Evidence for three-nucleon interaction in isotope shifts of $Z =$ magic nuclei

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Contents :

H.N. & T. Inakura, P.R.C 91, 021302(R) ('15)
H.N., P.R.C 92, 044307 ('15)

I. Introduction

Shell structure (→ **magic number**)

— **fundamental concept for nuclear structure**

astrophysical importance

- **waiting point in** s**- &** r**-processes**
- **constraint on EoS** ← **subtracting shell effects**
- **clusters in** n**-star inner crust (↔** ***e.g.*** **QPO)**

★ Z**- &** N**-dep.!** **("shell evolution")**

★ **central +** ℓs **potential**

ℓs **pot. ↔** ℓs **splitting** (→ **magic #'s in** $Z, N > 20$) ⋯ **origin?**

∘ $2N$ **LS int. — insufficient**

(⇒ **strong LS int.** **used in phenomenology)**

∘ **tensor int. (+** α**)** → Z**- &** N**-dep. (1st-order effects)**

contribution to overall strength (2nd-order effects)? ⋯ small

comprehension of its origin
→ **correct prediction of shell structure**

★ $3N$ **int. (χEFT) → ρ-dep. LS int.** Ref. : M. Kohno, P.R.C 86, 061301(R)

stronger LS int. at higher ρ (→ stronger ℓs pot.)

$k_F = 1.35\ \mathrm{fm}^{-1}$	AV18	NSC97	CD-B	$\mathrm{N^3LO}$	$\mathrm{N^3LO + 3NF}$
$B_S(T=0)$	2.0	1.9	3.1	2.5	7.0
$B_S(T=1)$	86.4	86.7	90.2	84.6	116.2
$B_S(\bar{q})$	88.4	88.6	93.3	87.1	123.2
$k_F = 1.07\ \mathrm{fm}^{-1}$	AV18	NSC97	CD-B	$\mathrm{N^3LO}$	$\mathrm{N^3LO + 3NF}$
$B_S(T=0)$	1.4	1.3	2.3	1.6	4.1
$B_S(T=1)$	88.1	88.7	92.2	86.5	106.7
$B_S(\bar{q})$	89.5	90.0	94.5	88.1	110.8

— **complementary to $2N$ LS int !**

⇒ **experimental evidence independent of ℓs splitting ?**

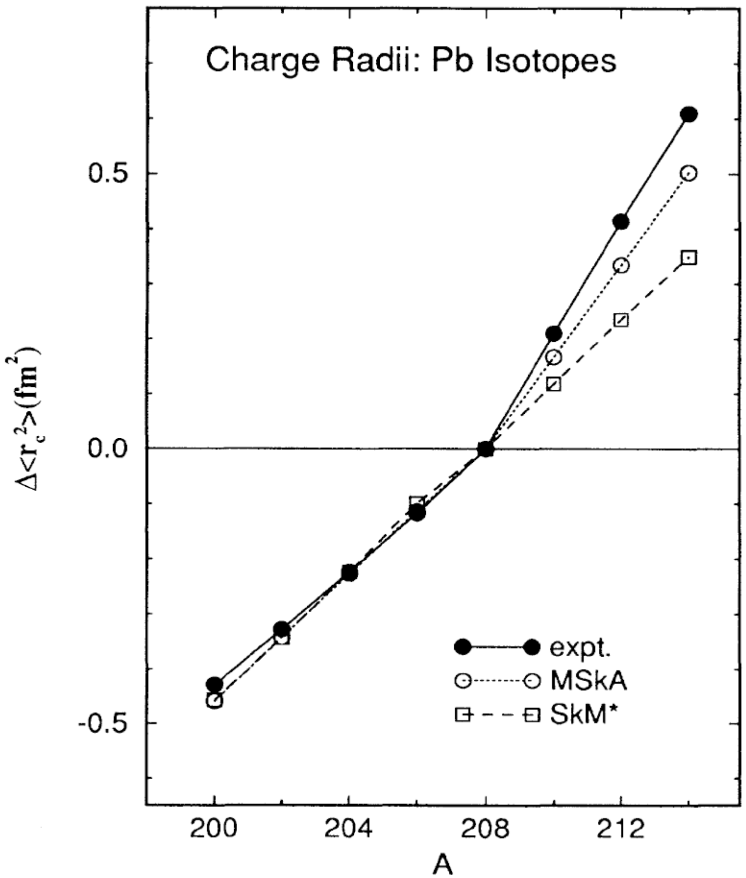
(→ good reliability)

Isotope shifts in Pb nuclei

$\Delta\langle r^2\rangle_p(^A\mathbf{Pb}) := \langle r^2\rangle_p(^A\mathbf{Pb}) - \langle r^2\rangle_p(^{208}\mathbf{Pb})$

exp. $\Rightarrow$ **kink at $N = 126$!**

- **electron scatt.**
- **X-ray freq. difference**
  - **(μ^- atom, $K\alpha$, OIS)**



Ref. : M.M. Sharma *et al.*, P.R.L. 74, 3744

reproduced by RMF, but not by Skyrme EDF up to '95

$\rightarrow$ **dep. on isospin content of LS int.** (**$\rightarrow$ extension of Skyrme EDF**)

$\cdots$ **but cannot be a complete solution !**

kink in $\Delta\langle r^2\rangle_p(^A\text{Pb})$ at $N = 126$ $\longleftarrow$ $n0i_{11/2}$ **occupation**

$\Uparrow$

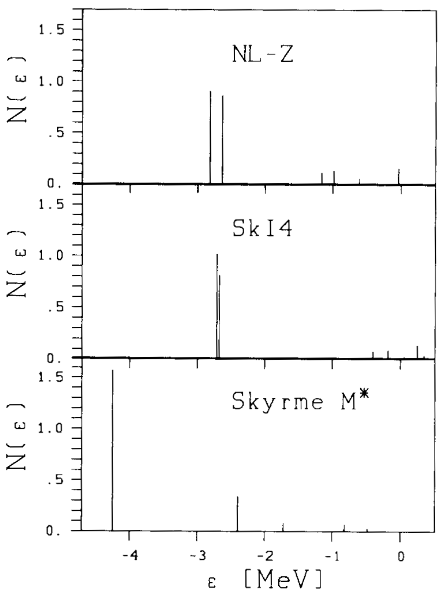
p-n **attraction**

larger $\langle r^2\rangle$ than neighboring orbits
$N < 126$ — unocc.
$N > 126$ — sizable occ. prob. ($\because$ pairing)

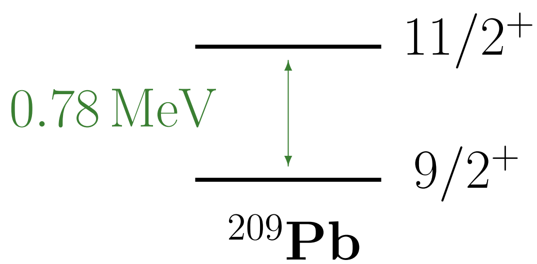
Ref.: P.-G. Reinhard & H. Flocard, N.P.A 584, 467

'understanding' in '90's $\cdots$ $\varepsilon_n(0i_{11/2}) - \varepsilon_n(1g_{9/2})$ is a key
$\leftrightarrow$ isospin content of LS int.

However, $\varepsilon_n(0i_{11/2}) \approx \varepsilon_n(1g_{9/2})$ required ! ($\leftrightarrow$ equal occ. prob.)



on the contrary $\cdots$



II. Mean-field approaches with semi-realistic interaction

"Semi-realistic" nucleonic interaction $\longleftarrow$ **microscopic** $2N$ **(**$+3N$**) int.**

$\uparrow$

phenomenological modification

$\begin{cases} \textbf{saturation properties} \\ \ell s \ \textbf{splitting (?)} \end{cases}$

$\Rightarrow$ **MF (HF, HFB) & RPA calculations**

nuclear reactions $\cdots$ **future project**

Effective Hamiltonian $H = H_N + V_C - H_{\rm c.m.}\,;\quad H_N = \sum_i \frac{\boldsymbol{p}_i^2}{2M} + \sum_{i<j} \hat{v}_{ij}$

(rotational & translational inv.)

$$\hat{v}_{ij} = \hat{v}_{ij}^{(\rm C)} + \hat{v}_{ij}^{(\rm LS)} + \hat{v}_{ij}^{(\rm TN)} + \hat{v}_{ij}^{(\rm C\rho)} \left(+ \hat{v}_{ij}^{(\rm LS\rho)} \right);$$

$$\hat{v}_{ij}^{(\rm C)} = \sum_n \left(t_n^{(\rm SE)} P_{\rm SE} + t_n^{(\rm TE)} P_{\rm TE} + t_n^{(\rm SO)} P_{\rm SO} + t_n^{(\rm TO)} P_{\rm TO} \right) f_n^{(\rm C)}(r_{ij}) \,,$$

$$\left(\begin{array}{ll} P_{\rm SE} := \left(\frac{1-P_\sigma}{2}\right)\left(\frac{1+P_\tau}{2}\right), & P_{\rm TE} := \left(\frac{1+P_\sigma}{2}\right)\left(\frac{1-P_\tau}{2}\right), \\ P_{\rm SO} := \left(\frac{1-P_\sigma}{2}\right)\left(\frac{1-P_\tau}{2}\right), & P_{\rm TO} := \left(\frac{1+P_\sigma}{2}\right)\left(\frac{1+P_\tau}{2}\right) \end{array} \right)$$

$$\hat{v}_{ij}^{(\rm LS)} = \sum_n \left(t_n^{(\rm LSE)} P_{\rm TE} + t_n^{(\rm LSO)} P_{\rm TO} \right) f_n^{(\rm LS)}(r_{ij}) \, \boldsymbol{L}_{ij} \cdot (\boldsymbol{s}_i + \boldsymbol{s}_j) \,,$$

$$\hat{v}_{ij}^{(\rm TN)} = \sum_n \left(t_n^{(\rm TNE)} P_{\rm TE} + t_n^{(\rm TNO)} P_{\rm TO} \right) f_n^{(\rm TN)}(r_{ij}) \, r_{ij}^2 S_{ij}$$

$$\hat{v}_{ij}^{(\rm C\rho)} = \left(C^{(\rm SE)}[\rho(\boldsymbol{r}_i)]\, P_{\rm SE} + C^{(\rm TE)}[\rho(\boldsymbol{r}_i)]\, P_{\rm TE} \right) \delta(\boldsymbol{r}_{ij}) \,;$$

$$\boldsymbol{L}_{ij} := \boldsymbol{r}_{ij} \times \boldsymbol{p}_{ij} \,, \quad S_{ij} := 3(\boldsymbol{\sigma}_i \cdot \hat{\boldsymbol{r}}_{ij})(\boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{ij}) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \,, \quad f_n(r) = e^{-\mu_n r}/\mu_n r$$

$$C^{(\rm Y)}[\rho] = t_\rho^{(\rm Y)} \rho^{\alpha^{(\rm Y)}} \quad (\mathrm{Y} = \mathrm{SE}\ \textbf{or}\ \mathrm{TE}\,; \quad \alpha^{(\rm SE)} = 1\,,\ \alpha^{(\rm TE)} = 1/3)$$

M3Y int. $\cdots$ **Yukawa function** $\rightarrow$ **fit to G-matrix (at $\rho \approx \rho_0/3$)**

- **OPEP** $\rightarrow$ **longest part of** $\hat{v}_{ij}^{(\mathrm{C})}$ $(\equiv \hat{v}_{\mathrm{OPEP}}^{(\mathrm{C})})$
- **popular in reaction problems**
- **no saturation (without modification)** $\rightarrow$ **add** $\hat{v}_{ij}^{(\mathrm{C}\rho)}$

'**M3Y-Pn**'

- **modifying M3Y-Paris** $\begin{cases} \text{replace short-range part of } \hat{v}^{(\mathrm{C})} \text{ by } \hat{v}^{(\mathrm{C}\rho)} \\ \text{enhance } \hat{v}^{(\mathrm{LS})} \quad (\leftrightarrow \ell s \text{ splitting}) \end{cases}$
- **keeping** $\hat{v}_{\mathrm{OPEP}}^{(\mathrm{C})}$
- **no change for** $\hat{v}^{(\mathrm{TN})}$ **from M3Y-Paris in M3Y-P5 to P7**

(— **realistic tensor force**)

$\cdots$ **leading order of chiral dynamics**

basic formulae : H.N., P.R.C 68, 014316 ('03)

M3Y-P6, P7 : H.N., P.R.C 87, 014336 ('13)

$\Rightarrow$ **nuclear matter & finite nuclei**

Numerical methods for finite nuclei — Gaussian expansion method

- **spherical HF** ··· H.N. & M. Sato, N.P.A 699, 511 ('02); 714, 696 ('03)
- **spherical HFB** ··· H.N., N.P.A 764, 117 ('06); 801, 169 ('08)
- **axial HF & HFB** ··· H.N., N.P.A 808, 47 ('08)
- **spherical RPA** ··· H.N., K. Mizuyama, M. Yamagami & M. Matsuo, N.P.A 828, 283 ('09)

Advantages of the method

(i) ability to describe ε-dep. exponential/oscillatory asymptotics

(ii) tractability of various 2-body interactions

(iii) basis parameters insensitive to nuclide

(iv) exact treatment of Coulomb & c.m. Hamiltonian

★ **Energies & radii of doubly magic nuclei :**

		SLy5	D1S	M3Y-P6	M3Y-P7	CCSD	Exp.
^{16}**O**	$-E$	128.6	129.5	126.3	125.9	107.5	127.6
	$\sqrt{\langle r^2\rangle}$	2.59	2.61	2.59	2.57	—	2.61
^{40}**Ca**	$-E$	344.3	344.6	335.9	334.3	308.8	342.1
	$\sqrt{\langle r^2\rangle}$	3.29	3.37	3.37	3.35	—	3.47
^{48}**Ca**	$-E$	416.0	416.8	413.8	414.9	355.2	416.0
	$\sqrt{\langle r^2\rangle}$	3.44	3.51	3.51	3.49	—	3.57
^{90}**Zr**	$-E$	782.4	785.9	781.1	780.8	—	783.9
	$\sqrt{\langle r^2\rangle}$	4.22	4.24	4.23	4.22	—	4.32
^{208}**Pb**	$-E$	1635.2	1639.0	1634.5	1635.5	—	1636.4
	$\sqrt{\langle r^2\rangle}$	5.52	5.51	5.53	5.51	—	5.49

CCSD $\cdots$ **G. Hagen *et al.*, P.R.L. 101, 092502 ('08)**

(chiral N^{3}LO without 3NF)

★ **Separation energies of proton- / neutron-magic nuclei :**

$\cdots$ **important in astrophysics**

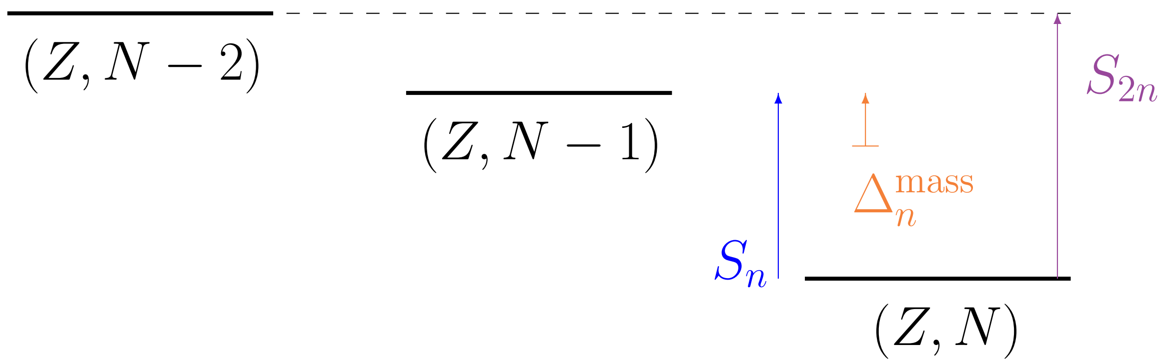
$$S_n(Z,N) := E(Z,N-1) - E(Z,N)$$

$$S_{2n}(Z,N) := E(Z,N-2) - E(Z,N) = S_n(Z,N) + S_n(Z,N-1)$$

$$\Delta_n^{\rm mass}(Z,N-1) := E(Z,N-1) - \frac{1}{2}\left[E(Z,N-2) + E(Z,N)\right]$$

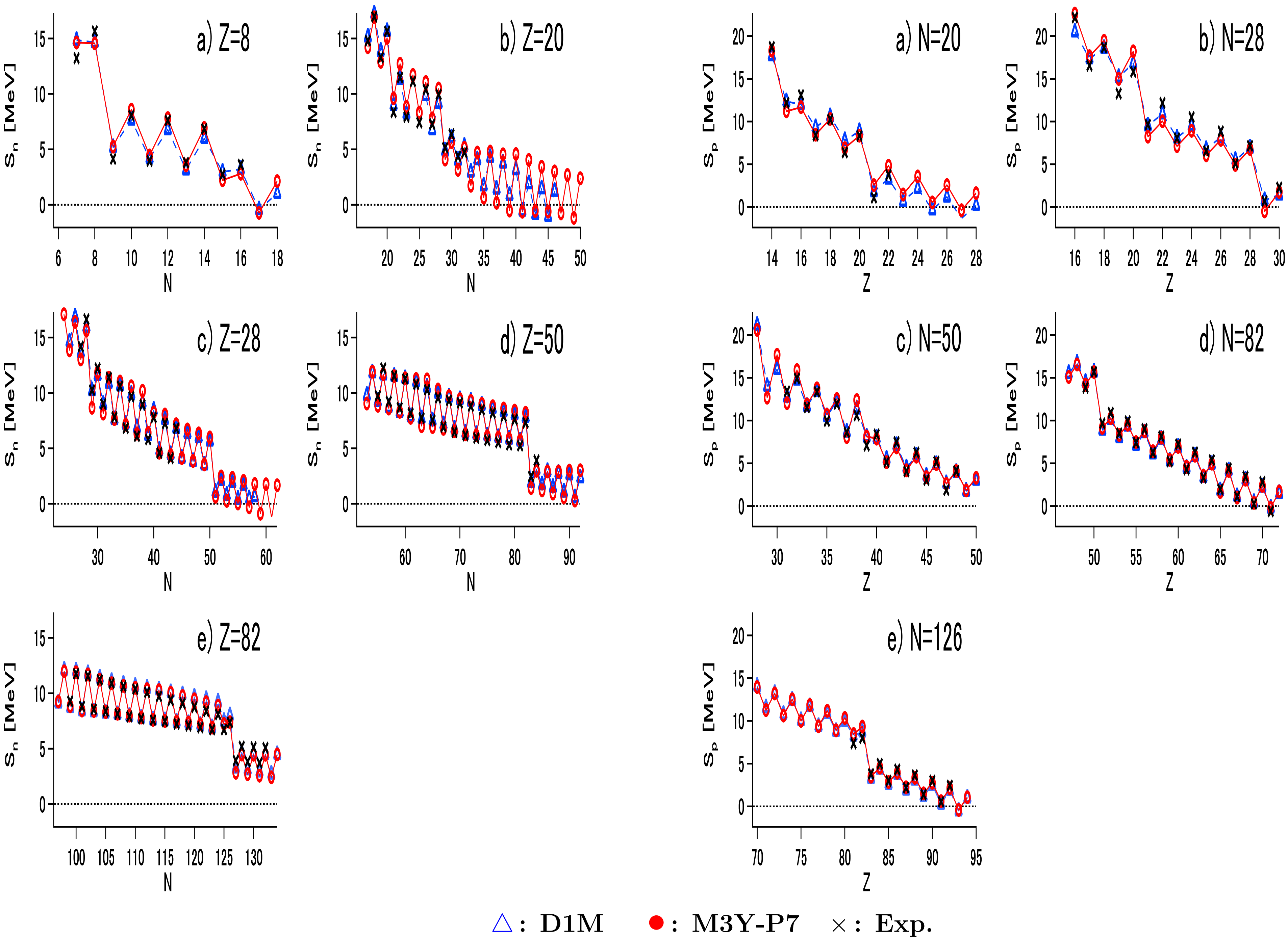
$$= \frac{1}{2}\left[S_n(Z,N) - S_n(Z,N-1)\right] \quad \textbf{(for } N-1 = \text{odd}\textbf{)}$$

$\leftrightarrow$ **pairing**



(S_p, S_{2p}, $\Delta_p^{\rm mass}$ $\rightarrow$ **obtained analogously)**

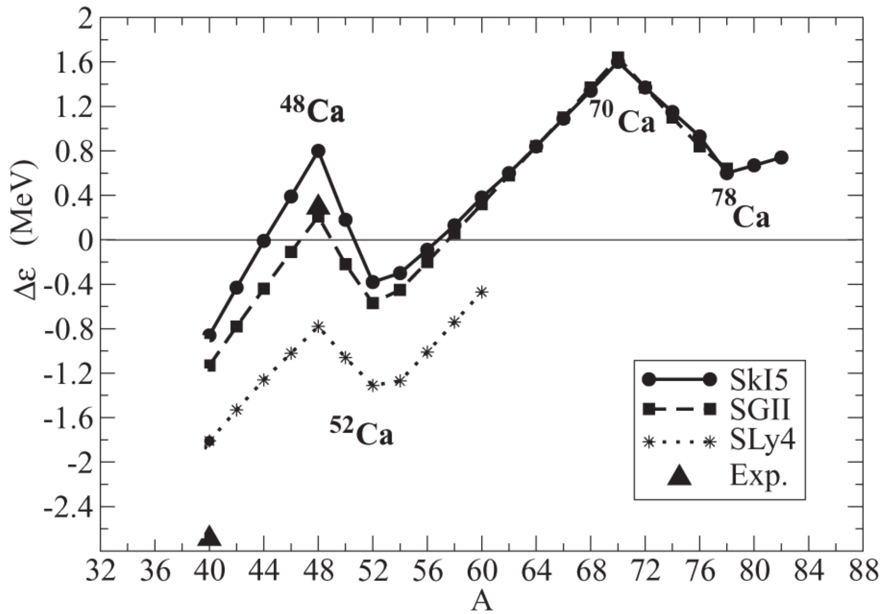
$\Delta_n^{\rm mass}$ **for** $Z=50$ **&** $\Delta_p^{\rm mass}$ **for** $N=82$ $\cdots$ **fitted**

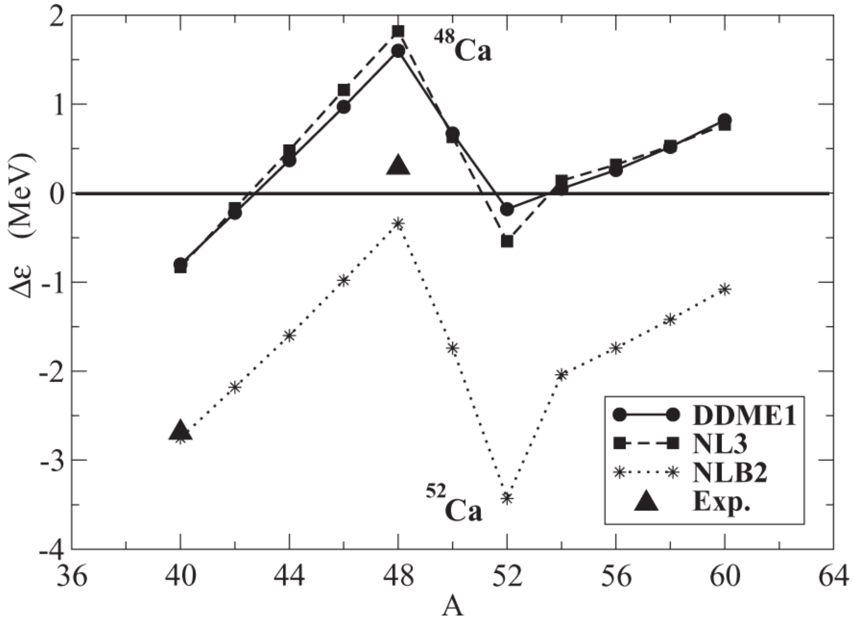

a) Z=8
b) Z=20
c) Z=28
d) Z=50
e) Z=82
a) N=20
b) N=28
c) N=50
d) N=82
e) N=126
S_n [MeV]
S_p [MeV]
N
Z
△ : D1M
● : M3Y-P7
× : Exp.

★ N-dependence of single-proton energies below $Z = 20$

— $1s_{1/2}$-$0d_{3/2}$ inversion

$\Delta\varepsilon_p = \varepsilon_p(1s_{1/2}) - \varepsilon_p(0d_{3/2})$

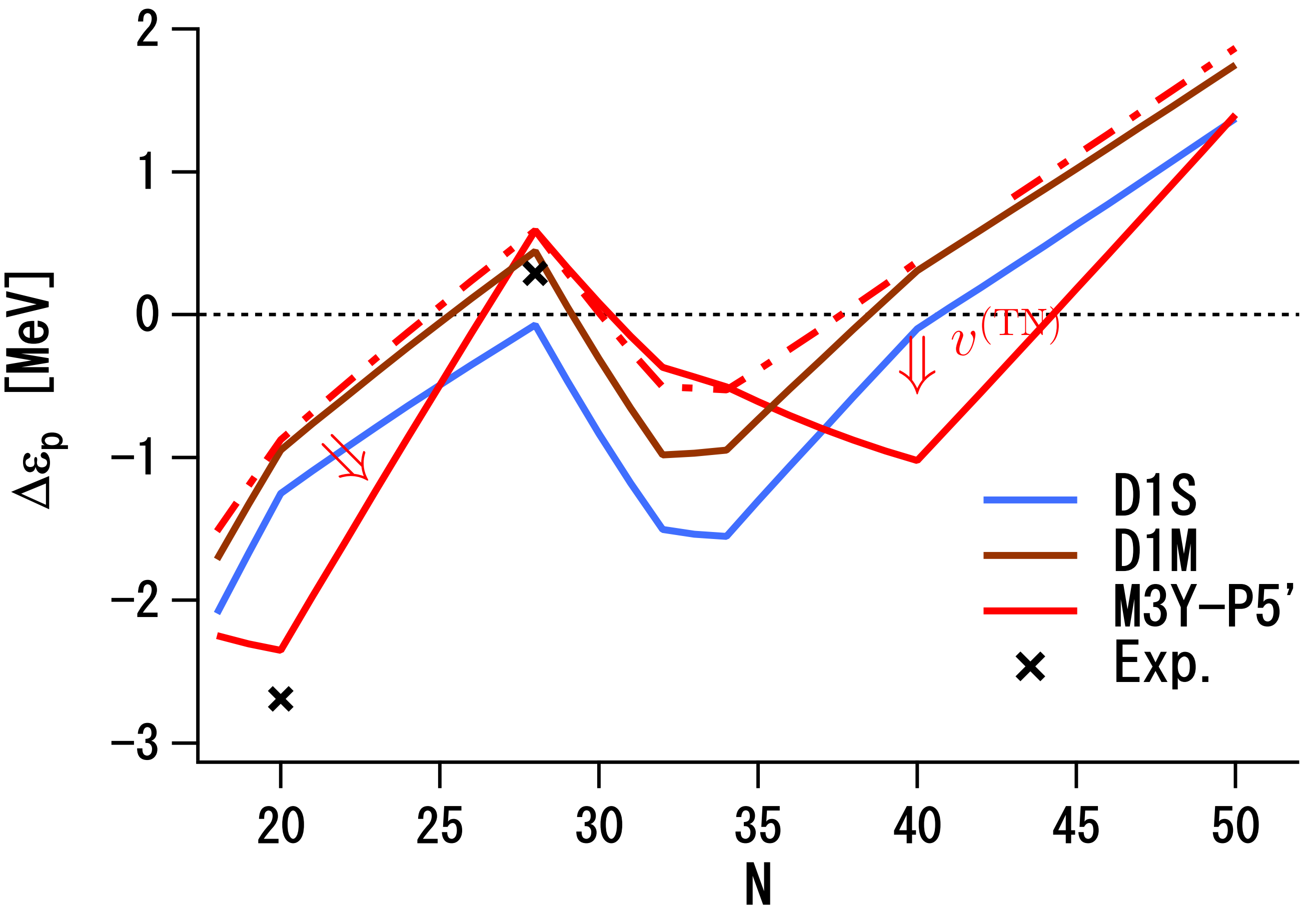




Ref: M. Grasso *et al.*, P.R.C 76, 044319 ('07)

(Exp.: average weighted by spectroscopic factor)

Case of semi-realistic int. (M3Y-P5′)

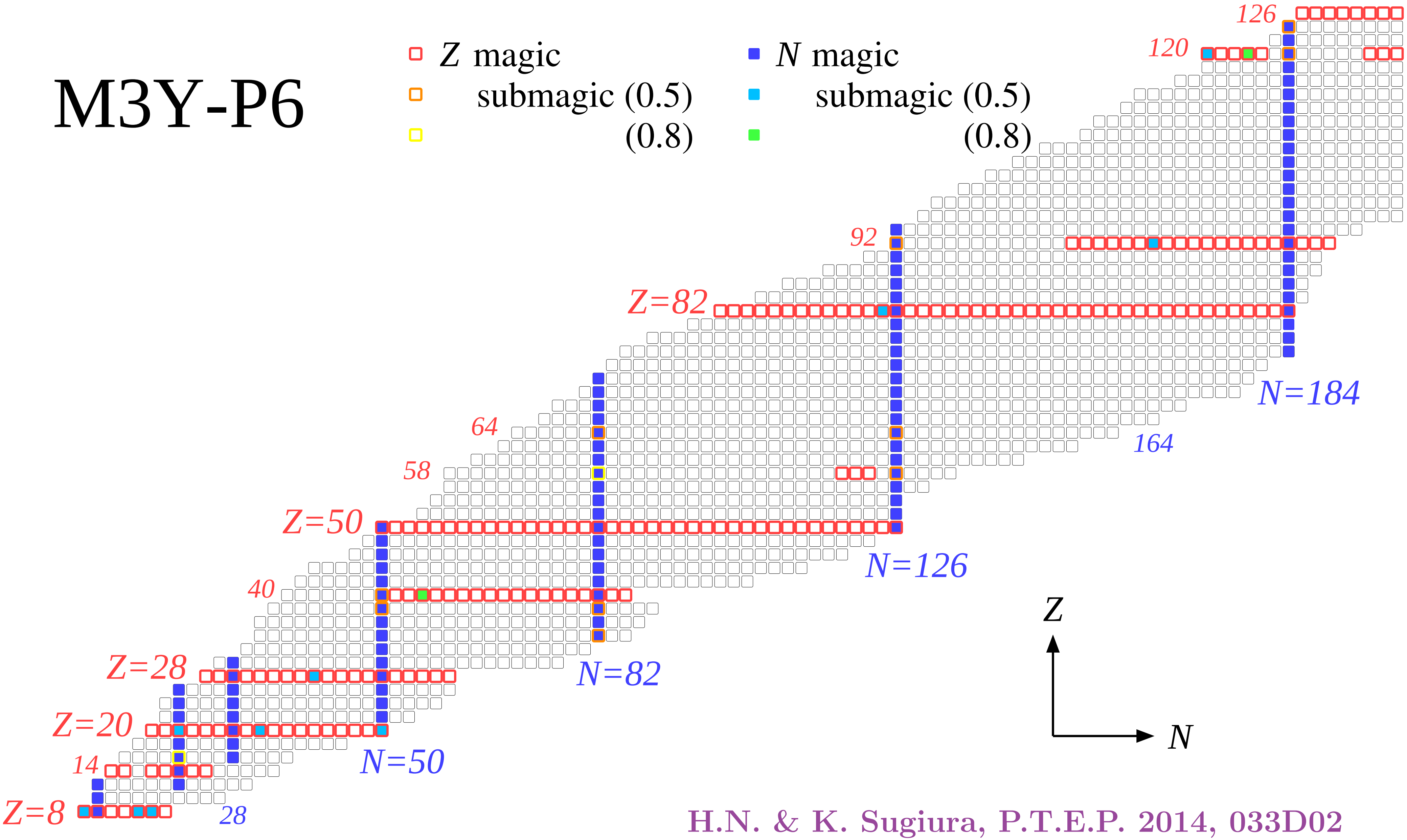


realistic $\hat{v}^{(\mathrm{TN})}$ → correct N-dep. of $\Delta\varepsilon_p$ (in $N = 20 - 28$) !

H.N., K. Sugiura & J. Margueron, P.R.C 87, 067305 ('13)

★ Magic numbers

M3Y-P6



H.N. & K. Sugiura, P.T.E.P. 2014, 033D02

III. Incorporating $3N$ LS interaction

Semi-realistic M3Y-P6 int. $\cdots$ **reasonable shell structure** $\Rightarrow$ **yardstick**

$3N$ **LS int.** $\leftrightarrow$ ρ**-dep. LS int.** ($\hat{v}^{(\mathrm{LS}\rho)}$) **Ref.: M. Kohno, P.R.C 86, 061301(R)**

$\Downarrow$

M3Y-P6 — $\hat{v}^{(\mathrm{LS})}_{\mathrm{M3Y}} \times 2.2$

vs. (**equate** $\varepsilon_n(0i_{11/2}) - \varepsilon_n(0i_{13/2})$ **at** ^{208}**Pb**)

M3Y-P6a — $\hat{v}^{(\mathrm{LS})}_{\mathrm{M3Y}} + \hat{v}^{(\mathrm{LS}\rho)}$

$$v^{(\mathrm{LS}\rho)} = 2i\, D[\rho(\boldsymbol{R}_{ij})]\, \boldsymbol{p}_{ij} \times \delta(\boldsymbol{r}_{ij})\, \boldsymbol{p}_{ij} \cdot (\boldsymbol{s}_i + \boldsymbol{s}_j)\,;$$

$$D[\rho(\boldsymbol{r})] = -w_1 \frac{\rho(\boldsymbol{r})}{1 + d_1 \rho(\boldsymbol{r})} \Big(\approx -w_1\, \rho(\boldsymbol{r}) \Big)$$

$d_1 = 1.0\,\mathrm{fm}^3$ **(prefix),** $w_1\,(>0)$ **: fitted**

★ Local current representation of contribution to HF energy

$$E_{ph}^{(\mathrm{LS}\rho)} = \frac{1}{4}\int d^3r\, D[\rho(\mathbf{r})]$$
$$\times \Big\{\rho(\mathbf{r})\,\nabla\cdot\mathbf{J}(\mathbf{r}) + \sum_{\tau=p,n}\rho_\tau(\mathbf{r})\,\nabla\cdot\mathbf{J}_\tau(\mathbf{r}) + i\,\mathbf{J}(\mathbf{r})\cdot\mathbf{j}^*(\mathbf{r}) + i\sum_{\tau=p,n}\mathbf{J}_\tau(\mathbf{r})\cdot\mathbf{j}_\tau^*(\mathbf{r})$$
$$-i\,\mathbf{J}^*(\mathbf{r})\cdot\mathbf{j}(\mathbf{r}) - i\sum_{\tau=p,n}\mathbf{J}_\tau^*(\mathbf{r})\cdot\mathbf{j}_\tau(\mathbf{r}) - 2\,\mathbf{Q}(\mathbf{r})\cdot\mathbf{s}(\mathbf{r}) - 2\sum_{\tau=p,n}\mathbf{Q}_\tau(\mathbf{r})\cdot\mathbf{s}_\tau(\mathbf{r})\Big\};$$

$$\rho(\mathbf{r}) = \sum_{\tau=p,n}\rho_\tau(\mathbf{r})\,,\qquad \rho_\tau(\mathbf{r}) = \sum_{\alpha,\beta\in\tau}\varrho_{\alpha\beta}\,\phi_\beta^\dagger(\mathbf{r})\,\phi_\alpha(\mathbf{r})\,,$$
$$\mathbf{j}(\mathbf{r}) = \sum_{\tau=p,n}\mathbf{j}_\tau(\mathbf{r})\,,\qquad \mathbf{j}_\tau(\mathbf{r}) = -i\sum_{\alpha,\beta\in\tau}\varrho_{\alpha\beta}\,\phi_\beta^\dagger(\mathbf{r})\,\nabla\phi_\alpha(\mathbf{r})\,,$$
$$\mathbf{Q}(\mathbf{r}) = \sum_{\tau=p,n}\mathbf{Q}_\tau(\mathbf{r})\,,\qquad \mathbf{Q}_\tau(\mathbf{r}) = i\sum_{\alpha,\beta\in\tau}\varrho_{\alpha\beta}\,\nabla\phi_\beta^\dagger(\mathbf{r})\times\nabla\phi_\alpha(\mathbf{r})\,,$$
$$\mathbf{s}(\mathbf{r}) = \sum_{\tau=p,n}\mathbf{s}_\tau(\mathbf{r})\,,\qquad \mathbf{s}_\tau(\mathbf{r}) = \sum_{\alpha,\beta\in\tau}\varrho_{\alpha\beta}\,\phi_\beta^\dagger(\mathbf{r})\,\mathbf{s}\,\phi_\alpha(\mathbf{r})\,,$$
$$\mathbf{J}(\mathbf{r}) = \sum_{\tau=p,n}\mathbf{J}_\tau(\mathbf{r})\,,\qquad \mathbf{J}_\tau(\mathbf{r}) = 2i\sum_{\alpha,\beta\in\tau}\varrho_{\alpha\beta}\,\phi_\beta^\dagger(\mathbf{r})\,\mathbf{s}\times\nabla\phi_\alpha(\mathbf{r})\,.$$

spherical $\rightarrow$ $\mathbf{Q}_\tau(\mathbf{r}) = \mathbf{s}_\tau(\mathbf{r}) = 0$ **,** $i[\mathbf{j}_\tau(\mathbf{r}) - \mathbf{j}_\tau^*(\mathbf{r})] = \nabla\rho_\tau(r)$ **,** $\mathbf{J}_\tau(\mathbf{r}) = \mathbf{J}_\tau^*(\mathbf{r})$

★ **Contribution to ℓs potential**

$$-\frac{1}{r}\left[D[\rho(r)]\,\frac{d}{dr}\Big(\rho(r)+\rho_\tau(r)\Big) + \frac{1}{2}\,\frac{\delta D}{\delta\rho}[\rho(r)]\,\Big(\rho(r)+\rho_\tau(r)\Big)\,\frac{d\rho(r)}{dr}\right]\boldsymbol{\ell}\cdot\mathbf{s}\,. \quad (\tau = p, n)$$

★ **Influence on s.p. wave functions :**

presence of $D[\rho]$ → { **stronger LS for interior (higher ρ)**; **weaker LS for extetior (lower ρ)** }

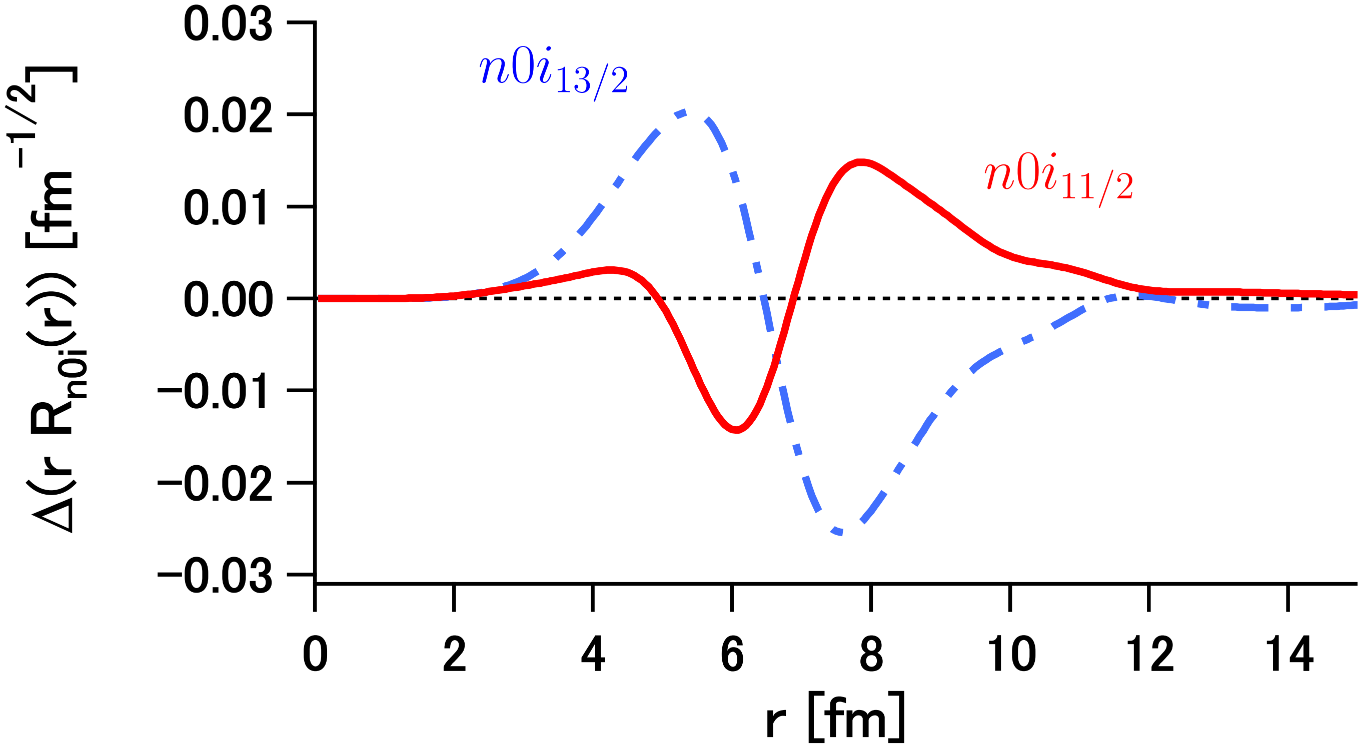
→ { $j = \ell + 1/2$ **orbitals shrink**; $j = \ell - 1/2$ **orbitals extends** } **($\because$ variation)**

e.g. **larger $\langle r^2\rangle$ for $n0i_{11/2}$ for Pb**

→ **influence on isotope shifts of Pb ?**

★ **Confirming effects on s.p. functions**

$\Delta[r\,R_j(r)] := [r\,R_j(r)]_{\mathrm{M3Y-P6a}} - [r\,R_j(r)]_{\mathrm{M3Y-P6}}$ @ ^{208}**Pb**



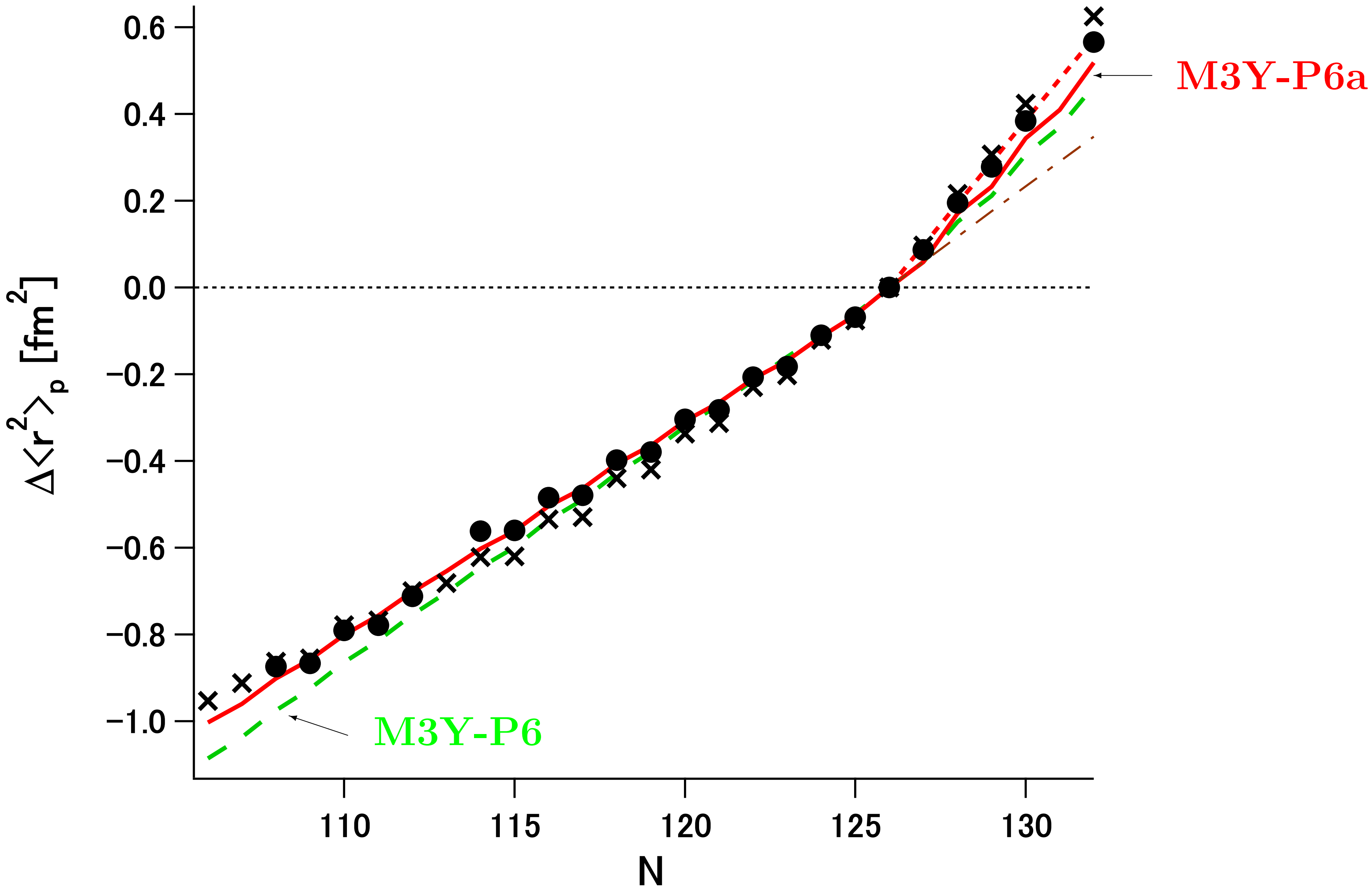
$\begin{cases} j = \ell + 1/2 \text{ orbitals shrink} \\ j = \ell - 1/2 \text{ orbitals extends} \end{cases}$

$\langle r^2 \rangle_j\ [\mathrm{fm}^2]$:

	M3Y-P6	**M3Y-P6a**
$n1g_{9/2}$	32.3	31.8
$n0i_{11/2}$	40.2	40.7

IV. $3N$ **LS interaction & isotope shifts**

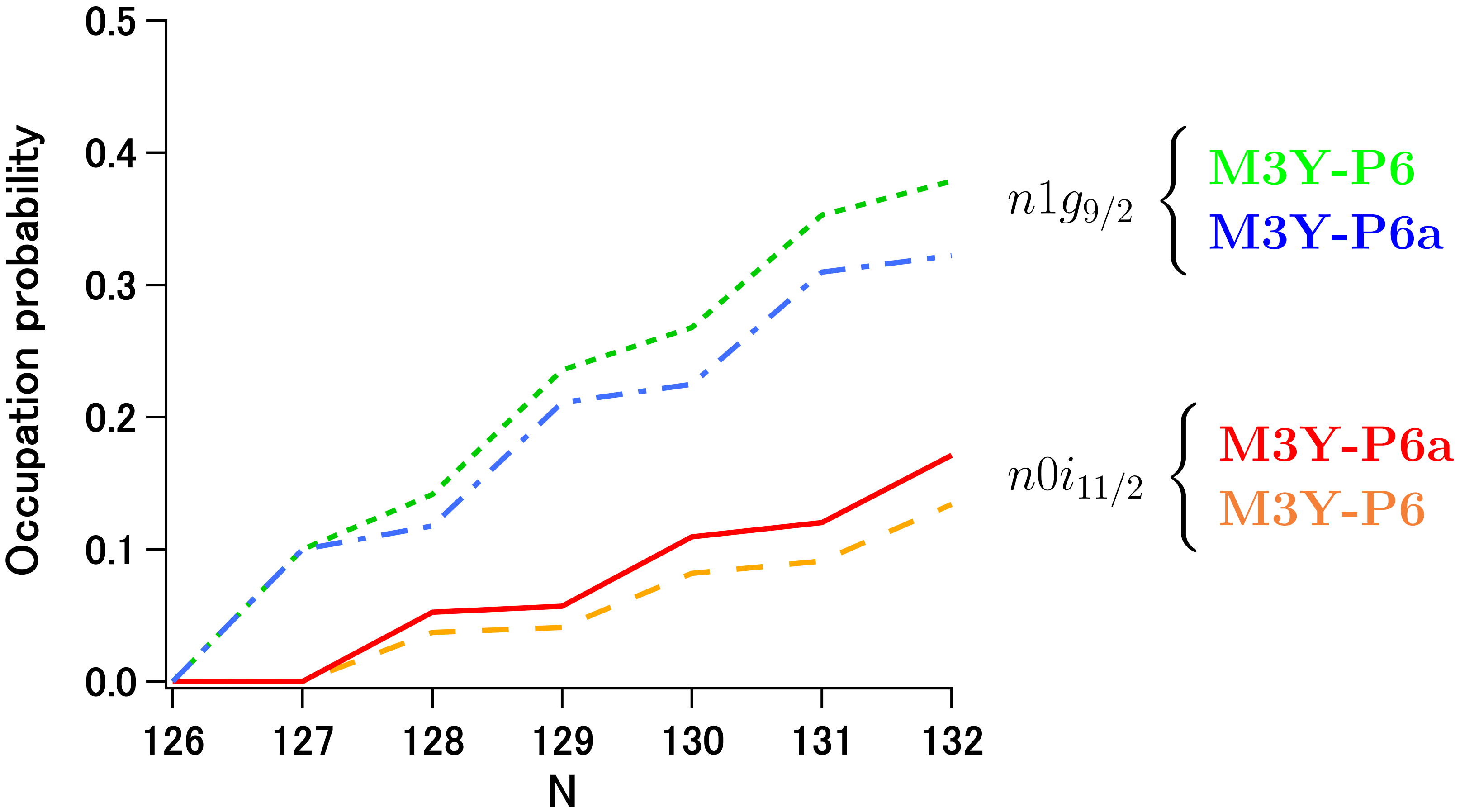
★ **Isotope shifts of Pb nuclei** $\Delta\langle r^2\rangle_p(^A\mathbf{Pb}) := \langle r^2\rangle_p(^A\mathbf{Pb}) - \langle r^2\rangle_p(^{208}\mathbf{Pb})$



★ **S.p. energies & occ. prob.**

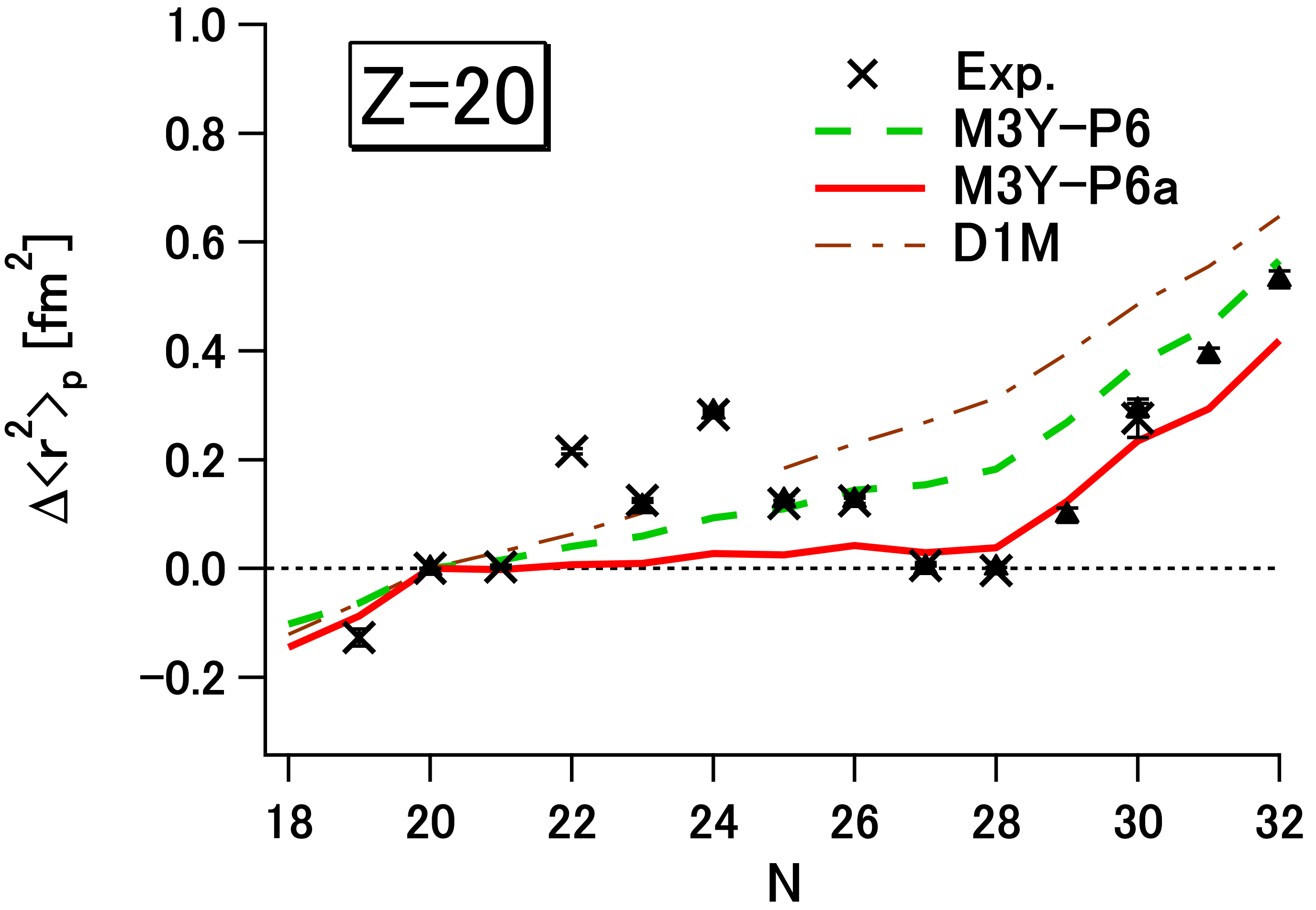
$\varepsilon_n(0i_{11/2}) - \varepsilon_n(1g_{9/2})$: { **exp. @ ^{209}Pb** → 0.78 MeV; **M3Y-P6a** → 0.72 MeV }

occ. prob.



⇒ **kink at $N = 126$ reproduced without $n1g_{9/2}$-$n0i_{11/2}$ degeneracy !**

★ **Isotope shifts of Ca nuclei** $\Delta\langle r^2\rangle_p(^A\mathbf{Ca}) := \langle r^2\rangle_p(^A\mathbf{Ca}) - \langle r^2\rangle_p(^{40}\mathbf{Ca})$

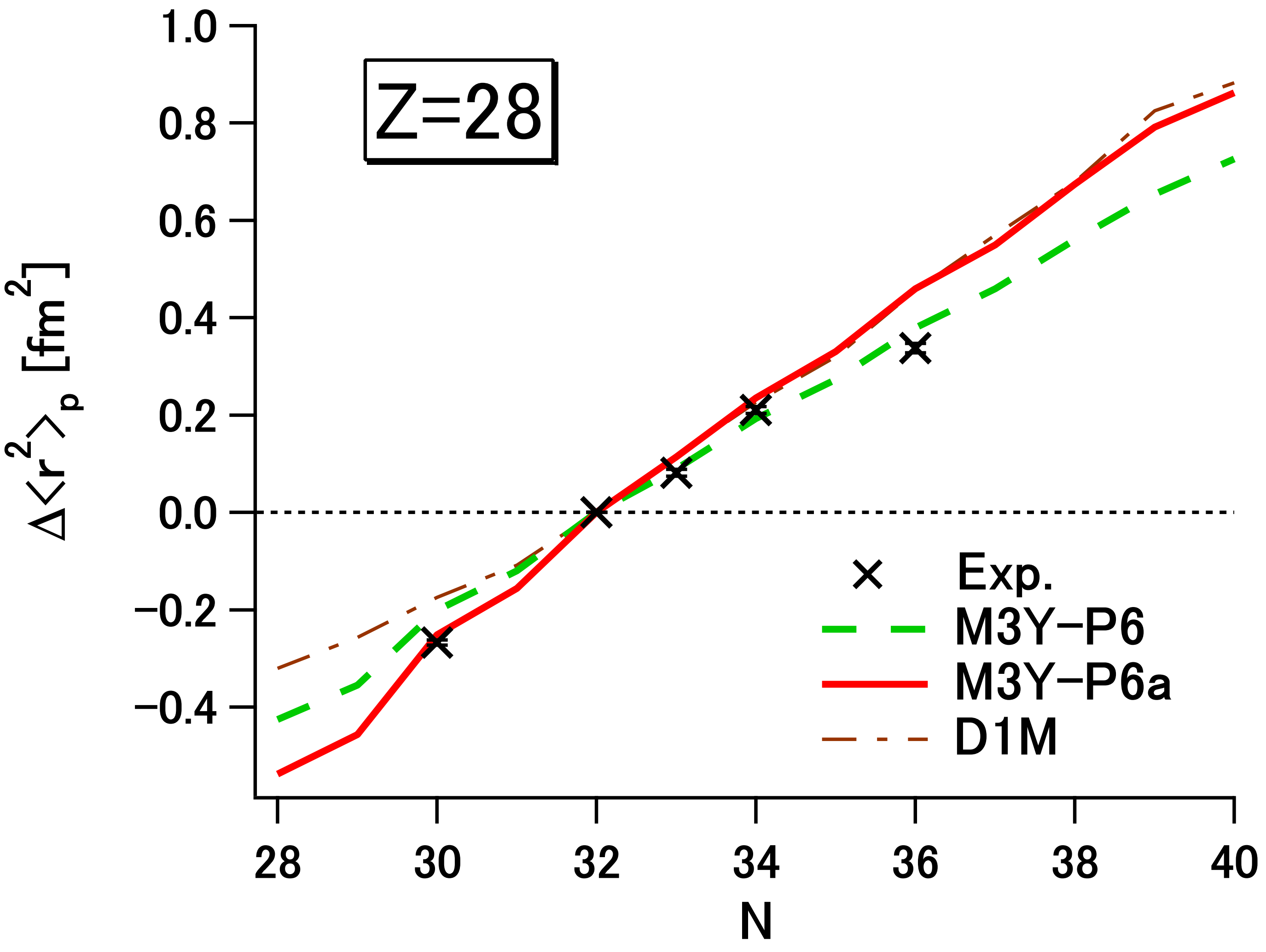


$\sqrt{\langle r^2\rangle_p}(^{40}\mathbf{Ca}) \approx \sqrt{\langle r^2\rangle_p}(^{48}\mathbf{Ca})$ **!**

$\cdots$ difficult to be reproduced so far

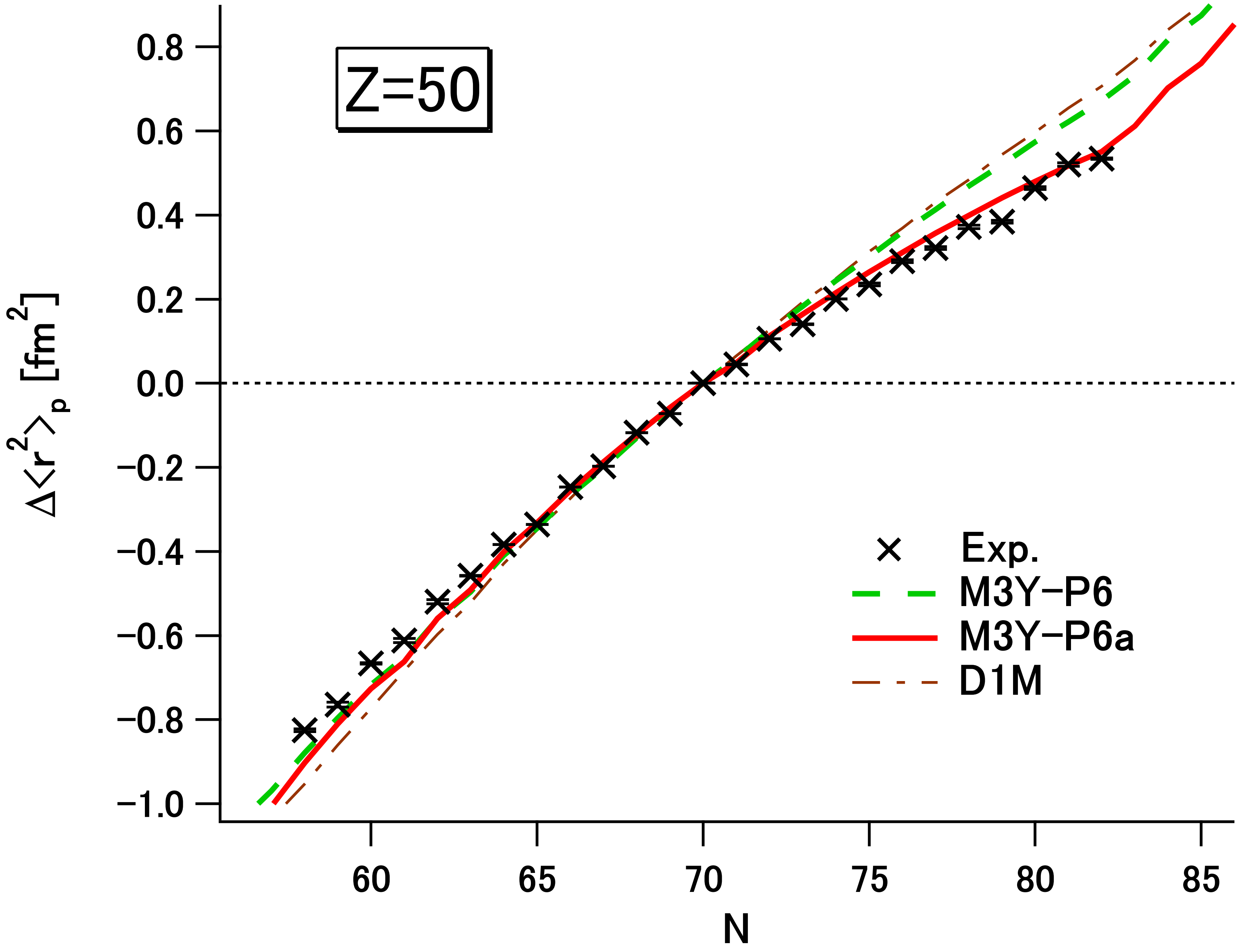
(▲: new exp., Nat. Phys. AOP 3645)

★ **Isotope shifts of Ni nuclei** $\Delta\langle r^2\rangle_p(^{A}\mathbf{Ni}) := \langle r^2\rangle_p(^{A}\mathbf{Ni}) - \langle r^2\rangle_p(^{60}\mathbf{Ni})$



Data on ^{56}Ni ?

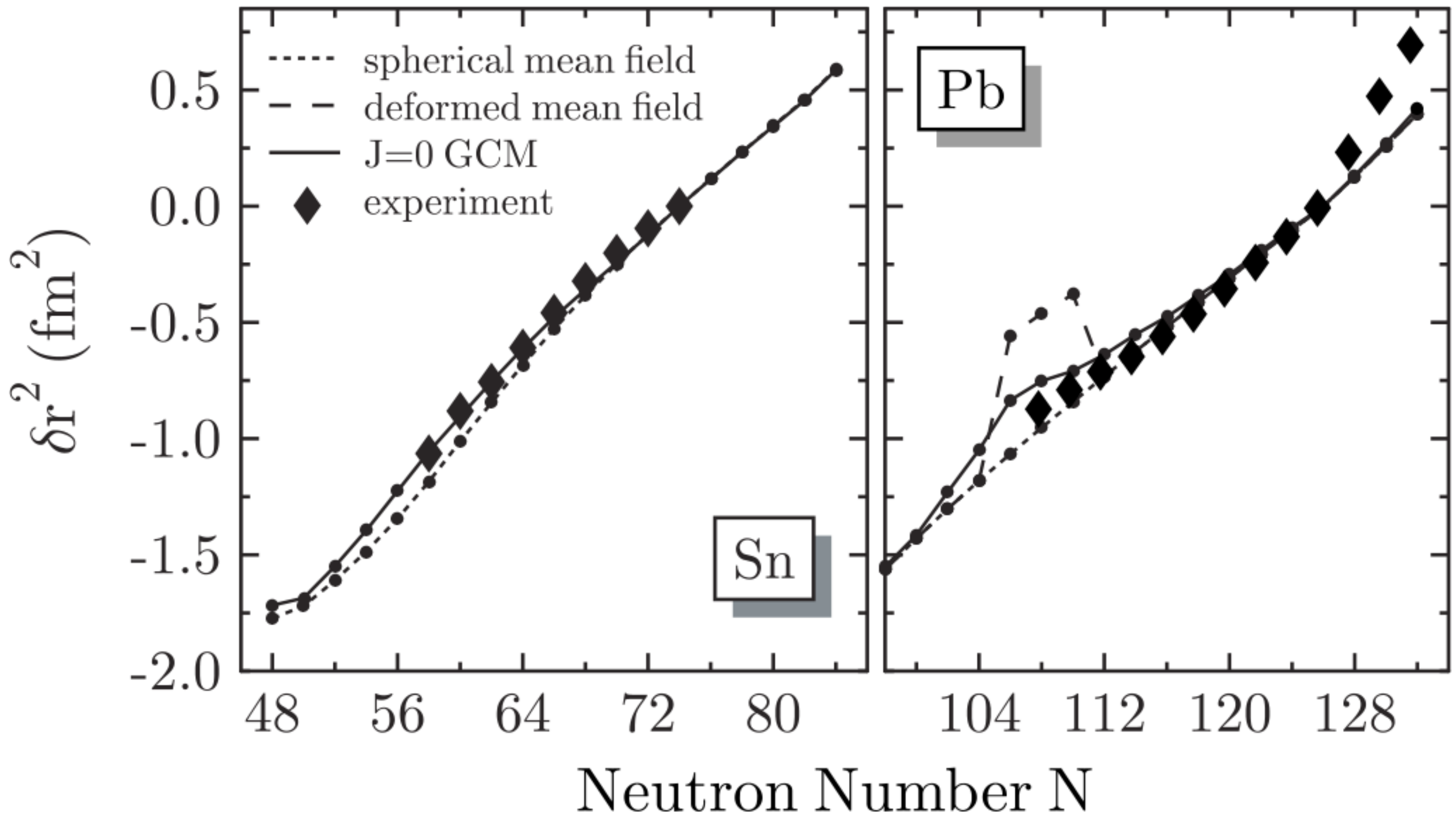
★ **Isotope shifts of Sn nuclei** $\Delta\langle r^2\rangle_p(^A\mathbf{Sn}) := \langle r^2\rangle_p(^A\mathbf{Sn}) - \langle r^2\rangle_p(^{120}\mathbf{Sn})$



A kink predicted at $N = 82$!

— **in sharp contrast to int. without ρ-dep. LS** $\Rightarrow$ **data ?**

cf. **Influence of deformation & correlations beyond mean field** (SLy4)



Ref. : M. Bender *et al.*, P.R.C 73, 034322

- **No improvement on the kink at $N = 126$ for Pb isotopes**
- **Not in good agreement at $N \approx 80$ for Sn isotopes (not shown)**

— **no kink at $N = 82$ (?)**

V. Summary

1. **We have investigated effects of $3N$ LS int. on isotope shifts of nuclei.**
   $\leftarrow$ sph. HFB with semi-realistic int. M3Y-P6 & its variant M3Y-P6a

2. **With M3Y-P6a which contains ρ-dep. LS channel,**

   - **isotope shifts of the Pb nuclei are described fairly well**
     without fictitious $n1g_{9/2}$-$n0i_{11/2}$ degeneracy,
   - **almost equal charge radii between ^{40}Ca and ^{48}Ca are reproduced,**
   - **isotope shifts of the Sn nuclei are in agreement with available data,**
     and a kink is predicted at $N = 82$. $\rightarrow$ data ?

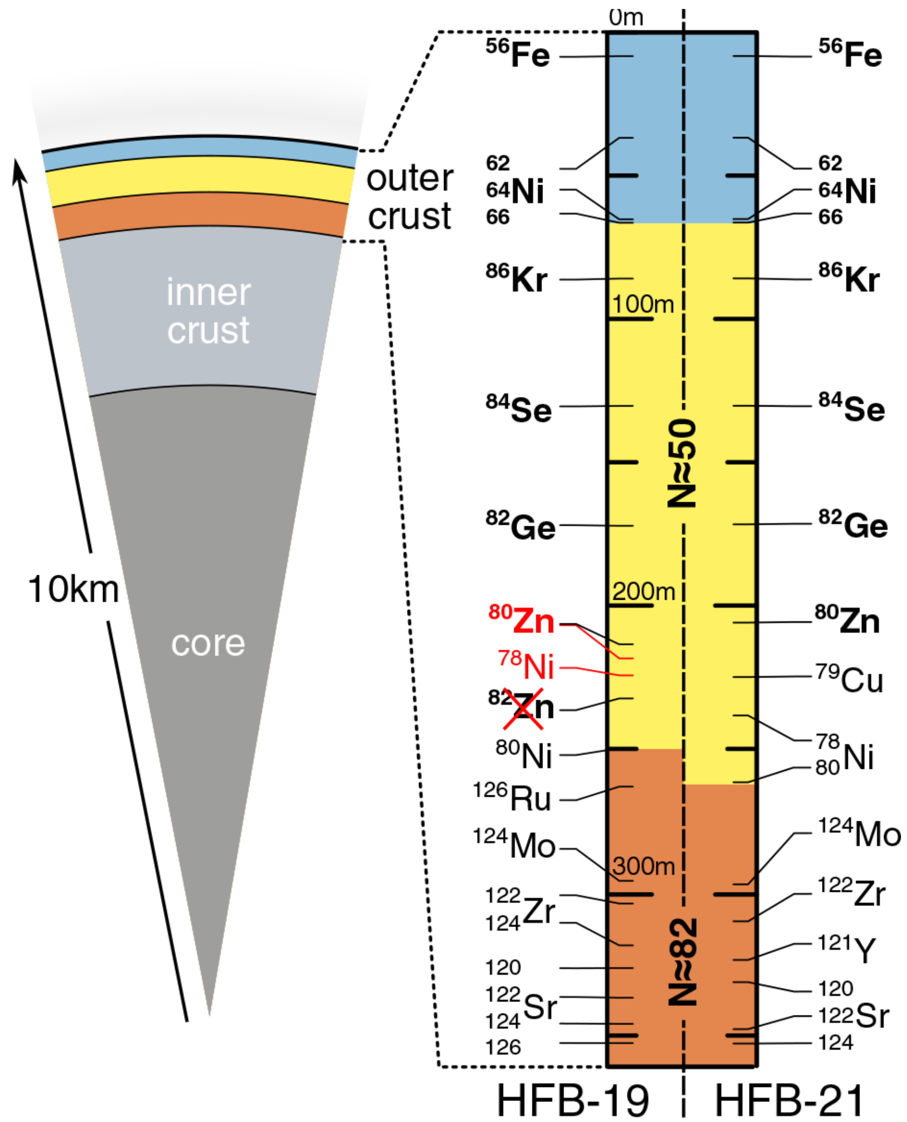
3. **Results may be regarded as evidence for $3N$ LS interaction**
   based on χEFT, indep. of ℓs splitting.

   — qualitative evidence for $3N$ interaction !

Perspectives :

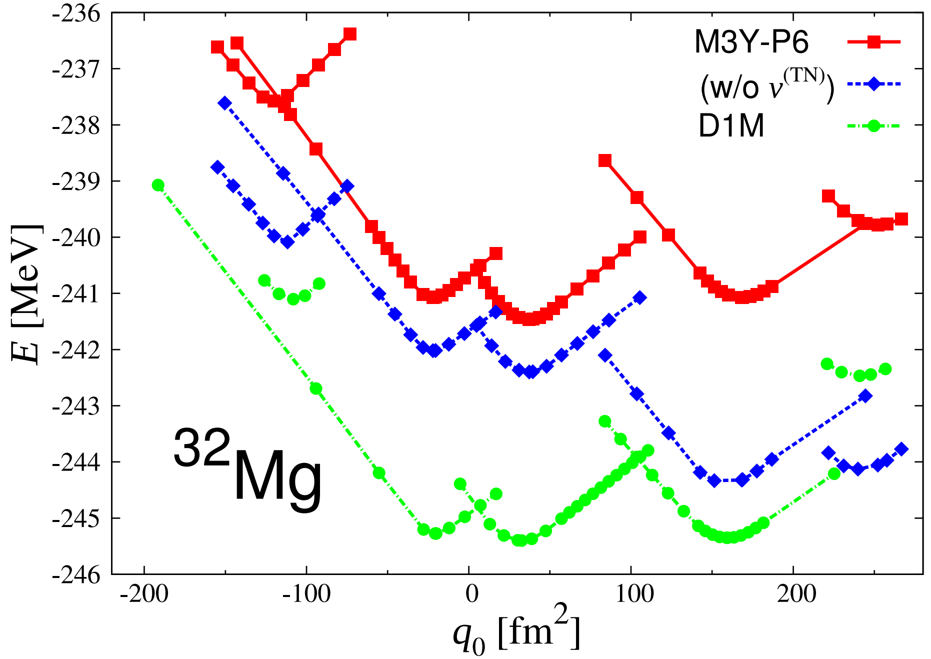
- **Influence on drip line, shell structure ?**
- **Effects on deformation & excitation (*e.g.* spin excitation) ?**

★ **nuclear mass & structure of n-star crust**



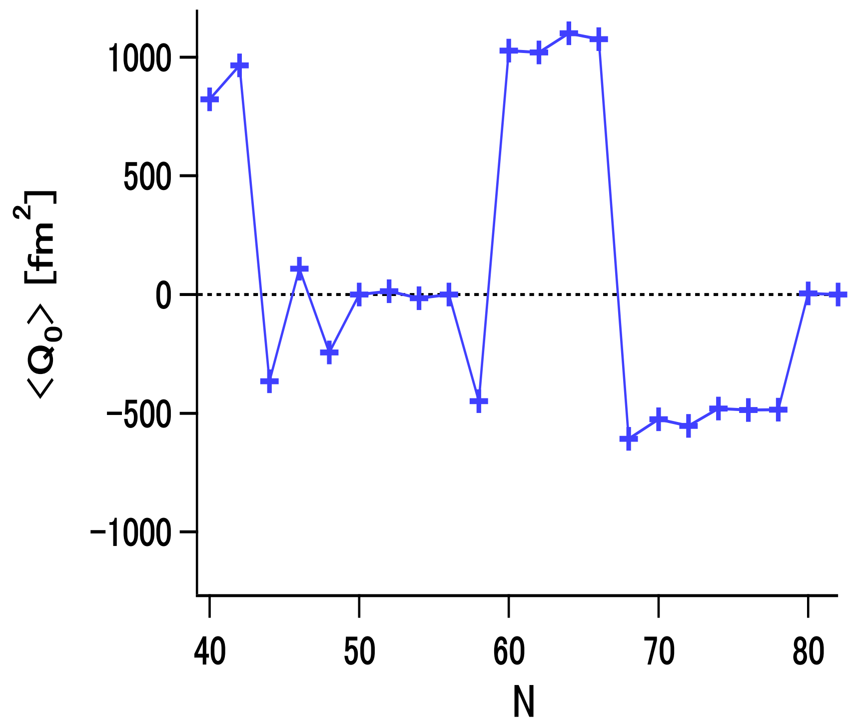
R.N. Wolf *et al.*, P.R.L. 110, 041101 ('13)

★ Quadrupole deformation of ^{32}Mg (← axial HF)



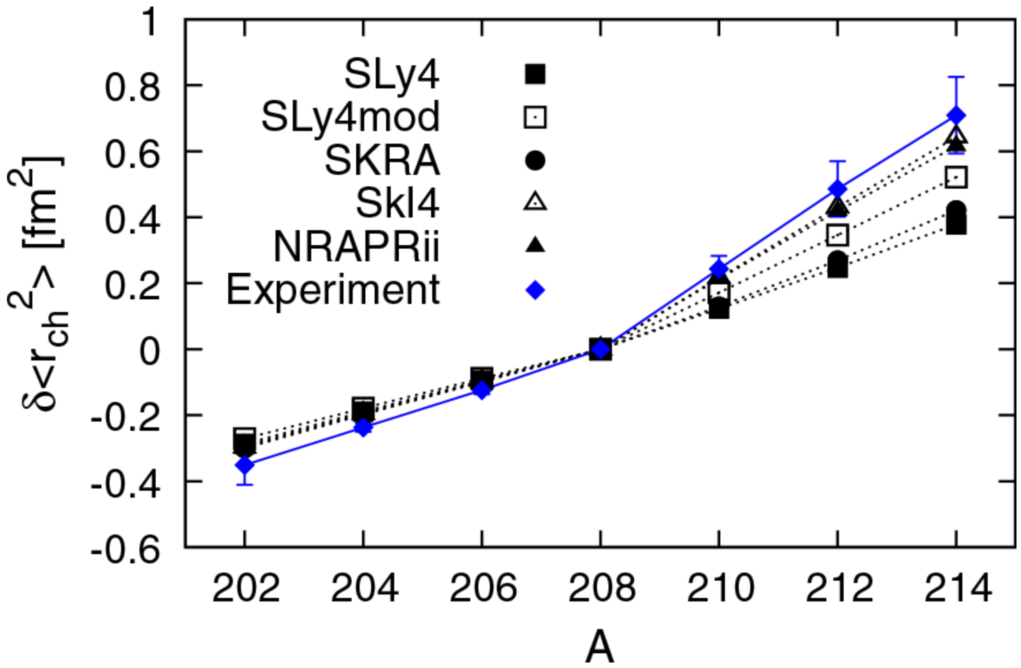
Ref. : Y. Suzuki, H.N. & S. Miyahara, arXiv: 1604.03202

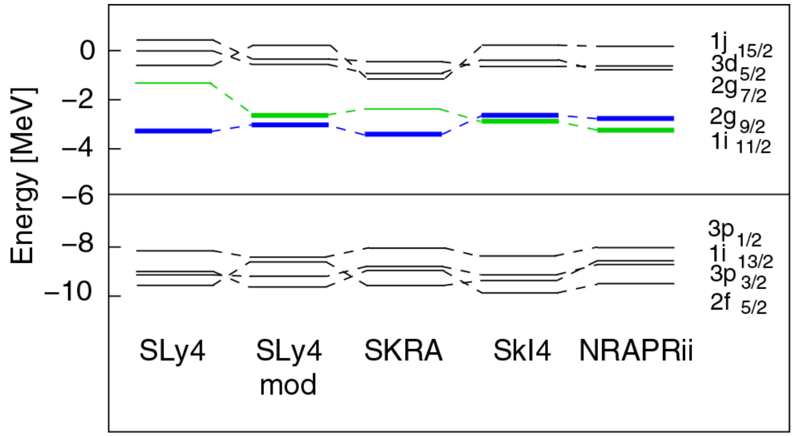
★ Quadrupole deformation of Zr isotopes (← axial HF)



Ref. : S. Miyahara & H.N., in preparation

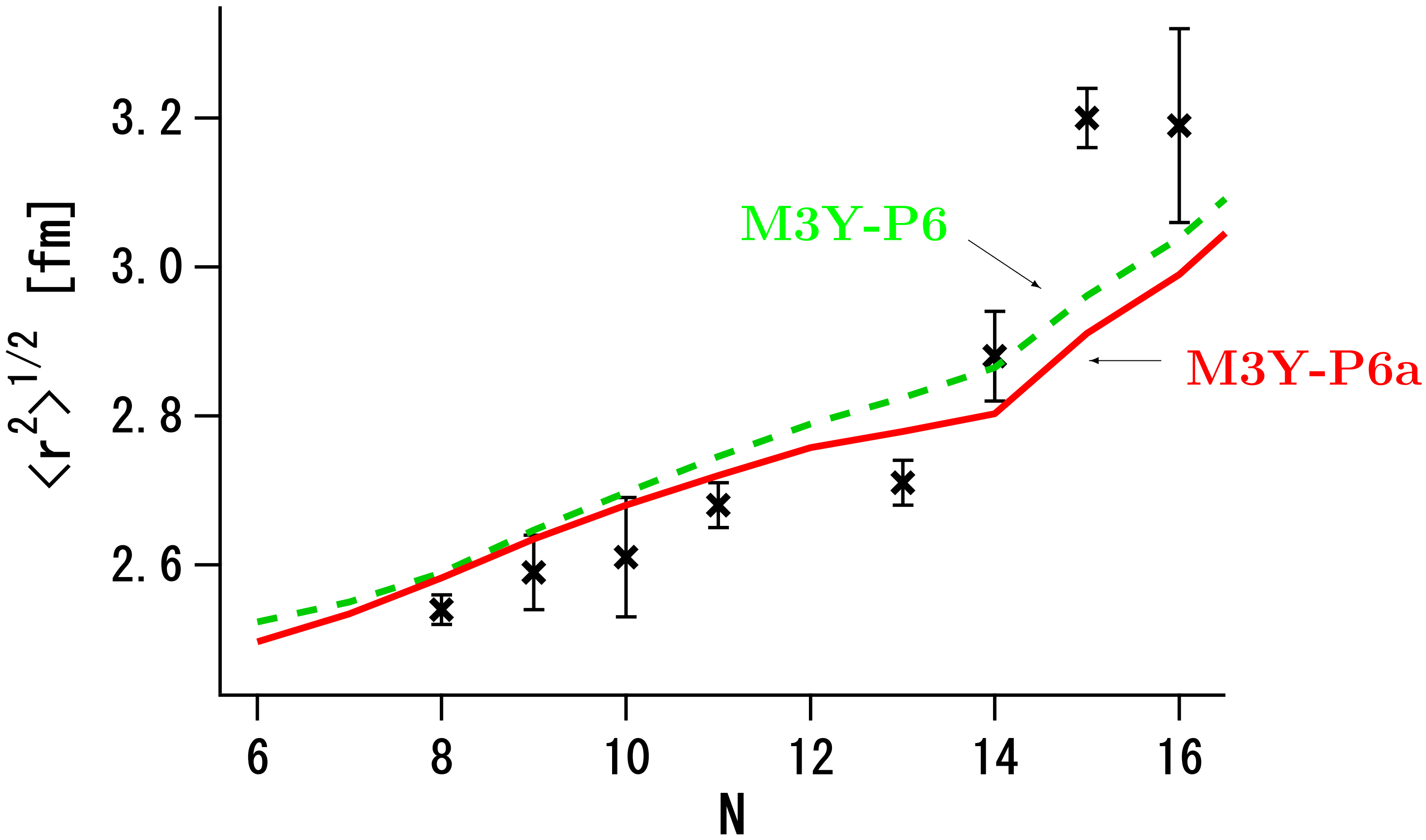
★ Isotope shifts of Pb nuclei with Skyrme interactions





Ref. : P.M. Goddard *et al.*, P.R.L. 110, 032503

★ $\sqrt{\langle r^2 \rangle}$ **in O isotopes**



improved, but still discrepant in ^{23}O