FRW/CFT duality: Holographic formulation of eternal inflation, and its applications

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hep-th/0606204, 0908.3844[hep-th], work in progress
Outline

• Holographic dual description for eternal inflation
  – Open universes are created by bubble nucleation (tunneling from an “ancestor” vacuum).
  – The dual theory: 2D CFT defined at the boundary (spatial infinity) of an open universe.

• Observational consequences of bubble nucleation
  – Possible effects of the ancestor vacuum on the CMB spectrum (assuming the inflation after tunneling has minimal number of e-foldings).
  – The concept of “scaling dimension” is useful.
The setup

- Gravity + a scalar field ("a model for the landscape")

- Existence of a holographic dual theory will give support for the existence of string landscape.
  - At the moment, we don’t know the meaning of the meta-stable vacua beyond the semi-classical level. The dual theory will provide one.

\[
V(\Phi) \quad \Phi_F \quad \Phi_T \quad \Phi
\]

False ("ancestor") vacuum: positive c.c. (de Sitter)
- Hubble parameter \( H_A \)

True vacuum: zero c.c.
- (3+1) D spacetime
CDL instanton

• The false vacuum decays through bubble nucleation.

• Described by Coleman-De Luccia (CDL) instanton
  – Euclidean (“bounce”) solution with SO(4) symmetry, which interpolates the false and true vacuum
    – Geometry: right figure.

• Evolution after nucleation:
  – Given by analytic continuation.
  – The spacetime has SO(3,1) sym.
Causal structure

• **Open FRW (shaded region) in the bubble**
  
  \[ ds^2 = a^2(T)(-dT^2 + dR^2 + \sinh^2 R d\Omega_2^2) \]

  – Constant time slices (dashed lines): \( H^3 \) (isometry: SO(3,1))
  – Boundary (at \( R \to \infty \)): \( \Sigma \)
  – The beginning of the FRW time \( T \to -\infty \) is smooth (locally flat)
    \[ a(T) = H_A^{-1} e^T, \quad (a(t) \sim t) \]

• We are assuming the final c.c. is zero. The whole FRW is in causal contact with a single observer (“census taker”).
Eternal Inflation

• If nucleation rate is small \( \Gamma \lesssim H^4_A \), true vacuum does not “percolate”. Physical volume continues to be dominated by false vacuum.

• Infinite number of bubbles are nucleated eventually, and infinite number of bubbles collide. (Guth-Weinberg ’83)

• Late time asymptotics depends on the type of colliding bubble. E.g. collision with a bubble of the same vacuum: bulk space is smoothed out (one time-like infinity).
Holographic duality

- Dual theory is a CFT on $S^2$
  (at the boundary $\Sigma$)
- $\text{SO}(3,1)$: conformal sym in 2D.
- The dual has 2 less dimensions than the bulk.
- The dual theory contains gravity (Liouville field).
  (2D gravity coupled to matter with $c>25$)
- In the semi-classical regime, Liouville field represents time (as in the Wheeler-DeWitt theory).
- One field in the bulk corresponds to an infinite tower of the CFT operators.
- (Different from dS/CFT correspondence)
Fluctuations in the CDL background

• In open universe, normalizable fluctuations decay at scales larger than curvature radius (R>>1).
  – Harmonics on \( H^3 \):
    \[
    \nabla_H^2 q^{(k)}(R) = -(k^2 + 1)q^{(k)}(R), \quad q^{(k)}(R) = \frac{\sin kR}{\sinh R}
    \]

• There could be non-normalizable mode (super-curvature mode), which has imaginary k. (Fluctuations generated in the ancestor vacuum)
  – Exists when mass in the ancestor vacuum is small; c.f. log correlation for massless field in de Sitter.

• Initial condition: determined by Euclidean prescription.

• Earlier studies: Garriga, Montes, Sasaki, Tanaka, ’99, See also Gratton, Hertog, Turok
Euclidean prescription

• We compute the correlator in Euclidean space, and analytically continue it to FRW.

• Euclidean geometry: \((-\infty \leq X \leq \infty)\)

\[
ds^2_E = a^2(X) \left( dX^2 + d\theta^2 + \sin^2 \theta d\Omega_2^2 \right)
\]

\[
a(X) = \tilde{H}_A^{-1} e^X \quad (\text{flat}), \quad a(X) = \frac{H_A^{-1}}{\cosh X} \quad (\text{de Sitter})
\]

• Analytic continuation to the FRW universe:

\[
X \rightarrow T + \frac{\pi}{2}i, \quad \theta \rightarrow iR
\]
Euclidean correlator

• e.o.m. for minimally coupled scalar:

\[
-\partial^2_X + \frac{a''}{a} + \nabla^2_S + m^2 a^2 \left( a\phi \right) = 0
\]

• Calculation of the correlator is essentially a 1-dimensional scattering problem:

\[
\left[ -\partial^2_X + \frac{a''}{a} + m^2 a^2 \right] u_k(X) = (k^2 + 1)u_k(X)
\]

• Massless case:
  bound state at \( k = i \)
• B.s. exists when mass is small compared to \( H_A \)
• There is at most one b.s. (in 3+1D).

\[
V(X) = \frac{a''}{a} \text{ (in the thin-wall limit)}
\]

- Flat
  \( V(X) = 1 \)
- de Sitter (sphere)
  \( V(X) = 1 - 2/cosh^2 X \)

Delta fn at domain wall
Euclidean correlator

- In the thin-wall limit, on the flat side,

\[
\langle \phi(X, \theta) \phi(X', 0) \rangle = \tilde{H}_A^2 e^{-(X+X')} \int_{C_1} dk \left( e^{ik(X-X')} + \mathcal{R}(k)e^{-ik(X+X')} \right) \frac{\sinh k(\pi - \theta)}{\sinh k\pi \sin \theta}
\]

- The last factor: Green’s fn. on $S^3$ with mass $(k^2 + 1)$
  It has poles at $k = ni$.

- If there is a bound state, the reflection coefficient $\mathcal{R}(k)$ has a pole at $k = ib$ ($0 < b \leq 1$). The contour goes over this pole (when there is a bound state).
Correlator in the FRW

• In the thin-wall limit (or in the early time limit),

\[
\langle \phi(T, R) \phi(T', 0) \rangle = \tilde{H}_A^2 e^{-(T+T')} \int_{C_1} dk \left( e^{ik(T-T')} \cosh k\pi \right)
\]

\[
+ R(k) e^{-ik(T+T')} \right) \frac{\sin kR}{\sinh k\pi \sinh R}
\]

(R: geodesic distance on $H^3$)

– The first term: “flat space piece”: Minkowski correlator written in open slicing

– The second term: “ancestor piece”: May contain a non-normalizable mode. Modifies the spectrum of normalizable mode.
Near-boundary limit

• We do the integral by contour deformation (assuming T+R>0, T−R<0):

\[
\langle \phi(T, R)\phi(T', 0) \rangle = \sum_{\Delta} G^{(1)}_{\Delta} e^{-\Delta(R_1+R_2)} e^{-\Delta(T_1+T_2)} (1 - \cos \Omega)^{-\Delta}
\]

\[
+ \sum_{\Delta'=2}^{\infty} G^{(2)}_{\Delta'} e^{-\Delta'(R_1+R_2)} e^{(\Delta'-2)(T_1+T_2)} (1 - \cos \Omega)^{-\Delta'}
\]

- The geodesic distance:

\[
R \sim R_1 + R_2 + \log(1 - \cos \Omega) \quad \text{(when } R \to \infty)\]

- \((1 - \cos \Omega)^{-\Delta} : 2D \text{ correlator with dimension } \Delta\)

- Integer dimensions starting from 2 appear.

- There could be lower dimension depending on the mass. (Massless field: dims. 0, 1+\alpha (0< \alpha<1)).
Interpretation of the scales

• We identify
  – UV cutoff: \( a \leftrightarrow e^{-(T+R)} \)
  – Reference scale: \( \delta \leftrightarrow e^{-R} \)
  (Late time in the bulk: UV cutoff becomes finer)

• Wave function renormalization (the prefactors)
  – Operators in the 1\(^{\text{st}}\) sum: “RG-invariant” operators
    (defined at the UV scale); scale like \( e^{-\Delta(T+R)} \)
  – Operators in the 2\(^{\text{nd}}\) sum: “RG-covariant” operators
    (defined at the reference scale: e.g. effective action or
    energy momentum tensor); scale like \( e^{(\Delta-2)T-\Delta R} \)
Graviton correlator

- The transverse-traceless mode (on $H^3$) is essentially equivalent to the massless scalar.
- There is a non-normalizable mode ($\Delta=0$).
  - This is pure gauge in the bulk, but has physical effects on the boundary. Boundary (2D) curvature correlator:
    \[
    \langle R^{(2)} R^{(2)} \rangle = \frac{1}{(1 - \cos \Omega)^2}
    \]
    Gravity is not decoupled at the boundary.
- Dimension 2 piece of graviton is transverse (conserved)-traceless in 2D.
  - Identified as energy-momentum tensor of the 2D CFT
Three-point functions

(work in progress /w Daniel Park)

• **Graviton-scalar-scalar 3-point function will tell us:**
  Central charge ( ~ de Sitter entropy, from dim. analysis)

Operator algebra:

\[ T(w) \mathcal{O}(0) \sim \frac{\Delta \mathcal{O}(0)}{w^2} + \frac{\partial \mathcal{O}(0)}{w} + \ldots \]

• **Computation:** Analytic continuation from Euclidean CDL.
  We can deform the contour for the k-integrations, and write the answer as AdS 3-point functions with the dimensions for external lines summed over.

\[ \sum_{\Delta_1 \Delta_2 \Delta_3} \int dH_3 \]
Comment on bubble collisions

- Universe (true vacuum region) with a boundary with arbitrary genus (connected to a single observer) can arise. (Bousso, Freivogel, YS, Shenker, Susskind, Yang, Yeh, ‘08)

- There could be multiple boundaries (separated by singularity) (Kodama, Maeda, Sasaki, Sato ‘82)

- Characterization of “Phases” of eternal inflation: work in progress (w/ Shenker and Susskind)
Observational consequences

• Slow-roll inflation after tunneling:
  We assume \( H_I \ll H_A \)

• Observational bound on the curvature:
  \[
  \Omega_0 > 0.98 \quad \Rightarrow \quad 7H_0^{-1} < R_{\text{curv}}
  \]

• Let us assume there is negative curvature close to this bound. Then, the radius of the last scattering surface is not much different from the curvature radius (Freivogel et al ‘05):
  \[
  R_{\text{l.s.}} = \int_{z=1100}^{z=0} \frac{dt}{a} \sim 0.5R_{\text{curv}}
  \]

• We might be able to see the effect of the ancestor vacuum in the low-l spectrum of the CMB.
Curvature dominated era

- Curvature dominated until $t \sim H_I^{-1}$, \quad ($T \sim \log(H_A/H_I)$)

After that, vacuum energy dominates:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_I^2 + \frac{1}{a^2}$$

- Naïve estimate of the amplitude:

$$\ddot{\phi}_k + \frac{3}{t}\dot{\phi}_k - (k^2 + 1)\phi_k = 0, \quad \phi_k \sim t^{-1 \pm ik}$$

(Fluctuations of order $H_A$ at $t \sim H_A^{-1}$ decay to order $H_I$ by the end of curvature domination, if there is no NNM.)

- To get more precise estimate of the amplitude (especially of the effect of the ancestor vacuum), we evaluate the correlator by contour deformation (in the “late time” limit $T \sim \log(H_A/H_I)$).
Amplitude at the end of curvature domination

• Correlator during curvature domination:

\[ \langle \phi(T, R)\phi(T', 0) \rangle = \tilde{H}^2_A e^{-(T+T')} \int_{C_1} dk \left( e^{ik(T-T')} \cosh k\pi \right) \]

\[ + R(k) e^{-ik(T+T')} \left( \frac{\sin kR}{\sinh k\pi \sinh R} \right) \]

• The 1\textsuperscript{st} term (the part usually considered to be the initial condition for SR inflation): \( H^2_A e^{-(T+T')} \sim H^2_I \)

• The 2\textsuperscript{nd} term (the effect of the ancestor): A pole at \( k = i(1 - \Delta) \) yields

\[ H^2_I e^{2(1-\Delta)(T+T')} = H^2_I \left( \frac{H_A}{H_I} \right)^{2(1-\Delta)} \]

• The lowest dimension gives the leading contribution. If there is NNM (0 < \( \Delta < 1 \)), larger than \( H^2_I \)
The spectrum

- We evolve forward the initial condition into slow-roll regime using the e.o.m.

- The first term (fluctuations generated after tunneling):
  \[
  \langle \phi(T, R) \phi(T', 0) \rangle = H_I^2 \int_{-\infty}^{\infty} dk \frac{1}{k^2 + 1} \frac{\cosh k\pi \sin kR}{\sinh k\pi \sinh R}
  \]
  \[
  = -H_I^2 \left\{ \log(\cosh R - 1) - R \frac{\cosh R}{\sinh R} \right\}
  \]

- Due to curvature, large scale fluctuations are suppressed, but for \( R_{l.s.} \sim 0.5 \), the angular spectrum is almost indistinguishable from scale invariance, \( C_l^{(\text{scale inv.})} = \frac{H_I^2}{l(l+1)} \) (1% suppression at \( l = 2 \))
The effect of the ancestor vacuum

• Characteristic behavior: exponential decrease in $l$.

$$C_l^{\text{ancestor}} \sim \left( \frac{R}{2} \right)^{2l} \quad (\text{when } R < 1)$$

Because the radial function for the $H^3$ harmonics satisfies

$$\left[ \frac{1}{\sinh^2 R} \frac{\partial}{\partial R} (\sinh^2 R) \frac{\partial}{\partial R} + \frac{l(l+1)}{\sinh^2 R} + (k^2 + 1) \right] q^{(kl)}(R) = 0$$

When $R < 1$, $q^{(kl)}(R) \sim R^l$ (suppression near $R=0$ due to centrifugal barrier).

• If there is a non-normalizable mode, we will see this type of enhancement at low-$l$. 
How large is the effect of the ancestor?

Scalar mode (curvature perturbation)

• Small effect:
  – There will be no non-normalizable mode, since the tunneling field should have large mass in the ancestor in order for the CDL instanton to exist.

For extra fields ("isocurvature" perturbations)

• The effect could be large. (There could be fields with small mass in the ancestor, but we should understand how they contribute to observable quantities.)
How large is the effect of the ancestor?

**Tensor modes**

- **Effect on the temperature fluctuations**

\[
\frac{\delta T(\Omega)}{T} = \int dT \partial_T h_{RR}(T, R(T), \Omega)
\]

- There is no NNM (it is pure gauge), and the lowest
dimension is \( \Delta = 1 + \alpha \) where \( \alpha \sim (r_c H_A)^2 \)

- The ancestor effect is small:

\[
H_I^2 \left( \frac{H_I}{H_A} \right)^{2\alpha}
\]

- (When \( \alpha \ll 1 \) (the bubble is very small), prefactor is important; work in progress)
Open questions

On holographic duality

- Description of the de Sitter side (and application to the measure problem)
- Problems with non-zero final c.c.?
- Matter content of the dual theory (possibly related to D-brane world volume theory)

On observational consequence

- Curvature perturbations in two-field models (when the tunneling field is different from the slow-roll field) could be large?