CW beam physics analysis

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With materials from
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Acknowledgements

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Mini-workshop on Accelerator Physics: Key Beam Physics and Technologies Issues for Colliders
Jan. 13, 2022, HKUST IAS

Outline

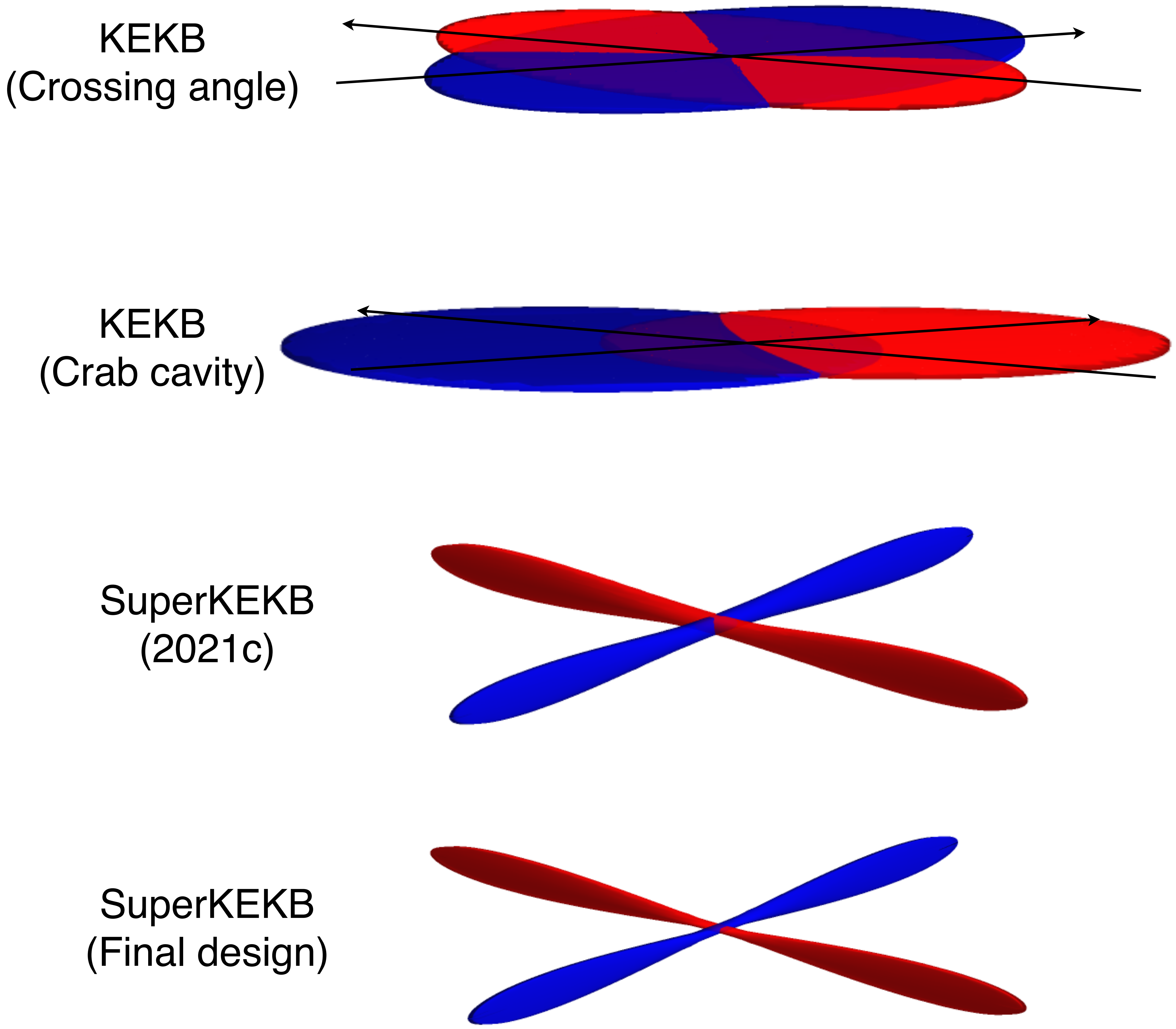
- Ideal crab waist
- Crab waist applied to SuperKEKB
- Crab waist optics design for Super Charm Tau factory
- Summary

Status of SuperKEKB

- Collision scheme (KEKB → SuperKEKB)
  - SuperKEKB: A “green” collider

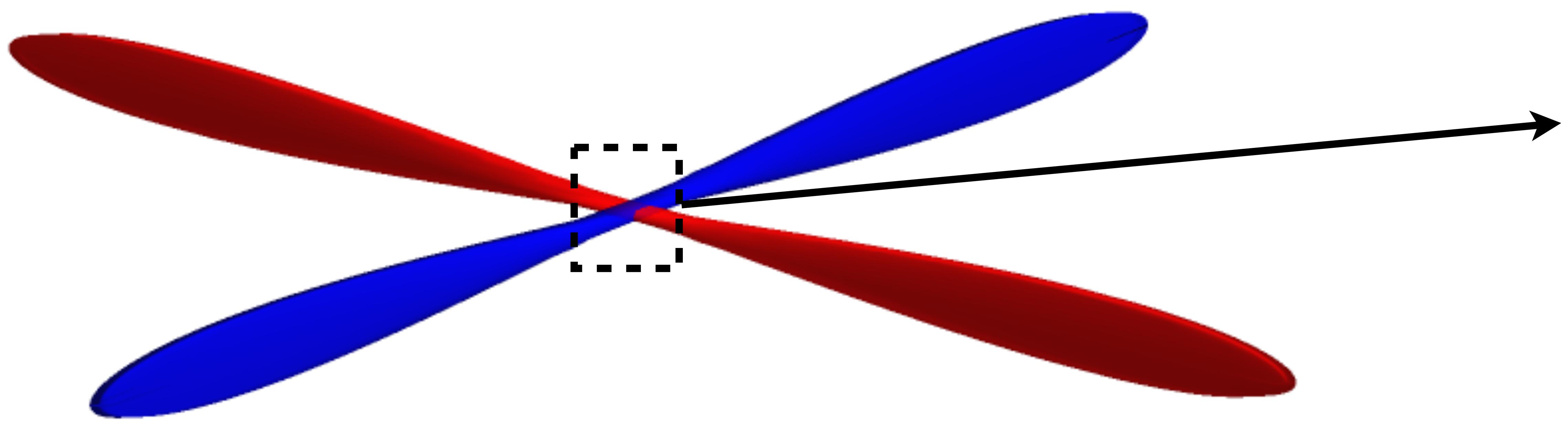
	KEKB (2009.06.17)		SKEKB (2021.12.23)		SKEKB (Final design)	
	HER	LER	HER	LER	HER	LER
I_{beam} (A)	1.2	1.0	0.8	1.02	2.6	3.6
# bunch	1585		1370		2500	
ε_x (nm)	24	18	4.6	4.0	4.6	3.2
ε_y (pm)	150	150	~50	~50	12.9	8.64
β_x (mm)	1200	1200	60	80	25	32
β_y (mm)	5.9	5.9	1	1	0.3	0.27
σ_z (mm)	6	6	5	6	5	6
ν_x	44.511	45.506	45.53	44.524	45.53	44.53
ν_y	41.585	43.561	43.52	46.589	43.57	46.57
ν_s	0.0209	0.0246	0.0272	0.0233	0.028	0.0245
Crab waist	-		40%	80%	-	
Crossing angle (mrad)	0 (22)		83		83	
Luminosity (10^{34} $cm^{-2}s^{-1}$)	2.1		3.81		80	

Schematic view of collision schemes

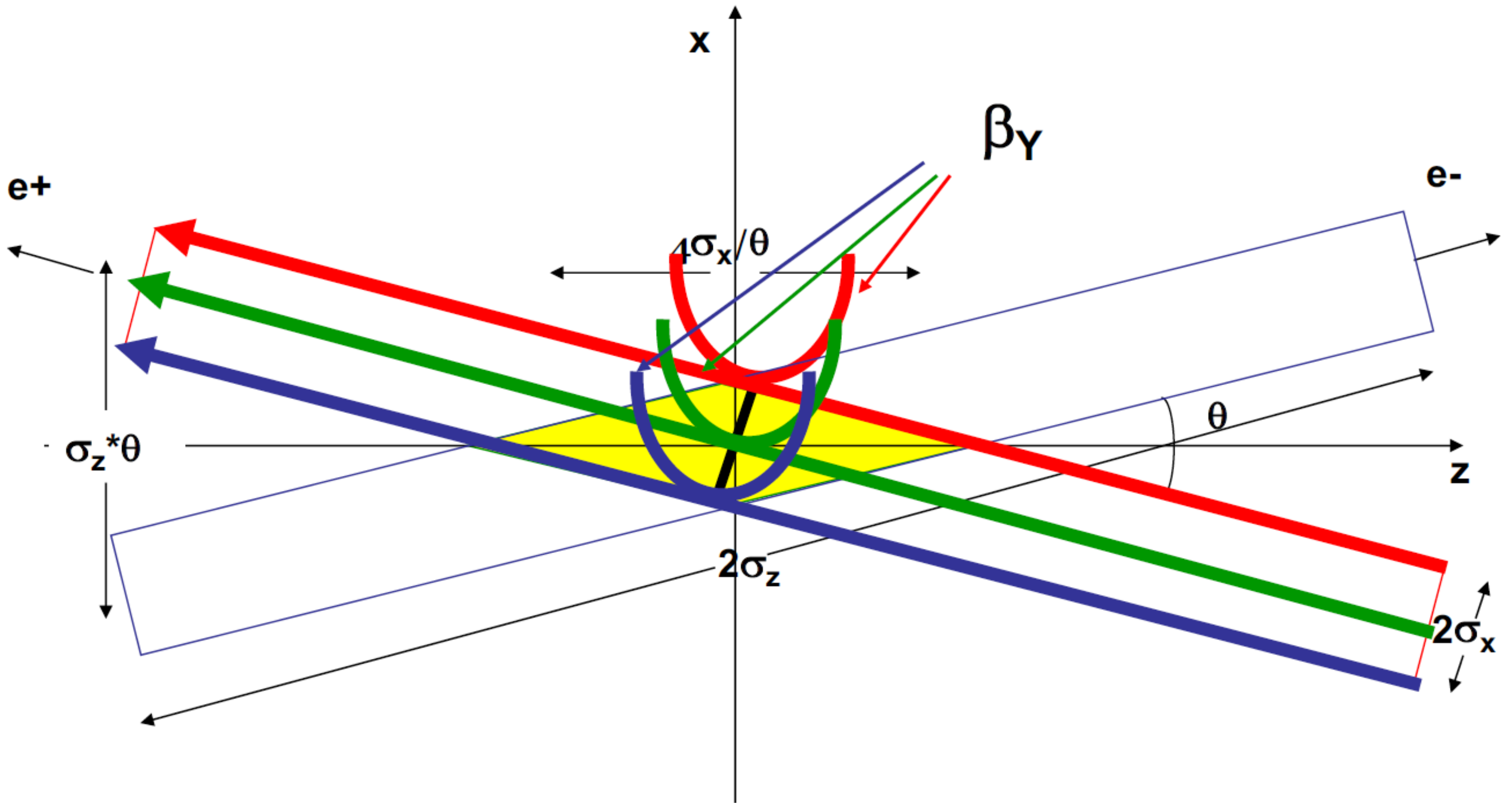


Ideal crab waist

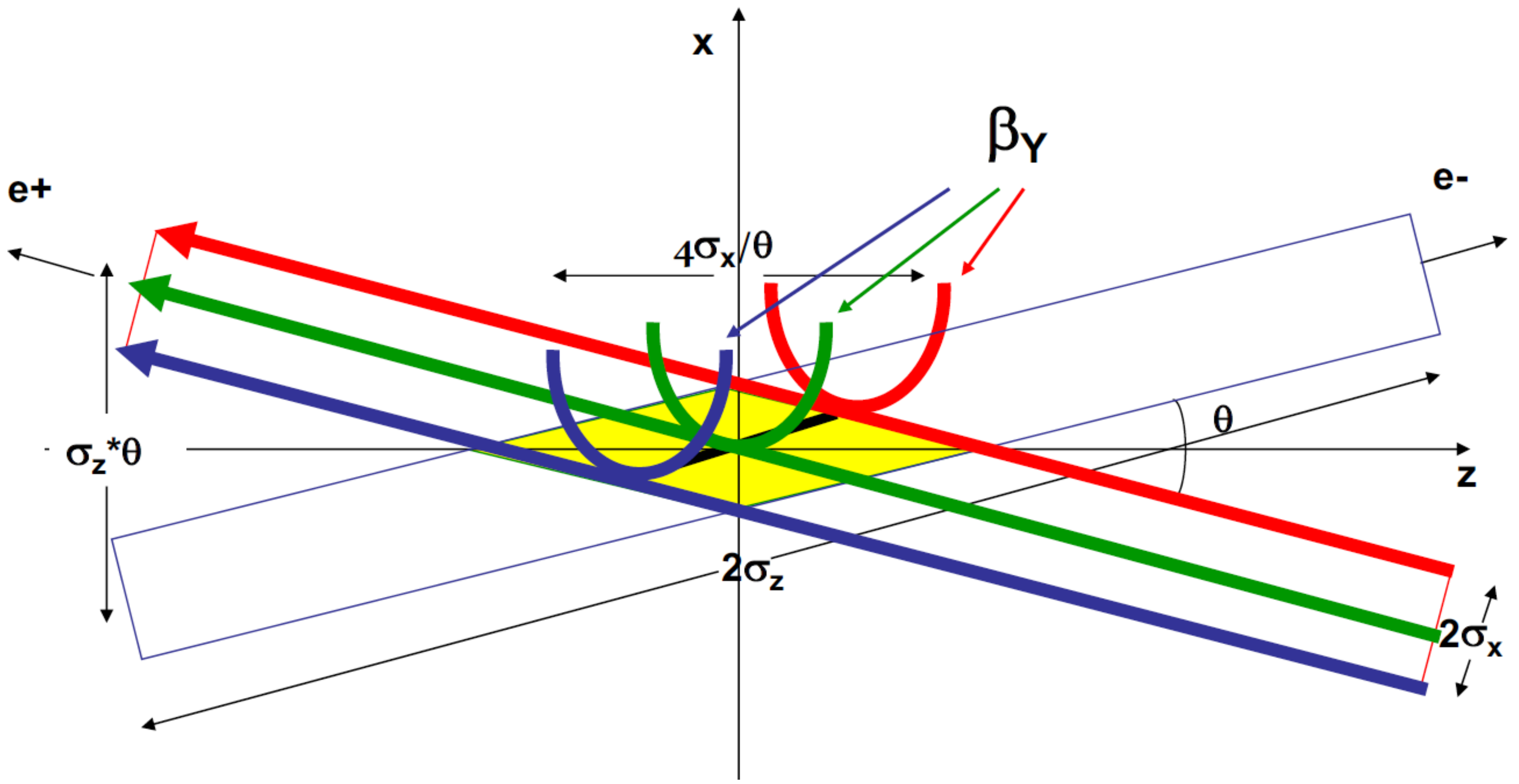
- Crab waist is used to manipulate the overlapped region of colliding beams
  - To alleviate nonlinear beam-beam effects (specially beam-beam resonances)



w/o crab waist



w/ crab waist

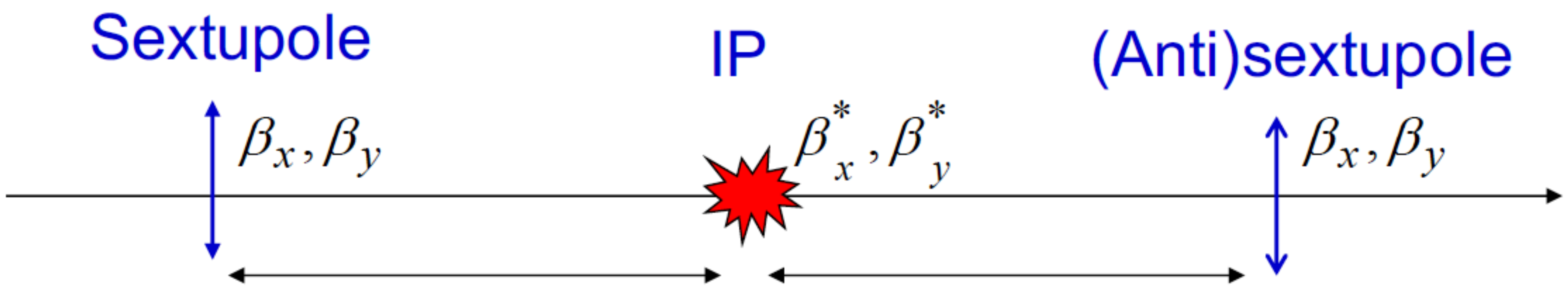


Ref.[1] P. Raimondi, 2nd SuperB Workshop, March 2006

Ref.[2] M. Zobov, EIC Workshop, Oct. 8, 2020.

Ideal crab waist

- Ideal crab waist requires a transparent interaction region
  - Linear transfer map with perfect phase advance between the CW sextupole and anti-sextupole.



$$\Delta\mu_y = \frac{\pi}{2} \qquad \Delta\mu_y = \frac{\pi}{2}$$

$$\Delta\mu_x = \pi \qquad \Delta\mu_x = \pi$$

Sextupole strength

$$K = \frac{1}{2\theta}\frac{1}{\beta^*_y \beta_y}\sqrt{\frac{\beta^*_x}{\beta_x}}$$

Equivalent Hamiltonian

$$H = H_0 + \frac{1}{2\theta} x p_y^2$$

$$\boxed{\beta_y = \beta^*_y + \frac{(s - x/\theta)^2}{\beta^*_y}}$$

Ref.[2] M. Zobov, EIC Workshop, Oct. 8, 2020.

Ideal crab waist

- Beam-beam resonances
  - X-Y resonances can be well suppressed by ideal crab waist.
  - Note that X-Z synchro-betatron resonances cannot be fully suppressed by crab waist when $\Phi \gg 1$.

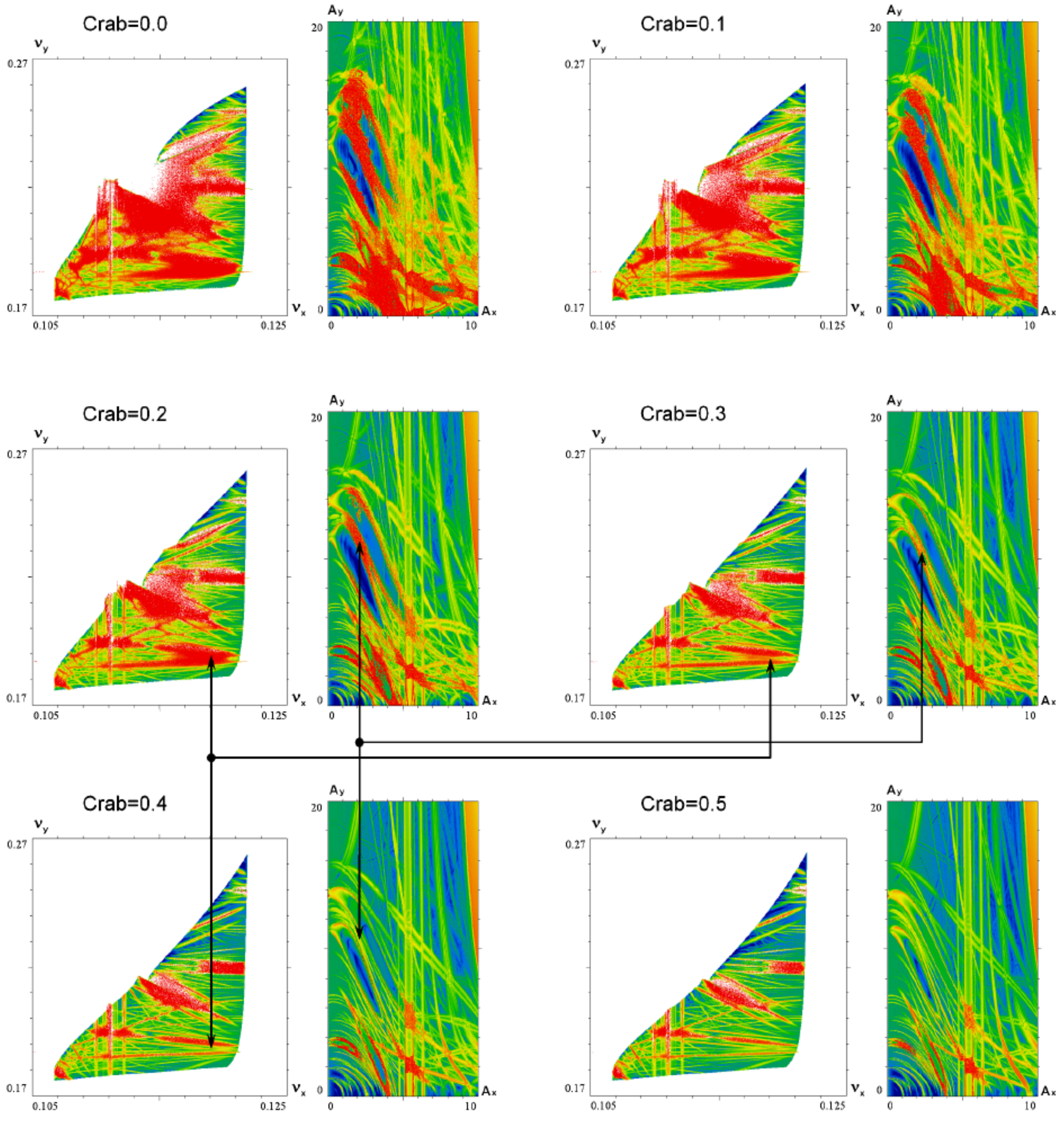
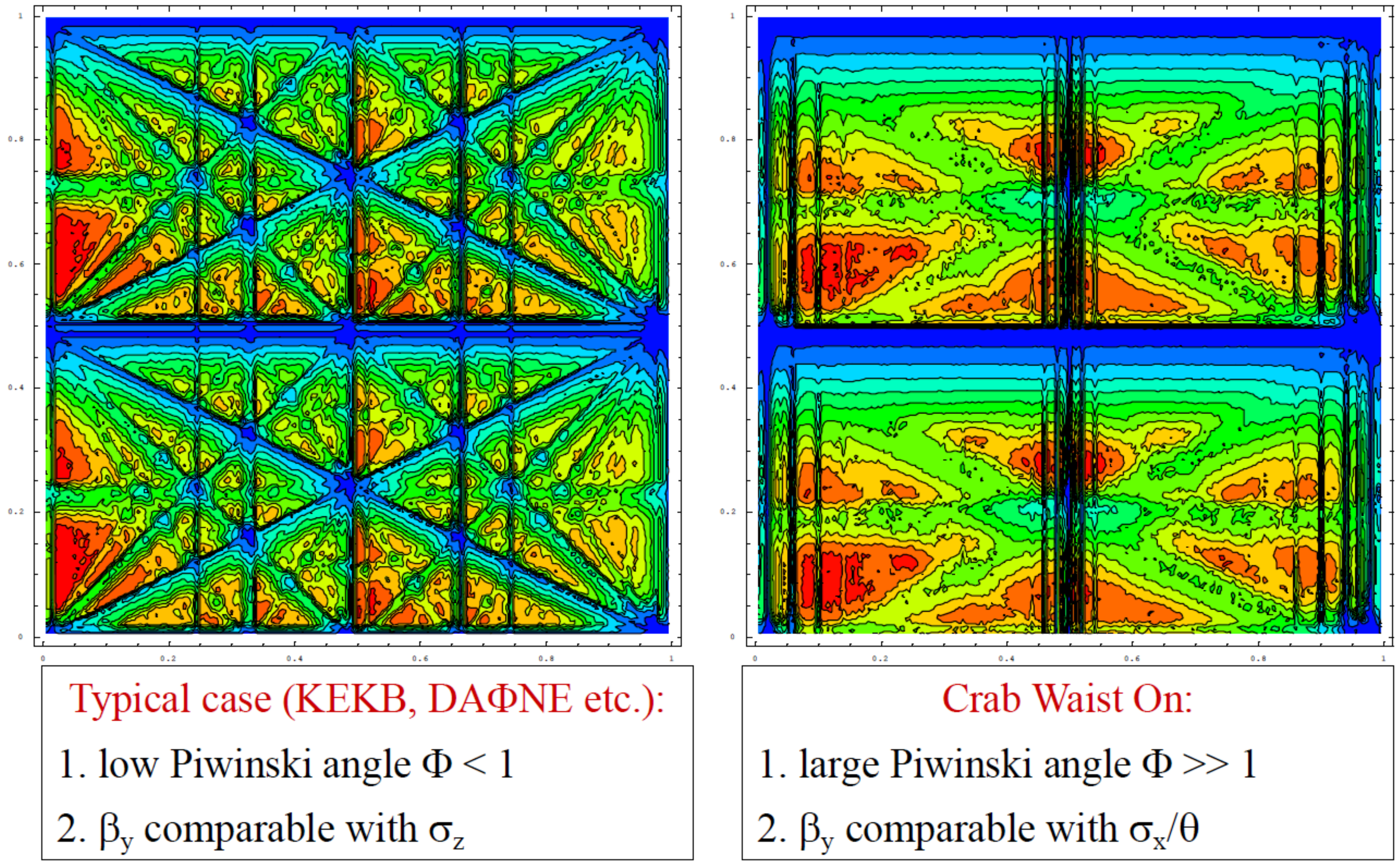


FIG. 7. Beam-beam resonances versus the crab value for DAΦNE (part 1).

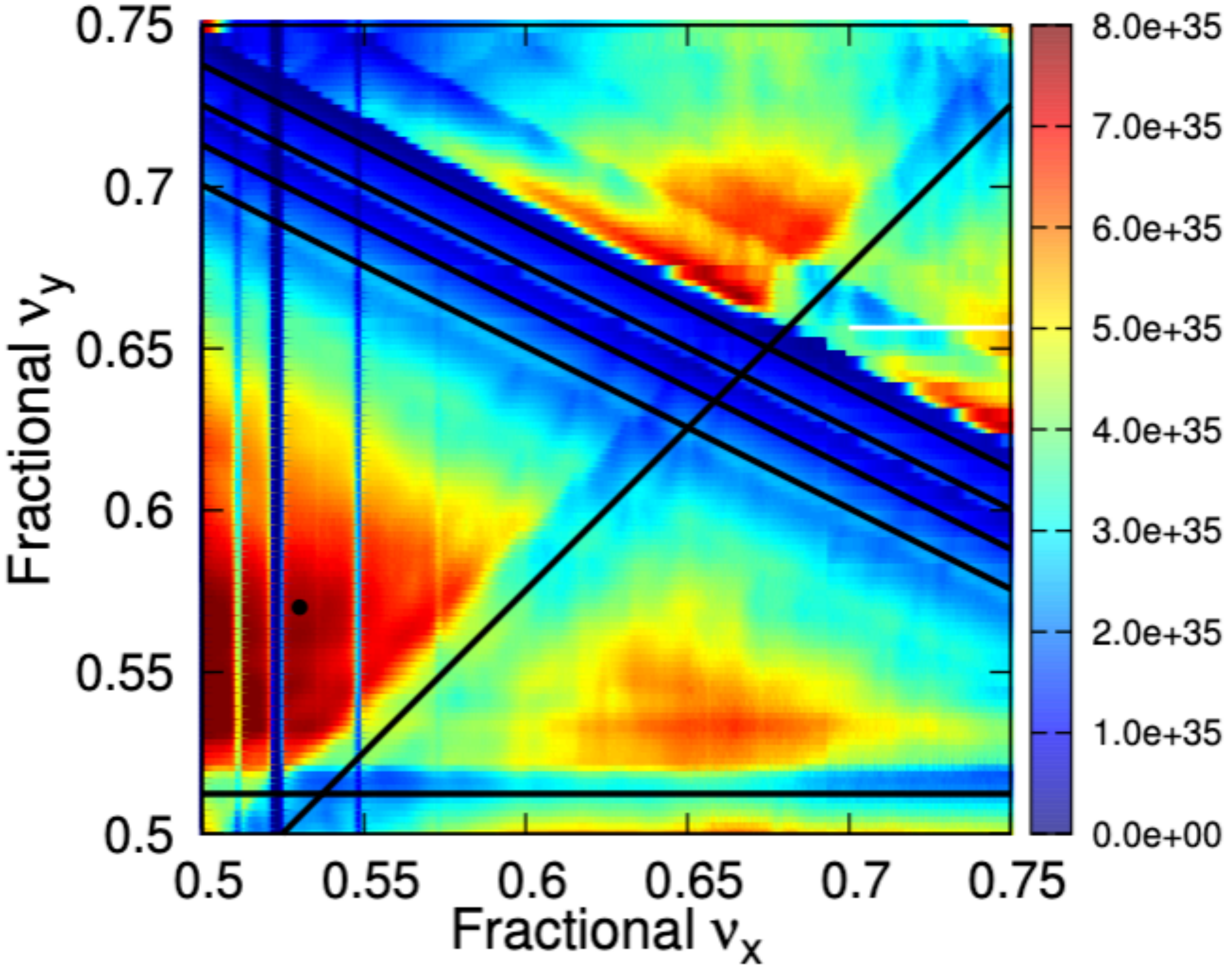
Ref.[2] D. Shatilov et al., PRST-AB 14, 014001 (2011).

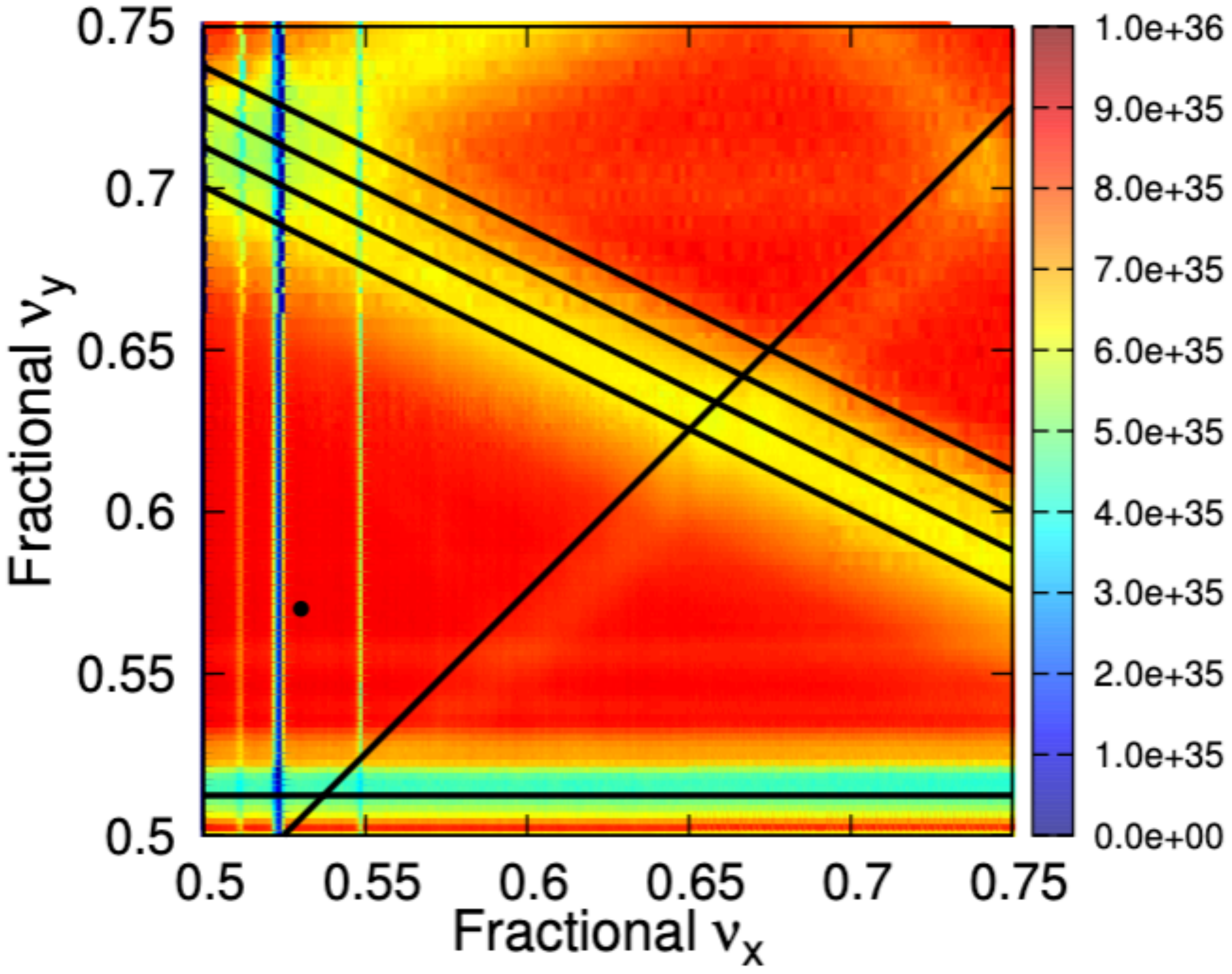


Ref.[2] M. Zobov, EIC Workshop, Oct. 8, 2020.

Ideal crab waist

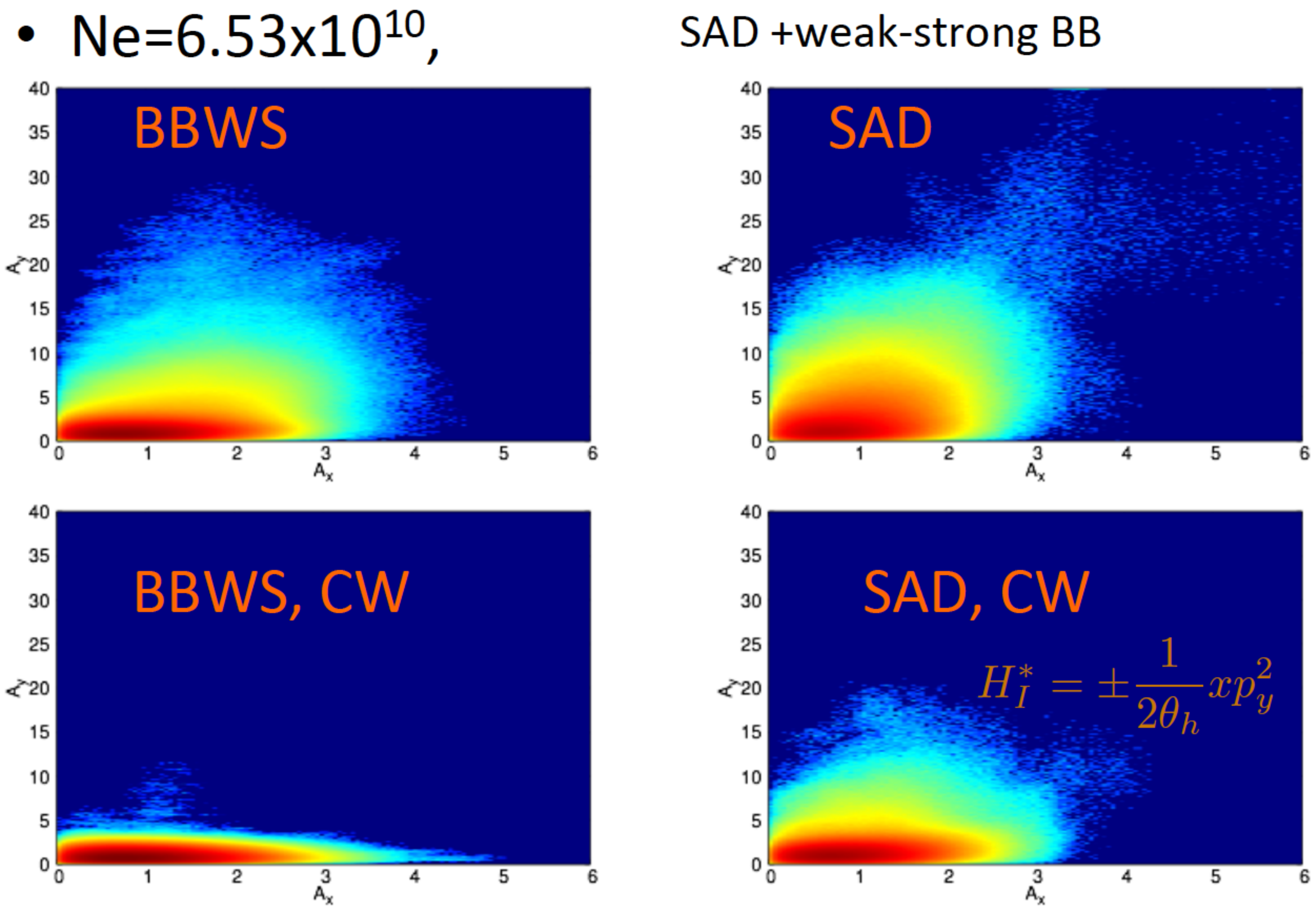
- SuperKEKB final design ($\beta_y^* = 0.3/0.27$ mm) with ideal crab waist
  - Crab waist creates large area in tune space for choice of working point





Ideal crab waist

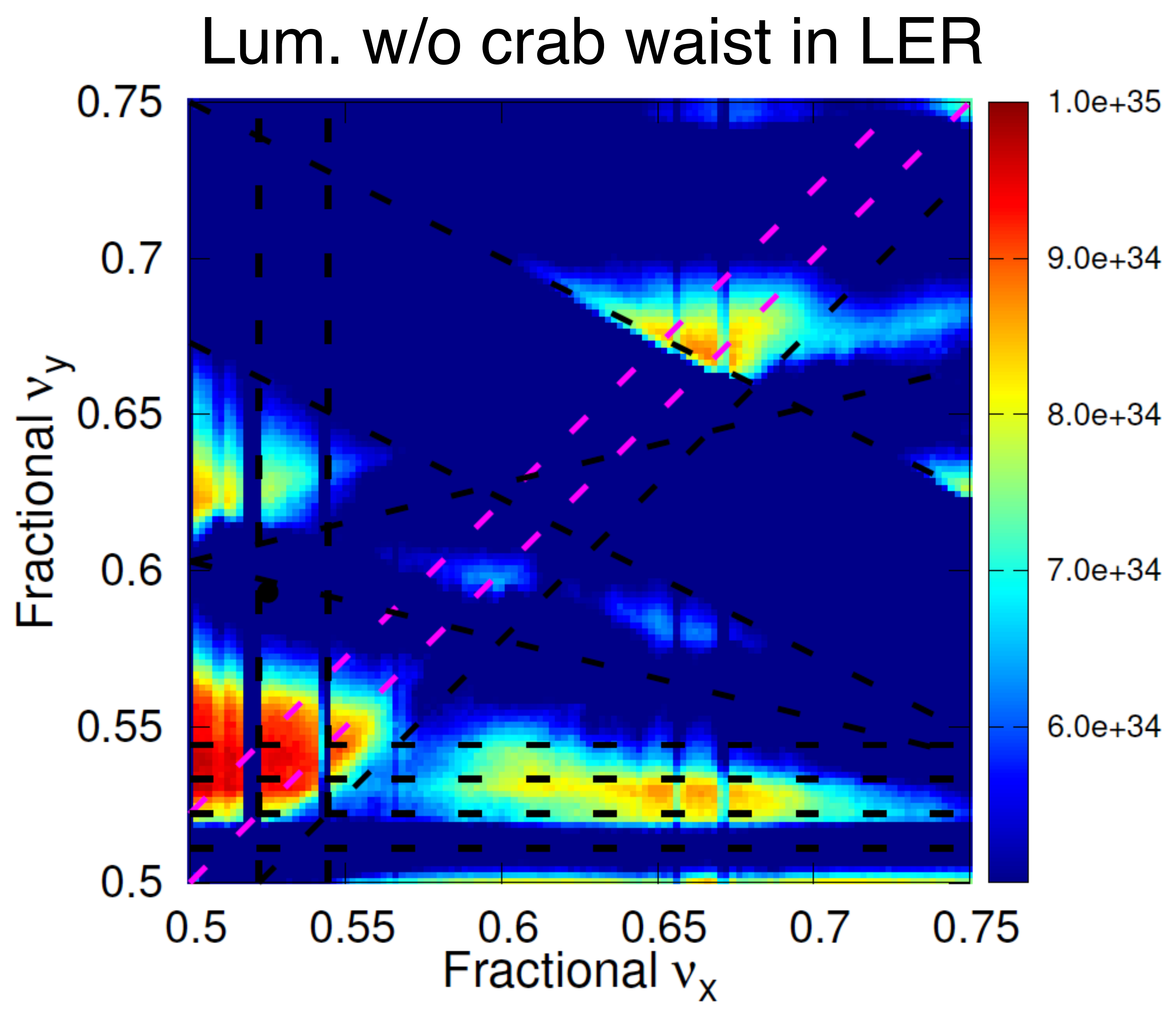
- SuperKEKB final design ($\beta_y^* = 0.3/0.27$ mm) with ideal crab waist
  - Beam-beam driven halo can be suppressed

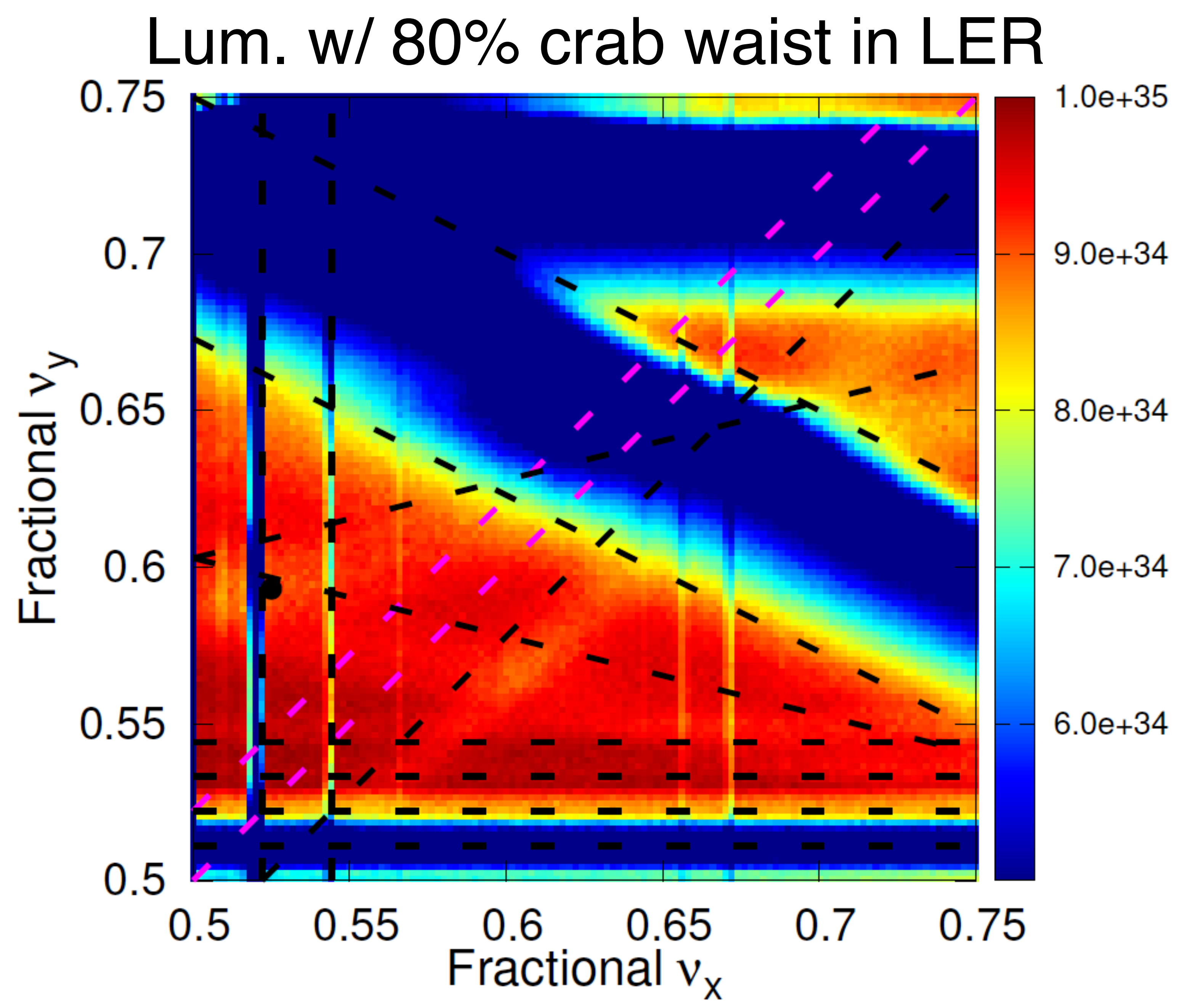


Ideal crab waist

- SuperKEKB 2021b run ($\beta_y^* = 1$ mm) with ideal crab waist
  - Tune scan using BBWS showed that 80% crab waist ratio in LER is effective in suppressing vertical blowup caused by beam-beam resonances (mainly $\nu_x \pm 4\nu_y + \alpha = N$).

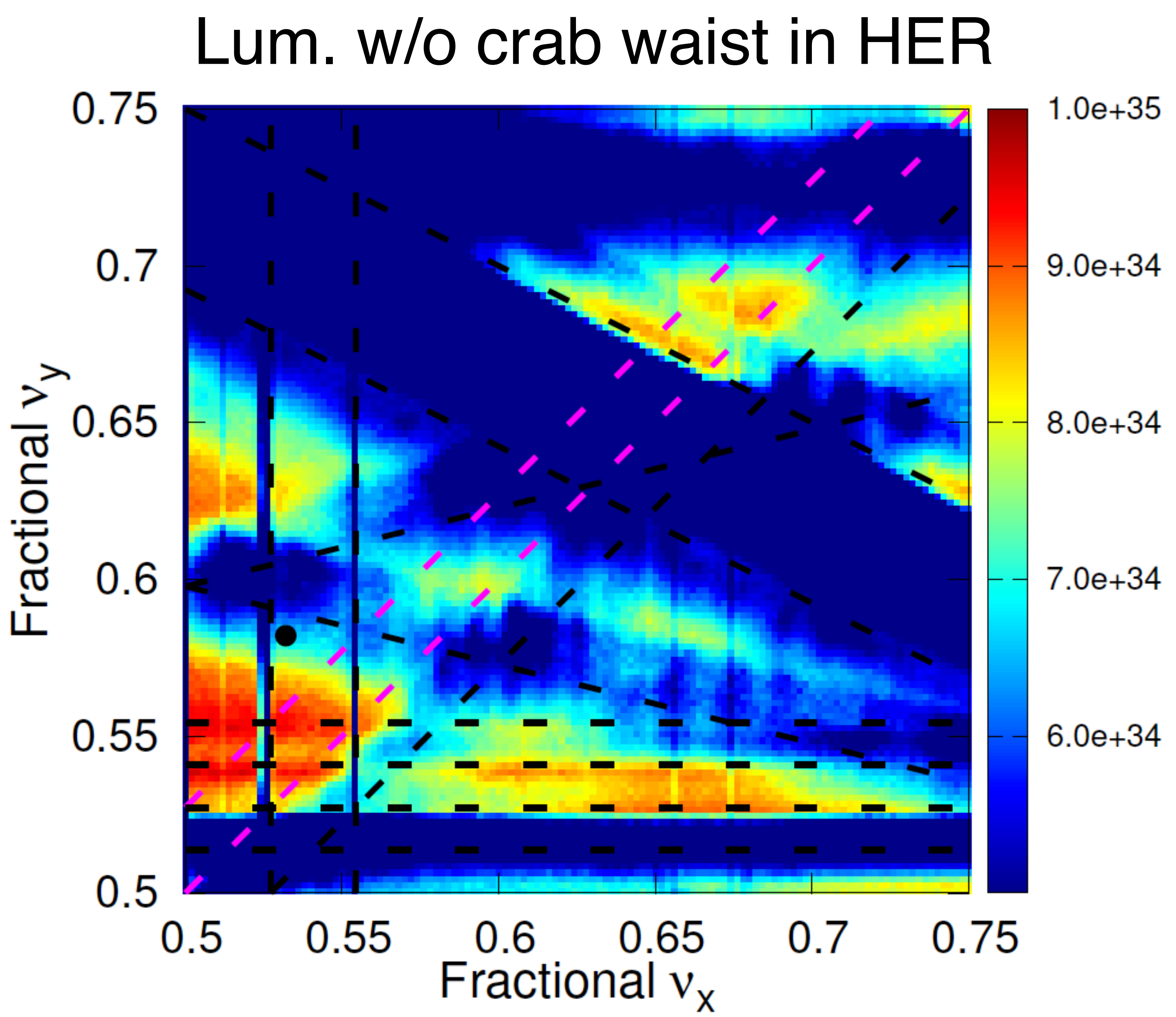
	2021.07.01		Comments
	HER	LER	
I_{bunch} (mA)	0.80	1.0	
# bunch	1174		Assumed value
ε_x (nm)	4.6	4.0	w/ IBS
ε_y (pm)	23	23	Estimated from XRM data
β_x (mm)	60	80	Calculated from lattice
β_y (mm)	1	1	Calculated from lattice
σ_{z0} (mm)	5.05	4.84	Natural bunch length (w/o MWI)
ν_x	45.532	44.525	Measured tune of pilot bunch
ν_y	43.582	46.593	Measured tune of pilot bunch
ν_s	0.0272	0.0221	Calculated from lattice
Crab waist	40%	80%	Lattice design

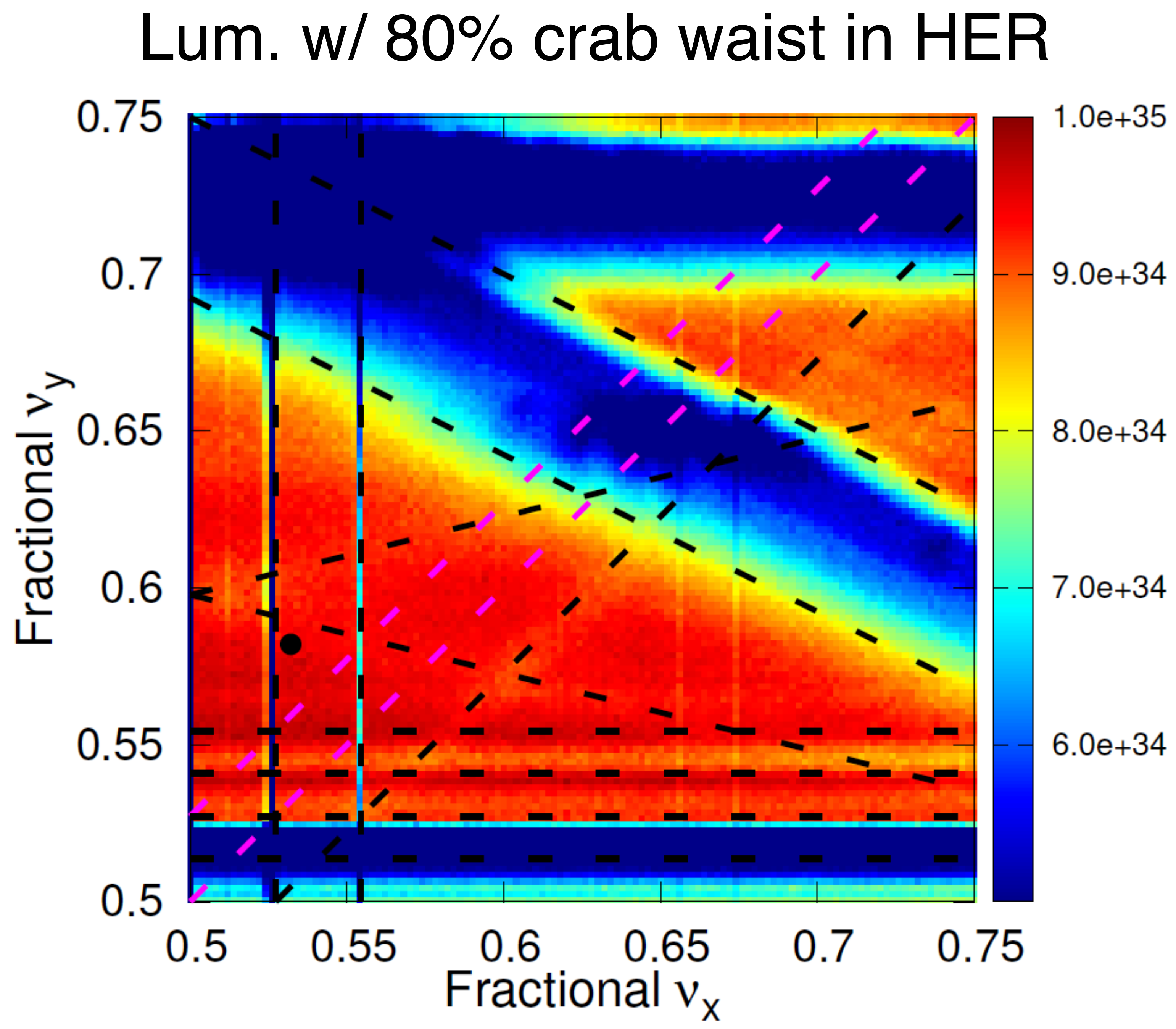


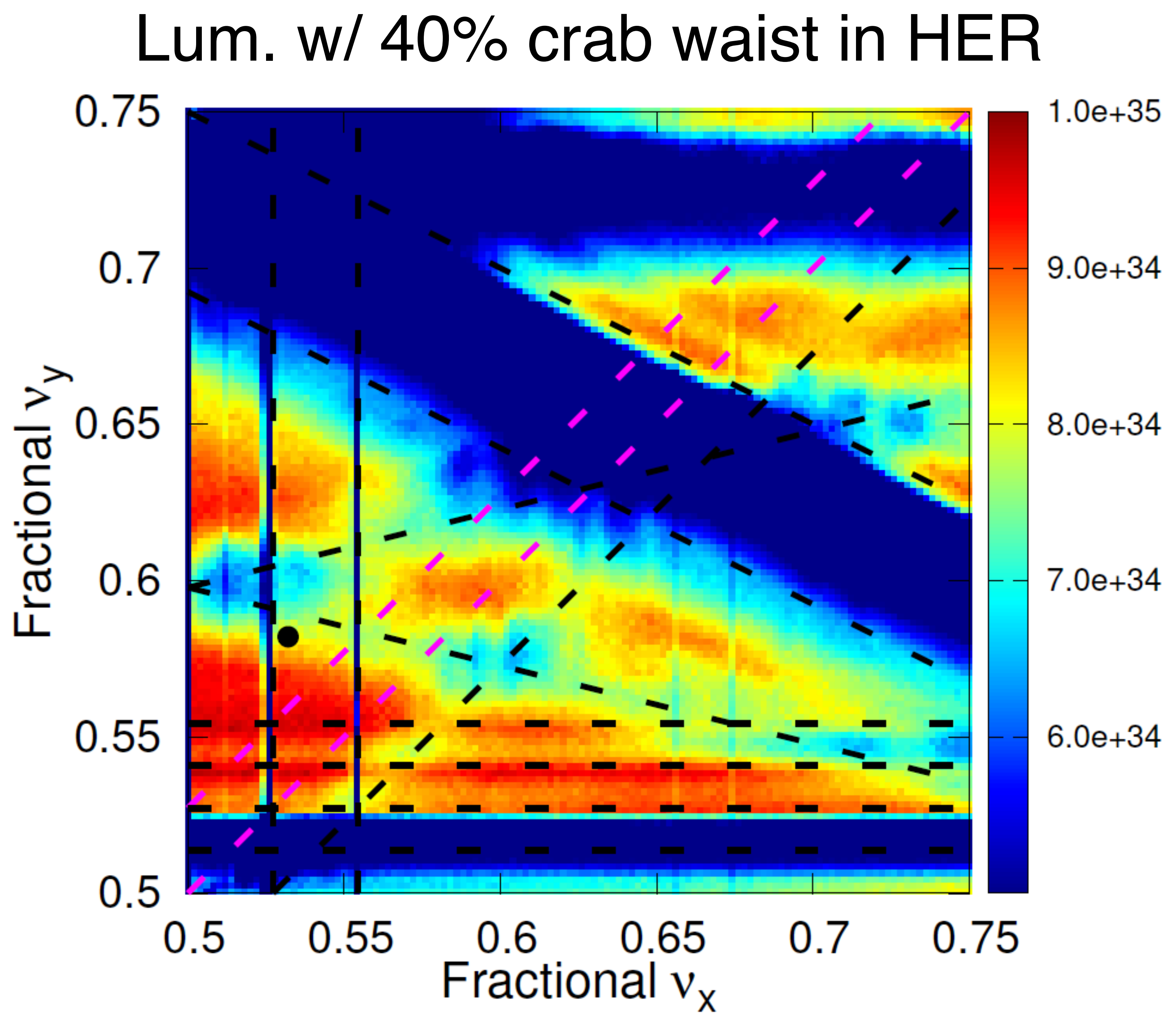


Ideal crab waist

- SuperKEKB 2021b run ($\beta_y^* = 1$ mm) with ideal crab waist
  - Tune scan using BBWS showed that 40% crab waist ratio (current operation condition) in HER might not be enough for suppressing vertical blowup caused by beam-beam resonances (mainly $\nu_x \pm 4\nu_y + \alpha = N$).

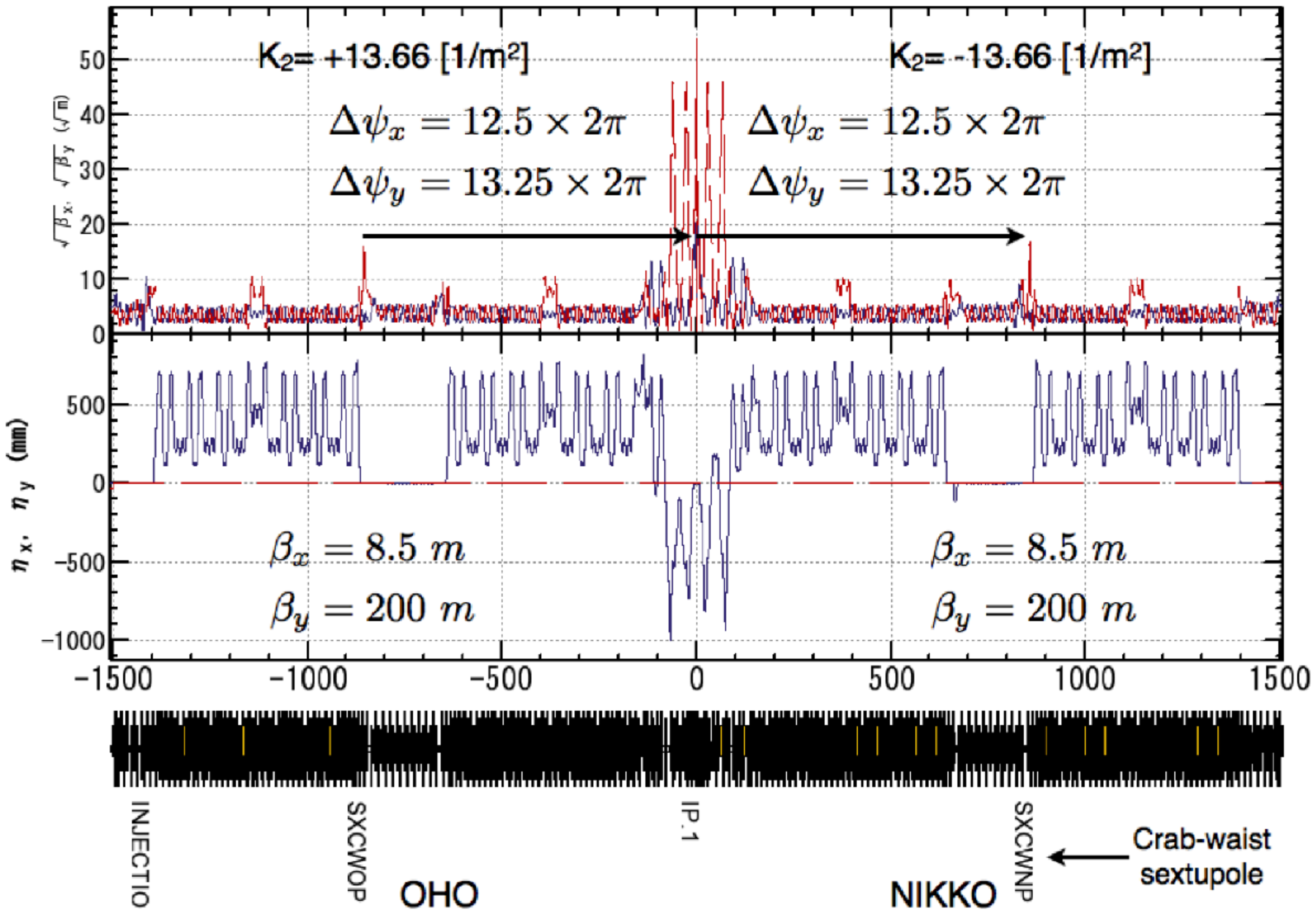






Crab waist applied to SuperKEKB

- SuperKEKB final design ($\beta_y^* = 0.3/0.27$ mm) with practical crab waist
  - CW scheme with CW sextupoles outside IR
  - CW reduces dynamic aperture and Touschek lifetime, and was not chosen as baseline for TDR



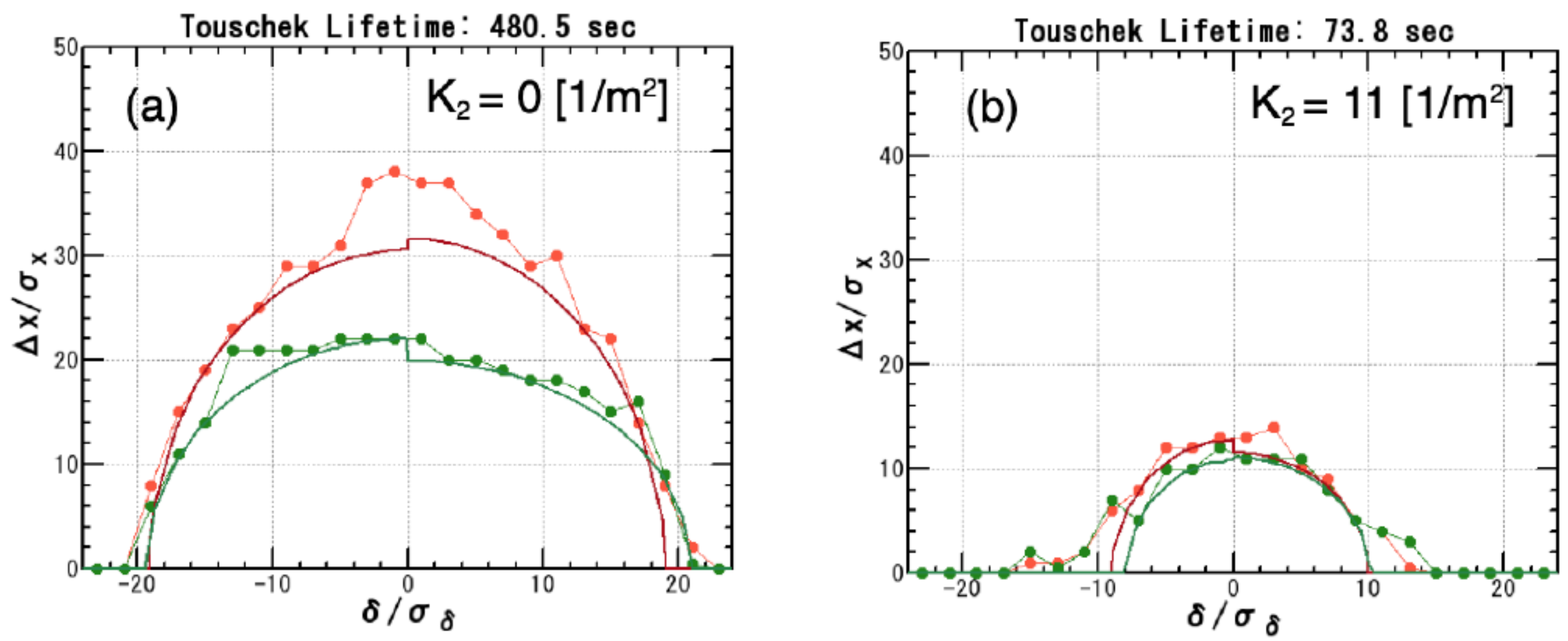
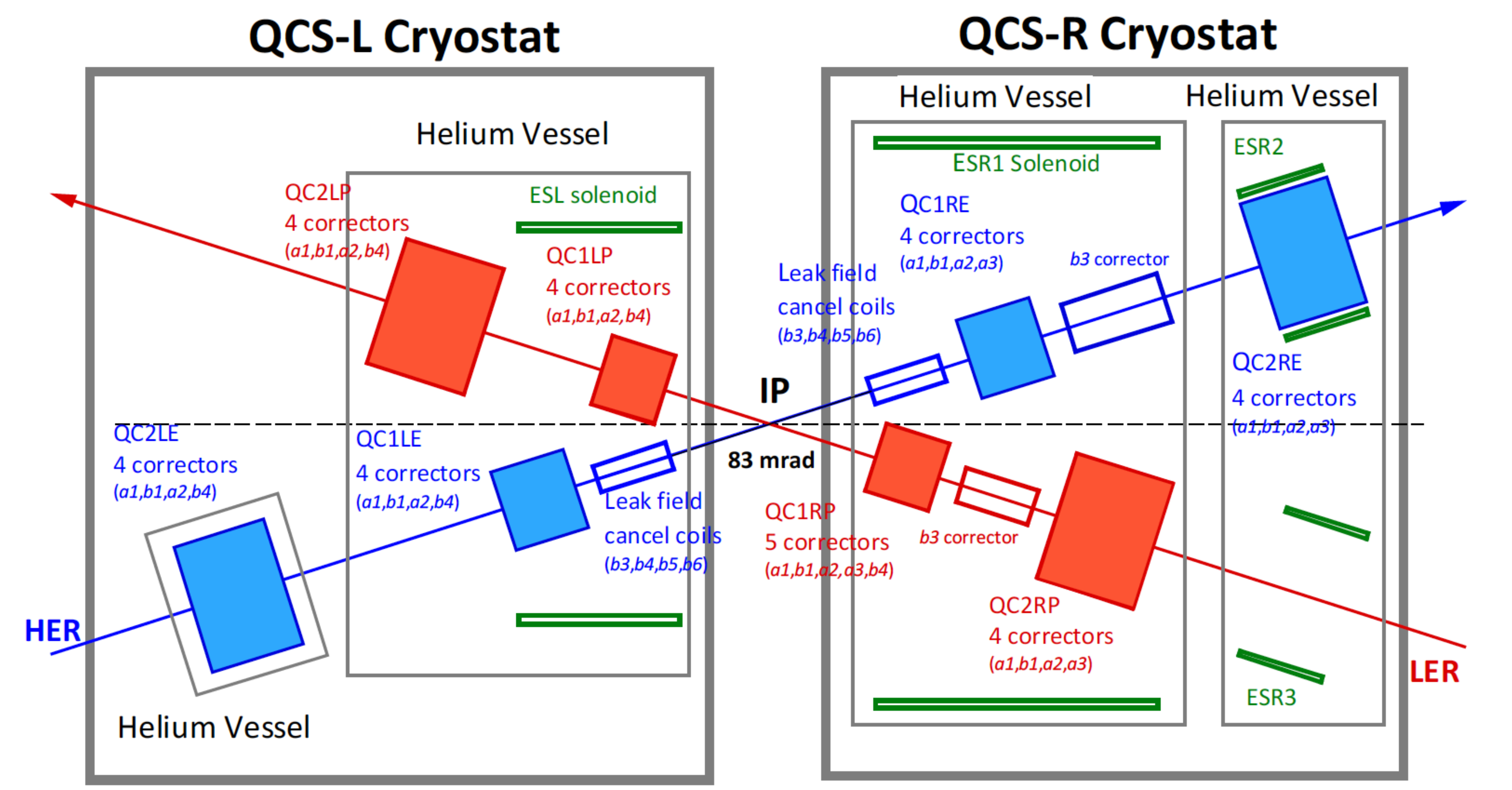


Figure 4.28: Dynamic aperture in the LER crab-waist lattice without beam-beam effect. Initial ratio of the vertical to the horizontal amplitude is 0.27 %. (a) $K_2 = 0$ $[1/m^2]$, (b) $K_2 = 11$ $[1/m^2]$.

Ref.[3] SuperKEKB TDR.

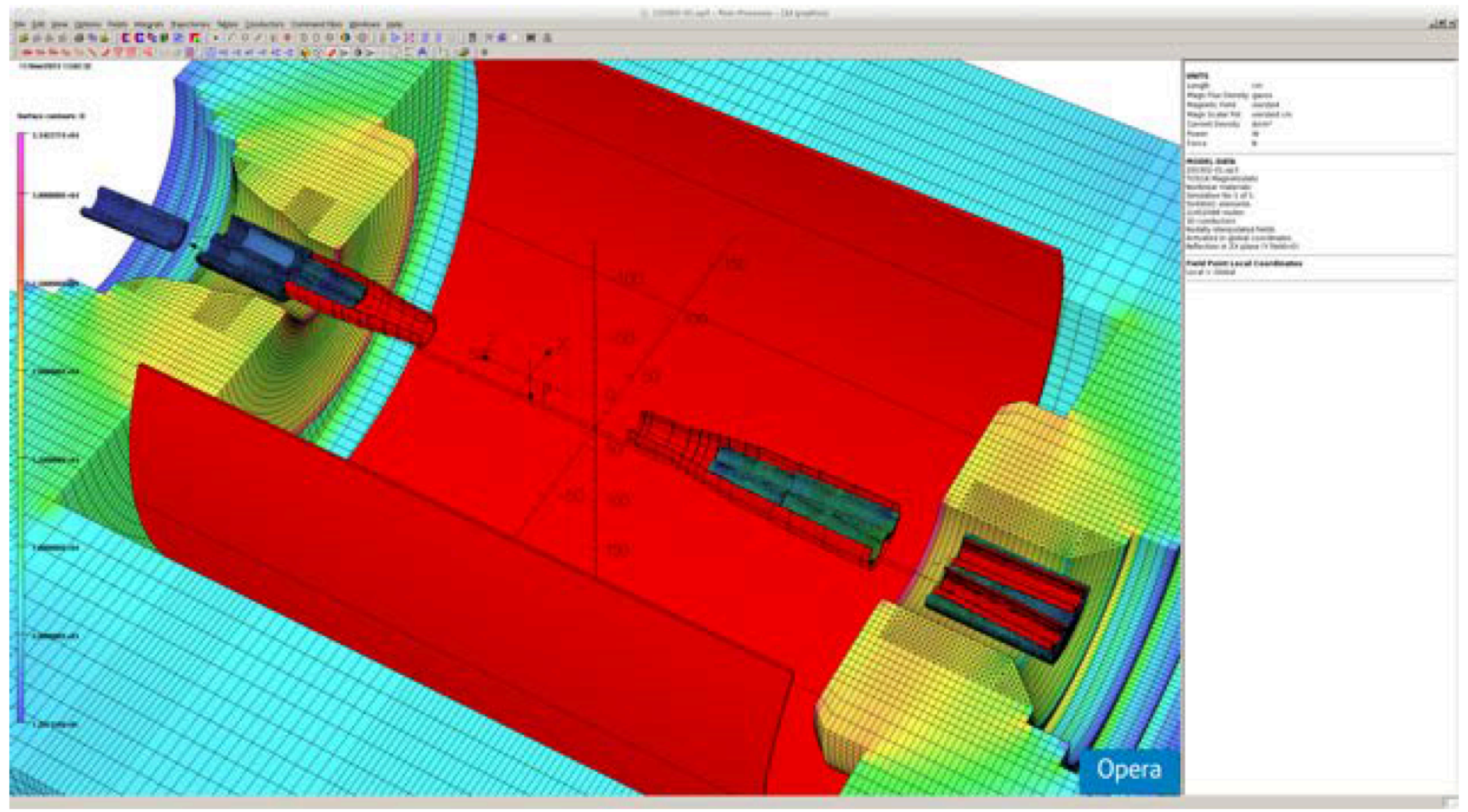
Crab waist applied to SuperKEKB

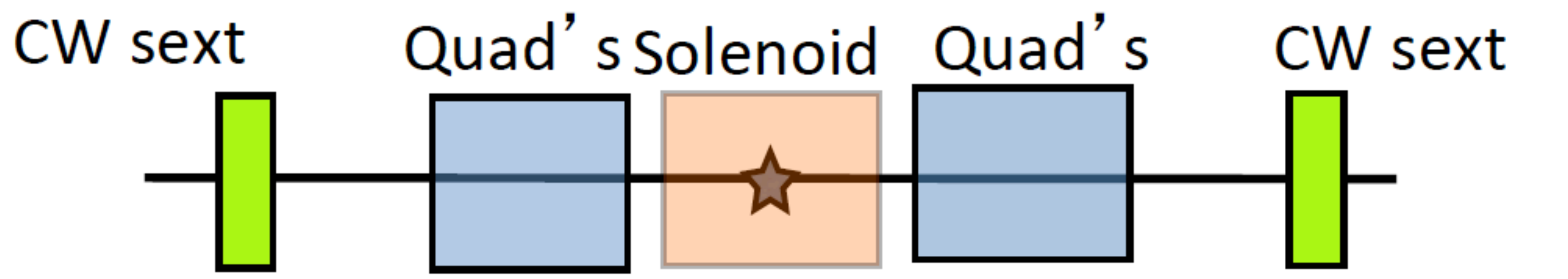
- SuperKEKB final design ($\beta_y^* = 0.3/0.27$ mm) with practical crab waist
  - CW does not work well because of the nonlinear IR. The nonlinearity scales as $1/\beta_y^*$.
  - SuperKEKB design lattice include nonlinear fields extracted from 3D model



4 SC main quadrupole magnets: 1 collared magnet, 3 yoked magnets
16 SC correctors: a1, b1, a2, b4
4 SC leak field cancel magnets: b3, b4, b5, b6
1 compensation solenoid

4 SC main quadrupole magnets: 1 collared magnet, 3 yoked magnets
19 SC correctors: a1, b1, a2, a3, b3, b4
4 SC leak field cancel magnets: b3, b4, b5, b6
3 compensation solenoid





$$\mathcal{M}_{IR} = \underbrace{e^{-axy^2}}_{\text{crab waist sext}} e^{-H_{Q's}} e^{-H_{Sol}} e^{-H_{BB}} e^{-H_{Sol}} e^{-H_{Q's}} \underbrace{e^{-axy^2}}_{\text{crab waist sext}}$$

$$\neq$$

$$e^{-H_{Q's}} e^{-H_{Sol}} \overbrace{e^{-xp_y^2/2\phi}}^{\text{ideal crab waist}} e^{-H_{BB}} \overbrace{e^{-xp_y^2/2\phi}}^{\text{ideal crab waist}} e^{-H_{Sol}} e^{-H_{Q's}}$$

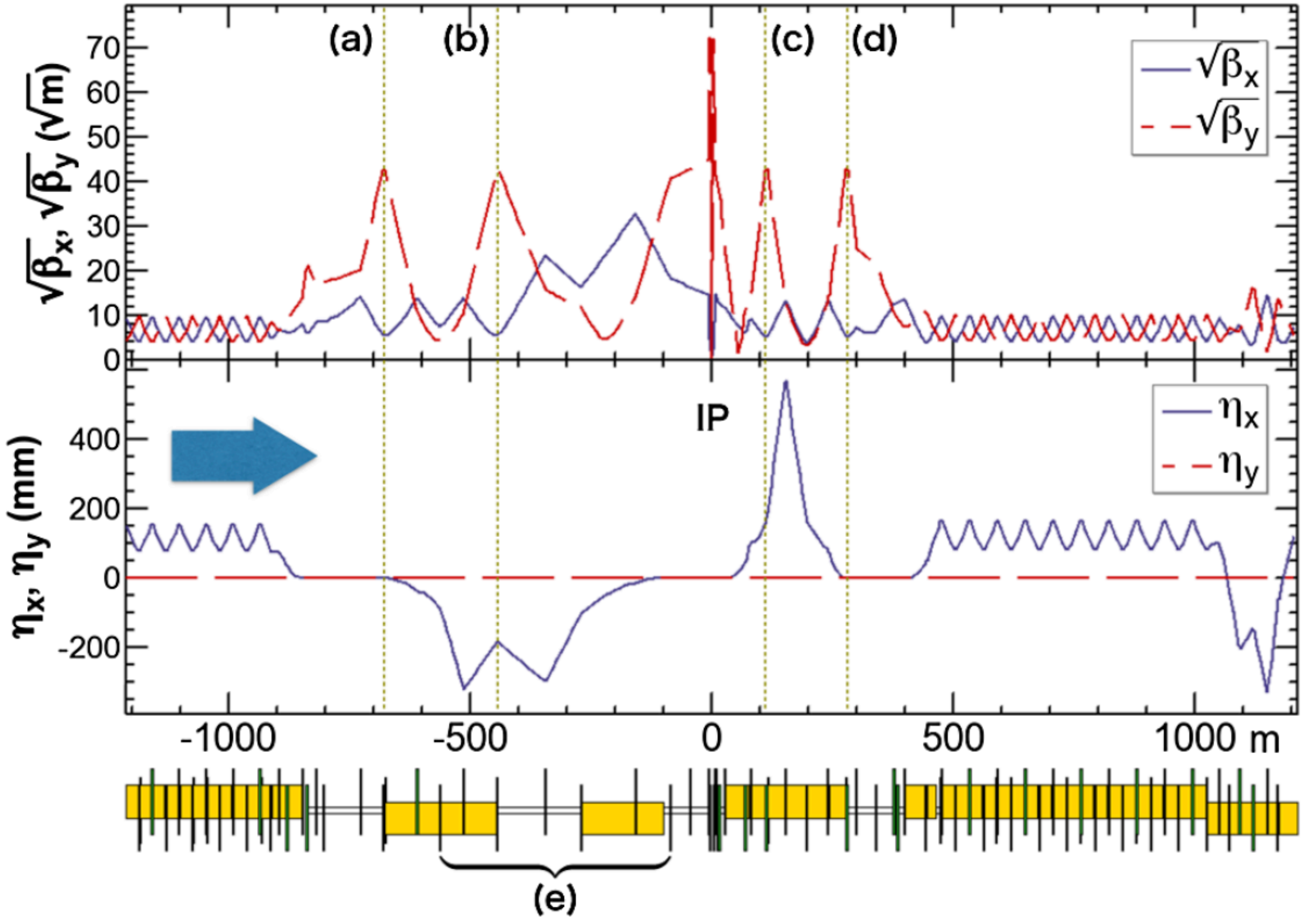
Ref.[4] K. Ohmi, EIC workshop, March, 2014.

Ref.[5] N. Ohuchi, SuperKEKB ARC, 2018.

Crab waist applied to SuperKEKB

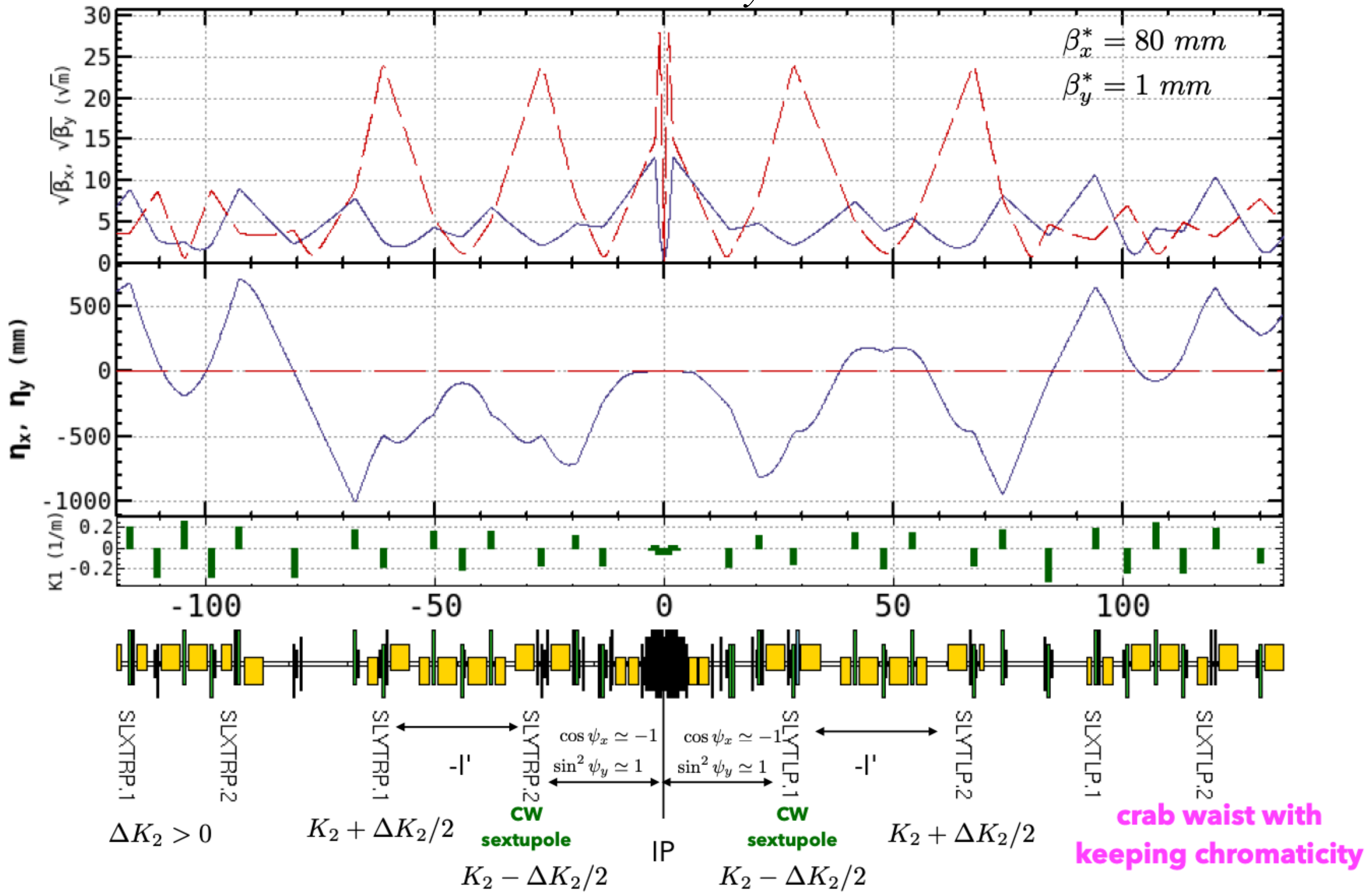
- Optics design with crab waist for $\beta_y^* = 1$ mm
  - In 2020, K. Oide introduced the FCC-ee CW scheme to SuperKEKB.
  - FCC-ee CW scheme utilizes the sextupoles (a-d) for local chromaticity correction and crab waist.

FCC-ee



Ref.[6] K. Oide et al., PRAB 19, 111005 (2016).

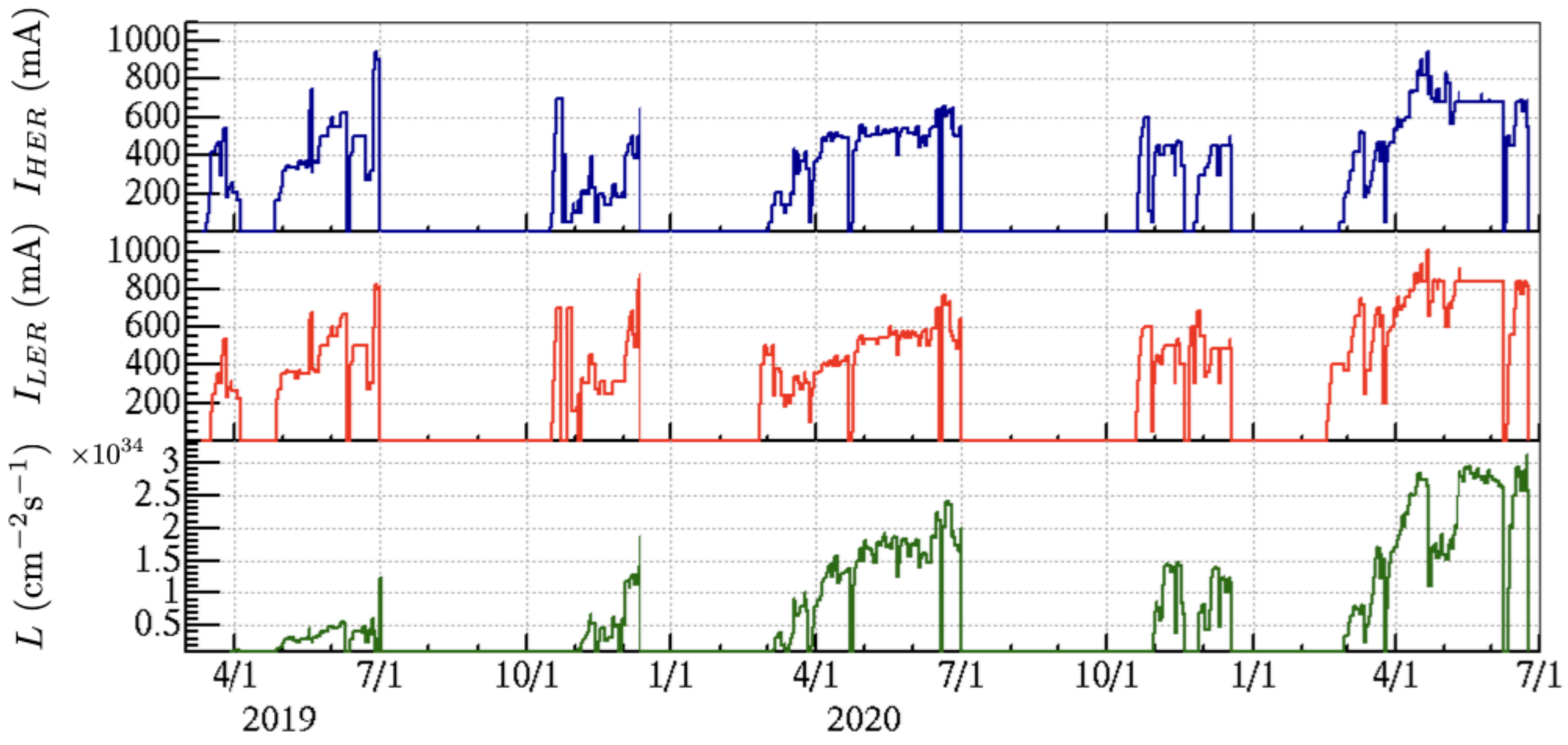
SuperKEKB w/ $\beta_y^* = 1$ mm

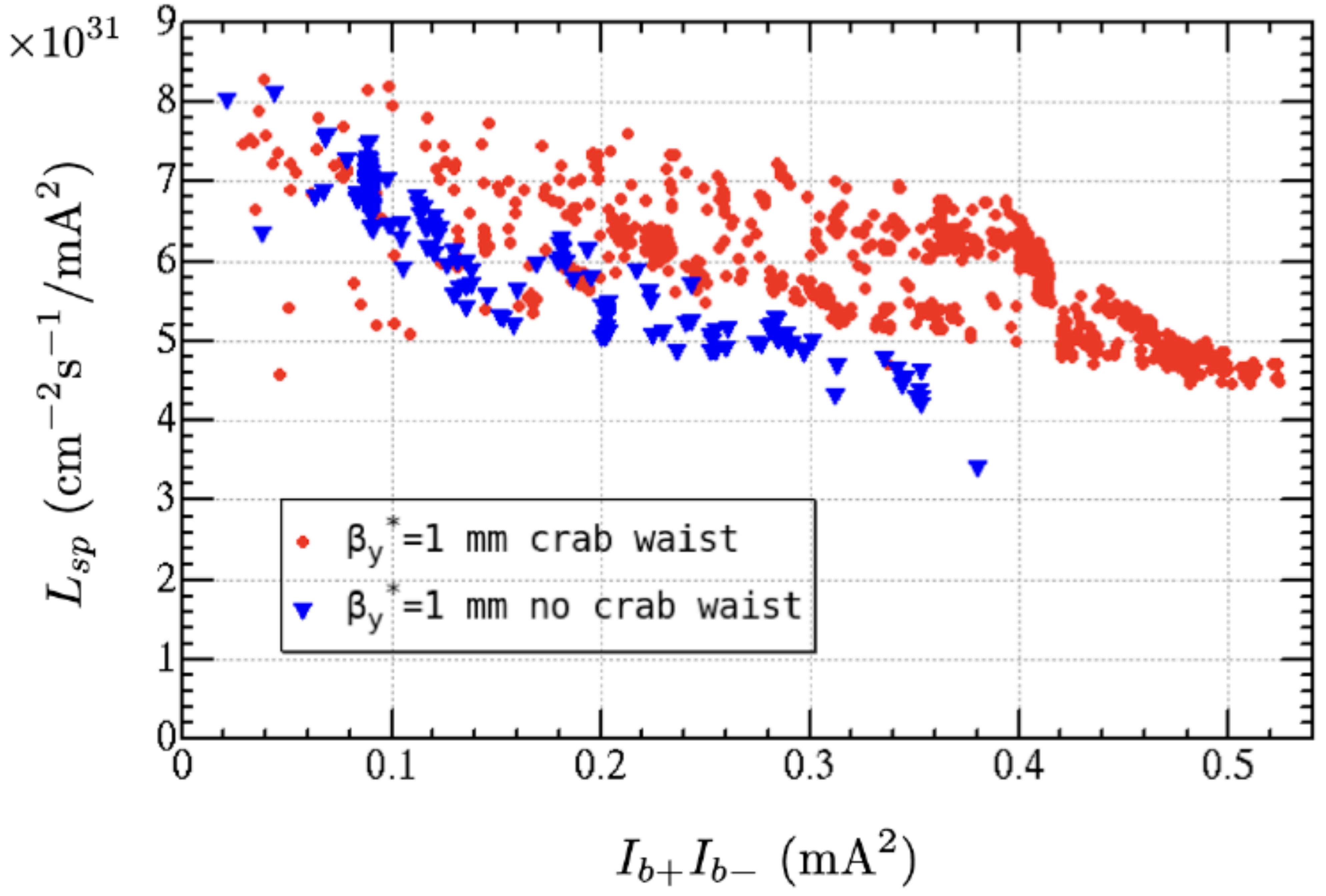


Ref.[7] Y. Ohnishi, SuperKEKB ARC 2020.

Crab waist applied to SuperKEKB

- SuperKEKB beam operation with crab waist for $\beta_y^* = 1$ mm
  - Operation with CW has been successful.





Ref.[8] Y. Ohnishi, The European Physical Journal Plus volume 136, 1023 (2021).

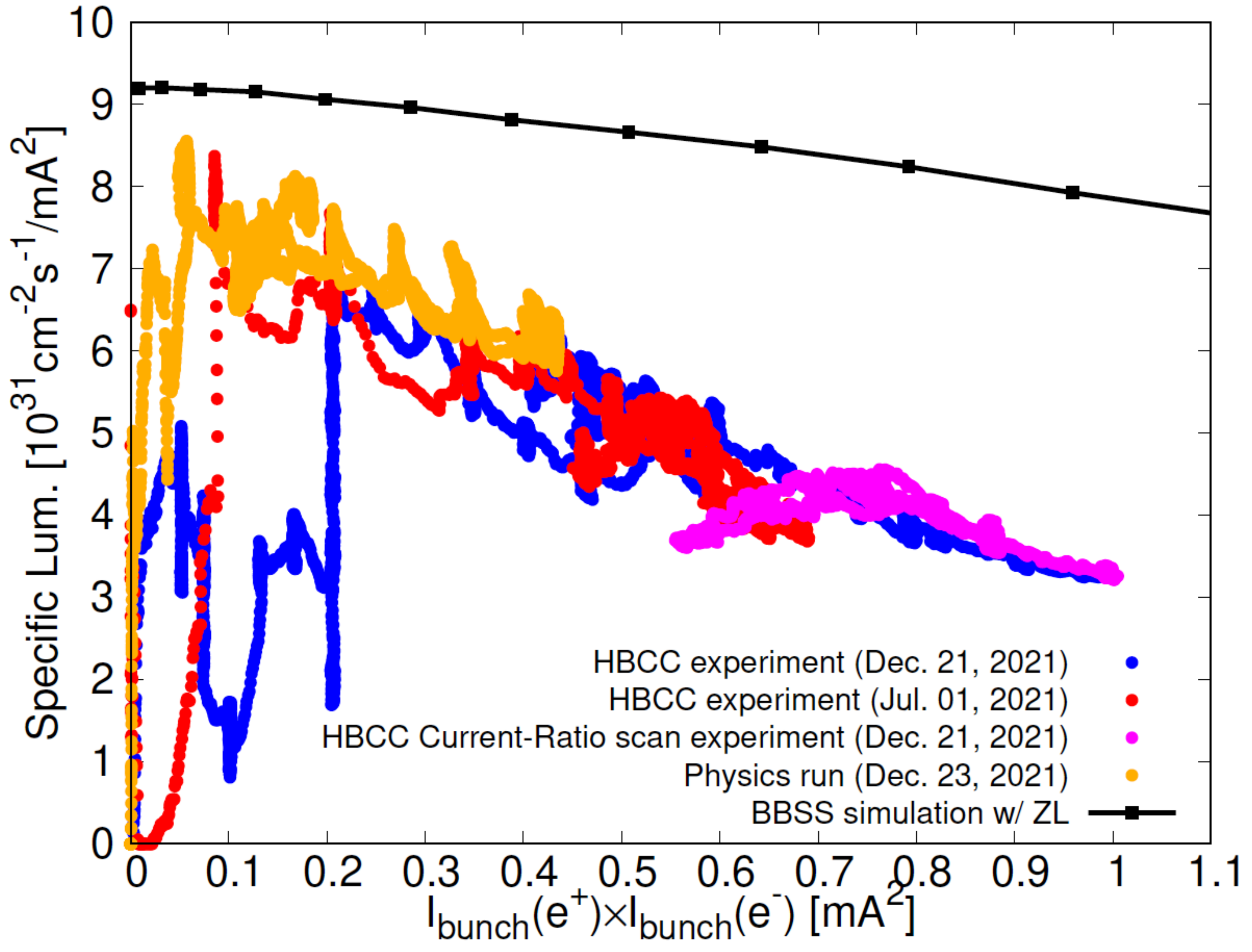
Crab waist applied to SuperKEKB

- SuperKEKB beam operation with crab waist for $\beta_y^* = 1$ mm
  - The discrepancy between simulated and observed luminosity became large when bunch currents increase.
  - With optimized working point and fine IP tuning knobs, slightly better luminosity performance can be achieved.
  - Sources of luminosity degradation at high bunch currents are to be investigated.

Operation parameter set for BBSS simulation

	2021.12.21		Comments
	HER	LER	
I_{bunch} (mA)	Ie	1.25*Ie	
# bunch	393		Assumed value
ε_x (nm)	4.6	4.0	w/ IBS
ε_y (pm)	20	35	Estimated from XRM data
β_x (mm)	60	80	Calculated from lattice
β_y (mm)	1	1	Calculated from lattice
σ_{z0} (mm)	5.05	4.60	Natural bunch length (w/o MWI)
ν_x	45.532	44.525	Measured tune of pilot bunch
ν_y	43.582	46.593	Measured tune of pilot bunch
ν_s	0.0272	0.0233	Calculated from lattice
Crab waist	40%	80%	Lattice design

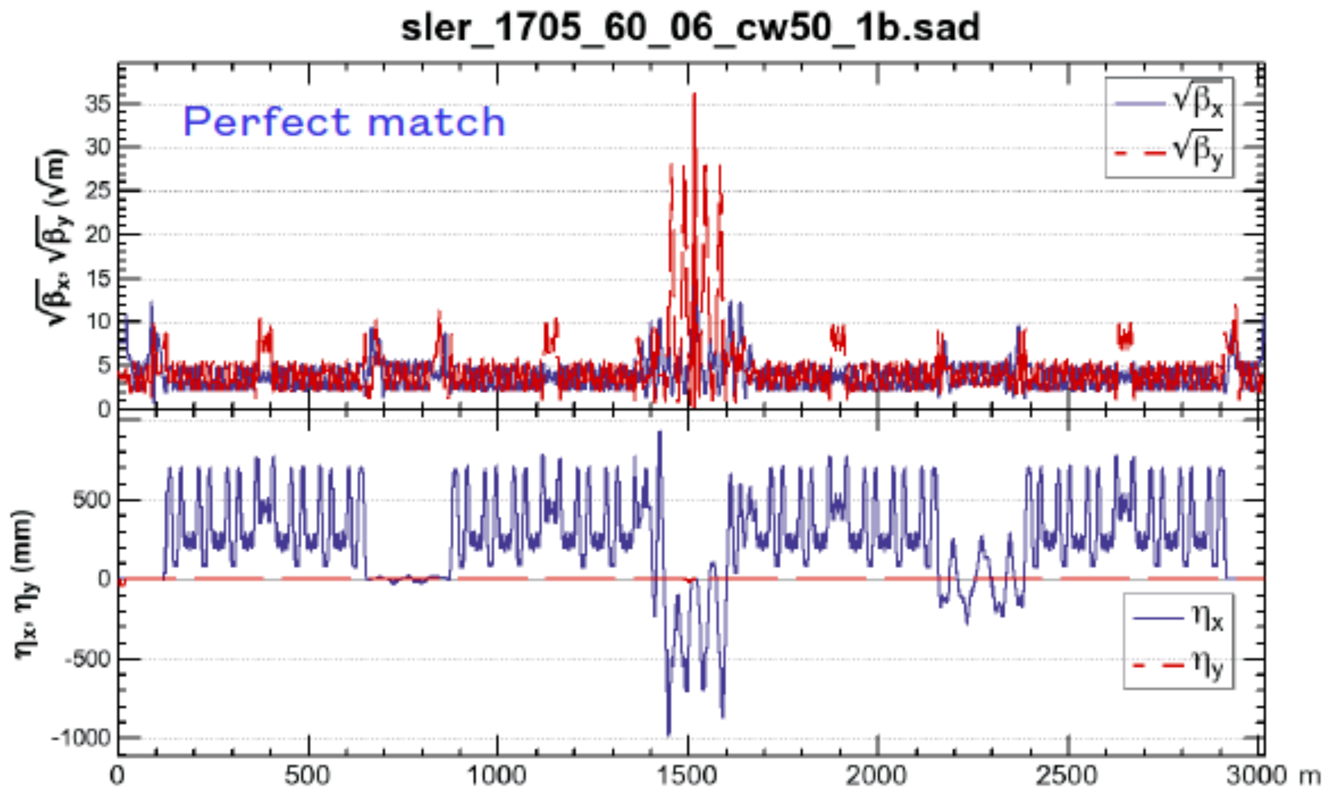
Lsp: BBSS simulations vs observation

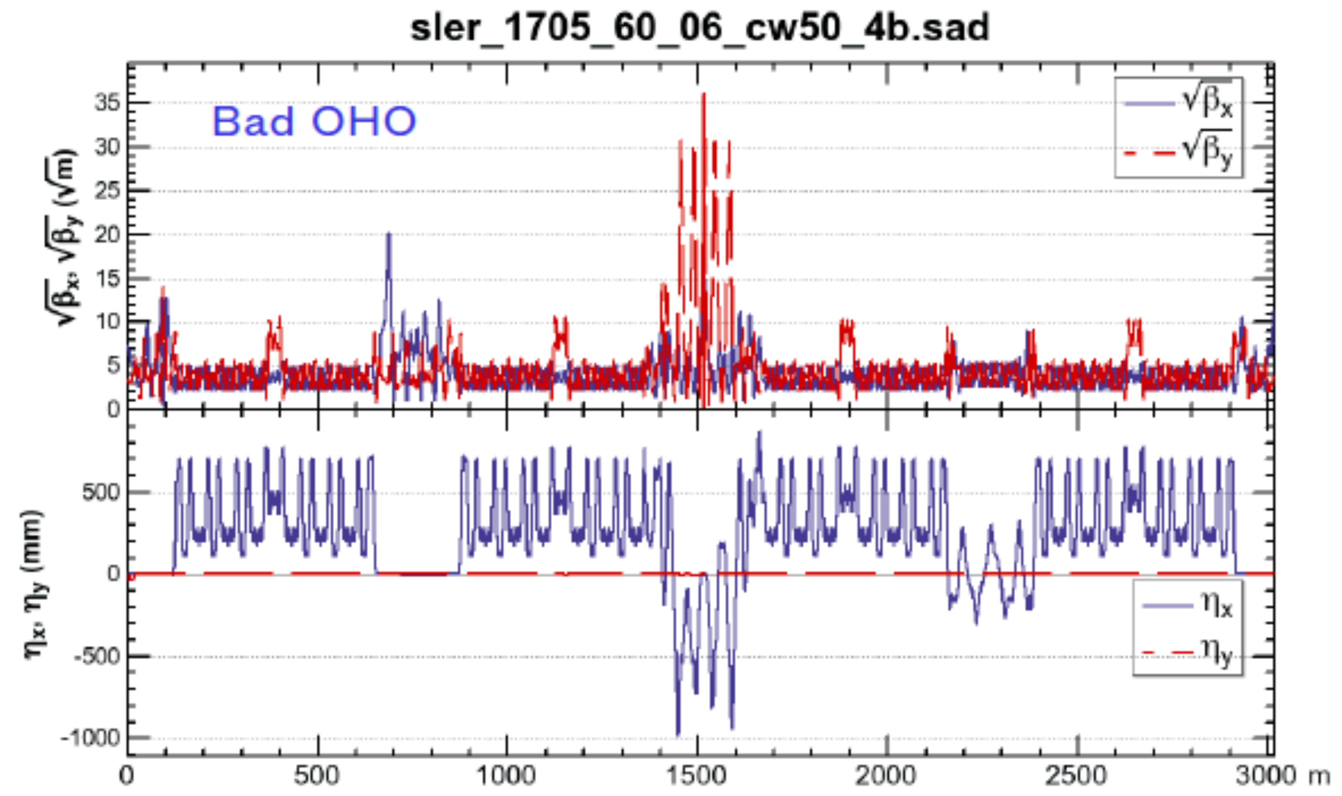


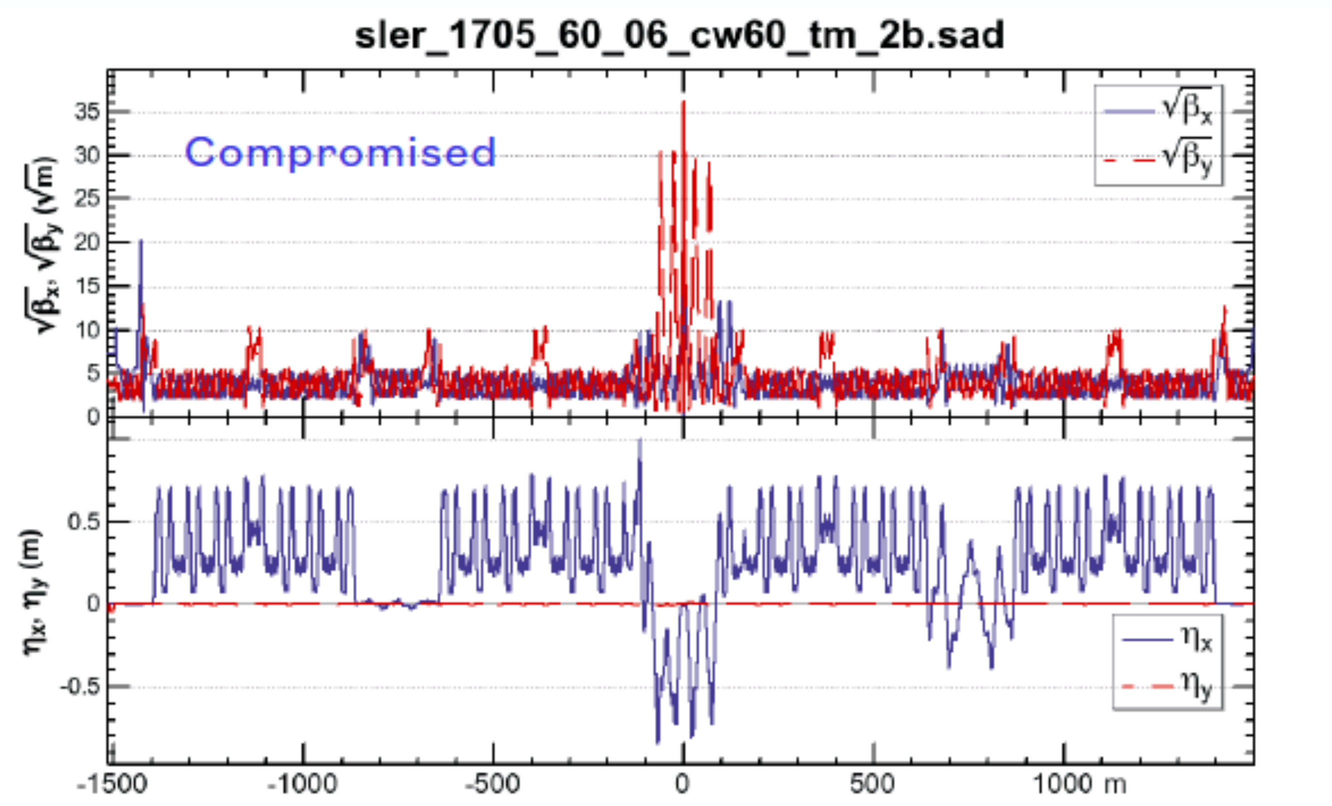
Crab waist applied to SuperKEKB

- Optics design with crab waist for $\beta_y^* = 0.6$ mm by K. Oide

Comparison of optics







Parameters		"Bad OHO"	"compromised"	"perfect"
β^*x/y, LER=HER	mm		60 / 0.6	
Hor. emittance LER/HER	nm	4.2 / 4.5	4.4 / 4.5	3.6 / 4.5
$\varepsilon_{y0}/\varepsilon_x$, LER=HER	%		1.5	
Bunch current LER/HER	mA		1.0 / 0.81	
Bunches/ring			2200	
Bunch length[1] LER/HER	mm		4.8 / 5.5	
Energy spread[1] LER/HER	10^{-4}		7.6 / 6.3	
Luminosity	$10^{34}/\mathrm{cm^2 s}$	11.4	11.1	12.2
$\Delta\psi_{x,y}/2\pi$ btw. IP & SLY		0.003	0.009	0.001
$\Delta\beta/\beta$ btw. SLY	%	0.13	3.6	0.011
$\Delta\eta_{px}$ @PQLY1C		0.01	0.01	0.0002
R_1, R_4 @IP		1×10^{-8}	0.012	1×10^{-10}
R_2 @IP	m	8×10^{-14}	7×10^{-6}	2×10^{-14}
R_3 @IP	1/m	5×10^{-6}	0.21	3×10^{-10}
η_y @IP	m	1×10^{-14}	2×10^{-6}	5×10^{-15}
η_{py} @IP		3×10^{-14}	1×10^{-5}	6×10^{-13}

3

[1]@zero impedance

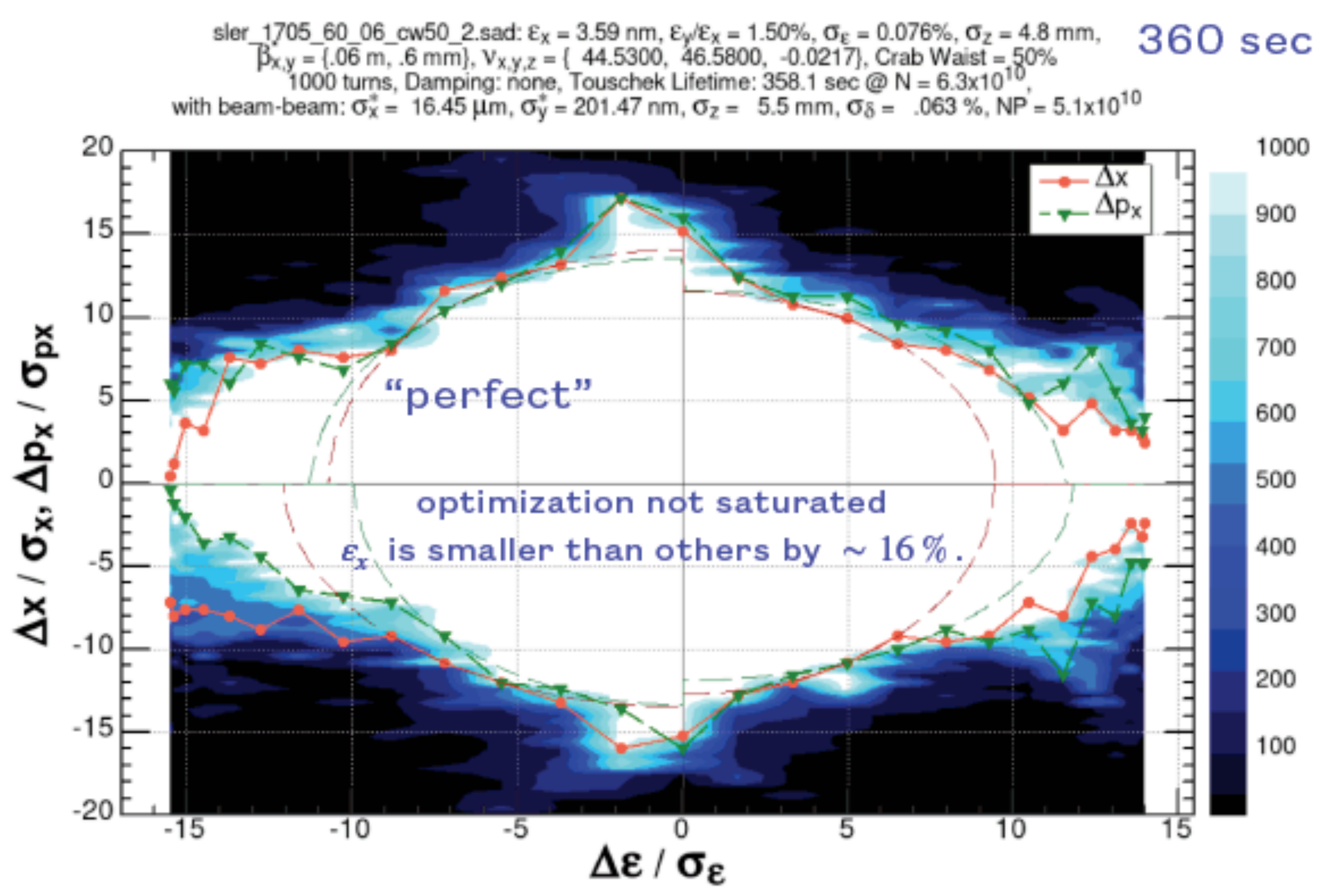
Aug. 27, 2021 K. Oide

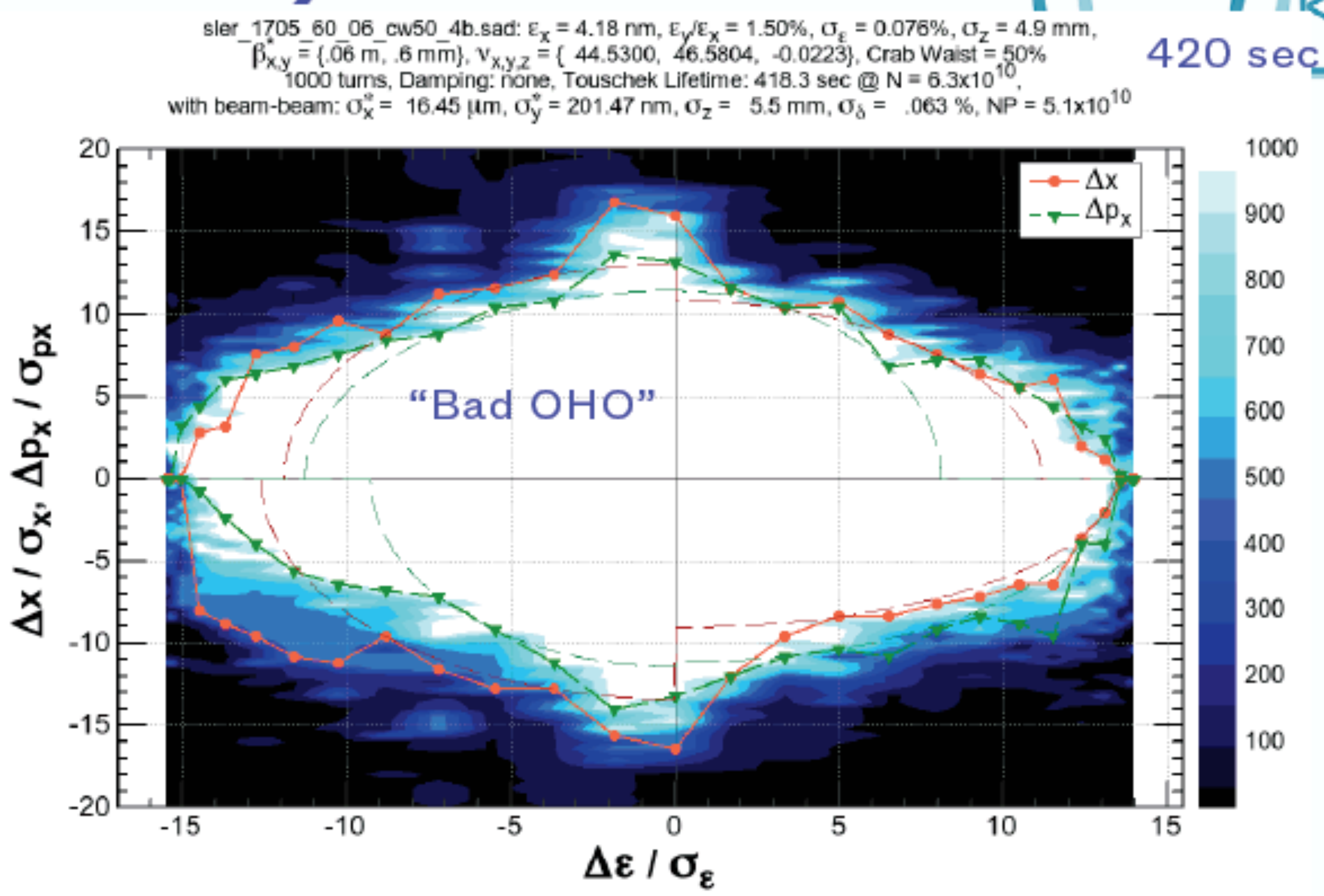
Ref.[9] K. Oide, SuperKEKB ARC, 2021.

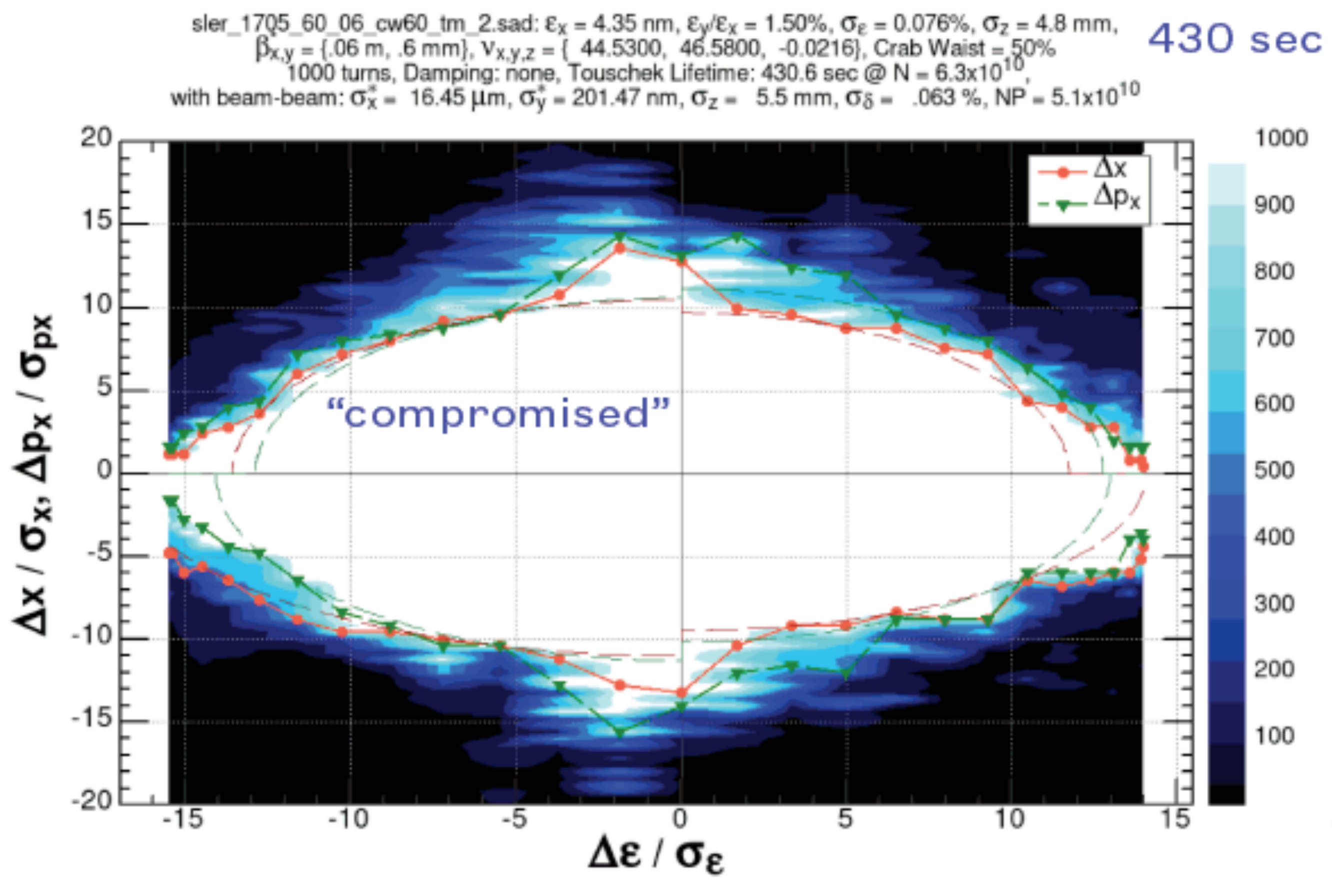
Crab waist applied to SuperKEKB

- Optics design with crab waist for $\beta_y^* = 0.6$ mm by K. Oide
  - With 50% CW strength, lifetime is acceptable for beam operation

Dynamic aperture (with beam-beam, CW = 50%)







Parameters		"Bad OHO"	“compromised”	“perfect”
$\beta^* x/y$, LER=HER	mm		60 / 0.6	
Hor. emittance LER/HER	nm	4.2 / 4.5	4.4 / 4.5	3.6 / 4.5
$\varepsilon_{y0}/\varepsilon_x$, LER=HER	%		1.5	
Bunch current LER/HER	mA		1.0 / 0.81	
Bunches/ring			2200	
Bunch length[1] LER/HER	mm		4.8 / 5.5	
Energy spread[1] LER/HER	10^{-4}		7.6 / 6.3	
Luminosity	$10^{34}/\mathrm{cm}^2\mathrm{s}$	11.4	11.1	12.2
$\Delta\psi_{x,y}/2\pi$ btw. IP & SLY		0.003	0.009	0.001
$\Delta\beta/\beta$ btw. SLY	%	0.13	3.6	0.011
$\Delta\eta_{px}$ @PQLY1C		0.01	0.01	0.0002
R_1, R_4 @IP		1×10^{-8}	0.012	1×10^{-10}
R_2 @IP	m	8×10^{-14}	7×10^{-6}	2×10^{-14}
R_3 @IP	1/m	5×10^{-6}	0.21	3×10^{-10}
η_y @IP	m	1×10^{-14}	2×10^{-6}	5×10^{-15}
η_{py} @IP		3×10^{-14}	1×10^{-5}	6×10^{-13}

4

[1] @zero impedance

Aug. 27, 2021 K. Oide

Ref.[9] K. Oide, SuperKEKB ARC, 2021.

Crab waist optics design for Super Charm Tau Factory

- The Super Charm Tau Factory under design utilizes CW as baseline

Design parameters

E(MeV)	1500	2000	2500	3000	3500
Π(m)			632.94		
F_{RF}(MHz)			350		
2θ(mrad)			60		
$\varepsilon_y/\varepsilon_x$(%)			0.5		
β_x^*/β_y^* (mm)			100/1		
I(A)	2	2	2	2	2
$N_{e/bunch}\times10^{-10}$	9	9	8	9	[illegible]
N_b	292	292	328	29[illegible]	[illegible]
U_0(keV)	130	260	465	773	1220
V_{RF}(kV)	1600	2000	2500	3500	5000
ν_s	0.0164	0.0159	0.0158	0.017	0.019
δ_{RF}(%)	1.9	1.8	1.7	1.7	1.9
$\sigma_e\times10^3$ (SR/IBS+WG)	0.28/1	0.4/1.1	0.5/1.1	0.6/1.1	0.7/1.1
σ_s(mm) (SR/IBS+WG)	4/15	7/15	7/15	[illegible]	[illegible]
ε_x(nm) (SR/IBS+WG)	2.7/8.8	5/5.5	7/4.6	[illegible]	[illegible]
$L_{HG}\times10^{-35}(cm^{-2}s^{-1})$	0.8	1	1	1	1
ξ_x/ξ_y	0.007/0.15	0.005/0.14	0.003/0.1	0.003/0.09	0.004/0.08

SuperKEKB 03.12.2019 $\beta_y^* = 1\ mm$

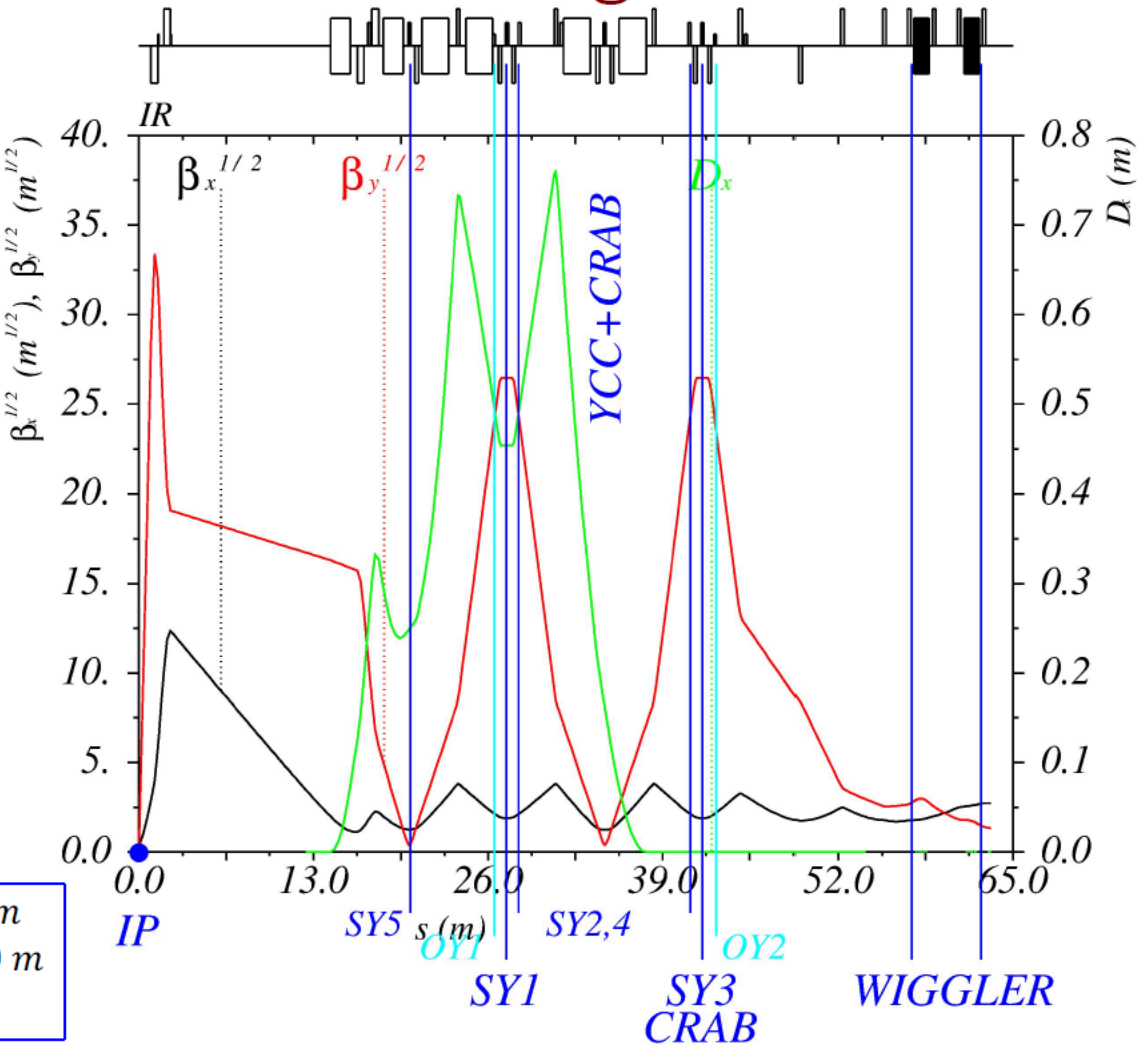
PEPII: I(e+)=3.2 A PEPII
DAFNE : I(e-)=2.45A

SuperKEKB: $L = 3.8\times10^{34}$

From A. V. Bogomyagkov

Crab waist optics design for Super Charm Tau Factory

K.Oide style interaction region



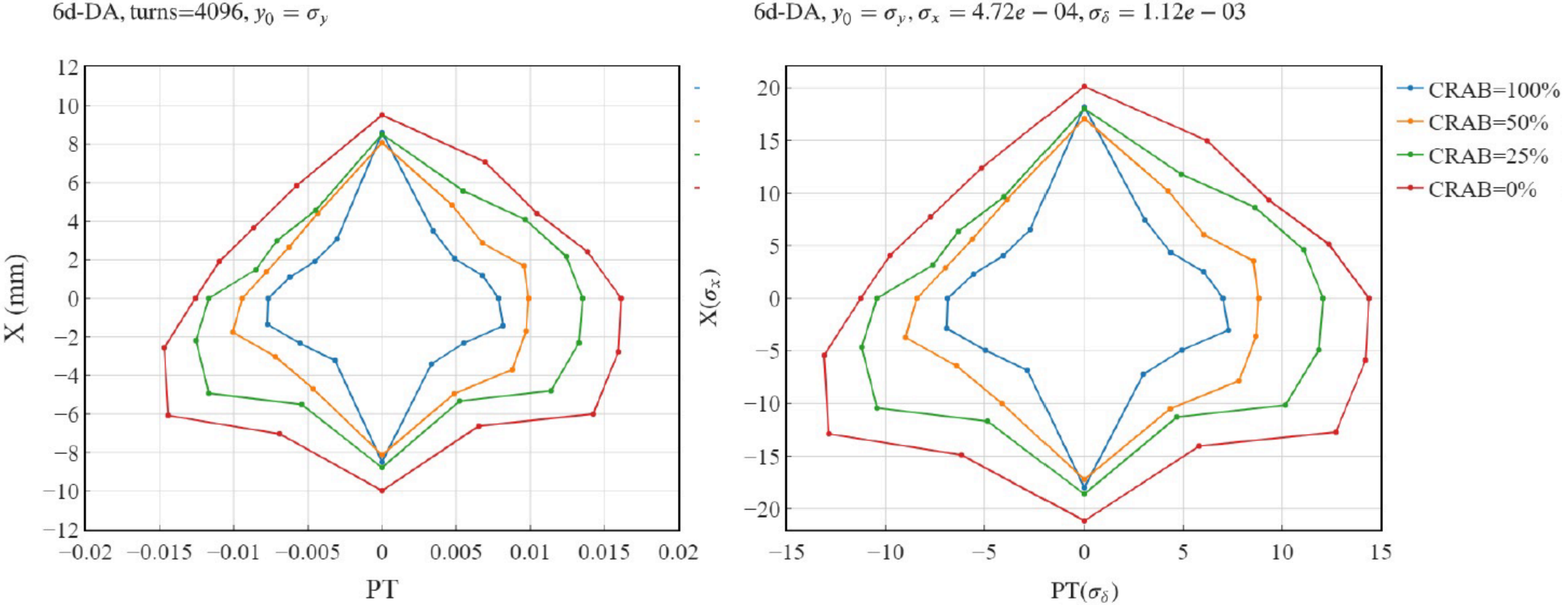
Wiggler at E=1.5 GeV
$\tau_x = 0.3\ s \rightarrow 0.05\ s$

CRAB: $\mu_x = 3\pi, \beta_x = 3.62\ m$
$\mu_y = 2.5\pi, \beta_y = 700\ m$
$K2 = 19.8\ m^{-2}$

K.Oide FCC-ee

From A. V. Bogomyagkov

CRAB sextupole influence on 6d DA, E=1.5 GeV

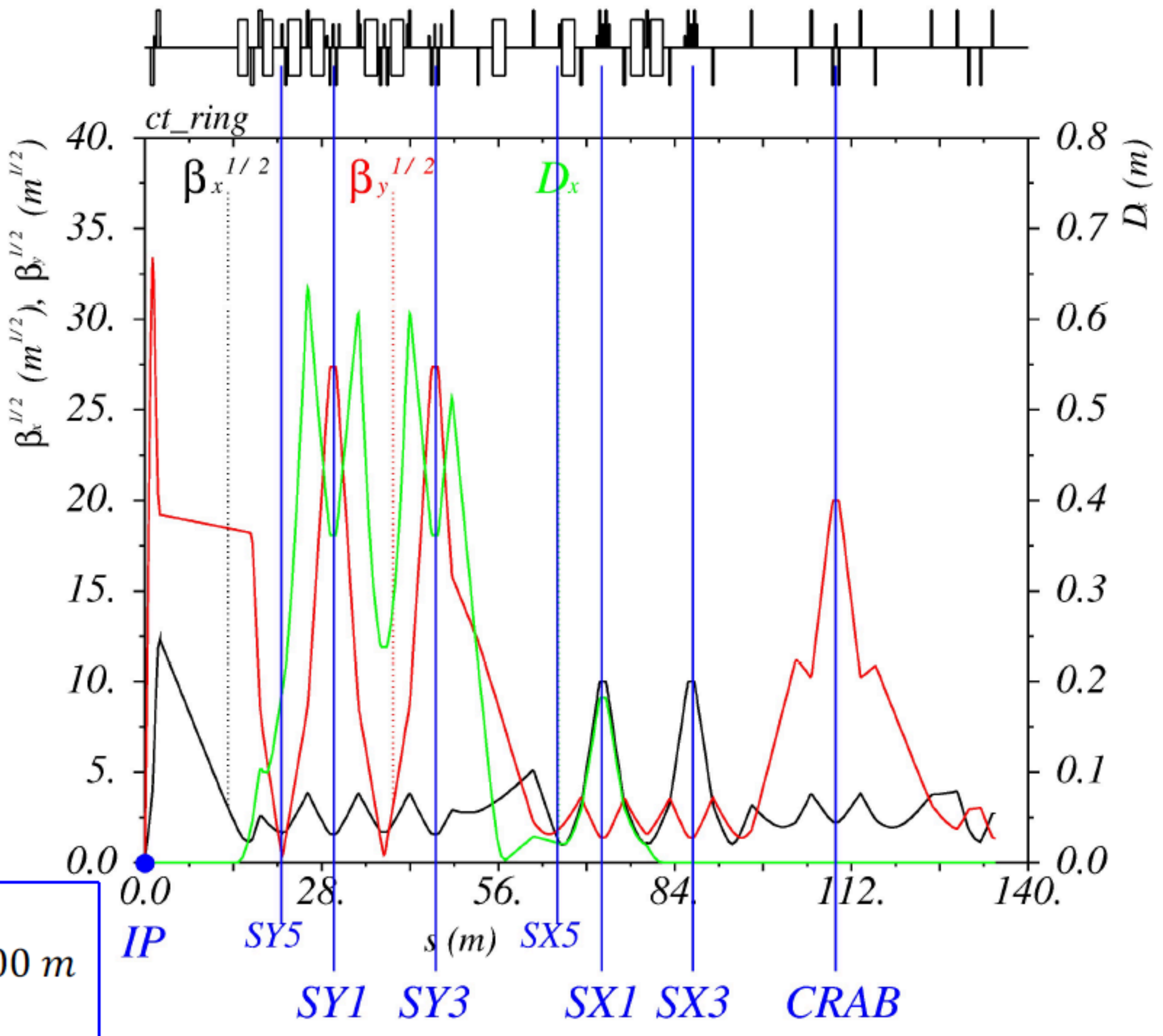


	CRAB=0%	CRAB=25%	CRAB=50%	CRAB=100%
N_{part}	9×10^{10}	9×10^{10}	9×10^{10}	9×10^{10}
$\tau_{Touschek}$(s)	265	155	80	39

From A. V. Bogomyagkov

Crab waist optics design for Super Charm Tau Factory

New Interaction region + CCSX + CRAB (classical)

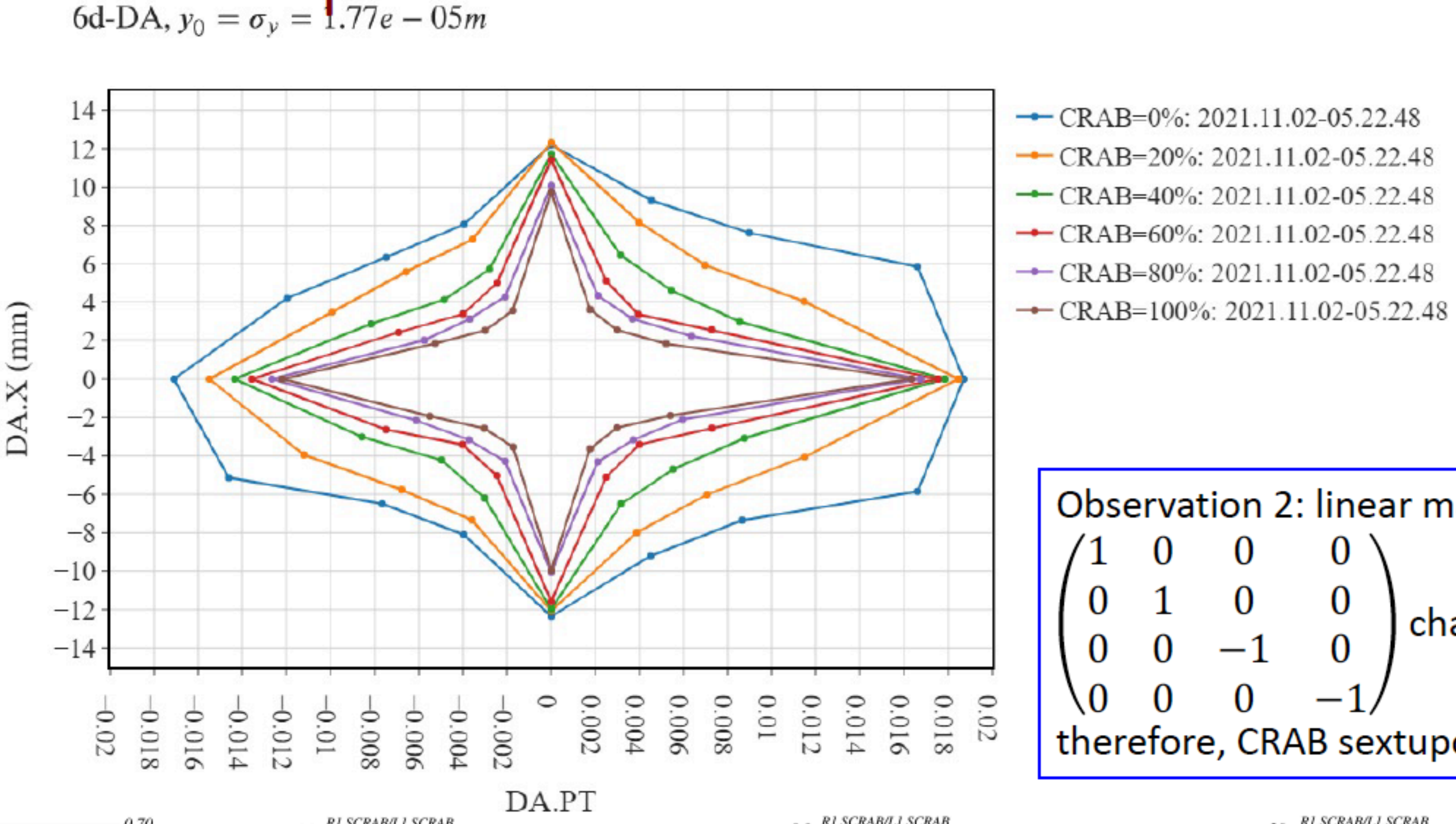


CRAB: $\mu_x = 7\pi, \beta_x = 5\ m$
$\mu_y = 5.5\pi, \beta_y = 400\ m$
$K2 = 29.46\ m^{-2}$

From A. V. Bogomyagkov

Crab waist optics design for Super Charm Tau Factory

CRAB sextupole influence: 6d-DA

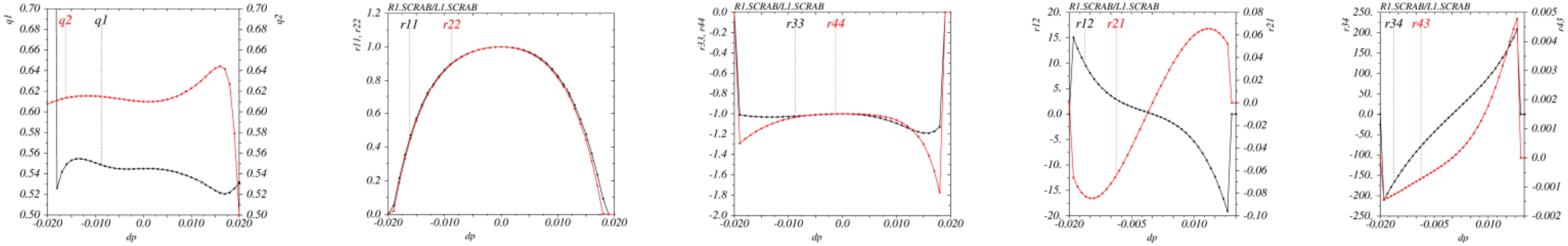


Observation 1: addition of CCSX increased energy acceptance from [-0.8;+1.1] to [-1.2;+1.6]

Observation 2: linear map between CRAB sextupoles

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$ changes with δ;

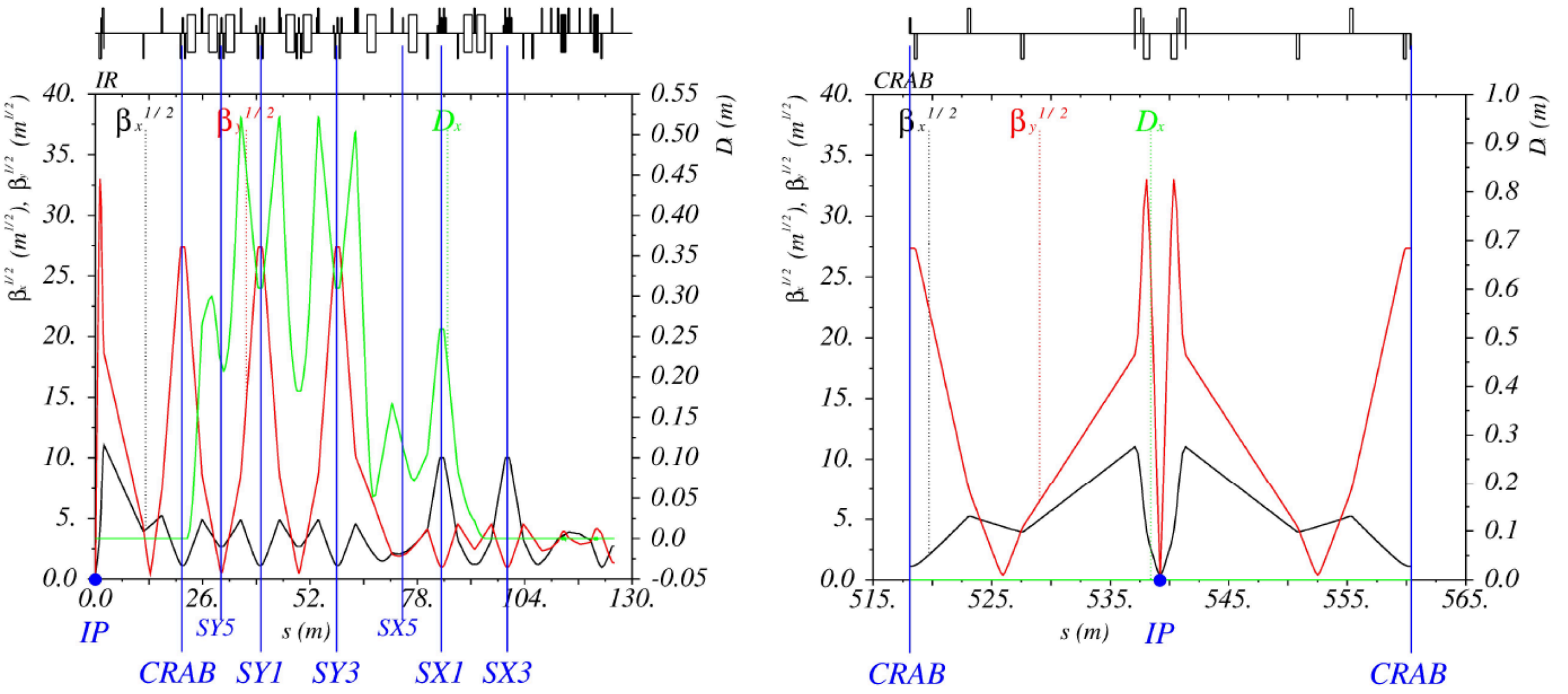
therefore, CRAB sextupoles reduce 6d DA



From A. V. Bogomyagkov

Crab waist optics design for Super Charm Tau Factory

Breaking the rules interaction region

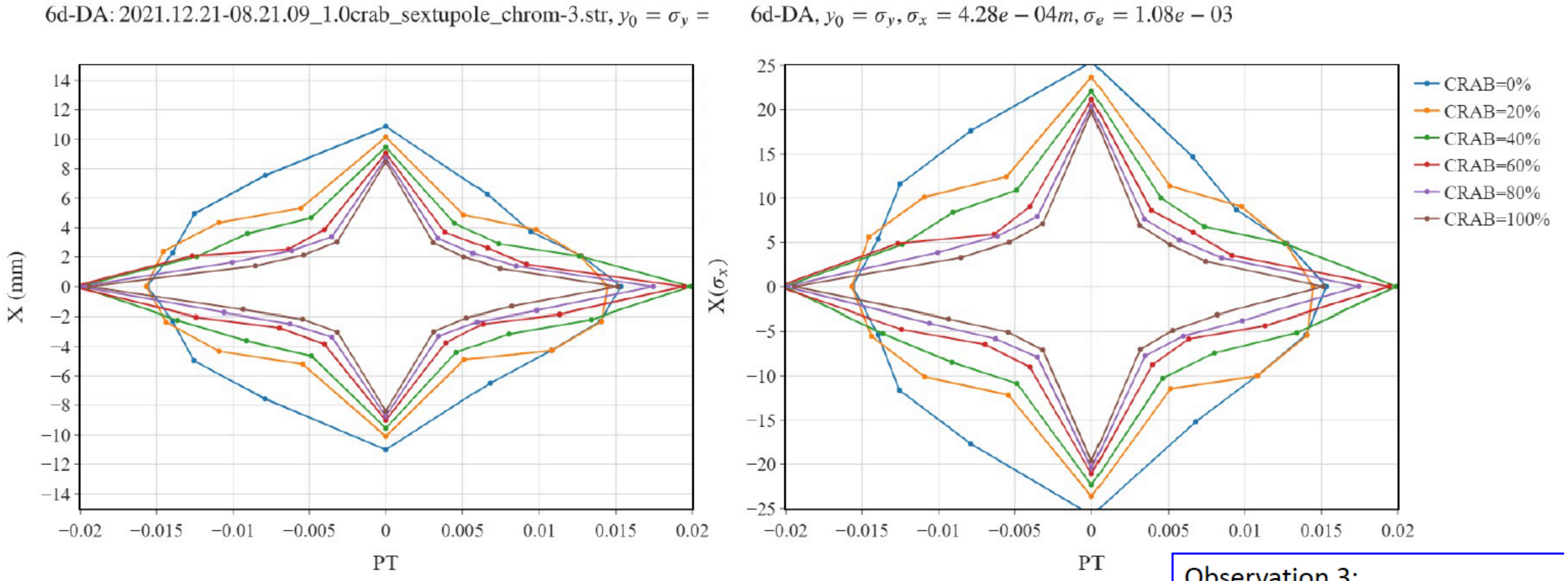


CRAB: $\mu_x = \pi, \beta_x = 1.23\ m$ $\mu_y = 1.5\pi, \beta_y = 750\ m$ $K2_{crab} = -31.72\ m^{-2}$

From A. V. Bogomyagkov

Crab waist optics design for Super Charm Tau Factory

CRAB sextupole influence on 6d DA, E=1.5 GeV



	CRAB=0%	CRAB=60%	CRAB=80%	CRAB=100%
N_{part}	9×10^{10}	9×10^{10}	9×10^{10}	9×10^{10}
$\tau_{Touschek}$(s)	450	126	84	66

Observation 3:
Moving CRAB in front:
increased energy
acceptance from
[-0.8;+1.1] to [-1.9%;+1.5%]
Touschek life time doubled

From A. V. Bogomyagkov

Summary

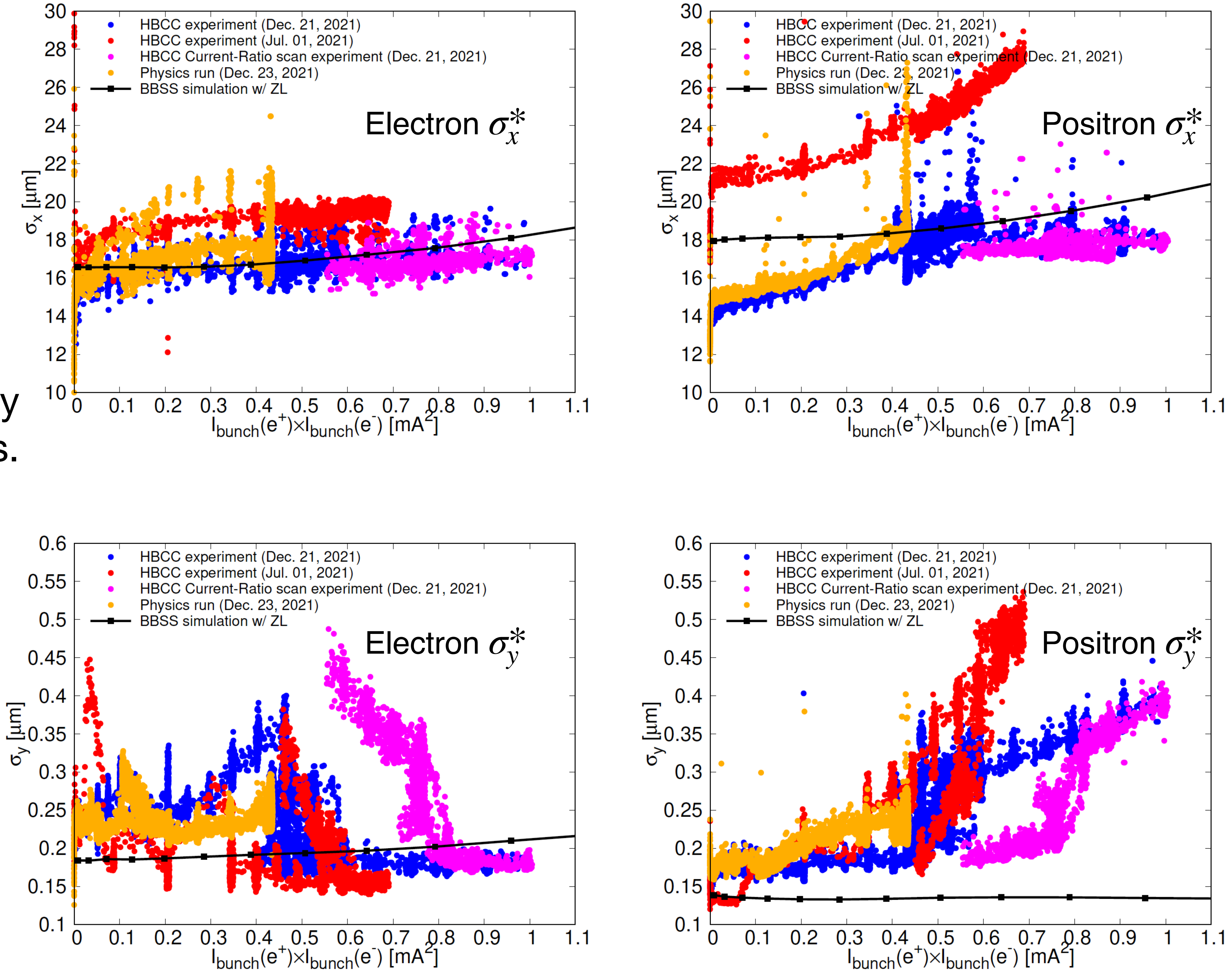
- Crab waist is powerful in suppression of nonlinear beam-beam effects
- The nonlinear IR breaks the crab waist transformation, resulting in loss of dynamic aperture and lifetime
- SuperKEKB and SCTF share similar challenges in optics optimization with crab waist

- International collaborations are welcome!

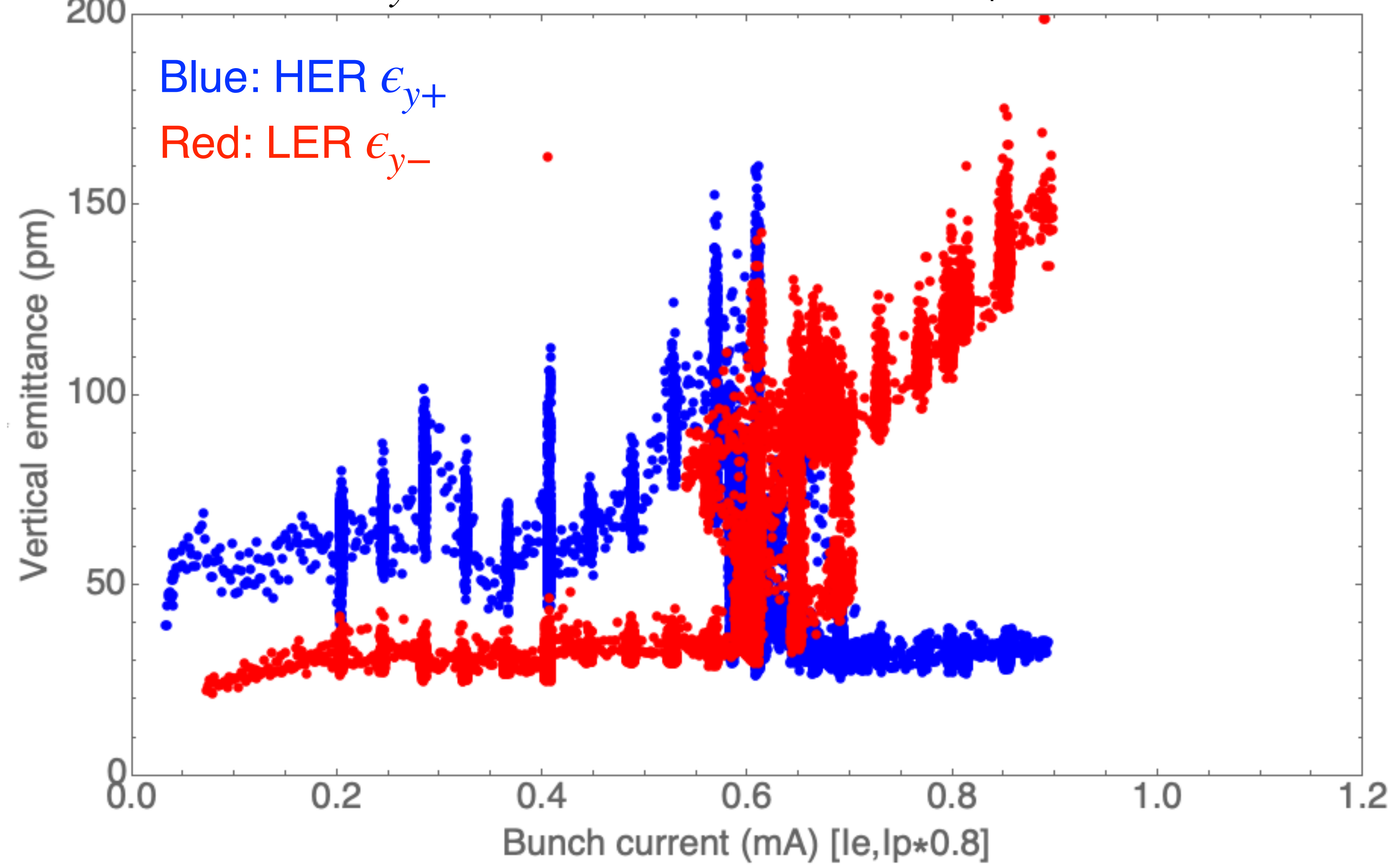
Backup

Beam-beam: simulations vs observations

- Dec. 21-22, 2021: HBCC study
  - LER σ_x^* blowup was partially mitigated by reducing LER ν_x.
  - It was hard to achieve balanced collision ($\sigma_{y+}^* \approx \sigma_{y-}^*$) when $I_{b+}I_{b-} > 0.45$ mA².
  - When bunch current ratio is fixed with I_{b+}/I_{b-}=1.25, a "flip-flop" phenomenon appeared: At lower bunch currents, HER beam seems to be weaker; At higher bunch currents, LER beam is weaker. But balanced collision could be achieved by tune optimization and IP knob tunings at low bunch currents.

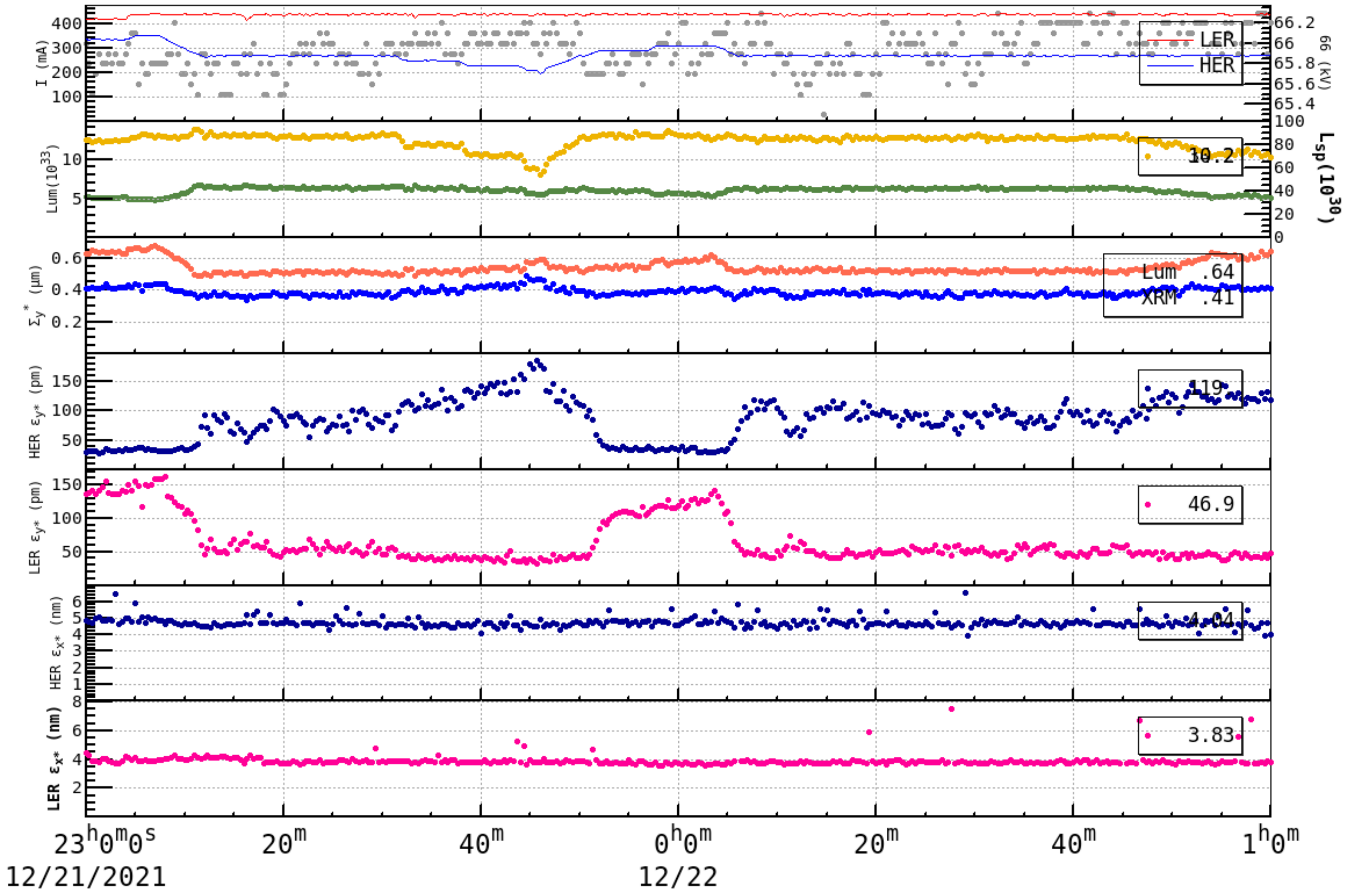


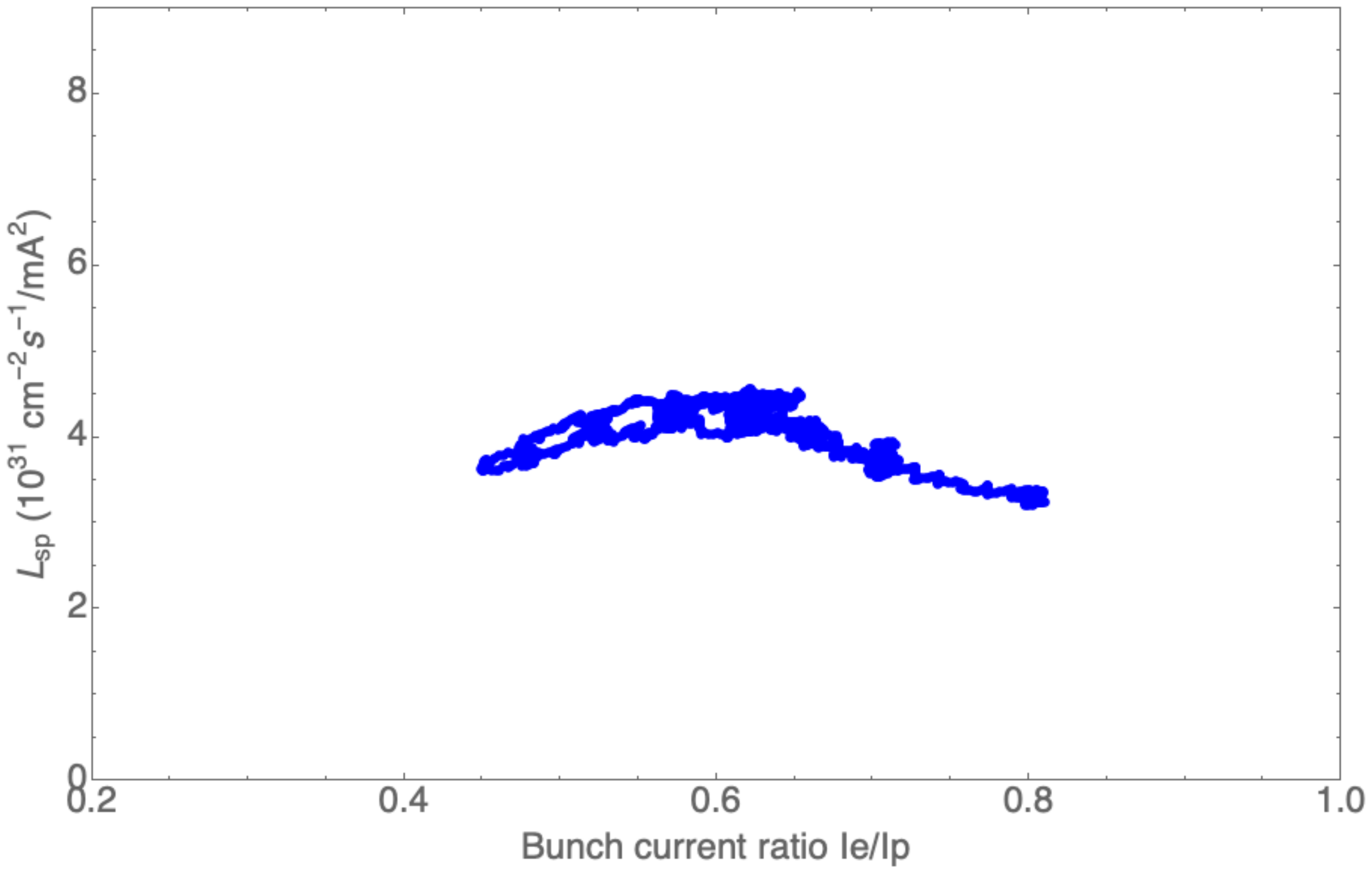
Vertical ϵ_y for HBCC study with fixed I_{b+}/I_{b-}=1.25

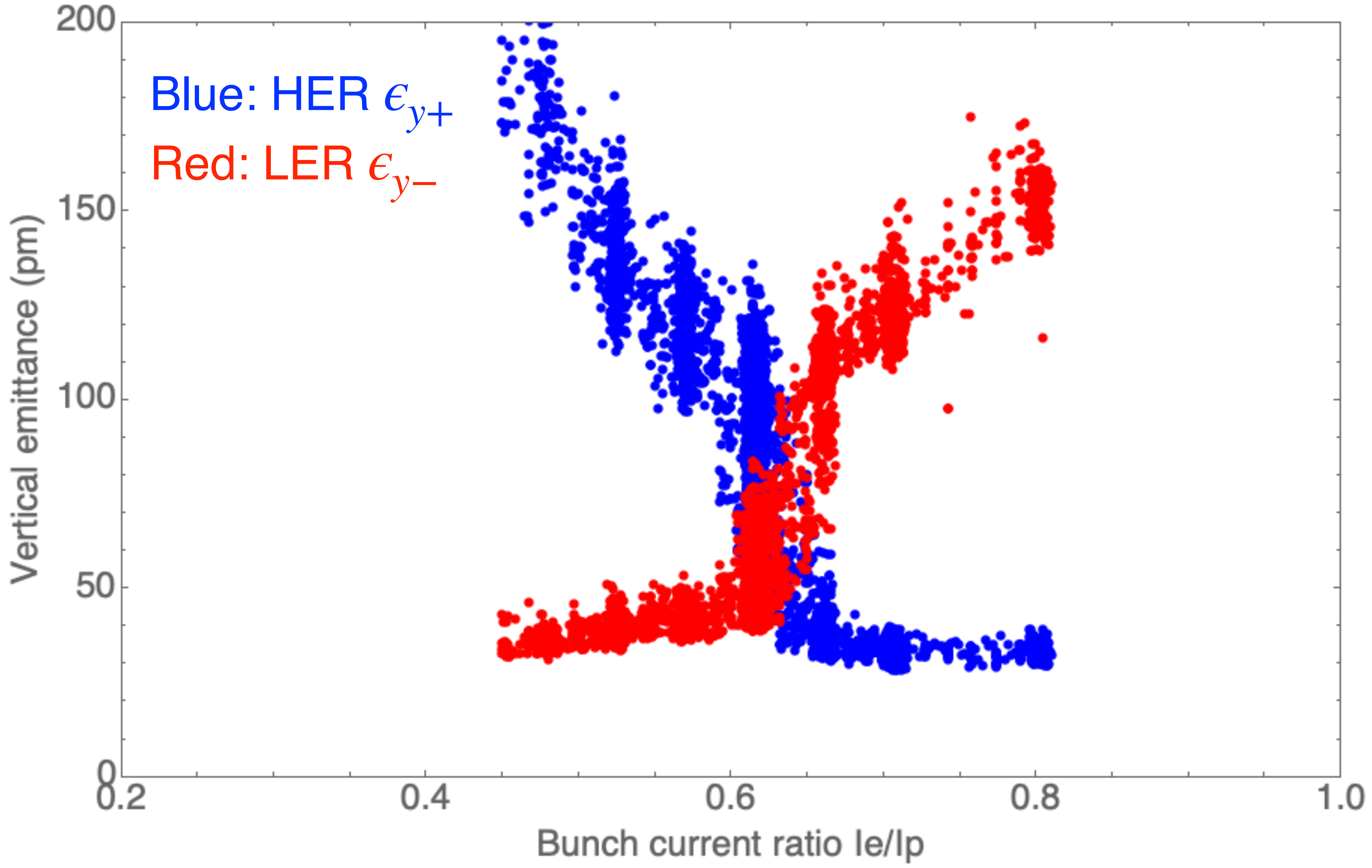


Beam-beam: simulations vs observations

- Dec. 21-22, 2021: HBCC current-ratio study
  - When the LER beam current is fixed at 440 mA (393 bunches), the optimum current ratio (“optimum” means maximum Lsp with $\sigma^*_{y+} \approx \sigma^*_{y-}$) was found at $I_{b+}/I_{b-} \approx 1.7$, close to the energy transparency condition $I_{b+}/I_{b-} = \gamma_-/\gamma_+$.

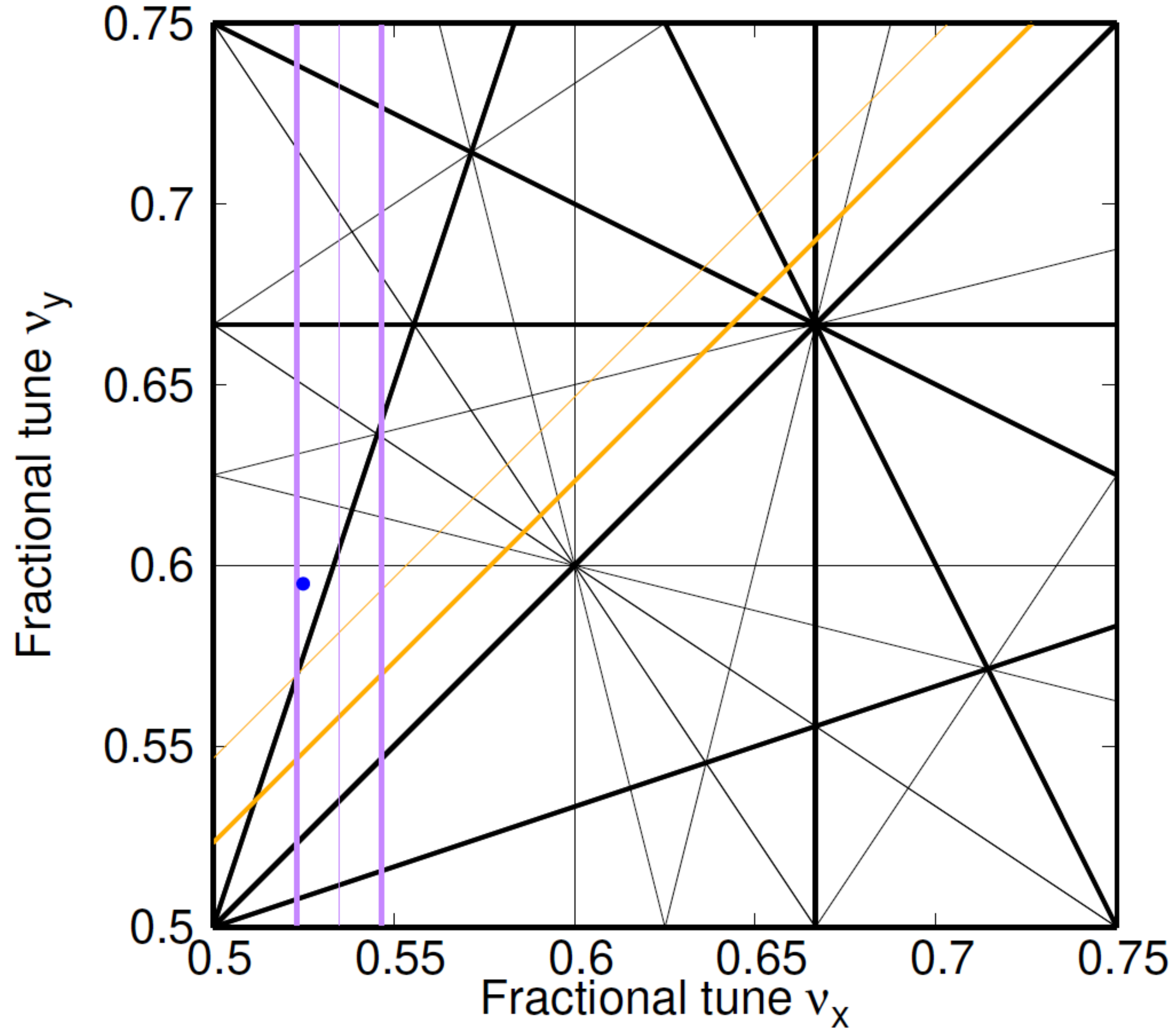


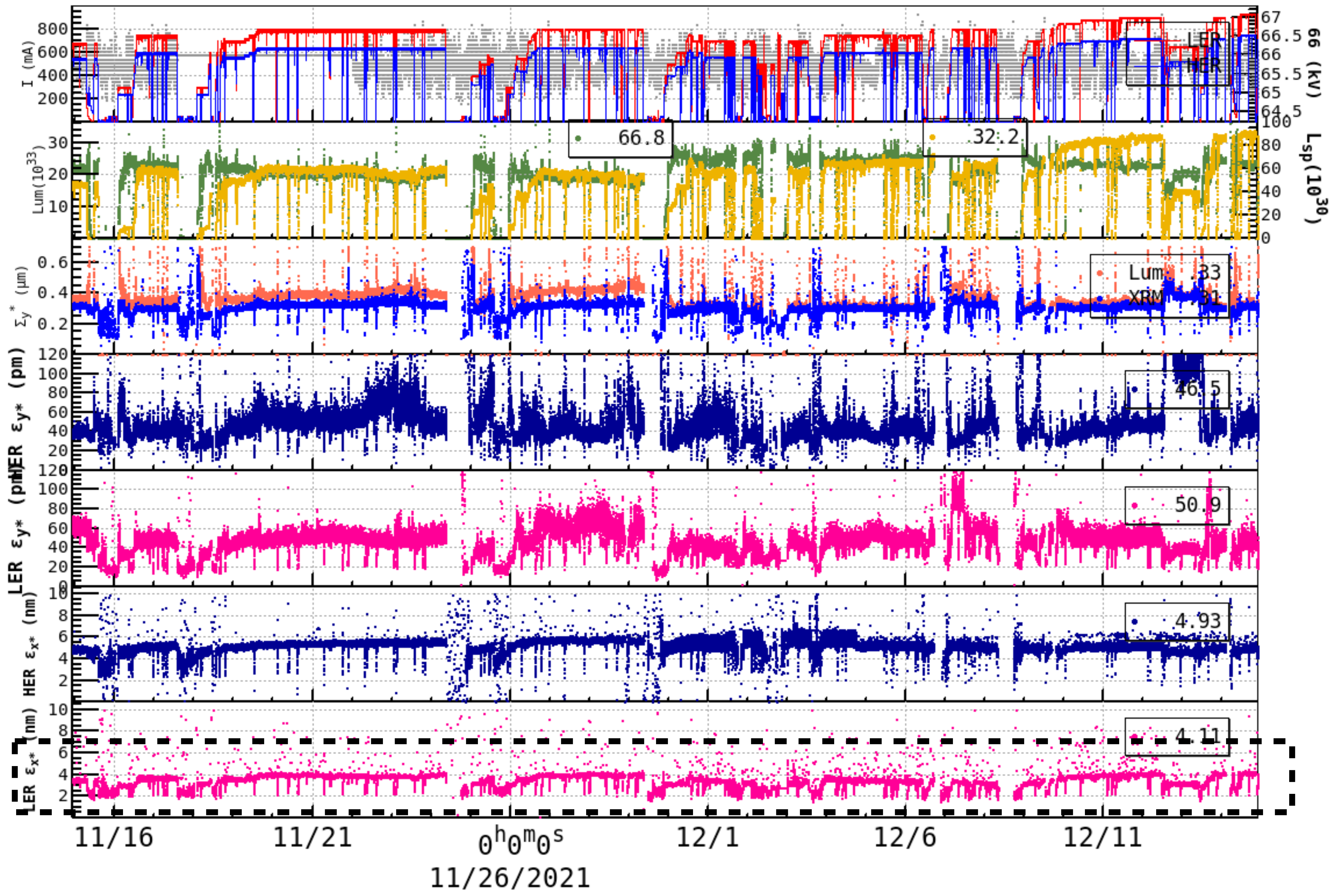




Discussion on candidates for vertical emittance blowup

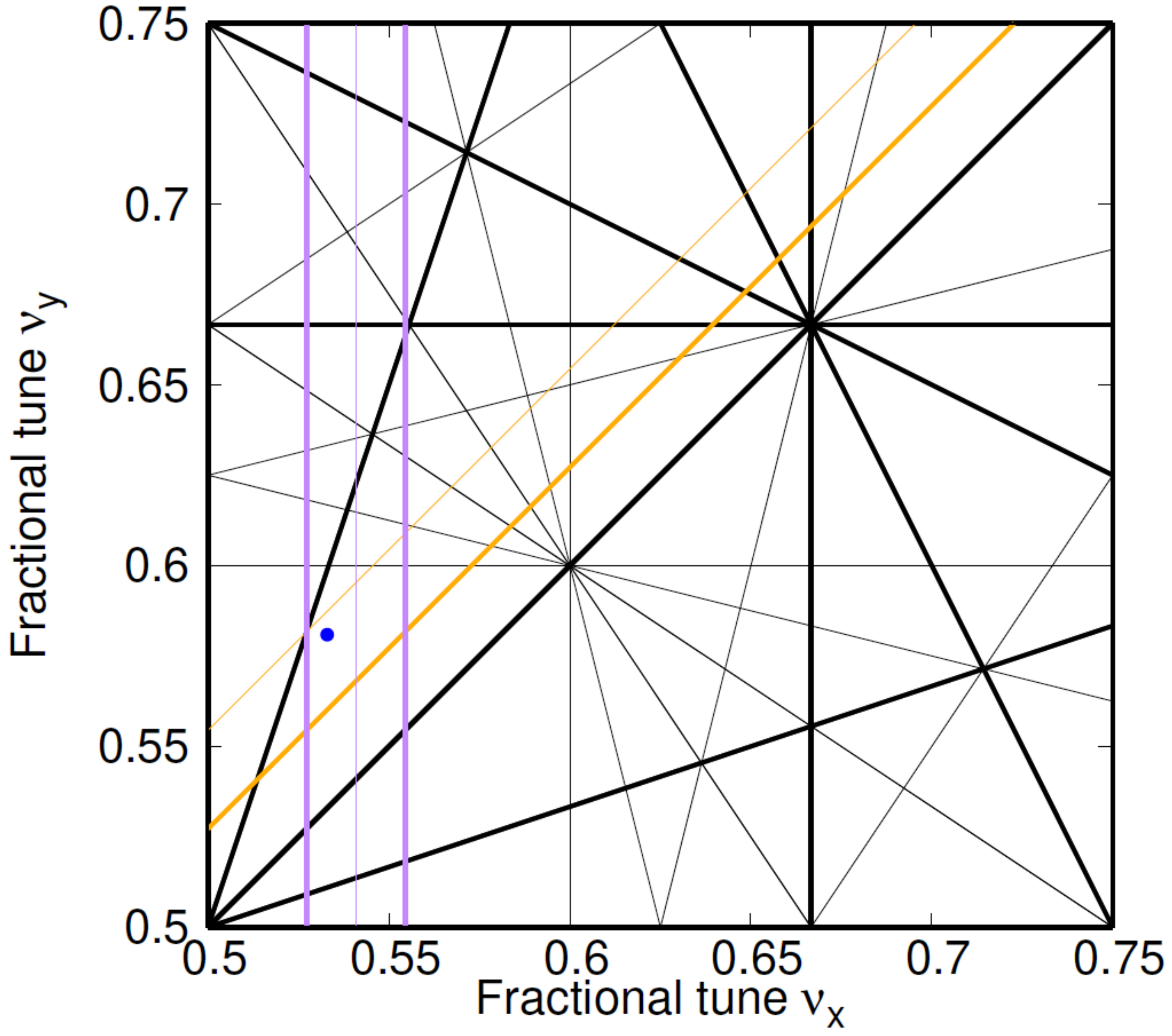
- LER
  - Beam-beam driven synchro-betatron resonance (it means single-beam effect, not BBHTI or X-Z instability which means coherent blowup of both beams. Potential-well distortion cause ν_s spread and increase width of $2\nu_x - k\nu_s = N$ resonances.).
  - “TMCI”: Interplay of beam-beam, impedance and lattice nonlinearity.
  - Imperfect CW (imperfect phase-advance between SLY* magnets, non-perfect CW for off-momentum particles)
  - Others?

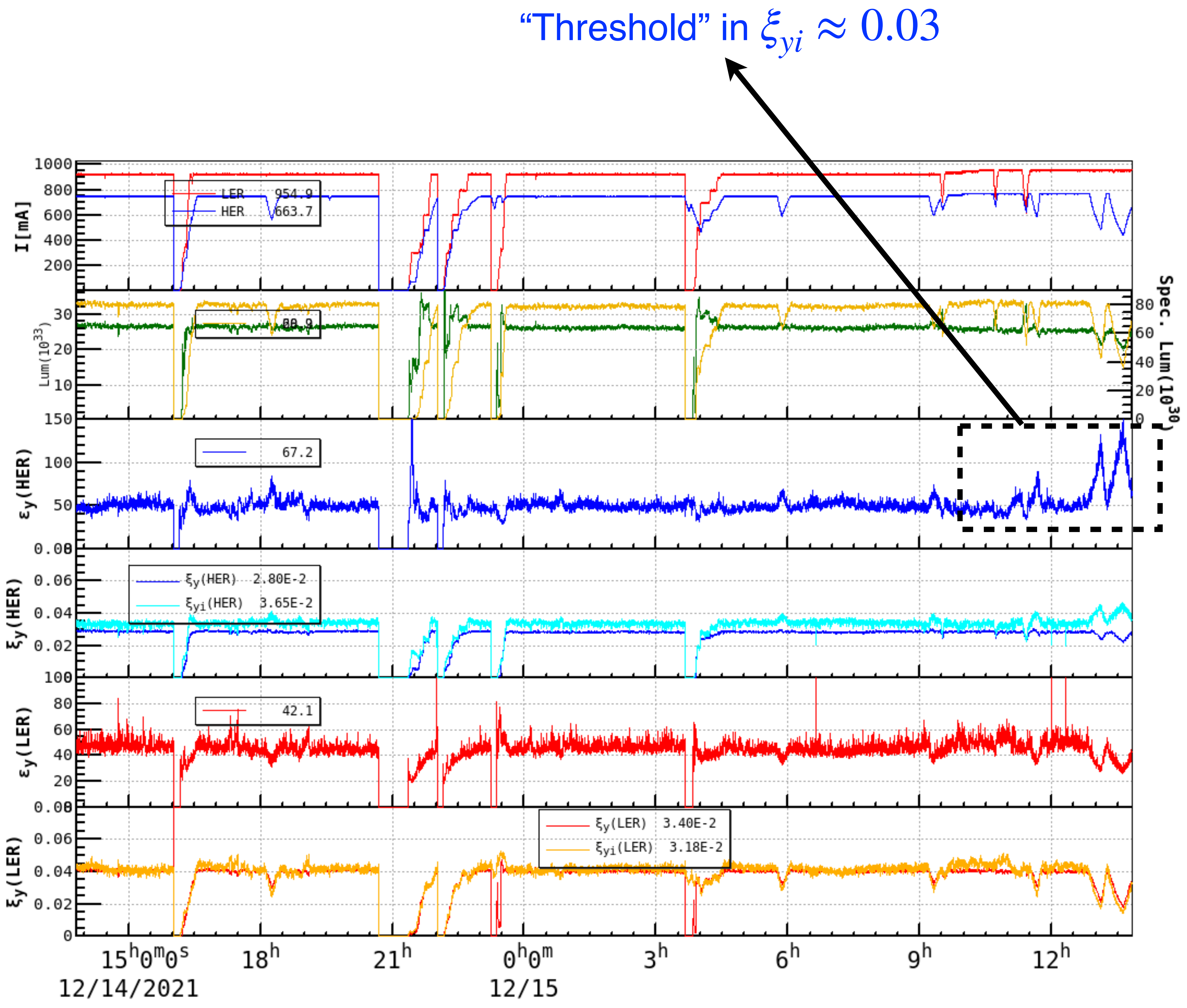




Discussion on candidates for vertical emittance blowup

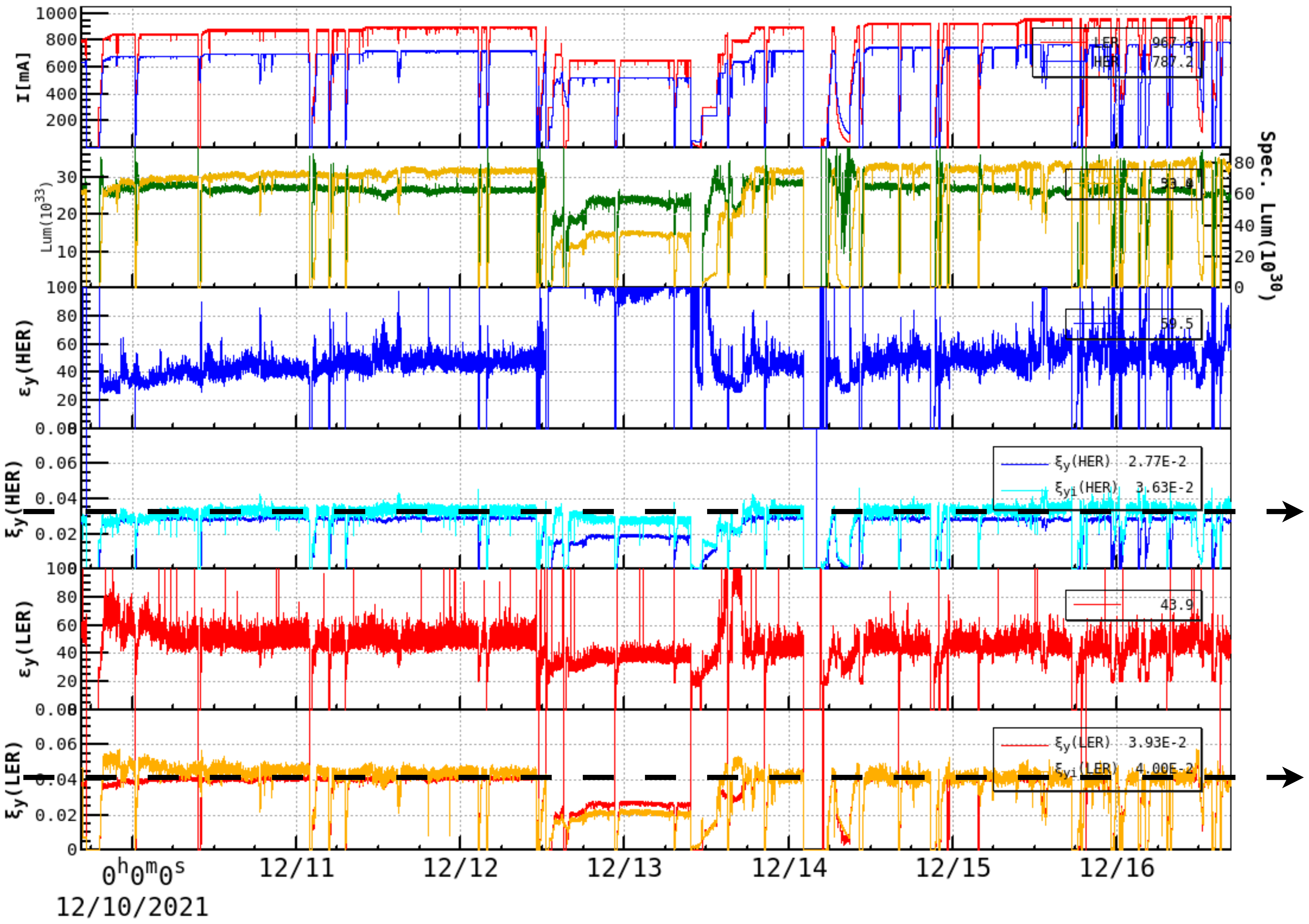
- HER
  - Chromatic coupling ($\nu_x - \nu_y + \nu_s = N$ and $\nu_x - \nu_y + 2\nu_s = N$)
  - $3\nu_x - \nu_y =$ N?
  - Insufficient CW (now 40%, limited by SLY* strengths).
  - Imperfect CW (imperfect phase-advance between SLY* magnets, non-perfect CW for off-momentum particles)
  - Others?





Scaling laws of luminosity

- Beam-beam parameter (tune shift)
  - Under balanced collision ($\sigma^*_{y+} \approx \sigma^*_{y-}$), the two methods for beam-beam parameter (tune shift) are almost equivalent.
  - The currently achieved beam-beam parameters are $\xi_{y+} \approx 0.04$ and $\xi_{y-} \approx 0.03$ (w/ crab waist), which are much lower than the design values of ~0.09 (w/o crab waist). This is the most important challenge at SuperKEKB.



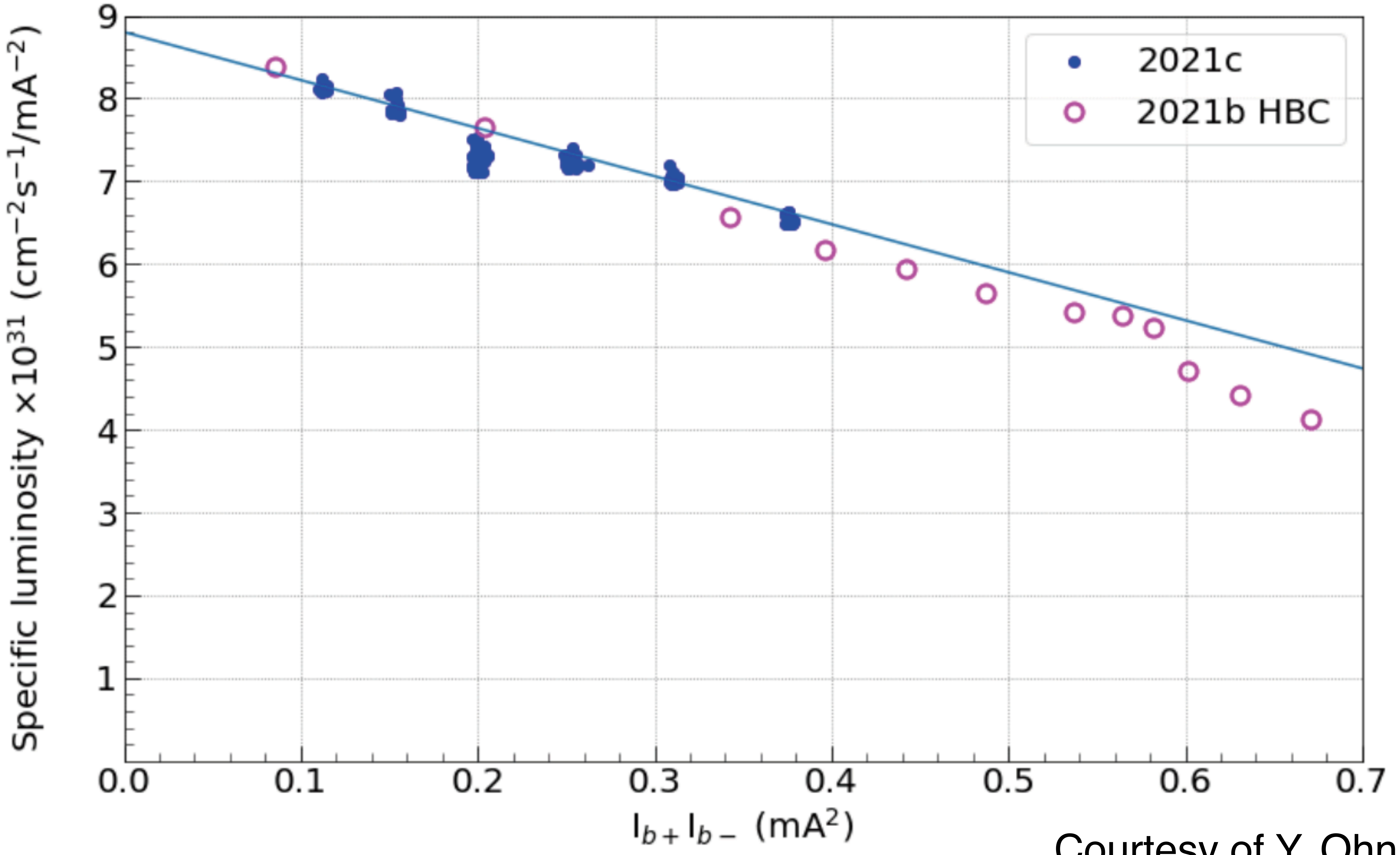
$$L = \frac{1}{2er_e}\frac{\gamma_+ I_+}{\beta^*_{y+}}\xi_{y+} = \frac{1}{2er_e}\frac{\gamma_- I_-}{\beta^*_{y-}}\xi_{y-}$$

$$\xi^i_{y+} \approx \frac{r_e}{2\pi\gamma_+}\frac{N_-\beta^*_{y+}}{\sigma^*_{y-}\sqrt{\sigma^2_{z-}\tan^2\frac{\theta_c}{2} + \sigma^{*2}_{x-}}}$$

$$\xi^i_{y-} \approx \frac{r_e}{2\pi\gamma_-}\frac{N_+\beta^*_{y-}}{\sigma^*_{y+}\sqrt{\sigma^2_{z+}\tan^2\frac{\theta_c}{2} + \sigma^{*2}_{x+}}}$$

Scaling laws of luminosity

- Specific luminosity
  - Specific luminosity L_{sp} is “the last piece of the puzzle” for discussion of reaching 1E35 luminosity at SuperKEKB.
  - The best scenario is: L_{sp} is a constant. It means there are no beam-size blowup.
  - But in the realistic machine, L_{sp} drops when bunch currents increase due to “collective effects”.



$$L_{sp} \approx \frac{1}{2\pi e^2 f \sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}} \sqrt{\sigma_{z+}^2 + \sigma_{z-}^2} \tan\frac{\theta_c}{2}}$$

Courtesy of Y. Ohnishi

Outlook of reaching 1E35 luminosity

- Scenario-1: Constant beam-beam parameter
  - When the machine hits a “beam-beam limit”, the beam-beam parameter will saturate and cannot increase furthers. This is an empirical observation based on experiences from colliders.
  - Let us tentatively accept $\xi_{y+} \approx 0.04$ and $\xi_{y-} \approx 0.03$ which are taken from the current SuperKEKB observation. Then we can simply find the necessary beam currents to achieve 1E35 luminosity. The results are summarized in the table.
  - Note that we achieved 3.815E34 luminosity wit β_y^*=1 mm (Dec. 23, 2021).

β_y (mm)	3.5E+34		6E+34		1E+35	
	HER	LER	HER	LER	HER	LER
1	0.77	1.01	1.32	1.73	2.20	2.88
0.8	0.61	0.81	1.05	1.38	1.76	2.31
0.6	0.46	0.61	0.79	1.04	1.32	1.73
0.4	0.31	0.4	0.53	0.69	0.88	1.15
0.3	0.23	0.3	0.40	0.52	0.66	0.87

$$L = \frac{1}{2er_e}\frac{\gamma_+ I_+}{\beta^*_{y+}}\xi_{y+} = \frac{1}{2er_e}\frac{\gamma_- I_-}{\beta^*_{y-}}\xi_{y-}$$

Outlook of reaching 1E35 luminosity

- Scenario-2: Given specific luminosity slope
  - From the observed specific luminosity slope (see page.13), we can estimate the total luminosity with given beam currents.
  - We can assume $L_{sp}[10^{31}$ cm^{-2}s^{-1}/mA$^2] = 8.8 - 5.8 I_{b+} I_{b-}$[mA^2]. Note that this scaling law is only valid for for β_y^*=1 mm.
  - Also I assume bunch current ratio of $I_{b-}/I_{b+} = 0.8$ which is currently used at SuperEKKB. The possible bunch current products and number of bunches are listed in the table and resulting luminosity [scaled by 1E35].
  - Squeezing β_y^* is effective to increase L_{sp}, but has many other side effects (not discussed here).

Bunch number	$I_{b+}I_{b-}$ [mA^2]		
	0.5	0.7	1
1270	0.41	0.49	0.53
1370	0.44	0.53	0.57
1565	0.51	0.61	0.65
2000	0.65	0.78	0.83
2500	0.81	0.97	1.04

$$L_{sp} = \frac{L}{N_b N_+ N_- (ef)^2} \approx \frac{1}{2\pi e^2 f \sqrt{\sigma_{y+}^{*2} + \sigma_{y-}^{*2}} \sqrt{\sigma_{z+}^2 + \sigma_{z-}^2} \tan\frac{\theta_c}{2}}$$