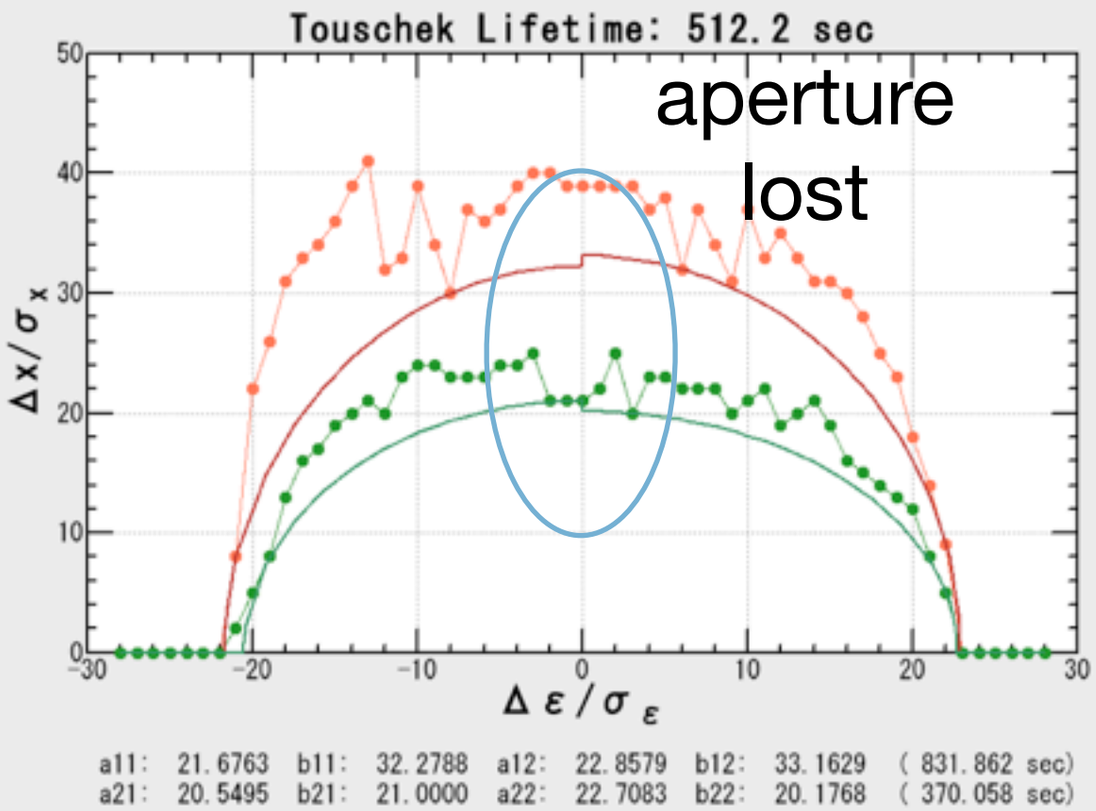
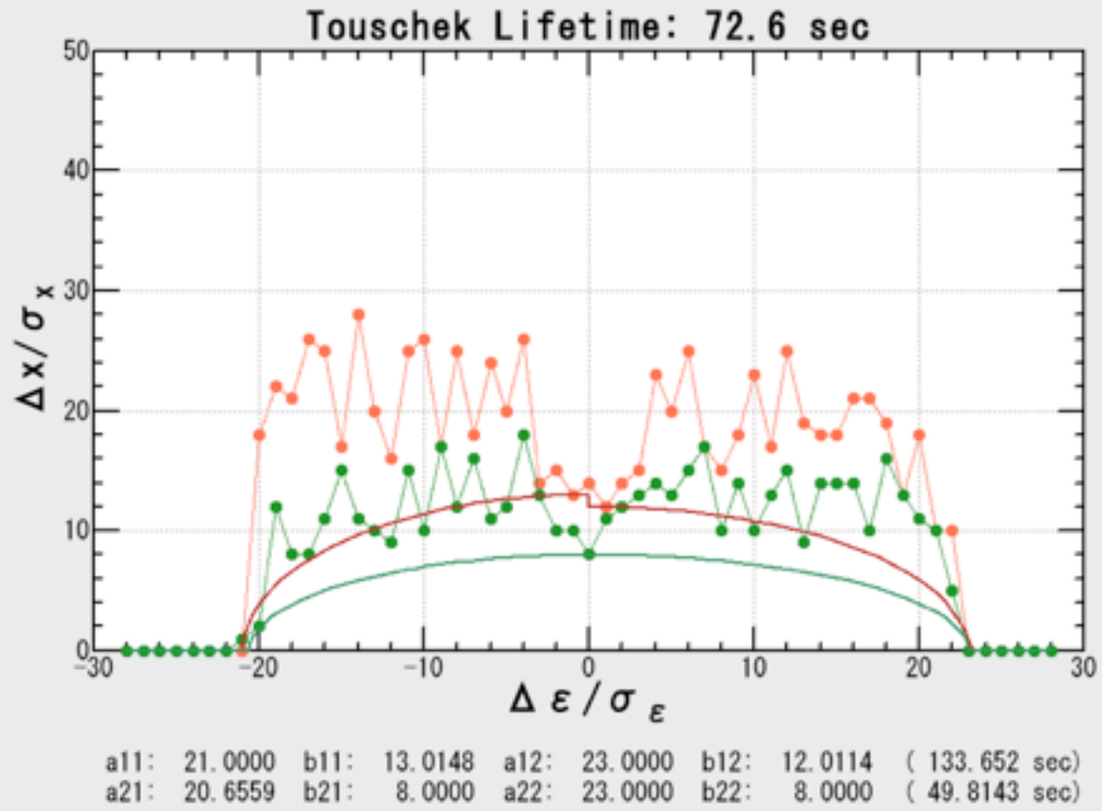
Dynamic Aperture with Beam-Beam

Y. OHNISHI

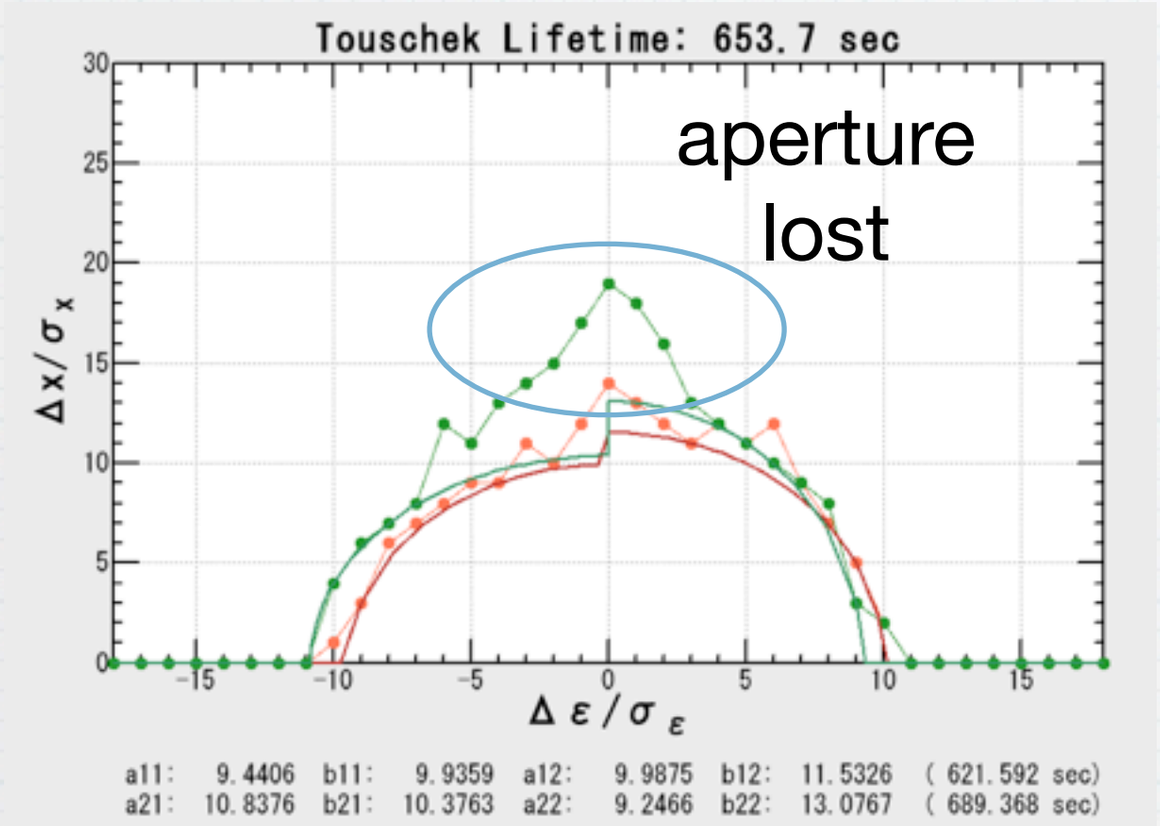
LER Dynamic Aperture

beam-beam

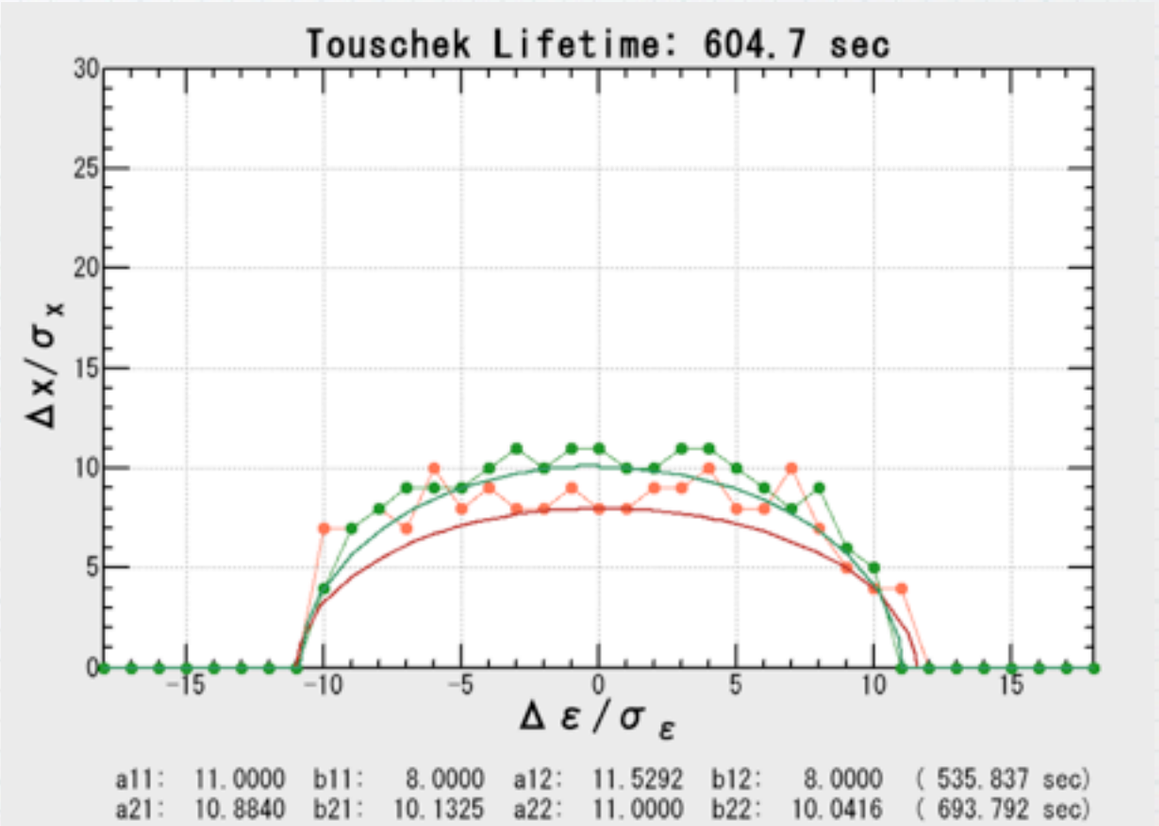




HER Dynamic Aperture



beam-beam



Crab Waist

- Hamiltonian

$$H_3 = k_x X^3 + \boxed{k_y X Y^2}$$

$$k_x = \frac{\beta_{x,s}^{3/2} K_2}{6}$$

$$k_y = -\frac{\sqrt{\beta_{x,s}}\beta_{y,s} K_2}{2}$$

$$\begin{aligned} H_{3,crab} &= k_y X Y^2 \\ &= k_y (X^* \cos\Delta\psi_x + P_X^* \sin\Delta\psi_x)(Y^* \cos\Delta\psi_y + P_Y^* \sin\Delta\psi_y)^2 \\ &= k_y \cos(m\pi) X^* P_Y^{*2} \end{aligned}$$

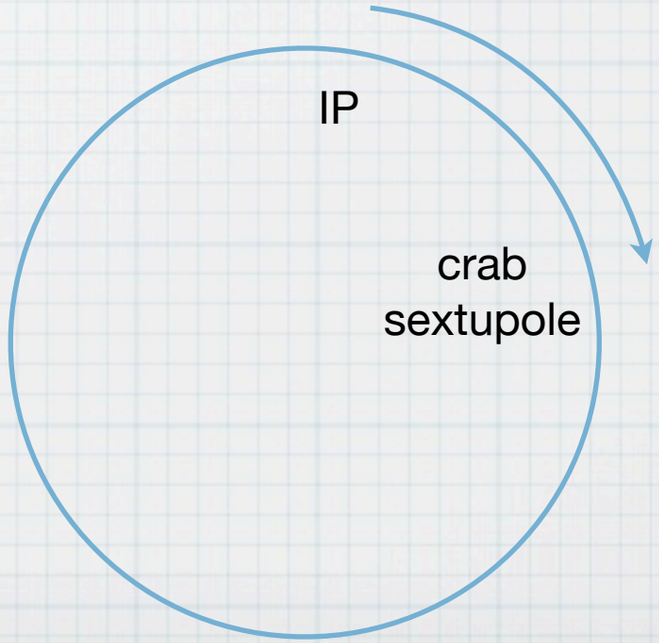
$$= -\frac{K_2 \cos(m\pi)}{2} \beta_{y,s}\beta_y^* \sqrt{\frac{\beta_{x,s}}{\beta_x^*}} x^* p_y^{*2}$$

$$= -\frac{x^* p_y^{*2}}{2\tan 2\phi_x}$$

$$K_2 = \frac{1}{\cos(m\pi)\tan 2\phi_x \beta_{y,s}\beta_y^*} \sqrt{\frac{\beta_x^*}{\beta_{x,s}}}$$

$$\Delta\psi_x = m\pi$$

$$\Delta\psi_y = \frac{\pi}{2} + n\pi$$



Crab Waist

- 衝突点にHamiltonianによる変換を入れる。

$$H_{IP} = -\frac{a}{2} x p_y^2$$

$$a = -\frac{1}{\tan 2\phi} \qquad \text{(nominal)}$$

- Hamilton 方程式

$$\frac{dx}{ds} = \frac{\partial H_{IP}}{\partial p_x} = 0 \qquad \frac{dy}{ds} = \frac{\partial H_{IP}}{\partial p_y} = -axp_y$$

$$\frac{dp_x}{ds} = -\frac{\partial H_{IP}}{\partial x} = \frac{a}{2} p_y^2 \qquad \frac{dp_y}{ds} = -\frac{\partial H_{IP}}{\partial p_y} = 0$$

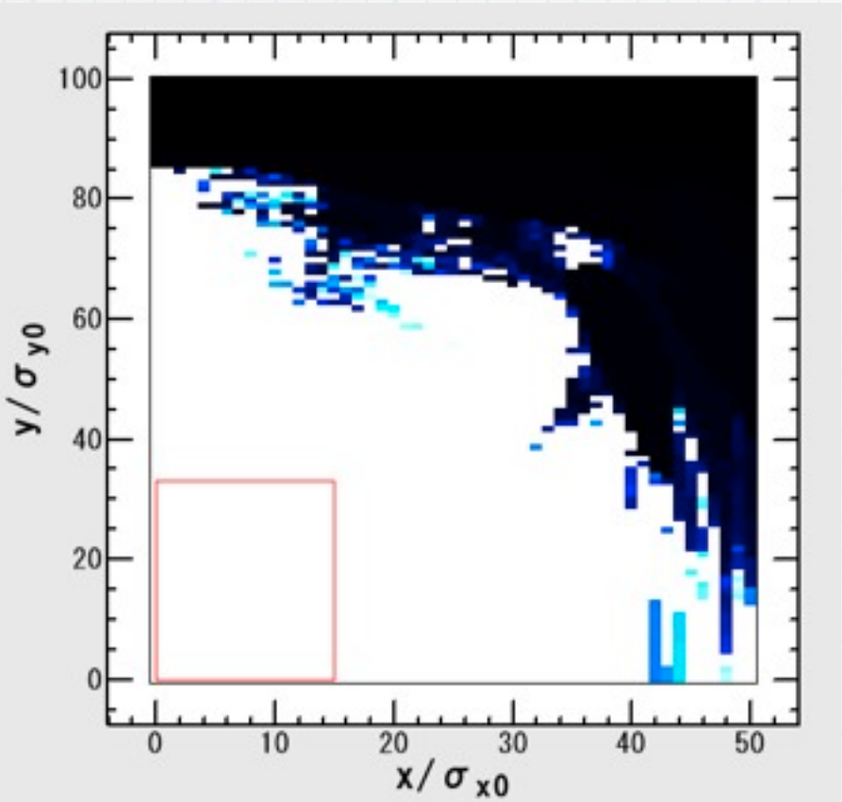
$$p_x \to p_x + \frac{a}{2} p_y^2$$

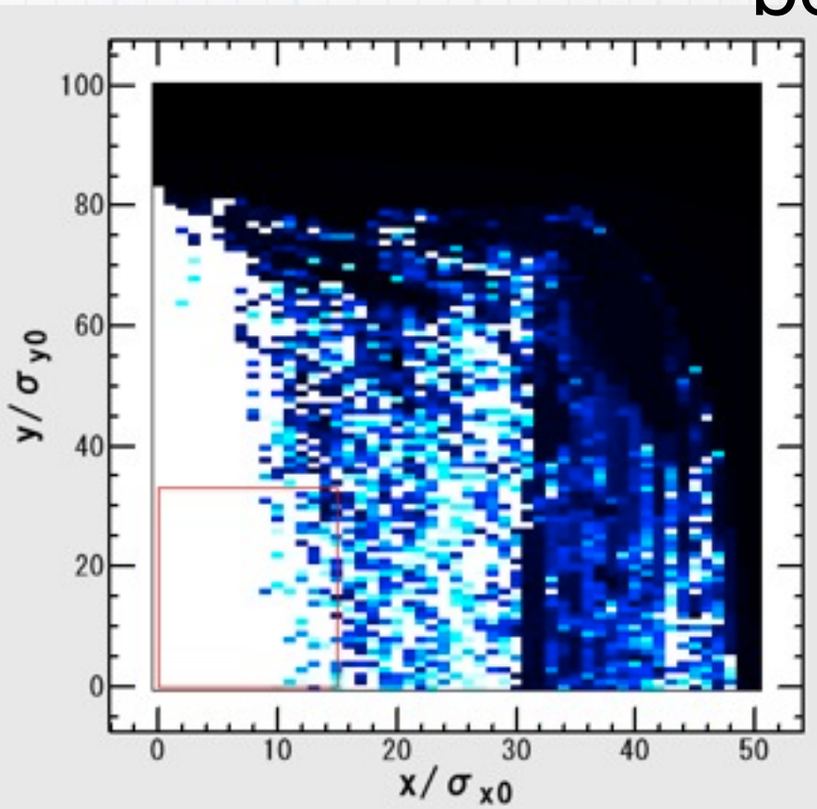
$$y \to y - axp_y$$

左の変換をビームビーム相互作用の前に、
その逆変換を後に置く。

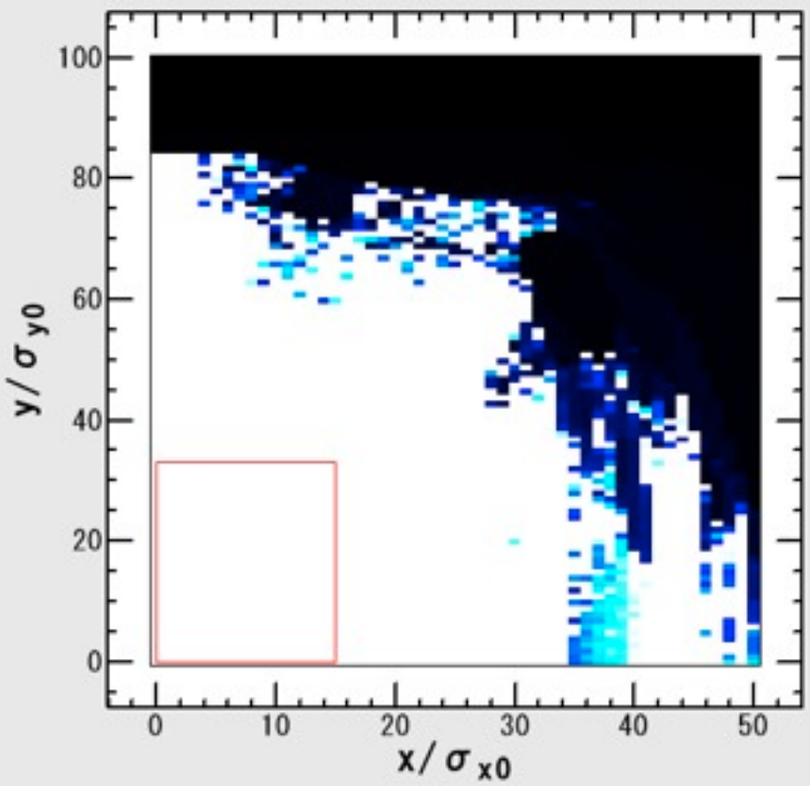
LER Dynamic Aperture

beam-beam

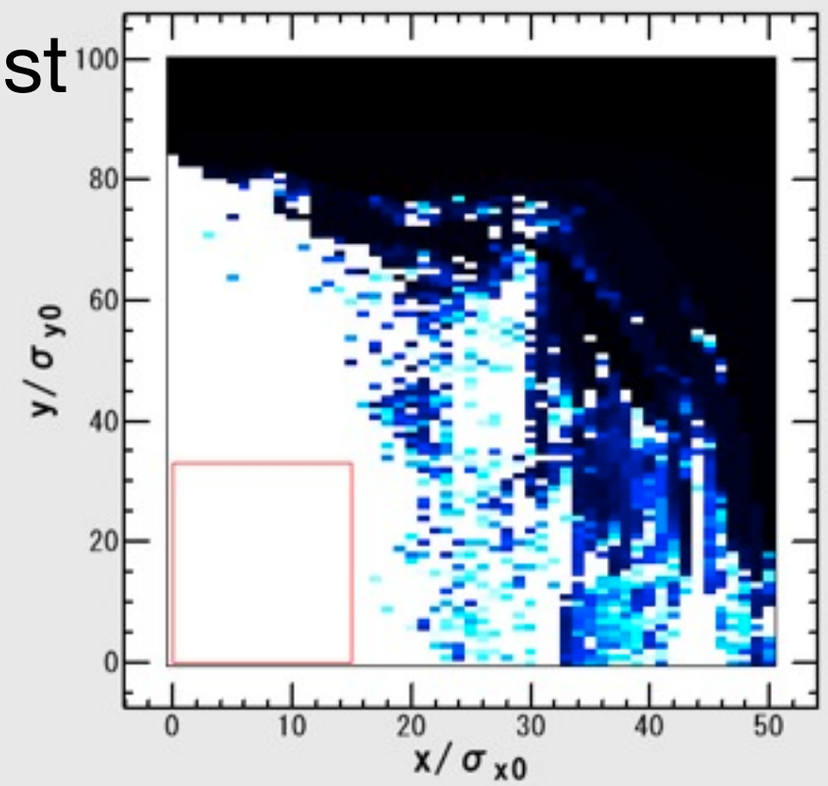




crab waist



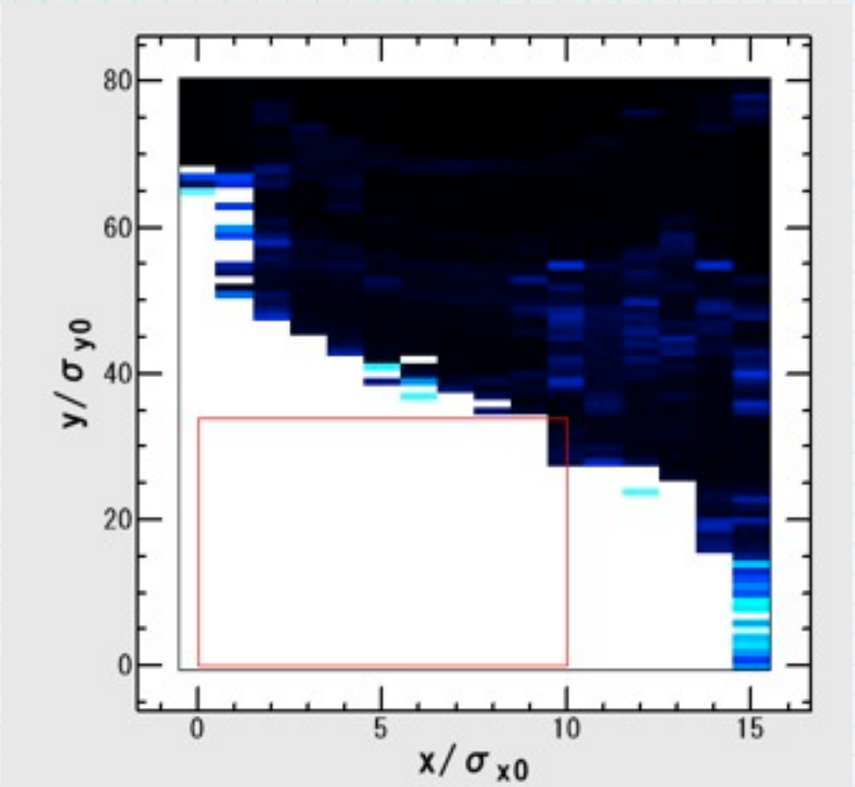
full

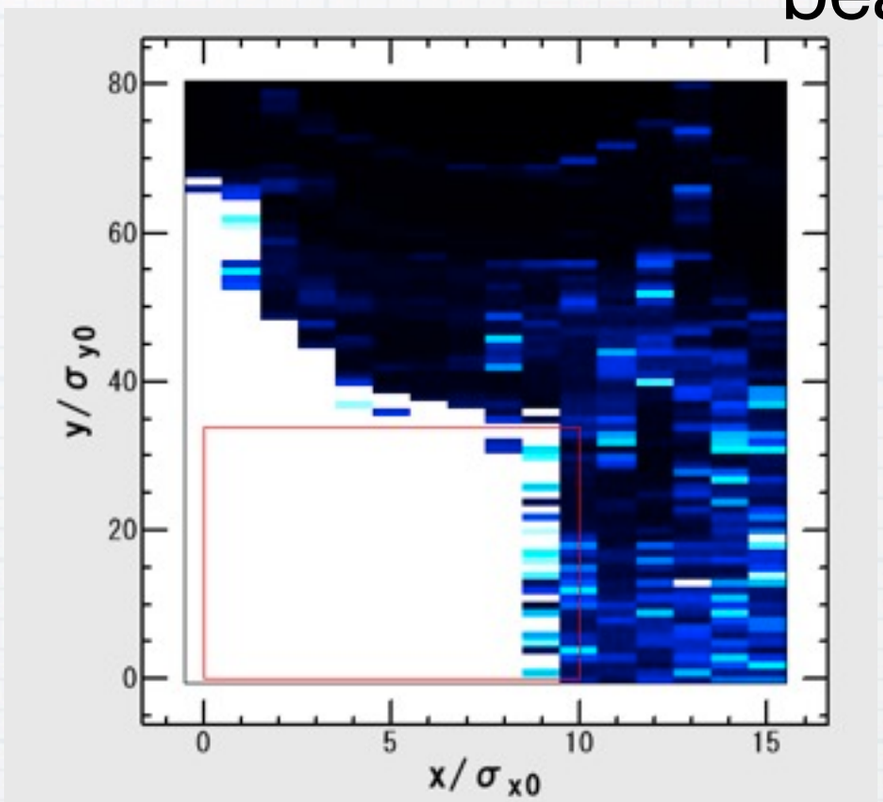


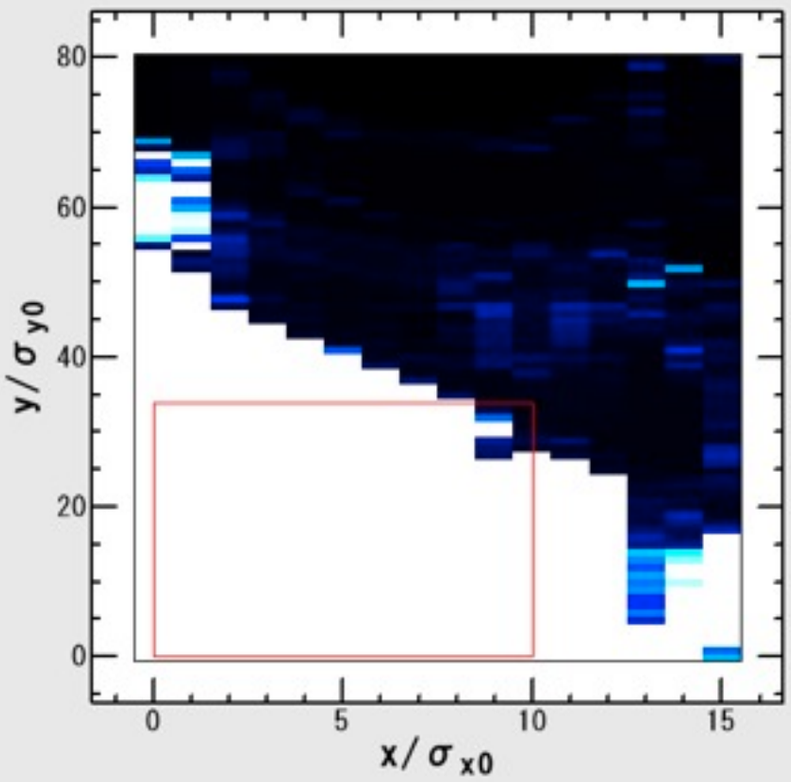
half

HER Dynamic Aperture

beam-beam







crab waist

full

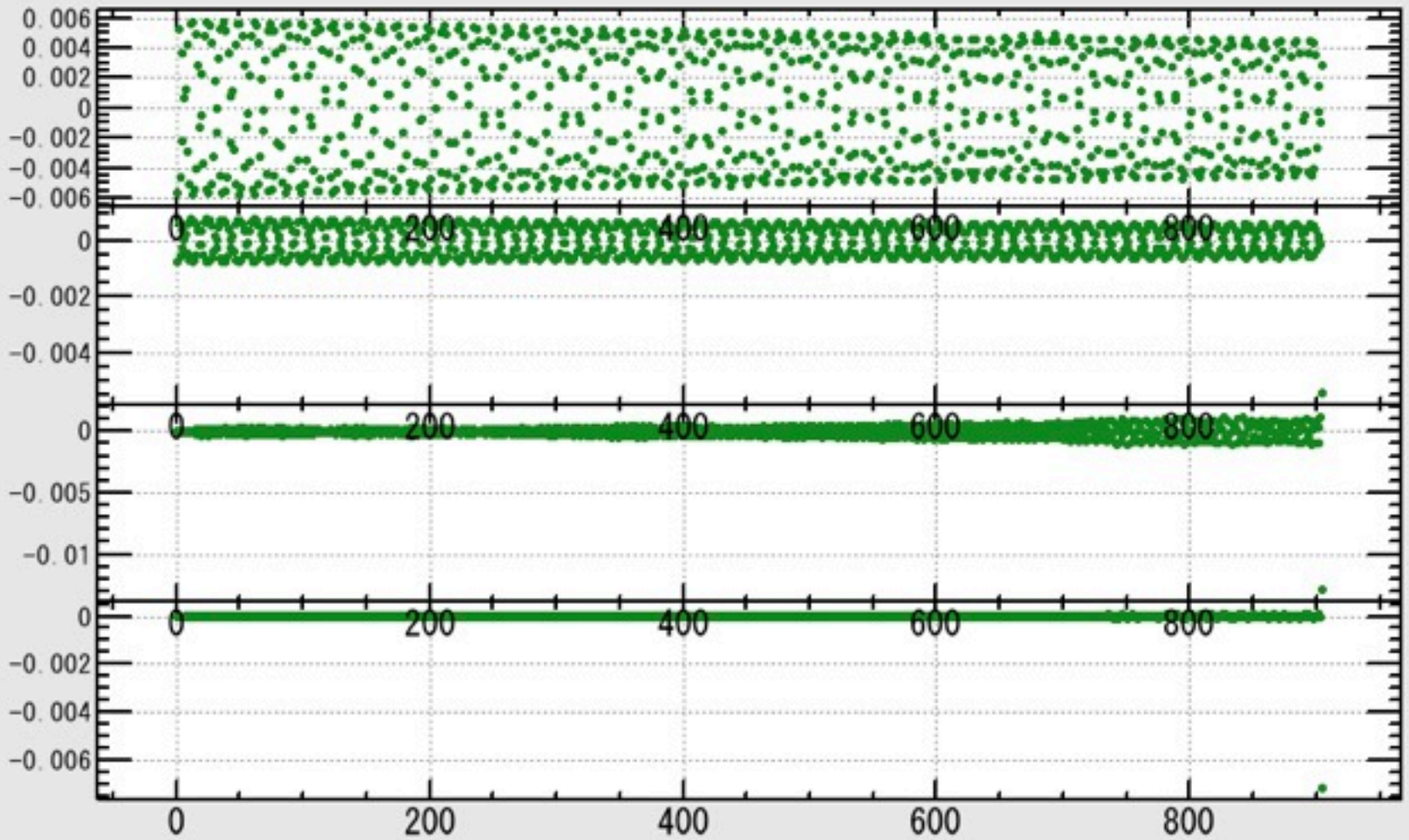
x_0 = 10 sigma, y_0 = 0

x

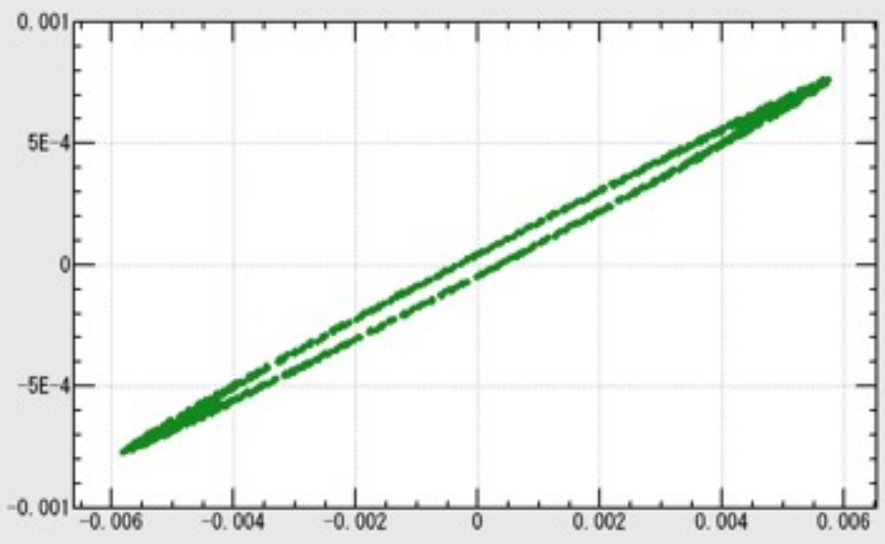
px

y

py

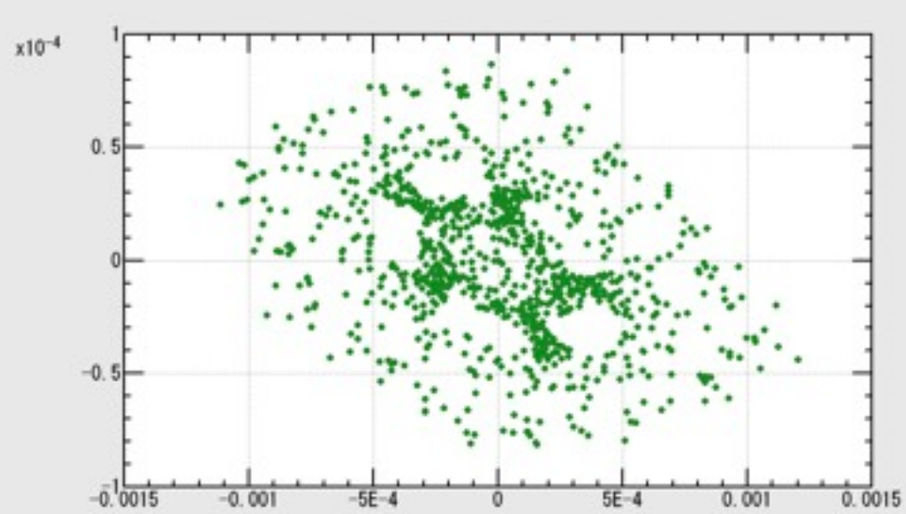


px



x

py



y

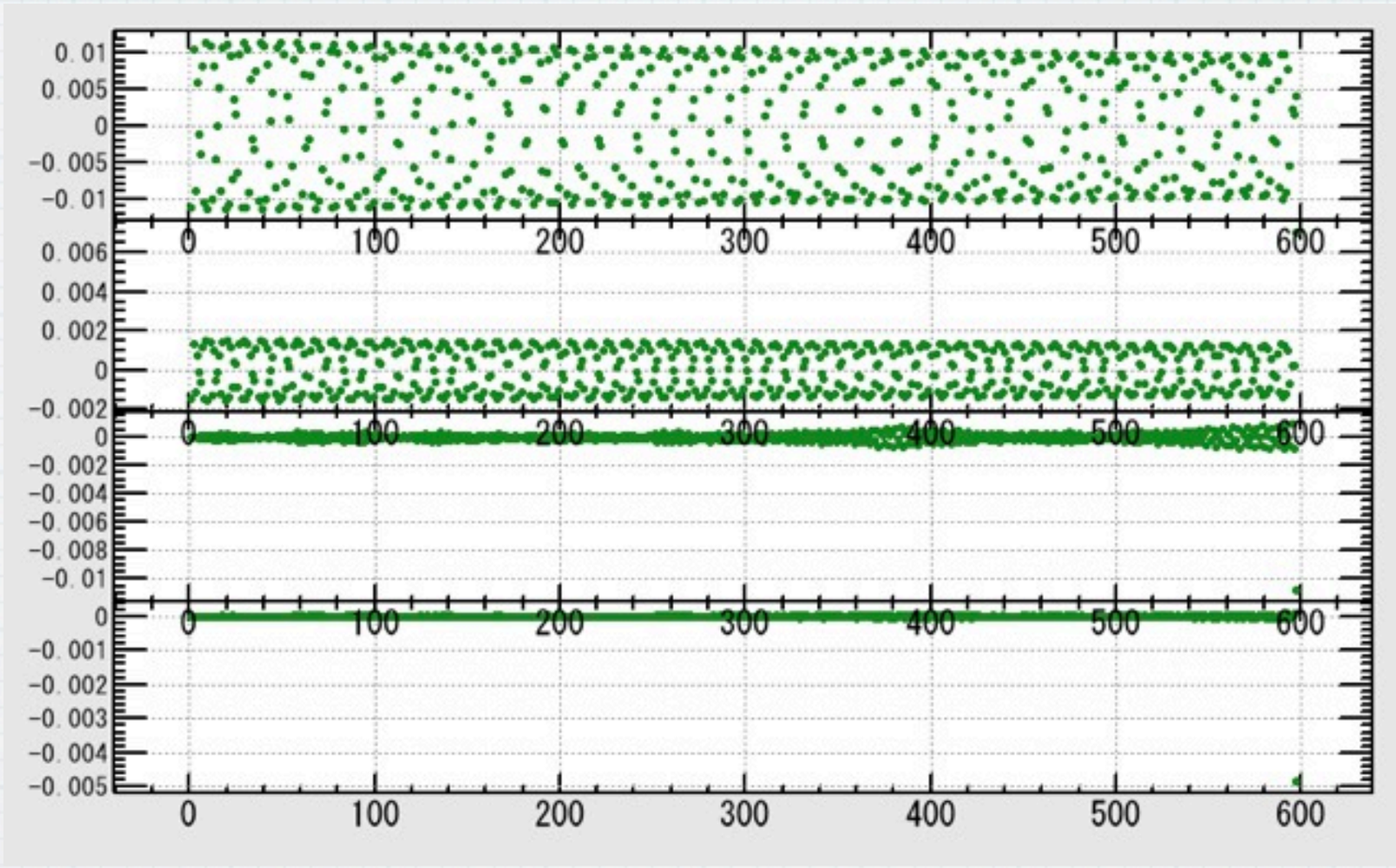
x_0 = 20 sigma, y_0 = 0

x

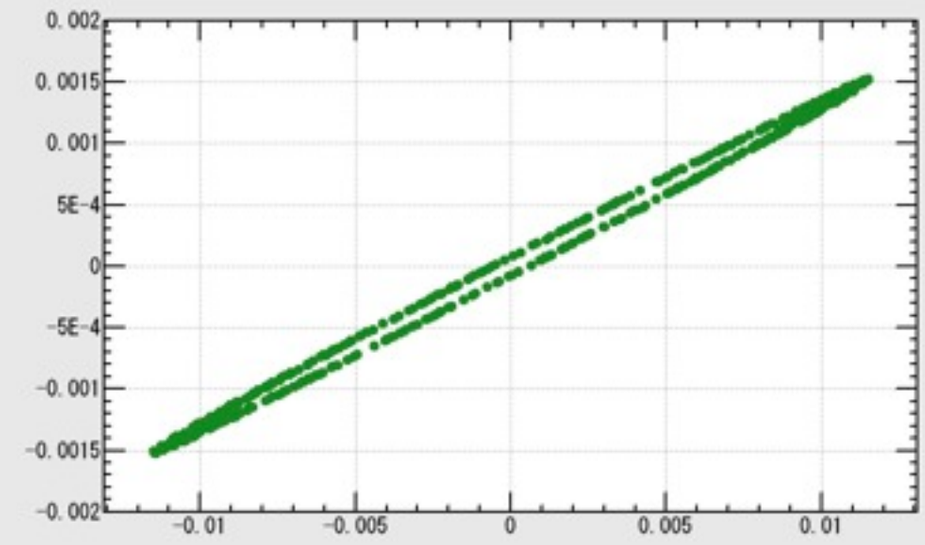
px

y

py

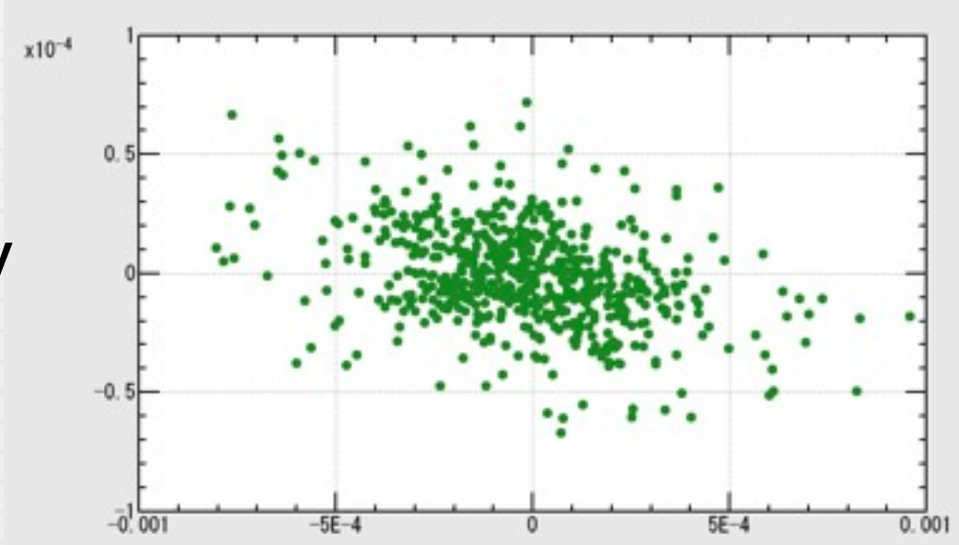


px



x

py



y

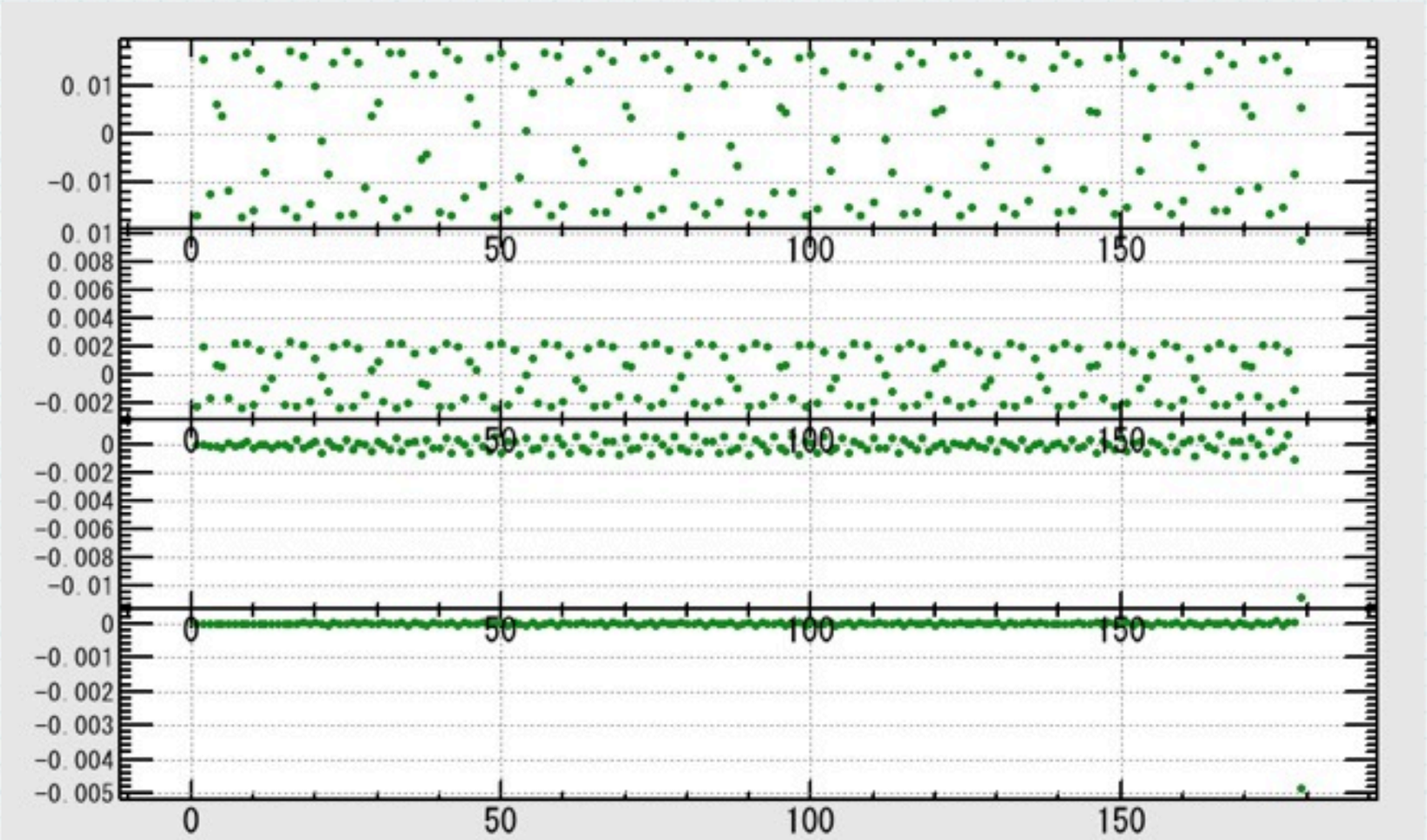
x_0 = 30 sigma, y_0 = 0

x

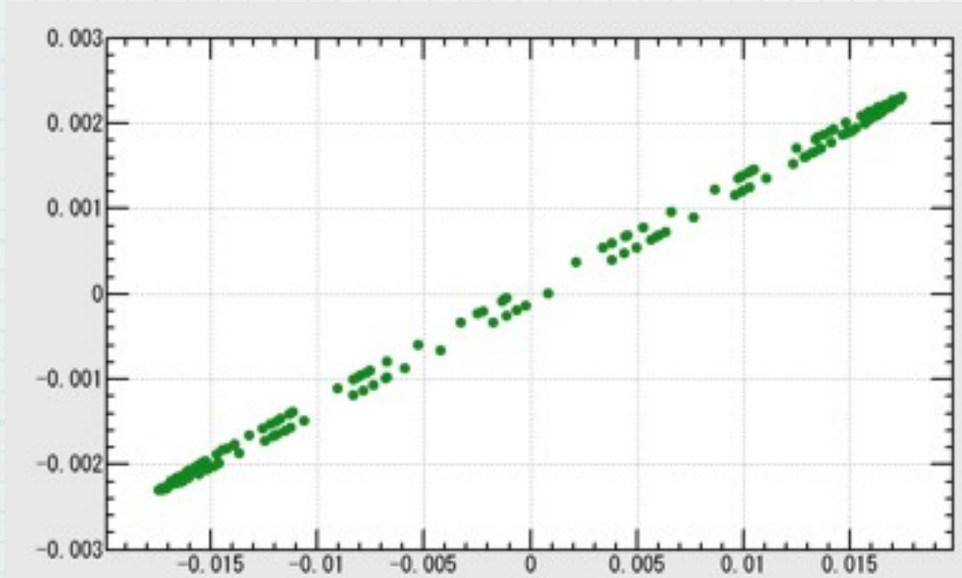
px

y

py

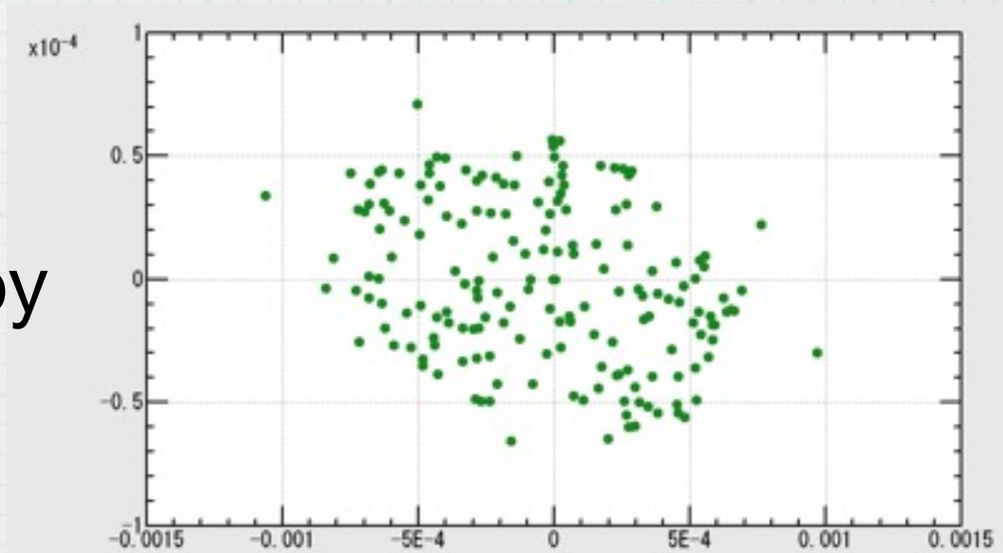


px



x

py



y

Summary

- Nano-beamにおけるビーム・ビーム付きのトラッキングでは、水平振幅は進行方向の位置のずれに変換される。

$$\Delta z = \frac{\Delta x}{\phi_x} \qquad \Delta x = 30\sigma_{x0} \rightarrow \Delta z = 7.3\ mm$$

$$\beta_y(\Delta z) = 0.2\ m$$

- 垂直ベータ関数の大きいところでキックを受けると垂直振動が誘起されて増大する。アパーチャーから外れると失われる。
- クラブ・ウエストは、この現象を緩和できる（が、別の問題がある）。