1.1 Study of various collision schemes for Super KEKB

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1.1.1 **Introduction**

KEKB will achieve the integrated luminosity of 1 ab^{-1} this or next year. We are planning an upgrade of KEKB. The integrated luminosity should target 10 ab^{-1} next 5-10 years. The peak luminosity should be 10 times higher than the present value of 1.7×10^{34} $cm^{-2}s^{-1}$. Various collision schemes are proposed to boost the luminosity performance. Every collision schemes should be studied for the upgrade. Here we present the trials for the collision schemes.

1.1.2 **Collision schemes for B factories**

1.1.2.1 ***Crossing angle***

Various collision schemes are proposed for high luminosity B factories. In recent colliders, multi-bunch collision is crucial to get gain the multiplicity of the number of bunches. The crossing angle is introduced to avoid parasitic encounters.

An essential of crossing angle is expressed by transformations as shown in Figure 1. The electro-magnetic field is formed perpendicular to the traveling direction. The transformation which particles in the beam experience is expressed by [1,2]

$$\begin{aligned} \Delta p_x &= -F_x(x+2s\phi, y) \\ \Delta p_y &= -F_y(x+2s\phi, y) \\ \Delta\delta &= -\phi F_x(x+2s\phi, y) \end{aligned} \tag{1}$$

where $s=(z-z_c)/2$ and ϕ is the half crossing angle. The transformation is separated by three parts.

$$e^{\phi p_x z} \circ e^{-:H_{bb}:} \circ e^{-\phi p_x z} \tag{2}$$

where H_{bb} is Hamiltonian for the beam-beam interaction. The first transformation is given by

$$\begin{aligned} e^{-\phi:p_x z:}x &= x - \phi[p_x z, x] = x + \phi z \\ e^{-\phi:p_x z:}\delta &= \delta - \phi[p_x z, \delta] = x - \phi p_x \end{aligned} \tag{3}$$

The residual of the first and third transformations gives the transformation for δ in Eq.(1). This expression, which is called Lie operator expression, is presented in [3]. Note the operator order; $\circ$ denotes the multiplication of transformations, which is inverse order of Lie operator multiplication.

Both beams are transferred by the same transformation. The term ϕz_c appears from $2s\phi$ in Eq.(1). This transformation is actually equivalent to the appearance of z dependent dispersion (ζ_x) at the collision point: i.e., the revolution matrix including the crossing transformation is expressed by

$$M = e^{\phi p_x z} \circ M_0 \circ e^{-\phi p_x z} \tag{4}$$

where M_0 is the revolution matrix of the lattice. Now the beam envelope matrix has a finite element of $\langle xz\rangle = \zeta_x\sigma_z = \phi\sigma_z$ [4], for the weak limit of the beam-beam interaction. The collision is now regarded as head-on collision with tilt beams in x-z plane as shown in Figure 2. Electro-magnetic field is the perpendicular to the moving direction now.

Another important point of the crossing angle is that the collision area in the two beams is limited. For long bunch compare than beta function, tune shift enlargement due to the hourglass of the beta function is avoidable. The long bunch scheme is called superbunch scheme [5]. This feature is great merit for collision with extreme small beta function. The bunch length and beta function can be chosen independently in this scheme. The relations of optics and beam-beam parameters are summarized in as follows.

Table 1:

	Short bunch	***Long bunch***
Requirement 1	$\sigma_x/\phi > \sigma_z$	$\sigma_x/\phi < \sigma_z$
Requirement 2	$\sigma_z < \beta_y$	$\sigma_x/\phi < \beta_y$
$L\left(\frac{f_{rep}}{4\pi}\times\right)$	$\frac{N^2}{\sqrt{\varepsilon_x\beta_x\varepsilon_y\beta_y}}$	$\frac{N^2}{\phi\sigma_z\sqrt{\varepsilon_y\beta_y}}$
$\xi_x\left(\frac{r_e}{2\pi\gamma}\times\right)$	$\frac{N}{\varepsilon_x}$	$\frac{N\beta_x}{(\phi\sigma_z)^2}$
$\xi_y\left(\frac{r_e}{2\pi\gamma}\times\right)$	$N\sqrt{\frac{\beta_y}{\varepsilon_x\beta_x\varepsilon_y}}$	$\frac{N}{\phi\sigma_z}\sqrt{\frac{\beta_y}{\varepsilon_y}}$

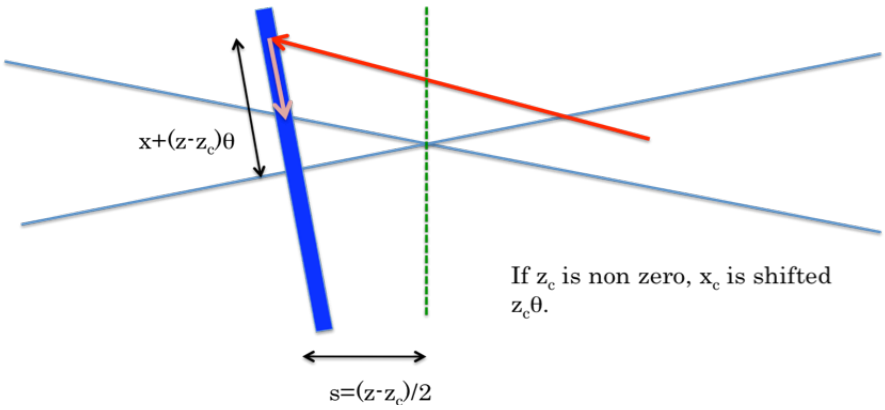

Figure 1: Transformation for crossing angle.

Figure 2: Collision with crossing angle is equivalent to head-on collision with tilt beam.

1.1.2.1 *Crab crossing*

The crab crossing [1,6] is basically meaningful for the short bunch scheme. A transformation, which is equivalent to the crossing angle, is applied before and after the collision,

$$e^{-\phi p_x z} \circ e^{\phi p_x z} \circ e^{-:H_{bb}:} \circ e^{-\phi p_x z} \circ e^{\phi p_x z} = e^{-:H_{bb}:}, \tag{5}$$

thus the effective transformation is the same as that for the head-on collision. To realize the transformation, crab cavities, which gives the transformation, $e^{-V':xz:/E_0}$, are placed at locations where linear transformation T_A is satisfied to,

$$e^{\phi p_x z} = T_A \circ e^{-V':xz:/E_0} \circ T_A^{-1} = e^{-(V'/E):T\circ xz:}, \tag{6}$$

where

$$T_A \circ x = \sqrt{\frac{\beta_x^*}{\beta_{x,c}}}\, x\cos\varphi_x + \sqrt{\beta_x^*\beta_{x,c}}\, p_x \sin\varphi_x \tag{7}$$

φ_x is the horizontal betatron phase difference between the collision point and crab cavity position, and β_x* and $\beta_{x,c}$ are horizontal beta functions at the collision point and crab cavity position.

The well-known formula for crab angle and voltage is given by choosing the betatron phase difference of $\pi/2$.

$$\phi = \frac{\omega_{crab} V}{cE_0}\sqrt{\beta_{x,c}\beta_x^*} \tag{8}$$

Only one crab cavity can be possible to realize the transformation

$$e^{\phi p_x z} \circ M_0 \circ e^{-\phi p_x z} = T_B \circ e^{-V':xz:/E_0} \circ T_B^{-1} \circ M_0 \tag{9}$$

Basically this procedure is really 6x6 optics matching for the dispersion ζ_x.

1.1.2.2 *Crab waist scheme*

A transformation with the form $\exp(-a:p_y^2:/2)$ controls the vertical waist position with keeping minimum beta function. The transformation is represented by matrix as

$$\begin{pmatrix} y \\ p_y \end{pmatrix} = T_a \begin{pmatrix} y \\ p_y \end{pmatrix} \qquad T_a = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \tag{10}$$

Twiss parameters at the collision point are transferred by

$$\begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = T\begin{pmatrix} \beta & 0 \\ 0 & 1/\beta \end{pmatrix} T_a^t = \begin{pmatrix} \beta + a^2/\beta & a/\beta \\ a/\beta & 1/\beta \end{pmatrix} \tag{11}$$

This transformation is equivalent to shift the waist position of *a*.

The waist position is shifted so as to linearly depend on the horizontal coordinate x under the presence of the crossing angle in the crab waist scheme [7]. The transformation at the collision point is expressed by

$$e^{axp_y^2} \circ e^{\phi p_x z} \circ e^{-:H_{bb}:} \circ e^{-\phi p_x z} \circ e^{-axp_y^2} \tag{12}$$

The transformation is rewritten as

$$\begin{aligned} &e^{\phi p_x z} \circ e^{-\phi p_x z} \circ e^{axp_y^2} \circ e^{\phi p_x z} \circ e^{-:H_{bb}:} \circ e^{-\phi p_x z} \circ e^{-axp_y^2} \circ e^{\phi p_x z} \circ e^{-\phi p_x z} \\ &= e^{\phi p_x z} \circ e^{a(x-\phi z)p_y^2} \circ e^{-:H_{bb}:} \circ e^{-a(x-\phi z)p_y^2} \circ e^{-\phi p_x z} \end{aligned} \tag{13}$$

Choosing a=1/2ϕ, the waist position is s=z/2-x/2ϕ. Beam particles satisfying x=zϕ, which collide with central axis of another beam at s=0, have the waist at s=0. Particles with x=zϕ +Δx, which collide with the centre at s=–Δx/2ϕ, have the waist s=–Δx/2ϕ. This feature minimizes the beam-beam effect for colliding particles. The transformation $\exp(-:xp_y^2:/2\phi)$ is realized by sextupole magnet: that is, at least two sextupole magnets are located at both side of the collision point. The betatron phase difference is nπ for x and (1/2+n)π for y, and the strength is determined by Eq. (7).

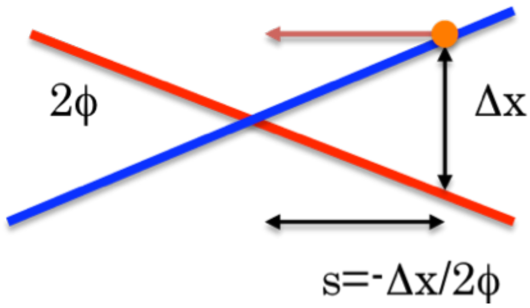

Figure 3: Deviation of collision point for x and waist position in crab waist scheme.

Characteristic of the crab waist scheme can be seen following picture. Coordinates x and y are transferred by the crab waist action near the collision point as follows,

$$y(s) = y_0 + xp_y/2\phi + p_y s \approx xp_y/2\phi + p_y s$$
$$x(s) = x_0 + p_x s \approx x_0 \tag{14}$$

Beam distribution is Gaussian except for the collision point. Near the collision point, the distribution is distorted by Eq.(14). The distribution is roughly given by

$$\exp\left[-\frac{x^2}{2\varepsilon_x\beta_x} - \frac{\beta_y p_y^2}{2\varepsilon_y}\right] = \exp\left[-\frac{x^2}{2\varepsilon_x\beta_x} - \frac{\beta_y y^2}{2\varepsilon_y(x/2\phi + s)^2}\right], \tag{15}$$

where $\exp(-y_0^2/2\varepsilon_y\beta_y)$ is neglected, because p_y is dominant for $s>\beta_y$. Figure 3 shows the contour of the distribution. Particles located at x collide with another beam at their waist position as shown in Figure 3.

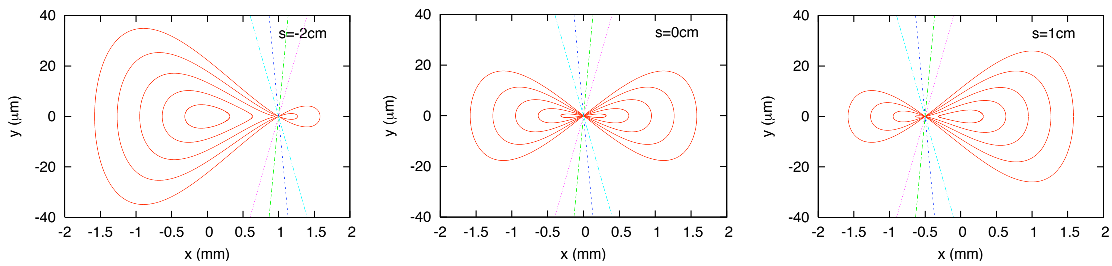

Figure 3: Particle distribution of colliding beam in the crab waist scheme. Collision arises at the point with the minimum y size. Another beam distributes symmetric for x.

1.1.2.3 ***Travel focus scheme***

Beam particles with z collide with the center of another beam at s=z/2 in the travel focus scheme [8]. The particles with z should have the waist position at s=z/2 to minimize the beam-beam effect. The transformation exp(-p_y^2z/4) realizes the travel focusing:

$$e^{zp_y^2/4} \circ e^{-:H_{bb}:} \circ e^{-zp_y^2/4} \tag{16}$$

RF focusing is used for the transformation. However heavy development works are necessary for the RF device. We know the crab cavity exchanges x and z.

$$\begin{aligned} &e^{-\phi p_x z} \circ e^{xp_y^2/4\phi} \circ e^{\phi p_x z} \circ e^{-:H_{bb}:} \circ e^{-\phi p_x z} \circ e^{-xp_y^2/4\phi} \circ e^{\phi p_x z} \\ &= e^{(x-\phi z)p_y^2/4\phi} \circ e^{-:H_{bb}:} \circ e^{-(x-\phi z)p_y^2/4\phi} \end{aligned} \tag{17}$$

The first and last operator exp(+-ϕp_xz) at the first line of Eq.(17) are actions of the crab cavities, while 3rd and 5th are the crossing angle. The 2nd and 6th operators are from two sextupole magnets located at the both sides of the collision point. Additional two sextupole magnets in both sides are added to cancel the residual nonlinear term [9].

$$\begin{aligned} &e^{xp_y^2/4\phi} \circ e^{-(x-\phi z)p_y^2/4\phi} \circ e^{-:H_{bb}:} \circ e^{-(x-\phi z)p_y^2/4\phi} \circ e^{xp_y^2/4\phi} \\ &= e^{zp_y^2/4} \circ e^{-:H_{bb}:} \circ e^{-zp_y^2/4} \end{aligned} \tag{18}$$

Realistic arrangement of IR is given by chosen betatron phase so as to realize the transformation as is done in Eq. (7). Two pairs of crab cavities, which are inserted between two sextupole magnets, are located at the horizontal betatron phase difference of (1/2+n)π. The sextupole magnets are located at the vertical betatron phase difference of (1/2+n)π. The phase difference of two sextupole magnets is π or 2π depending on the sign of magnets. In this scheme two crab cavity is necessary.

In the travel focus scheme, the waist position shifts for z but does not for s: that is, particles at z have waist position for the variation of s, and the waist position is located at the centre (z=0) of the colliding beam. The hourglass effect is not avoidable even in the travel waist scheme.

1.1.3 **Study of the collision schemes in Super KEKB**

These collision schemes has to be studied to upgrade KEKB. The crab cavity has been studied since 2007 at KEKB. The crab cavity was expected to boost the luminosity twice higher [10]. Figure 4 shows the beam-beam parameter as a function of the bunch population of HER, where the transparency condition is assumed.

The beam-beam parameter, which is regarded as a normalized luminosity, is defined by

$$\xi_n = \frac{2r_e\beta_{y,\pm}L}{N_\pm\gamma_\pm f_{col}} . \tag{19}$$

The luminosity is 4.5x10^{35} $cm^{-2}s^{-1}$ for the nominal parameter, N(HER)=5.5x10^{10} and 2ns collision repetition.

Figure 5 shows the beam-beam parameter as a function of the travel focus strength. The beam-beam parameter does little depend on the strength. The vertical beam size, which is simple 2nd order moment, has a minimum at the optimum strength. The tail distribution should be improved by the travel focus, while the luminosity performance is not remarkable.

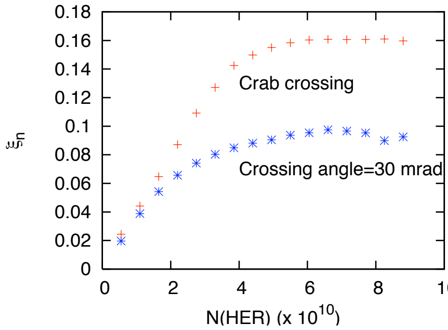

Figure 4: Beam-beam parameter with and without crab cavity given by a strong-strong simulation. Number of the longitudinal slice in the simulation is 5.

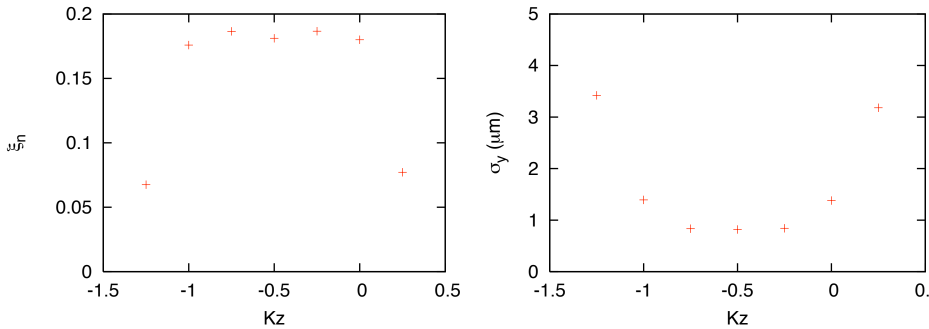

Figure 5: Effect of travel focusing in the simulation. The optimum is Kz=-0.5, where exp(Kz:p_y^2z/2:)

Using the travel-focusing scheme, higher luminosity is targeted. Figure 6 shows the beam-beam parameters as a function of the bunch population of HER beam given by both of strong-strong and weak-strong simulations. The corresponding luminosity is 8.0×10^{35} $cm^{-2}s^{-1}$ for the nominal parameter, N(HER)=5.5×10^{10} and 2ns collision repetition.

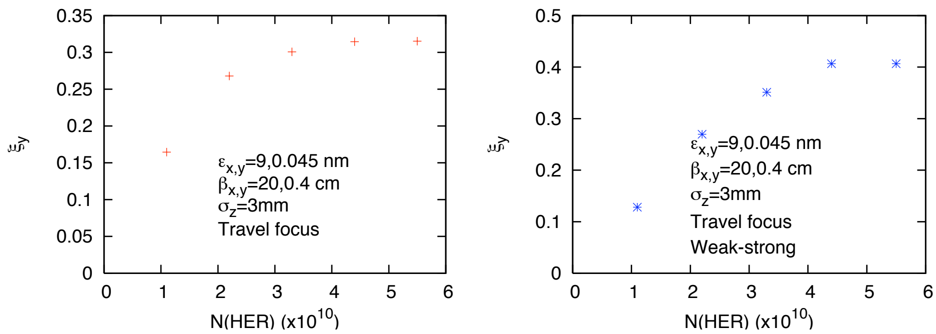

Figure 6: Beam-beam parameters given by strong-strong and weak-strong simulations.

Superbunch and crab waist scheme have been studied. The simulation for superbunch scheme is very hard; because a bunch has to be sliced into many pieces and a number of collisions between slices, square of the number of slices, have to be calculated per one revolution. Figure 7 shows the luminosity evolution in a strong-strong simulation for Super B parameters [11]. This luminosity is given for the collision repetition of 2 ns. Since it is 4 ns for the present design of Super B, the luminosity is 0.7×10^{36} $cm^{-2}s^{-1}$. The simulation, which is preliminary, is very hard and can contain numerical difficulties. Since better simulations will give better luminosity, this result can be considered as a lower bound for an ideal machine. The beam-beam parameter is 0.08 using Eq. (19), where β_y=0.22/0.39 mm, N=5.52x10^{10} and E=4/7 GeV.

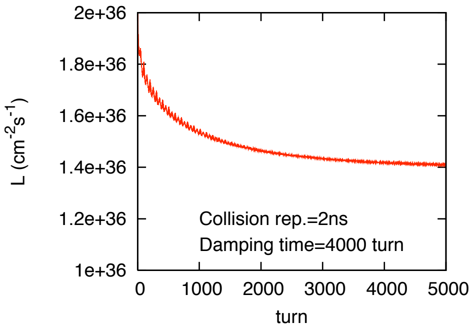

Figure 7: Luminosity evolution in a strong-strong simulation for Super B parameters.

1.1.4 **References**

1. K. Oide and Y. Yokoya, “Beam-beam collision scheme for storage-ring colliders”, Phys. Rev. A40, 315 (1989).
2. K. Hirata, "Analysis of beam-beam interactions with a large crossing angle", Phys. Rev. Lett., 74, 2228 (1995).
3. A. J. Dragt, “Lecture on nonlinear orbit dynamics”, AIP Conf. Proc. 87:147-313 (1982).
4. K. Ohmi, K. Hirata and K. Oide, “From the beam envelope matrix to synchrotron radiation integrals”, Phys. Rev. E49, 751 (1994).
5. K. Takayama, “Superbunch hadron colliders”, Phys. Rev. Lett. 88, 144801 (2002).
6. R. B. Palmer, “Energy scaling, crab crossing and the pair problem”, SLAC-PUB-4707 (1988).
7. P. Raimondi, “Suppression of beam-beam resonances in crab waist collision”, Proceedings of PAC2007, WEPP045.
8. V.E. Balakin and N.A. Solyak, “Status of electron accelerators for linear colliders”, Nucl. Instru. & Methods in Phys. Res. A355, 142 (1995).
9. K. Oide and H. Koiso, private communications.
10. K. Ohmi et al., “Luminosity limit due to the beam-beam interactions with or without crossing angle”, Phys Rev. ST-AB. 7, 104401 (2004).
11. SuperB conceptual Design report, INFN/AE-07/2, SLAC-R-856, LAL 07-15, March 2007.