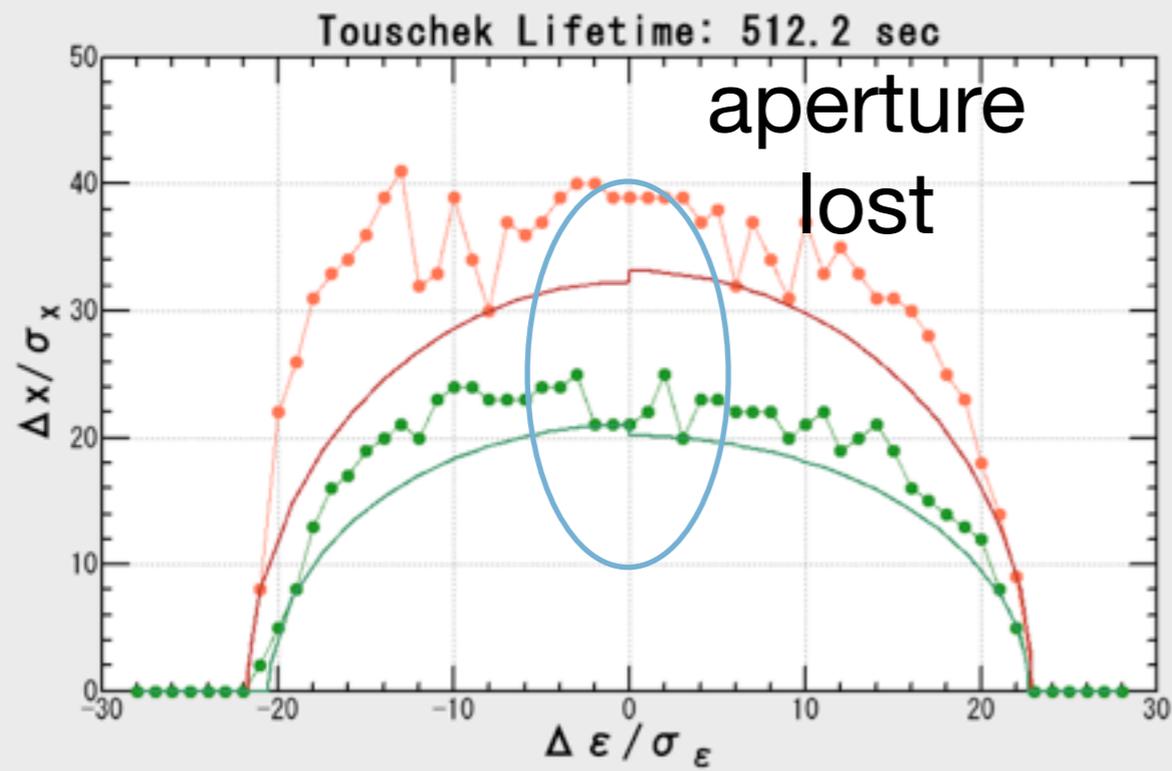


Dynamic Aperture with Beam-Beam

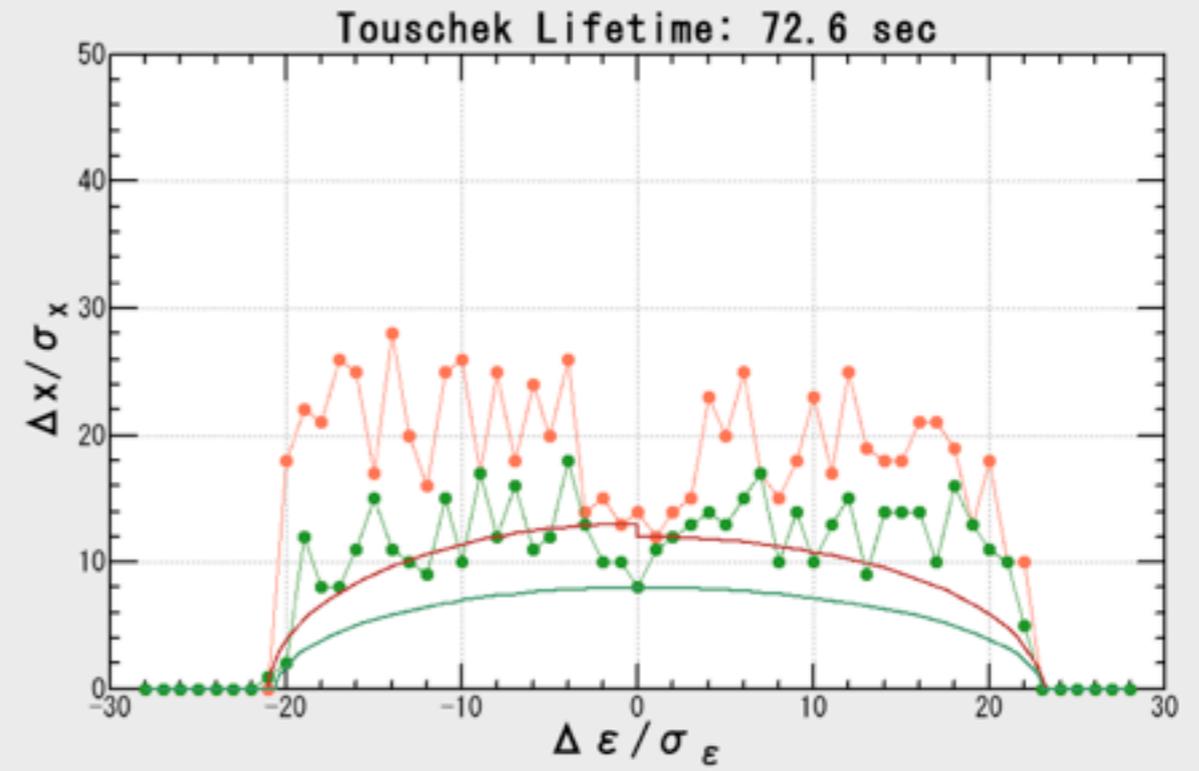
Y. OHNISHI

LER Dynamic Aperture

beam-beam



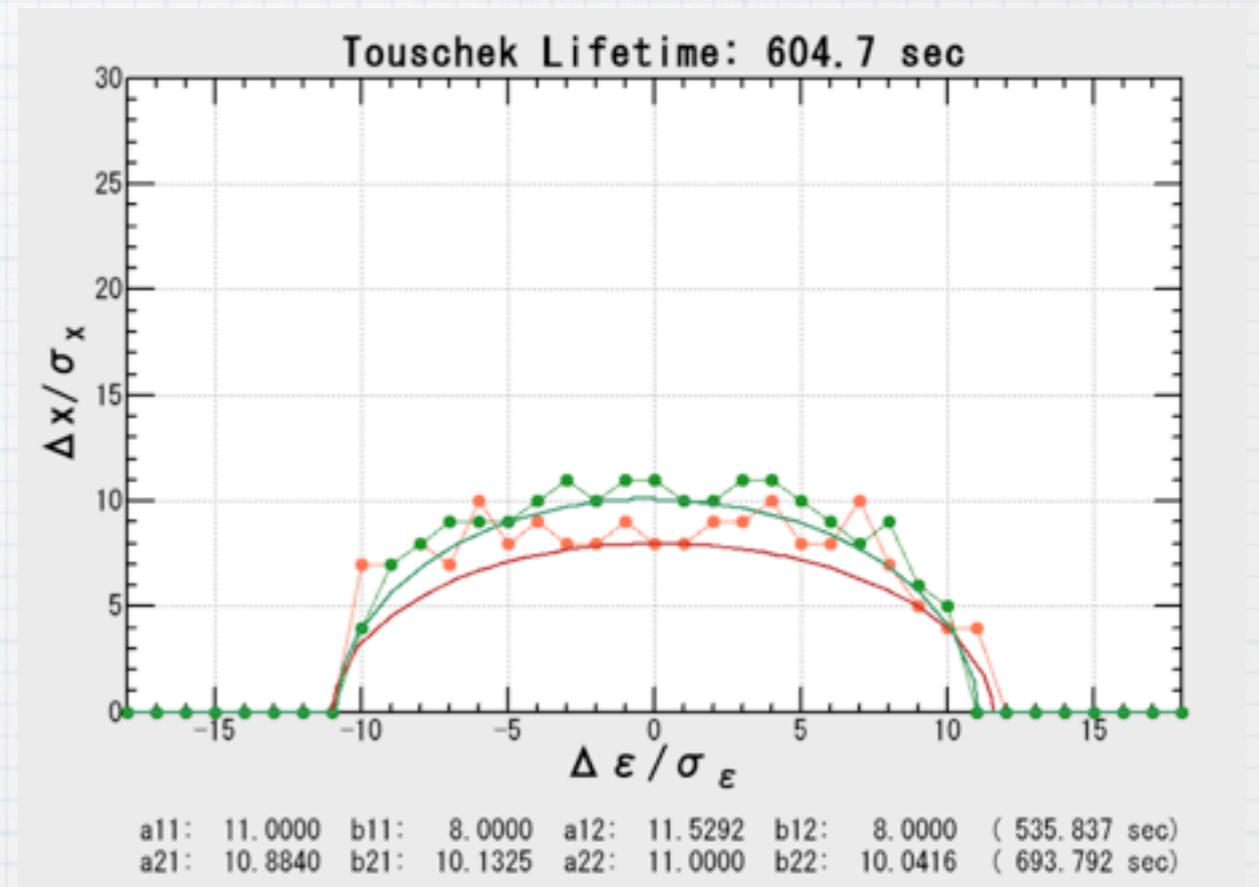
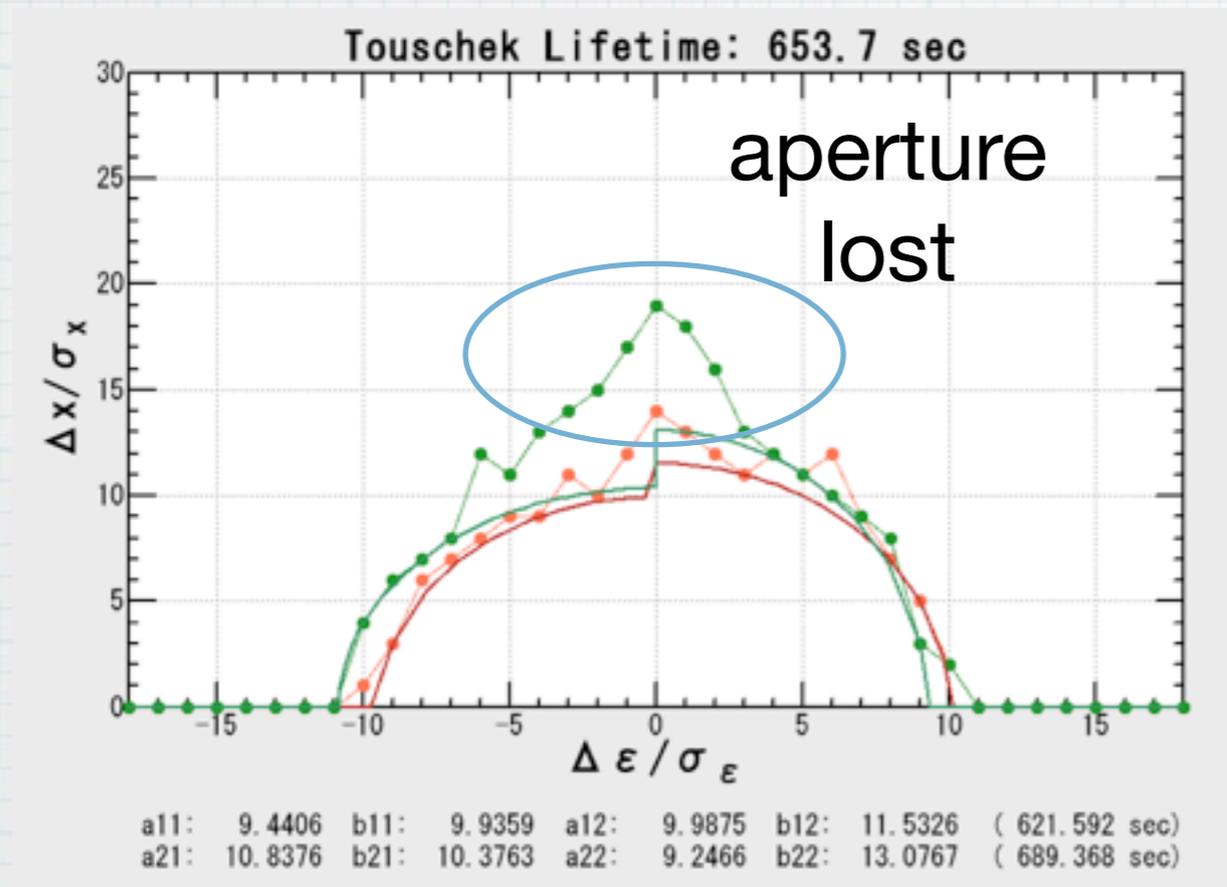
a11:	21.6763	b11:	32.2788	a12:	22.8579	b12:	33.1629	(831.862 sec)
a21:	20.5495	b21:	21.0000	a22:	22.7083	b22:	20.1768	(370.058 sec)



a11:	21.0000	b11:	13.0148	a12:	23.0000	b12:	12.0114	(133.652 sec)
a21:	20.6559	b21:	8.0000	a22:	23.0000	b22:	8.0000	(49.8143 sec)

HER Dynamic Aperture

beam-beam



Crab Waist

- Hamiltonian

$$H_3 = k_x X^3 + k_y XY^2$$

$$H_{3,crab} = k_y XY^2$$

$$= k_y (X^* \cos \Delta\psi_x + P_X^* \sin \Delta\psi_x) (Y^* \cos \Delta\psi_y + P_Y^* \sin \Delta\psi_y)^2$$

$$= k_y \cos(m\pi) X^* P_Y^{*2}$$

$$= -\frac{K_2 \cos(m\pi)}{2} \beta_{y,s} \beta_y^* \sqrt{\frac{\beta_{x,s}}{\beta_x^*}} x^* p_y^{*2}$$

$$= -\frac{x^* p_y^{*2}}{2 \tan 2\phi_x}$$

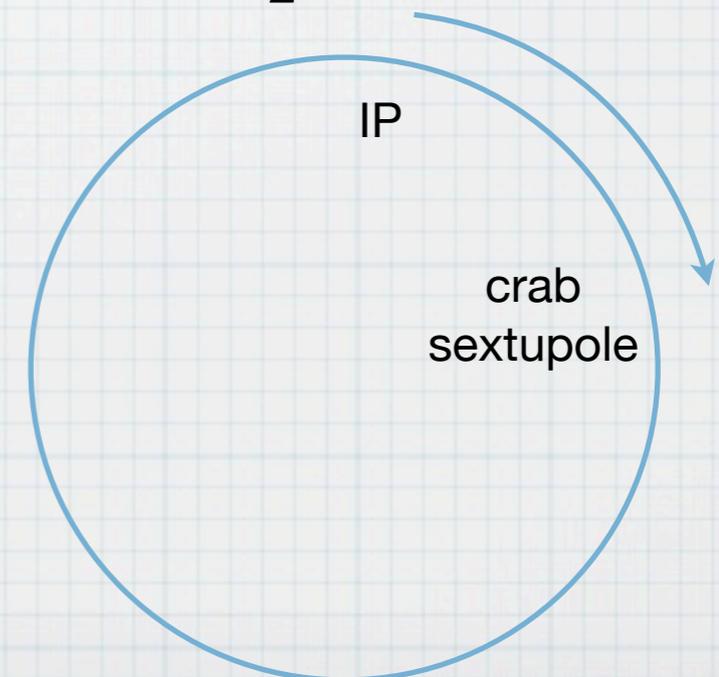
$$K_2 = \frac{1}{\cos(m\pi) \tan 2\phi_x \beta_{y,s} \beta_y^*} \sqrt{\frac{\beta_x^*}{\beta_{x,s}}}$$

$$k_x = \frac{\beta_{x,s}^{3/2} K_2}{6}$$

$$k_y = -\frac{\sqrt{\beta_{x,s} \beta_{y,s}} K_2}{2}$$

$$\Delta\psi_x = m\pi$$

$$\Delta\psi_y = \frac{\pi}{2} + n\pi$$



Crab Waist

- 衝突点にHamiltonianによる変換を入れる。

$$H_{IP} = -\frac{a}{2}xp_y^2$$
$$a = -\frac{1}{\tan 2\phi} \quad (\text{nominal})$$

- Hamilton 方程式

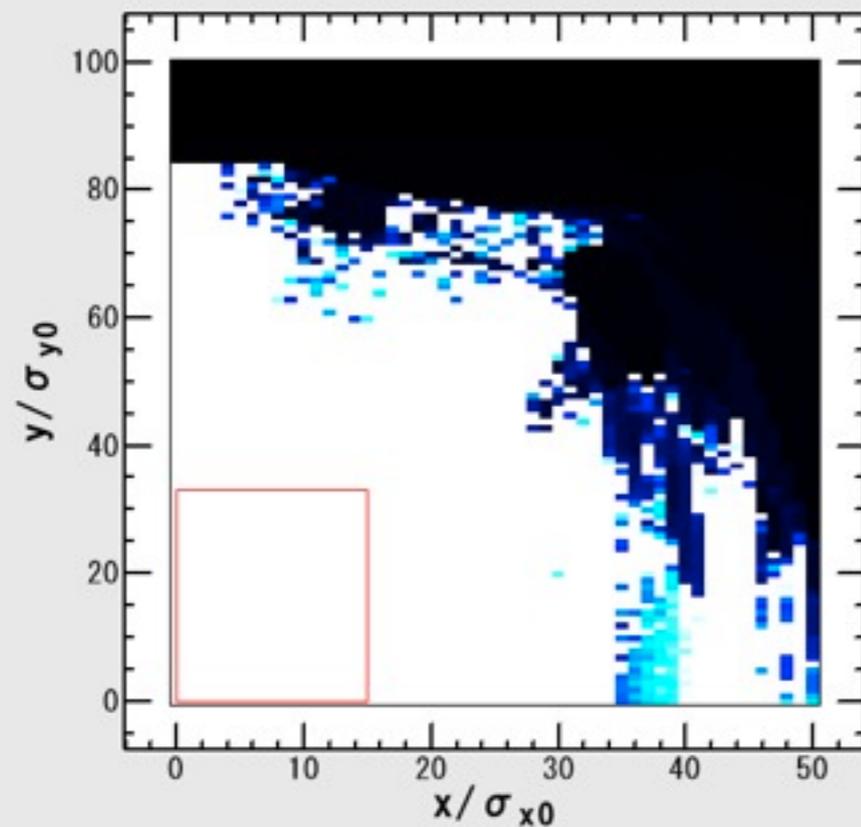
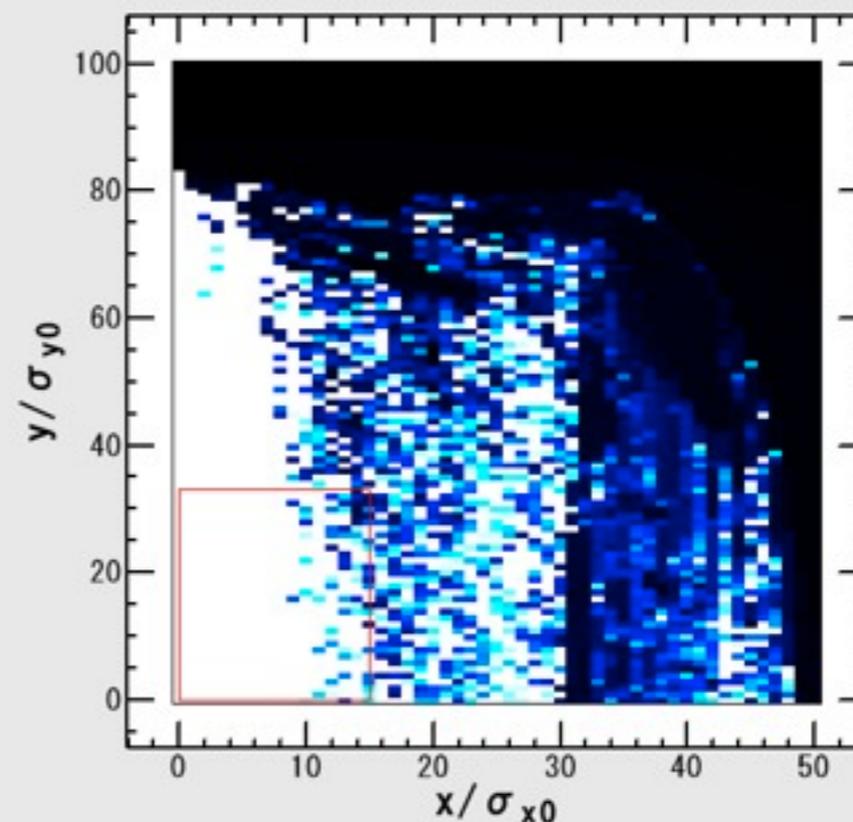
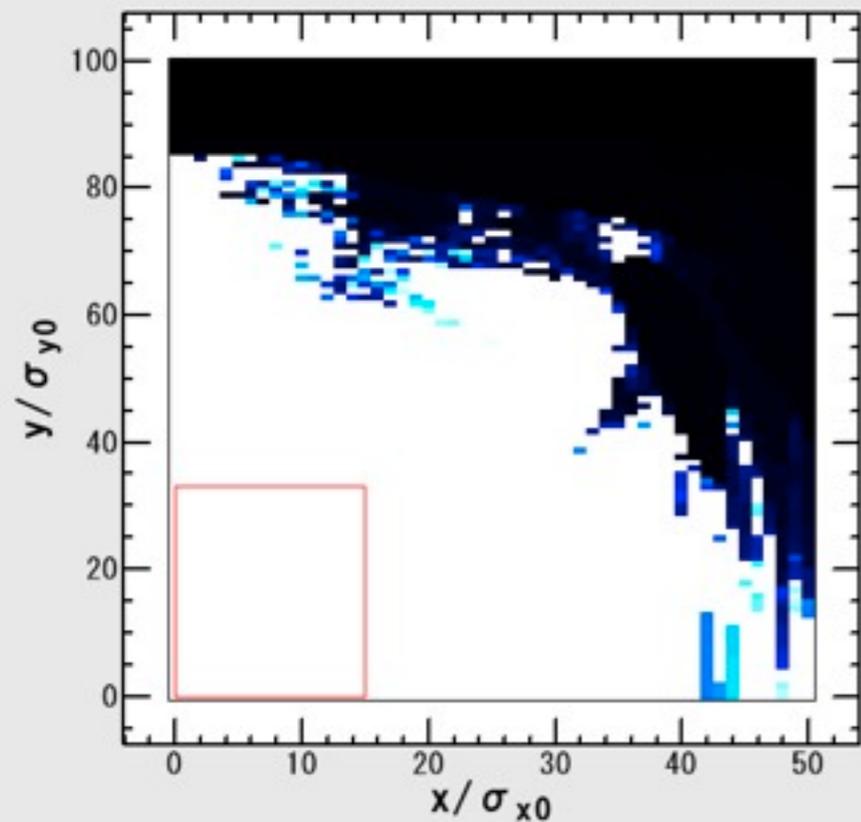
$$\frac{dx}{ds} = \frac{\partial H_{IP}}{\partial p_x} = 0$$
$$\frac{dp_x}{ds} = -\frac{\partial H_{IP}}{\partial x} = \frac{a}{2}p_y^2$$
$$\frac{dy}{ds} = \frac{\partial H_{IP}}{\partial p_y} = -axp_y$$
$$\frac{dp_y}{ds} = -\frac{\partial H_{IP}}{\partial p_y} = 0$$

$$p_x \rightarrow p_x + \frac{a}{2}p_y^2$$
$$y \rightarrow y - axp_y$$

左の変換をビームビーム相互作用の前に、
その逆変換を後に置く。

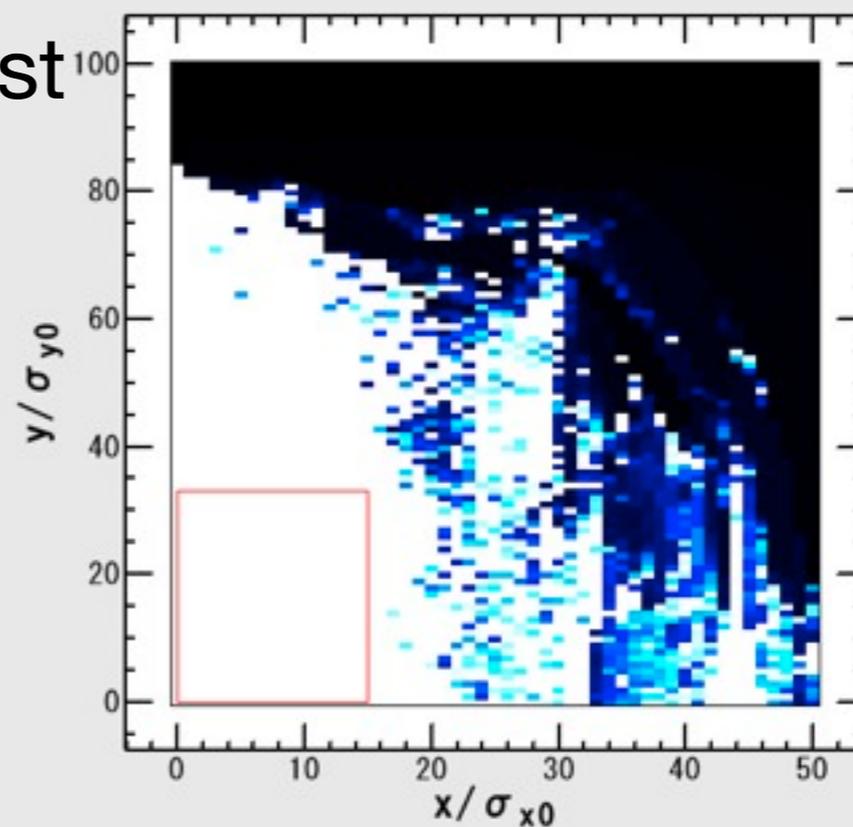
LER Dynamic Aperture

beam-beam



crab waist

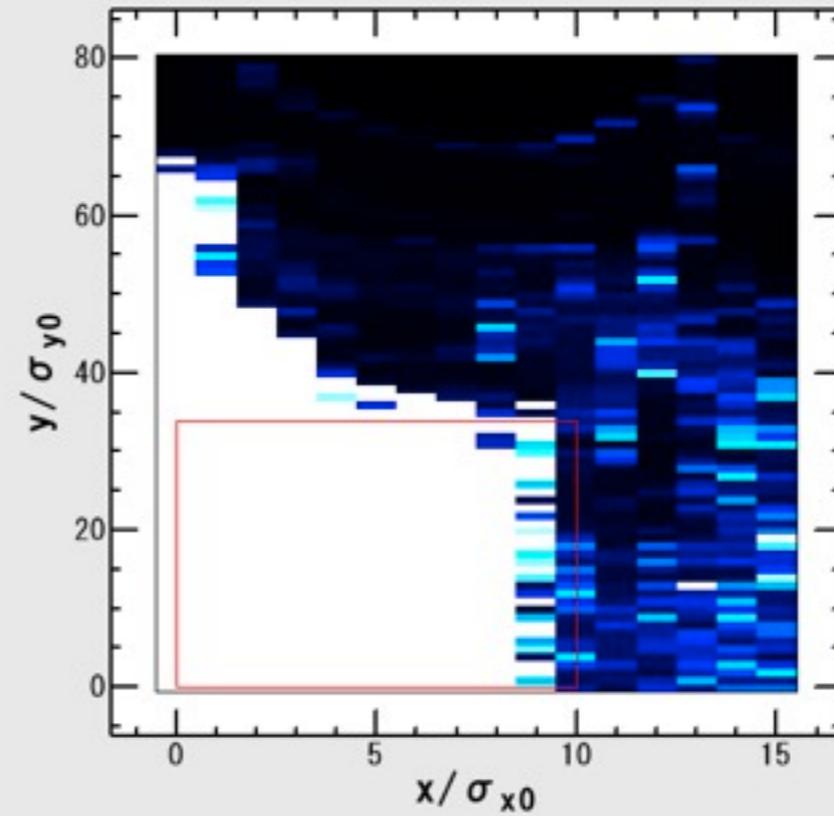
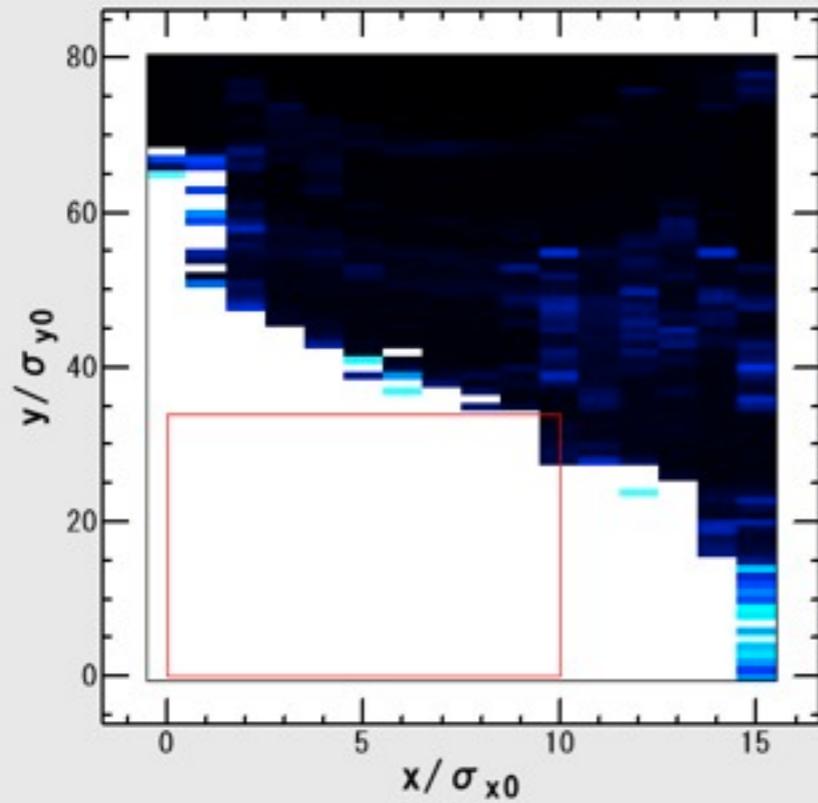
full



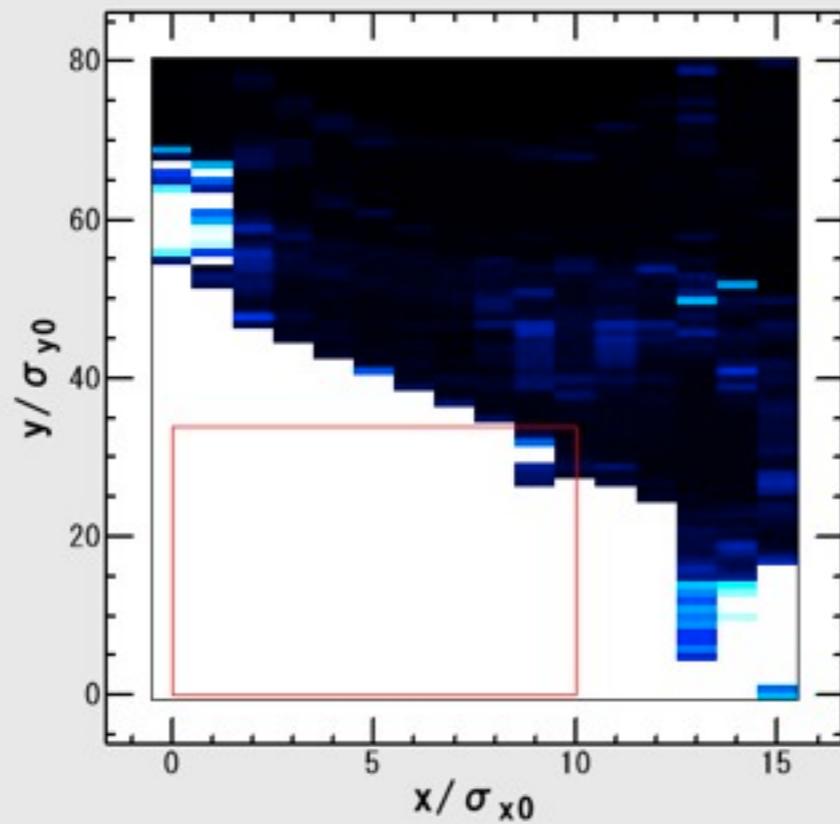
half

HER Dynamic Aperture

beam-beam



crab waist



full

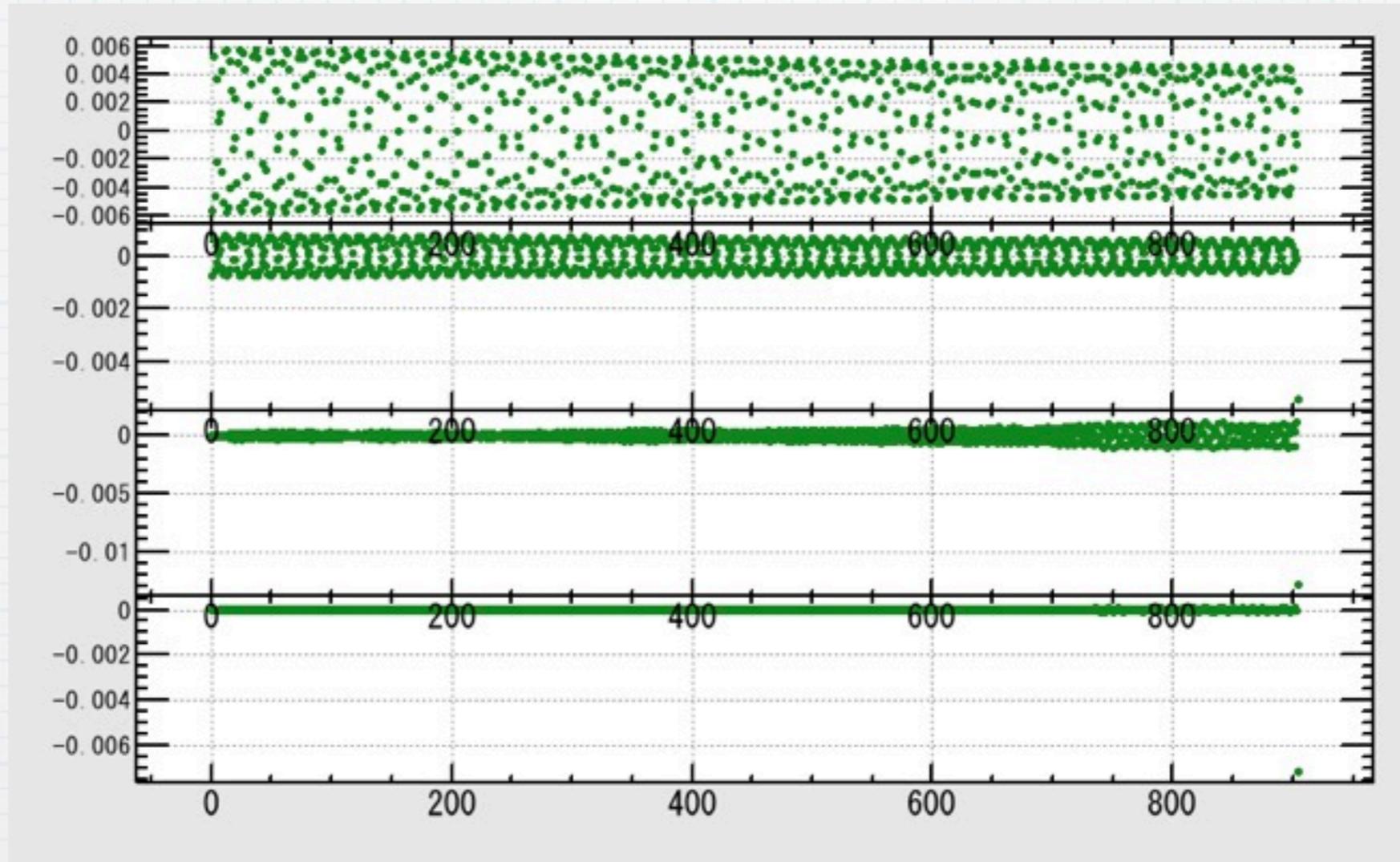
$x_0 = 10 \text{ sigma}, y_0 = 0$

x

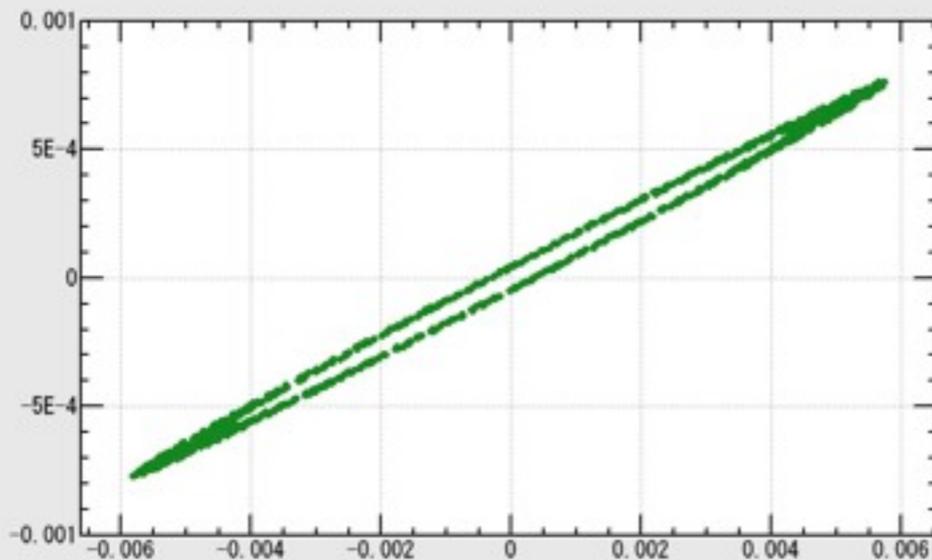
px

y

py

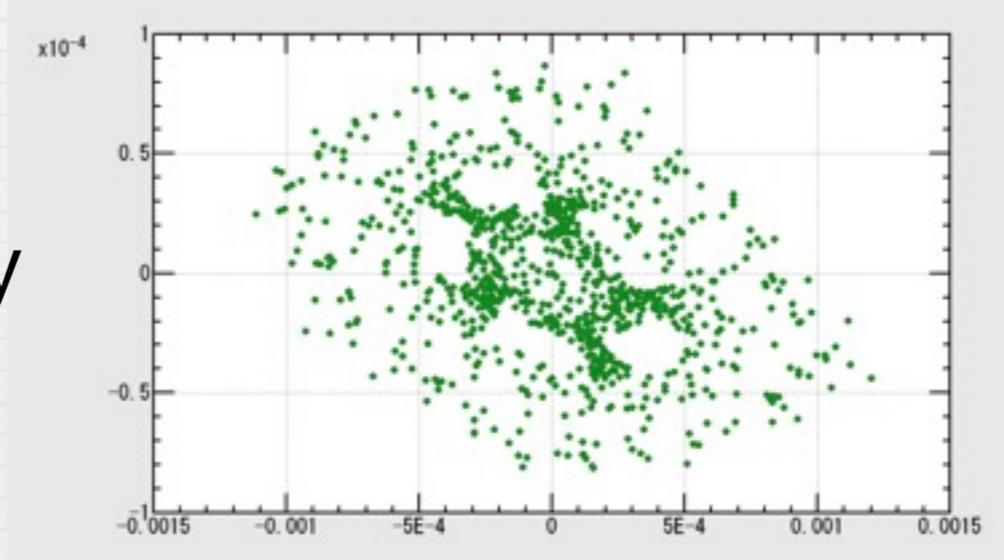


px



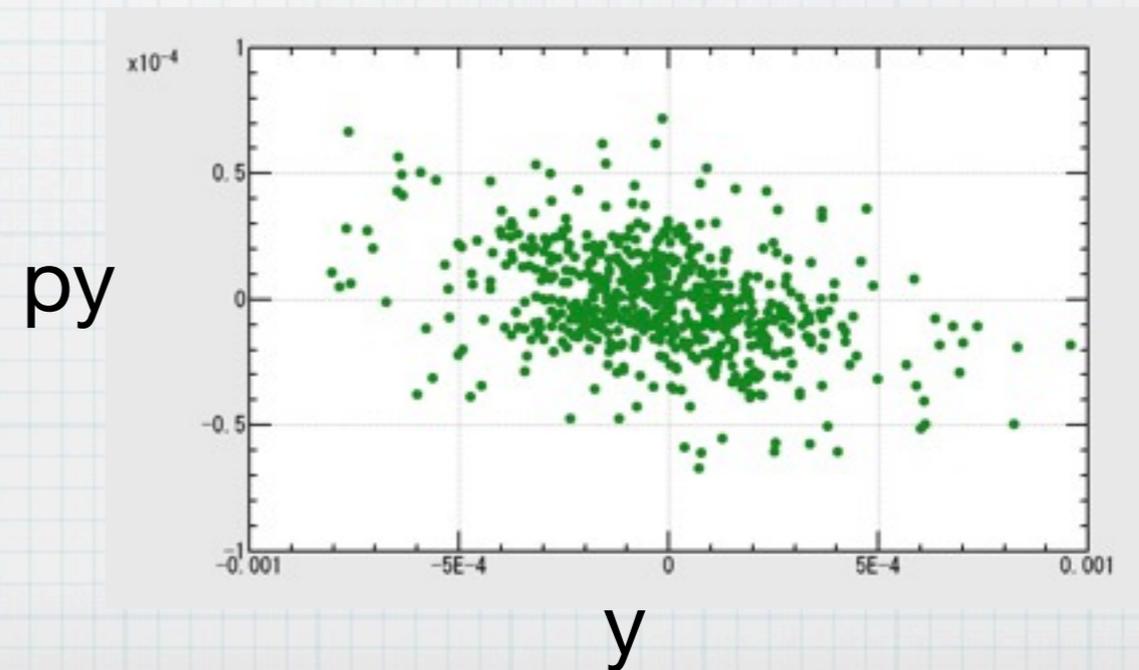
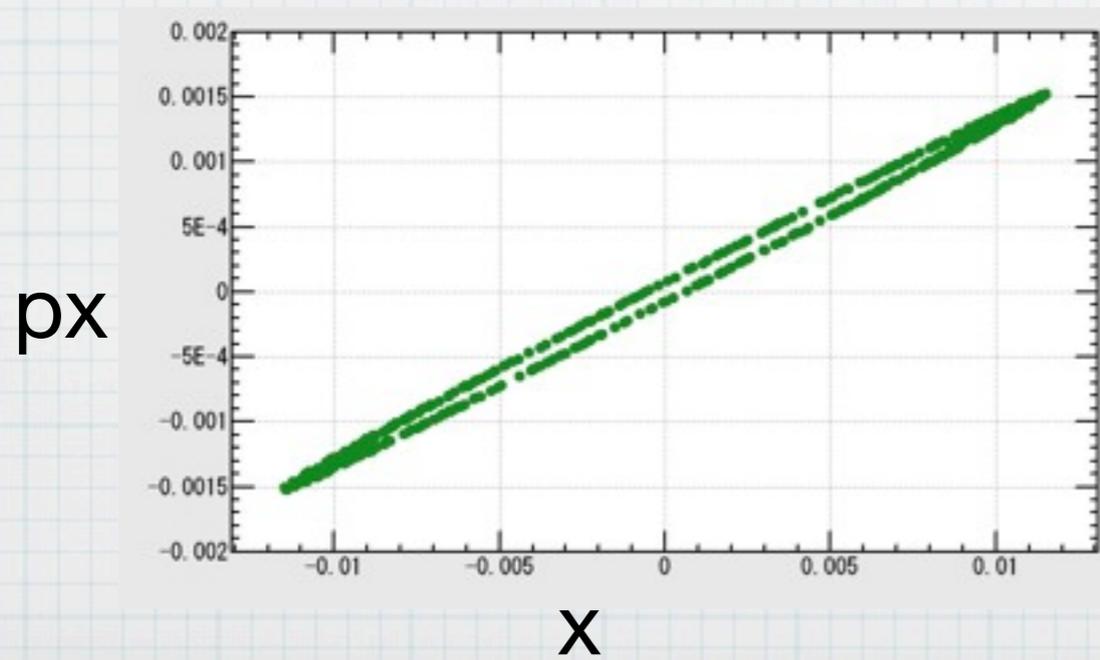
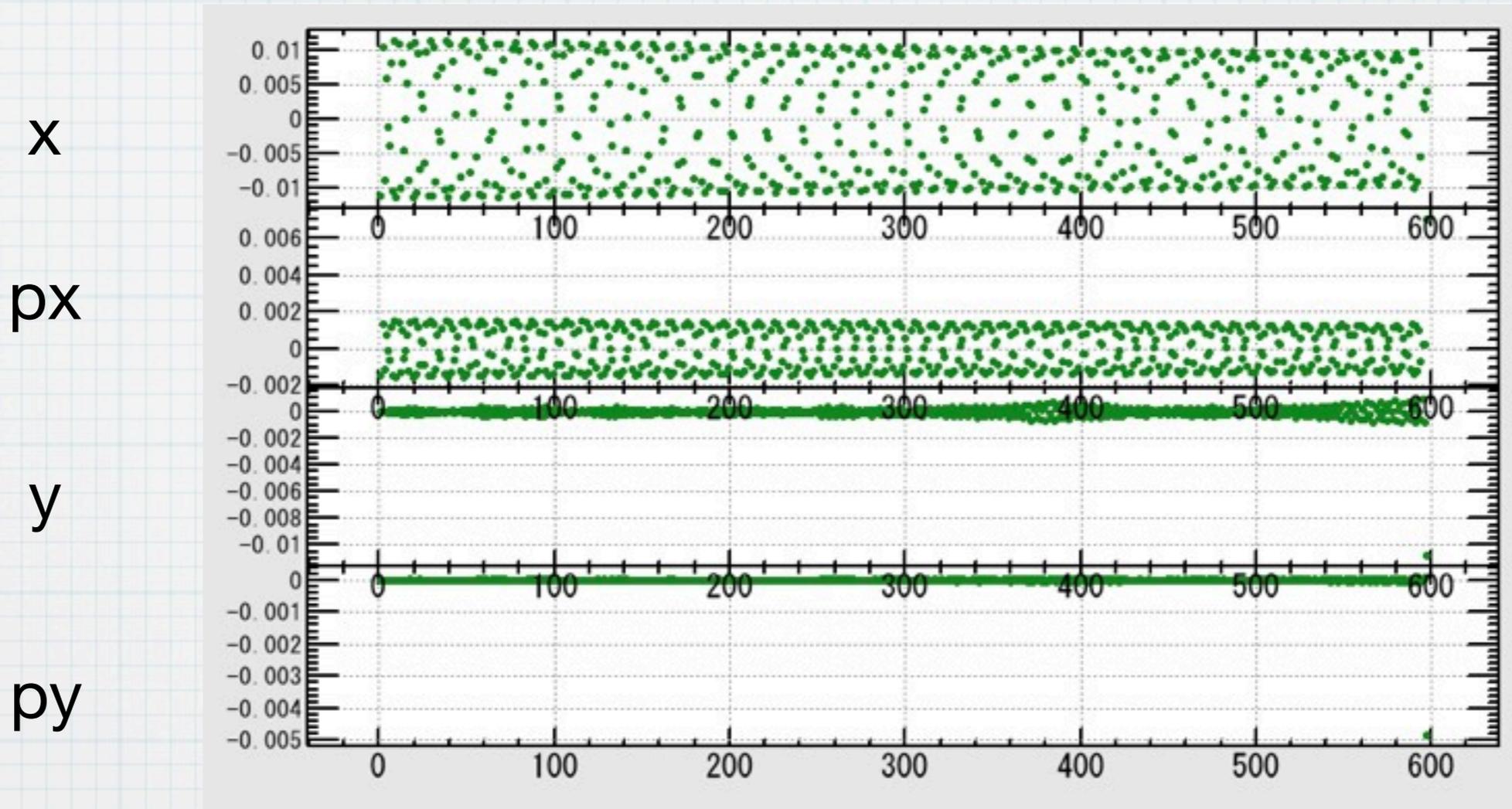
x

py

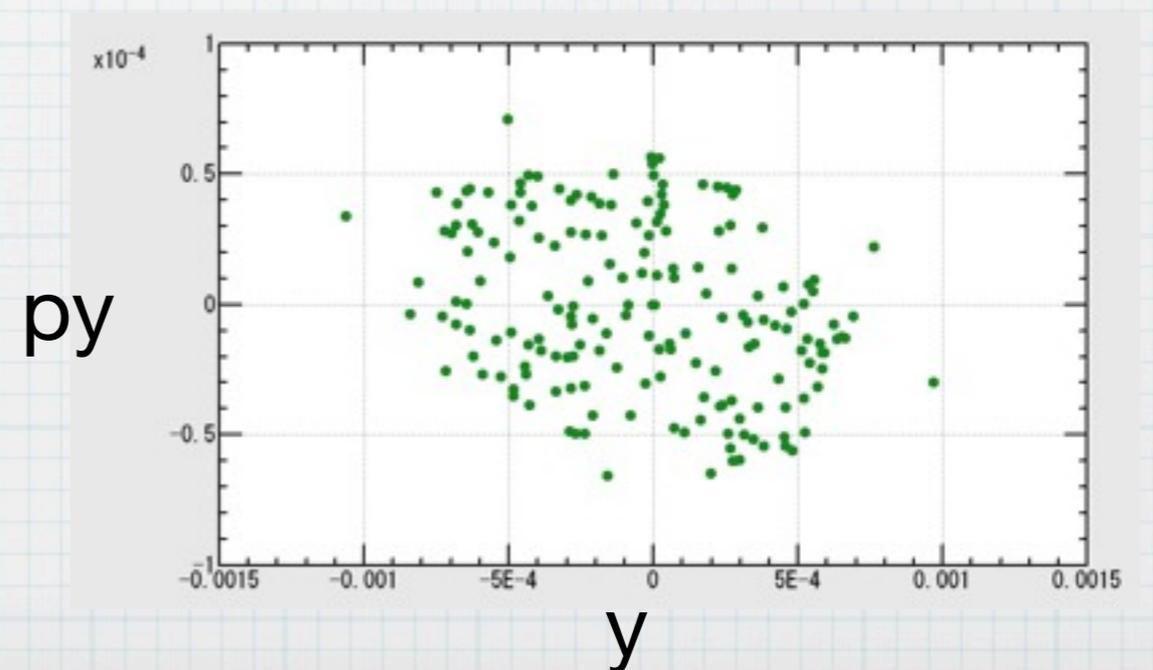
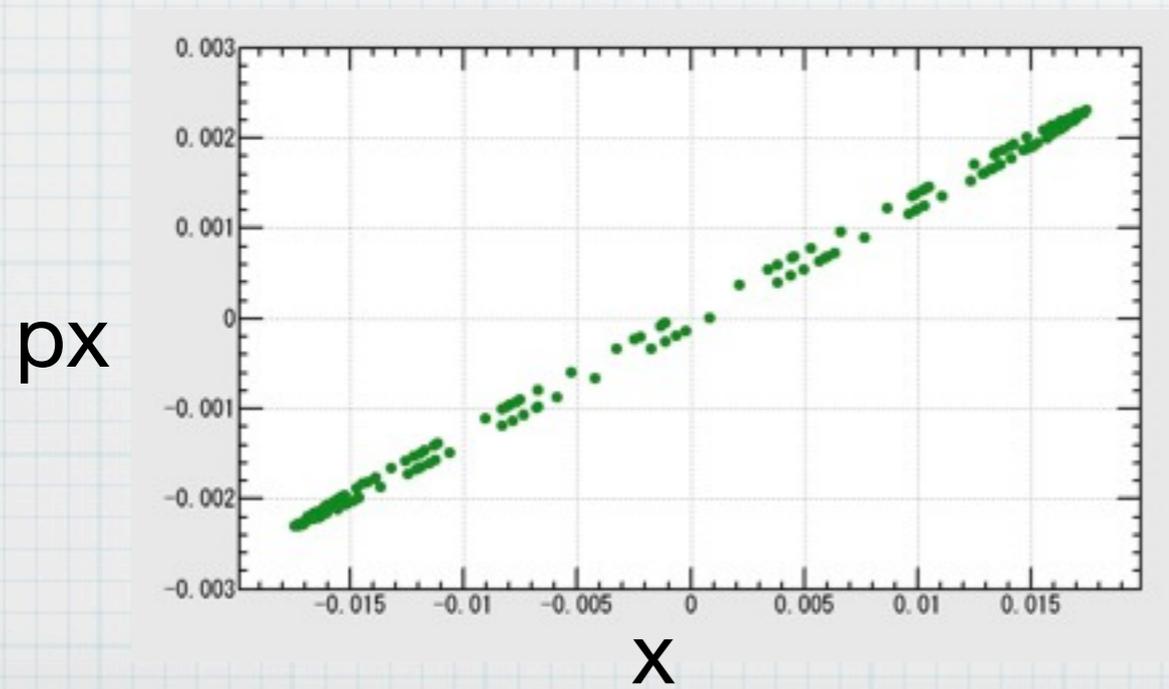
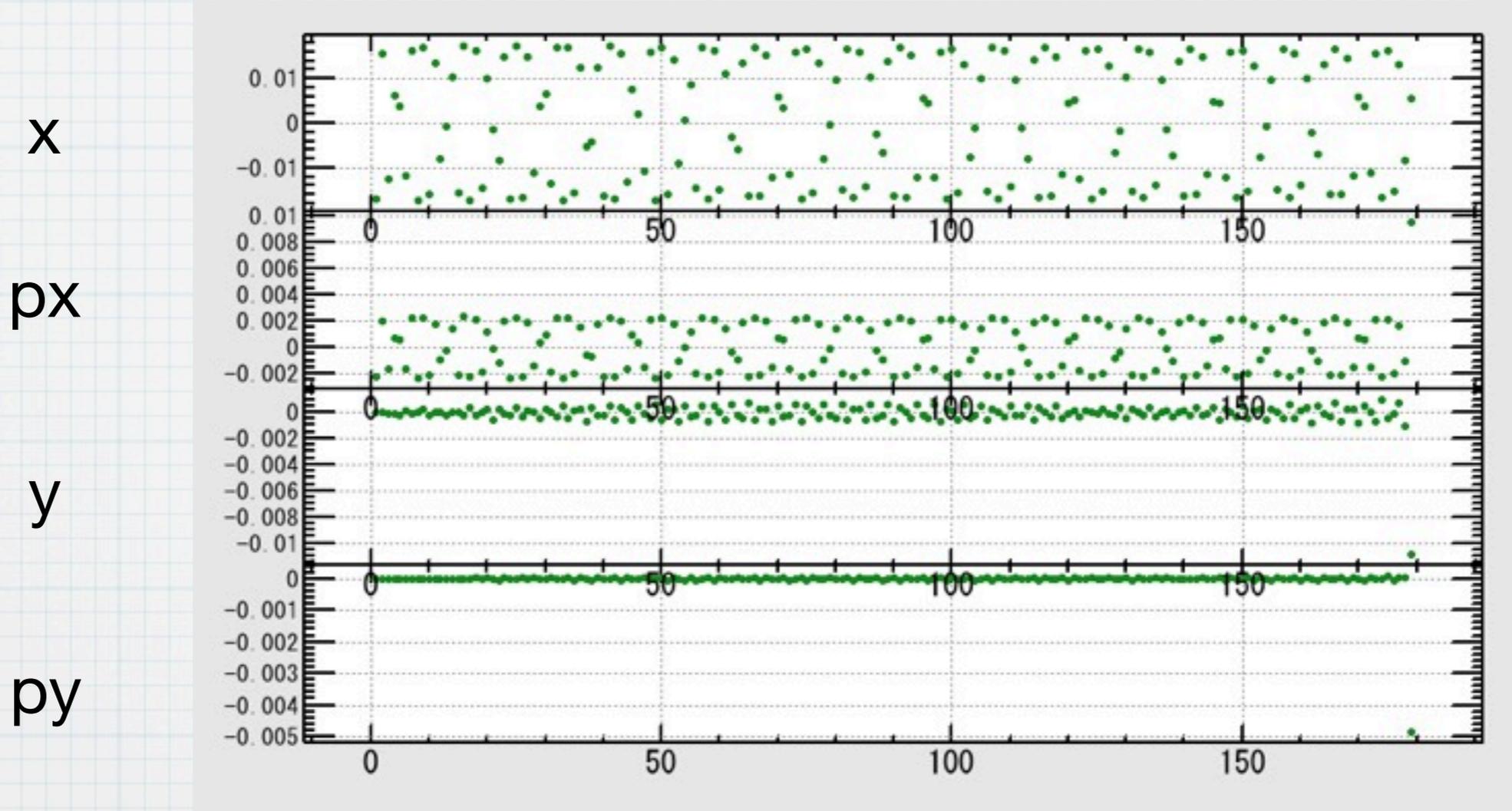


y

$x_0 = 20 \text{ sigma}, y_0 = 0$



$x_0 = 30 \text{ sigma}, y_0 = 0$



Summary

- Nano-beamにおけるビーム・ビーム付きのトラッキングでは、水平振幅は進行方向の位置のずれに変換される。

$$\Delta z = \frac{\Delta x}{\phi_x} \quad \Delta x = 30\sigma_{x0} \rightarrow \Delta z = 7.3 \text{ mm}$$
$$\beta_y(\Delta z) = 0.2 \text{ m}$$

- 垂直ベータ関数の大きいところでキックを受けると垂直振動が誘起されて増大する。アパーチャーから外れると失われる。
- クラブ・ウエストは、この現象を緩和できる（が、別の問題がある）。