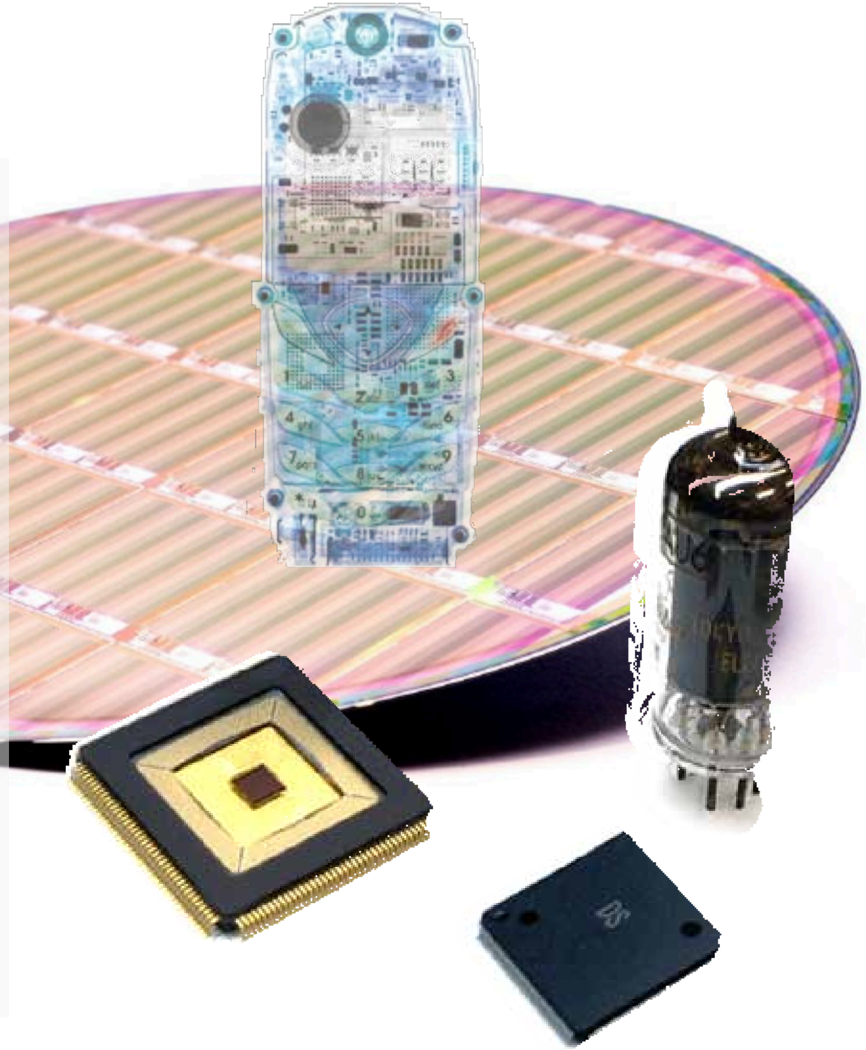
筑波大学数理物質科学研究科 物理学専攻

エレクトロニクス・データ処理
（放射線計測とLSI）
#3

2016年7月1日
高エネルギー加速器研究機構
素粒子原子核研究所
新井康夫　(yasuo.arai@kek.jp)



- Lecture Schedule（金曜第3時限 12:15-13:30）
  - June　17, 24
  - July　1, 8, *15(No lecture)*, 22, 29
  - Total 6 times
- Contents
  (http://research.kek.jp/people/araiy/16lecture/)
  - ❑ Radiation Measurement and LSI
  - ❑ Analog CMOS Circuit for Radiation Measurement
  - ❑ Digital LSI Circuit
  - ❑ Semiconductor Radiation Detector
- 単位認定(Credit)
  講義出席及びレポート
  （田中氏の分と合算）

Lecture Plan

1. Radiation measurement and LSI
   高エネルギー物理実験とLSI、
   LSI技術の変遷
   半導体放射線検出器

2. Basic low and tools
   オームの法則、
   信号伝送、
   信号規格 …

3. Semiconductor devices
   半導体の基礎、
   半導体プロセス、
   MOSデバイス基礎、
   …

4. Analog CMOS circuit
   1段増幅回路、差動増幅回路、
   カレントミラー、
   アナログ・シミュレーション、
   オペアンプ回路、
   雑音, …

5. Digital LSI circuit
   CMOS論理回路、メモリー、
   演算器、カウンター、同期回路, 順序回路、ADC, TDC、…
   論理合成

6. Semiconductor radiation detector
   放射線の相互作用
   検出器の動作原理、
   実例、…

3. Semiconductor Devices

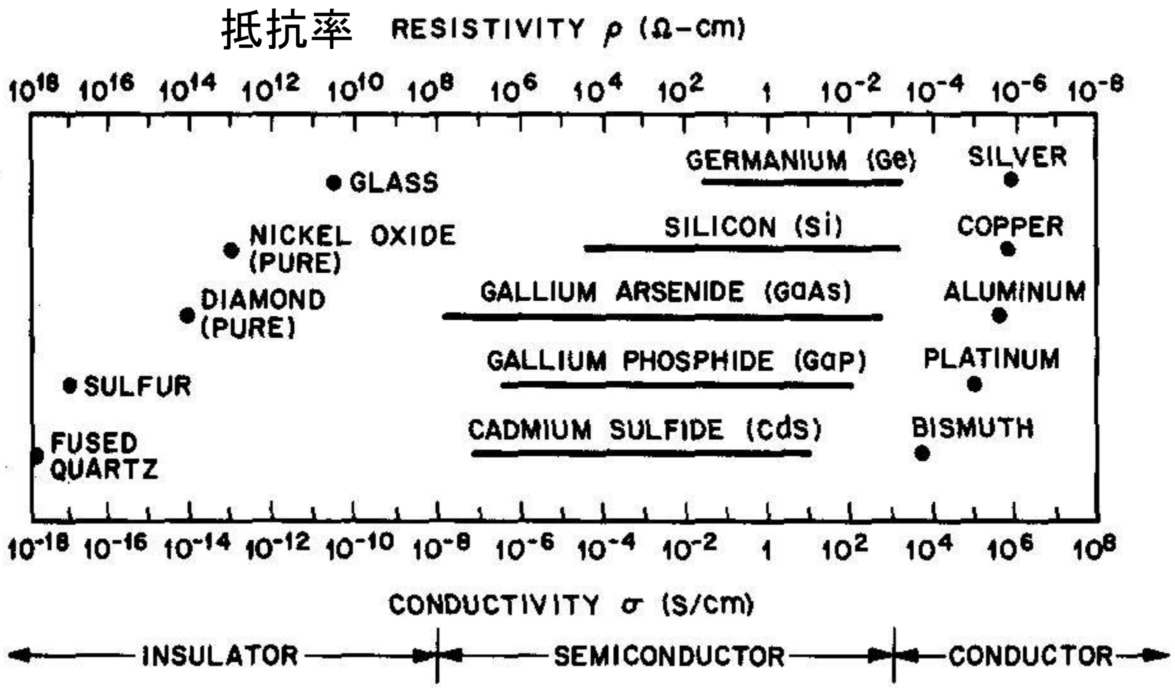
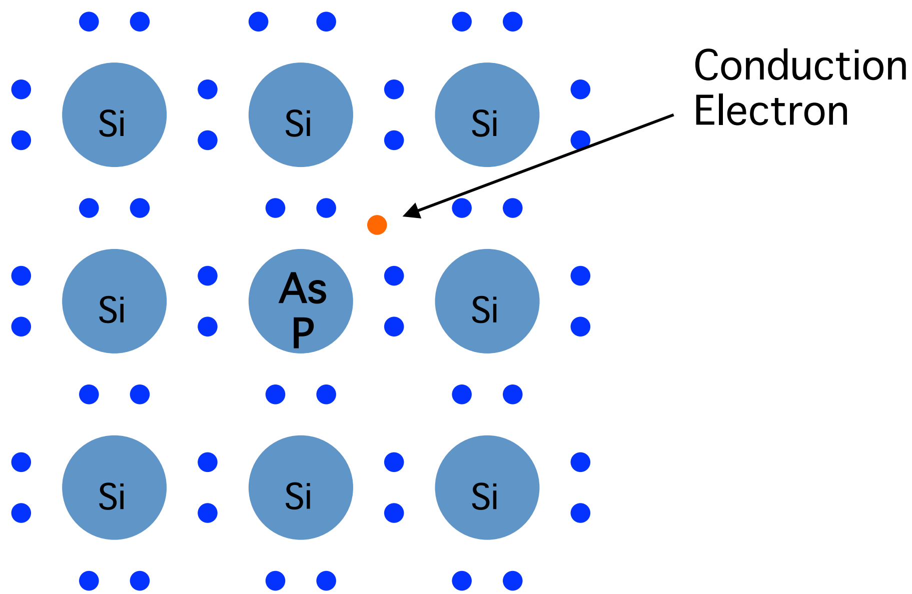
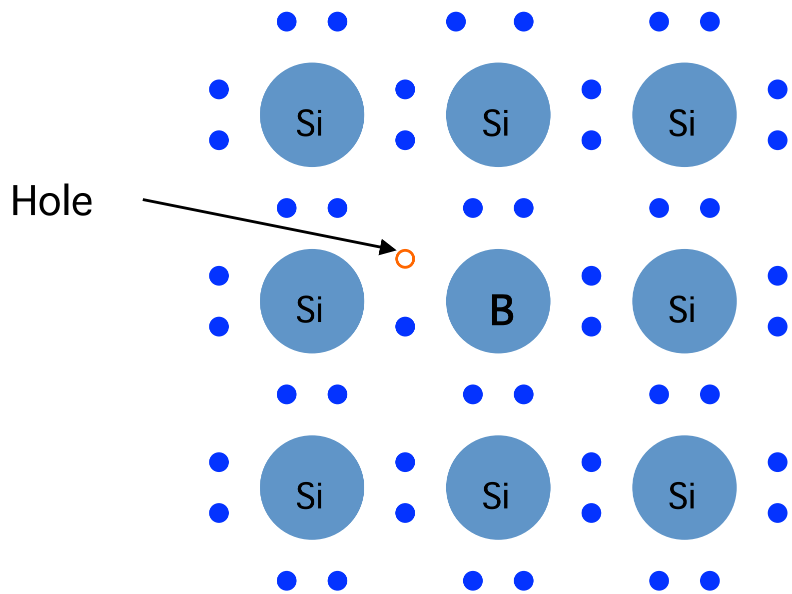


Fig. 1 Typical range of conductivities for insulators, semiconductors, and conductors.

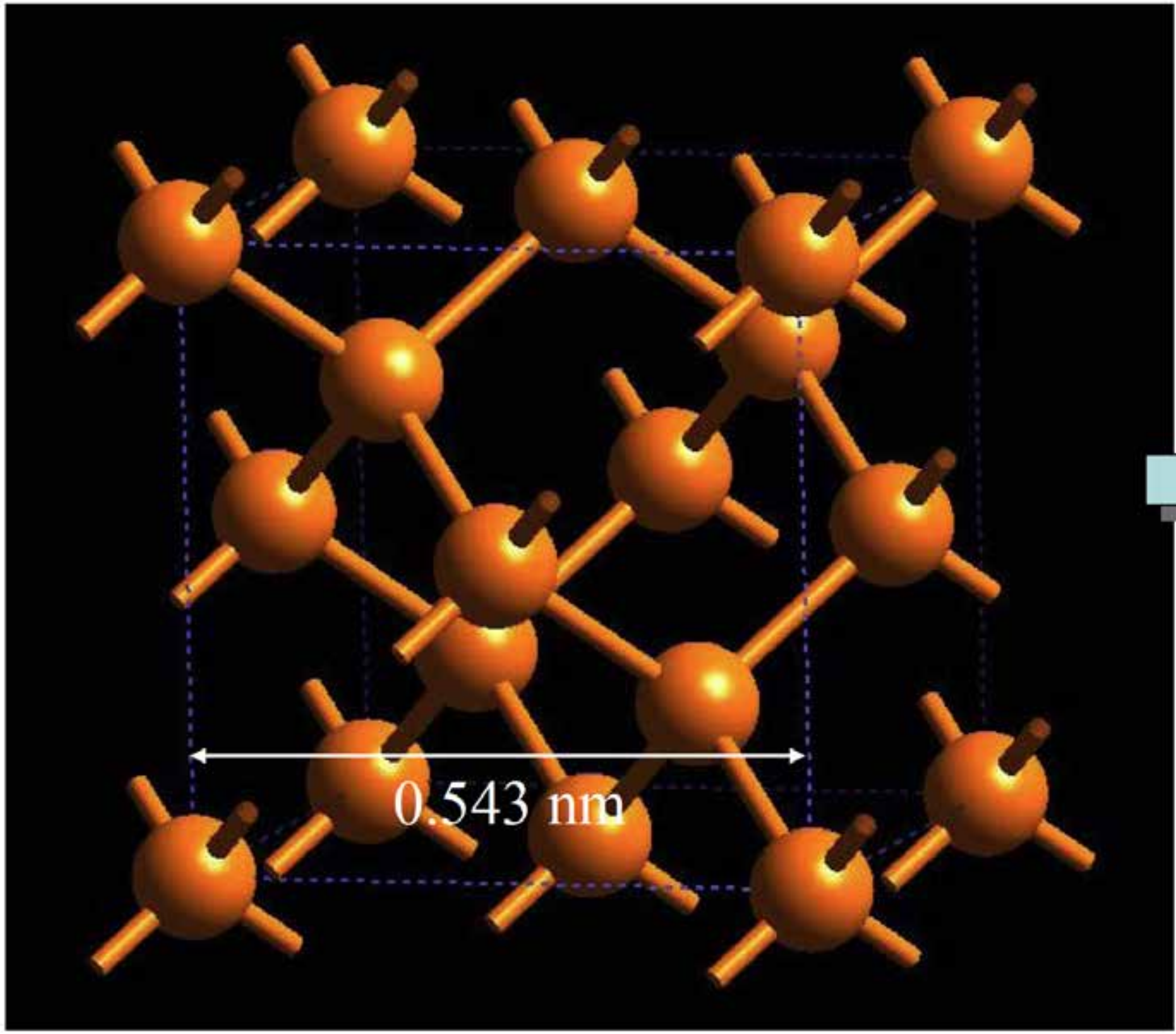
n-type Si



p-type Si

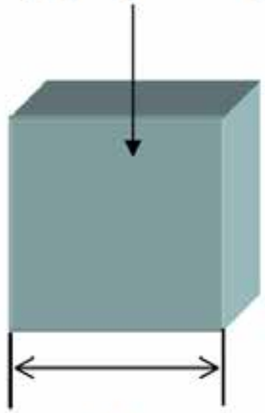


シリコンの結晶構造



Diamond Structure

Si原子　8個



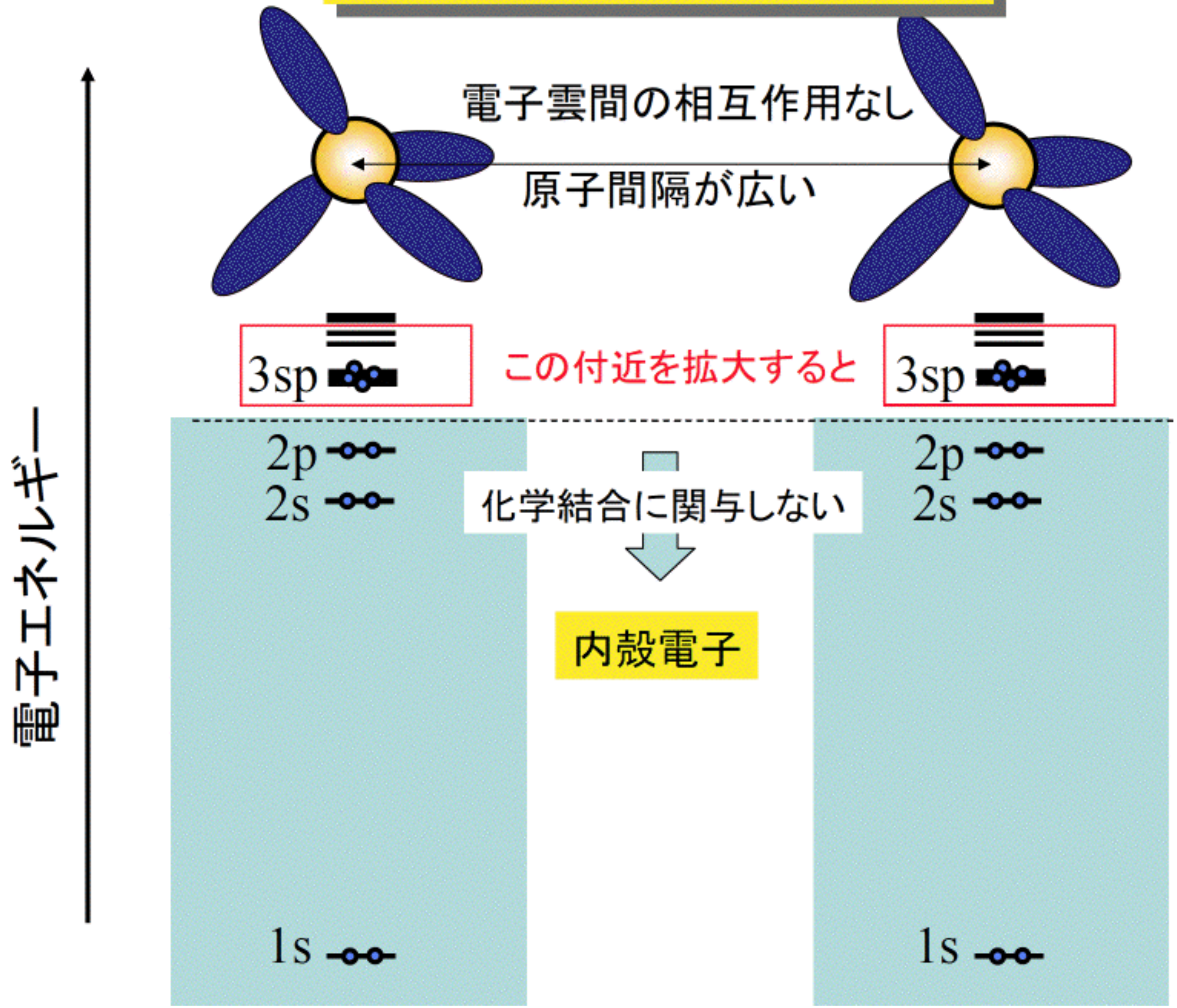
(覚え易い数字)

5×10^{22} 個$/cm^3$

（阪大　谷口研二）

On the other hand, Impurity density is $10^{12}\sim10^{20}$ cm^{-3}

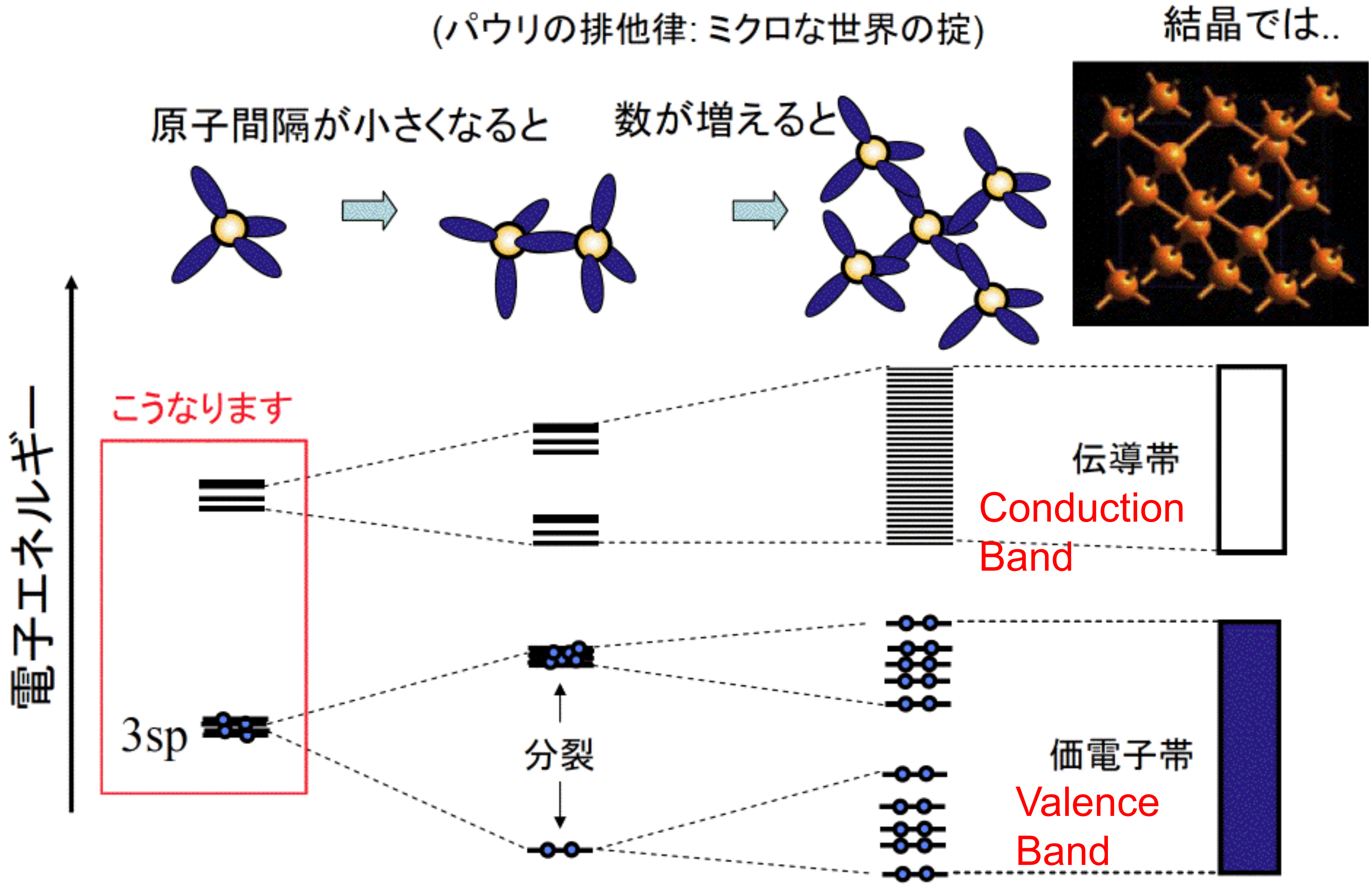
電子軌道とエネルギー(1)



（阪大　谷口研二）

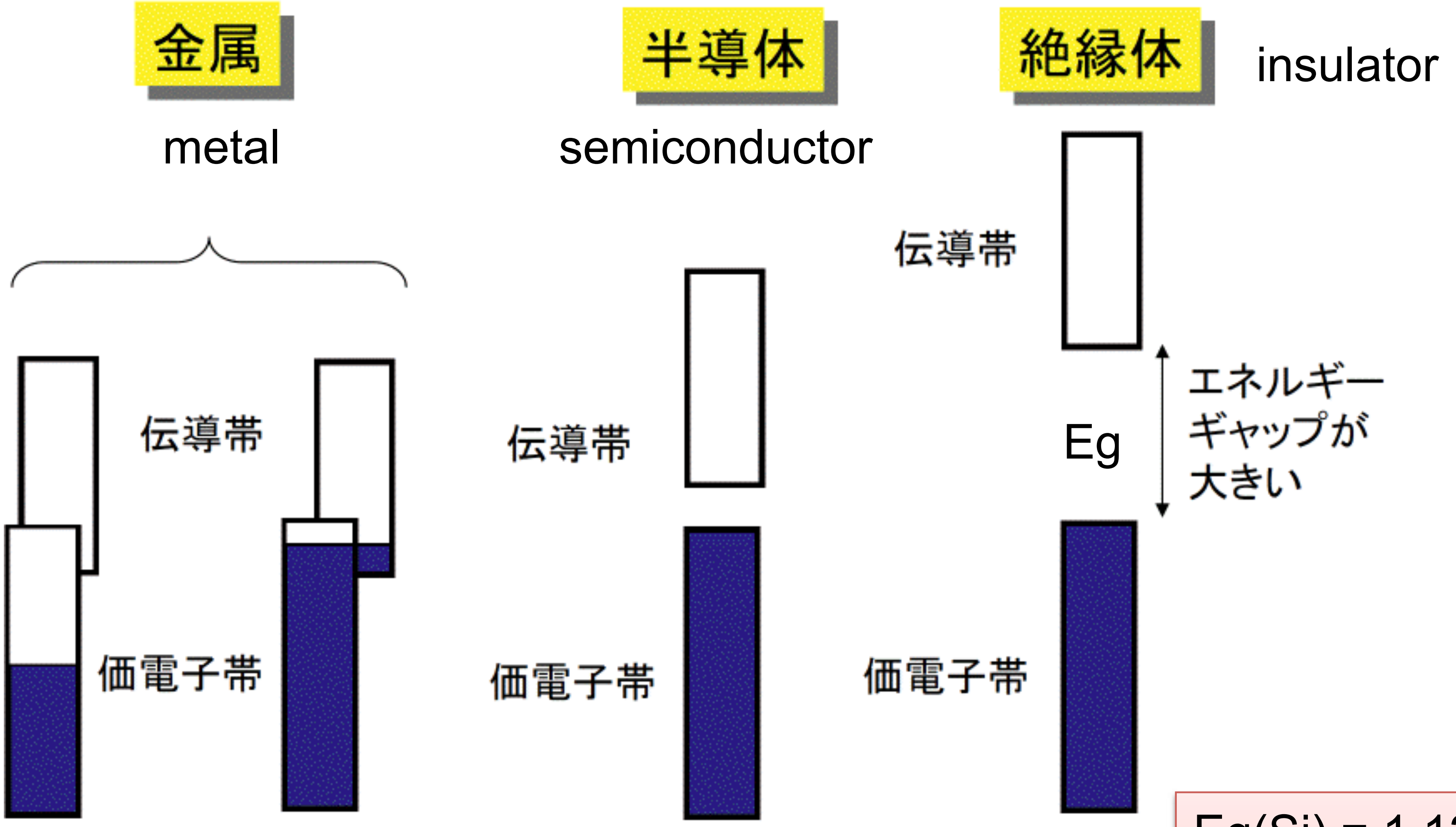
電子軌道とエネルギー(2)

(パウリの排他律: ミクロな世界の掟)



（阪大　谷口研二）

電子帯構造 Band Structure

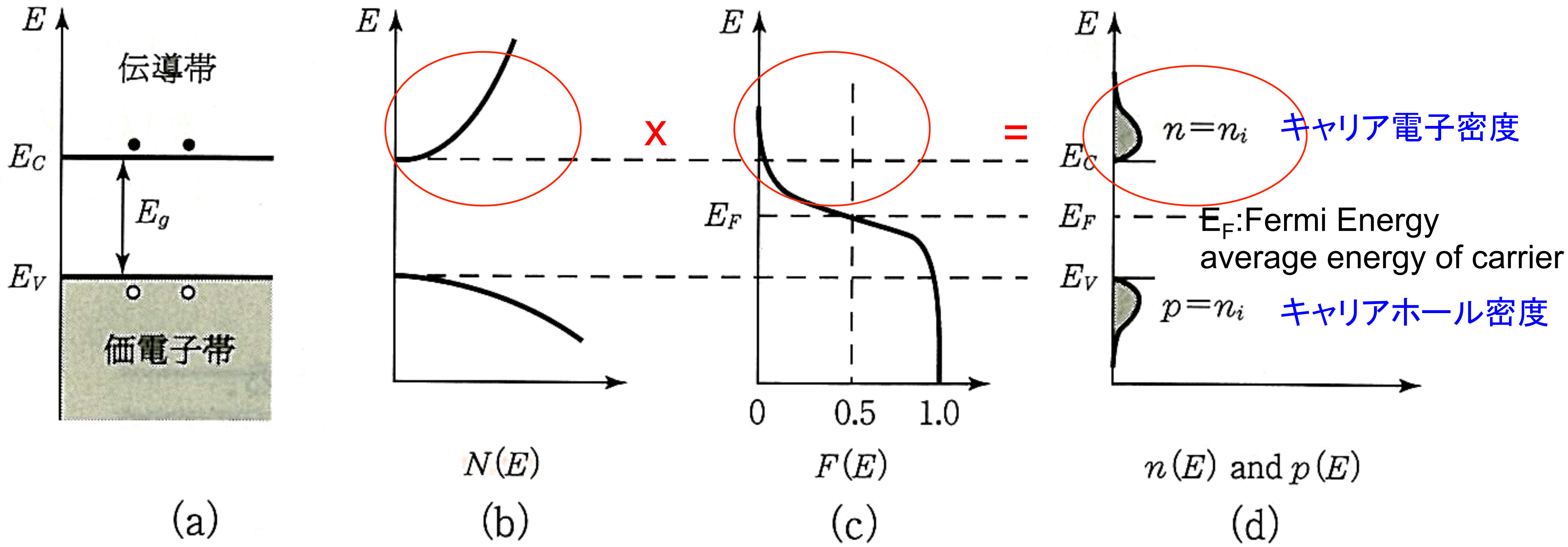


電子帯構造: ①結晶構造、②構成元素、③原子間距離
光学的性質や電気伝導に大きく影響する

Eg(Si) = 1.12 eV
Eg(Ge) = 0.66 eV
Eg(GaAs)=1.42 eV
kT/q ~ 25 meV @室温

（阪大　谷口研二）

Carrier density of Intrinsic semiconductor



真性半導体における，(a) バンド図，(b) 状態密度，(c) フェルミ分布関数，(d) キャリア密度.

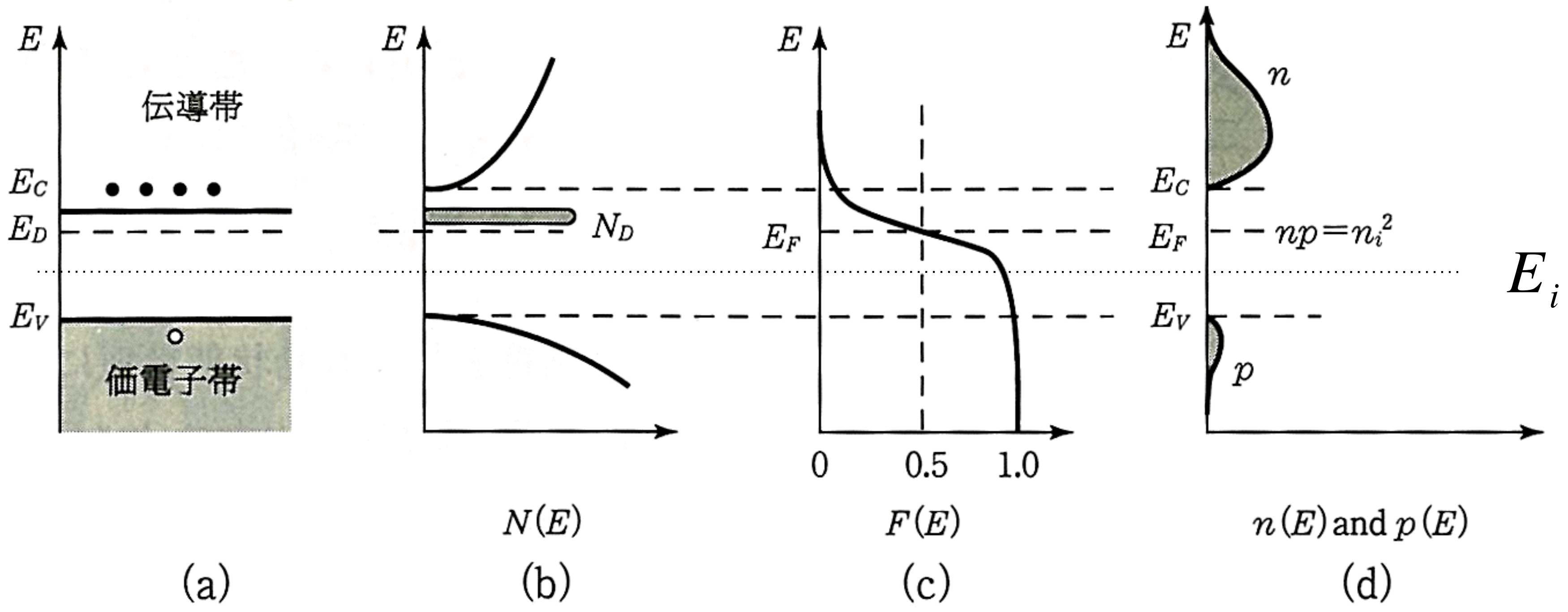
Intrinsic carrier density :

$$n_i = n = p \approx 10^{10}\ cm^{-3}\ @300\mathrm{K}$$

$$\propto \exp\left(-Eg/2kT\right)$$

Intrinsic semiconductor:
No impurity in the semiconductor
（not p-type nor n-type）

Carrier density of Extrinsic semiconductor



n 形半導体の（a）バンド図，（b）状態密度，（c）フェルミ分布関数，（d）キャリア密度．
$np=n_i^2$ であることに注意．

$$n = n_i \exp\left((E_F - E_i) / 2kT \right)$$

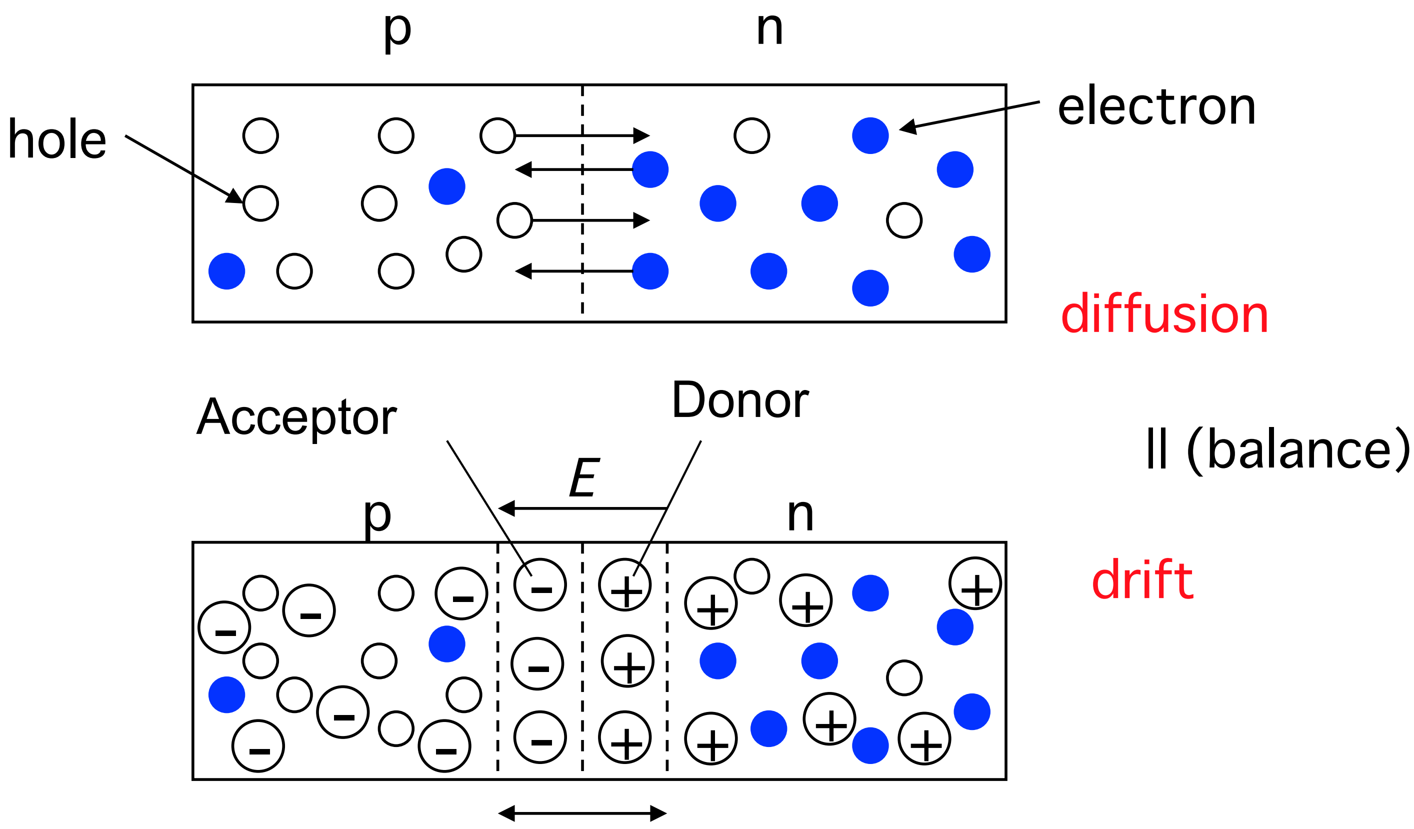
$$p = n_i \exp\left((E_i - E_F) / 2kT \right)$$

E_i : Fermi Level of the Intrinsic Semiconductor

$$np = n_i^2$$

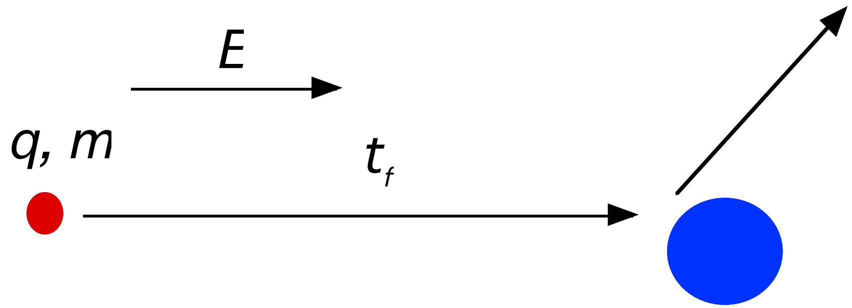
mass action law
(質量作用則
誤訳? → 多数作用則)

p-n junction



空乏層(depletion layer)

Drift



$$f = m \cdot a$$

$$\delta x = \frac{1}{2} a \cdot t_f^2 = \frac{f}{2m} t_f^2$$

$$\therefore v_{drift} = \frac{\delta x}{t_f} = \frac{f \cdot t_f}{2m}$$

ここで $f = q \cdot E$

$$\therefore v_{drift} = \frac{q \cdot t_f}{2m} E = \mu \cdot E$$

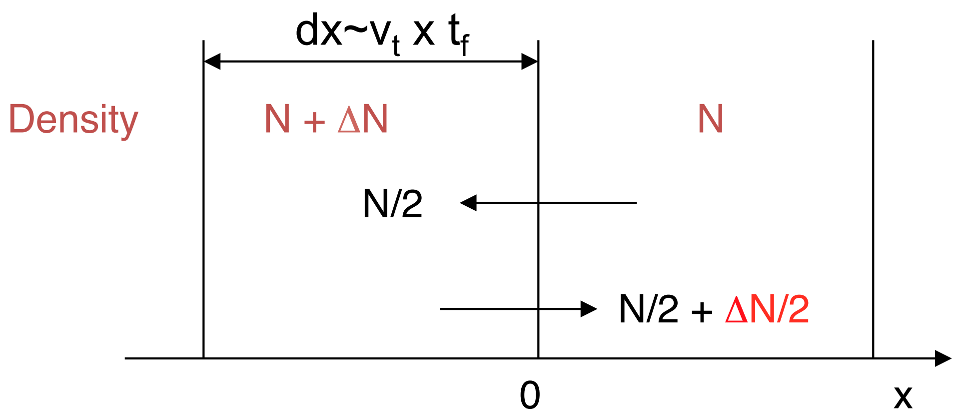
$v_{drift} = \mu E$ --- drift velocity

$\mu = \dfrac{q t_f}{2m}$ --- mobility

t_f = mean free time, E = electric field

Drift velocity is proportional to electric field.
The constant of the proportionality is called mobility.

Diffusion （拡散）



If there is density difference in adjacent region, particles of ΔN/2 will move v_t in a unit time. Thus average velocity of the N particles will be

$$\cdot\, v_{diff} = \frac{\Delta N}{2} v_T \frac{1}{N}$$

$$\cdot\, \frac{dN}{dx} \approx \frac{-\Delta N}{v_T \cdot t_f}$$

$$\therefore v_{diff} = -\frac{1}{2N}\frac{dN}{dx} t_f v_T^2$$

$$\frac{1}{2} m v_T^2 = \frac{1}{2} kT$$ を代入すると

$$\therefore v_{diff} = -\frac{kT \cdot t_f}{2m}\frac{1}{N}\frac{dN}{dx} = -D\frac{1}{N}\frac{dN}{dx}$$

$$D = \frac{kTt_f}{2m}$$ --- diffusion constant

k = Boltzman's constant, v_T = thermal velocity

Diffusion velocity is proportional to density difference.
The constant of the proportionality is called diffusion constant.

Dとμの関係

$$D = \frac{kT \cdot t_f}{2m}, \quad \mu = \frac{q \cdot t_f}{2m}$$

$$\therefore D = \frac{kT}{q}\mu \quad --- \quad \text{Einstein relation}$$

Boltzmann Distribution

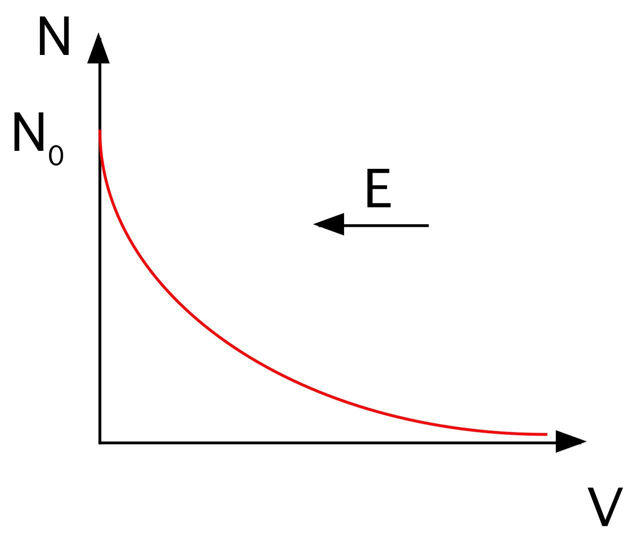
in equilibrium

$$v_{drift} = v_{diff}$$

$$\frac{q \cdot t_f}{2m} E = -\frac{kT \cdot t_f}{2m} \frac{1}{N} \frac{dN}{dx}$$

$$\frac{1}{N}\frac{dN}{dx} = -\frac{q \cdot E}{kT}$$

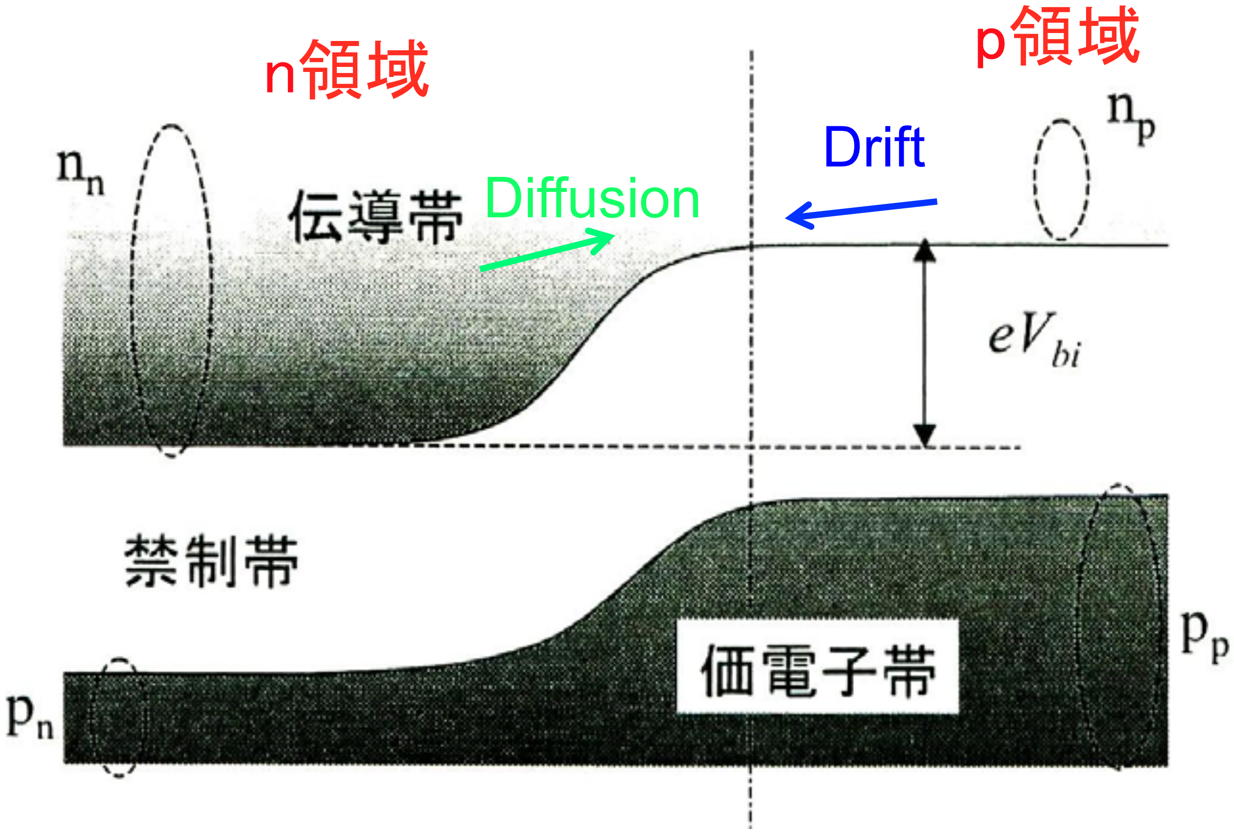
$$\therefore N = N_0 e^{-\frac{q \cdot V}{kT}}$$



Density distribution of particles of average energy qV.

Band structure

Vbi:拡散電位(Built In Potential)



$$\frac{n_p}{n_n} = \exp\left(-\frac{eV_{bi}}{k_B T}\right)$$

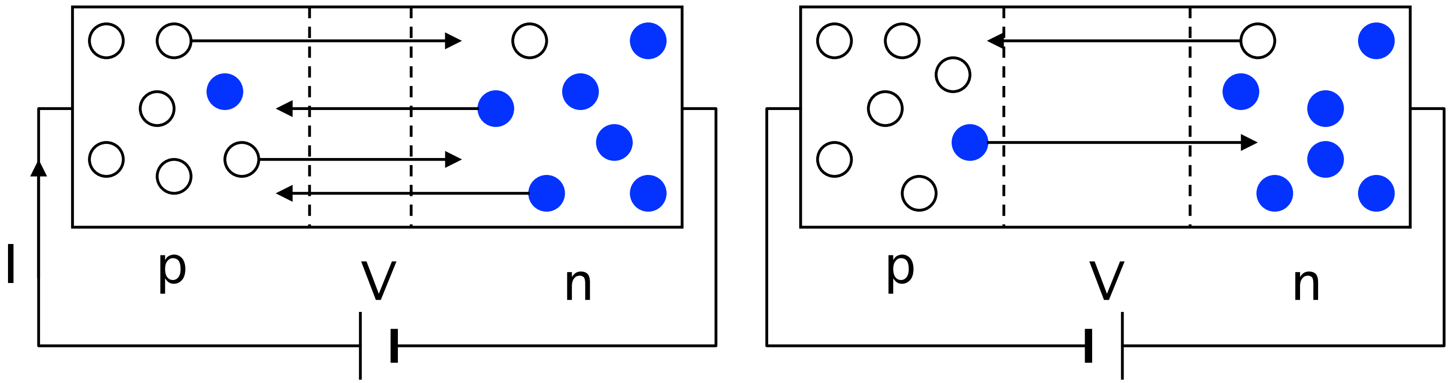
電圧印加なし

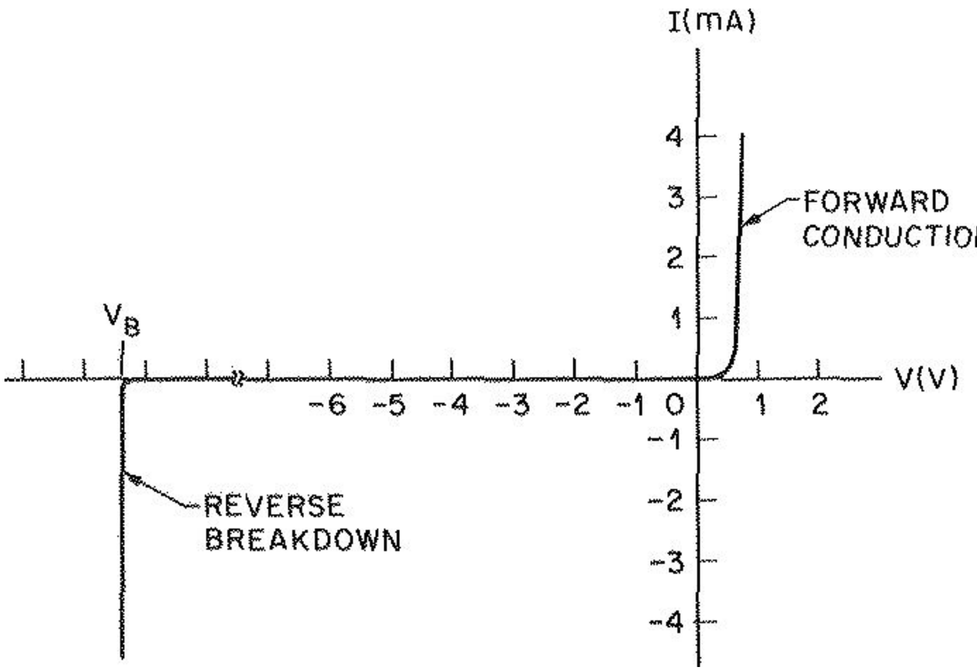
n_n : n領域中のキャリア電子密度
n_p : p領域中のキャリア電子密度
p_n : n領域中のキャリア正孔密度
p_p : p領域中のキャリア正孔密度

p−n junction and bias voltage

順方向(Forward)バイアス

逆方向(Reverse)バイアス





One-sided abrupt junction

At p-n junction, one side is highly doped than the other is called one-side abrupt junction.

p-n junction部が電場が強い。

Depletion region

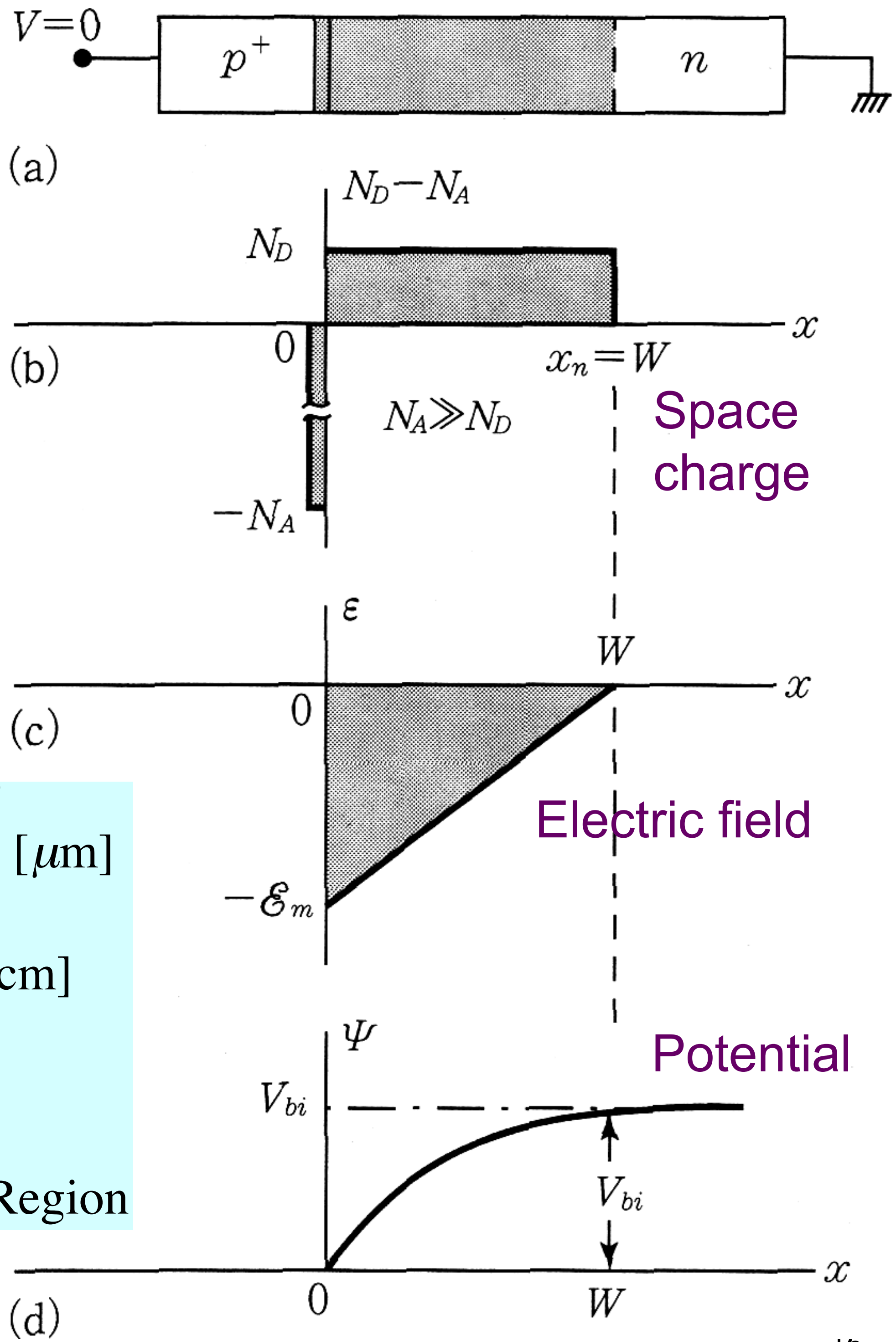
$$W = \sqrt{\frac{2\varepsilon_{Si}(V_{bi} - V_{bias})}{qN_B}} \approx 36\sqrt{\frac{V_{bias}[V]}{N_B[10^{12}cm^{-3}]}}\ [\mu m]$$

ε_{Si} = Dielectric Constant of Si $\approx 10^{-12}$ [F/cm]

V_{bi} = Builtin Potential $\approx$ 0.7 [V]

q = electron charge

N_B = Impurity Density of Lightly Doped Region



Characteristic of Ideal Diode

$$J = J_S\left(\exp\left(qV/kT\right) - 1\right)$$

J_S : Saturation Current Density

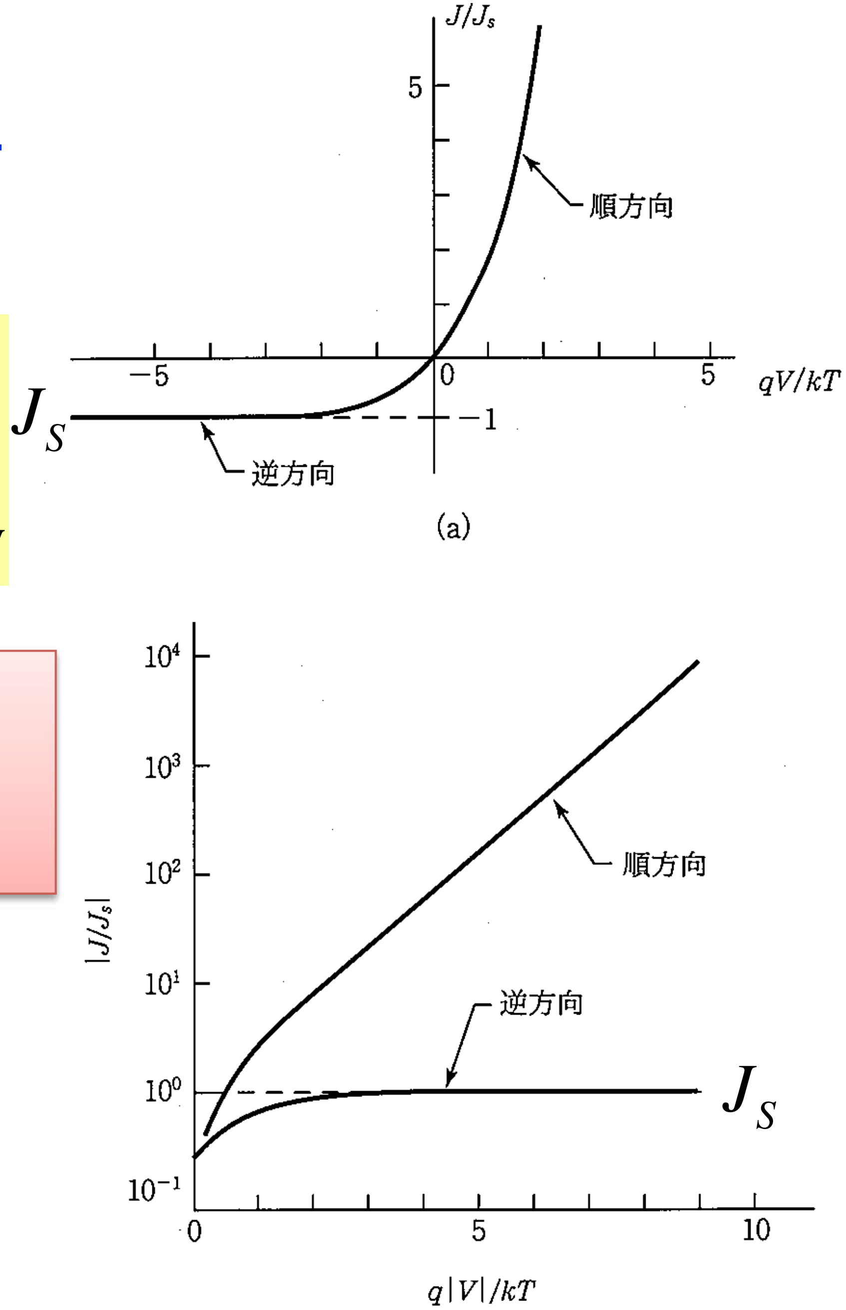
Forward current:

magnitude increases e(~2.7) times at every 25mV(=q/kT) (@300K)

at high current region

$$I \approx I_S \exp\left(q\left(V - IR\right)/kT\right)$$

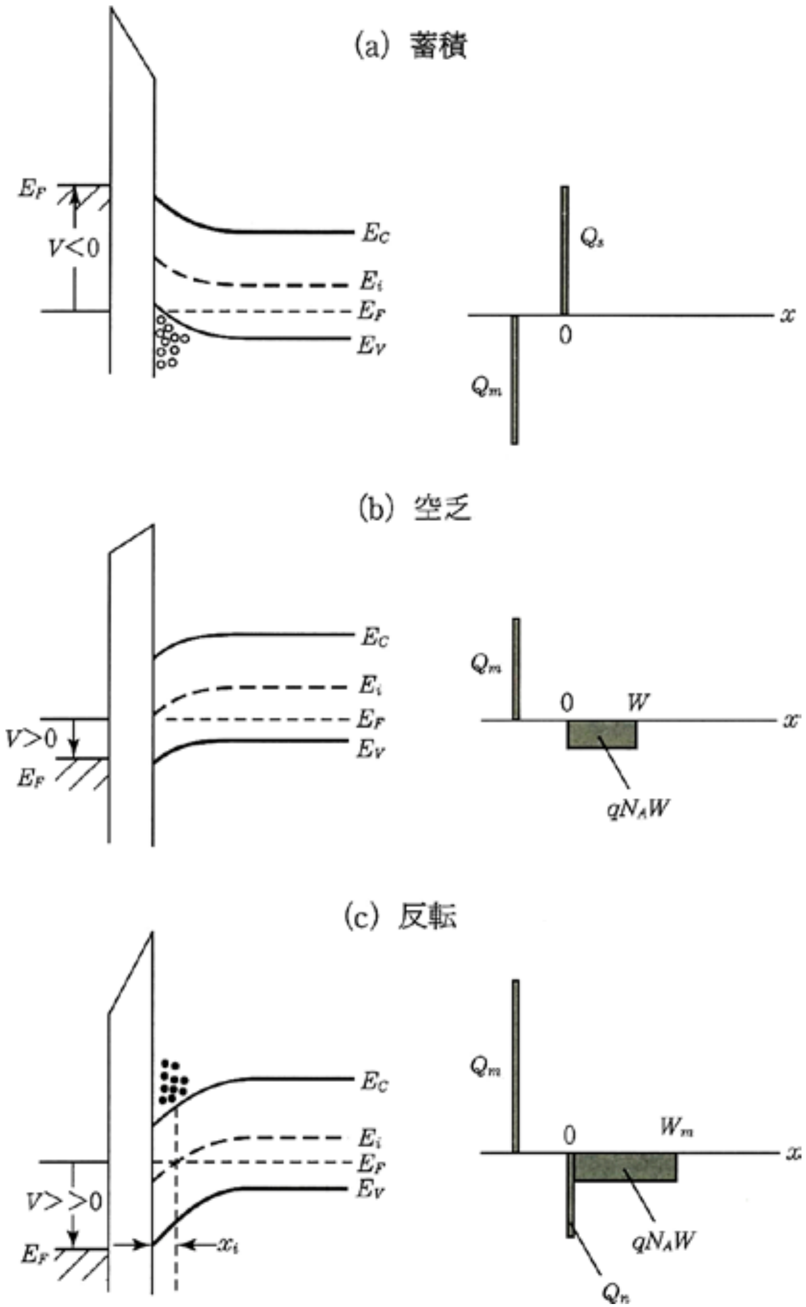
R : Internal Resistance



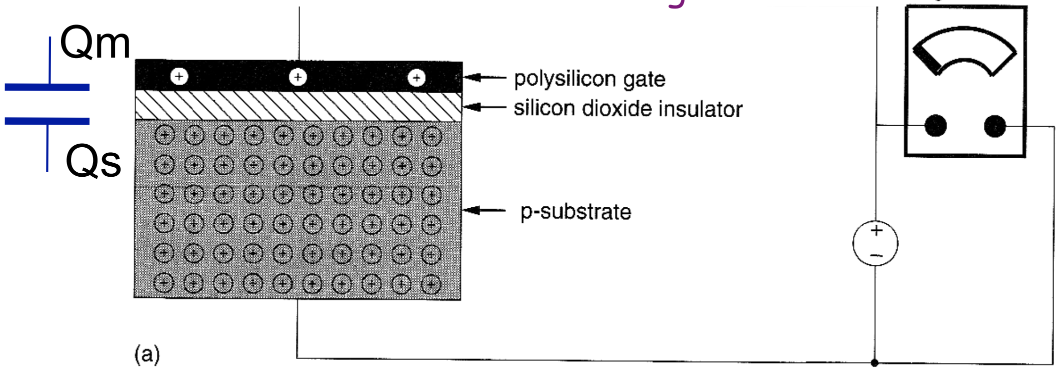
(b)

理想電流−電圧特性. (a) 線形座標プロット, (b) 片対数座標プロット.

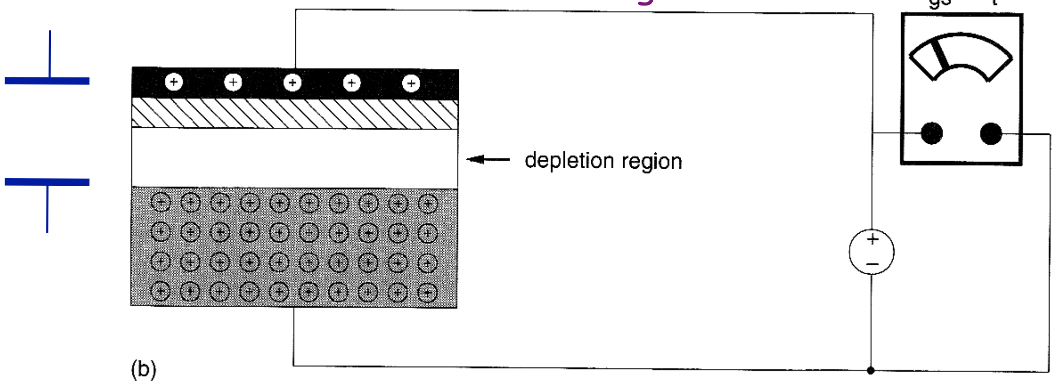
MOS Capacitor (Vds=0)



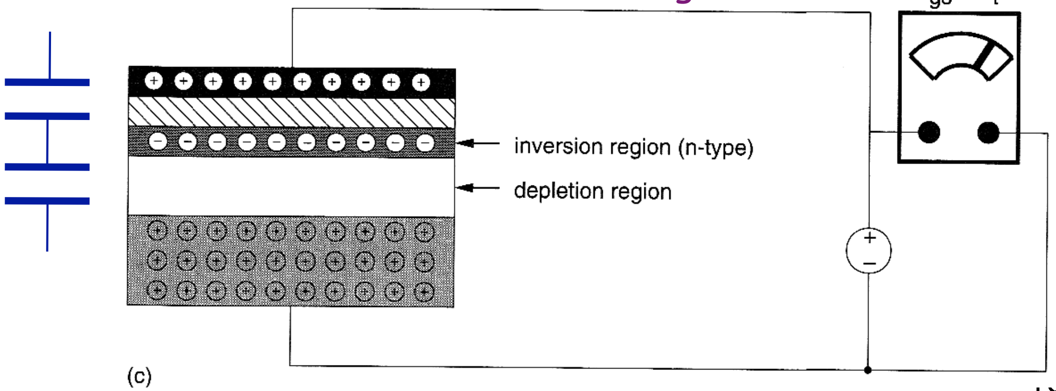
Accumulation (V_{gs} << V_{th})



Depletion (V_{gs} ~ V_{th})

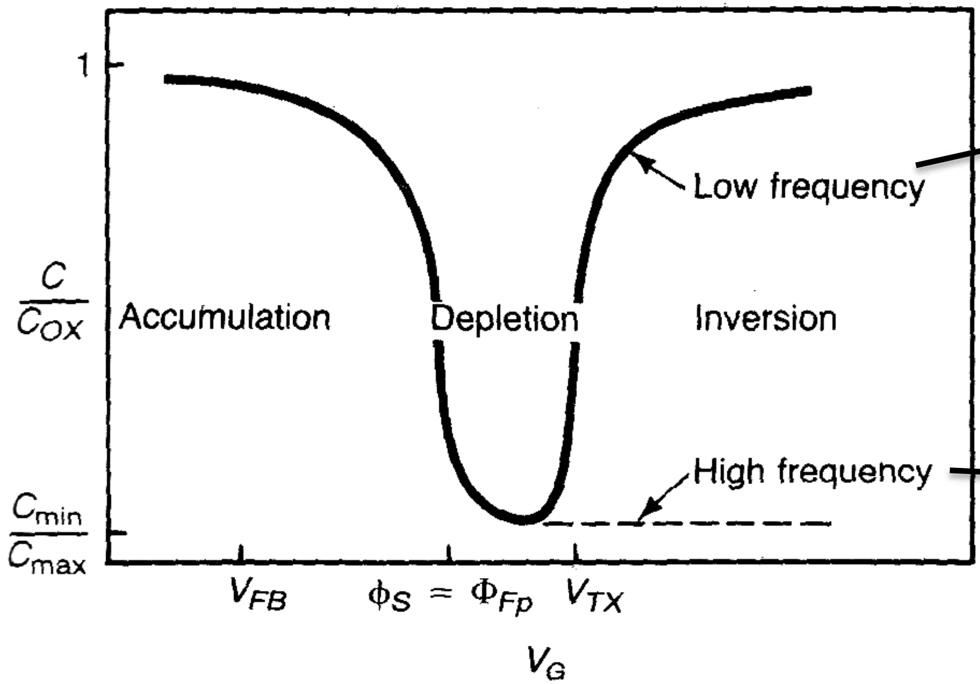


Inversion (V_{gs} > V_{th})



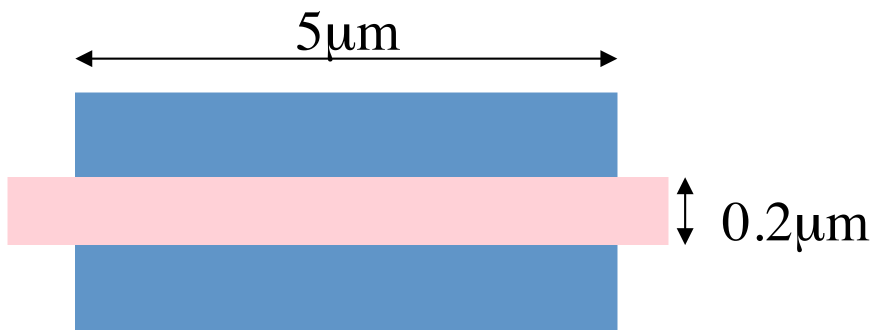
MOS Capacitor

Capacitance characteristics of a CMOS capacitor.



又は drain/sourceを接地した場合

drain/sourceがopenの場合



Transistor of W/L=5um/0.2um → 7 fF

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

Permittivity

Oxide thickness

$$= \frac{3.9 \times 8.85 \times 10^{-12}[F/m]}{5[nm]}$$

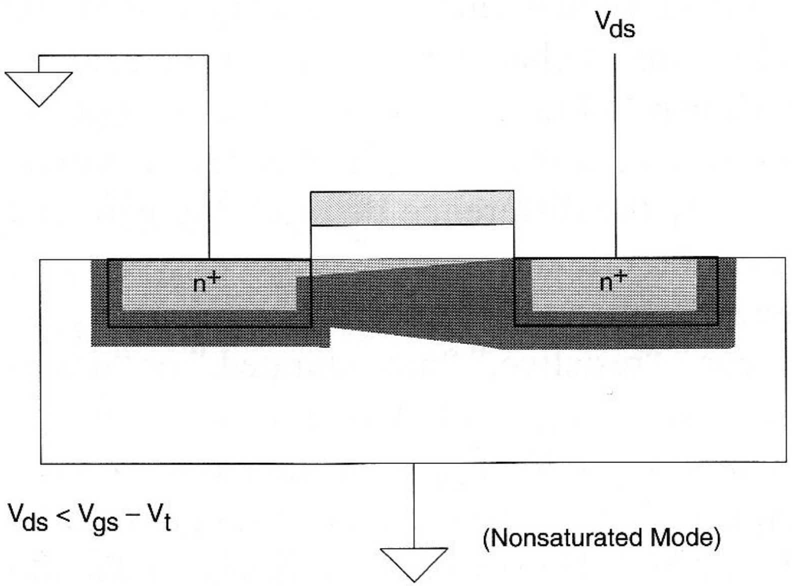
$$\approx 7\ [fF/\mu m^2]$$

MOS Transistor

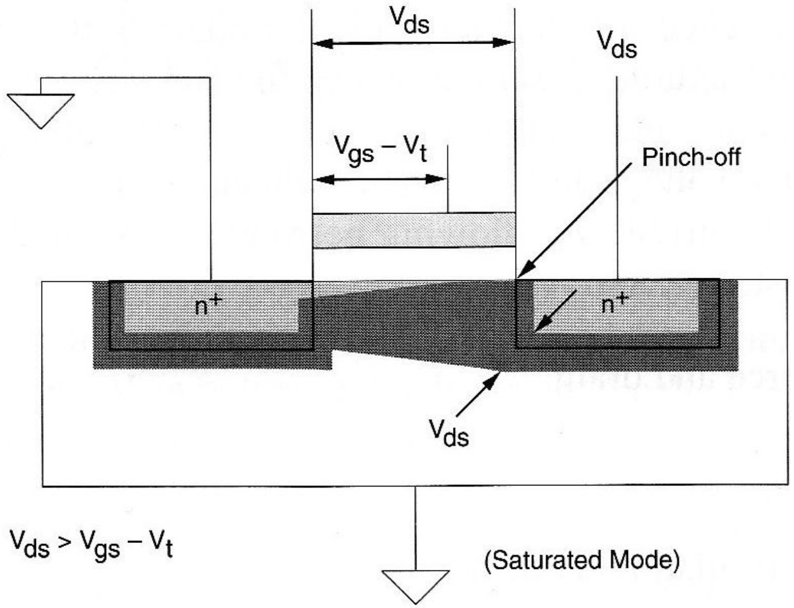
Vgs < Vth, Cut Off Ids = 0

Vgs > Vth

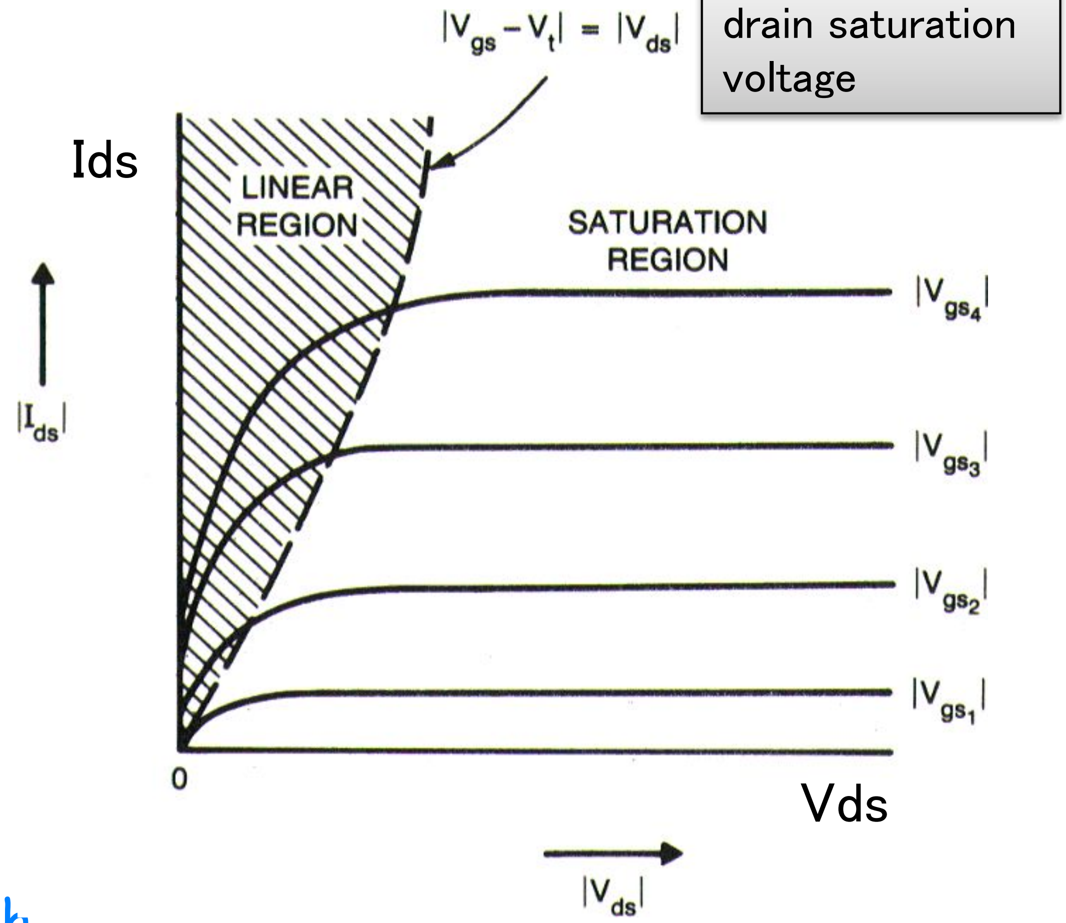
Linear : Vds < Vgs - Vth



Saturated : Vds > Vgs - Vth



Ids-Vds Curve



Cut Off Region : Vgs < Vth

$$I_{ds} = 0$$

Linear Region : Vds < Vgs - Vth

$$I_{ds} = \beta\left[\left(V_{gs} - V_{th}\right)V_{ds} - \frac{V_{ds}^2}{2}\right]$$

Saturated Region: Vds > Vgs - Vth

$$I_{ds} = \beta\frac{\left(V_{gs} - V_{th}\right)^2}{2}$$

$$\beta = \mu \cdot \mathrm{C}_{\mathrm{ox}}\left(\frac{W}{L}\right)$$ --- Gain Factor

μ --- Carrier mobility

$$\mu_n \approx 3 \times \mu_p$$

$V_{gs} - V_{th} = \Delta_{OV}$ --- Overdrive Voltage

Linear Region :

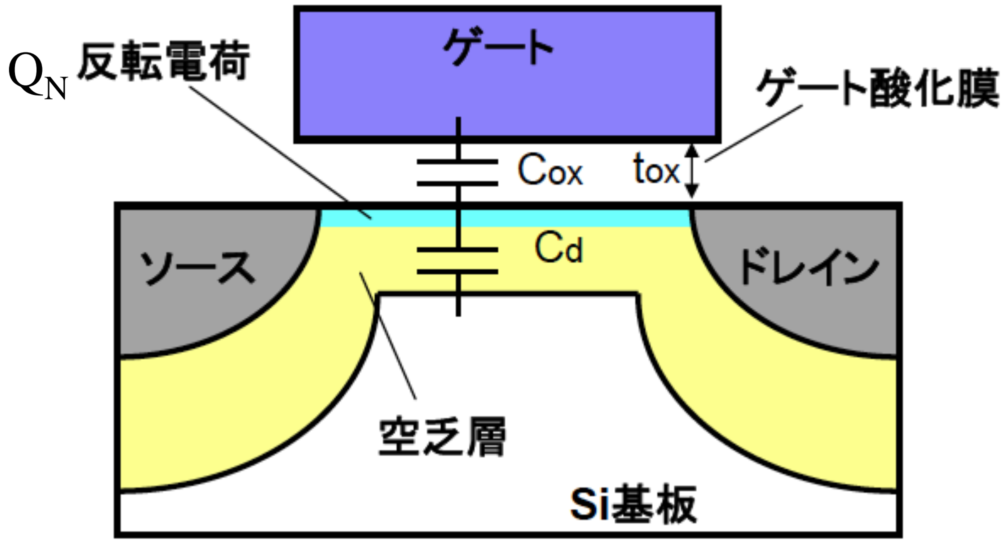
$$I_{ds} = Q_N \cdot W \cdot v_{drift}$$

$$v_{drift} = \mu \cdot E, \quad E = \frac{V_{ds}}{L}$$

$$I_{ds} = \mu \cdot Q_N \cdot \frac{W}{L} \cdot V_{ds}$$

$$Q_N \approx C_{ox}\left(V_{gs} - V_{th}\right)$$ なので

$$\therefore I_{ds} = \mu \cdot C_{OX}\left(\frac{W}{L}\right)\left(V_{gs} - V_{th}\right)V_{ds}$$



$$V_{gs} \rightarrow V_{gs} - \frac{V_{ds}}{2}$$ (since average channel potential is not 0 but Vds/2.

$$\therefore I_{ds} = \beta\left(V_{gs} - V_{th} - \frac{V_{ds}}{2}\right) \cdot V_{ds}$$

$$I_{ds} = \beta\left[\left(V_{gs} - V_{th}\right)V_{ds} - \frac{V_{ds}^2}{2}\right]$$

Saturated Region:

At Vds > Vgs – Vth, additional voltage is applied between pinch-off point and drain, so there is no current increase. Therefore Vds of linear region is replaced with (Vgs-Vth).

$$I_{ds} = \beta\left[\left(V_{gs} - V_{th}\right)V_{ds} - \frac{V_{ds}^2}{2}\right]$$

$$= \beta\left[\left(V_{gs} - V_{th}\right)\left(V_{gs} - V_{th}\right) - \frac{\left(V_{gs} - V_{th}\right)^2}{2}\right]$$

$$= \beta\frac{\left(V_{gs} - V_{th}\right)^2}{2}$$

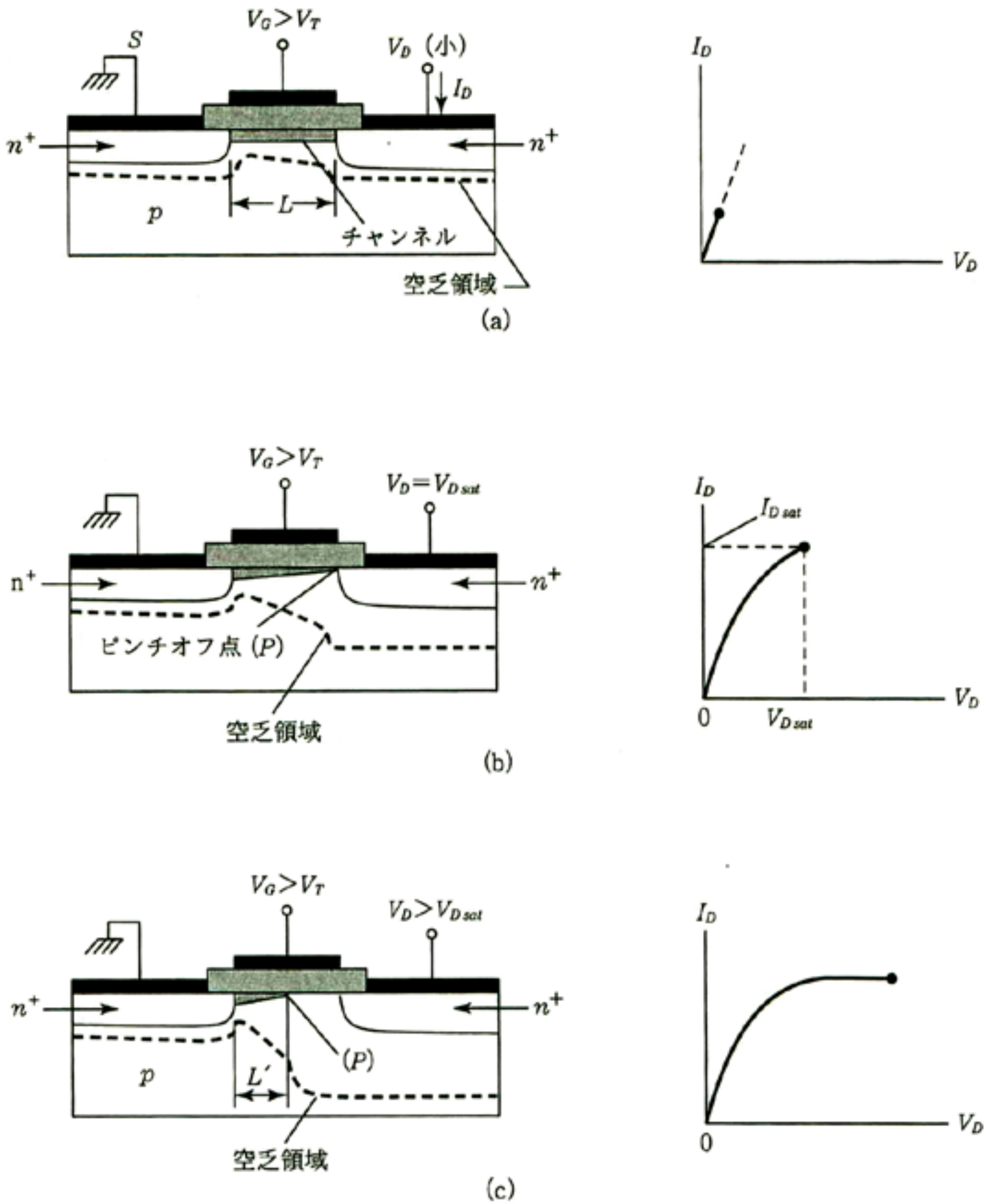


図 15　MOSFET の動作と I-V 出力特性．(a) 低ドレイン電圧，(b) 飽和の開始，点 P はピンチオフ点を示す，(c) 飽和後．

Modulation of Channel Length

At saturated region we get

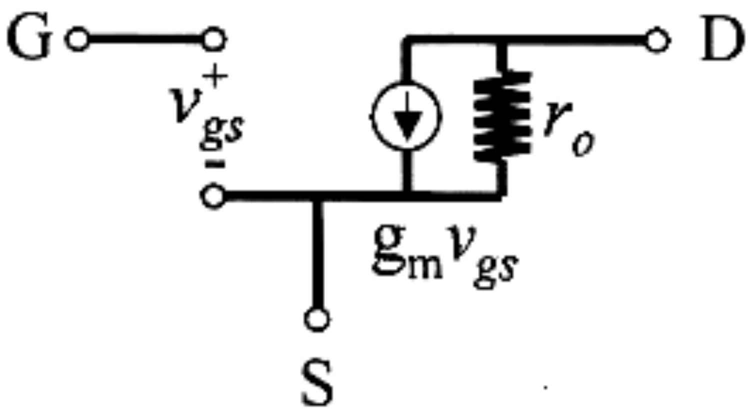
$$I_{ds} = \frac{1}{2}\mu \cdot C_{ox} \frac{W}{L}\left(V_{gs} - V_{th}\right)^2$$

but the channel length L will be changed due to the pinch-off.

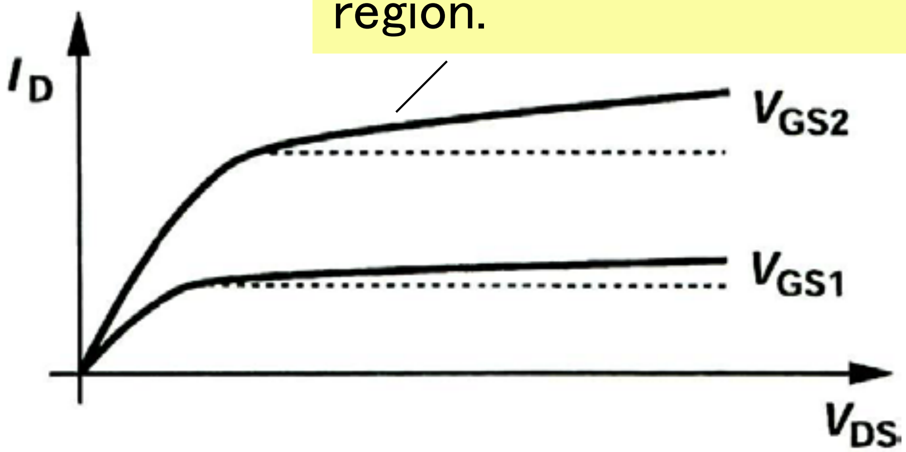
$$\frac{1}{L} \rightarrow \frac{1}{L - \Delta L} \approx \frac{1}{L}(1 + \frac{\Delta L}{L}) \approx \frac{1}{L}(1 + \lambda V_{ds}) \qquad \left(\frac{\Delta L}{L} = \lambda V_{ds}\right)$$

$$\therefore I_{ds} = \frac{1}{2}\mu \cdot C_{ox} \frac{W}{L}\left(V_{gs} - V_{th}\right)^2\left(1 + \lambda V_{ds}\right)$$

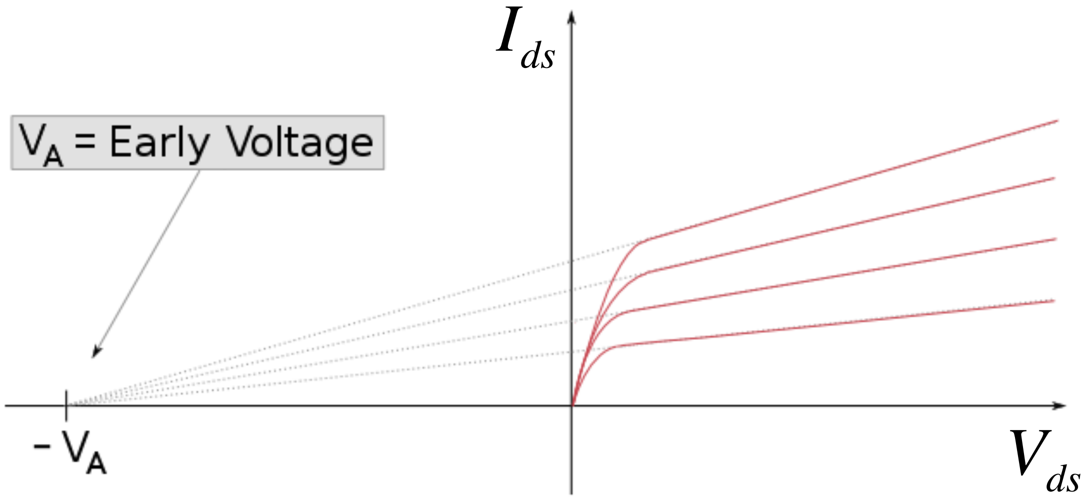
$$g_m = \mu \cdot C_{ox} \frac{W}{L}\left(V_{gs} - V_{th}\right)\left(1 + \lambda V_{ds}\right)$$



This slope corresponds to the output resistance r_o of a transistor in the saturated region.



Early Voltage



$$I_{ds} = \frac{1}{2}\beta\left(V_{gs} - V_{th}\right)^2\left(1 + \lambda V_{ds}\right)$$

$$= \frac{1}{2}\beta\left(V_{gs} - V_{th}\right)^2\left(1 + \frac{V_{ds}}{V_A}\right)$$

$$\left(V_A = \frac{1}{\lambda} : \text{ Early Voltage}\right)$$

Large Early Voltage means better constant current characteristic and higher output resistance.

Output resistance

$$r_o = \frac{V_{ds} + V_A}{I_{ds}(sat)} \approx \frac{V_A}{I_{ds}(sat)}$$

Transconductance

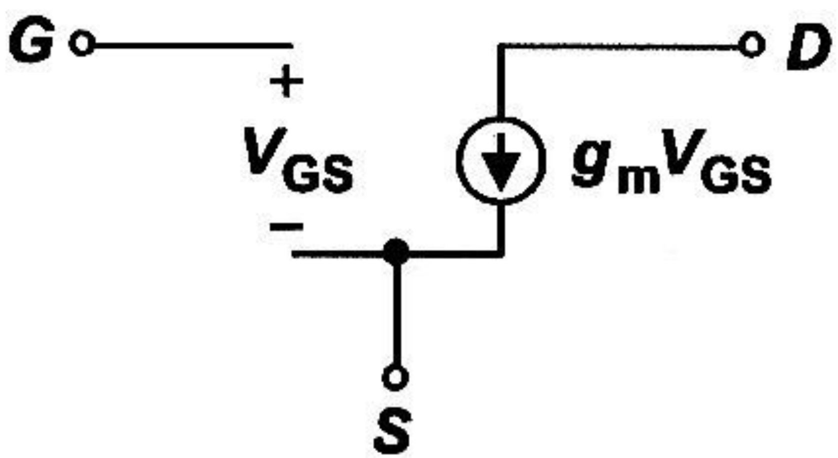
Drain current in the saturated region is determined by the over drive voltage (Vgs-Vth).

Transconductance (gm) is defined by the ΔIds/ΔVgs, and this indicate the performance of the transistor.

$$g_m = \left.\frac{\partial I_{ds}}{\partial V_{gs}}\right|_{Vds=const.}$$

$$= \beta\left(V_{gs} - V_{th}\right)$$

$$= \mu \cdot C_{ox} \frac{W}{L}\left(V_{gs} - V_{th}\right)$$

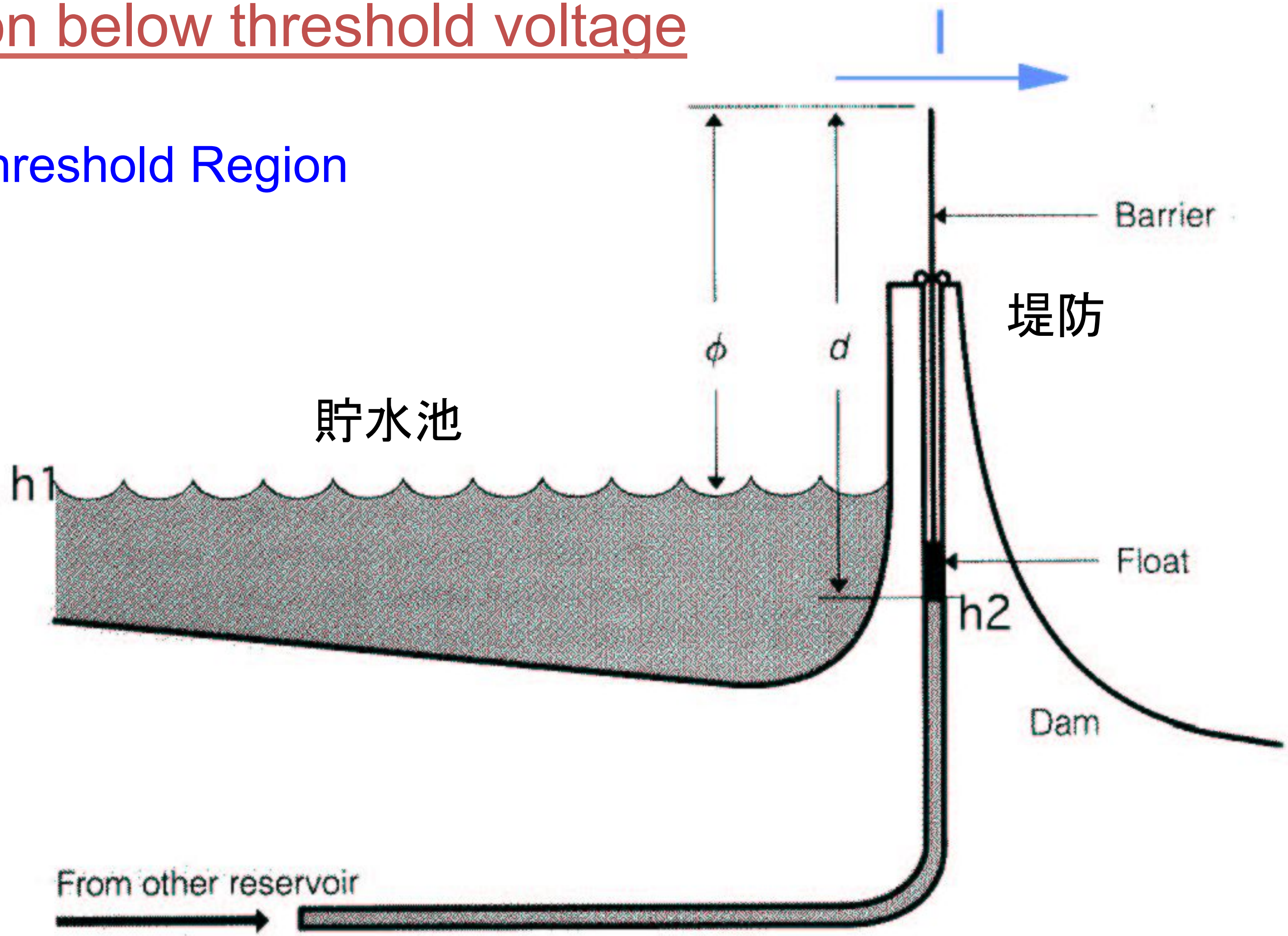


At small signal model, drain current becomes gm times Vgs.

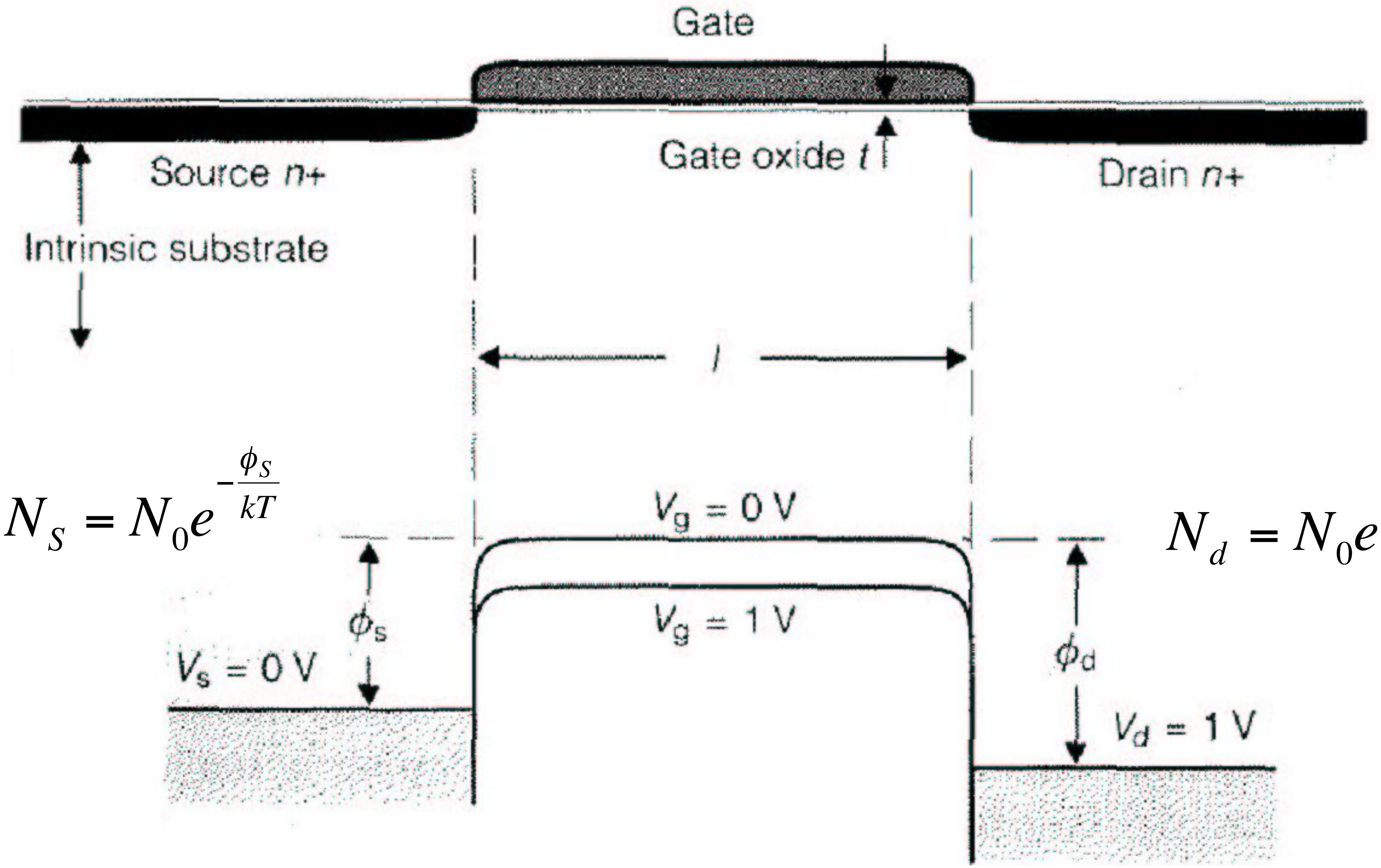
MOS Tr 小信号モデル

Operation below threshold voltage

Sub-Threshold Region



$$I = I_0 e^{-\frac{w}{kT}(h2 - h1 + d)}, \quad w = \text{weight of a molecule}$$

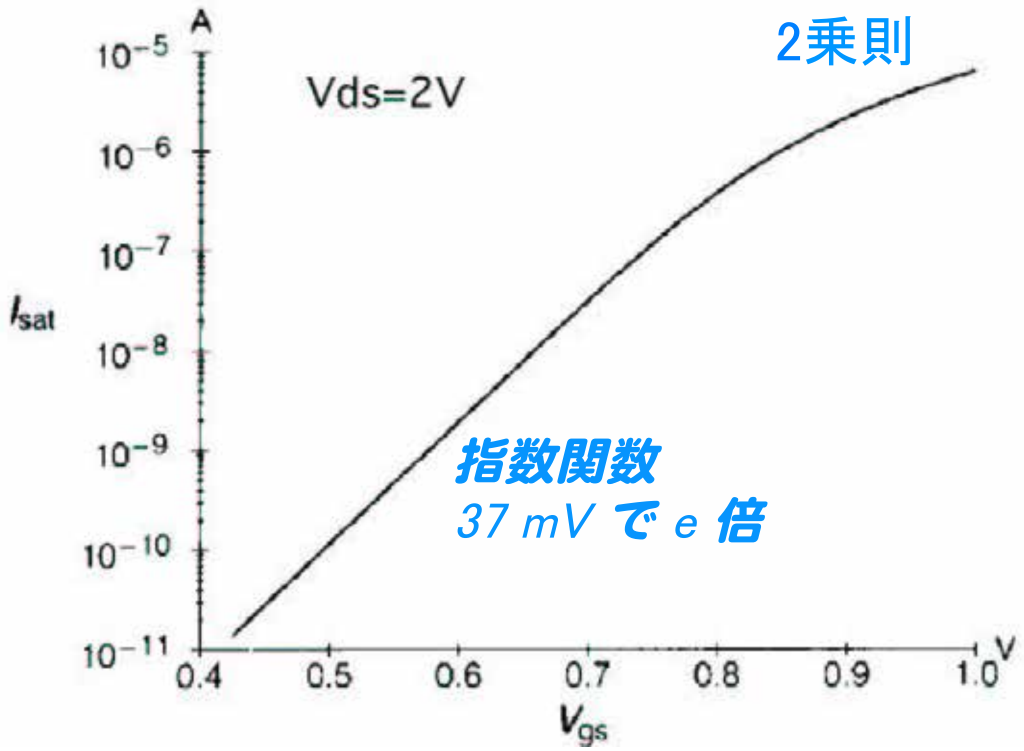


$$N_S = N_0 e^{-\frac{\phi_S}{kT}}$$

$$N_d = N_0 e^{-\frac{\phi_d}{kT}}$$

Sub-Threshold Region
(弱反転領域)

In previous slide, we set Ids=0 @Vgs<Vth, but actually a small current will flow. From the Boltzman distribution



$$I = I_s - I_d$$

$$= I_0 e^{\frac{qV_{gs}}{kT}}\left(1 - e^{-\frac{qV_{ds}}{kT}}\right), \quad \frac{kT}{q} = 25\mathrm{mV}(\text{intrinsic substarate})$$

↓

g_m is large in the sub-threshold region but Ids is very small.

$$\mathrm{I} = I_0 e^{\frac{q\kappa V_{gs}}{kT}}\left(1 - e^{-\frac{qV_{ds}}{kT}}\right), \quad \frac{kT}{q\kappa} = 37\mathrm{mV}(\text{p or n substarate})$$

How to extract the threshold voltage

The gate voltage necessary to create the inversion layer is called the threshold voltage.

a) Constant current method

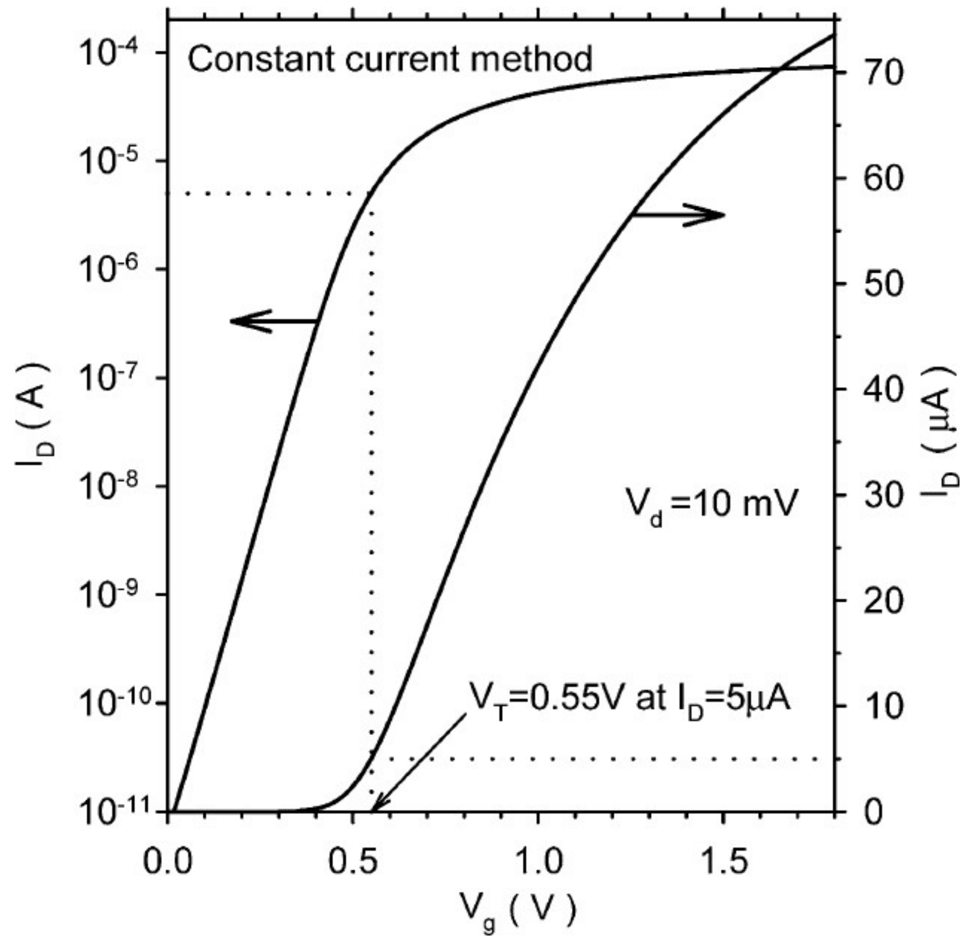
Vth is defined as the gate voltage where Id becomes some constant value in linear region.

Normally the constant value of
$Id = (W/L) \times 10^{-7}$ [A]
is used.

•simple

•widely used in industry

•depend on the constant value.



ref.) A. Ortiz-Conde, et al., 'A review of recent MOSFET threshold voltage extraction methods', Microelectronics Reliability 42 (2002) 583-596.

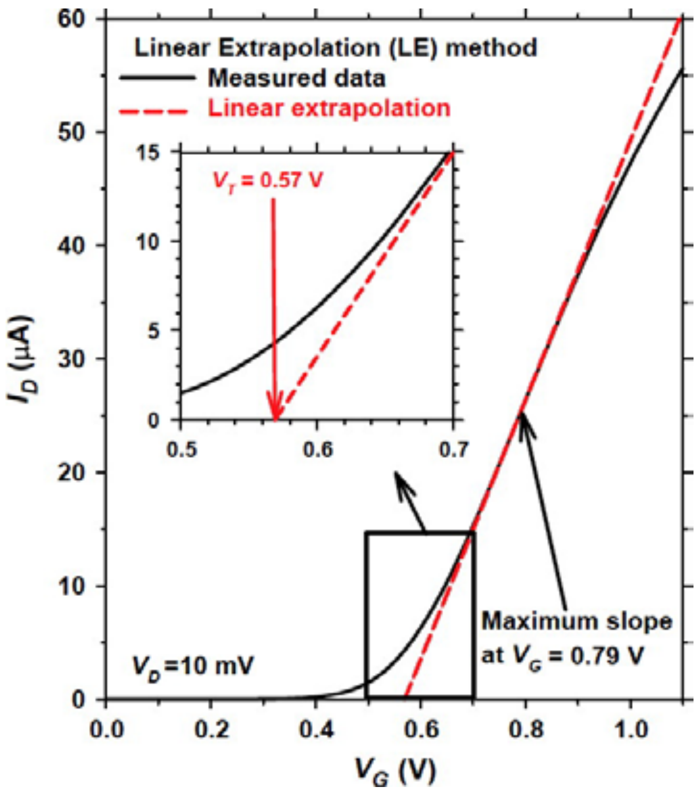
How to extract the threshold voltage (cont.)

b) Linear extrapolation method

Find the gate-voltage axis intercept (i.e., I_D=0) of the linear extrapolation of the I_D–Vg curve at its maximum slope (gm) point in linear region. *(Then subtract Vd/2 to the resulting value.)*

Or, from $I_D^{0.5}$–Vg curve in saturated region.

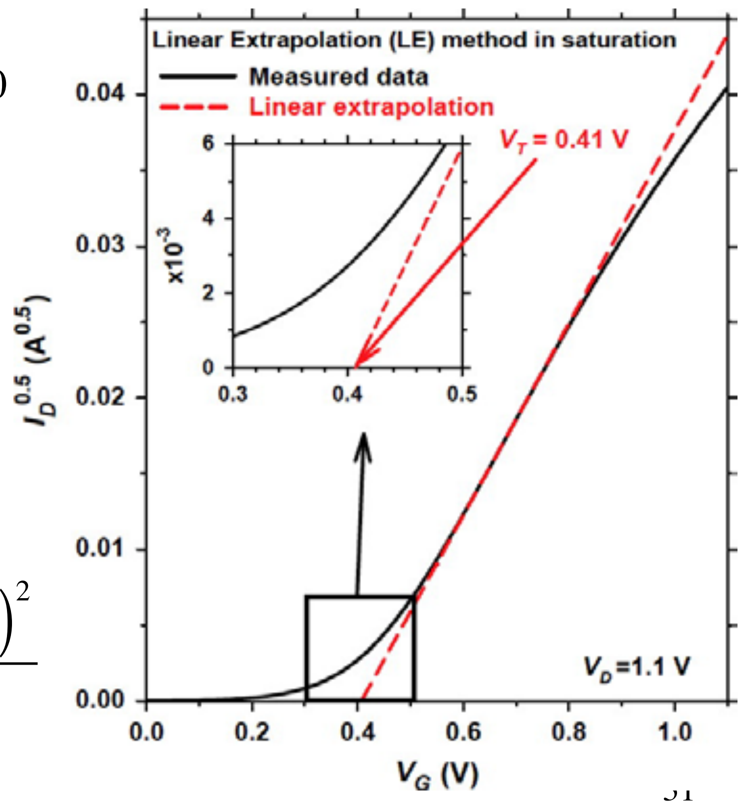
Drawback of this method is that the maximum slope point might be uncertain.



$$I_d = \beta\left[\left(V_{gs0} - V_{th}\right)V_{ds} - \frac{V_{ds}^2}{2}\right] = 0$$

$$\therefore V_{th} = V_{gs0} - \frac{V_{ds}}{2}$$

$$I_{ds} = \beta\frac{\left(V_{gs} - V_{th}\right)^2}{2}$$



Summary

- MOS capacitance is divided to 3 different regions depend on gate voltage; Accumulation, Depletion, Inversion regions.
- Operation of MOS Transistor is divided to Cut Off, Linear, Saturation Regions depend on gate voltage and drain voltage.

$$I_{ds} = \beta\left[\left(V_{gs} - V_{th}\right)V_{ds} - \frac{V_{ds}^2}{2}\right] \quad \text{Linear Region}$$

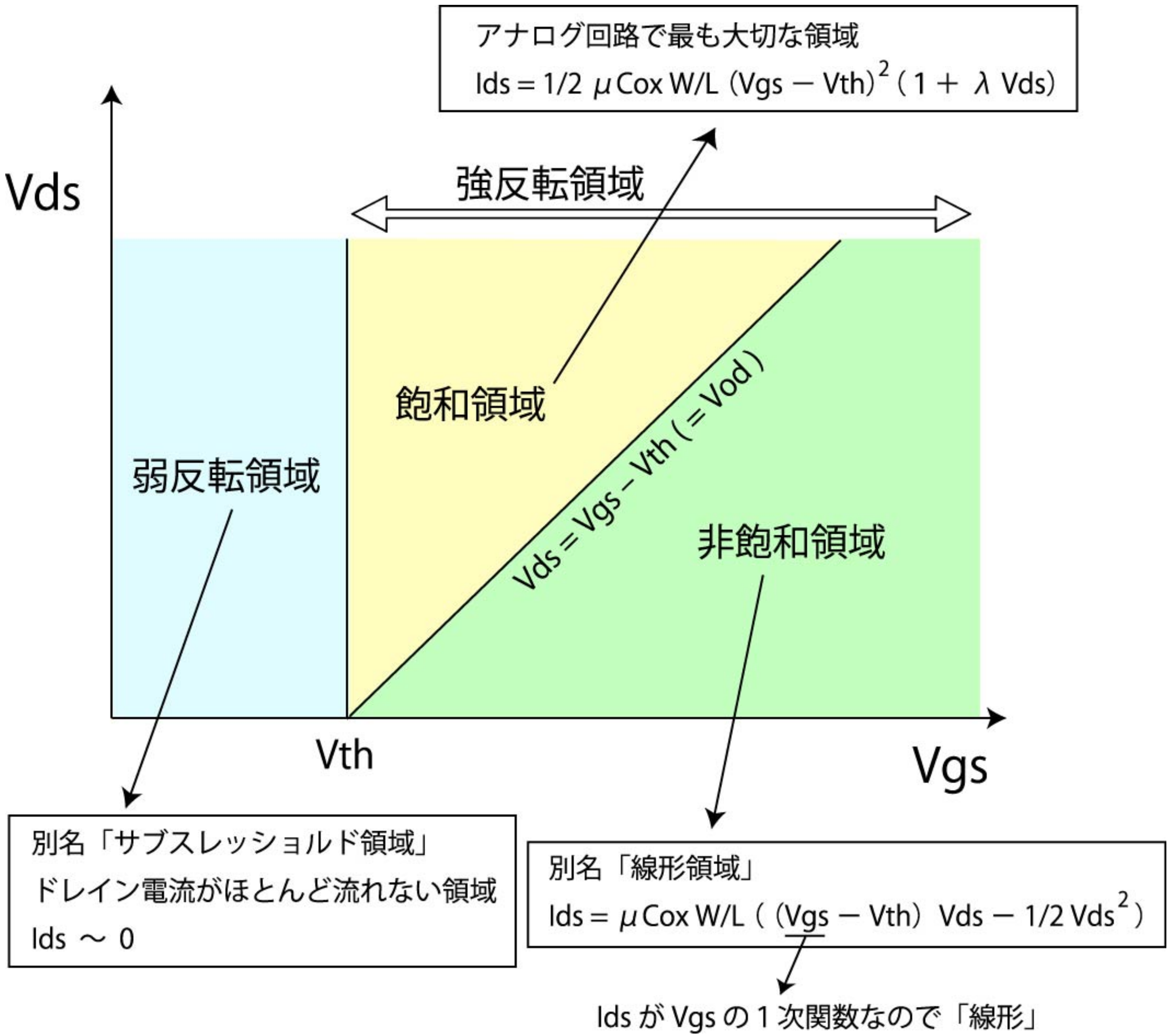
$$I_{ds} = \beta\frac{\left(V_{gs} - V_{th}\right)^2}{2} \quad \text{Saturation Region}$$

$$\left(\beta = \mu \cdot \mathrm{C}_{\mathrm{ox}}\left(\frac{W}{L}\right) \text{ Gain Factor}\right)$$

- NMOS has 2~3 times larger gain factor β than PMOS.
- Transconductance is defined as $g_m = \left.\frac{\partial I_d}{\partial V_{gs}}\right|_{Vds=const.}$

- Sub-Threshold region
  In digital circuit this region is assumed that there is no drain current, but actually small current will flow. This region is sometime used for low-power analog circuit.

- $$\mathrm{I} = I_0 e^{\frac{q\kappa V_{gs}}{kT}} \left(1 - e^{-\frac{qV_{ds}}{kT}}\right), \quad \frac{kT}{q\kappa} = 37\mathrm{mV}(\text{p or n substarate})$$



from http://homepage.mac.com/mosfet/cmos/region.html