Black Hole microstate and supersymmetric gauge theories

KEK workshop 2014

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with S. Shiba (KEK), T. Wiseman (Imperial College), B. Withers (Southampton)

cf) Shiba-san’s poster
Introduction
Introduction

◆ Black Hole Thermodynamics

- Hawking Radiation
- BH entropy

\[ S = \frac{\text{Area}}{4G_N} \]

The Microscopic Origin would exist.

Black Hole would be a key object connecting classical gravity and quantum gravity.
Introduction

◆ Black Hole Thermodynamics

Many Proposals for the microscopic description of the black hole

- Asymptotic Virasoro algebra (BTZ BH, Strominger)
- Near horizon Virasoro algebra (Carlip)
- Loop Quantum Gravity
- Fuzzball conjecture (Mathur)
- Entanglement Entropy via AdS2/CFT1 (Azeyamagi-Nishioka-Takayanagi)
- Kerr/CFT correspondence (Strominger et al)
- Semi-classical BH solution (Kawai-Matsuo-Yokokura)
- ...

(This list must be incomplete...)


Introduction

◆ Black Hole Thermodynamics

In string theory, the microscopic calculations for the extremal BPS black holes have been done by the following groups

- Strominger-Vafa
- Callan-Maldacena
- Dijkgraaf-Verlinde-Verlinde
- Ooguri-Strominger-Vafa
- Dabholkar
- Sen

(This list must be incomplete...)
Introduction

◆ Black Hole Thermodynamics

Numerical calculations of supersymmetric gauge theories based on the gauge/gravity correspondence

- Catterall-Wiseman
- Anagnostopoulos-Hanada-Nishimura-Takeuchi
- Catterall-Joseph-Wiseman
- Kadoh-Kamata
- Hanada-Hyakutake-Ishiki-Nishimura (Hyakutake-san’s talk)
I introduce our new model for a microscopic description of near extremal BHs/black branes, "warm p-soup" model.

So far we have just estimated the thermodynamical properties of our model and have not completed the rigorous calculation, but our model captures the black hole area law:

\[ S \propto \frac{\text{Area}}{G_N} \]

But our model may provide some physical insights about the BH.
Plan of today's talk

1. Introduction
2. Our proposal: warm p-soup
3. p-soup and gauge/gravity correspondence
4. Summary
Near extremal black p-brane
in D-dimensional Maxwell-Einstein-Dilaton gravity

e.g. black Dp/F1/NS5 brane solutions in 10 dim SUGRA
black M2/M5 brane solutions in 11 dim SUGRA

Let’s us focus on an 4 dim near extremal black hole as an example
Our proposal “p-soup”

Example: 4dim Maxwell-Einstein-Dilaton gravity

No forces between the branes

extremal N (separated) 0-brane solution
Our proposal “p-soup”

Example: 4dim Maxwell-Einstein-Dilaton gravity

No forces between the branes

extremal N (separated) 0-brane solution

pump a little energy (It breaks the extremality)

The branes attract each other and form a strongly coupled bound state, “soup of 0-branes”.

T.M-Shiba-Wiseman-Withers (2013)
Our proposal “p-soup”

- Example: 4dim Maxwell-Einstein-Dilaton gravity

  extremal \( N \) (separated) 0-brane solution

  pump a little energy (It breaks the extremality)

  No forces between the branes

  Near extremal black hole with the same charge and energy

  The branes attract each other and form a strongly coupled bound state, “soup of 0-branes”.

T.M-Shiba-Wiseman-Withers (2013)
Our proposal “p-soup”

Example: 4dim Maxwell-Einstein-Dilaton gravity

\[ ds_{\text{Einstein}}^2 = -H^{-1/2} dt^2 + H^{1/4} \left( \sum_{I=1}^{3} (dx^I)^2 \right) \]

extremal N (separated) 0-brane solution

pump a little energy
(It breaks the extremality)

★ low energy effective action for the branes in the gravity

\[ (|\partial_t \vec{x}_i| \ll 1) \]

\[ S_{\text{moduli}} = \int dt \frac{m}{2} \sum_{i=1}^{N} \partial_t \vec{x}_i \cdot \partial_t \vec{x}_i + \frac{\kappa^2 m^2}{4} \sum_{i<j} \frac{|\partial_t (\vec{x}_i - \vec{x}_j)|^4}{4\pi |\vec{x}_i - \vec{x}_j|} + \cdots \]

We call this action as “moduli action”.

T.M-Shiba-Wiseman-Withers (2013)
Our proposal “p-soup”

Example: 4dim Maxwell-Einstein-Dilaton gravity

Gravitational interaction:
(attractive)

\[
\sum_{i<j} \frac{\kappa^2 m^2}{16\pi} \frac{1}{|\vec{x}_i - \vec{x}_j|} + \frac{\kappa^2 m^2}{4} \frac{1}{4\pi |\vec{x}_i - \vec{x}_j|} + \ldots
\]

Coulomb interaction:
(repulsive)

\[
- \sum_{i<j} \frac{\kappa^2 m^2}{16\pi} \frac{1}{|\vec{x}_i - \vec{x}_j|}
\]

\[
|\partial_t \vec{x}_i| \ll 1
\]

\[
S_{\text{moduli}} = \int dt \frac{m}{2} \sum_{i=1}^{N} \partial_t \vec{x}_i \cdot \partial_t \vec{x}_i + \frac{\kappa^2 m^2}{4} \sum_{i<j} \frac{|\partial_t (\vec{x}_i - \vec{x}_j)|^4}{4\pi |\vec{x}_i - \vec{x}_j|} + \ldots
\]

\[
\sim (\text{kinetic energy})^2
\]

We call this action as “moduli action”.

\[\kappa: \text{gravitational coupling} \]
\[m: \text{mass of the 0-brane} \]
\[q = \sqrt{2\kappa m}: \text{charge (BPS)} \]
\[\vec{x}_i: \text{position of the i-th brane} \]
Our proposal “p-soup”

- Example: 4dim Maxwell-Einstein-Dilaton gravity

\[ ds^2_{\text{Einstein}} = -H^{-1/2}dt^2 + H^{1/4} \left( \sum_{I=1}^{3} (dx^I)^2 \right) \]

extremal N (separated) 0-brane solution

- pump a little energy
  (It breaks the extremality)

- low energy effective action for the branes in the gravity
  \[ (|\partial_t \vec{x}_i| \ll 1) \]

\[ S_{\text{moduli}} = \int dt \frac{m}{2} \sum_{i=1}^{N} \vec{\partial}_t \vec{x}_i \cdot \vec{\partial}_t \vec{x}_i + \frac{\kappa^2 m^2}{4} \sum_{i<j} \frac{|\partial_t (\vec{x}_i - \vec{x}_j)|^4}{4\pi |\vec{x}_i - \vec{x}_j|} + \ldots \]

We call this action as “moduli action”.

\sim (\text{kinetic energy})^2

T.M-Shiba-Wiseman-Withers (2013)
Our proposal “p-soup”

- Example: 4dim Maxwell-Einstein-Dilaton gravity

- Relativistic corrections:
  \[ |\partial_t \vec{x}_i| \ll 1 \]

- Multi-graviton exchanges:
  \[ S_{2l} \sim \int dt \sum_{i_1, i_2, \ldots, i_{l+1}} \kappa^{2l} \frac{m(\partial_t \vec{x})^2}{|\vec{x}|^l} (l+1) \]

\[ \text{exchanges of } l \text{ gravitons between } (l + 1) \text{ 0-branes.} \]

\[ \star \text{low energy effective action for the branes in the gravity} \]

\[ S_{\text{moduli}} = \int dt \frac{m}{2} \sum_{i=1}^{N} \partial_t \vec{x}_i \cdot \partial_t \vec{x}_i + \frac{\kappa^2 m^2}{4} \sum_{i<j} \frac{\partial_t (\vec{x}_i - \vec{x}_j)^4}{4\pi |\vec{x}_i - \vec{x}_j|} + \cdots \]

We call this action as “moduli action”.  
\[ \sim (\text{kinetic energy})^2 \]
Our proposal “p-soup”

**Estimation of the thermodynamics**

**low energy effective action for the branes in the gravity**

\[ S_{\text{moduli}} = \int dt \frac{m}{2} \sum_{i=1}^{N} \partial_t \vec{x}_i \cdot \partial_t \vec{x}_i + \frac{\kappa^2 m^2}{4} \sum_{i<j} \frac{\partial_t (\vec{x}_i - \vec{x}_j)^4}{4\pi |\vec{x}_i - \vec{x}_j|^4} + \ldots \]

\[ \equiv L_0 \]

\[ \equiv L_2 \]

**assumptions:**

\[ \begin{cases} |\partial_t \vec{x}_i| \sim |\partial_t (\vec{x}_i - \vec{x}_j)| \equiv v \\ |\vec{x}_i| \sim |\vec{x}_i - \vec{x}_j| \equiv x \end{cases} \]

\[ L_0 \sim Nmv^2 \]

\[ L_2 \sim \frac{\kappa^2 N^2 m^2 v^4}{x} \]

**Virial theorem:** (assume the branes compose a bound state)

\[ L_0 \sim L_2 \]

\[ v^2 \sim \frac{x}{\kappa^2 Nm} \]

(The velocity relates to the size of the bound state.)

The total energy of the bound state is estimated as,

\[ E \sim L_0 \sim L_2 \sim \frac{x}{\kappa^2} \]
Our proposal “p-soup”

- Estimation of the thermodynamics

To consider the temperature dependence of the bound state, we employ another assumption: temperature controls the derivatives.

\[ v^2 \sim \frac{x}{\kappa^2 N m} \quad - ① \]
\[ E \sim \frac{x}{\kappa^2} \quad - ② \]

From ①, we obtain
\[ x \sim \frac{1}{\kappa^2 N m T^2} \]

From ②, we obtain
\[ E \sim \frac{1}{\kappa^4 N m T^2} \quad \Rightarrow \quad S_{\text{entropy}} \sim \frac{1}{\kappa^4 N m T^3} \]
Our proposal “p-soup”

Estimation of the thermodynamics

bound state of the branes

\[
\begin{align*}
E & \sim \frac{x}{\kappa^2} \\
x & \sim \frac{1}{\kappa^2 N m T^2} \\
S_{\text{entropy}} & \sim \frac{1}{\kappa^4 N m T^3}
\end{align*}
\]

In the same gravity, we have a black hole solution carrying the same charge in the near extremal limit.

\[
\begin{align*}
E & = \frac{3\pi r_H}{\kappa^2} \\
r_H & = \frac{1}{2^3 \pi \kappa^2 N m T^2} \\
S_{\text{entropy}} & = \frac{1}{4\kappa^4 N m T^3}
\end{align*}
\]

\[
\left( \frac{1}{\kappa^2 N m T} \ll 1 \right)
\]

near extremal limit
Our proposal “p-soup”

- Estimation of the thermodynamics

\[
\begin{align*}
E & \sim \frac{x}{\kappa^2} \\
x & \sim \frac{1}{\kappa^2 N m T^2} \\
S_{\text{entropy}} & \sim \frac{1}{\kappa^4 N m T^3}
\end{align*}
\]

\[ (v \sim T x) \]

In the near extremal limit, the physical parameter \((\kappa, N, m, T)\) dependences agree through the identification \(x \sim r_H \).

\[
\begin{align*}
E &= \frac{3\pi r_H}{\kappa^2} \\
r_H &= \frac{1}{2^3 \pi \kappa^2 N m T^2} \\
S_{\text{entropy}} &= \frac{1}{4\kappa^4 N m T^3}
\end{align*}
\]

\( \left( \frac{1}{\kappa^2 N m T} \ll 1 \right) \) near extremal limit.
Our proposal “p-soup”

Estimation of the thermodynamics

\[ S_{\text{moduli}} = \int dt \frac{m}{2} \sum_{i=1}^{N} \dot{x}_i \cdot \dot{x}_i + \frac{\kappa^2 m^2}{4} \sum_{i<j} \frac{|\partial_t (x_i - x_j)|^4}{4\pi|x_i - x_j|} + \cdots \]

The moduli action may explain the microscopic origin of the black hole thermodynamics in the near extremal limit.

\[
\begin{align*}
E &= \frac{3\pi r_H}{\kappa^2} \\
1 &= \frac{2^3\pi\kappa^2 N m T^2}{r_H} \\
S_{\text{entropy}} &= \frac{1}{4\kappa^4 N m T^3}
\end{align*}
\]

\[
\left( \frac{1}{\kappa^2 N m T} \ll 1 \right)
\]

equal near extremal limit
Our proposal “p-soup”

- Estimation of the thermodynamics

\[
S_{\text{moduli}} = \int dt \frac{m}{2} \sum_{i=1}^{N} \partial_t \vec{x}_i \cdot \partial_t \vec{x}_i + \frac{\kappa^2 m^2}{4} \sum_{i<j} \frac{|\partial_t (\vec{x}_i - \vec{x}_j)|^4}{4\pi |\vec{x}_i - \vec{x}_j|} + \ldots
\]

The moduli action may explain the microscopic origin of the black hole thermodynamics in the near extremal limit.

- Corrections for the moduli action.
  - Relativistic corrections:
    \[ |\partial_t \vec{x}_i| \ll 1 \]
  - Multi-graviton exchanges:
    \[
    S_{2l} \sim \int dt \sum_{i_1,i_2,\ldots,i_{l+1}} \kappa^{2l} \left\{ \frac{m (\partial_t \vec{x})^2}{|\vec{x}|} \right\}^{(l+1)} \left( \text{exchanges of } l \text{ gravitons between } (l + 1) \text{ 0-branes.} \right)
    \]
Our proposal “p-soup”

◆ Relativistic corrections

\[ |\partial_t \vec{x}_i| \ll 1 \]

We should check when this condition is satisfied in the bound state.

\[ |\partial_t \vec{x}_i| \sim Tx \sim \frac{1}{\kappa^2 N m T} \]

This condition corresponds to the near extremal limit in the black hole solution.

Non-relativistic approximation in our model \( \ll \) Near extremal limit in black hole solution
Our proposal “p-soup”

◆ Multi-graviton exchanges

\[ S_{2l} \sim \int dt \sum_{i_1, i_2, \ldots, i_{l+1}} \kappa^{2l} \left\{ \frac{m(\partial_t \vec{x})^2}{|\vec{x}|^l} \right\}^{(l+1)} \]

exchanges of \( l \) gravitons between \( (l + 1) \) 0-branes.

\[
\begin{align*}
E &\sim \frac{x}{\kappa^2} \\
x &\sim \frac{1}{\kappa^2 N m T^2} \\
S_{\text{entropy}} &\sim \frac{1}{\kappa^4 N m T^3} \\
(v &\sim |\partial_t \vec{x}| \sim T x)
\end{align*}
\]

We estimate the magnitude of this term in the bound state.

\[ S_{2l} \sim \int dt \sum_{i_1, i_2, \ldots, i_{l+1}} \kappa^{2l} \left( \frac{m(\partial_t \vec{x})^2}{|\vec{x}|^l} \right)^{(l+1)} \sim \int dt N m v^2 \]

\[ S_{\text{moduli}} = \int dt \frac{m}{2} \sum_{i=1}^{N} \partial_t \vec{x}_i \cdot \partial_t \vec{x}_i + \frac{\kappa^2 m^2}{4} \sum_{i<j} \frac{|\partial_t (\vec{x}_i - \vec{x}_j)|^4}{4\pi |\vec{x}_i - \vec{x}_j|} + \sum_{l=2}^{\infty} S_{2l} \]

\[ \sim N m v^2 \quad \leftrightarrow \quad \sim N m v^2 \quad \sim N m v^2 \]

The multi-graviton exchange processes are the same order to the first two terms.
(Consistent with the virial theorem by which we argued the first two terms are the same order.)

→ The 0-branes are strongly coupled in the bound state. → “soup” of 0-branes
Our proposal “p-soup”

Summary of our proposal

\[
S_{\text{moduli}} = \int dt \frac{m}{2} \sum_{i=1}^{N} \partial_t \vec{x}_i \cdot \partial_t \vec{x}_i + \frac{\kappa^2 m^2}{4} \sum_{i<j} \frac{\left| \partial_t (\vec{x}_i - \vec{x}_j) \right|^4}{4\pi |\vec{x}_i - \vec{x}_j|} + \sum_{l=2}^{\infty} S_{2l} \]

Non-relativistic N 0-branes at low energy

→ Compose a strongly coupled bound state. “soup” of the 0-branes

Near extremal black hole

\[
\left( \frac{1}{\kappa^2 N m T} \ll 1 \right) \]

The bound state would provide the microscopic description of the black hole.
Our proposal “p-soup”

◆ Generalization to black p-brane solution in D-dimensional gravity

\[
I = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} \left( R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2(p+2)!} e^{a \phi} F^2_{2+p} \right)
\]

Maxwell-Einstein-dilaton

\[
a^2 = 4 - \frac{2(p+1)(n-2)}{D-2} \\
D = p + n + 1 \quad (n > 2) \text{ case}
\]

Extremal N parallel p-brane solutions, which satisfy the no force condition, exist.

★ moduli effective action of the branes \(||\partial X|| \ll 1\)

\[
S_{\text{moduli}} = -\int dt dx^p \frac{\mu}{2} \sum_{i=1}^N \partial_\mu \bar{X}_i \cdot \partial^\mu \bar{X}_i \\
+ \frac{\mu^2 \kappa^2}{4(n-2)\Omega_{n-1}} \int dt dx^p \sum_{a < b} \frac{2 \left( \partial_\mu \bar{X}_{ab} \cdot \partial^\nu \bar{X}_{ab} \right)^2 - \left( \partial_\mu \bar{X}_{ab} \cdot \partial^\mu \bar{X}_{ab} \right)^2}{|\bar{X}_{ab}|^{n-2}} \\
+ \sum_{l=2}^\infty S_{2l}
\]

\[\mu : \text{tension of the p-brane}, \quad \Omega_{n-1} = \frac{2\pi^{\frac{n}{2}}}{\Gamma \left( \frac{n}{2} \right)}\]
Our proposal “p-soup”

- Generalization to black $p$-brane solution in $D$-dimensional gravity

\[
S_{\text{moduli}} = - \int dt dx^p \frac{\mu}{2} \sum_{i=1}^{\tilde{N}} \partial_\mu \vec{X}_i \cdot \partial^\mu \vec{X}_i
\]

\[
+ \frac{\mu^2 \kappa^2}{4(n-2)\Omega_{n-1}} \int dt dx^p \sum_{a<b} \frac{2(\partial_\mu \vec{X}_{ab} \cdot \partial_\nu \vec{X}_{ab})^2 - (\partial_\mu \vec{X}_{ab} \cdot \partial^\mu \vec{X}_{ab})^2}{|\vec{X}_{ab}|^{n-2}} + \sum_{l=2}^\infty S_{2l}
\]

- Virial theorem
- Assumption for the derivative and temperature

\[
|\partial_\mu X| \sim TX
\]

Strongly coupled bound state of $p$-brane, “p-soup”

The thermodynamics of the bound states agrees with the corresponding black $p$-brane solutions.

e.g. black $Dp/F1/NS5$ brane solutions in 10 dim SUGRA
black $M2/M5$ brane solutions in 11 dim SUGRA
Our proposal “p-soup”

Generalization to black p-brane solution in D-dimensional gravity

\[
S_{\text{moduli}} = - \int dt dx^p \frac{\mu}{2} \sum_{i=1}^{N} \partial_{\mu} \vec{X}_i \cdot \partial^\mu \vec{X}_i \\
+ \frac{\mu^2 \kappa^2}{4(n-2)\Omega_{n-1}} \int dt dx^p \sum_{a<b} 2 \left( \partial_{\mu} \vec{X}_{ab} \cdot \partial_{\nu} \vec{X}_{ab} \right)^2 - \left( \partial_{\mu} \vec{X}_{ab} \cdot \partial^\mu \vec{X}_{ab} \right)^2 \left| \vec{X}_{ab} \right|^{n-2} + \sum_{l=2}^{\infty} S_{2l}
\]

- Virial theorem
- Assumption for the derivative and temperature

\[|\partial_{\mu} X| \sim TX \rightarrow |\partial_{\mu} X| \sim \pi TX\]

\[X(t) \sim \sum_n X_n e^{2\pi T n \Omega_{n-1}}\]

\[r_H = \left( \frac{2^5 \pi^2 NT^2 \mu \kappa^2}{(n-2)^3 \Omega_{n-1}} \right)^{\frac{1}{n-4}}\]

Horizon of the black brane

\[X \sim \left( \frac{\pi^2 NT^2 \mu \kappa^2}{\Omega_{n-1}} \right)^{\frac{1}{n-4}}\]

size of the bound state

If we use ↑ assumption, we can reproduce the π dependence too.

example)
1. Introduction

2. Our proposal: warm p-soup

3. p-soup and gauge/gravity correspondence

4. Summary

Smilga 2009 (D0)
Wiseman 2013 (D0,D1,D2)
T.M and Shiba 2013 (D3,D4,D6, M2, M5)
T.M-Shiba-Wiseman-Withers 2013 (D5)

cf. Matrix black hole
Horowitz and Martinec (1998)
Banks-Fischler-Klebanov-Susskind (1998)
Li and Martinec (1998)
**p-soup and supersymmetric gauge theory**

**black p-brane**

**thermal bound state of p-brane**

\[ S_{\text{moduli}} = -\int dt dx^p \frac{\mu}{2} \sum_{i=1}^{N} \partial_{\mu} \tilde{X}_i \cdot \partial^{\mu} \tilde{X}_i \]

\[ + \frac{\mu^2 \kappa^2}{4(n-2)\Omega_{n-1}} \int dt dx^p \sum_{a<b} \frac{2 \left( \partial_{\mu} \tilde{X}_{ab} \cdot \partial_{\nu} \tilde{X}_{ab} \right)^2 - \left( \partial_{\mu} \tilde{X}_{ab} \cdot \partial^{\mu} \tilde{X}_{ab} \right)^2}{|\tilde{X}_{ab}|^{n-2}} + \sum_{l=2}^{\infty} S_{2l} \]

**gauge/gravity correspondence**

(Dp/M2/M5)

**Thermal state of p+1 dim SYM/ABJM/6d (2,0) SCFT**

cf. Hyakutake-san’s talk

Yoneya-san’s talk

**Q. Are these two microscopic descriptions related?**

**A. Yes. The moduli action may appear as the low energy effective action of these gauge theories.**
**p-soup and supersymmetric gauge theory**

◆ p+1 dim SYM/ABJM/6d (2,0) SCFT

\[
S_{\text{SYM}} = \frac{N}{\lambda_p^p} \int_0^\beta d\tau \int d^p x \, \text{Tr} \left[ \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \Phi^I)^2 - \frac{1}{4} [\Phi^I, \Phi^J]^2 + \cdots \right]
\]

\[
S_{\text{ABJM}} = k \int d\tau d^2 x \, \text{Tr} \left[ \frac{1}{2} (D_\mu \Phi^I)^2 + (\Phi^I)^6 + \cdots \right]
\]

\[
S_{6d\text{SCFT}} = \int_0^\beta d\tau d^5 x \, [??] \, \text{(Action unknown)}
\]

**Conjecture: The low energy effective theory.**

\[
S_{\text{moduli}} = -\int dt dx^p \frac{\mu}{2} \sum_{i=1}^N \partial_\mu \vec{X}_i \cdot \partial^\mu \vec{X}_i
\]

\[
+ \frac{\mu^2 \kappa^2}{4(n-2)\Omega_{n-1}} \int dt dx^p \sum_{a<b} \frac{2 \left( \partial_\mu \vec{X}_{ab} \cdot \partial_\nu \vec{X}_{ab} \right)^2 - \left( \partial_\mu \vec{X}_{ab} \cdot \partial^\mu \vec{X}_{ab} \right)^2}{|\vec{X}_{ab}|^{n-2}} + \sum_{l=2}^\infty S_{2l}
\]

\[n = D - p - 1, \ (D = 10/11)\]

We can explicitly show this conjecture in SYM/ABJM for the first several terms. We can also argue it by using a dimensional analysis which works 6d SCFT too.

→ Today I will show the dimensional analysis.
p-soup and supersymmetric gauge theory

\[ S_{\text{SYM}} = \frac{N}{\chi_p} \int_0^\beta d\tau \int d^p x \text{Tr} \left[ \frac{1}{4} F_{\mu \nu}^2 + \frac{1}{2} (D_\mu \Phi^I)^2 - \frac{1}{4} [\Phi^I, \Phi^J]^2 + \cdots \right] \]

\[ \Phi^I = \begin{pmatrix} \phi_1^I \\ \vdots \\ \phi_N^I \end{pmatrix} \quad \phi_i^I = \frac{1}{\alpha'} X_i^I : \text{scalar moduli, the position of the i-th Dp-brane.} \]

**Conjecture:** The low energy effective theory.

\[ S_{\text{moduli}} = -\int dtdx^p \frac{\mu}{2} \sum_{i=1}^N \partial_\mu \vec{X}_i \cdot \partial^\mu \vec{X}_i \]

\[ + \frac{\mu^2 \kappa^2}{4(n-2)\Omega_{n-1}} \int dtdx^p \sum_{a<b} \frac{2 \left( \partial_\mu \vec{X}_{ab} \cdot \partial_\nu \vec{X}_{ab} \right)^2 - \left( \partial_\mu \vec{X}_{ab} \cdot \partial^\mu \vec{X}_{ab} \right)^2}{|\vec{X}_{ab}|^{n-2}} + \sum_{l=2}^{\infty} S_{2l} \]

**Goal:** Derive the moduli action from the SYM theory by integrating the off-diagonal components.
p-soup and supersymmetric gauge theory

- p+1 dim SYM

\[ S_{\text{SYM}} = \frac{N}{\lambda_p} \int_0^\beta d\tau \int d^p x \, \text{Tr} \left[ \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \Phi^I)^2 - \frac{1}{4} [\Phi^I, \Phi^J]^2 + \cdots \right] \]

- Estimation of the effective action after the integral of the off-diagonal components.

\[ S_{\text{eff}} = \frac{N}{\lambda_p} S_0 + S_1 + \lambda S_2 + \lambda^2 S_3 \cdots S_k : k\text{-loop} \]

\[ \Phi^I = \begin{pmatrix} \phi_1^I \\ \vdots \\ \phi_N^I \end{pmatrix} \]

Leading term (classical term)

\[ S_0 = \int_0^\beta d\tau \int d^p x \sum_{i=1}^N \frac{1}{2} (\partial_\mu \phi_i^I)^2 \]

(We will ignore diagonal components of the gauge and fermion fields today)
p-soup and supersymmetric gauge theory

- p+1 dim SYM

\[ S_{\text{SYM}} = \frac{N}{\lambda_p} \int_0^\beta d\tau \int d^p x \, \text{Tr} \left[ \frac{1}{4} F_{\mu \nu}^2 + \frac{1}{2} (D_\mu \Phi^I)^2 - \frac{1}{4} [\Phi^I, \Phi^J]^2 + \ldots \right] \]

- Estimation of the effective action after the integral of the off-diagonal components.

\[ S_{\text{eff}} = \frac{N}{\lambda_p} S_0 + S_1 + \lambda S_2 + \lambda^2 S_3 \ldots \quad S_k : k\text{-loop} \]

★ One-loop: two types of corrections

- **Thermal corrections** (depending on temperature)

  \[ \propto e^{-\beta |\phi_a - \phi_b|} \quad \rightarrow \quad \text{Irrelevant if } \beta |\phi_a - \phi_b| \ll 1 \]
  
  \[ \uparrow \quad \text{(mass of the off-diagonal components.)} \]
  
  (Boltzmann factor of the off-diagonal components.)

- **Non-thermal corrections** (independent of temperature)

  SUSY will constrain the interactions.
p-soup and supersymmetric gauge theory

◆ p+1 dim SYM

\[ S_{\text{SYM}} = \frac{N}{\lambda_p} \int_0^\beta d\tau \int d^p x \, \text{Tr} \left[ \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \Phi^I)^2 - \frac{1}{4} [\Phi^I, \Phi^J]^2 + \ldots \right] \]

◆ Estimation of the effective action after the integral of the off-diagonal components.

\[ S_{\text{eff}} = \frac{N}{\lambda_p} S_0 + S_1 + \lambda S_2 + \lambda^2 S_3 \ldots \quad S_k : k\text{-loop} \]

★ Estimation of the non-thermal corrections (SUSY) non-normalization theorem:

\[ \begin{cases} 
\text{No } (\phi)^n \text{ potentials without derivatives } (\partial \phi)^m \text{ (moduli at } T=0) \\
\text{No } (\partial \phi)^2 \text{ potentials} 
\end{cases} \]

★ dimensions:

t'Hooft coupling: \([\lambda_p] = 3 - p\), adjoint scalar: \([\Phi^I] = 1\), temperature: \([T] = 1\)

\[ S_1 \sim - \int d\tau d^p x \sum_{a,b=1}^N \frac{(\partial \phi_a - \partial \phi_b)^4}{|\phi_a - \phi_b|^{7-p}} \left( 1 + \frac{(\partial \phi)^2}{\phi^4} + \ldots \right) \]

... dimensional analysis
\textbf{p-soup and supersymmetric gauge theory}

\begin{itemize}
  \item \textbf{p+1 dim SYM}
  \begin{align*}
  S_{\text{SYM}} &= \frac{N}{\lambda_p} \int_0^\beta d\tau \int d^p x \, \text{Tr} \left[ \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \Phi^I)^2 - \frac{1}{4} [\Phi^I, \Phi^J]^2 + \cdots \right] \\
  \beta |\phi_a - \phi_b| &\ll 1 \quad \text{(Assumption: suppress the thermal corrections.)}
  \\
  S_{\text{eff}} &= \frac{N}{\lambda_p} S_0 + S_1 + \lambda S_2 + \lambda^2 S_3 \cdots
  \\
  \begin{cases}
  S_0 = \int_0^\beta d\tau \int d^p x \sum_{i=1}^N \frac{1}{2} (\partial_\mu \phi^I_i)^2 \\
  S_1 \sim - \int d\tau d^p x \sum_{a,b=1}^N \frac{(\partial \phi_a - \partial \phi_b)^4}{|\phi_a - \phi_b|^{7-p}} \left(1 + \frac{(\partial \phi)^2}{\phi^4} + \cdots \right)
  \end{cases}
  \\
  \item \textbf{The moduli action for Dp-brane calculated in the gravity}
  \begin{align*}
  S_{\text{moduli}} &= - \int dt dx^p \frac{\mu}{2} \sum_{i=1}^N \partial_\mu \vec{X}_i \cdot \partial^\mu \vec{X}_i \\
  &\quad + \frac{\mu^2 \kappa^2}{4(n-2)\Omega_{n-1}} \int dt dx^p \sum_{a<b} \frac{2 \left( \partial_\mu \vec{X}_{ab} \cdot \partial_\nu \vec{X}_{ab} \right)^2 - \left( \partial_\mu \vec{X}_{ab} \cdot \partial^\mu \vec{X}_{ab} \right)^2}{|\vec{X}_{ab}|^{7-p}} + \sum_{l=2}^{\infty} S_{2l}
  \end{align*}
\end{itemize}
p-soup and supersymmetric gauge theory

◆ p+1 dim SYM

\[ S_{\text{SYM}} = \frac{N}{\lambda_p} \int_0^\beta d\tau \int d^p x \ Tr \left[ \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \Phi^I)^2 - \frac{1}{4} [\Phi^I, \Phi^J]^2 + \cdots \right] \]

\[ \beta |\phi_a - \phi_b| \ll 1 \quad \text{(Assumption: suppress the thermal corrections.)} \]

\[ S_{\text{eff}} = \frac{N}{\lambda_p} S_0 + S_1 + \lambda S_2 + \lambda^2 S_3 \cdots \]

\[ S_0 = \int_0^\beta d\tau \int d^p x \sum_{i=1}^N \frac{1}{2} (\partial_\mu \phi_i^I)^2 \]

\[ S_1 \sim -\int d\tau d^p x \sum_{a,b=1}^N \frac{(\partial \phi_a - \partial \phi_b)^4}{|\phi_a - \phi_b|^{7-p}} \left( 1 + \frac{(\partial \phi)^2}{\phi^4} + \cdots \right) \]

They agrees when \( \beta |\phi_a - \phi_b| \ll 1 \)
and \( (\partial \phi)^2 / \phi^4 \ll 1 \).

◆ The moduli action for Dp-brane calculated in the gravity

\[ S_{\text{moduli}} = -\int dt d^p x^\mu \frac{\mu}{2} \sum_{i=1}^N \partial_\mu \bar{X}_i \cdot \partial^\mu \bar{X}_i \]

\[ + \frac{\mu^2 \kappa^2}{4(n-2)\Omega_{n-1}} \int dt d^p x \sum_{a<b} 2 \left( \partial_\mu \bar{X}_{ab} \cdot \partial_\nu \bar{X}_{ab} \right)^2 - \left( \partial_\mu \bar{X}_{ab} \cdot \partial^\mu \bar{X}_{ab} \right)^2 \left\| \bar{X}_{ab} \right\|^{7-p} \]

\[ + \sum_{l=2}^\infty S_{2l} \]
p-soup and supersymmetric gauge theory

◆ thermodynamics of p+1 dim SYM

\[ S_{\text{SYM}} = \frac{N}{\lambda_p} \int_{0}^{\beta} d\tau \int d^p x \ Tr \left[ \frac{1}{4} F_{\mu \nu}^2 + \frac{1}{2} (D_{\mu} \Phi^I)^2 - \frac{1}{4} [\Phi^I, \Phi^J]^2 + \ldots \right] \]

Assumption: \( \beta |\phi_a - \phi_b| \ll 1 \) and \( (\partial \phi)^2 / \phi^4 \ll 1 \)

\[ S_{\text{moduli}} = - \int dtdx^p \frac{\mu}{2} \sum_{i=1}^{N} \partial_{\mu} \vec{X}_i \cdot \partial^\mu \vec{X}_i \]

+ \( \frac{\mu^2 \kappa^2}{4(n-2)\Omega_{n-1}} \int dtdx^p \sum_{a < b} 2 \left( \partial_{\mu} \vec{X}_{ab} \cdot \partial_{\nu} \vec{X}_{ab} \right)^2 - \left( \partial_{\mu} \vec{X}_{ab} \cdot \partial^\mu \vec{X}_{ab} \right)^2 \frac{1}{|\vec{X}_{ab}|^{7-p}} + \sum_{l=2}^{\infty} S_{2l} \)

Repeat the same calculations in the previous section.

Black Dp brane

\[ \begin{aligned}
E &\sim N^2 T^{\frac{2(7-p)}{5-p}} \lambda_p^{-\frac{3-p}{5-p}} \\
\phi &\sim T^{\frac{2}{5-p}} \lambda_p^{\frac{1}{5-p}} \\
(\partial \phi) &\sim T \phi
\end{aligned} \]

We should check the above assumptions are self-consistently satisfied.

\[ \begin{aligned}
\beta |\phi_a - \phi_b| &\ll 1 \\
(\partial \phi)^2 / \phi^4 &\ll 1
\end{aligned} \]

\( \Rightarrow \) are OK if \( \lambda_p / T^{3-p} \gg 1 \)

SYM would describe BH at the strong coupling \( \lambda_p / T^{3-p} \gg 1 \).
\textbf{p-soup and supersymmetric gauge theory}

\begin{itemize}
  \item \textbf{ABJM}
  \end{itemize}

\[
S_{\text{ABJM}} = k \int d\tau d^2 x \ Tr \left[ \frac{1}{2} (D_\mu \Phi^I)^2 + (\Phi^I)^6 + \cdots \right] \quad [\Phi^I] = 1/2
\]

\textbf{Assumption:} \( \beta |\phi_a - \phi_b|^2 \ll 1 \) and \( (\partial \phi)^2/\phi^6 \ll 1 \)

\[
S_0 = k \int_0^\beta d\tau \int d^2 x \sum_{i=1}^N \frac{1}{2} (\partial_\mu \phi^I_i)^2 \quad S_1 \sim -\int d\tau d^p x \sum_{a,b=1}^N \frac{(\partial \phi_a - \partial \phi_b)^4}{|\phi_a - \phi_b|^6}
\]

\textbf{Repeat the same calculations in the previous section.}

\textbf{Black M2 brane}

\[
\begin{cases}
  E_{\text{M2}} \sim N^{\frac{3}{2}} \sqrt{k} T^3 \\
  \phi \sim N^{\frac{1}{4}} k^{-\frac{1}{4}} T^{\frac{1}{2}} \\
  (\partial \phi \sim T \phi)
\end{cases}
\]

\[
\begin{cases}
  \beta |\phi_a - \phi_b|^2 \ll 1 \\
  (\partial \phi)^2/\phi^6 \ll 1
\end{cases} \implies \quad \text{are OK if } N/k \gg 1
\]

\textbf{We should check the above assumptions are self-consistently satisfied.}

\textbf{ABJM would describe BH at large-N with finite k.}
p-soup and supersymmetric gauge theory

◆ 6dim (2,0) SCFT (M5 theory)

\[ S_{6\text{dSCFT}} = \int_{0}^{\beta} d\tau \int d^{5}x \sum_{i=1}^{N} \frac{1}{2}(\partial_{\mu}\phi_{i}^{I})^{2} + \text{???} \]

Assumption: \( \beta^{2}\left|\phi_{a} - \phi_{b}\right| \ll 1 \) and \( (\partial\phi)^{2}/\phi^{3} \ll 1 \)

non-renormalization theorem

\[ [\phi_{i}^{I}] = 2 \]

The details of the action are unknown but the scalar moduli must exist.

Repeat the same calculations in the previous section.

\[ S_{0} = \int_{0}^{\beta} d\tau \int d^{5}x \sum_{i=1}^{N} \frac{1}{2}(\partial_{\mu}\phi_{i}^{I})^{2} \quad S_{1} \sim -\int d\tau d^{p}x \sum_{a,b=1}^{N} \frac{(\partial\phi_{a} - \partial\phi_{b})^{4}}{|\phi_{a} - \phi_{b}|^{3}} \]

Black M5 brane

\[ \begin{align*}
E_{M5} & \sim N^{3}T^{6} \\
\phi & \sim NT^{2} \\
(\partial\phi & \sim T\phi)
\end{align*} \]

We should check the above assumptions are self-consistently satisfied.

\[ \begin{cases}
\beta^{2}\left|\phi_{a} - \phi_{b}\right| \ll 1 \\
(\partial\phi)^{2}/\phi^{3} \ll 1
\end{cases} \]

are OK if \( N \gg 1 \)

6dSCFT would describe BH at large-N.
p-soup and supersymmetric gauge theory

Thermal bound state of p-brane

\[ S_{\text{moduli}} = - \int dt dx^p \frac{\mu}{2} \sum_{i=1}^{N} \partial_{\mu} X_i \cdot \partial^{\mu} X_i \]

\[ + \frac{\mu^2 \kappa^2}{4(n-2) \Omega_{n-1}} \int dt dx^p \sum_{a<b} \frac{2 \left( \partial_{\mu} \tilde{X}_{ab} \cdot \partial_{\nu} \tilde{X}_{ab} \right)^2 - \left( \partial_{\mu} \tilde{X}_{ab} \cdot \partial^{\mu} \tilde{X}_{ab} \right)^2}{|\tilde{X}_{ab}|^{n-2}} + \sum_{l=2}^{\infty} S_{2l} \]

Low energy effective action

(if \( \lambda_p / T^{3-p} \gg 1 \) : SYM)

gauge/gravity correspondence

(Dp/M2/M5)

Thermal state of p+1 dim SYM/ABJM/6d (2,0) SCFT

cf. Hyakutake-san’s talk
Yoneya-san’s talk
p-soup and supersymmetric gauge theory

◆ Key point: Open-closed duality     (M2? M5?)

\[ S_{\text{SYM}} = \frac{N}{\lambda_p} \int_0^\beta d\tau \int d^p x \text{ Tr} \left[ \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \Phi^I)^2 - \frac{1}{4} [\Phi^I, \Phi^J]^2 + \cdots \right] \]

\[ S_{\text{eff}} = \frac{N}{\lambda_p} S_0 + S_1 + \lambda S_2 + \lambda^2 S_3 \cdots \]

\[ S_0 = \int_0^\beta d\tau \int d^p x \sum_{i=1}^N \frac{1}{2} (\partial_\mu \phi^I_i)^2 \]

\[ S_1 \sim - \int d\tau d^p x \sum_{a,b=1}^N \frac{(\partial \phi_a - \partial \phi_b)^4}{|\phi_a - \phi_b|^{7-p}} \]

\[ \frac{1}{r^{7-p}} \]

\[ \phi^I_a \rightarrow \phi^I_b \rightarrow \phi^I_a \] one-loop in gauge theory (open string)

\[ \text{tree level in SUGRA (closed string)} \]

cf. Yoneya-san’s talk

Kawai, Lewellen and Tye 1986
Banks, Fischler, Shenker, and Susskind 1997
Ishibashi, Kawai, Kitazawa, Tsuchiya 1997
Becker, Becker, Polchinski and Tseytlin 1997
Okawa and T. Yoneya 1999
Kazama and Muramatsu 2000
Baek, Hyun, Jang and Yi 2008
Wiseman 2013
SUMMARY
Summary

black p-brane

Thermal bound state of p-brane

Thermal state of p+1 dim SYM/ABJM/6d (2,0) SCFT

gauge/gravity correspondence (Dp/M2/M5)

Warm p-soup conjecture

Low energy effective action (if $\lambda_p / T^{3-p} \gg 1$ : SYM)

\[ S_{\text{moduli}} = -\int dt dx^p \mu \sum_{i=1}^{N} \partial_i \vec{X}_i \cdot \partial^i \vec{X}_i \]

\[ + \frac{\mu^2 \kappa^2}{4(n-2) \Omega_{n-1}} \int dt dx^p \sum_{a<b} \frac{2 (\partial_a \vec{X}_{ab} \cdot \partial_b \vec{X}_{ab})^2 - (\partial_a \vec{X}_{ab} \cdot \partial^a \vec{X}_{ab})^2}{|\vec{X}_{ab}|^{n-2}} + \sum_{i=2}^{\infty} S_{2i} \]
Summary

One Important message:
Dynamics of M2 and M5-branes at large-N are similar to Dp-branes, although the N-dependences look different.

\[
E_{Dp} \sim N^2 T^{\frac{2(7-p)}{5-p}} \lambda_p^{\frac{3-p}{5-p}}
\]

\[
E_{M2} \sim N^{\frac{3}{2}} \sqrt{k} T^3 \quad E_{M5} \sim N^3 T^6
\]

Thermal state of p+1 dim SYM/ABJM/6d (2,0) SCFT

Low energy effective action
(if \( \lambda_p / T^{3-p} \gg 1 :\text{SYM} \))
Future directions

• Exact computation: 
  \[ S \propto \frac{\text{Area}}{G_N} \rightarrow S = \frac{\text{Area}}{4G_N} \]

• 1/N and \(1/(\lambda_p \beta^{3-p})\) corrections. 
  cf. Hyakutake-san’s talk

• Understand the universal viscosity ratio.

• Understand the Information paradox based on our model 
  cf. Yoneya-san and Sekino-san’s talks

• Understand the various dynamics of black brane:
  GL transitions, M-theory transitions, etc

(Shiba-san’s poster presentation)

work in progress with Shiba-Wiseman-Withers