FRW/CFT duality: A holographic framework for eternal inflation

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Based on collaborations with:
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The problem addressed in this work:

What happens when gravity is coupled to a theory with metastable vacuum?

- e.g. scalar field which has a false vacuum and a true vacuum
  \[ V(\Phi_F) > 0, \ V(\Phi_T) = 0 \]

- If we ignore gravity, first order phase transition:
  - Nucleation of bubbles of true vacuum (Callan, Coleman, ...)
  - The whole space eventually turns into true vacuum.

When there is gravity, inflation in the false vacuum leads to qualitatively different picture.
Motivation

• “String Landscape”: There is evidence that string theory has de Sitter (positive vacuum energy) vacua.

• We do not have non-perturbative formulation of string theory to study the landscape yet.

• We try to gain as much information as possible from semi-classical approximation.

Goals:

• Construct holographic formulation

• Find observational signatures (our universe: true vacuum)
Outline

Part I: Phases of eternal inflation
• R. Bousso, B. Freivogel, YS, S. Shenker, L. Susskind, I.-S. Yang, C.-P. Yeh, PRD78 (2008) 063538;

Part II: Holographic description (FRW/CFT duality)
• B. Freivogel, L. Susskind, YS, C.-P. Yeh, PRD 74, 086003 (2006);
• YS, Susskind, PRD80 (2009) 083531;

Part III: Observational consequence
• B. Freivogel, YS, L. Susskind, C.-P. Yeh, (unpublished)
False vacuum: de Sitter space

• de Sitter space: hyperboloid in $R^{4,1}$

$$ds^2 = -dt^2 + H^{-2} \cosh^2(Ht)d^2\Omega_3$$

Hubble parameter (expansion rate)

$$H^2 = \frac{8\pi G}{3} V(\phi_F)$$

• Causal structure: Two points separated by $> H^{-1}$ are causally disconnected.
Bubble of true vacuum

Described by Coleman-De Luccia instanton

• Generalization of “bounce” solution (c.f. Coleman’s textbook)

• Field configuration in Euclidean space which contributes to imaginary part of energy (decay rate) of false vacuum:
Coleman-De Luccia instanton

Euclidean geometry:
• Deformed $S^4$ (Euclidean de Sitter is $S^4$)
• Rotationally symmetric: SO(4)
• Nucleation rate: $\Gamma \sim e^{-(S_{cl} - S_{deSitter})}$

Lorentzian geometry:
• Given by analytic continuation
• Describes the nucleation and subsequent expansion of a bubble.
Penrose diagram: right figure (flat space patched to de Sitter)

• The whole geometry has SO(3,1)
• Bubble wall is constantly accelerated.
• Open FRW universe (shaded region) inside the bubble (: our universe).
  Spatial slice (dotted lines) : $H^3$

$$ds^2 \sim -dt^2 + t^2(dR^2 + \sinh^2 R d\Omega^2)$$

• Beginning of FRW universe: non-singular (similar to Rindler horizon)
Eternal inflation

• A single bubble does not cover the whole space. (Fills only the horizon volume.)

• Many bubbles will form in the de Sitter region with the rate $\Gamma$ (per unit physical 4-volume). (Bubble collisions are inevitable.)

• But if $\Gamma \ll H^4$, bubble nucleation cannot catch up the expansion of space, and false vacuum exists forever: “Eternal Inflation” [Guth, Linde, Vilenkin, ...]
Part I: Phases of eternal inflation
Outline of Part I

• There are three phases of eternal inflation, depending on the nucleation rate.

• Phases are characterized by the existence of percolating structures (lines, sheets) of bubbles in de Sitter space. (First proposed by Winitzki, ’01.)

• The cosmology in the true vacuum region is qualitatively different in each phase.
View from the future infinity

• Consider conformal future of de Sitter. (future infinity in comoving coordinates)

\[ ds^2 \sim -d\eta^2 + \frac{d\vec{x}^2}{H^2 \eta^2} \quad (-\infty < \eta < 0) \]

• A bubble: represented as a sphere cut out from de Sitter.

• “Scale invariant” distribution of bubbles

  Bubbles nucleated earlier:
  appear larger: radius \( \sim H^{-3} |\eta|^3 \)
  are rarer: volume of nucleation sites \( \sim |\eta|^{-3} \)
Model for eternal inflation

- **Mandelbrot model (Fractal percolation)**
  - Start from a white cell.  
    (White: inflating, Black: non-inflating;  
    Cell: One horizon volume)
  - Divide the cell into cells  
    with half its linear size.  
    (The space grows by a factor of 2.  
    Time step:  \( \Delta t = H^{-1} \ln 2 \)  
    )
  - Paint each cell in black with probability \( P \).  
    (\( P \sim GV_{\text{hor}} \Delta t \) = nucleation rate per horizon volume: constant)  
  - Subdivide the surviving (white) cells, and paint cells  
    in black w/ probability \( P \). Repeat this infinite times.
Properties of Mandelbrot model

• If $P > 1 - (1/2)^3 = 7/8$, the whole space turns black, since (the rate of turning black) > (the rate of branching). (No eternal inflation)

• If $P < 7/8$, white region is not an empty set (is a fractal). Non-zero fractal dimension $d_F$ (rate of growth of # of cells):

$$N_{\text{cells}} = 2^{n_d_F}, \quad d_F = 3 - |\log(1 - P)|/\log 2$$

($n$: # of steps)

Physical volume of de Sitter region (# of cells) grows. (Eternal inflation)

[Fractals in eternal inflation: noted by Vilenkin, Winitzki, ..]
Three phases of eternal inflation

From the result on the 3D Mandelbrot model


In order of increasing $P$ (or $\Gamma$), there are

(white = inflating, black = non-inflating)

- **Black island phase**: Black regions form isolated clusters; $\exists$ percolating white sheets.
- **Tubular phase**: Both regions form tubular network; $\exists$ percolating black and white lines.
- **White island phase**: White regions are isolated; $\exists$ percolating black sheets.
Geometry of the true vacuum region

• Mandelbrot model: the picture of the de Sitter side. (de Sitter region outside the light cone of the nucleation site is not affected by the bubble.)

• To find the spacetime in the non-inflating region inside (the cluster of) bubbles, we need to understand the dynamics of bubble collisions.

• Exact analysis is difficult, but the intuition gained from simple examples of bubble collisions is helpful.
Black island phase (isolated cluster of bubbles)

Small deformations of open FRW universe.

• Basic fact: Collision with a bubble (of the same vacuum) does not destroy the bubble [c.f. Bousso, Freivogel, Yang, ‘07]

• Analysis of bubble collision: Solving junction condition assuming a domain wall forms after collision.
• Symmetry for the case of two-bubble collision: SO(2,1)
  
  – De Sitter space: hyperboloid in $\mathbb{R}^{4,1}$
    \[-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = \ell^2\]
  
  – Bubbles: planes at $X_3 = \text{const.} = \sqrt{\ell^2 - r_0^2}$
    and at $X_4 = \text{const.} = \sqrt{\ell^2 - r_0^2}$
  
  – Two bubbles collide at spacelike $H^2$

• Parametrization of de Sitter:

$$ds^2 = -f^{-1}(t)dt^2 + f(t)dz^2 + t^2dH^2_2$$

$$f(t) = 1 + t^2/\ell^2, \quad (0 \leq z \leq 2\pi\ell)$$

$$\left(X_a = tH_a \ (a = 0, 1, 2), \quad X_3 = \sqrt{t^2 + \ell^2} \cos(z/\ell), \quad X_4 = \sqrt{t^2 + \ell^2} \sin(z/\ell) \right)$$
• Parametrization of flat space:
\[ ds^2 = -dt^2 + dz^2 + t^2 dH_2^2 \]
• World volume of domain wall:
\[ ds_{DW}^2 = -d\tau^2 + R^2(\tau)dH_2^2 \quad (R(\tau) = t(\tau)) \]
• Energy on the domain wall decays at late time. (Negative curvature of $H^2$ dilutes the energy.)
\[ \rho = \rho_0/t^2 \quad \text{(for dust wall)} \]
• The spatial geometry approach smooth $H^3$
• In black island phase, local geometry near a collision will be similar to the two-bubble case, and the space will be smoothed at late times.
Tubular phase (tube-like structure of bubbles)

In the late time limit, approaches flat spacetime whose spatial slice has a boundary with infinite genus.

- True vacuum with higher genus boundary: torus case
  [Bousso, Freivogel, YS, Shenker, Susskind, Yang, Yeh, ‘08]
  - Ring-like initial configuration of bubbles.
  - If the hole in the middle is larger than horizon size, it cannot shrink.
– Solve a sequence of junction conditions

\[ ds^2 = -f(t)dt^2 + f^{-1}(t)dz^2 + t^2dH_2^2 \]
\[ f(t) = 1 + t^2/\ell^2 \quad \text{(de Sitter)} \]
\[ f(t) = 1 - t_n/t \quad \text{(in region } n; t_n: \text{ const.)} \]

– Spacetime approaches flat at late time.

\[ ds^2 = -dt^2 + t^2ds_{H/\Gamma}^2 \]

– Spatial slice: negatively curved space with toroidal boundary. Constructed as an orbifold of \( H^3 \).

\[ ds_{H^3}^2 = \frac{dx^2 + dy^2 + dz^2}{z^2} \]
\[ (x, y, z) \sim \lambda(x, y, z) \]

• Negatively curved space with arbitrary genus can be constructed similarly. The geometry of true vacuum region in the tubular phase will be such a space.
White island phase (isolated inflating region)

An observer in the true vacuum is “surrounded” by inflating region (contrary to the intuition from Mandelbrot model).

- Simple case: two white islands (with $S^2$ symmetry)
  [Kodama et al ’82, BFSSSSYY ’08]

  Global slicing ($S^3$) of de Sitter

  Penrose diagram

  – An observer can see only one boundary; the other boundary is behind the black hole horizon. [c.f. “non-traversability of a wormhole”]
• In the white island phase, a white region will split.
• Also when there are more than two white islands, singularity and horizon will form so that the boundaries are causally disconnected.

– Late time geometry for the three white island case:
Summary of Part I

Three phases of eternal inflation and their cosmology:

• Black island phase:
  Small deformation of an open FRW

• Tubular phase:
  Negatively curved space with an infinite genus boundary

• White island:
  Observer sees one boundary and one or more black hole horizons (behind which there are other boundaries).
Part II: Holographic duality
FRW/CFT duality [Freivogel, Sekino, Susskind, Yeh, ’06]

• Proposal for holographic duality for a universe created by bubble nucleation. (Ignore bubble collisions.)

• Different from “dS/CFT correspondence”. Formulated using the d.o.f. that a single observer (“Census Taker”) can see.

• Dual theory: CFT on $S^2$ at the boundary of $H^3$.
  – $\text{SO}(3,1)$: 2D conformal symmetry
  – Dual theory has two less dimensions than the bulk
• The dual theory contains 2D gravity (Liouville field)
  – Non-decoupling of gravity: due to the compactness of the Euclidean space.
  – Liouville plays the role of time (similarly to Wheeler-DeWitt formalism)

• We study correlation functions in the CDL background.
  – Harmonics on hyperboloid has continuous spectrum; Normalizable modes decay as \( e^{-R} \) as \( R \to \infty \)  
    e.g. \( \frac{\sin kR}{\sinh R} \)
  – Non-normalizable modes can appear (e.g. for massless fields) [Sasaki, Tanaka,..., ‘90s].
  – Alternative representation: Organize terms w.r.t. the “scaling dimensions”, \( \Sigma \Delta \exp(-\Delta R) \)
Correlation functions

- Hartle-Hawking prescription: Analytic continuation from Euclidean
- Essentially a 1D scattering problem:

\[
\left[-\partial_X^2 + \frac{a''}{a} + m^2 a^2\right] u_k(X) = (k^2 + 1) u_k(X)
\]

- Euclidean correlator: Written in terms of the reflection coefficient.

\[
\langle \phi(X, \theta) \phi(X', 0) \rangle = \frac{1}{a(X)a(X')} \int dk u_k(X) u_k^*(X') G_k(\theta) + \text{(bound state)}
\]

\[
\langle \phi(X, \theta) \phi(X', 0) \rangle = \tilde{H}_A^2 e^{-(X+X')} \int_{C_1} dk \left( e^{ik(X-X')} + R(k) e^{-ik(X+X')} \right) \frac{\sinh k(\pi - \theta)}{\sinh k\pi \sin \theta}
\]
Correlator in open FRW

• Analytic continuation: \( X \to T + \frac{\pi}{2}i, \quad \theta \to iR \)

\[
\langle \phi(T, R)\phi(T', 0) \rangle = \tilde{H}_A^2 e^{-(T+T')} \int_{C_1} dk \left( e^{ik(T-T')} \cosh k\pi \right.

\left. + \mathcal{R}(k)e^{-ik(T+T')} \right) \frac{\sin kR}{\sinh k\pi \sinh R}
\]

\( (R: \text{Geodesic distance on } H^3) \)

• 1st term: “Flat space piece” (Minkowski correlator written in hyperbolic slicing)
• 2nd term (which depends on \( R(k) \)): Effect of the false vacuum.
• Bound state: non-normalizable mode
Decomposition in terms of scaling dimensions

• Correlation function can be written as discrete sum (over contributions from the poles)

\[
\langle \phi(T, R)\phi(T', 0) \rangle = \sum_{\Delta} G_{\Delta}^{(1)} e^{-\Delta (R_1 + R_2)} e^{-\Delta (T_1 + T_2)} (1 - \cos \Omega)^{-\Delta} \\
+ \sum_{\Delta' = 2}^{\infty} G_{\Delta'}^{(2)} e^{-\Delta' (R_1 + R_2)} e^{(\Delta' - 2) (T_1 + T_2)} (1 - \cos \Omega)^{-\Delta'}
\]

(geodesic distance: \( R \sim R_1 + R_2 + \log(1 - \cos \Omega) \))

• Each term has definite scaling dimension \( \Delta \).
  \((\Delta = 2, 3, 4, \ldots; \text{ and } 0, 1+\alpha \ (0<\alpha<1) \text{ for massless fields})\)

• This is an expansion valid near the boundary \( R \to \infty \).
Interpretation in the dual theory

- One bulk field corresponds to a tower of CFT operators.

\[ \phi \sim \sum_{\Delta} q_{\Delta} e^{-\Delta R + (\Delta - 2)T} \mathcal{O}_{\Delta} + \sum_{\Delta'} \tilde{q}_{\Delta'} e^{-\Delta' (R + T)} \tilde{\mathcal{O}}_{\Delta'}, \]

- “UV-IR relation” (by identifying T= \( \phi_L \)).
  - \( e^{-R} \): “reference” (length) scale
  - \( e^{-(T+R)} \): UV cutoff (lattice) scale
- Late time corresponds to dual theory with finer cutoff. (At late time, Census Taker sees more and more stuff.)
- The prefactors of the correlator is interpreted as wave function renormalization.
Graviton correlator

• Graviton correlator remains finite at $R \to \infty$ (Graviton correlator has non-normalizable mode).
  – 2D Curvature correlator:
    \[ \langle R^{(2)} R^{(2)} \rangle = \frac{1}{(1 - \cos \Omega)^2} \]
    – Gravity is not decoupled at the boundary (unlike in AdS/CFT): Dual theory contains gravity.

• $\Delta=2$ piece of graviton is transverse traceless along $S^2$:
  – Identified as energy-momentum tensor of 2D CFT.
  – Central charge: of order de Sitter entropy
Three-point functions (work in progress)

• Needed for computation of central charge (order de Sitter entropy), and for checks of operator product expansion.

• Euclidean prescription:
  – Use $\langle \phi \phi \phi \rangle$ as propagator, integrate vertex (e.g. $\phi^3$) over Euclidean space, and analytically continue to FRW.

• Strategy:
  – Do the integral over $X$ (corresponding to FRW time) first.
  – Rotate external points to Lorentzian: $\theta_I \rightarrow iR_I, \ X_I \rightarrow T_I + \frac{\pi}{2}i$
  – Integral over $S^3$ can be rotated to integral over $H^3$.
  – Each propagator can be written as a sum over $\Delta$.
  – Final expression: Sum of 3-pt fn on $H^3$ (summed over $\Delta$‘s)
Summary of Part II

• Holographic dual for universe created by bubble nucleation: 2D CFT on $S^2$ at the boundary of $H^3$
  – Liouville plays the role of time.
  – Evidence: dimension 2 energy-momentum tensor.
  – There is well-defined method of calculation of 3-point functions. This will test OPE, and tell us central charge.

• Implication of results of Part I: Need to sum over topology of 2D space, but need not to consider multiple boundaries. (Phase transitions: instability of 2D gravity?)
Part III: Observational consequences

- Freivogel, YS, Susskind, Yeh (unpublished)
- See also Yamauchi et al, Phys.Rev. D84 (2011) 043513
Basic picture

- Bubble nucleation predicts negative spatial curvature.
- The scale that we are looking at (roughly, the present Hubble scale) is $< 0.1 \, R_c$ (curvature radius).
- Slow-roll inflation after tunneling:
  - Vacuum energy will be lower than in the ancestor vacuum $H_I \ll H_A$
  - Einstein equation: 
    \[
    \left( \frac{\dot{a}}{a} \right)^2 = H_I^2 + \frac{1}{a^2}
    \]
  - Just after the tunneling, there is a period of curvature domination. $t < H_I^{-1}$ ($T < T_1 = \log(H_A/H_I)$)
  
  Evolution of fluctuations during this period is important.
Evolution during curvature domination

• Gives the initial condition for slow-roll inflation.
  \[
  \langle \phi(T, R)\phi(T', 0) \rangle = \tilde{H}_A^2 e^{-(T+T')} \int_{C_1} dk \left( e^{i k (T-T')} \cosh k\pi \right. \\
  \left. + \mathcal{R}(k)e^{-i k (T+T')} \right) \frac{\sin kR}{\sinh k\pi \sinh R}
  \]
  – 1\textsuperscript{st} term: usually assumed to be the initial condition.
  – 2\textsuperscript{nd} term: effect of the ancestor vacuum.

• We want to find the 2\textsuperscript{nd} term in the CMB fluctuations, assuming the duration of slow-roll inflation is minimal.

• The 1\textsuperscript{st} term: order \( H_I^2 \) (agree with the standard estimate).
Classification in terms of scaling dimensions

• Useful for the study of time dependence.
• Close the contour in the lower half plane, and write the correlator as a sum.
  – Pole at $k = -ib$ ($b > -1$) gives $H_I^2 e^{-b(T+T')} = H_I^2 \left( \frac{H_I}{H_A} \right)^{2b}$
  
  – The term with the smallest $b$ is the largest.
  – In particular, non-normalizable mode ($b < 0$), if exists, is enhanced relative to $H_I^2$
  – The angular spectrum: $C_l^{(\text{ancestor})} \sim \left( \frac{R}{2} \right)^{2l}$ (when $R < 1$)
    (Exponentially peaked at low $l$)
Prospects for observation

• Curvature perturbation (scalar mode):
  – Small effect (at least in the single-field model)
  – No non-normalizable mode (since the field has large mass in the ancestor vacuum)

• Tensor fluctuations (gravitons):
  – Effect on the B-mode polarization will be small.
  – Could have large contribution to $\delta T/T$

• Extra fields (isocurvature perturbation):
  – There could be large effect if we have field with small mass in the ancestor vacuum (e.g. axions),
  – But we have to understand how they contribute to measurable quantities.
Summary of Part III

- Bubble nucleation predicts negative spatial curvature: We are looking at a scale smaller than $R_c$.
- The presence of the ancestor vacuum could affect the IR part of the CMB spectrum.
- (Of course, study of the generation of CMB fluctuations after tunneling is important. [See Habara, Kawai, Ninomiya, YS])
- But the prospects for observability do not seem good: In most cases, the effect of the ancestor is smaller than the fluctuations generated during inflation after tunneling.
- It might be necessary to think about “measure problem” seriously to find predictions of string theory.