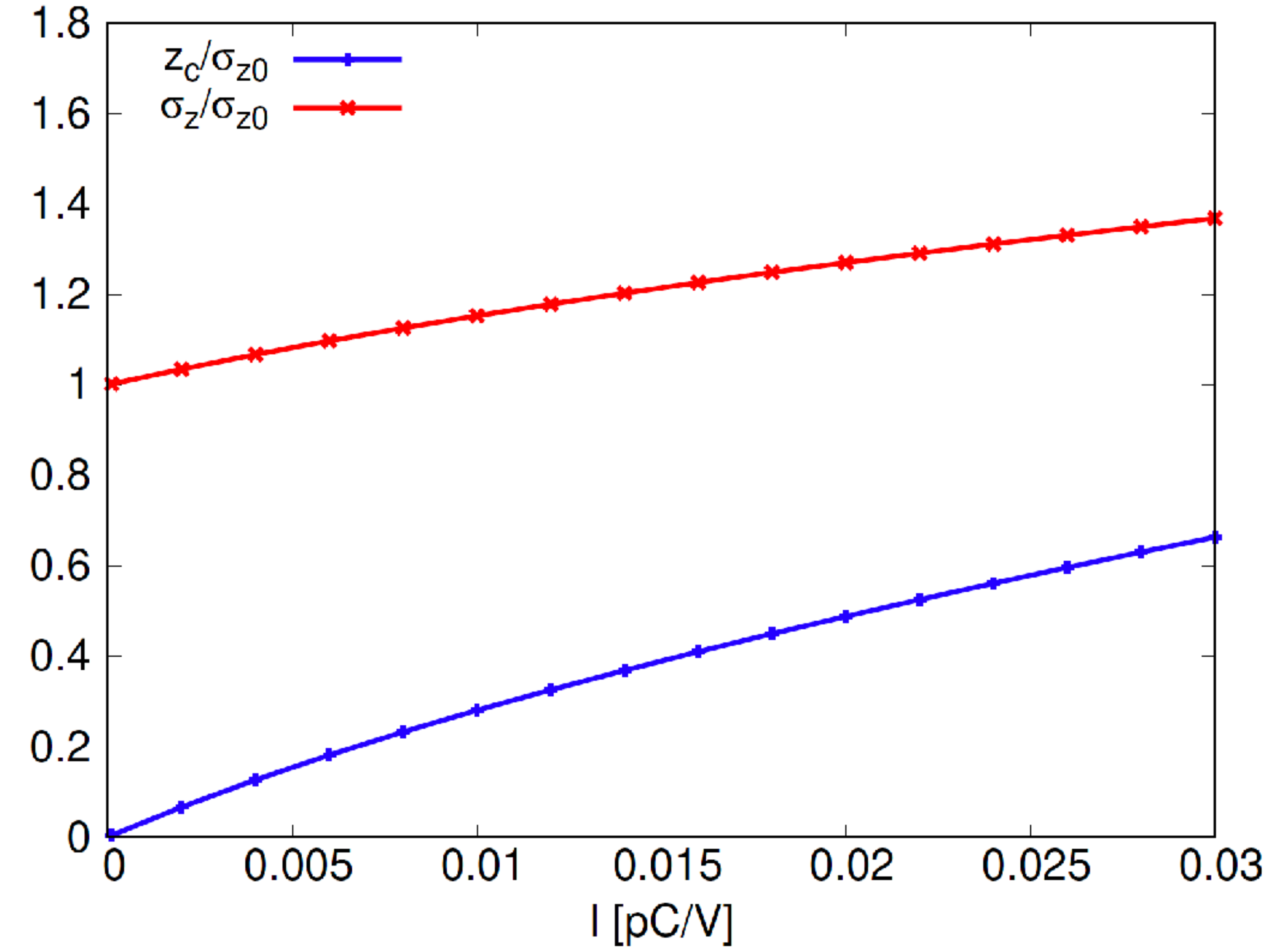
Haissinski distribution

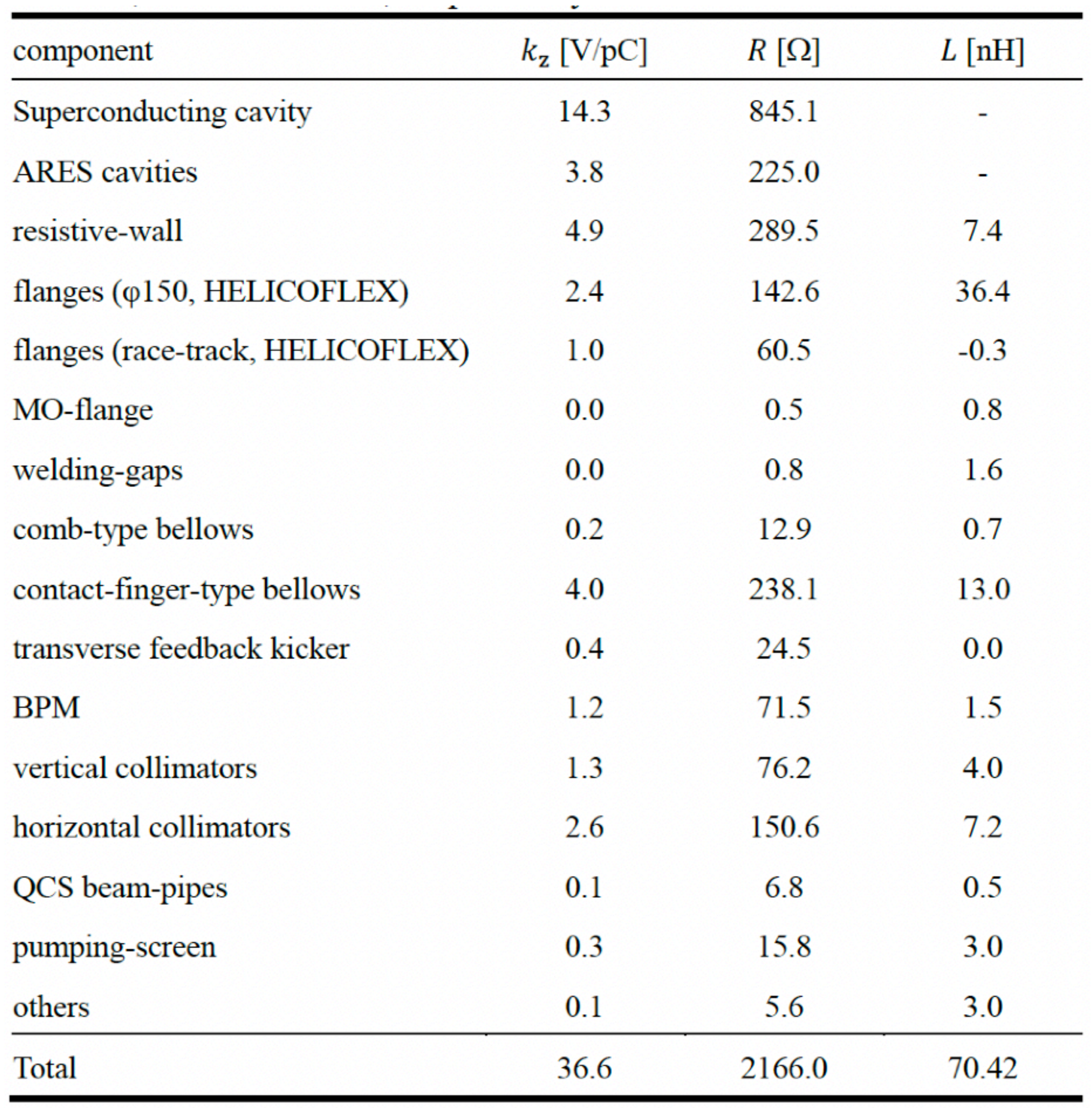


Effective impedance





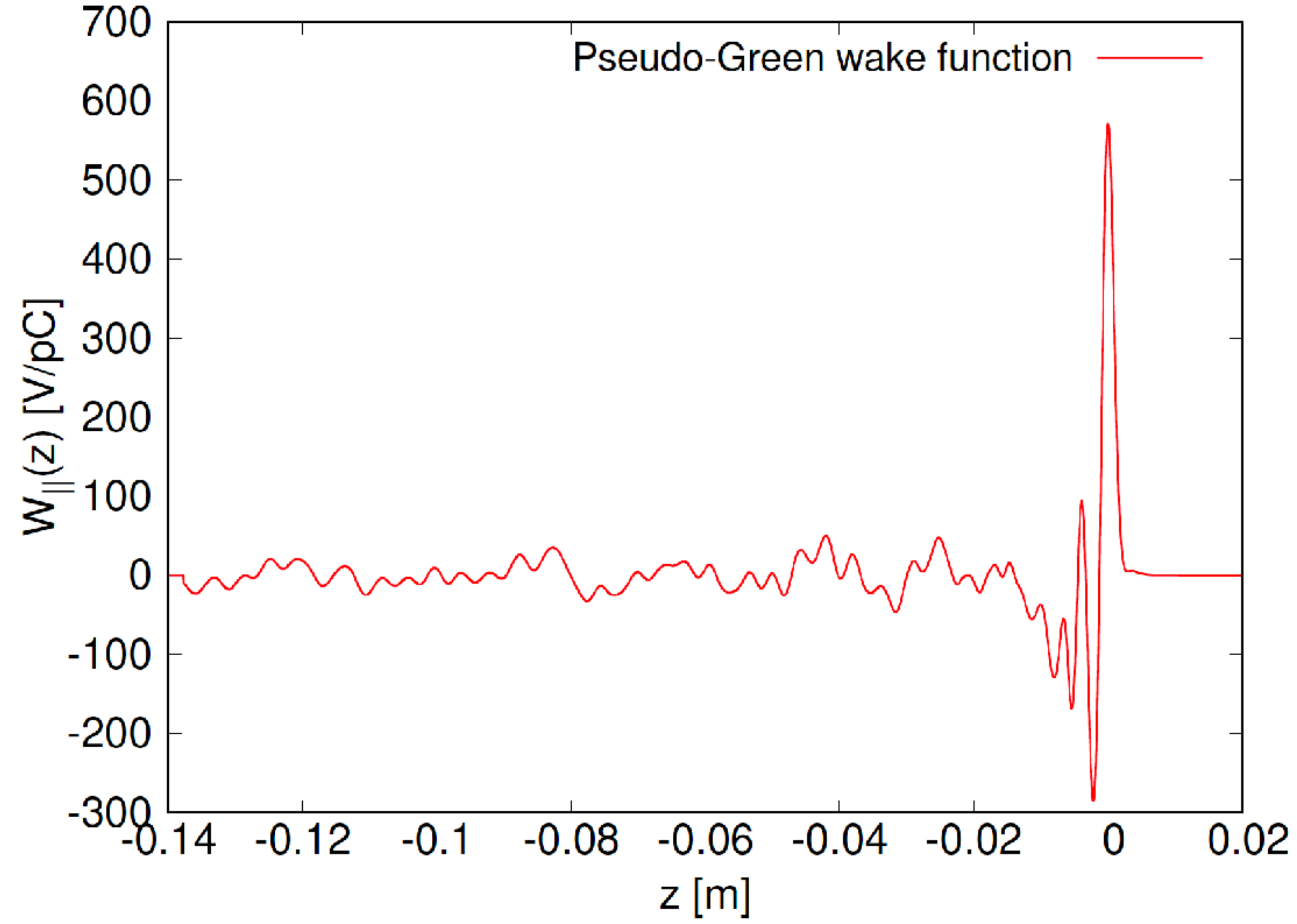




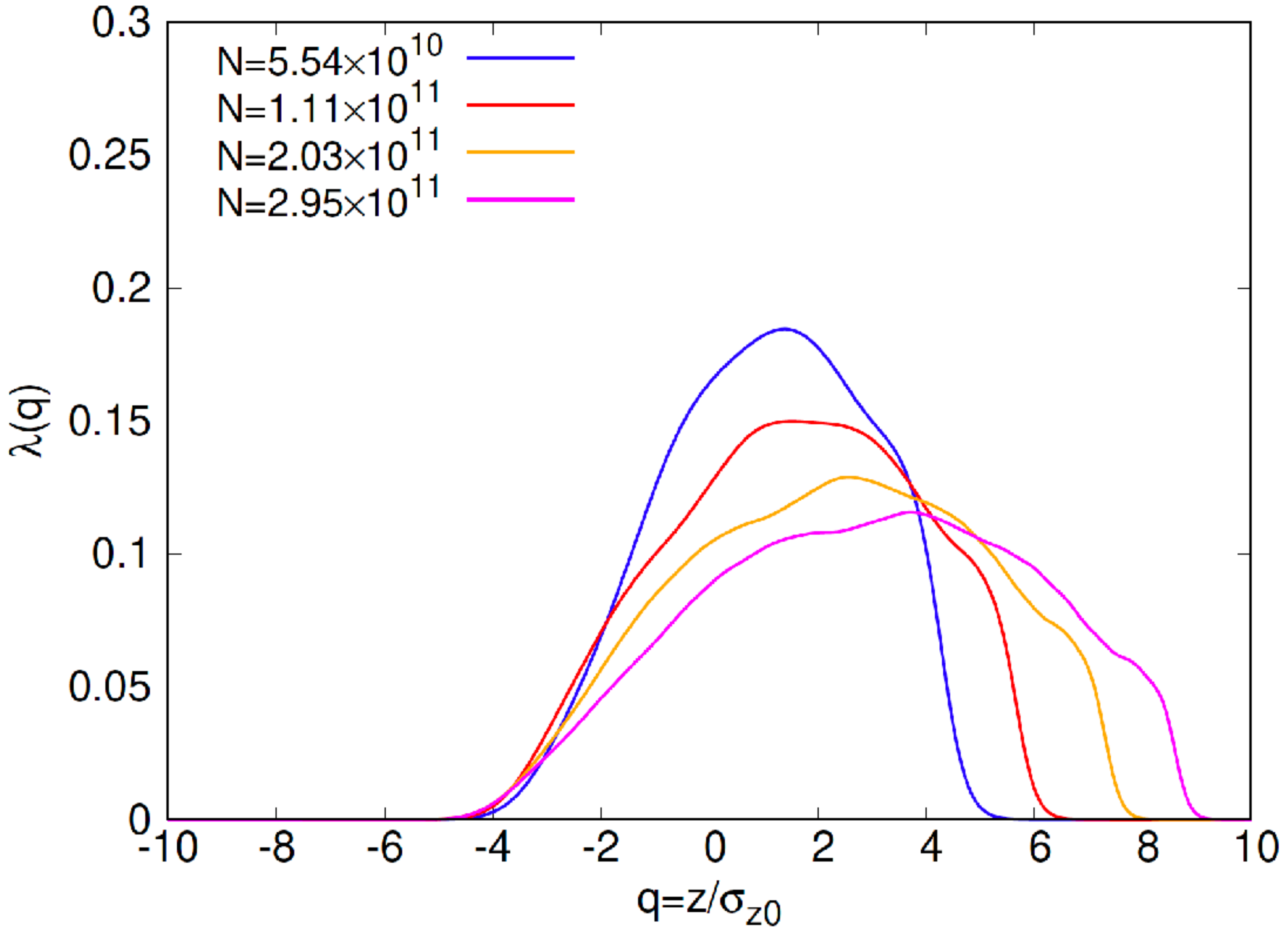
component	k_z [V/pC]	R [Ω]	L [nH]
Superconducting cavity	14.3	845.1	-
ARES cavities	3.8	225.0	-
resistive-wall	4.9	289.5	7.4
flanges (φ150, HELICOFLEX)	2.4	142.6	36.4
flanges (race-track, HELICOFLEX)	1.0	60.5	-0.3
MO-flange	0.0	0.5	0.8
welding-gaps	0.0	0.8	1.6
comb-type bellows	0.2	12.9	0.7
contact-finger-type bellows	4.0	238.1	13.0
transverse feedback kicker	0.4	24.5	0.0
BPM	1.2	71.5	1.5
vertical collimators	1.3	76.2	4.0
horizontal collimators	2.6	150.6	7.2
QCS beam-pipes	0.1	6.8	0.5
pumping-screen	0.3	15.8	3.0
others	0.1	5.6	3.0
Total	36.6	2166.0	70.42

T. Ishibashi et al., ICFA Newsletter #85, 2023

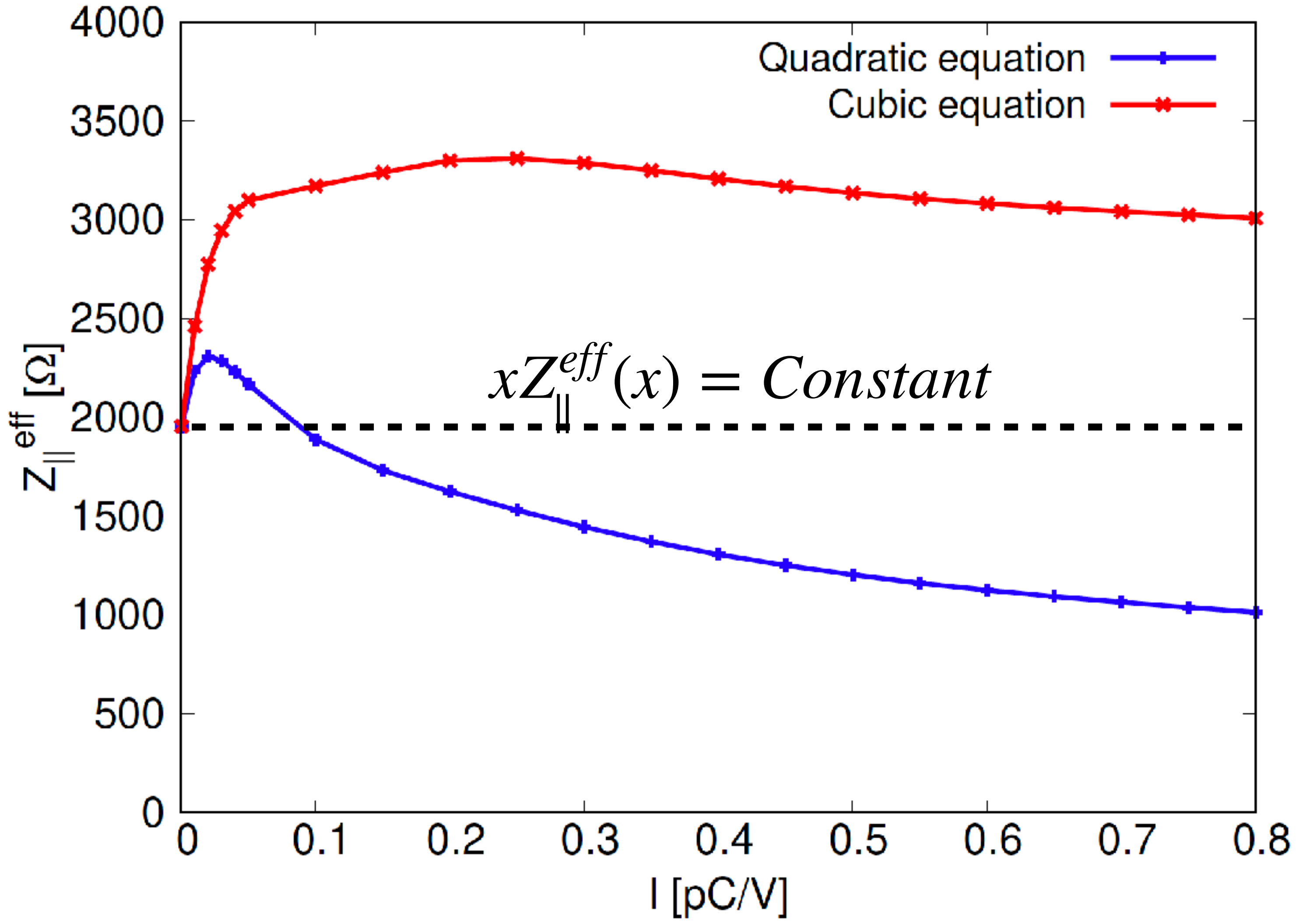
Example 3: SLC damping ring with original vacuum chamber [1]

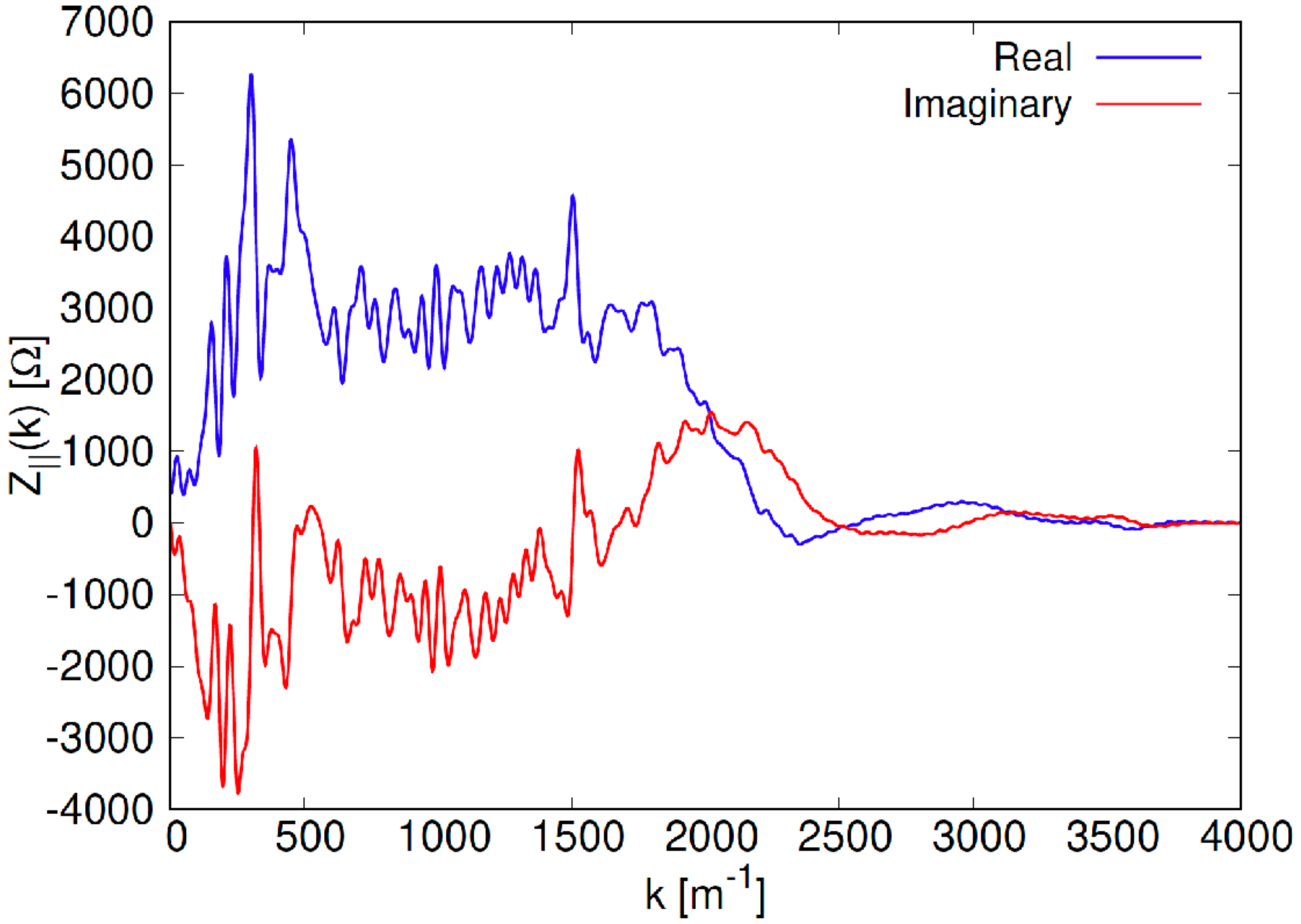


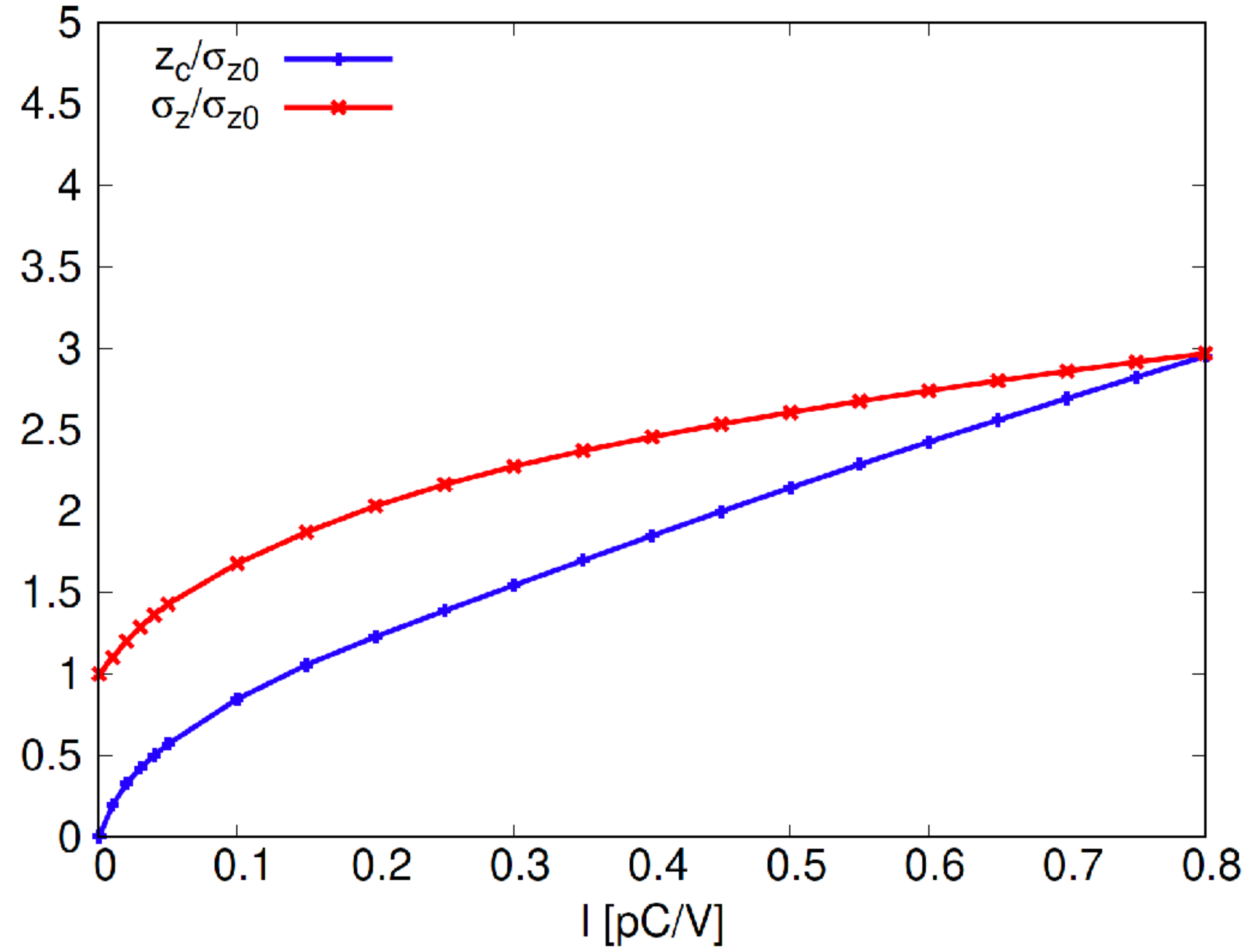
Haissinski distribution



Effective impedance



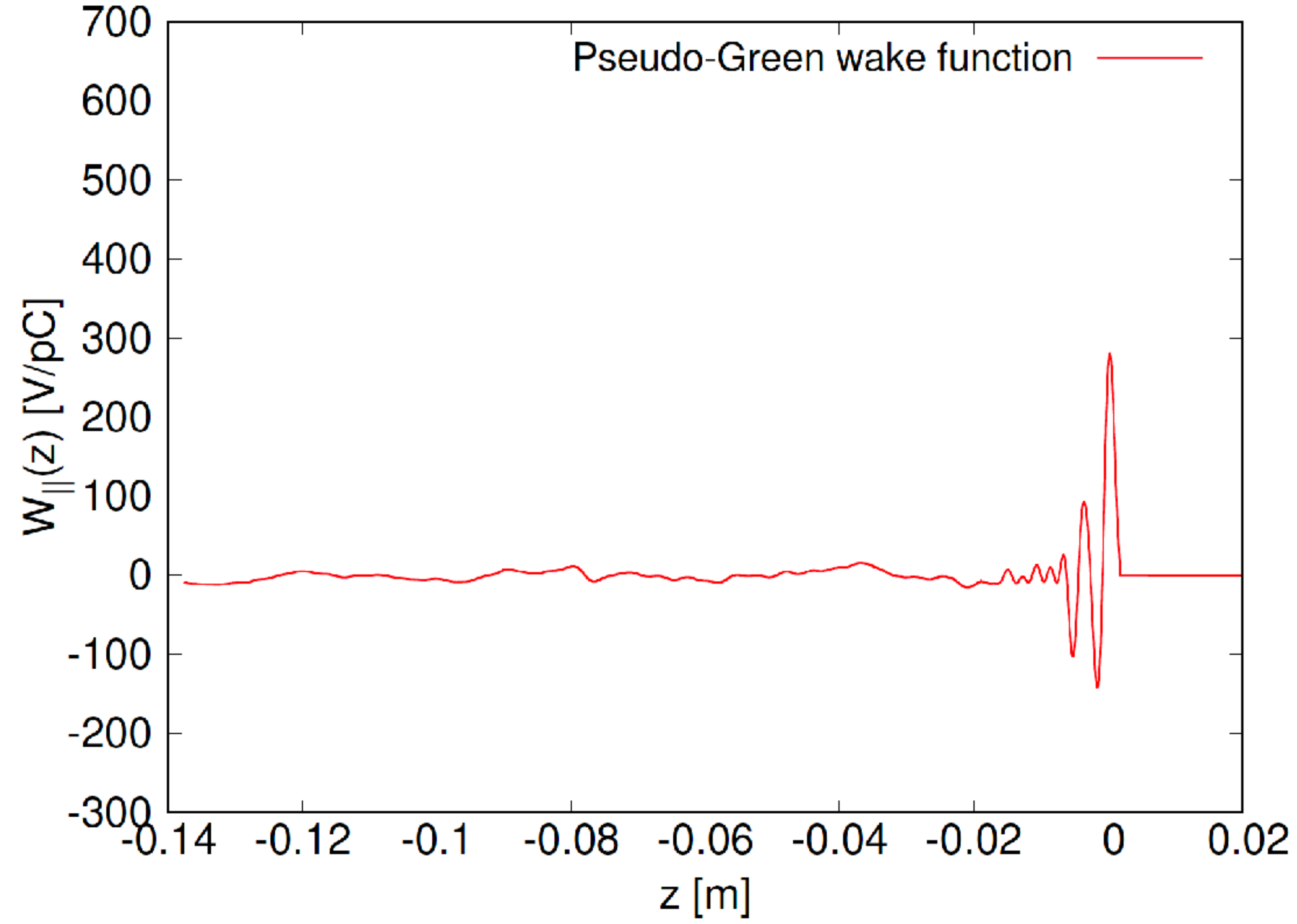




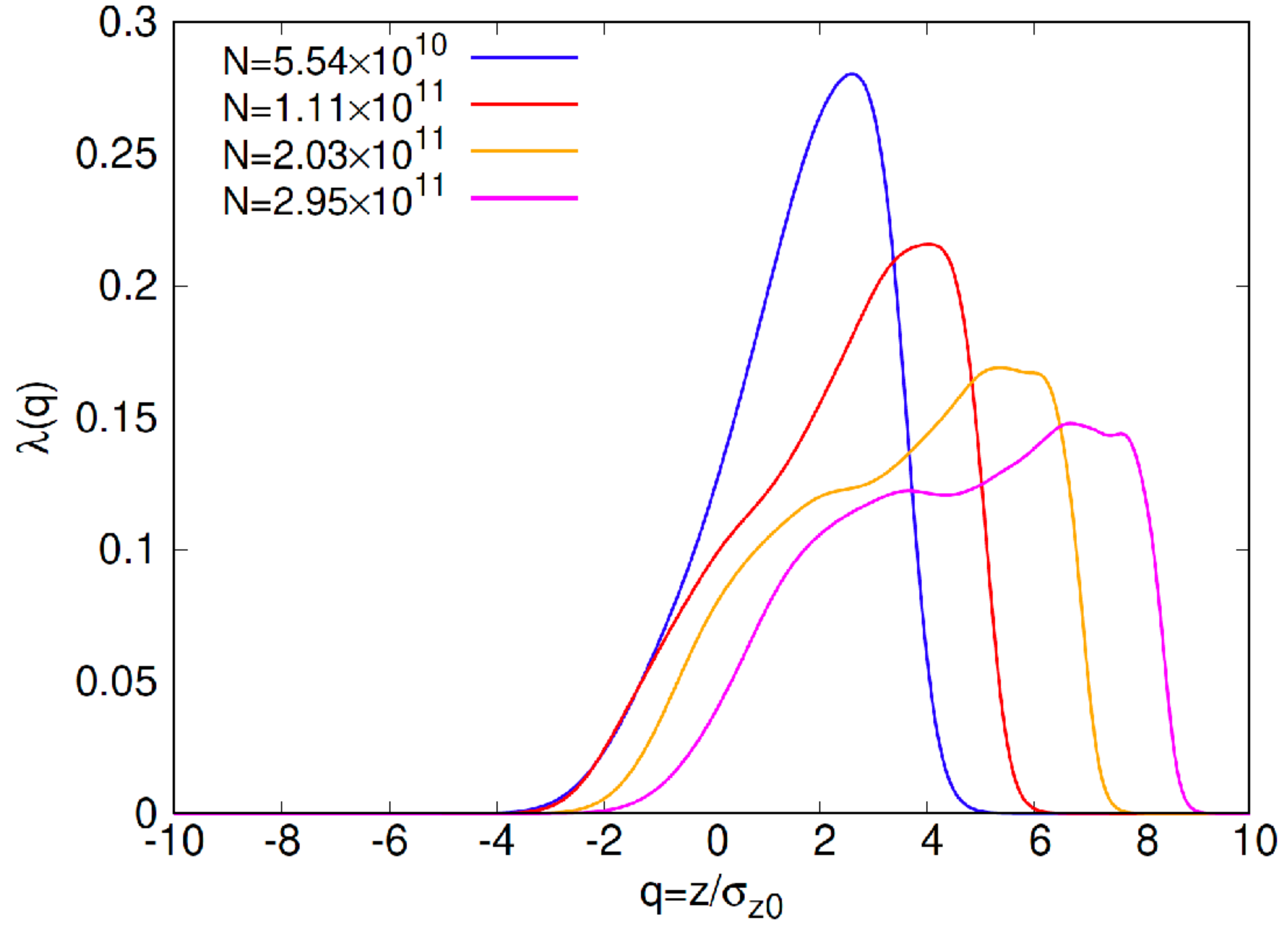
SLC is a special case:
* With original vacuum chamber, inductive character is dominant.
* Zotter's equation does not apply because of large resistive impedance

[1] R. Warnock and K. Bane, Phys. Rev. Accel. Beams 21, 124401 (2018).

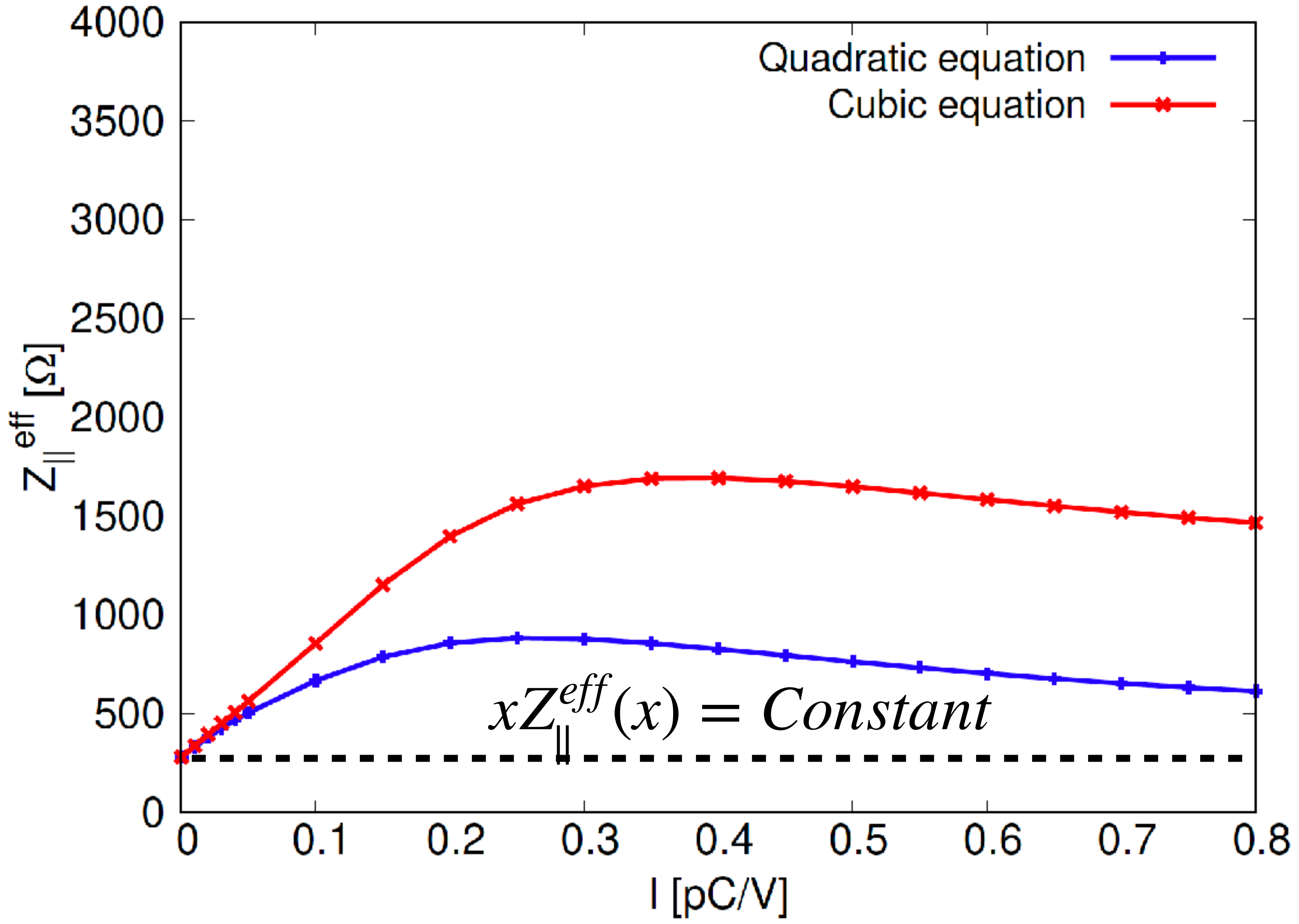
Example 4: SLC damping ring with improved vacuum chamber [1]

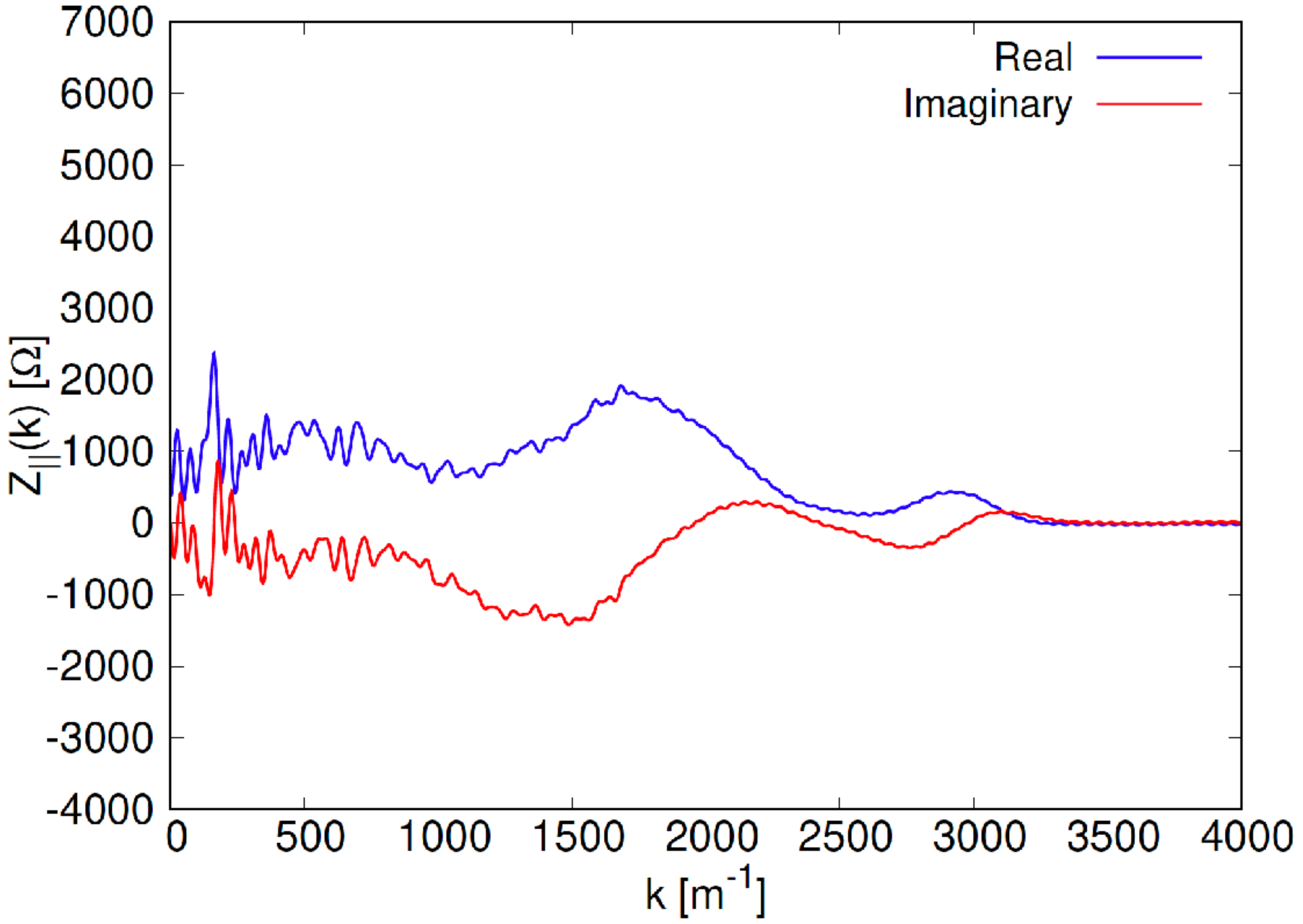


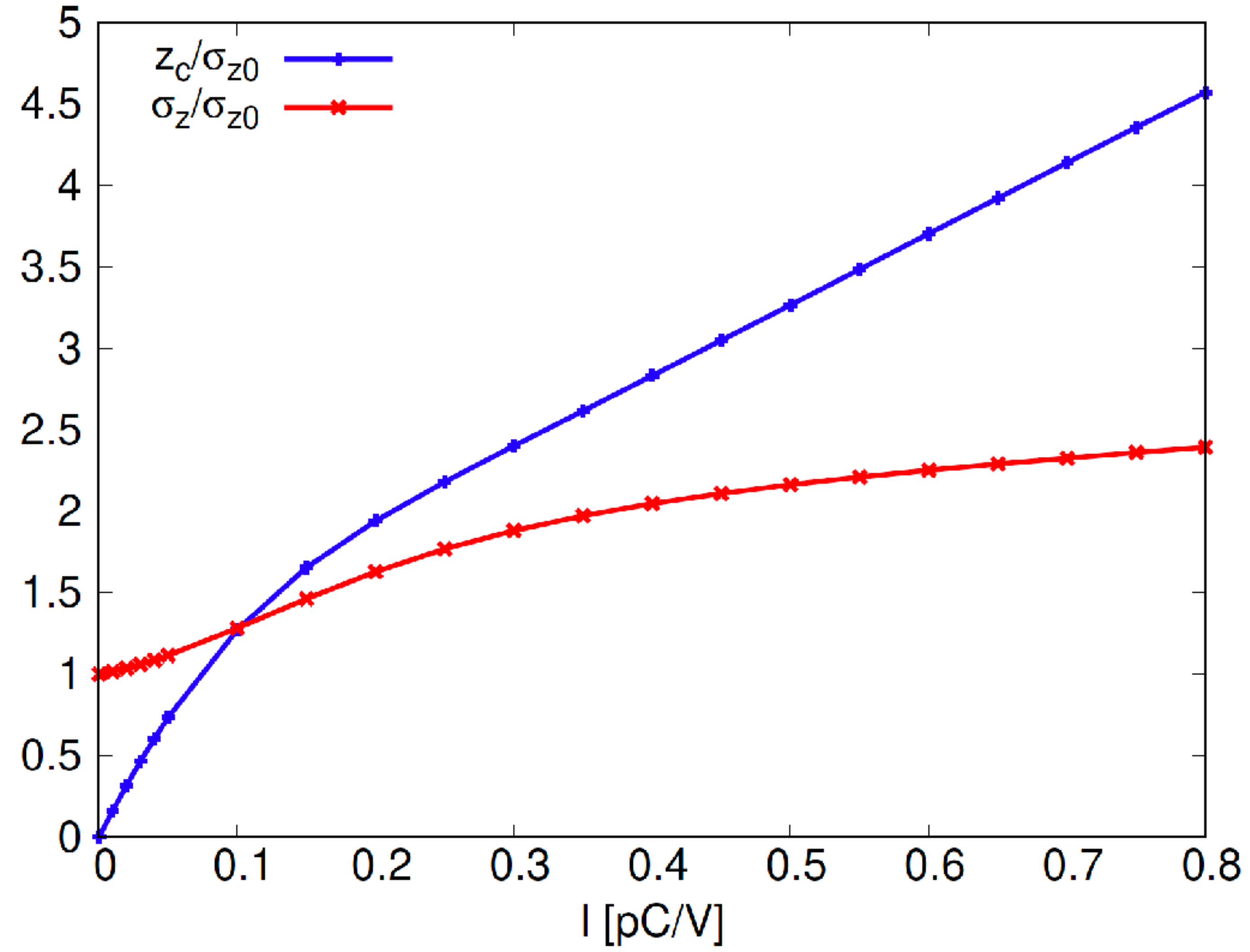
Haissinski distribution



Effective impedance







SLC is a special case:
* With improved vacuum chamber, resistive character is dominant.
* Zotter's equation does not apply because of relative large resistive impedance

[1] R. Warnock and K. Bane, Phys. Rev. Accel. Beams 21, 124401 (2018).

A global picture

VFP equation: $$\frac{\partial\psi}{\partial s}+\frac{dz}{ds}\frac{\partial\psi}{\partial z}+\frac{d\delta}{ds}\frac{\partial\psi}{\partial\delta}=\frac{2}{ct_d}\frac{\partial}{\partial\delta}\left[\delta\psi+\sigma_{\delta0}^2\frac{\partial\psi}{\partial\delta}\right]$$

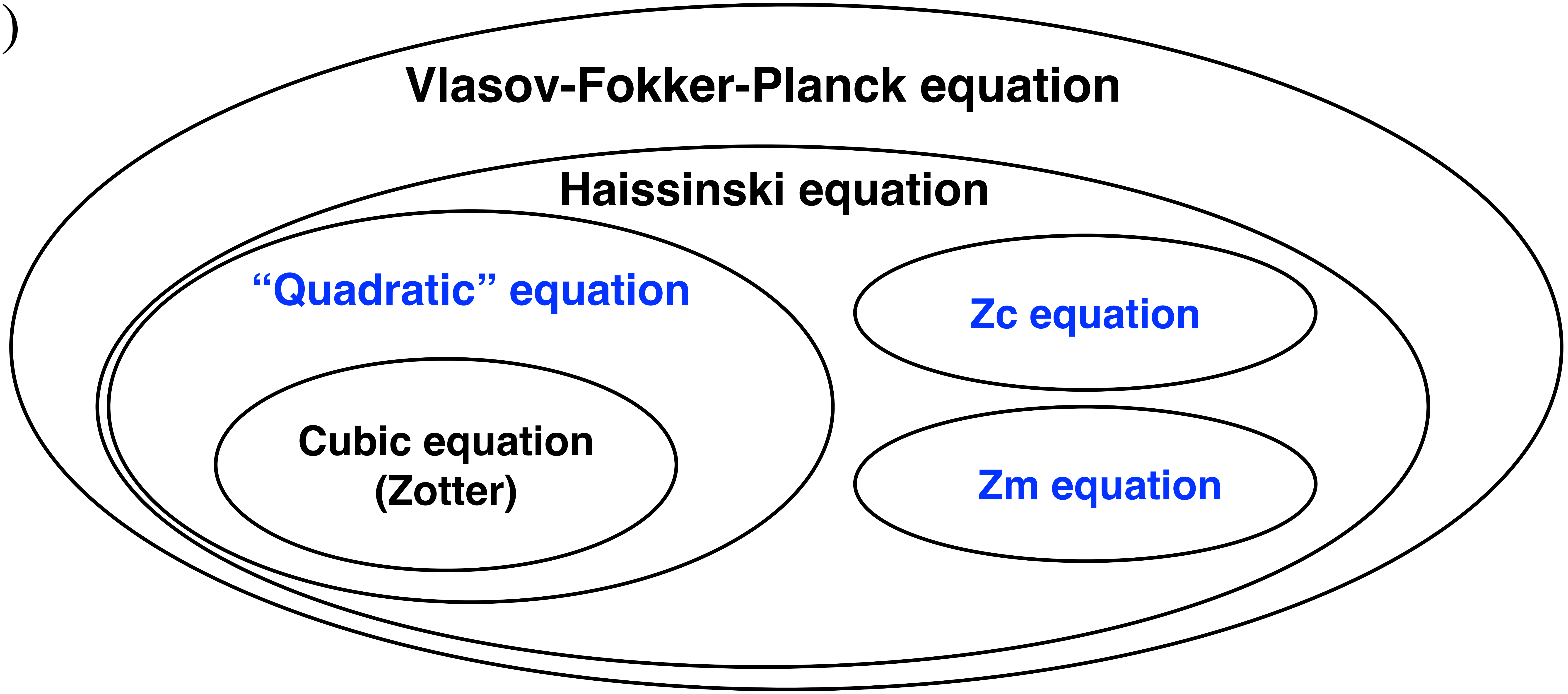
Haissinski equation: $$\lambda_0(z)=Ae^{-\frac{z^2}{2\sigma_{z0}^2}-\frac{I}{\sigma_{z0}}\int_z^\infty dz'\mathbb{W}_\parallel(z')}$$

“Quadratic” equation: $$x^2-1-\frac{cI}{2\pi\sigma_{z0}}Z_\parallel^{eff}(x)=0$$

Zotter’s equation: $$x^3-x+\frac{cI_b}{\kappa\eta\omega_0\sigma_{z0}\sigma_{\delta0}^2(E/e)}\textbf{Im}\left(\frac{Z_\parallel}{n}\right)_{eff}^{m=1}=0$$

Zc equation: $$z_c(I)=I\sigma_{z0}\kappa_\parallel$$

Zm equation: $$z_m=I\sigma_{z0}\mathbb{W}_\parallel(z_m)$$



Vlasov-Fokker-Planck equation

Haissinski equation

“Quadratic” equation

Cubic equation (Zotter)

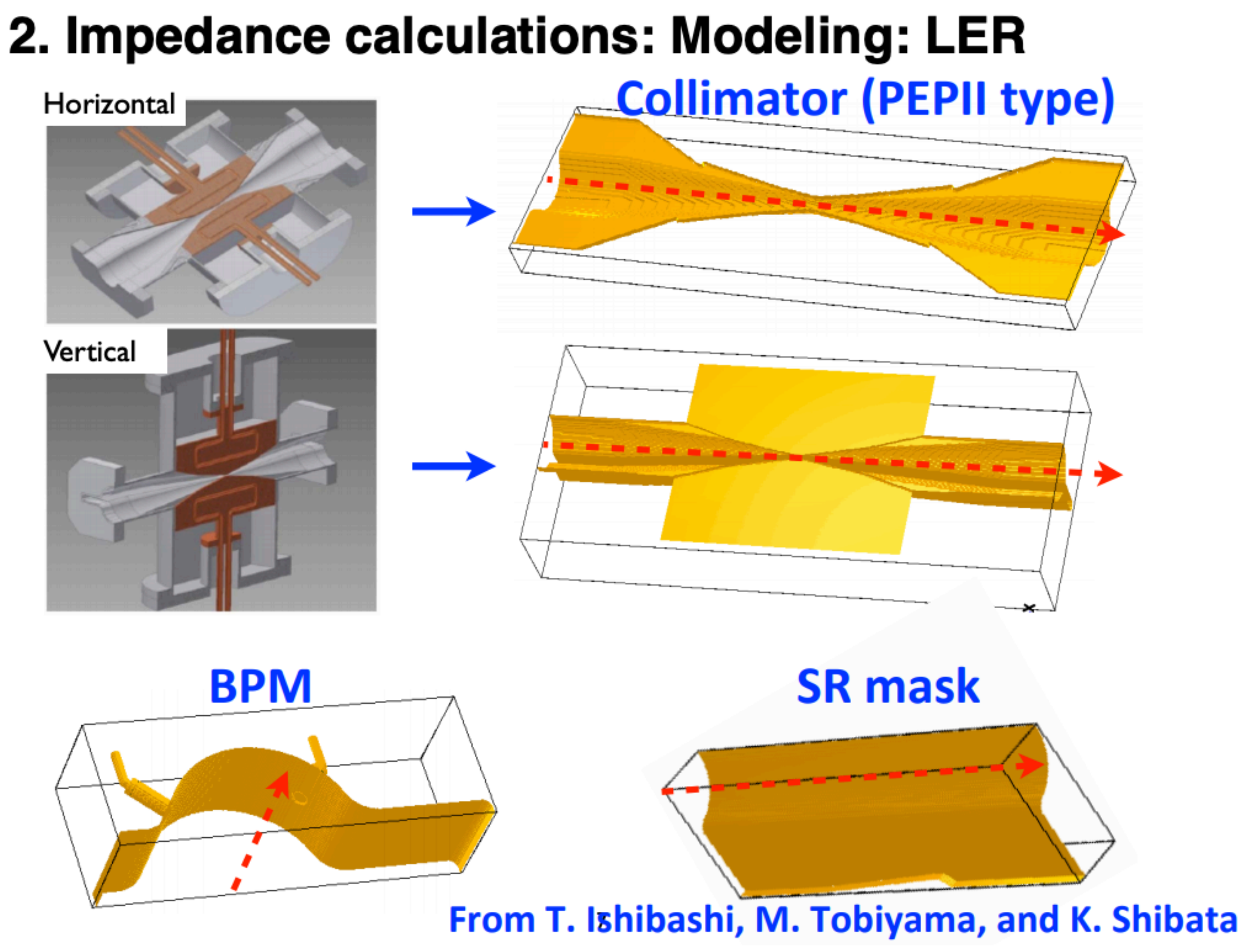
Zc equation

Zm equation

Subset diagram

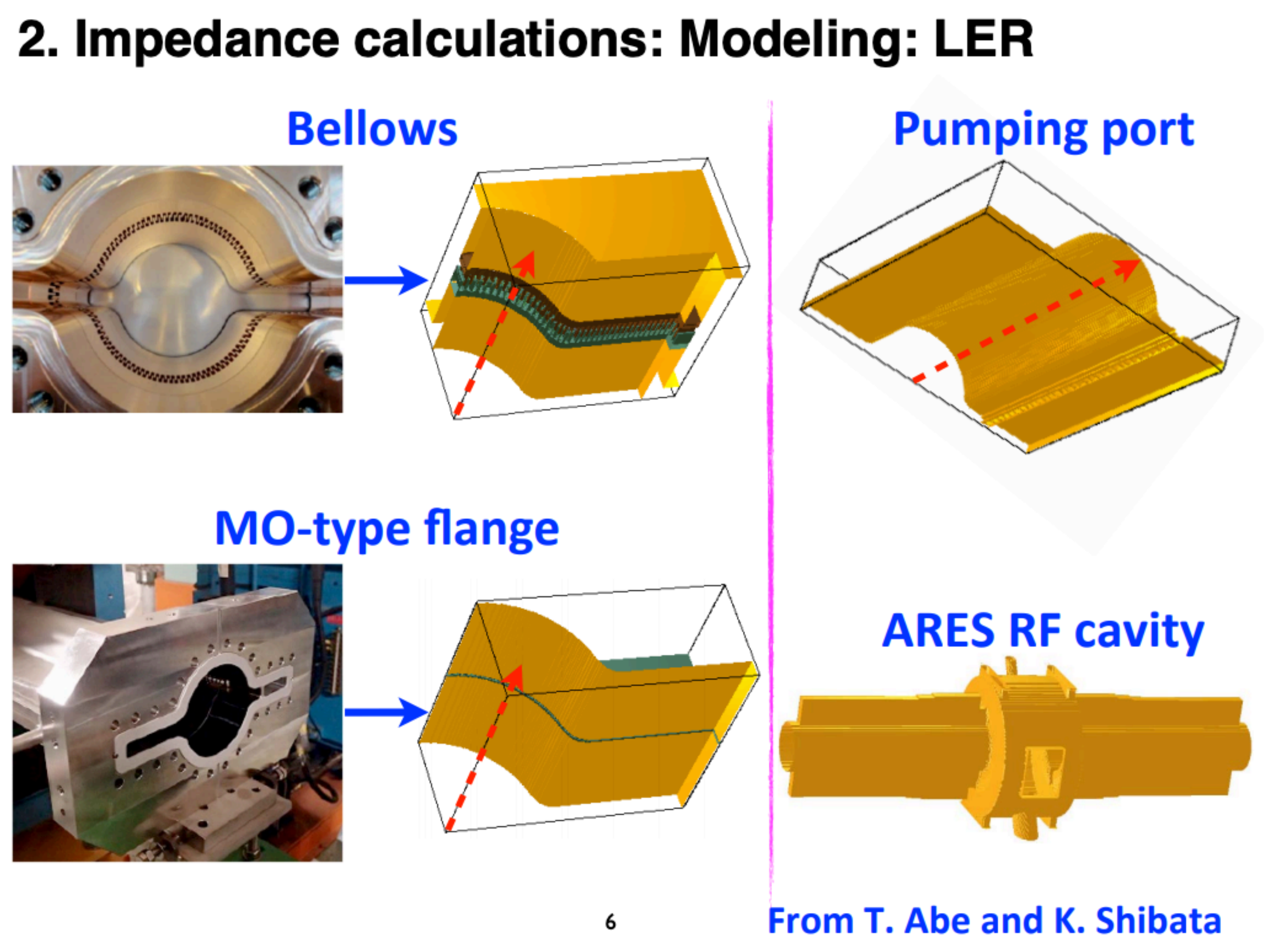
Longitudinal pseudo-Green's function impedance model for SuperKEKB LER and HER

- Long. wakes for SuperKEKB [1]
  - T. Abe, T. Ishibashi, M. Tobiyama, and others calculated the wakes with 0.5 mm Gaussian bunch. I collected wake data for MWI simulations using VFP solver.
  - Most of the wake calculations were done using GdfidL.
  - Compare with KEKB: Most of LER chambers were new; most of HER chambers were reused.



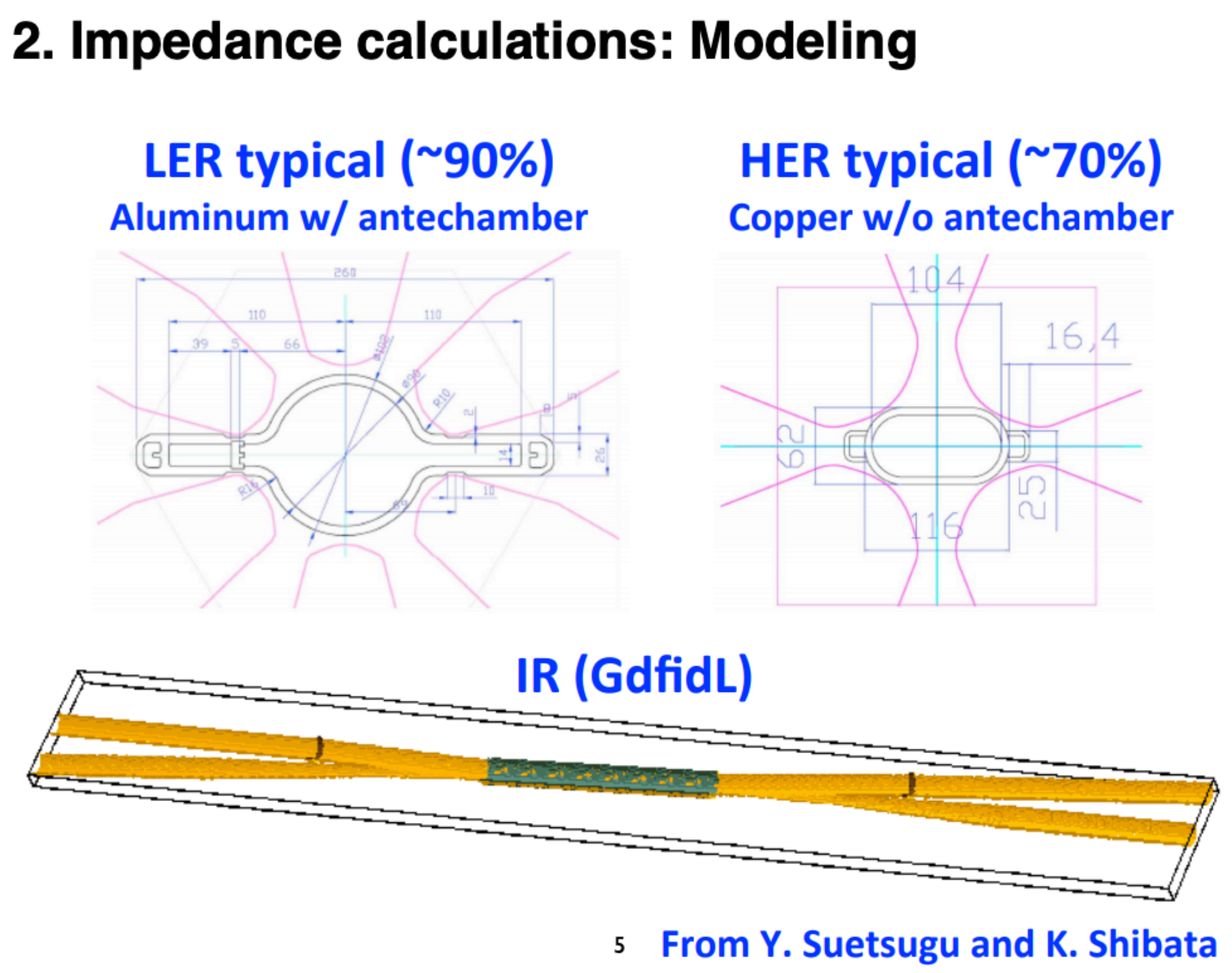
2. Impedance calculations: Modeling: LER

From T. Ishibashi, M. Tobiyama, and K. Shibata



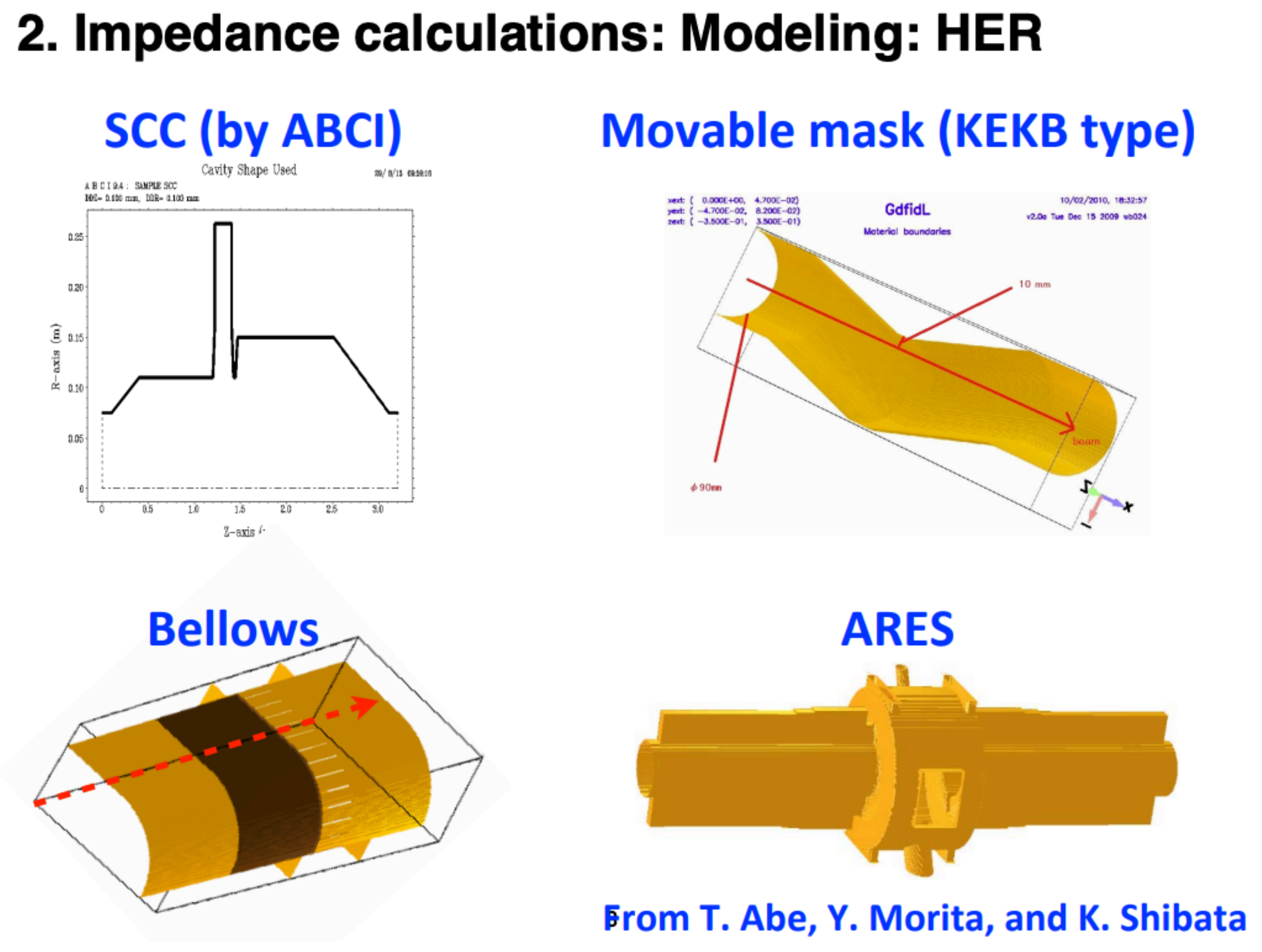
2. Impedance calculations: Modeling: LER

From T. Abe and K. Shibata



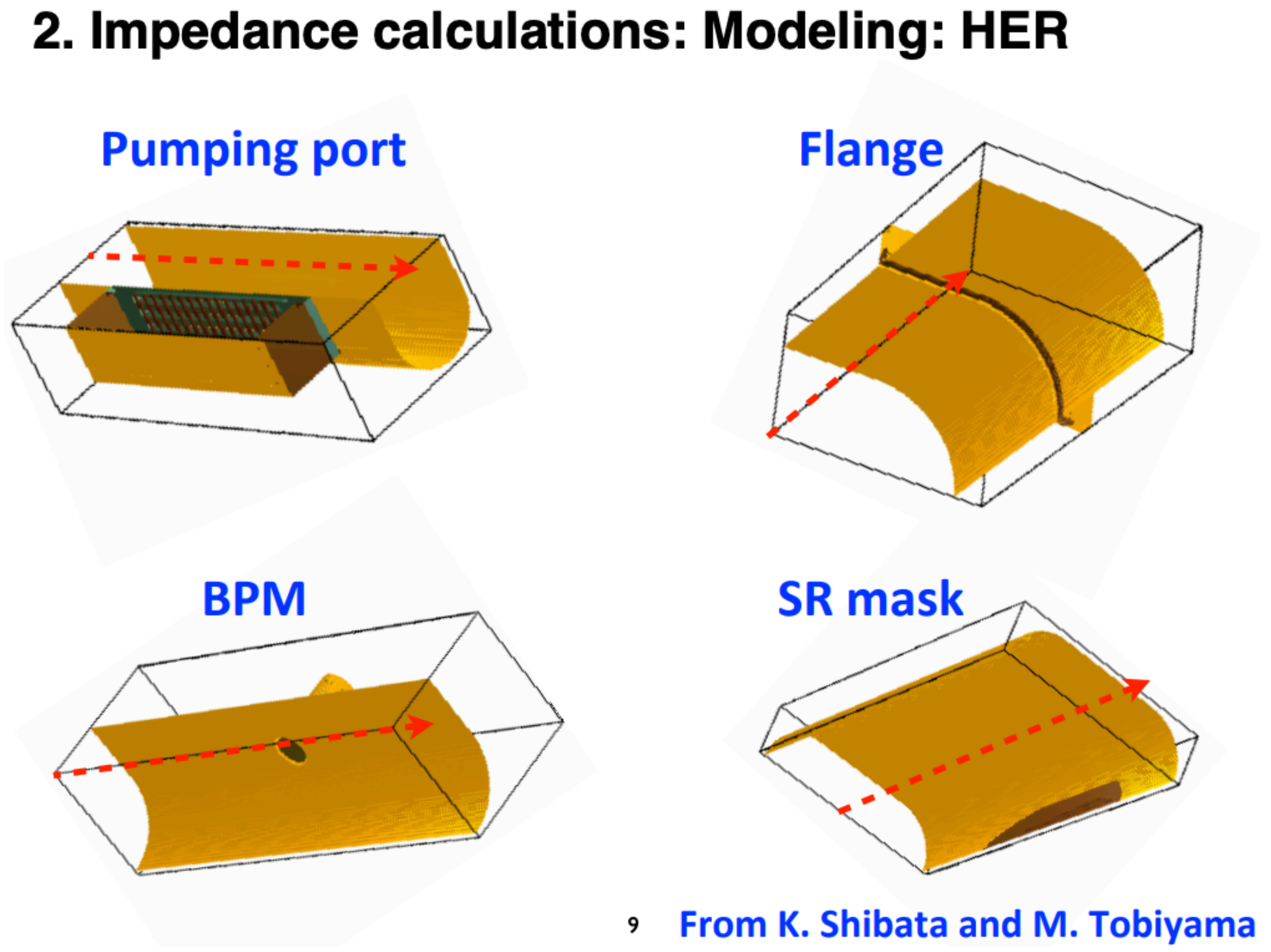
2. Impedance calculations: Modeling

From Y. Suetsugu and K. Shibata



2. Impedance calculations: Modeling: HER

From T. Abe, Y. Morita, and K. Shibata



2. Impedance calculations: Modeling: HER

From K. Shibata and M. Tobiyama

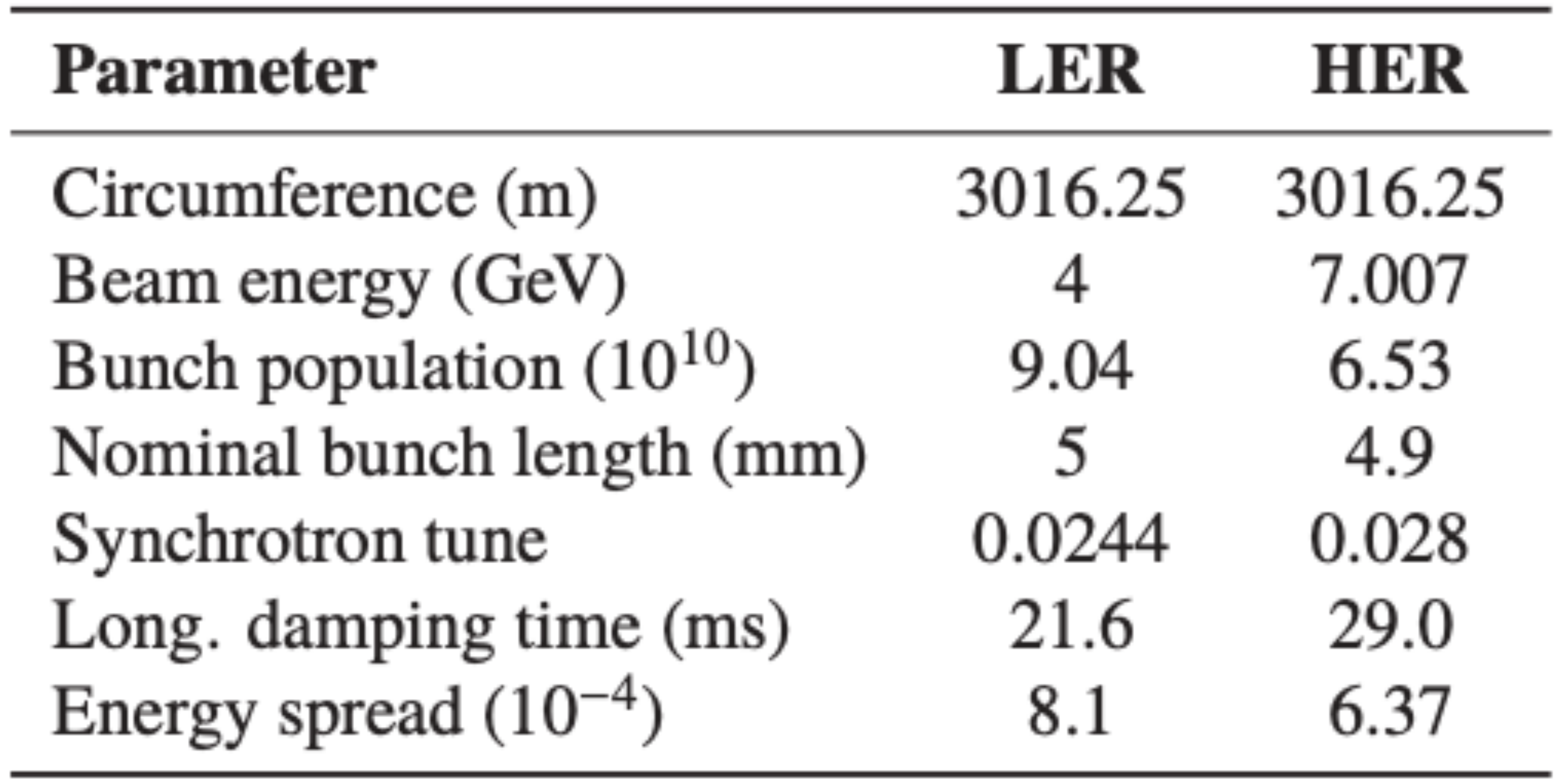
---

[1] D. Zhou, Impedance Issues in SuperKEKB, Talk presented at the 19th KEKB Accelerator Review Committee, KEK, Mar. 3, 2014.

Longitudinal pseudo-Green's function impedance model for SuperKEKB LER and HER

- MWI simulations for SuperKEKB [1]
  - MWI simulations were done with design configurations.
  - For LER, CSR is important in defining the MWI threshold.
  - For HER, CSR effect is negligible.

Table 2: Key Parameters of SuperKEKB Main Rings for MWI Simulations



Parameter	LER	HER
Circumference (m)	3016.25	3016.25
Beam energy (GeV)	4	7.007
Bunch population (10^{10})	9.04	6.53
Nominal bunch length (mm)	5	4.9
Synchrotron tune	0.0244	0.028
Long. damping time (ms)	21.6	29.0
Energy spread (10^{-4})	8.1	6.37

MWI/microbunching driven by impedance at high frequencies $k \gg 1/\sigma_{z0}$ (CSR, SC, RW) is a rich topic at both storage rings and FELs.

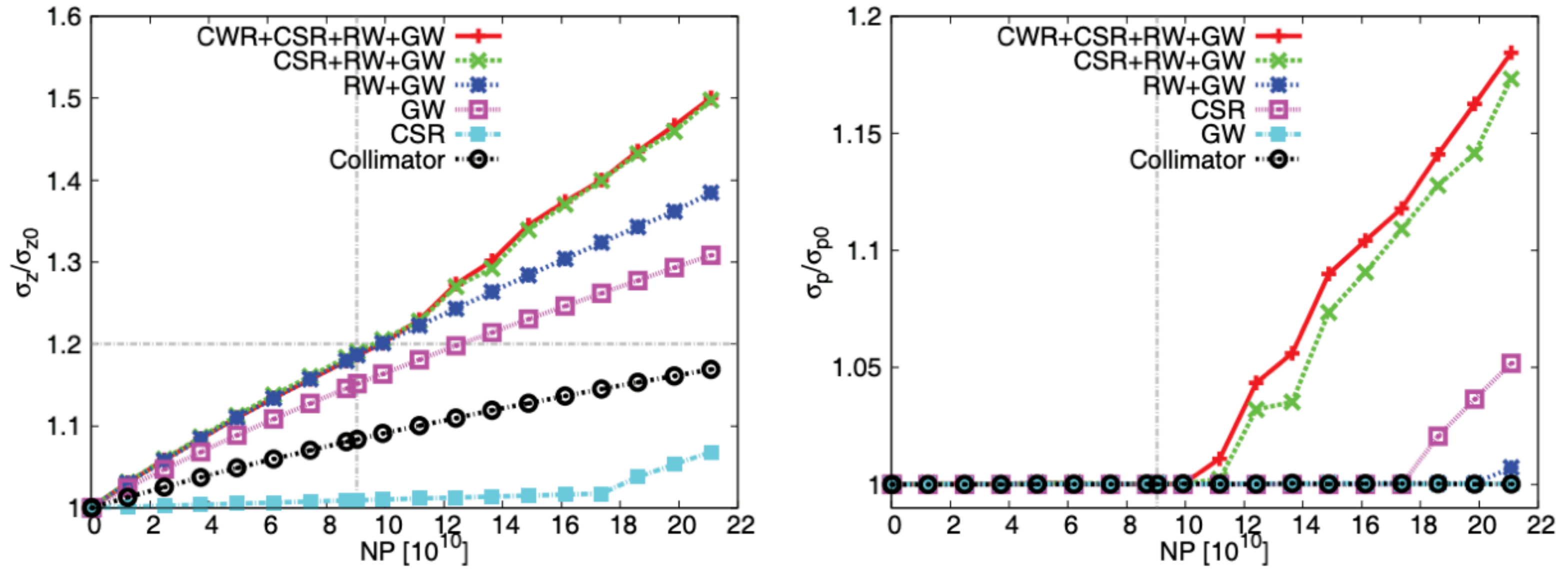


Figure 6: Normalised bunch length and energy spread as a function of the bunch population in LER. The vertical and horizontal dashed grey lines indicate the nominal bunch population and bunch length, respectively. Simulations are performed by using VFP solver with various impedance sources. Left: Bunch length. Right: Energy spread.

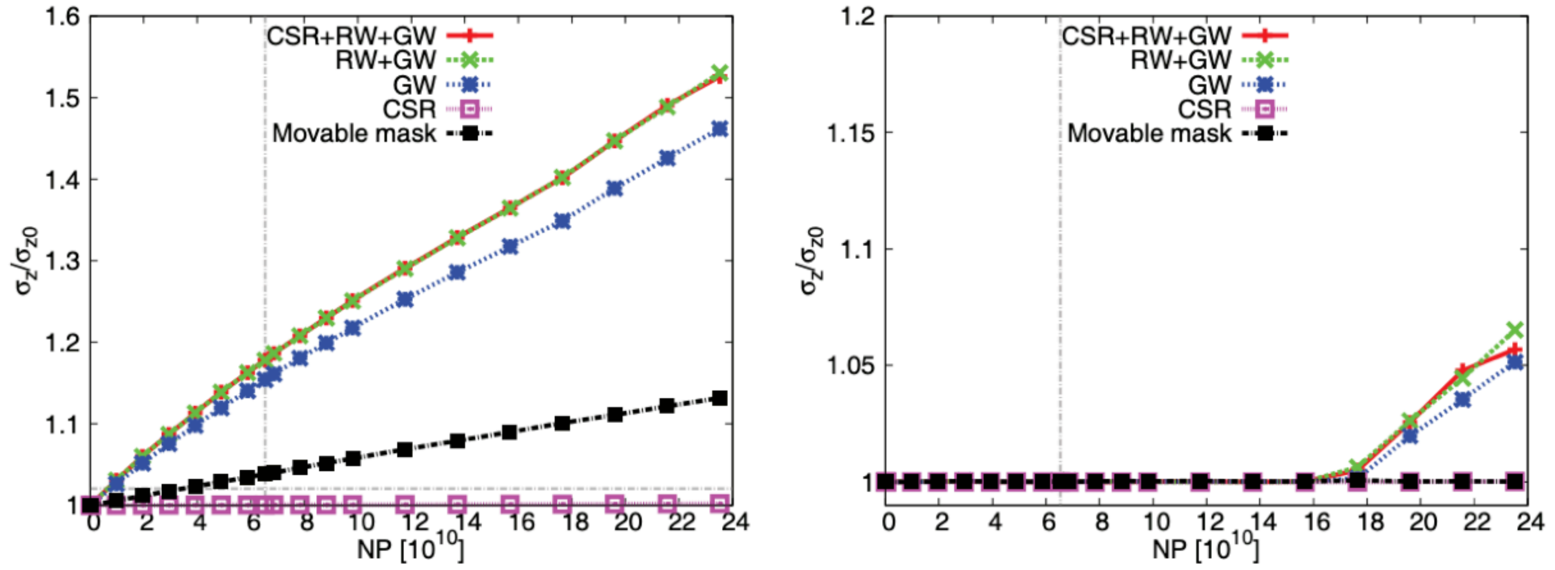


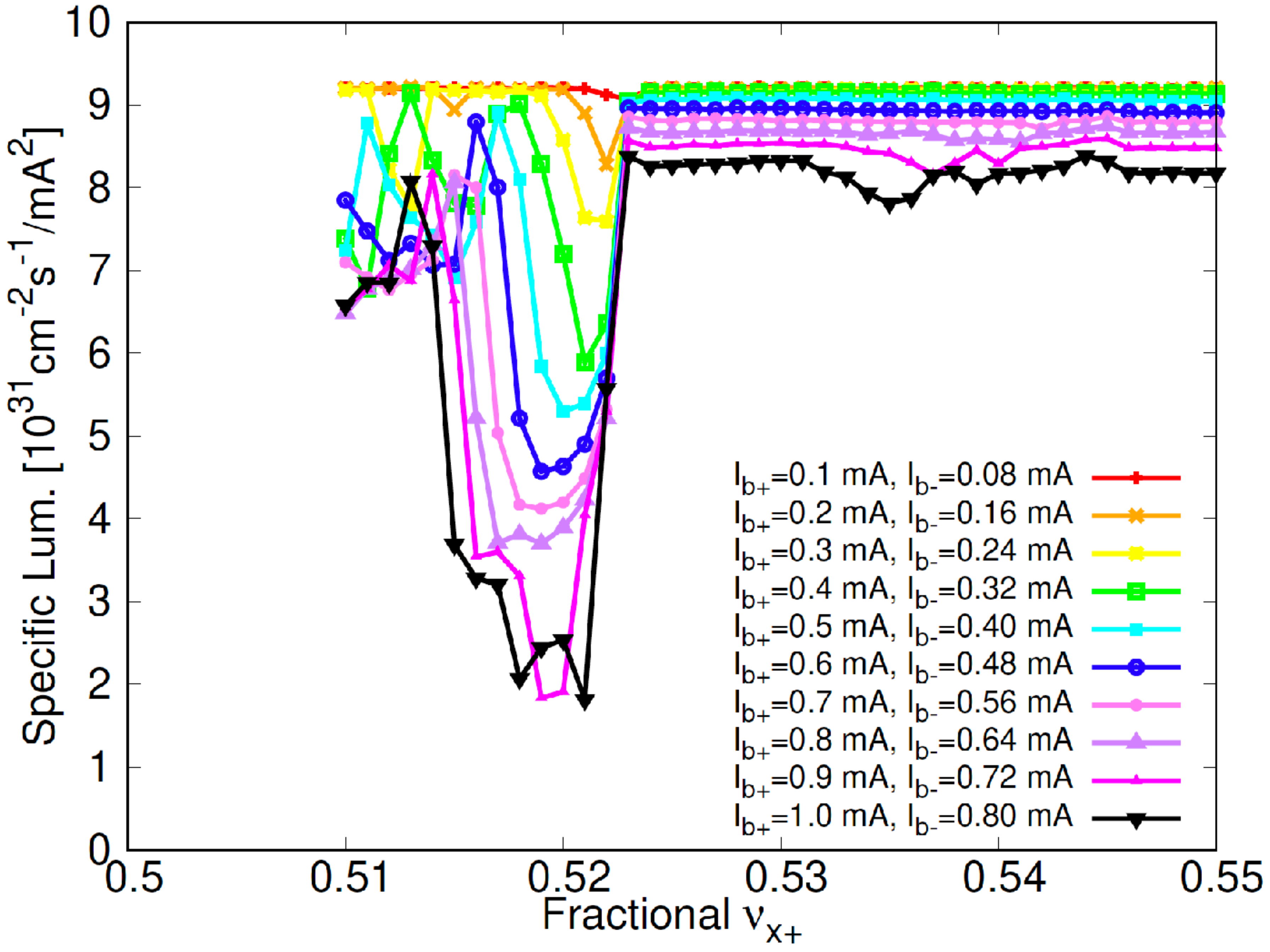
Figure 7: Normalised bunch length and energy spread as a function of the bunch population in HER. The vertical and horizontal dashed grey lines indicate the nominal bunch population and bunch length, respectively. Left: Bunch length. Right: Energy spread.

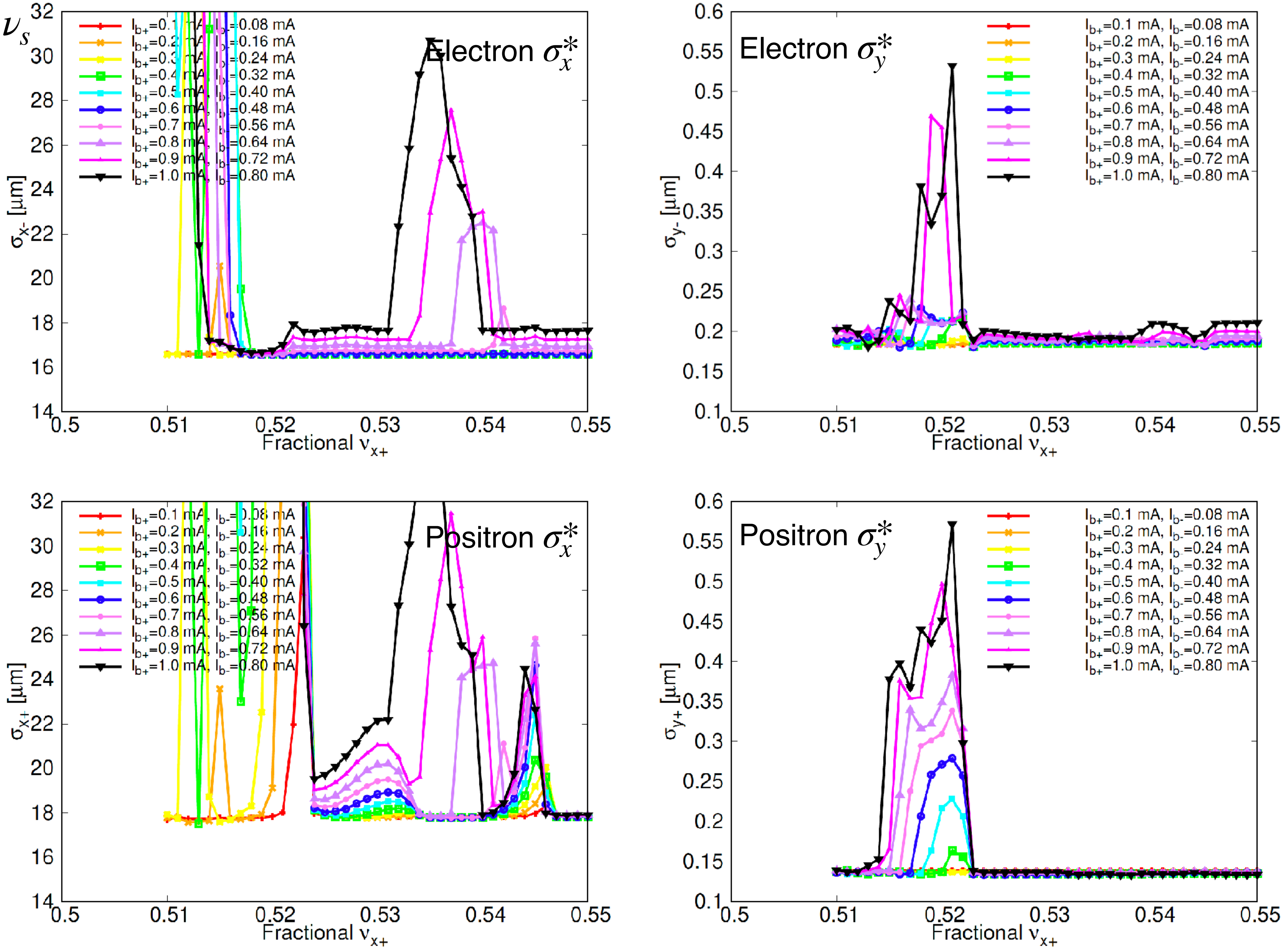
[1] D. Zhou et al., Impedance calculation and simulation of microwave instability for the main rings of SuperKEKB, in Proceedings of IPAC'14, Dresden, Germany.

Interplay of beam-beam and impedance effects at SuperKEKB

- Scan LER ν_x (with LER ν_y and HER $\nu_{x,y}$ fixed as the values of the parameter table of 2021.12.21)
  - Coupling impedances included
  - Weak horizontal blowup when $0.5 + \nu_s < [\nu_x] < 0.5 + 1.5\nu_s$

X-Z instability is sensitive to ν_x.



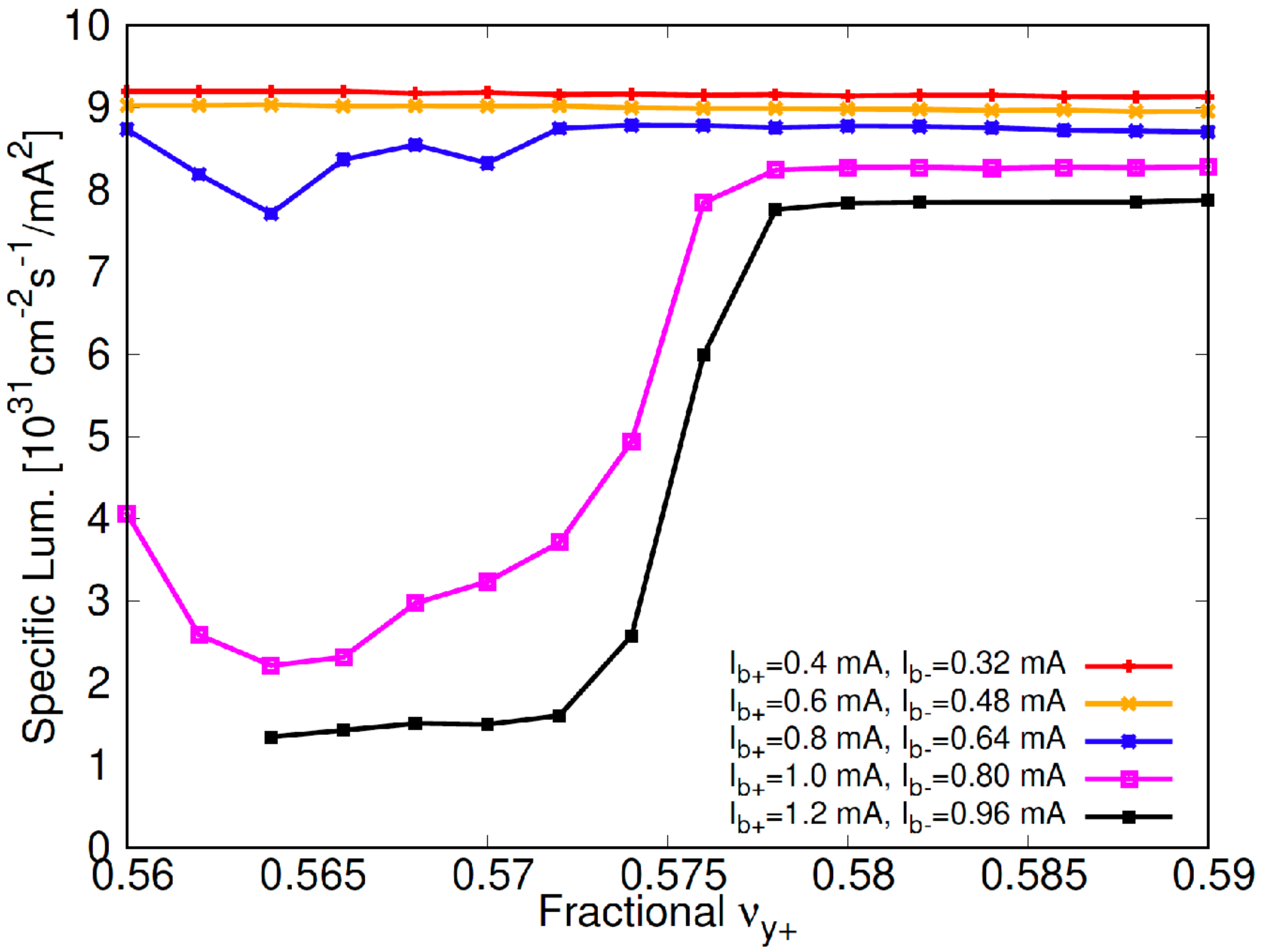


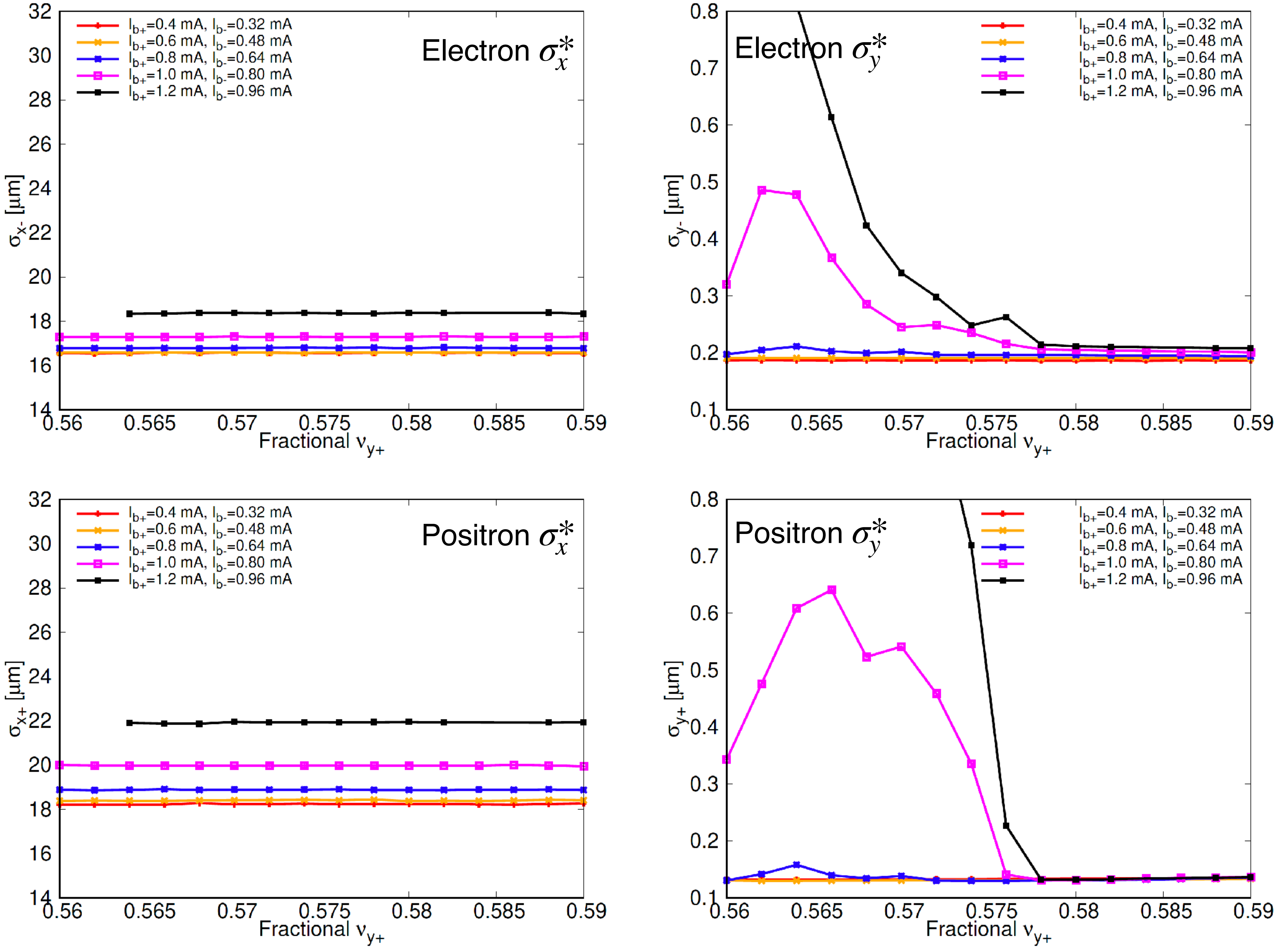
Interplay of beam-beam and impedance effects at SuperKEKB

- BBSS simulations: Scan LER ν_y with bunch currents varied (with LER ν_x and HER $\nu_{x,y}$ fixed as the values of the parameter table of 2021.12.21, BB+Wxy+Wz)

* The interplay of BB+Wx,y+Wz causes instability, consistent with Y. Zhang and K. Ohmi’s findings.

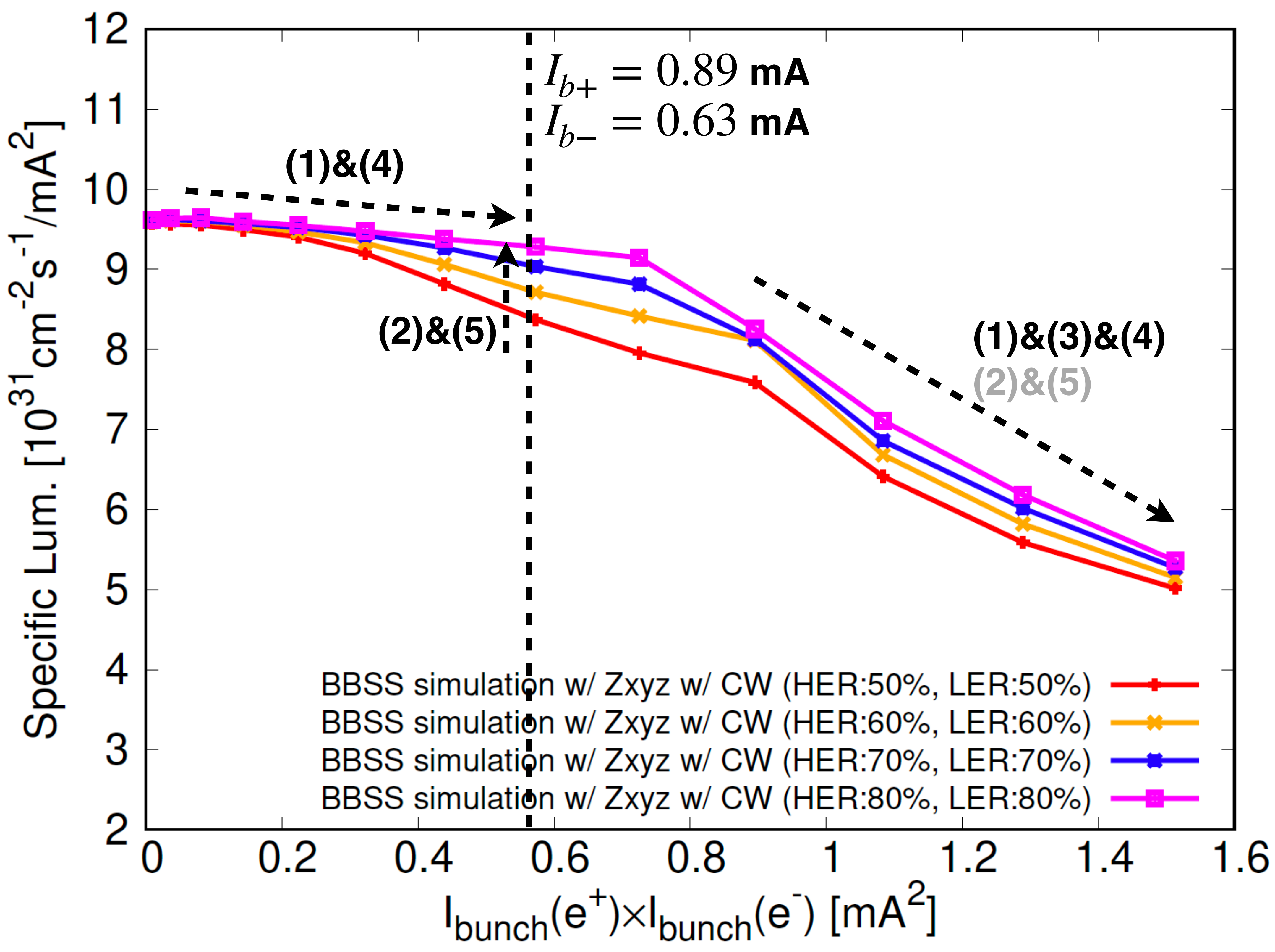
* This instability has a threshold that is ν_y-dependent.





Interplay of beam-beam and impedance effects at SuperKEKB

- Beam-beam simulations for post-LS1 operation (1E35 luminosity). Factors affecting luminosity:
  - (1) Bunch lengthening and synchrotron tune spread caused by longitudinal impedance → Unavoidable
  - (2) Beam-beam-driven fifth-order betatron resonances $\nu_x \pm 4\nu_y + \alpha = N$ → Cured by crab waist
  - (3) Vertical TMCI-like instability driven by the interplay of beam-beam and vertical impedance [1]
  - (4) Dynamic beta and dynamic emittance caused by linear transverse beam-beam force ($\beta_y^* \searrow$, $\epsilon_y \nearrow$)
  - (5) Crab waist (CW) suppresses the fifth-order beam-beam resonances

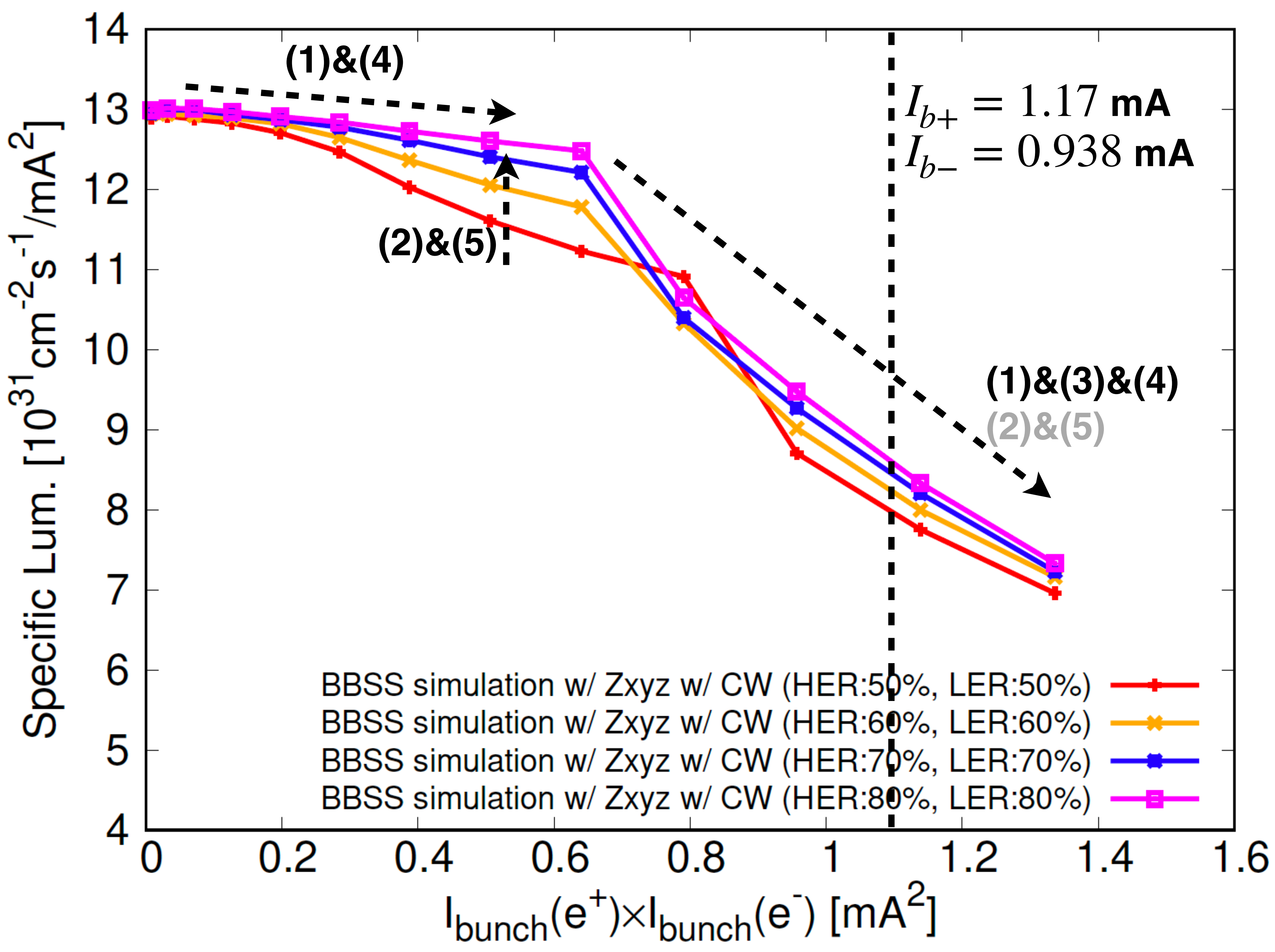


	post-LS1 1E35		Comments
	HER	LER	
I_{bunch} (mA)	0.63	0.89	
# bunch	2345		2022a operation value
ε_x (nm)	4.6	4.0	w/o IBS
ε_y (pm)	30	30	Single-beam emittance
β_x (mm)	60	60	Lattice design value
β_y (mm)	0.8	0.8	Lattice design value
σ_{z0} (mm)	5.1	4.6	Natural bunch length (w/o MWI)
ν_x	45.532	44.524	2022a operation value
ν_y	43.574	46.589	2022a operation value
ν_s	0.0272	0.0222	Calculated from lattice
$\tau_{x,y}$ (ms)	58.0	53.1	Transverse damping time (w/ NLC)
τ_z (ms)	29.0	26.6	Longitudinal damping time
Crab waist	80%	80%	Lattice design

[1] Y. Zhang et al., PRAB 26, 064401 (2023)

Interplay of beam-beam and impedance effects at SuperKEKB

- Beam-beam simulations for post-LS1 operation (2.4E35 luminosity). Factors affecting luminosity:
  - (1) Bunch lengthening and synchrotron tune spread caused by longitudinal impedance → Unavoidable
  - (2) Beam-beam-driven fifth-order betatron resonances $\nu_x \pm 4\nu_y + \alpha = N$ → Cured by crab waist
  - (3) Vertical TMCI-like instability driven by the interplay of beam-beam and vertical impedance [1]
  - (4) Dynamic beta and dynamic emittance caused by linear transverse beam-beam force ($\beta_y^* \searrow$, $\epsilon_y \nearrow$)
  - (5) Crab waist (CW) suppresses the fifth-order beam-beam resonances



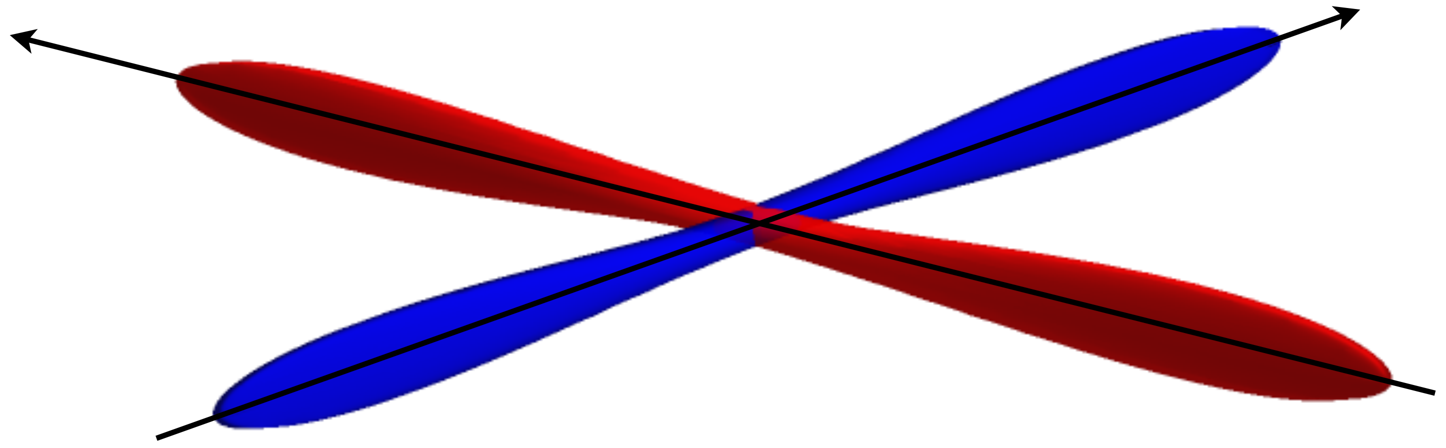
	post-LS1 2.4E35		Comments
	HER	LER	
I_{bunch} (mA)	0.938	1.17	
# bunch	2345		2022a operation value
ε_x (nm)	4.6	4.0	w/o IBS
ε_y (pm)	21	21	Single-beam emittance
β_x (mm)	60	60	Lattice design value
β_y (mm)	0.6	0.6	Lattice design value
σ_{z0} (mm)	5.1	4.6	Natural bunch length (w/o MWI)
ν_x	45.532	44.524	2022a operation value
ν_y	43.574	46.589	2022a operation value
ν_s	0.0272	0.0222	Calculated from lattice
$\tau_{x,y}$ (ms)	58.0	53.1	Transverse damping time (w/ NLC)
τ_z (ms)	29.0	26.6	Longitudinal damping time
Crab waist	80%	80%	Lattice design

[1] Y. Zhang et al., PRAB 26, 064401 (2023)

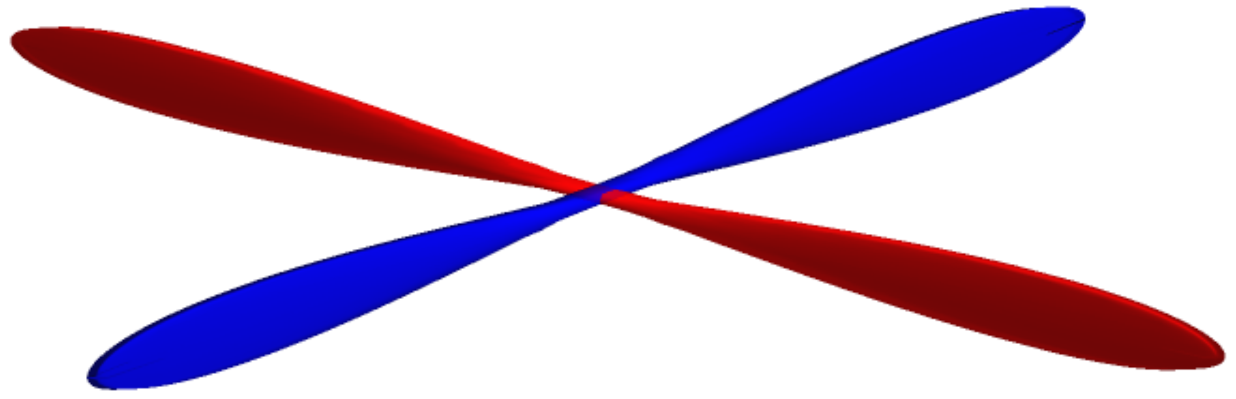
Interplay of beam-beam and impedance effects at SuperKEKB

- From the luminosity viewpoint, we list some important issues [1]:
  - Issue 1: Limits on bunch currents
  - Issue 2: Multi-bunch effects
  - Issue 3: Optics distortion at high beam currents
  - Issue 4: Impedance effects
  - Issue 5: Lsp injection correlation

$$\overset{\#5}{\boxed{L}} \approx \frac{\overset{\#1,2,3,4,5}{\boxed{N_b N_+ N_-}} f}{2\pi \sqrt{\underset{\#1,2,3,4,5}{\boxed{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}}}} \sqrt{\underset{\#4}{\boxed{\sigma_{z+}^{2} + \sigma_{z-}^{2}}}} \tan \underset{\text{BB, CW, …}}{\boxed{\frac{\theta_c}{2}}}} e^{-\frac{\overset{\#2,5}{\boxed{\Delta^2}}}{2(\sigma_{y+}^{*2} + \sigma_{y-}^{*2})}}$$



SuperKEKB (2021c)



SuperKEKB (Final design)

[1] D. Zhou et al., PRAB 26, 071001 (2023).

Summary

- An introduction to fundamental formulations of impedance and wakes, and impedance modeling for components in accelerators.
- An introduction to impedance effects in electron storage rings, with practical examples of SuperKEKB.