Calculation of resonance driving terms for the SuperKEKB using PTC

Demin Zhou

Acknowledgements:

E. Forest, D. Sagan (Cornell)

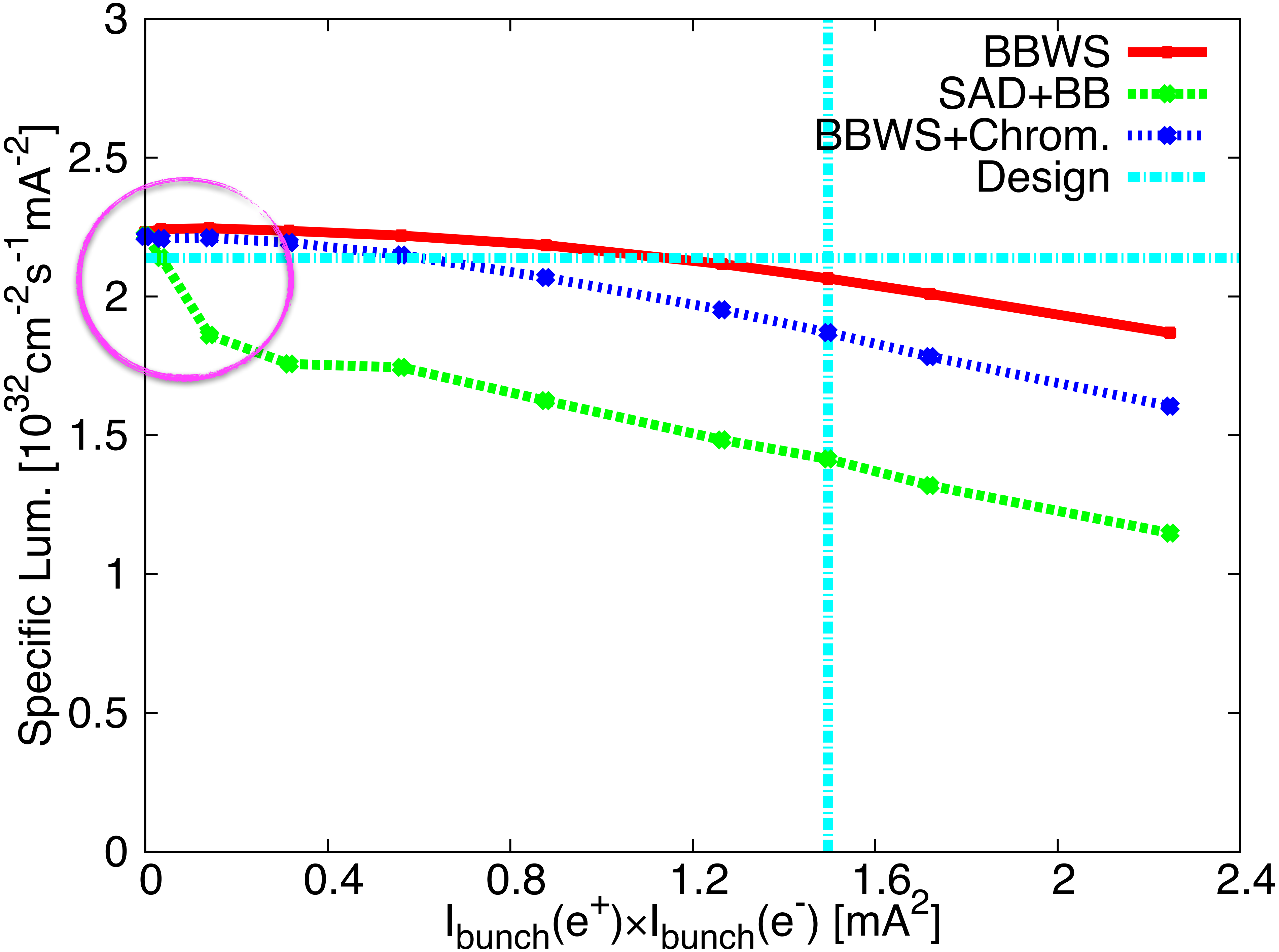
SuperKEKB mini optics meeting, May. 26, 2016

Outline

- **Introduction**
  - **Previous findings**
- **Theory for resonance driving terms**
- **Results by PTC**
  - **3rd and 4th order RDTs**
- **Summary**

1. Previous findings

➤ **BB+LN cause significant lum. loss in SuperKEKB**



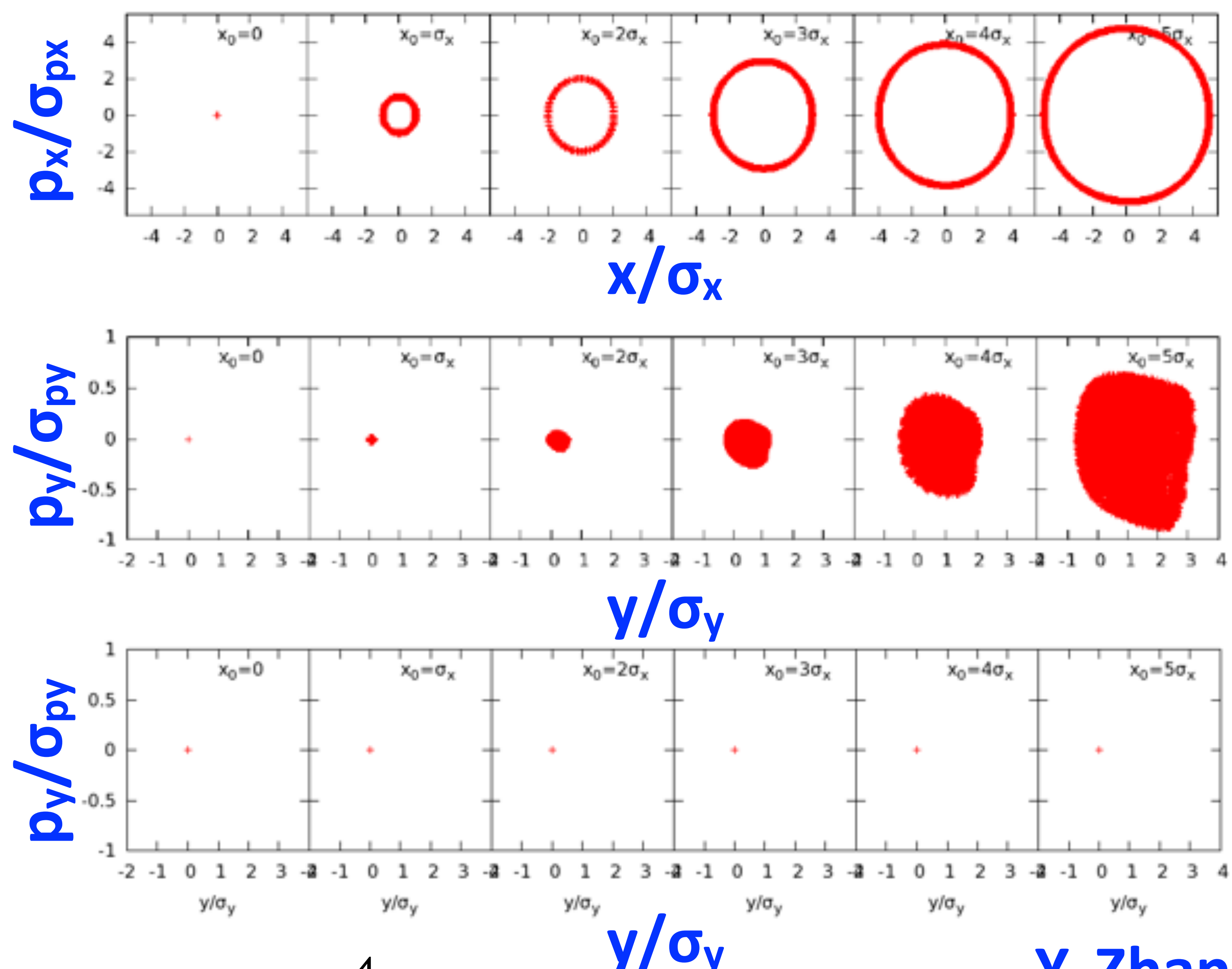
1. Previous findings

➤ **Strong nonlinear X-Y coupling in LER (baseline lattice w/ solenoids)**

➤ **Simplified lattice: LER w/o solenoids, FF magnets simplified: no offset, no rotation, dipole and skew-quad removed**

Baseline lattice →

Simplified lattice →

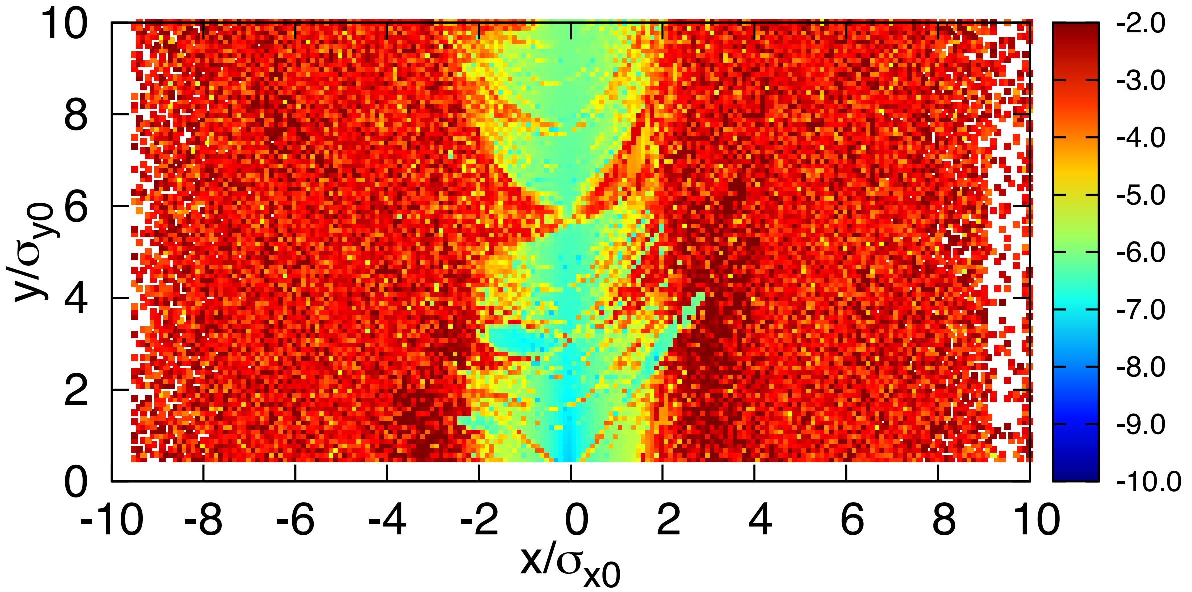


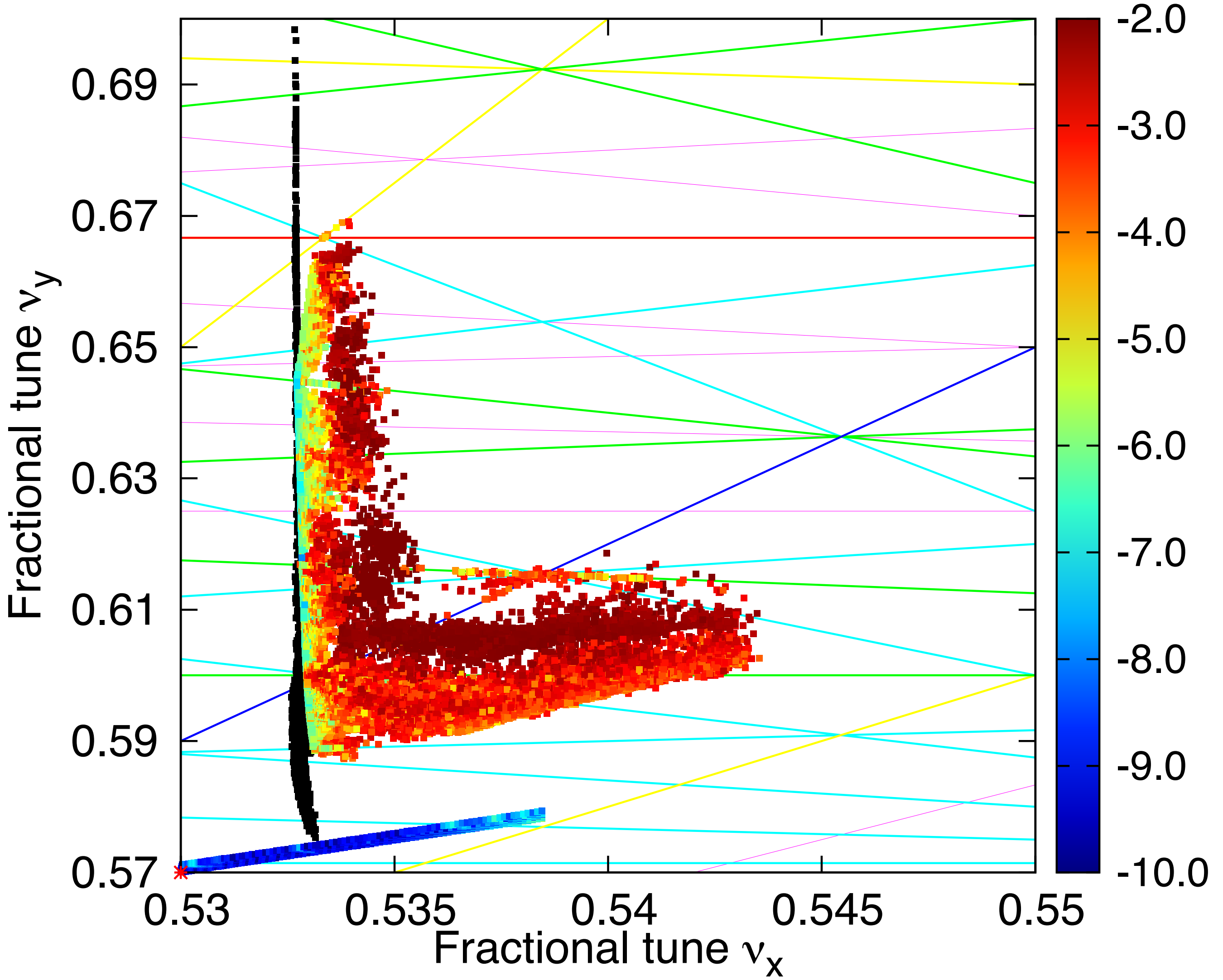
Y. Zhang

1. Previous findings

➤ **FMA with beam distribution for LER: $10\sigma_x \times 10\sigma_y$**

- **Footprint in the tune space extended by BB+LN**





1. Previous findings

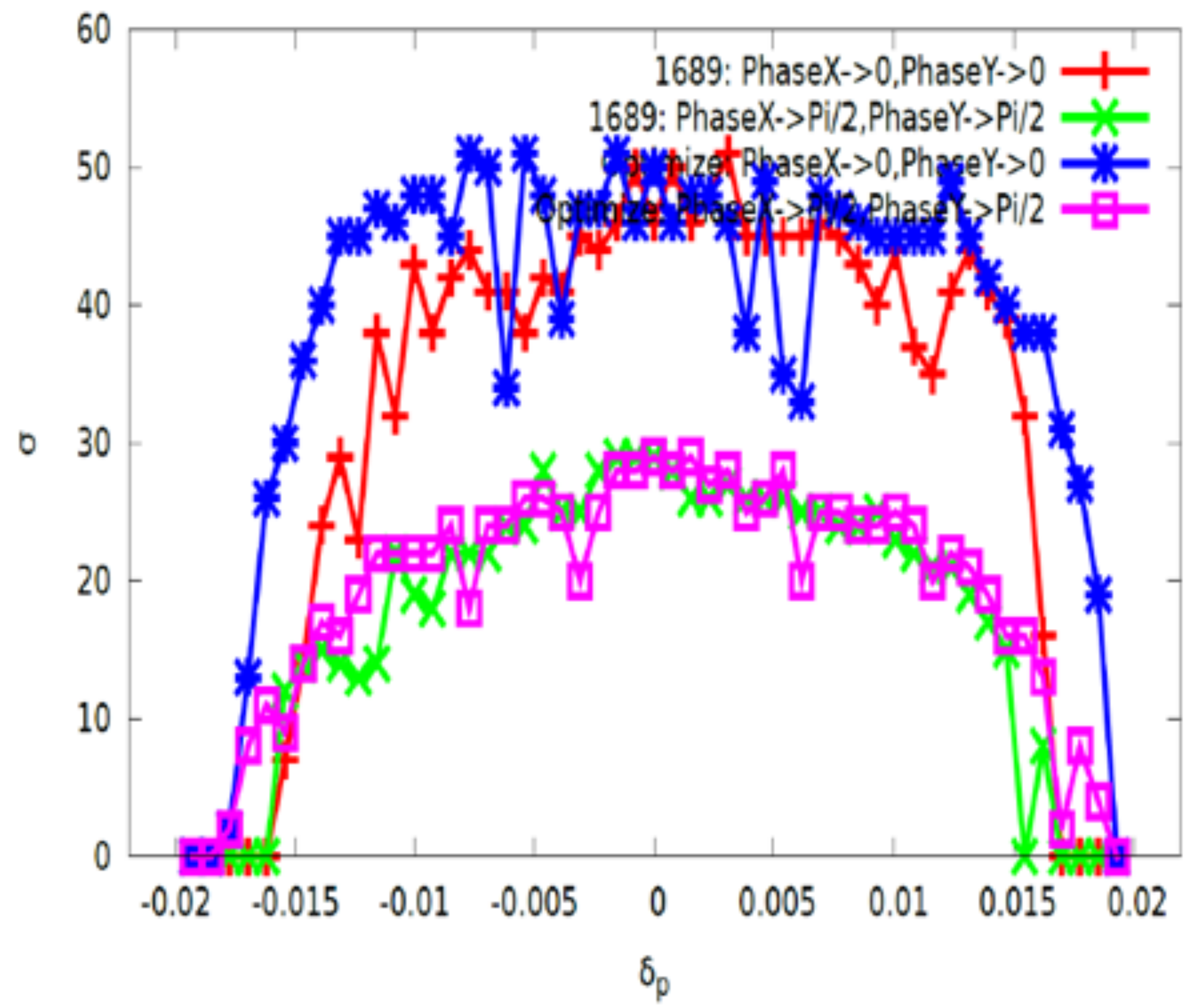
➤ **Optimization of DA using multi-objective Differential Evolution Algorithm (by Y. Zhang)**

- **Alternative for Downhill simplex method used in SAD**

DA Optimization of LER

- Objectives:
  - $\nu_x \in (0.53, 0.66), \nu_y \in (0.55, 0.66)$, for $\delta_p \in (-0.019, 0.019)$
  - $\frac{x^2}{50^2} + \frac{z^2}{26^2} = 1$, for z=Range[-24,24,3], $\epsilon_{x,0} = 1.89$ nmrad, $\delta_{p,0}$=7.7e-4
- Variables: 68
  - 2 Octupoles
  - 54 sextupole pairs
  - 12 skew sextupole pairs

Momentum aperture is increased.



Y. Zhang, TUOBA03, IPAC'16

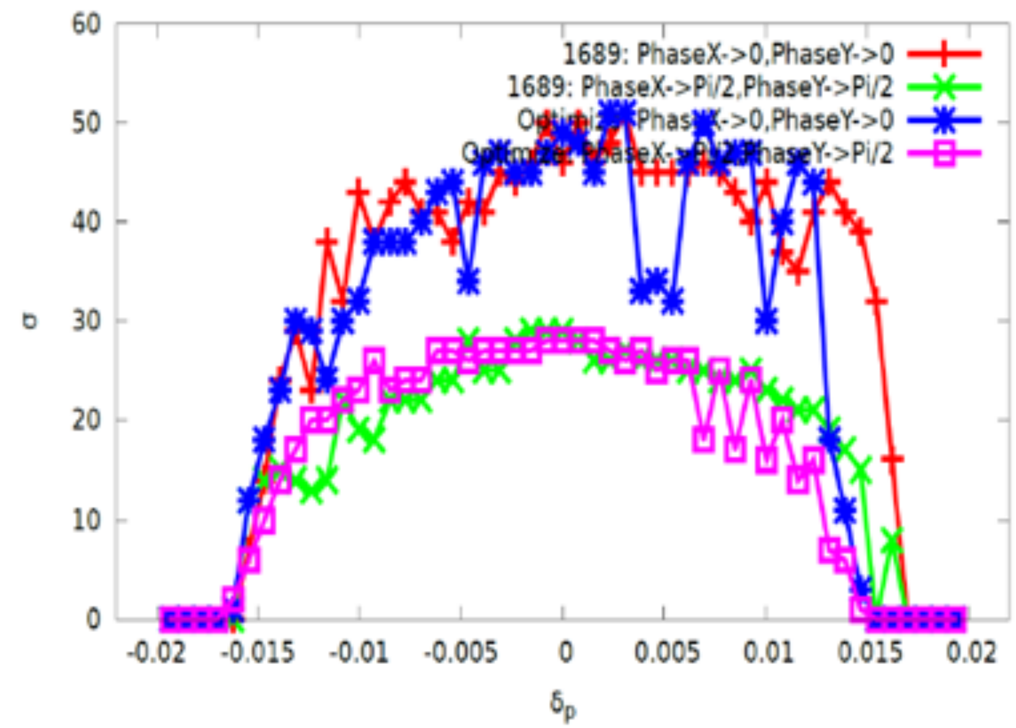
1. Previous findings

➤ Optimization of DA using multi-objective Differential Evolution Algorithm (by Y. Zhang)

- **Alternative for Downhill simplex method used in SAD**

Optimization of LER

- Objectives:
  - $\nu_x \in (0.53, 0.66), \nu_y \in (0.55, 0.66)$,
  - $\frac{x^2}{50^2} + \frac{z^2}{26^2} = 1$, for z=Range[-24,24,4],
  - **Suppression of skew sextupole resonance**:
    - $\frac{\langle y \rangle}{\sigma_y}$ for a particle with initial coordinate ($5\sigma_x$,0,0,0,0,0)
    - $\frac{|y - \langle y \rangle|}{\sigma_y}$ for a particle with initial coordinate ($5\sigma_x$,0,0,0,0,0)
- Variables: 80
  - 2 Octupoles
  - 54 sextupole pairs
  - 24 skew sextupole(symmetry of skew sextupole pair is broken)



Y. Zhang, TUOBA03, IPAC'16

1. Previous findings

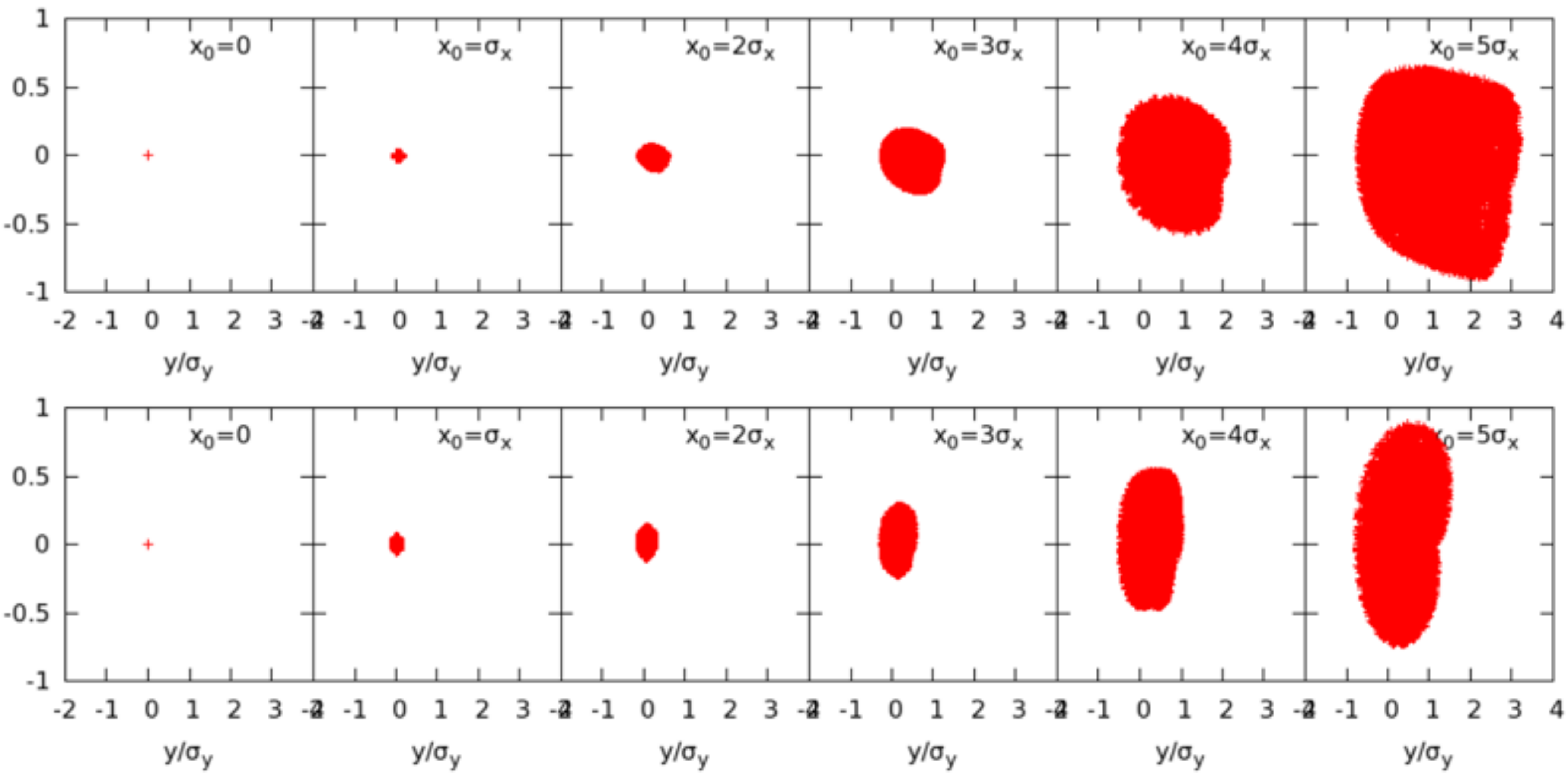
➤ **Optimization of DA using multi-objective Differential Evolution Algorithm (by Y. Zhang)**

- **Alternative for Downhill simplex method used in SAD**

Suppression of skew sextupole resonance

Before optimization:

After optimization:



Y. Zhang, TUOBA03, IPAC'16

2. Theory for RDTs

➤ Resonance driving terms (RDTs) indicate lattice nonlinearity

The effective Hamiltonian of a ring can be normalized in resonance bases [Ref. E. Forest, *Beam Dynamics – A New Attitude and Framework*, 1998].

For a ring with n elements, one can normalize the one turn map $\mathcal{M}_{1\to n}$ as [Ref. L. Yang *et al.*, Phys. Rev. ST Accel. Beams 14, 054001 (2011)]

$$\mathcal{M}_{1\to n} = \mathcal{A}_1^{-1} e^{:h:} \mathcal{R}_{1\to n} \mathcal{A}_1,$$

with $\mathcal{R}$: rotation, $e^{:h:}$: nonlinear Lie map, $\mathcal{A}_1$: normalizing map. Assume no coupling (the theory can be generalized for nonzero coupling), $\mathcal{A}_i$ in x plane at the ith element can be approximated in perturbation theory as

$$\mathcal{A}_i x = \sqrt{\beta_{x,i}} x + \eta_{x,i}\delta,$$

$$\mathcal{A}_i p_x = \frac{-\alpha_{x,i} x + p_x}{\sqrt{\beta_{x,i}}} + \eta'_{x,i}\delta.$$

2. Theory for RDTs

➤ RDTs indicate lattice nonlinearity

In the resonance basis, using action-angle variables (J, ϕ) one can write

$$h_x^{\pm} \equiv \sqrt{2J_x} e^{\pm i\phi_x} = X \mp iP_x,$$

$$\mathcal{R}_{i\to j} h_x^{\pm} = \mathcal{R}_{i\to j} \sqrt{2J_x} e^{\pm i\phi_x} = e^{\pm i\mu_{i\to j,x}} h_x^{\pm},$$

where $\mu_{i\to j,x}$ is the phase advance of $i \to j$. Consequently, the potential of a multipole magnetic field can be expanded in the resonance bases of h_{abcde} as

$$h = \sum h_{abcde} h_x^{+a} h_x^{-b} h_y^{+c} h_y^{-d} \delta^e.$$

Each h_{abcde} (a complex number in general) drives a certain resonance, and is an explicit function of magnet strengths, beta functions and dispersions.

2. Theory for RDTs

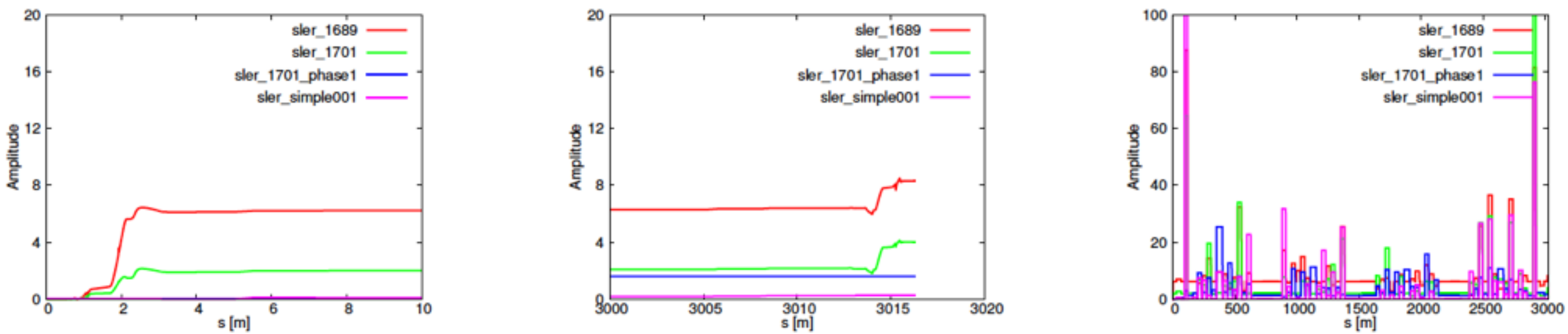
➤ RDTs indicate lattice nonlinearity

h_{abcde}	Driving effects
h_{11001}, h_{00111}	Linear chromaticity ζ_x, ζ_y
$h_{21000}, h_{12000} \| h_{10110}, h_{01110}$	ν_x $[(J_x)^{3/2}] \| [(J_x)^{1/2}(J_y)]$
$h_{30000}, h_{03000} \| h_{00300}, h_{00030}$	$3\nu_x$ $[(J_x)^{3/2}] \| 3\nu_y$ $[(J_y)^{3/2}]$
$h_{10020}, h_{01200} \| h_{10200}, h_{01020}$	$\nu_x - 2\nu_y \| \nu_x + 2\nu_y$ $[(J_x)^{1/2}(J_y)]$
$h_{20010}, h_{02100} \| h_{20100}, h_{02010}$	$2\nu_x - \nu_y \| 2\nu_x + \nu_y$ $[(J_x)(J_y)^{1/2}]$
$h_{00210}, h_{00120} \| h_{11100}, h_{11010}$	ν_y $[(J_y)^{3/2}] \| [(J_x)(J_y)^{1/2}]$
$h_{22000}, h_{00220}, h_{11110}$	$d\nu_x/dJ_x, d\nu_y/dJ_y, d\nu_{x,y}/dJ_{y,x}$
$h_{40000}, h_{04000} \| h_{00400}, h_{00040}$	$4\nu_x$ $[(J_x)^2] \| 4\nu_y$ $[(J_y)^2]$
$h_{31000}, h_{13000} \| h_{20110}, h_{02110}$	$2\nu_x$ $[(J_x)^2] \| [(J_x)(J_y)]$
$h_{00310}, h_{00130} \| h_{11200}, h_{11020}$	$2\nu_y$ $[(J_y)^2] \| [(J_x)(J_y)]$
$h_{20020}, h_{02200} \| h_{20200}, h_{02020}$	$2\nu_x - 2\nu_y \| 2\nu_x + 2\nu_y$ $[(J_x)(J_y)]$
$h_{30010}, h_{03100} \| h_{30100}, h_{03010}$	$3\nu_x - \nu_y \| 3\nu_x + \nu_y$ $[(J_x)^{3/2}(J_y)^{1/2}]$
$h_{10030}, h_{01300} \| h_{10300}, h_{01030}$	$\nu_x - 3\nu_y \| \nu_x + 3\nu_y$ $[(J_x)^{1/2}(J_y)^{3/2}]$

Table : Low-order driving terms.

3. Results by PTC

- **PTC applied to SuperKEKB**
  - **$3\nu_x$ $[(J_x)^{3/2}]$ resonance**
  - **3D fields near IP [Solenoid and FF quad fringe fields] generate lots of low-order nonlinearities**
  - **Simplified lattices show less nonlinearity**



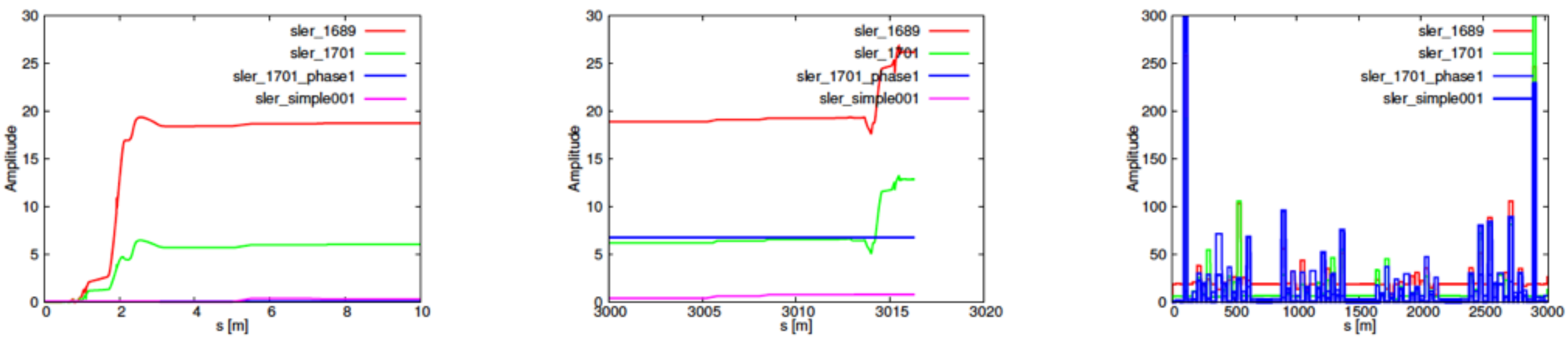
(a) IP→s=10m (b) s=3000m→IP (c) Whole ring

Figure : $|h_{30000}|$ accumulated along the ring.

3. Results by PTC

➤ PTC applied to SuperKEKB

- ν_x $[(J_x)^{3/2}]$ resonance



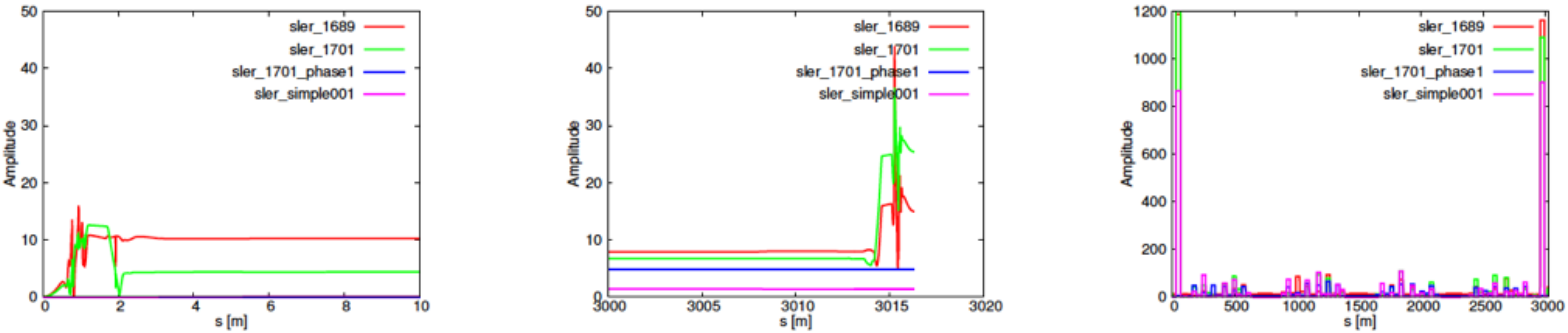
(a) IP$\rightarrow s$=10m (b) s=3000m$\rightarrow$IP (c) Whole ring

Figure : $|h_{21000}|$ accumulated along the ring.

3. Results by PTC

➤ **PTC applied to SuperKEKB**

- **ν_x-2ν_y [$(J_x)^{1/2}(J_y)$] resonance**



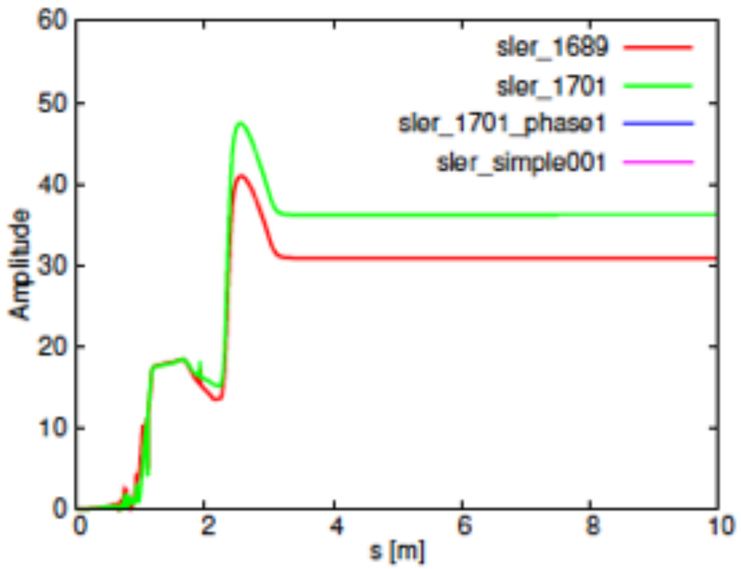
(a) IP→s=10m (b) s=3000m→IP (c) Whole ring

Figure : $|h_{10020}|$ accumulated along the ring.

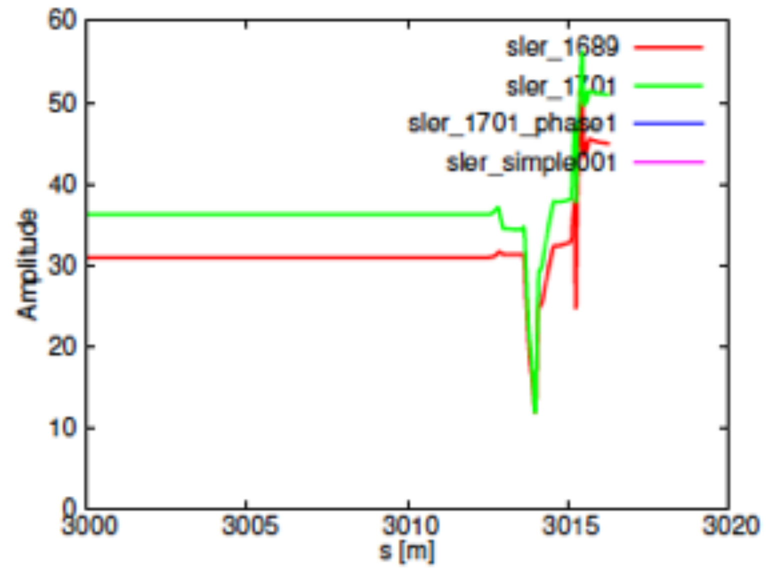
3. Results by PTC

➤ PTC applied to SuperKEKB

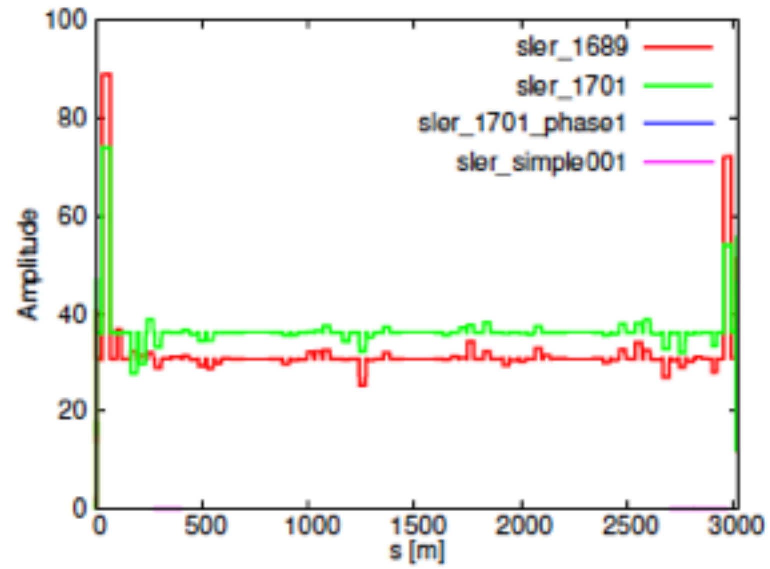
- $2\nu_x$-ν_y $[(J_x)(J_y)^{1/2}]$ resonance
- This term is hard to be compensated using arc multipoles [global correction]
- This term is almost invisible in simplified and phase-1 lattices



(a) IP→s=10m



(b) s=3000m→IP



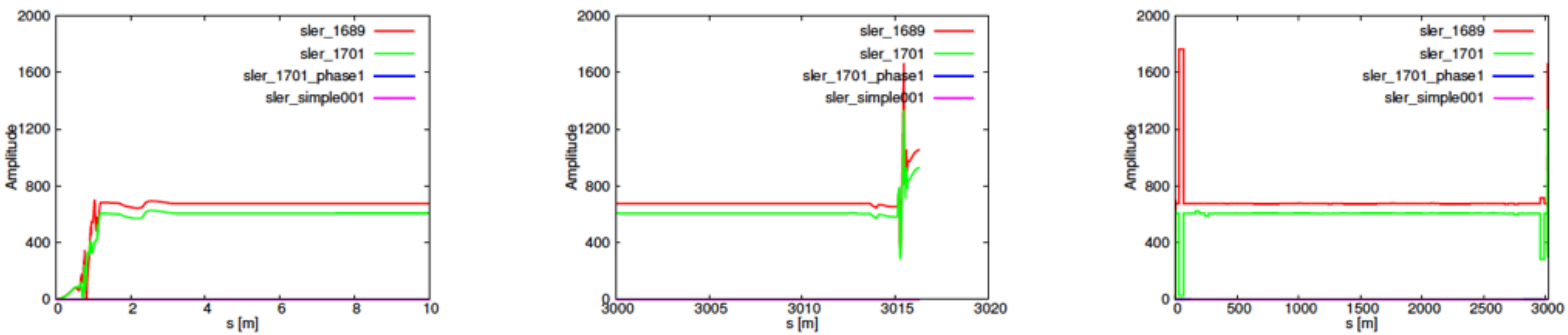
(c) Whole ring

Figure : $|h_{20010}|$ accumulated along the ring.

3. Results by PTC

➤ **PTC applied to SuperKEKB**

- **ν_y [$(J_y)^{3/2}$] resonance**
- **This term is hard to be compensated using arc multipoles [global correction]**
- **This term is almost invisible in simplified and phase-1 lattices**



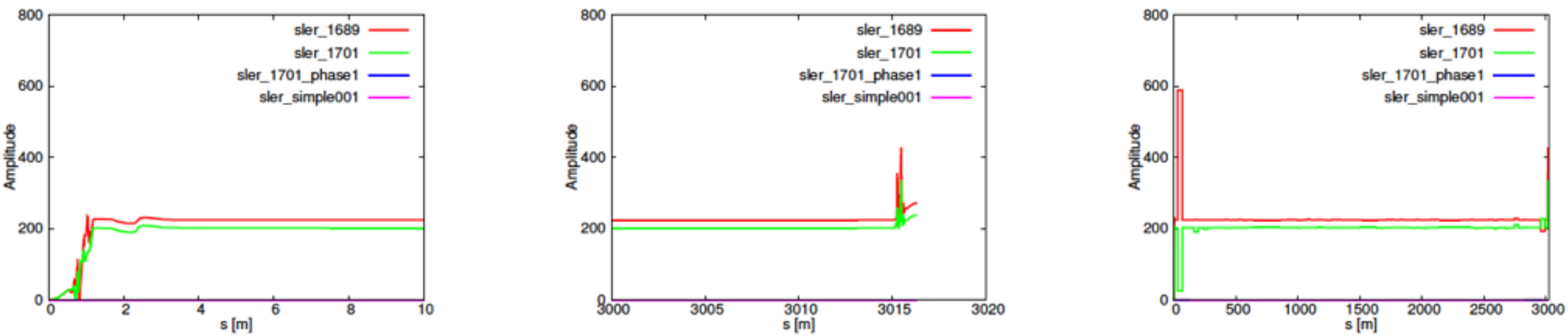
(a) IP→s=10m (b) s=3000m→IP (c) Whole ring

Figure : $|h_{00210}|$ accumulated along the ring.

3. Results by PTC

➤ PTC applied to SuperKEKB

- $3\nu_y$ $[(J_y)^{3/2}]$ resonance
- This term is hard to be compensated using arc multipoles [global correction]
- This term is almost invisible in simplified and phase-1 lattices



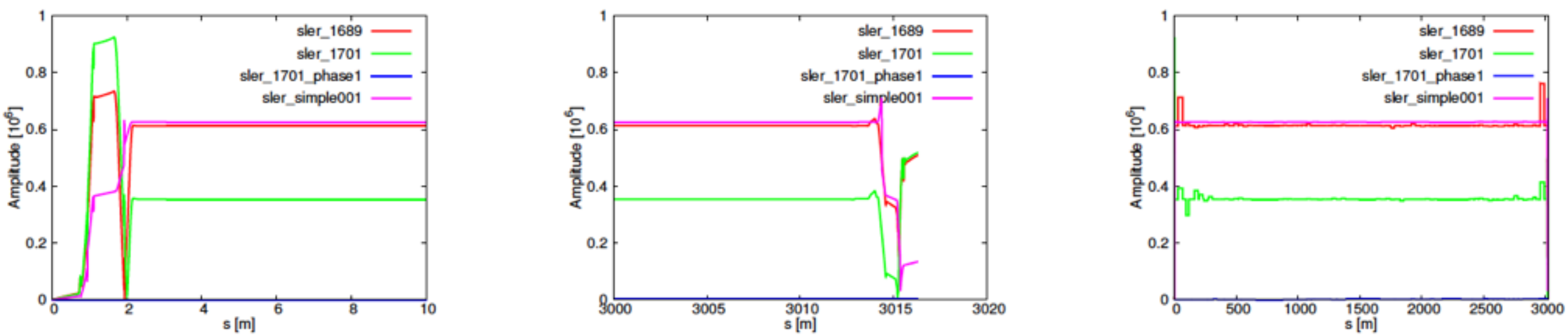
(a) IP→s=10m (b) s=3000m→IP (c) Whole ring

Figure : $|h_{00300}|$ accumulated along the ring.

3. Results by PTC

➤ PTC applied to SuperKEKB

- **Amplitude-dependent tune shift $d\nu_{x,y}/dJ_{y,x}[(J_x)(J_y)]$**
- **This term is hard to be compensated using arc multipoles [global correction]**
- **This term is almost invisible in Phase-1 lattices**



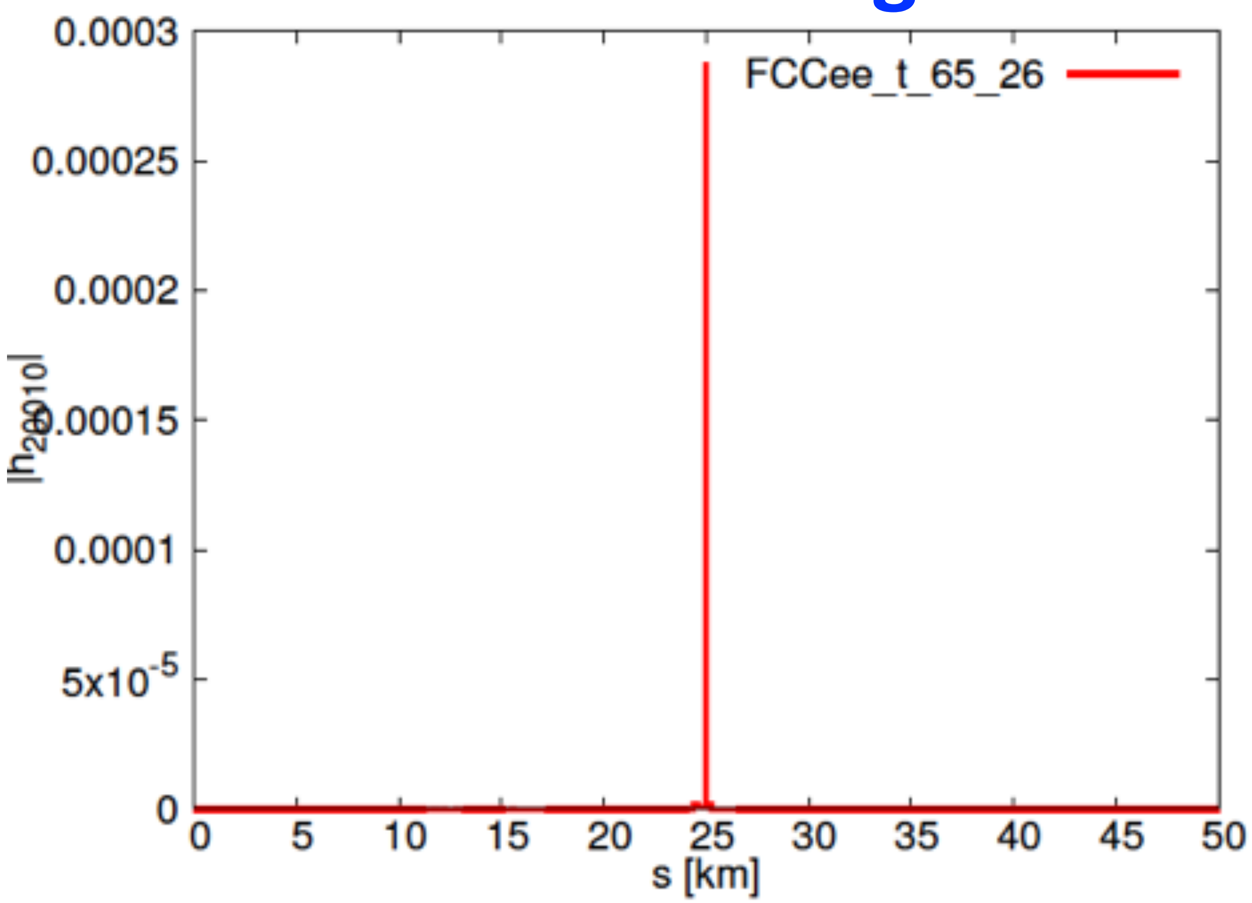
(a) IP→s=10m (b) s=3000m→IP (c) Whole ring

Figure : $|h_{11110}|$ accumulated along the ring.

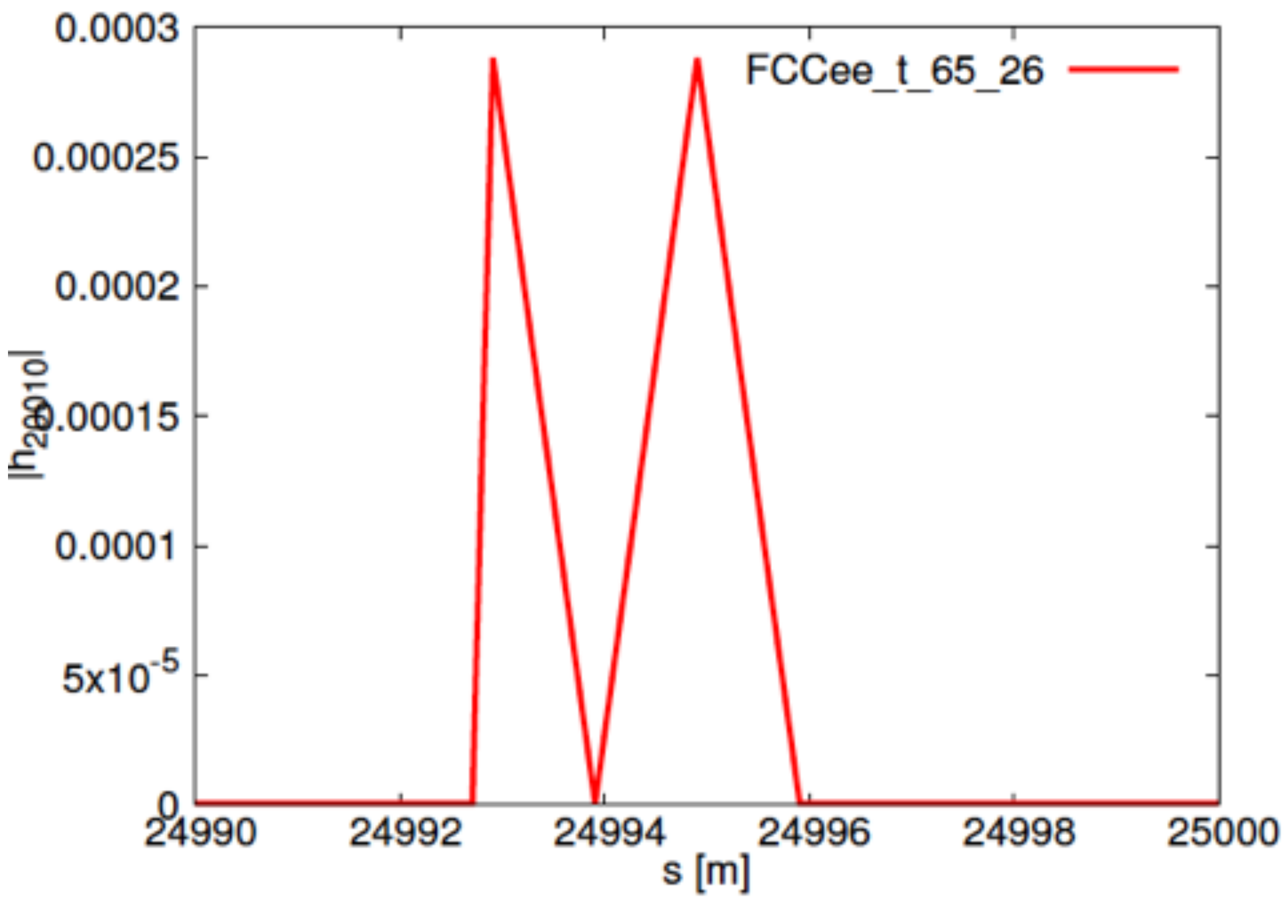
3. Results by PTC

- **PTC applied to FCC-ee t lattice: an example**
  - $2\nu_x$-ν_y $[(J_x)(J_y)^{1/2}]$ resonance for latt. ver. FCCee_t_65_26
  - In general, no significant 3rd resonances in FCC-ee lattices

Whole ring



Near IP

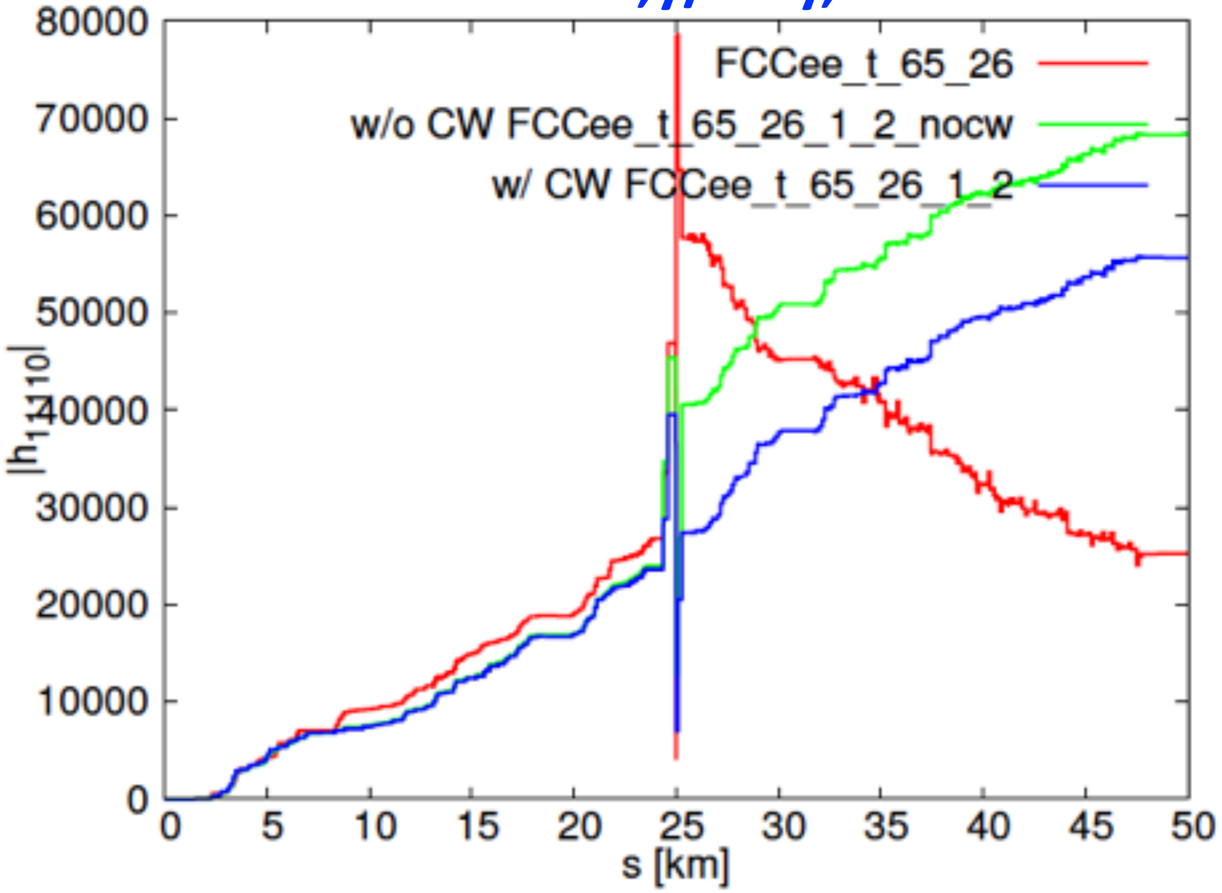


3. Results by PTC

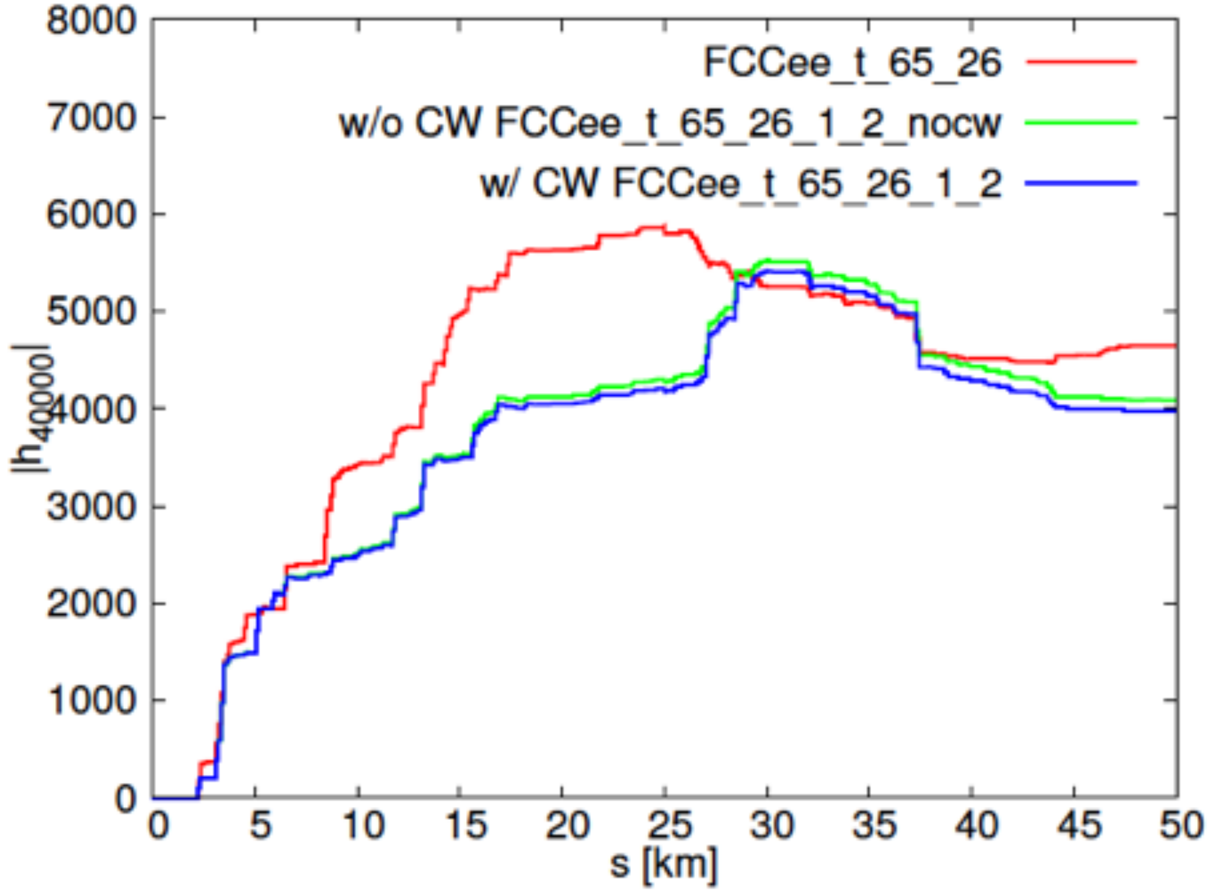
➤ PTC applied to FCC-ee t lattice: an example

- **4th order RDTs for latt. ver. FCCee_t_65_26**
- **Residual 4th order RDTs exist, and depend on lattice design/ optimization**

$d\nu_{x,y}/dJ_{y,x}$



$4\nu_x$



4. Summary

- ➤ RDTs as an indicator for evaluating a lattice design (good or not good)
- ➤ S-dependent RDTs as an indicator of source of nonlinearity along the ring
- ➤ Large 3rd resonance terms found in the SuperKEKB baseline lattice
- ➤ Global optimisation of DA is tried but not very successful yet
- ➤ Discussion
  - Is it possible to combine global and local correction schemes for DA optimization of SuperKEKB?
  - How to apply the nonlinear analysis using PTC for optics optimization of SuperKEKB?